Analytic bootstrap
of a conformal edge

(soon on arXiv)

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Theoretical physics
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Outline

1 A conformal edge

2 Analytic structure

3 Bootstrap at $\mathscr{O}(\varepsilon)$

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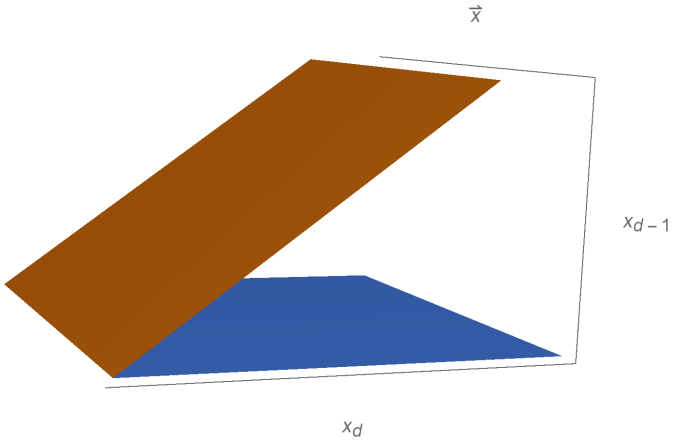
1. A conformal edge
2. Analytic structure
3. Bootstrap at $\mathscr{O}(\varepsilon)$

The setup



Four CFT's:

- Bulk: $SO(d+1,1)$
- Boundaries (wall and ramp): $SO(d,1)$
- Edge: $SO(d-1,1)$

Important in condensed matter.

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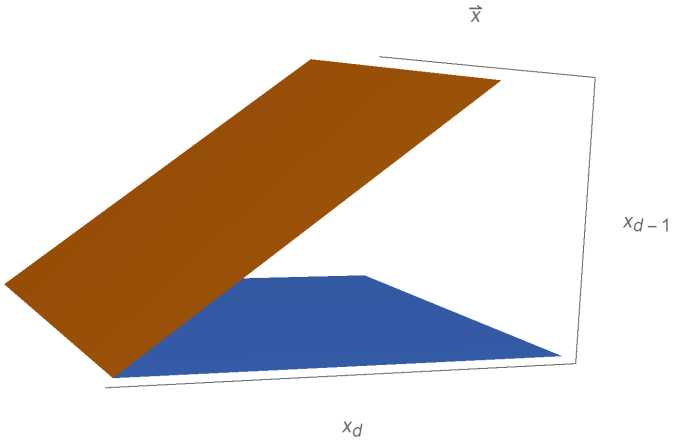

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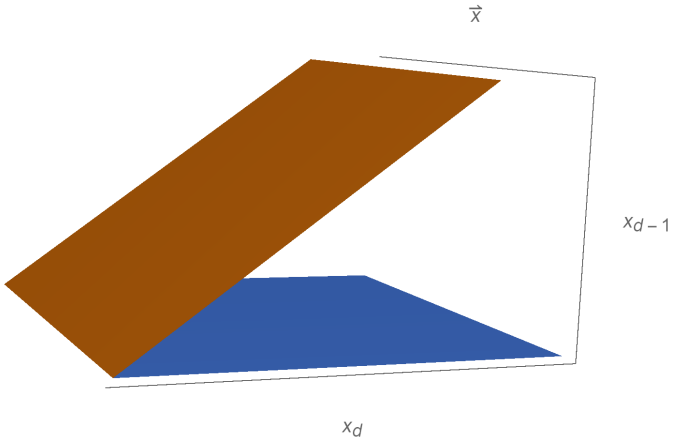

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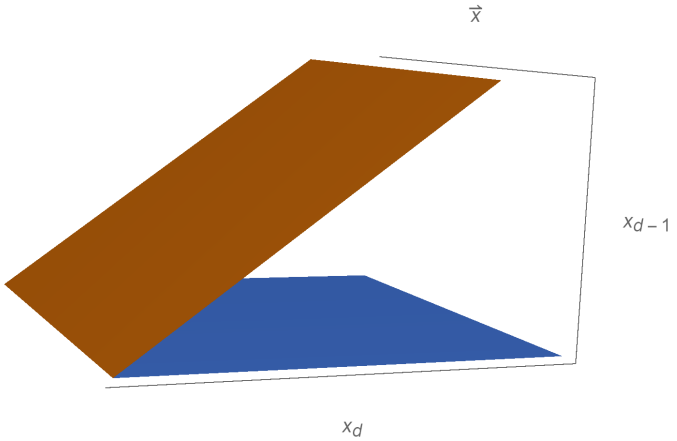

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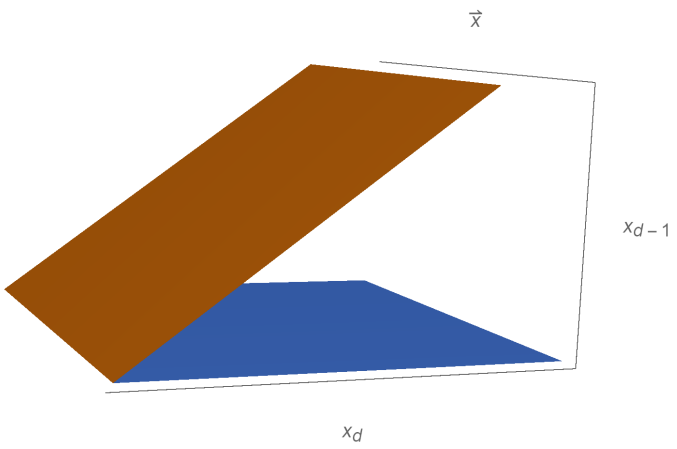

Additional OPE's

$$\phi(\vec{x}, x_{d-1}, x_d) = \sum_{\hat{\mathcal{O}}} \frac{\mu^{\phi}{}_{\hat{\mathcal{O}}}}{x_{d-1}^{\Delta_\phi - \hat{\Delta}}} \hat{\mathcal{O}}(\vec{x}, x_d) \ ,$$

$$\hat{\mathcal{O}}(\vec{x}, x_d) = \sum_{\hat{\hat{\mathcal{O}}}} \frac{\hat{\mu}^{\hat{\mathcal{O}}}{}_{\hat{\hat{\mathcal{O}}}}(\theta)}{x_d^{\hat{\Delta} - \hat{\hat{\Delta}}(\theta)}} \hat{\hat{\mathcal{O}}}(\vec{x}, \theta) \ .$$

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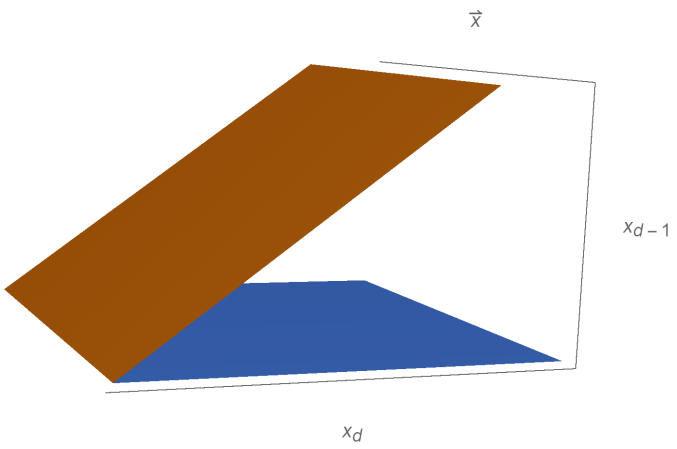

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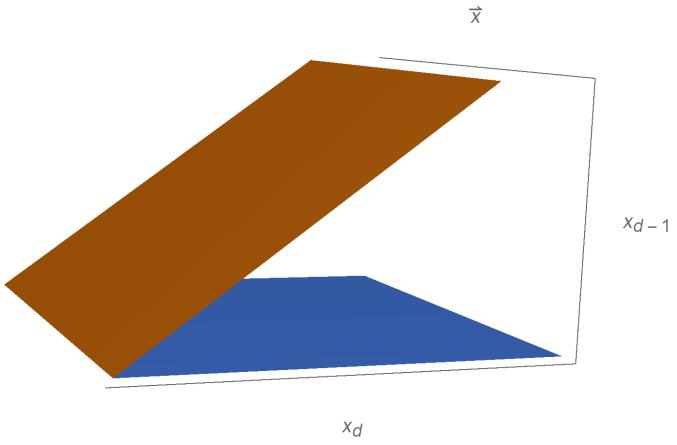
Bulk-edge correlator



Bootstrap equation [António Antunes, '21]

$$\langle \phi(\vec{x}, x_{d-1}, x_d) \hat{\hat{\mathscr{O}}}(\vec{y}) \rangle \propto f(\eta) \ , \quad \eta = \frac{x_{d-1}}{x_d} = \tan\varphi \ ,$$

$$f(\eta) = \sum_n c_n f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}, \eta) = p^{\Delta_\phi - \hat{\hat{\Delta}}} \sum_m c'_m f_{\text{wall}}(\hat{\Delta}'_m, \hat{\hat{\Delta}}, \zeta) \ .$$

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BOE Coefficients

$$c_n(\theta) = \mu^{\phi}_{w\,\hat{\mathscr{O}}_n} \hat{\mu}_w^{\hat{\mathscr{O}}_n}{}_{\hat{\hat{\mathscr{O}}}}(\theta) \ , \quad c'_m(\theta) = \mu^{\phi}_{r\,\hat{\mathscr{O}}'_m} \hat{\mu}_r^{\hat{\mathscr{O}}'_m}{}_{\hat{\hat{\mathscr{O}}}}(\theta) \ .$$

Prefactor and cross-ratio w.r.t. the ramp

$$p(\theta, \eta) = \frac{\eta}{\sin\theta - \eta\cos\theta} \ ,$$

$$\zeta(\theta, \eta) = \frac{\sin\theta - \eta\cos\theta}{\cos\theta + \eta\sin\theta} = \tan(\theta - \varphi) \ .$$

Conformal blocks

$$f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}, \eta) = \eta^{\hat{\Delta}_n - \hat{\hat{\Delta}}}\, {}_2F_1(..., -\eta^2) \ .$$

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Free theory

Only $\hat{\phi}$ and $\partial_\perp \hat{\phi}$ are exchanged

$$c'_{\hat{\phi}} = c'_{\hat{\phi}}(\theta, c_{\hat{\phi}}, c_{\partial_\perp \hat{\phi}}) \,, \quad c'_{\partial_\perp \hat{\phi}} = c'_{\partial_\perp \hat{\phi}}(\theta, c_{\hat{\phi}}, c_{\partial_\perp \hat{\phi}}) \,.$$

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- NN: $c_{\partial_\perp \hat{\phi}},\ c'_{\partial_\perp \hat{\phi}} = 0 \quad \Rightarrow \quad c'_{\hat{\phi}} = c'_{\hat{\phi}}(\theta, c_{\hat{\phi}})$.
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Edge scaling dimension

$$\hat{\hat{\Delta}}_k = \Delta_\phi + k \,, \quad k = \frac{\pi \alpha}{\theta} \,.$$

- α (or k) determines the combination of b.c.'s.
  - Integer for NN and DD.
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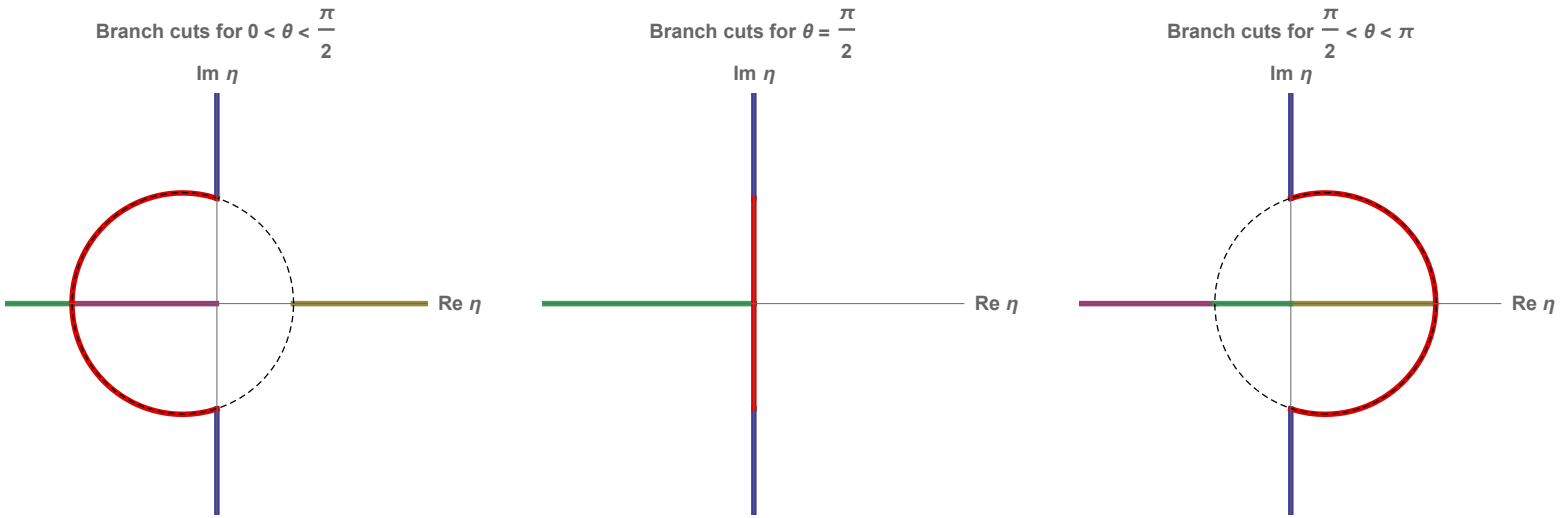

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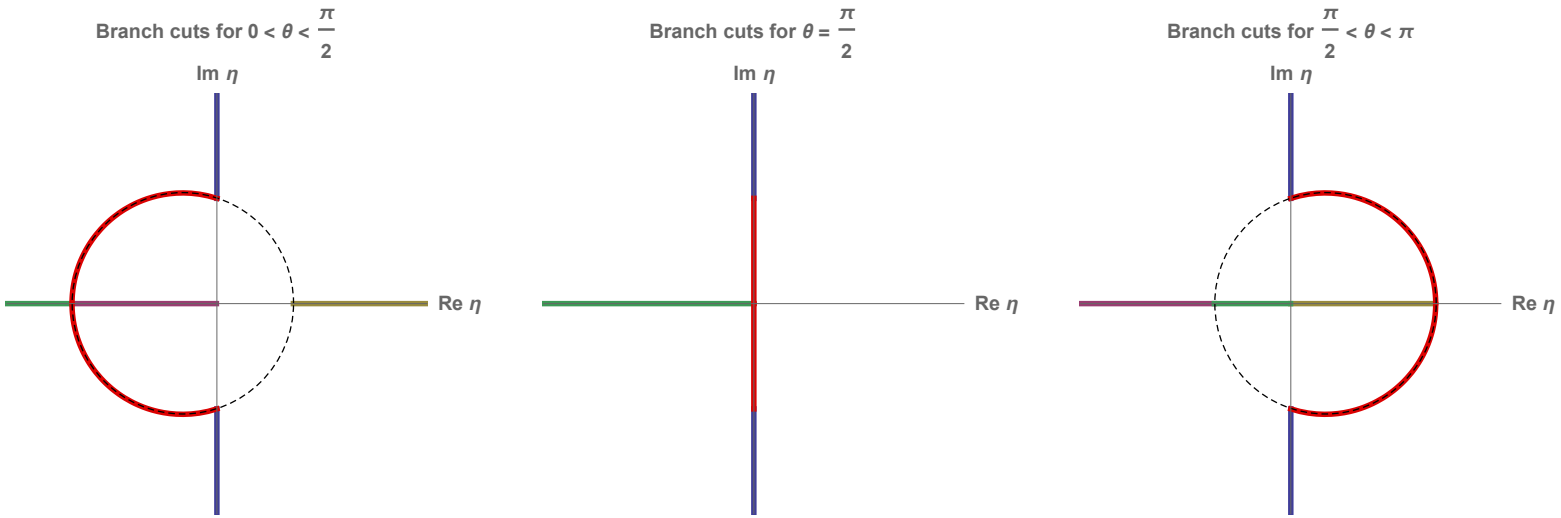

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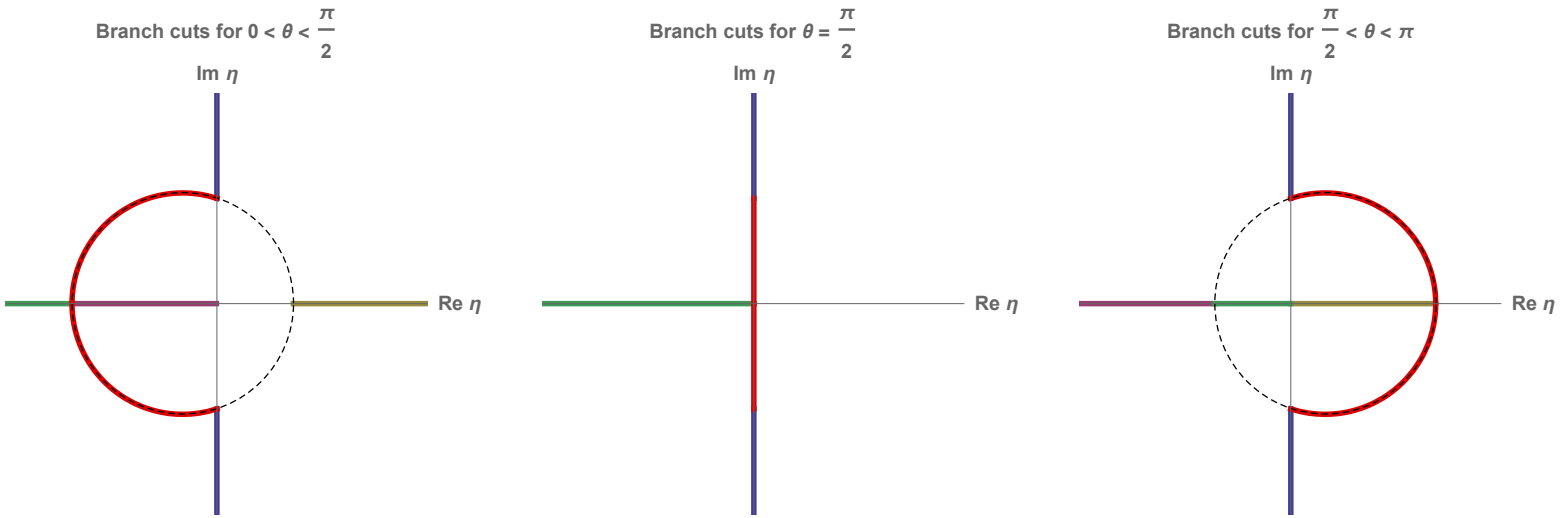

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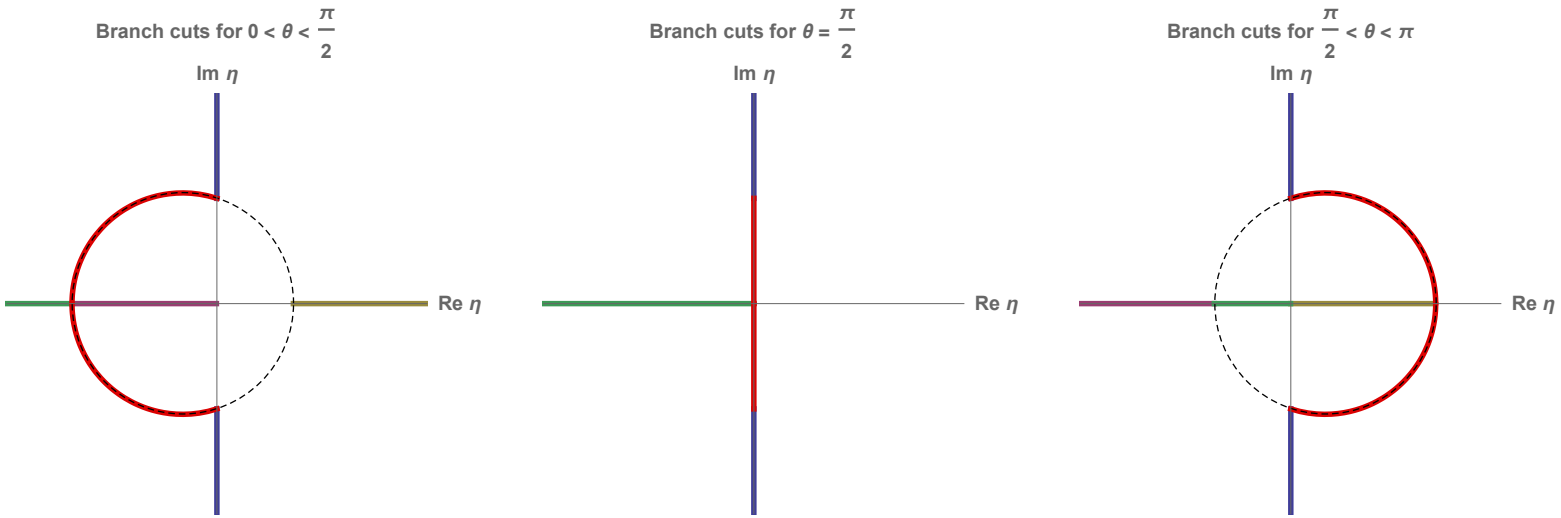

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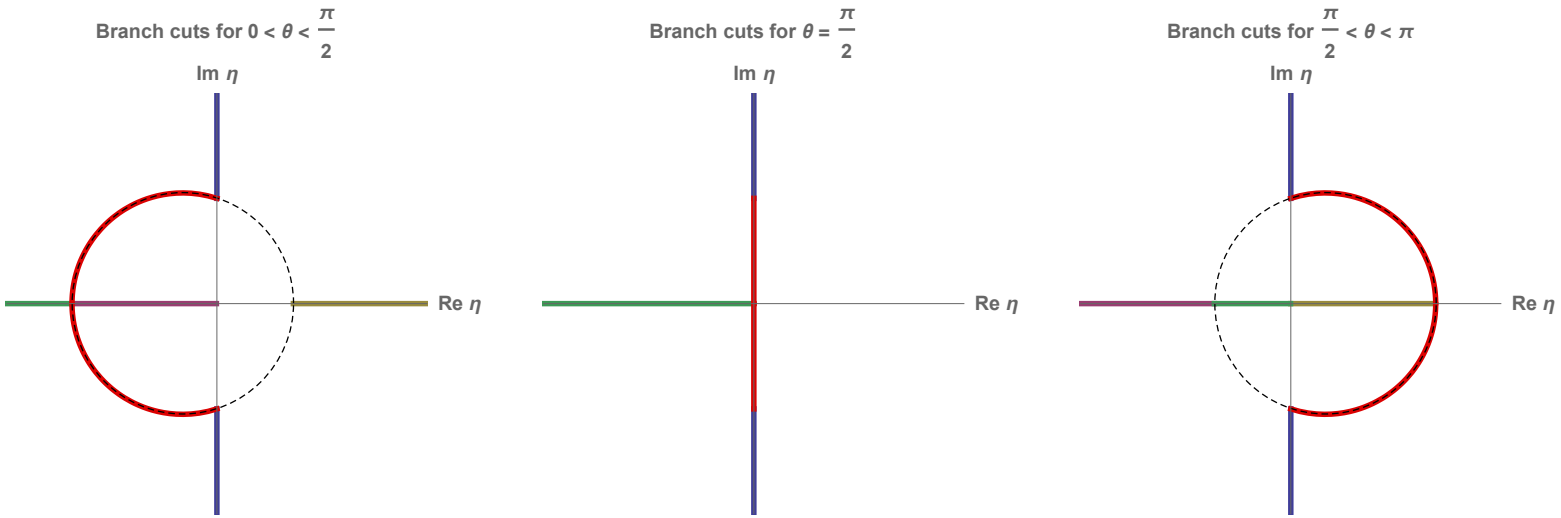

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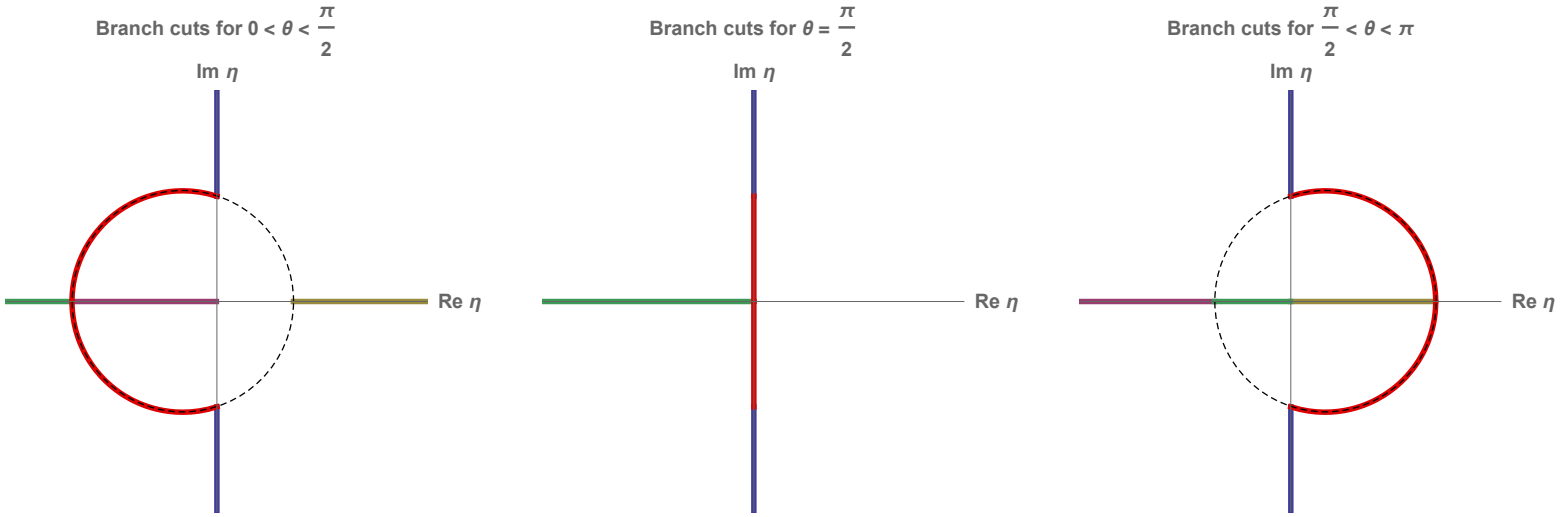

Strategy [Bissi, Hansen, AS, '18], [Dey, AS, '20]

- Study discontinuity along $\eta^2 < -1$

$$\operatorname*{disc}_{\eta^2<-1} f(\eta) = \lim_{\alpha\to 0^+} f(+ix+\alpha) - f(-ix-\alpha)\,, \quad x > 1\,.$$

- Find an orthogonality relation.
- Project out BOE coefficients, c_n.

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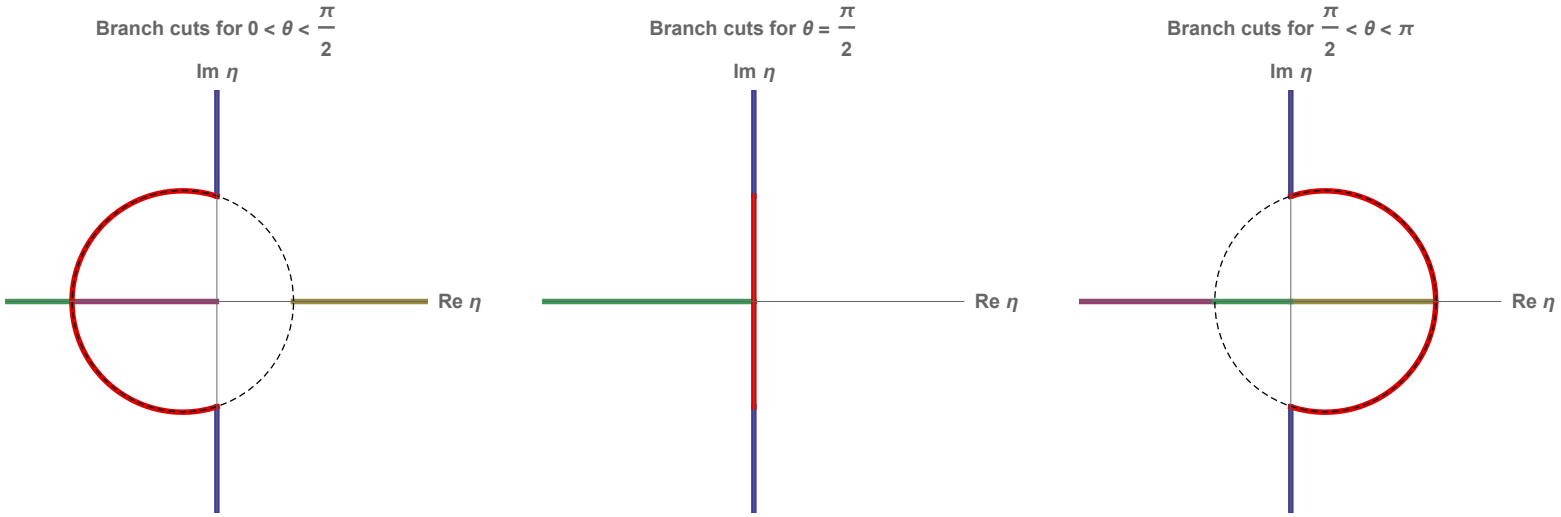

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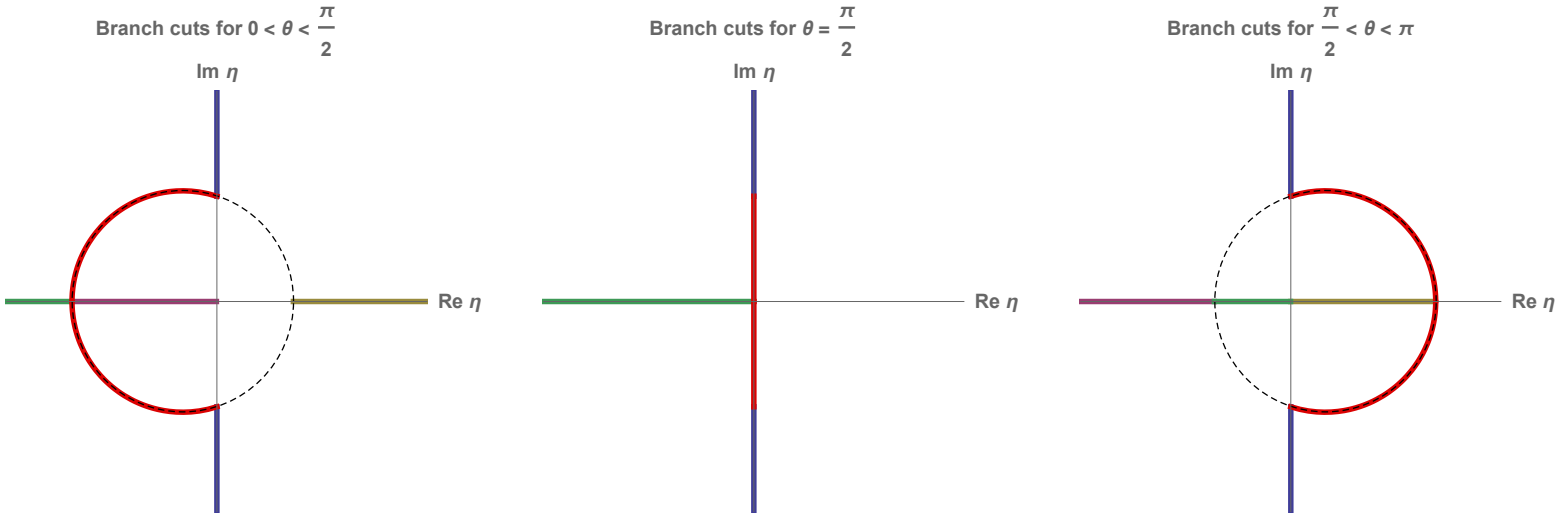

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Discontinuity along $\eta^2 < -1$

Only wall-channel contribute

$$\underset{\eta^2<-1}{\text{disc}}\, f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta) = (1 - e^{2\pi i(\Delta_\phi - \hat{\hat{\Delta}}_k)}) f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta)$$
$$+ \# \eta^{\hat{\Delta}_n - \hat{\hat{\Delta}}_k} {}_2F_1(..., \eta^2 + 1)\ .$$

- Two ${}_2F_1$'s!
- First term vanish for

$$\hat{\Delta}_n = \Delta_\phi + n\ , \quad n \in \mathbb{Z}\ ,$$
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The discontinuity is then a Jacobi polynomial

$$\underset{\eta^2<-1}{\text{disc}}\, f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta) \propto (1-t)^k \sqrt{1+t}\, P^{(k,\frac{1}{2})}_{\frac{n-k}{2}-1}(t)\ , \quad t = -1 - \frac{2}{\eta^2}\ .$$

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$$\hat{\hat{\Delta}}_k = \Delta_\phi + k \, , \quad k = \frac{\pi\alpha}{\theta} \in \mathbb{Z} \, .$$

The discontinuity is then a Jacobi polynomial

$$\underset{\eta^2<-1}{\text{disc}}\, f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta) \propto (1-t)^k \sqrt{1+t}\, P^{(k,\frac{1}{2})}_{\frac{n-k}{2}-1}(t) \, , \quad t = -1 - \frac{2}{\eta^2} \, .$$

Discontinuity along $\eta^2 < -1$

Only wall-channel contribute

$$\underset{\eta^2<-1}{\text{disc}}\, f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta) = (1 - e^{2\pi i(\Delta_\phi - \hat{\hat{\Delta}}_k)}) f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta)$$
$$+ \# \eta^{\hat{\Delta}_n - \hat{\hat{\Delta}}_k} {}_2F_1(..., \eta^2 + 1) \, .$$

- Two ${}_2F_1$'s!
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$$\int_{-1}^{+1} dt\, P^{(k,\frac{1}{2})}_{\frac{m-k}{2}-1}(t) \operatorname*{disc}_{\eta^2<-1} f_{\text{wall}}(\hat{\Delta}_n, \hat{\hat{\Delta}}_k, \eta) \propto \delta_{mn} \ .$$

- Can be used to project out the BOE coefficients c_m.
- Only orthogonal for even or odd m, n.
  - This is the case in $d = 4 - \varepsilon$ dimensions [Agnese Bissi, Tobias Hansen, AS, '18].

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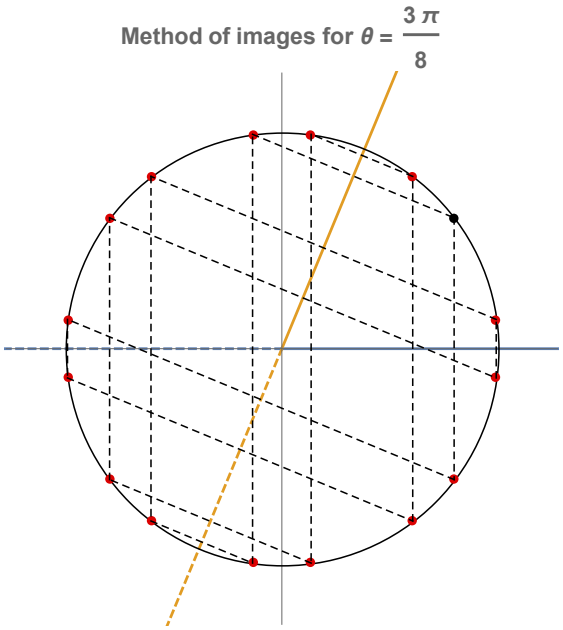
Method of images



Implication of $k \in \mathbb{Z}$

$$k = \frac{\pi\alpha}{\theta} \in \mathbb{Z} \quad \Leftrightarrow \quad \theta = \frac{\pi\alpha}{k} \in \pi\mathbb{Q} \ .$$

Finite number of images

$$\theta = \frac{\pi a}{b} \quad \Rightarrow \quad \#(\text{images}) = 2b - 1 \ .$$

Outline

1 A conformal edge

2 Analytic structure

3 Bootstrap at $\mathscr{O}(\varepsilon)$

The bootstrap equation

We expect an infinite amount of operators at $\mathscr{O}(\varepsilon)$.

Expand the bootstrap equation as

$$f(\eta) = G_w + H_w + \mathscr{O}(\varepsilon^2) = G_r + H_r + \mathscr{O}(\varepsilon^2) \ .$$

- G_w, G_r contain the anomalous dimensions

$$G_w(\eta) = c^{(f)}_{0/1} f_{\text{wall}}(\hat{\Delta}_{0/1}, \hat{\hat{\Delta}}_k, \eta) \ .$$

- H_w, H_r contain the BOE coefficients at $\mathscr{O}(\varepsilon)$.

$$H_w(\eta) = \varepsilon \sum_n \delta c_n f_{\text{wall}}(n+1, k+1, \eta) \ .$$

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$$\boxed{\delta c_n^{NN} = \frac{(-1)^p c_0^{(f)} (\hat{\gamma}_0 - \hat{\hat{\gamma}})}{(p+n+1)(2(p+n)+1)} \frac{\Gamma_{n+\frac{3}{2}} \Gamma_{2p+n+\frac{3}{2}}}{\sqrt{\pi}\Gamma_{2(p+n)+\frac{3}{2}}} .}$$

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Thank you!