Non-Lorentzian Supergravity in 10 Dimensions

Johannes Lahnsteiner

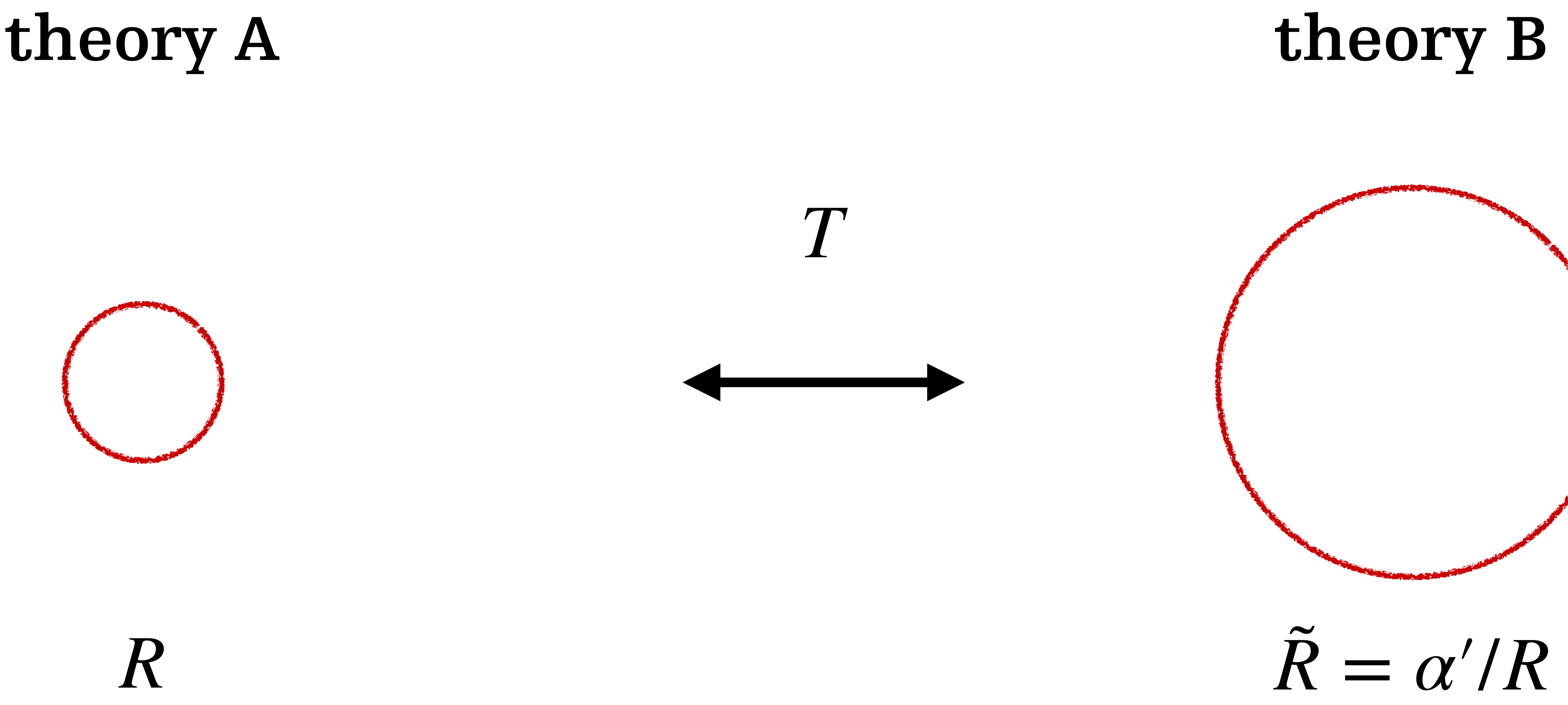
rijksuniversiteit groningen

30th Nordic Meeting on Strings, Field, and Branes
@ Nordita November 2021

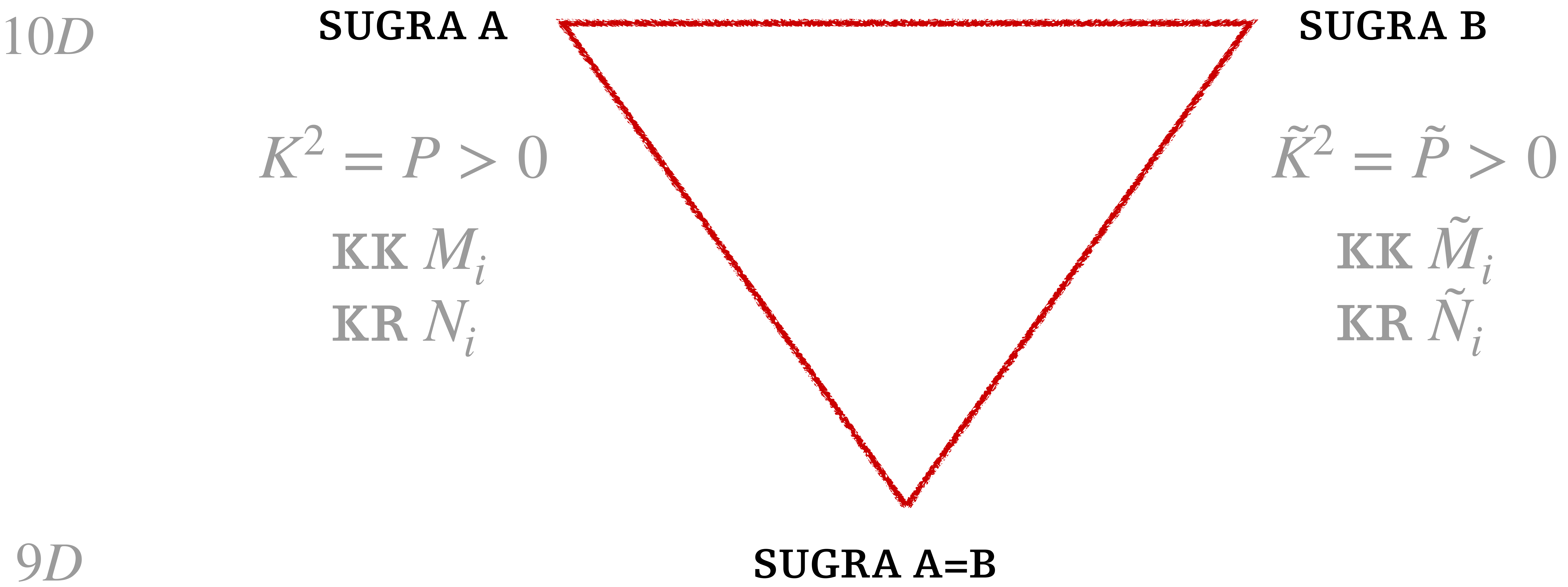
Based on 2102.06974, 2107.14636, and WIP with
E. Bergshoeff, L. Romano, J. Rosseel, and C. Simsek

T Duality

relates two different string theories A and B, compactified on a large and small volume, respectively:



T Duality: Target Space



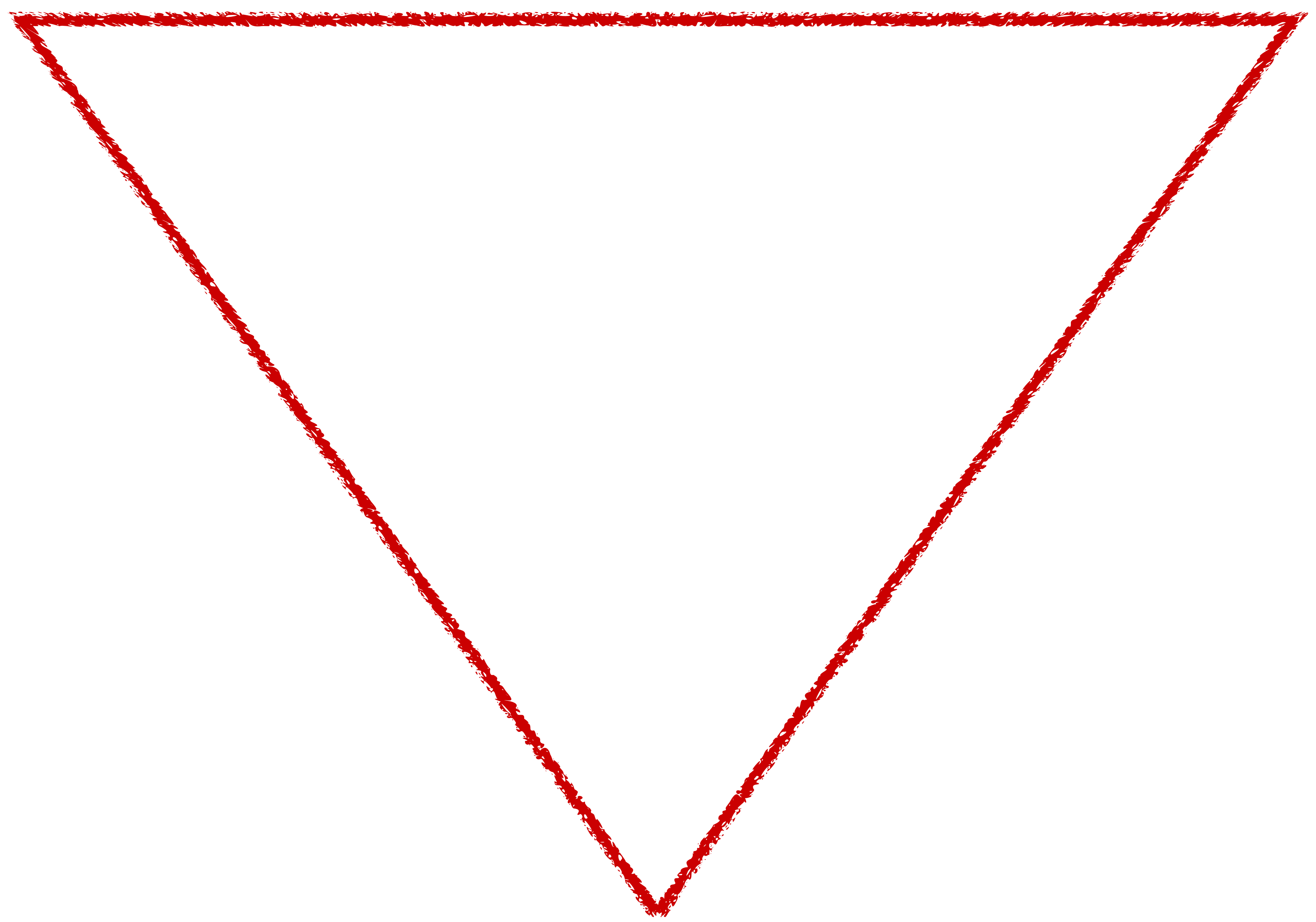
T Duality: Target Space

Buscher

$$\boxed{\begin{array}{ll} \tilde{P} = P^{-1}\,, & \mathrm{e}^{-2\,\tilde{\Phi}} = P\,\mathrm{e}^{-2\,\Phi} \\ \tilde{M}_i = P^{-1} N_i\,, & \tilde{N}_i = P^{-1} M_i\,, \cdots \end{array}}$$

$10D$

SUGRA A **SUGRA B**



$K^2 = P > 0$ $\tilde{K}^2 = \tilde{P} > 0$

KK M_i KK $\tilde{M}_i$

KR N_i KR $\tilde{N}_i$

$9D$

SUGRA A=B

Question: Is There a Notion of T Duality if the Isometry is Lightlike?

i.e., for a vanishing KK scalar $\tilde{K}^2 = \tilde{P} \to 0$

Regularization of Buscher

Buscher

$$\boxed{\begin{aligned} &\tilde{P} = P^{-1}\,, && e^{-2\tilde{\Phi}} = P\,e^{-2\Phi} \\ &\tilde{M}_i = P^{-1}N_i\,, && \tilde{N}_i = P^{-1}M_i\,, \cdots \end{aligned}}$$

Regularization of Buscher

Buscher

$$\boxed{\begin{aligned} \tilde{P} &= P^{-1}\,, & \mathrm{e}^{-2\tilde{\Phi}} &= P\,\mathrm{e}^{-2\Phi} \\ \tilde{M}_i &= P^{-1}N_i\,, & \tilde{N}_i &= P^{-1}M_i\,, \cdots \end{aligned}}$$

introduce a regularization parameter ϵ that will eventually $\epsilon \to 0$

$$P = p/\epsilon \qquad \longrightarrow \qquad \tilde{P} = \epsilon/p \to 0$$

Regularization of Buscher

Buscher

$$\boxed{\begin{array}{ll} \tilde{P} = P^{-1}\,, & e^{-2\tilde{\Phi}} = P\,e^{-2\Phi} \\ \tilde{M}_i = P^{-1}N_i\,, & \tilde{N}_i = P^{-1}M_i\,, \cdots \end{array}}$$

introduce a regularization parameter ϵ that will eventually $\epsilon \to 0$

$$P = p/\epsilon \qquad \longrightarrow \qquad \tilde{P} = \epsilon/p \to 0$$

ill-defined, unless also

$$\Phi = \phi - 1/2\,\log\epsilon\,, \qquad M_i = \epsilon^{-1}\tau_i^{\,1} + \cdots\,, \qquad N_i = \epsilon^{-1}\,\tau_i^{\,0} + \cdots$$

Gomis-Ooguri

the $\epsilon \to 0$ limit with

$$G_{\mu\nu} = \epsilon^{-1}\,\eta_{AB}\,\tau_\mu{}^A\tau_\nu{}^B + e_{\mu\nu}\,,$$

$$B = -\,\epsilon^{-1}\,\tau^0 \wedge \tau^1 + b\,,$$

$$\Phi = \phi - 1/2\,\log\epsilon$$

$A = 0{,}1$

leads to:

Gomis-Ooguri

the $\epsilon \to 0$ limit with

$$G_{\mu\nu} = \epsilon^{-1}\,\eta_{AB}\,\tau_\mu{}^A\tau_\nu{}^B + e_{\mu\nu}\,, \qquad A = 0,1$$

$$B = -\,\epsilon^{-1}\,\tau^0 \wedge \tau^1 + b\,,$$

$$\Phi = \phi - 1/2\,\log\epsilon$$

leads to:

- ★ **a non-Lorentzian geometric structure** $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi\right)$

[Andringa-Bergshoeff-Gomis-Roo ’12, Bergshoeff-JL-et.al. ’21, ...]

Gomis-Ooguri

the $\epsilon \to 0$ limit with

$$G_{\mu\nu} = \epsilon^{-1}\,\eta_{AB}\,\tau_\mu{}^A\tau_\nu{}^B + e_{\mu\nu}\,, \qquad A = 0,1$$

$$B = -\,\epsilon^{-1}\,\tau^0 \wedge \tau^1 + b\,,$$

$$\Phi = \phi - 1/2\,\log\epsilon$$

leads to:

- ★ **a non-Lorentzian geometric structure** $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi\right)$

  [Andringa-Bergshoeff-Gomis-Roo '12, Bergshoeff-JL-et.al. '21, ...]

- ★ **a unitary, UV complete string theory with NR spectrum**

  [Gomis-Ooguri '00, ..., Z.Yan lectures, Yan-Oling review: to appear]

Gomis-Ooguri

the $\epsilon \to 0$ limit with

$$G_{\mu\nu} = \epsilon^{-1}\eta_{AB}\,\tau_\mu{}^A\tau_\nu{}^B + e_{\mu\nu}\,, \qquad A = 0{,}1$$

$$B = -\,\epsilon^{-1}\,\tau^0 \wedge \tau^1 + b\,,$$

$$\Phi = \phi - 1/2\,\log\epsilon$$

leads to:

- ★ a non-Lorentzian geometric structure $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi\right)$

  [Andringa-Bergshoeff-Gomis-Roo '12, Bergshoeff-JL-et.al. '21, ...]

- ★ a unitary, UV complete string theory with NR spectrum

  [Gomis-Ooguri '00, ..., Z.Yan lectures, Yan-Oling review: to appear]

- ★ an effective non-Lorentzian target space gravity theory

  [Bergshoeff-JL-Rosseel-Romano-Simsek '21A/B]

Gomis-Ooguri String Theory is T Dual to the DLCQ of String Theory

Similarly for the target space theory. As shown, this can be seen as the regularized $\tilde{P} \to 0$ limit of ordinary T duality.

[Bergshoeff-Gomis-Yan '18, WIP]

Can This be Embedded in a Supersymmetric Theory?

concretely, in type $\mathcal{N} = (1{,}0)$ supergravity with minimal multiplet $(G_{\mu\nu}, B_{\mu\nu}, \Phi) \oplus (\Psi_\mu, \Lambda)$?

[Bergshoeff-JL-Romano-Rosseel-Simsek '21B, WIP]

$\mathcal{N} = (1{,}0)$ T Duality

imposing a null isometry $K = \partial_z$ is at odds with supersymmetry

$$\delta_\epsilon \tilde{K}^2 = \delta_\epsilon \tilde{G}_{zz} \neq 0$$

$\mathcal{N} = (1,0)$ T Duality

imposing a null isometry $K = \partial_z$ is at odds with supersymmetry

$$\delta_\epsilon \tilde{K}^2 = \delta_\epsilon \tilde{G}_{zz} \neq 0$$

instead: impose a multiplet of constraints

$$\tilde{G}_{zz} = 0\,, \qquad \tilde{\Psi}_z = 0 \qquad \partial_{[i}\tilde{Z}_{j]} = 0$$

where $\tilde{Z}_\mu = \tilde{G}_{z\mu} - \tilde{B}_{z\mu}$. Alternatively: $\{\tilde{P}, \tilde{\Psi}(K), \mathrm{d}\tilde{Z}\} = 0$

$\mathcal{N} = (1{,}0)$ T Duality

imposing a null isometry $K = \partial_z$ is at odds with supersymmetry

$$\delta_\epsilon \tilde{K}^2 = \delta_\epsilon \tilde{G}_{zz} \neq 0$$

instead: impose a multiplet of constraints

$$\tilde{G}_{zz} = 0\,, \qquad \tilde{\Psi}_z = 0 \qquad \partial_{[i}\tilde{Z}_{j]} = 0$$

where $\tilde{Z}_\mu = \tilde{G}_{z\mu} - \tilde{B}_{z\mu}$. Alternatively: $\{\tilde{P}, \tilde{\Psi}(K), \mathrm{d}\tilde{Z}\} = 0$

$\Longrightarrow \mathcal{N} = (1{,}0)$ multiplet in ten dimensions is reducible!! [?????? '??]

Longitudinal T Duality w/ SUSY

$10D$

Minimal
Gomis-Ooguri
SUGRA

[Bergshoeff-JL-Romano
Rosseel-Simsek '21B]

$\mathcal{N} = (1,0)/\{\tilde{P}, \tilde{\Psi}(K), \mathrm{d}\tilde{Z}\}$

[WIP]

$9D$

9D TNC SUGRA [WIP]

[see also: Harmark-Hartong-
Obers-Oling '18-,
Gursoy-Natale-Zinnato '20-]

Minimal Gomis-Ooguri Supergravity

Minimal Gomis-Ooguri Supergravity

A. NL minimal SUGRA multiplet $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi; \psi_{\mu+}, \psi_{\mu-}, \lambda_+, \lambda_-\right)$

Minimal Gomis-Ooguri Supergravity

A. NL minimal SUGRA multiplet $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi; \psi_{\mu+}, \psi_{\mu-}, \lambda_+, \lambda_-\right)$

B. Expected plus emergent symmetries:

   a. local Galilean

   b. supersymmetry

   c. fermionic S-/T-shift symmetry

   d. bosonic anisotropic dilatations

Minimal Gomis-Ooguri Supergravity

A. NL minimal SUGRA multiplet $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi; \psi_{\mu+}, \psi_{\mu-}, \lambda_+, \lambda_-\right)$

B. Expected plus emergent symmetries:

  a. local Galilean

  b. supersymmetry

  c. fermionic S-/T-shift symmetry

  d. bosonic anisotropic dilatations

C. Closure of the algebra $\left[\delta_1, \delta_2\right] = \delta_3$ iff $\tau^- \wedge \mathrm{d}\tau^- = 0$

Minimal Gomis-Ooguri Supergravity

A. NL minimal SUGRA multiplet $\left(\tau_\mu{}^A, e_{\mu\nu}, b_{\mu\nu}, \phi; \psi_{\mu+}, \psi_{\mu-}, \lambda_+, \lambda_-\right)$

B. Expected plus emergent symmetries:

a. local Galilean

b. supersymmetry

c. fermionic S-/T-shift symmetry $\quad \overset{\textbf{T}}{\longleftrightarrow} \quad \tilde{\Psi}(K) = \tilde{\Psi}_z = 0$

d. bosonic anisotropic dilatations $\quad \longleftrightarrow \quad \tilde{K}^2 = \tilde{P} = 0$

C. Closure of the algebra $\left[\delta_1, \delta_2\right] = \delta_3$ iff $\tau^- \wedge \mathrm{d}\tau^- = 0$ $\quad \longleftrightarrow \quad \mathrm{d}\tilde{Z} = \partial_{[i}\tilde{Z}_{j]} = 0$

Summary

I have presented an a new supergravity multiplet in ten dimensions that is manifestly non-Lorentzian.

The Gomis-Ooguri multiplet is smaller than the smallest relativistic multiplet and requires the constraint $\tau^- \wedge \mathrm{d}\tau^- = 0$ for consistency.

It is T dual to a relativistic multiplet—shortened by a multiplet of constraints $\{K^2 = 0, \Psi(K) = 0, \mathrm{d}Z = 0\}$.

The associated equations of motion are conjectured to capture the universal part of the NL superstring target space constraints following from Weyl anomaly cancellation.

Extended SUSY? Type SIIA/B?

Extending the supersymmetry to $8+8+8+8$ will introduce more constraints on the geometry and the structure of the multiplet. Work in progress.

RR T Duality is under control and relates SIIB to DLCQ of IIA

Still need to figure out:

- Fermions and SUSY consistency
- $SL(2,\mathbf{R})$ of SIIB
- embedding of SIIA in 11D

[see Blair-Gallegos-Zinnato '21 for the bosonic part]

Coupling to SYM Vectors?

Motivated by its relevance in relativistic SUGRA — i.e. heterotic SUGRA — we want to consider coupling to a new NL vector multiplet

$$\left(a_\mu{}^I, b^I, \chi_+{}^I, \chi_-{}^I\right)$$

and construct NL heterotic SUGRA. Work in progress.

- Noether coupling? Through a limiting procedure?
- Anomalies? constraints on the gauge group?
- relation to NL heterotic superstring theories?
- higher order α' correction?

Outlook

★ Obtain full understanding of space of NL SUGRAs in 10D:

- heterotic? type II theories? 11 D SUGRA?
- web of dualities?

★ Relation to NL superstring theories?

★ Solutions and compactifications

[Bergshoeff-Chatzistavrakidis-JL-Romano-Rosseel '20]

- Killing spinor equations
- (A)dS-like solutions? Horizons? Compactifications?

★ Towards NL Holography…

Thank You!

Action and Equations of Motion

Action and Equations of Motion

there is a set of differential equations

$$\langle X \rangle^I = 0$$

that is closed under all the symmetries of the theory

Action and Equations of Motion

there is a set of differential equations

$$\langle X \rangle^I = 0$$

that is closed under all the symmetries of the theory

A subset $\{\langle b \rangle^i\} \subset \{\langle X \rangle^I\}$ can be integrated into a (pseudo-)action S_{NR}

Action and Equations of Motion

there is a set of differential equations

$$\langle X \rangle^I = 0$$

that is closed under all the symmetries of the theory

A subset $\{\langle b \rangle^i\} \subset \{\langle X \rangle^I\}$ can be integrated into a (pseudo-)action S_{NR}

$\delta_Q S_{NR} \neq 0 \Leftrightarrow \{\langle b \rangle^i\}$ is not closed under supersymmetry,

only $\left\{\langle X \rangle^I\right\} = \left\{\langle b \rangle^i, \langle m \rangle^\alpha\right\}$ is.

[Vanhecke-Van Proeyen '17]

NL SUGRA from a Limit

Alternatively, the NL SUGRA theory can be obtained as an $\omega \to \infty$ limit of relativistic $\mathcal{N} = (1{,}0)$ SUGRA in ten dimensions, with

$$\tau_\mu{}^A = \omega^{-1} E_\mu{}^A\,, \qquad e_\mu{}^{A'} = E_\mu{}^{A'},$$

$$b_{\mu\nu} = B_{\mu\nu} + \epsilon_{AB} E_\mu{}^A E_\nu{}^B\,, \qquad \phi = \Phi - \log\omega$$

$$\psi_{\mu\pm} = \omega^{\mp 1/2}\Pi_\pm \Psi_\mu\,, \qquad \lambda_\pm = \omega^{\mp 1/2}\Pi_\pm \Lambda\,,$$

with $\Pi_\pm = 2^{-1}\big(\mathbf{1} \pm \Gamma_{01}\big)\,.$

Gomis-Ooguri SUGRA
$(\tau^A, e, b, \phi; \psi_\pm, \lambda_\pm)$**:**

$\mathcal{N} = (1{,}0)$ SUGRA
$(G, B, \Phi; \Psi, \Lambda)$**:**

one anisotropic dilatations

$\delta_D \phi = \lambda_D(x)\,, \qquad \cdots$

$\longleftrightarrow$

one null isometry

$\tilde{K}^2 = \tilde{P} = 0$

$8 + 8$ $S-/T-$shifts

$\delta_S \lambda_- = \eta_-\,, \quad \delta_T \psi_{\mu -} = \tau_\mu{}^+ \rho_-\,, \cdots$

$\longleftrightarrow$

16 constraints on gravitini

$\tilde{\Psi}(K) = \tilde{\Psi}_z = 0$

36 intrinsic torsion

$\tau^- \wedge \mathrm{d}\tau^- = 0$

$\longleftrightarrow$

36 additional constraints

$\mathrm{d}\tilde{Z} = \partial_{[i}\tilde{Z}_{j]} = 0$