SUSY enhancement on S_b^3

Charles Thull

Uppsala University

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A puzzle in ABJ(M) on S_b^3

ABJ(M) theory from an $\mathcal{N} = 2$ perspective:

> $U(N)_k \times U(N+M)_{-k}$ gauge theory + 4 bi-fundamental chirals and $U(1)_b \times SU(2)_R \times SU(2)_R$ flavor symmetry

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Can be put on the squashed sphere S_b^3 while preserving $\mathcal{N} = 2$ SUSY. Admits 3 mass parameters $m_1, m_2 + m_3, m_2 - m_3$.

Partition function can be computed using localization, complicated b dependence in general.

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Partition function can be computed using localization, complicated b dependence in general. However [Chester, Kalloor, Sharon '21] found:

$$\mathcal{Z}\left(b; m_1 = \frac{bm_+ - b^{-1}m_-}{2}, m_2 = \frac{bm_+ + b^{-1}m_-}{2}, m_3 = \mathrm{i}\frac{b - b^{-1}}{2}\right)$$
$$= \mathcal{Z}\left(1; m_1 = \frac{m_+ - m_-}{2}, m_2 = \frac{m_+ + m_-}{2}, m_3 = 0\right)$$

Outline

- $\mathcal{N} = 2$ supersymmetry on the squashed three-sphere
- Enhancement to $\mathcal{N} = 4$ supersymmetry
- Applications

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The squashed three-sphere

In this talk S_b^3 is given by the metric

$$ds^2 = \frac{r_3^2}{4}\left(\frac{b+b^{-1}}{2}\right)^2(\mathrm{d}\psi + \cos\theta\mathrm{d}\phi)^2 + \frac{r_3^2}{4}\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

$\rightarrow$ we stretch the Hopf fiber by a factor $\frac{b+b^{-1}}{2} \geq 1$.

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$\mathcal{N} = 2$ supersymmetry on S_b^3

To preserve SUSY: embed the metric into a background Sugra multiplet
$\Rightarrow$ Killing spinor equations (KSE)

In 3d $\mathcal{N} = 2$ (off-shell) conformal Sugra [Nishimura, Tanii '13]:

$$\nabla_\mu \zeta - \mathrm{i} A_\mu^{12} \zeta = \gamma_\mu \eta,$$
$$\nabla_\mu \widetilde{\zeta} + \mathrm{i} A_\mu^{12} \widetilde{\zeta} = \gamma_\mu \widetilde{\eta}.$$

(In the following I will not write the equations for the $\widetilde{\zeta}$.)
Correct choice of $SO(2)_R$-gauge field $A^{12} \Rightarrow$ 4 Killing spinors preserved.
[Imamura, Yokoyama '11]: b-dependent partition function from localization.

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Mass-deformed $\mathcal{N} = 2$

Supersymmetric mass: part of a background vector multiplet (A_μ, m, D):

$$\frac{1}{4}\gamma^{\mu\nu} F_{\mu\nu}\zeta - \mathrm{i}(D\zeta - m\eta) = 0 \qquad \text{\& corresp. } \widetilde{\zeta} \text{ equation.}$$

Fixes A, D in terms of m and the metric background.

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Embedding into $\mathcal{N} = 4$ conformal SUGRA

Assume the mass couples to a $U(1)$ of the R-symmetry: $A_\mu \to A_\mu^{34}$.

Then A^{12}, A^{34} are components of a background $SO(4)_R$ gauge field A^{IJ}
$\Rightarrow$ all backgound fields g, A^{IJ}, m, D part of an $\mathcal{N} = 4$ Weyl multiplet.

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$\Rightarrow$ all backgound fields g, A^{IJ}, m, D part of an $\mathcal{N} = 4$ Weyl multiplet.
$\mathcal{N} = 4$ conformal Killing spinor equations [Nishimura, Tanii '13]:

$$\begin{aligned} \nabla_\mu \zeta - \mathrm{i} A_\mu^{12} \zeta &= \gamma_\mu \eta, \\ \nabla_\mu \zeta' - \mathrm{i} A_\mu^{34} \zeta' &= \gamma_\mu \eta', \\ \frac{1}{4}\gamma^{\mu\nu} F_{\mu\nu}^{34} \zeta - \mathrm{i}(D\zeta - m\eta) &= 0, \\ \frac{1}{4}\gamma^{\mu\nu} F_{\mu\nu}^{12} \zeta' - \mathrm{i}(D\zeta' - m\eta') &= 0, \end{aligned}$$

& 4 corresponding equations for $\widetilde{\zeta}, \widetilde{\zeta}'$.

Twice as many equations as before: At general m no non-trivial $\zeta', \widetilde{\zeta}'$.

$\mathcal{N} = 4$ on the squashed sphere

For $m = \mathrm{i}\frac{b-b^{-1}}{2}$ the $\mathcal{N} = 4$ KSE admit additionaly non-trivial Killing spinors

$$\zeta' = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \widetilde{\zeta}' = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

i.e. we now have a total of 6 Killing spinors.

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Localization with only these two Killing spinors $\zeta', \widetilde{\zeta}'$ [Hama, Hosomichi, Lee '11]: squashing independent partition function.

$\Rightarrow$ Coupling R-symmetry to a mass $m = \mathrm{i}\frac{b-b^{-1}}{2}$ allows for SUSY enhancement, the squashing deformation is $\mathcal{Q}$-exact under these additional Killing spinors [Closset, Dumitrescu, Festuccia, Komargodski '12, '13]

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Note: Same happens for $m = -\mathrm{i}\frac{b-b^{-1}}{2}$

Back to ABJ(M) and beyond

For ABJ(M): m_3 coupled to a $U(1) \subset SU(2)_R \times SU(2)_R \subset SO(6)_R$

$\Rightarrow$ ABJ(M) features this SUSY enhancement for $m_3 = \mathrm{i}\frac{b-b^{-1}}{2}$ and squashing independence follows.

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Our conclusions hold for other theories as well.

Example: theories constructed of $\mathcal{N} = 4$ vector and hypers.
Fine-tuned mass couples to axial $U(1)$ of the $SU(2) \times SU(2) = SO(4)_R$.
We find then

$$\mathcal{Z}_{\mathcal{N}=4}(b; b^{\mp 1}\eta; b^{\pm 1}\vec{m}, m = \pm\mathrm{i}\frac{b-b^{-1}}{2}) = \mathcal{Z}_{\mathcal{N}=4}(1; \eta; \vec{m}, 0)$$

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Testing $\mathcal{N} = 4$ dualities on S_b^3

Wide variety of IR dualities for $3d$ $\mathcal{N} = 4$ theories, usually on conformally flat manifolds. E.g. [Kapustin, Willett, Yaakov '10]:

$$\mathcal{Z}_{\mathrm{ABJM}}(1; \zeta; 2\xi, 0) = \mathcal{Z}_{\mathcal{N}=8}(1; \xi + 2\zeta; \xi - 2\zeta, 0).$$

Here $\mathcal{N} = 8$ SYM: $\mathcal{N} = 4$ vector + 1 fund. hyper + 1 massive adj. hyper

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Putting things together with our results:

$$\mathcal{Z}_{\mathrm{ABJM}}\left(b;\zeta;2\xi,\mathrm{i}\frac{b-b^{-1}}{2}\right) = \mathcal{Z}_{\mathcal{N}=8}\left(b;\xi+2\zeta;\xi-2\zeta,-\mathrm{i}\frac{b-b^{-1}}{2}\right)$$

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Conclusion

What I have shown:

- Introducing a fine tuned mass in $\mathcal{N} = 2$ on S_b^3 leads to supersymmetry enhancement.
- This SUSY enhancement leads to squashing independence of partition functions.
- We have indications that $\mathcal{N} = 4$ dualities hold also on S_b^3.

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What I had no time to show:

- We also find an enhancement at $m=\mathrm{i}\frac{b+b^{-1}}{2}$. Coincides with a simplification of the partition function.
- 4d index perspective.
- THF and the ellipsoid.
- A similar story in 4d [Naseer, Thull '21].

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Outlook

Ongoing work and future directions:

- BPS-Wilson lines.
- Mirror symmetry on S_b^3.
- Brane constructions for the dualities on S_b^3.
- Seiberg-like dualities on S_b^3 ?
- What happens if we go away from this special point? Do the dualities break?
- Understand the $m = \mathrm{i}\frac{b+b^{-1}}{2}$ theories.

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