From Janus interfaces to holographic conformal manifolds

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Motivation

Realization of holography as the AdS/CFT correspondence is one of the most important discovery in modern theoretical physics.

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Recent developments in generalized geometry and exceptional field theory has opened a systematic way to derive consistent truncations to lower dimensions and even the mass matrices of truncated KK modes.

Malek, Samleben (2020)

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In some cases these backgrounds will be based on non-geometric compactifications of string theory.

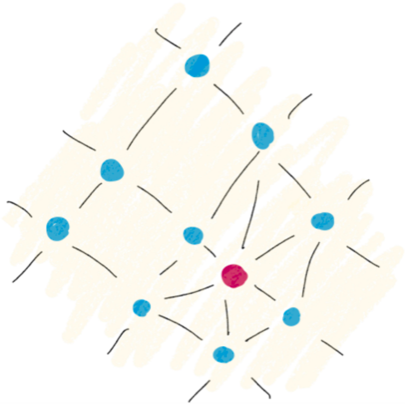
We will use these modern tools of holography to make quantitative predictions for observables of the dual field theory.

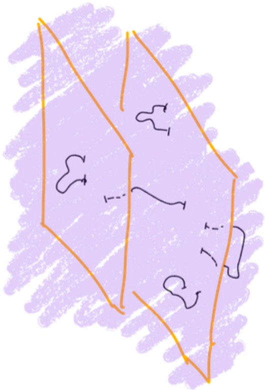
Outline

- Janus interfaces in $\mathcal{N} = 4$.
- Holographic Janus solutions.
- J-folds: new AdS_4 solutions.
- Conformal manifolds.
- Summary.

Janus interfaces

Interfaces and defects play an important role in many branches of physics





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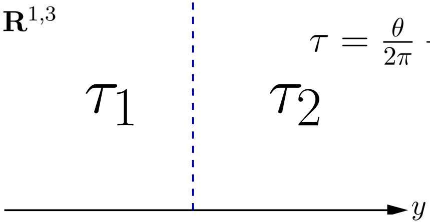
In $\mathcal{N} = 4$ SYM they arise from studying the theory with a position dependent coupling.



JANUS IN $\mathcal{N} = 4$

Start from the 4D SYM Lagrangian

$$\mathcal{L}_{\text{SYM}} = \frac{1}{2g_{\text{YM}}^2}\text{Tr}\Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - D_\mu\Phi_m D^\mu\Phi^m + \bar{\Psi}\gamma^\mu D_\mu\Psi - \frac{1}{2}[\Phi_m,\Phi_n][\Phi^m,\Phi^n] + \bar{\Psi}\Gamma^m[\Phi_m,\Psi]\Big] .$$

where the scalars Φ_m transform in the **6** and Ψ denote Weyl fermions transforming in the **4** of the R-symmetry group SU(4). All fields transform in the adjoint of the gauge group SU(N) with gauge field A_μ with field strength $F_{\mu\nu}$. I have omitted the θ-angle term.

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This deformation preserves $\text{SU}(4)$ but breaks all supersymmetry.

Janus in $\mathcal{N} = 4$

If we want to preserve supersymmetry, then we need to deform the Lagragian by introducing extra couplings .

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A systematic way to study the allowed couplings is to use 4D $\mathcal{N} = 4$ conformal supergravity. This is completely analogous to Festuccia-Seiberg.

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We can even preserve 3D conformal invariance by letting $\tau(y) \sim \Theta(y)$.

$\mathcal{N}$	supergroup	R-symmetry	Commutant
4	$\mathrm{OSp}(4\|4, \mathbf{R})$	$\mathrm{SU}(2) \times \mathrm{SU}(2)$	
2	$\mathrm{OSp}(2\|4, \mathbf{R})$	$\mathrm{U}(1)$	$\mathrm{SU}(2)$
1	$\mathrm{OSp}(1\|4, \mathbf{R})$		$\mathrm{SU}(3)$
0			$\mathrm{SU}(4)$

JANUS IN $\mathcal{N} = 4$

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This interesting and strongly coupled nature of the Janus interfaces in $\mathcal{N} = 4$ SYM warrants a careful holographic study of them.

Holographic realization

10D vs. 5D approach

The holographic dual geometry should be a solution to type IIB supergravity with $\mathrm{AdS}_5 \times S^5$ asymptotics. For highly symmetric examples such solutions were already constructed.

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Field content in one-to-one correspondence with the 4D conformal supergravity (table shows bosons only)

Δ	spin	Background field	SU(4) rep.
4	0	τ	$\mathbf{1}$
3	0	E_{ij}	$\mathbf{10} \oplus \bar{\mathbf{10}}$
2	0	$D^{ij}{}_{kl}$	$\mathbf{20'}$
2	1	$V_\mu{}^i{}_j$	$\mathbf{15}$
2	1	$T_{\mu\nu}{}^{ij}$	$\mathbf{6} \oplus \bar{\mathbf{6}}$
4	2	$g_{\mu\nu}$	$\mathbf{1}$

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$$\frac{E_{6(6)}}{\mathrm{USp}(8)}\,,$$

and can be expressed in terms of a 27×27 real matrix M with 42 independent components.

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The Lagrangian is

$$\mathcal{L} = \frac{\sqrt{|g_5|}}{16\pi G_N}\left(R_5 + \frac{1}{4}\mathrm{Tr}\big[\partial_\mu M \partial^\mu M^{-1}\big] - V\right)\,.$$

5D SUPERGRAVITY

The scalar potential V is a complicated function of M.

$$V = \frac{g^2}{168}\mathcal{M}^{\mathbb{MP}} X_{\mathbb{MN}}{}^{\mathbb{R}} X_{\mathbb{PQ}}{}^{\mathbb{S}} \left(\mathcal{M}^{\mathbb{NQ}}\mathcal{M}_{\mathbb{RS}} + 5\delta^{\mathbb{N}}_{\mathbb{S}}\delta^{\mathbb{Q}}_{\mathbb{R}}\right) ,$$

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Maximally supersymmetric vacuum at the origin of the scalar manifold. Uplifted to type IIB, this is just $\mathrm{AdS}_5 \times S^5$. The length scale of AdS is controlled by the gauge coupling $L = 2/g$ which is related to the rank N via

$$N = \frac{L^4}{4\pi\ell_s^4} .$$

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Parametrizing the manifold

$$\mathcal{M} = \frac{\mathrm{SL}(3, \mathbf{R})}{\mathrm{SO}(3)}\,.$$

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Supplemented by the five-dimensional metric

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For curved domain walls, however, the equations are slightly more complicated, e.g.

$$(A')^2 = \frac{1}{9}|W|^2 - \mathrm{e}^{-2A}\ .$$

5D BPS EQUATIONS

Bobev, FFG, Pilch, Suh, van Muiden (2020)

The remaining BPS equations are

$$35(\alpha' - \varphi')^2 = |\partial_\alpha W|^2 \,,$$
$$24(\alpha' - \varphi')(\chi') = \sinh 4\chi \, \mathrm{Re}\,(W \partial_\alpha W) \,,$$
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where

$$W = \frac{-3g}{2}\Big(\cosh 2\alpha \, \cosh 2\chi - i \sinh 2\alpha \, \sinh 2\chi\Big) \,.$$

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These equations can be reduced to

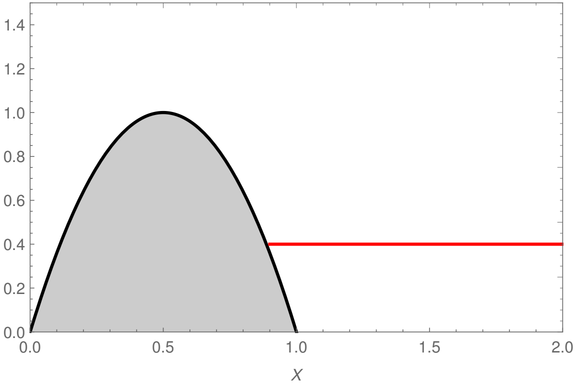
$$\cosh 4\alpha = \frac{2X^2 + \mathcal{I}}{2\sqrt{X^4 + \mathcal{I}X}} \,, \quad \sinh 4\chi = \sqrt{\mathcal{I}X^{-3}} \,, \quad \mathrm{e}^{2A} = \frac{4}{g^2} X \,.$$

The new function X determines the full background.
φ determined by an integral.

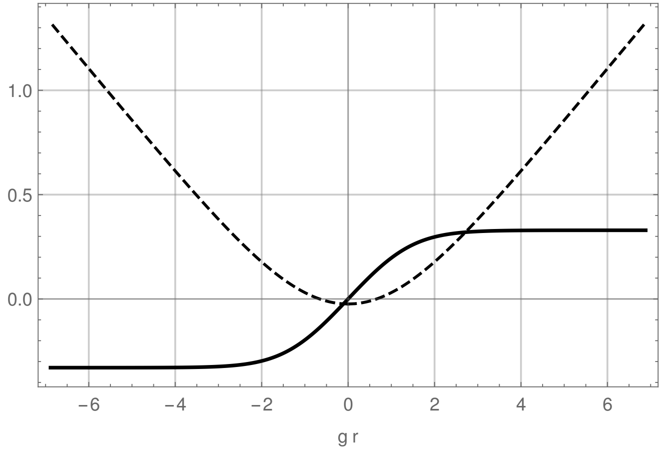
BPS EQUATIONS

One-dimensional scattering problem (Energy$= \mathcal{I}$, $0 \leq \mathcal{I} \leq 1$):

$$\frac{4}{g^2}(X')^2 + V_{\text{eff}} = \mathcal{I}\,, \quad V_{\text{eff}} = 4X(1-X)$$



TYPICAL SOLUTION



The function $(\varphi(r) - \varphi_0)$ (solid curve) for $\mathcal{I} = 1/3$ which determines the dilaton, and $(1/4)\log X$ (dashed curve) which determines the metric function A.

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Recently, more general Janus-type configurations have been considered.

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J-folds

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$$\varphi = \varphi_0 + \rho \,, \quad \rho = \frac{gr}{2} \,.$$

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This solutions looks singular, as the dilaton blows up when $\rho \to \pm\infty$.

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$$\tau' = \frac{a\tau + b}{c\tau + d}\,, \quad \tau = C_0 + i\mathrm{e}^{-\Phi}\,, \quad \Lambda = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{SL}(2,\mathbf{R})\,.$$

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It turns out that the matrix

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only has the effect of shifting the 5D dilaton $\varphi \mapsto \varphi' = \varphi - \rho_0$. This means that we can make the identification

Inverso, Samtleben, Trigiante (2017)

$$\rho \sim \rho + \rho_0\,.$$

if we simultaneously act with the $\mathrm{SL}(2,\mathbf{R})$ matrix $\mathfrak{J}_n$ on the supergravity fields.

J-FOLDS IN 5D

However, we can *compactify* the ρ-direction à la Scherk-Schwarz using the $\mathrm{SL}(2,\mathbf{R})$ symmetry of the theory. It turns out that the matrix

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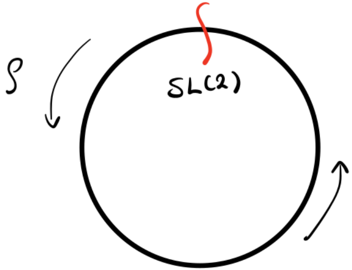

J-FOLDS

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This leads to a new 3D $\mathcal{N} = 4$ SCFTs with a close connection to the interfaces on $\mathcal{N} = 4$ SYM in four dimensions. The free energy on S^3 is given in terms of the 4rank N

$$\mathcal{F}_{S^3} = \frac{N^2}{2} \operatorname{arccosh}(n/2) \, .$$

J-FOLDS

I will give a brief interpretation of the 3D dual SCFT and its relation to the $T[\mathrm{U}(N)]$ theory living on the 3D interface in $\mathcal{N} = 4$ SYM.

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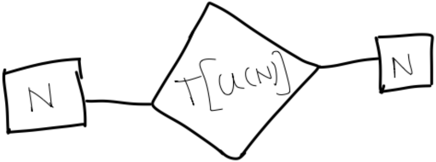
The 3D $\mathcal{N} = 4$ SCFTs of Gaiotto and Witten serve as new strongly coupled building blocks.



The two boxes denote two copies of $\mathrm{U}(N)$ flavor symmetry.

J-FOLDS

We can build new strongly coupled SCFTs in three dimensions using this building block. For example gauging the diagonal $\mathrm{U}(N)$ flavor symmetries and adding a Chern-Simons level n

Terashima, Yamazaki (2011)
Ganor, Moore, Sun, Torres-Chion (2014)
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In 3D language we can phrase this by adding a vector multiplet and the following superpotential

$$W_{\mathrm{UV}}^{\mathcal{N}=3} = -\frac{n}{4\pi}\mathrm{Tr}(\Phi^2) + \mathrm{Tr}(\Phi(\mu_H + \mu_C))\,.$$

Here $\mu_{H,C}$ are complex scalar primary operators in each of the two $\mathrm{U}(N)$ flavor multiplets (transform in the adjoint of $\mathrm{U}(N)$). Φ is a scalar primary of the $\mathcal{N}=4$ vector multiplet and n is the CS level.

J-FOLDS

Gang, Kim, Romo, Yamazaki (2015)
Assel, Tomasiello (2018)

Schematically we can integrate out Φ to obtain an IR superpotential

$$W_{\mathrm{IR}}^{\mathcal{N}=4} = -\frac{2\pi}{n}\mathrm{Tr}(\mu_H \mu_C)\,,$$

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supersymmetry enhancement argued in the IR to $\mathcal{N}=4$.

These theories were argued to be dual to the $\mathcal{N}=4$ AdS_4 J-fold solution. The partition function can be evaluated using supersymmetric localization. The matrix model is Gaussian and the free energy is.

$$F = \frac{N^2}{2}T + \sum_{j=1}^{N} \log(1-\mathrm{e}^{-jT}) \approx \frac{N^2}{2}T\,, \quad T = \text{arccosh}\,\frac{n}{2}\,.$$

Generalization

The holographic approach allows us to construct 5D J-fold solutions with less supersymmetry directly. We can compute the Free energy in hologarphy

$\mathcal{N}$	Flavor group	Free energy ($\mathcal{F}_{S^3}$)
4		$\frac{N^2}{2}\text{arccosh}(n/2)$
2	SU(2)	$\frac{N^2}{2}\text{arccosh}(n/2)$
1	SU(3)	$\frac{5^{5/2}}{3^3}\frac{N^2}{4}\text{arccosh}(n/2)$

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It is curious that the FE for $\mathcal{N}=4$ and $\mathcal{N}=2$ J-folds is the same. This is not an accident.

10D $\mathcal{N} = 1$ J-FOLDS

For concreteness I present here the $\mathcal{N} = 1$ J-fold metric

$$\mathrm{d}s^2_{10} = \sqrt{\frac{5}{6}}\frac{1}{6}\left(4\mathrm{d}\rho^2 + 5\mathrm{d}s^2_{\mathrm{AdS}_4} + 6\mathrm{d}s^2_{\mathbf{C}P^2} + \tfrac{36}{5}\zeta^2\right)\ ,$$

where

$$\mathrm{d}s^2_{S^5} = \mathrm{d}s^2_{\mathbf{C}P^2} + \zeta^2\ .$$

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$$\mathrm{d}s^2_{\mathrm{SE}^5} = \mathrm{d}s^2_{\mathrm{KE}_4} + \zeta^2 \ .$$

We can replace $\mathrm{d}s^2_{\mathbf{C}P^2}$ with $\mathrm{d}s^2_{\mathrm{KE}_4}$, a metric on an arbitrary Kähler-Einstein base, and the Einstein equation remain solved.

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We can replace $\mathrm{d}s^2_{\mathbf{C}P^2}$ with $\mathrm{d}s^2_{\mathrm{KE}_4}$, a metric on an arbitrary Kähler-Einstein base, and the Einstein equation remain solved.

This means that for any 4D $\mathcal{N} = 1$ SCFT dual to $\mathrm{AdS}_5 \times \mathrm{SE}_5$ we can construct a related $\mathcal{N} = 1$ Janus and J-fold.

Bobev, FFG, Pilch, Suh, van Muiden (2019)

$$\mathcal{F}_{S^3} = \frac{5^{5/2}}{3^3} a_{4D} \mathrm{arccosh}(n/2)$$

J-fold conformal manifolds

4D supergravity

We can describe the AdS_4 J-fold solutions in 4D supergravity instead of using the 5D we have used so far. This is not applicable to Janus configurations.

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The benefit of 4D approach is that classifying solutions is easier and the spectrum of fluctuations can be directly computed.

4D SUPERGRAVITY

Fields get rearranged in 4D

5D field	4D field	SU(4) rep.
τ	τ	$\mathbf{1}$
E_{ij}	E_{ij}	$\mathbf{10} \oplus \bar{\mathbf{10}}$
${D^{ij}}_{kl}$	${D^{ij}}_{kl}$	$\mathbf{20}'$
${V_\mu}^i{}_j$	${V_a}^i{}_j, {V_\rho}^i{}_j$	$\mathbf{15}$
${T_{\mu\nu}}^{ij}$	${T_{ab}}^{ij}, {T_{a\rho}}^{ij}$	$\mathbf{6} \oplus \bar{\mathbf{6}}$
$g_{\mu\nu}$	$g_{ab}, g_{a\rho}, g_{\rho\rho}$	$\mathbf{1}$

scalars: $\mathbf{1}_{\times 2} \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{20}' \oplus \mathbf{15} \oplus \mathbf{6} \oplus \bar{\mathbf{6}} \oplus \mathbf{1} = 70$
vectors: $\mathbf{15} \oplus \mathbf{6} \oplus \bar{\mathbf{6}} \oplus \mathbf{1} = 28$

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This is exactly the (bosonic) matter content of maximal supergravity in 4D.

J-folds in 4D sugra

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This approach has not been used for the $\mathrm{SO}(6) \times \mathrm{SO}(1,1)$ theory discussed here. Instead we use a consistent truncation.

Universal $\mathcal{N} = 1$ subsector

Using discrete symmetries we can identify a $\mathcal{N} = 1$ supergravity with only 7 complex scalar fields.

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The scalar potential is written in terms of a superpotential

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\begin{aligned}
K &= -\sum_i \log(2\mathrm{Im}\, z_i)\,, \\
V &= \mathrm{e}^K \left(K^{i\bar{\jmath}} D_i W D_{\bar{\jmath}} \overline{W} - 3W\overline{W} \right), \\
D_i W &= \partial_i W + W \partial_i K\,, \\
W &= z_1 z_5 z_6 + z_2 z_4 z_6 + z_3 z_4 z_5 + z_1 z_4 z_7 \\
&\quad + z_2 z_5 z_7 + z_3 z_6 z_7 + z_4 z_5 z_6 z_7 - 1\,,
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UNIVERSAL $\mathcal{N} = 1$ SUBSECTOR

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Supersymmetric critical points satisfy

$$D_i W = 0\,.$$

$\mathcal{N} = 4$ J-FOLD

We start by rediscovering the $\mathcal{N} = 4$ J-fold in this language

Gallerati, Samtleben, Trigiante (2014)

$$\mathbf{z} = \left(i, i, i, \mathrm{e}^{\pi i/4}, \mathrm{e}^{\pi i/4}, \mathrm{e}^{\pi i/4}, -\mathrm{e}^{-\pi i/4} \right) .$$

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Computing mass spectrum of fluctuations of all supergravity fields and arranging them in $\mathcal{N} = 4$ SCFT multiplets, we obtain (quantum numbers are $[\Delta; j; r_1, r_2]$ where $r_{1,2}$ are $\mathrm{SU}(2)_{1,2}$ R-symmetry labels)

$$A_2[1; 0; 0, 0], \qquad B_2[2; 0; 1, 1]$$

Notation from Cordova, Dumitrescu, Intriligator (2016)

$\mathcal{N} = 2$ J-FOLD

We find a two-parameter family of $\mathcal{N} = 2$ $\mathrm{U}(1)_R \times \mathrm{U}(1)_F$ invariant J-folds!

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$$A_1\overline{A}_1\,[2;1;0;0]\,, \qquad L\overline{A}_1\left[\tfrac{5}{2};\tfrac{1}{2};+1;0\right]\,, \qquad A_1\overline{L}\left[\tfrac{5}{2};\tfrac{1}{2};-1;0\right]\,,$$

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And long multiplets

$$L\overline{L}\left[\tfrac{1}{2}+\beta_1;0;0;\pm 2\right]\ ,\qquad L\overline{L}\left[\tfrac{1}{2}+\beta_2;0;0;0\right]\ ,\qquad L\overline{L}\left[\tfrac{1}{2}+\beta_3;0;0;0\right]$$
$$L\overline{L}\left[\tfrac{1}{2}+\beta_4;\tfrac{1}{2};0;\pm 1\right]\ ,\qquad L\overline{L}\left[\tfrac{1}{2}+\beta_5;\tfrac{1}{2};0;\pm 1\right]\ .$$

where β_i are non-trivial functions on the CM.

$\mathcal{N} = 2$ J-FOLD

Explicitly

$$\beta_1^2 = \frac{1}{4} + 2\varphi^2 + \frac{4\chi^2}{1+\varphi^2}\,,$$

$$\beta_2^2 = \frac{17+\varphi^2}{4(1+\varphi^2)}\,, \qquad \beta_3^2 = \frac{17+33\varphi^2}{4(1+\varphi^2)}\,,$$

$$\beta_4^2 = \frac{\varphi^2 + \left(\varphi^2+2\right)^2 + 2\chi^2 - 2\varphi\sqrt{\left(\varphi^2+2\right)^2 + 2\chi^2}}{2\left(1+\varphi^2\right)}\,,$$

$$\beta_5^2 = \frac{\varphi^2 + \left(\varphi^2+2\right)^2 + 2\chi^2 + 2\varphi\sqrt{\left(\varphi^2+2\right)^2 + 2\chi^2}}{2\left(1+\varphi^2\right)}\,.$$

GEOMETRY OF THE $\mathcal{N} = 2$ CM

Two special points

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\begin{aligned}
\varphi = \chi = 0 \quad &\Longrightarrow \quad \mathrm{SU}(2)_F \,. \\
\varphi = 1\,, \quad \chi = 0 \quad &\Longrightarrow \quad \mathcal{N} = 4\,.
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Scalar kinetic terms define a metric on the conformal manifold (Zamolodchikov metric)

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Partial compactification in 5D (or 10D)

Giambrone, Malek, Samtleben, Trigiante (2021)

$$\chi \sim \chi + \frac{2\pi}{\rho_0} \, .$$

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However, $\varphi \to \infty$ is a singular point in 5D.

Arav, Gauntlett, Roberts, Rosen (2021)

Field theory interpretation

Recall the 3D UV superpotential

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Lets introduce an arbitrary mass term for the adjoint scalar Φ.
This breaks supersymmetry to $\mathcal{N} = 2$.
Now integrating out Φ gives

$$W_{\mathrm{IR}}^{\mathcal{N}=2} = -\frac{2\pi}{n}\mathrm{Tr}(\mu_H\mu_C) + \lambda\mathrm{Tr}(\mu_H\mu_C)\,,$$

where λ is the putative marginal coupling.

$\mathcal{N} = 1$ Conformal manifold

$\mathcal{N} = 1$ conformal manifold with $\mathrm{U}(1)^2$ Flavor symmetry generalizes the $\mathrm{SU}(3)$ solution. Axions related to the cartan of $\mathrm{SU}(3)$ can be turned on $\chi_1 + \chi_2 + \chi_3 = 0$.

Guarino, Sterckx, Trigiante (2020)

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Zamolodchikov metric is just the flat metric on $\mathbf{T}^2$.

Axionic scalars in 5D

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5D field	4D field	SU(4) rep.
τ	τ	$\mathbf{1}$
E_{ij}	E_{ij}	$\mathbf{10} \oplus \bar{\mathbf{10}}$
$D^{ij}{}_{kl}$	$D^{ij}{}_{kl}$	$\mathbf{20}'$
$V_\mu{}^i{}_j$	$V_a{}^i{}_j$, $V_\rho{}^i{}_j$	$\mathbf{15}$
$T_{\mu\nu}{}^{ij}$	$T_{ab}{}^{ij}$, $T_{a\rho}{}^{ij}$	$\mathbf{6} \oplus \bar{\mathbf{6}}$
$g_{\mu\nu}$	g_{ab}, $g_{a\rho}$, $g_{\rho\rho}$	$\mathbf{1}$

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They are Wilson lines in the cartan of the preserved flavor group. Gauge symmetry in 5D guaranties that they are massless in 4D.

More $\mathcal{N} = 1$ Conformal manifold

In recent weeks we have been looking at a different gauged supergravity in 4D. The gauge group is

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$\mathcal{N} = 1$ subsector:

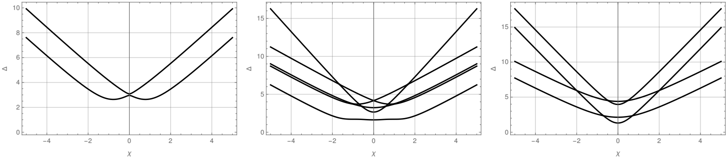
$$W = z_1 z_5 z_6 + z_2 z_4 z_6 + z_3 z_4 z_5 + z_1 z_4 z_7 + z_2 z_5 z_7 + z_3 z_6 z_7 + z_4 z_5 z_6 z_7 + 1 \,,$$

We find two distinct families of $\mathcal{N} = 1$ solutions (each is 1D) with only $\mathrm{U}(1)$ flavor symmetry.

Bobev, FFG, van Muiden (2021)

More $\mathcal{N} = 1$ Conformal manifold

We have computed the CFT spectrum (of low lying operators) and found that there is a complicated dependence on the marginal parameter χ (Figures show a part of the spectrum for one of the solutions.)



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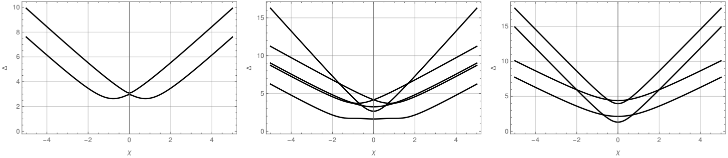

The free energy is

$$\mathcal{F}=\sqrt{\frac{5^5}{3^9}}\pi N^2\Big(k+\frac{1}{n}\Big), \quad \mathcal{F}=\frac{81\pi N^2}{16\sqrt{70+26\sqrt{13}}}\Big(k+\frac{1}{n}\Big).$$

Summary

Summary

- Five dimensional supergravity provides a systematic approach to constructing holographic Janus interfaces.
- For less supersymmetric Janus the 5D solutions have not yet been classified.

Arav, Cheung, Gauntlett, Roberts, Rosen (2020-1)

- J-folds are a new class of AdS_4 solutions in type IIB supergravity with $\mathcal{N} = 1, 2, 4$ supersymmetry.
- A universal feature of J-folds is that they have a conformal manifold. Even for $\mathcal{N} = 1$!
- The 3D SCFT dual to J-folds is really only properly understood for $\mathcal{N} = 4$ supersymmetry.
- Holography gives access to spectrum all over the conformal manifold. So far only low-lying operators, but new techniques allows for the full KK analysis.

Giambrone, Malek, Samtleben, Trigiante (2021)
Cesaro, Larios, Varela (2021)

SUMMARY

- New pair of U(1) invariant $\mathcal{N} = 1$ AdS_4 solution with 1D conformal manifold of type IIB. S-fold, but connection to Janus unclear.
- Can we trust the $\mathcal{N} = 1$ conformal manifolds? Is there a field theory explanation for their existence?

Summary

- New pair of U(1) invariant $\mathcal{N}=1$ AdS_4 solution with 1D conformal manifold of type IIB. S-fold, but connection to Janus unclear.
- Can we trust the $\mathcal{N}=1$ conformal manifolds? Is there a field theory explanation for their existence?

$\mathcal{N}$	Flavor	Free energy ($\mathcal{F}_{S^3}$)	$\mathrm{Dim}_{\mathbf{R}}$	Comment
2	U(1)	$\frac{N^2}{2}\mathrm{arccosh}(n/2)$	2	Contains $\mathcal{N}=4$
1	$U(1)^2$	$\frac{5^{5/2}}{3^3}a_{4D}\mathrm{arccosh}(n/2)$	2	Infinite class
1	U(1)	$\sqrt{\frac{5^5}{3^9}}\pi N^2\left(k+\frac{1}{n}\right)$	1	
1	U(1)	$\frac{81\pi N^2}{16\sqrt{70+26\sqrt{13}}}\left(k+\frac{1}{n}\right)$	1	

Thank you!