Quarter-BPS operators in $\mathcal{N} = 4$ SYM: a case study

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Based on 2111.06857

[Bissi, GF, Manenti]

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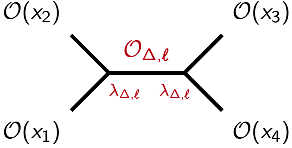

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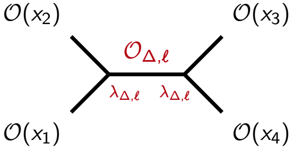

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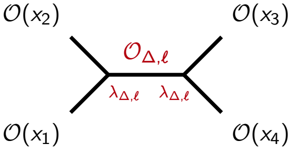

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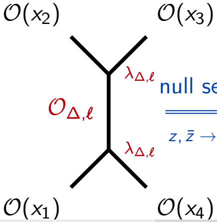

null sep. $\xRightarrow[z,\bar{z}\to 0]{}$ singularities $\updownarrow$ OPE data

[Rattazzi et al. (2008); Fitzpatrick et al. (2013)]

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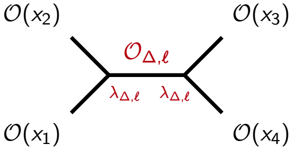

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Stronger constraints if we add supersymmetry $\rightarrow$ SCFT

We focus on 4d $\mathcal{N}=4$ SYM with SU(N)
- per se: "bootstrap" non protected contributions
- AdS/CFT correspondence

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4d $\mathcal{N} = 4$ SYM: SU(N) gauge group, g_{YM} coupling, $SU(4)_R$ $\longleftrightarrow$ IIB string theory on $AdS_5 \times S^5$ $\sqrt{\alpha'}$ string length, g_s coupling

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$\mathcal{O}_2$ stress tensor — SUGRA excitations, graviton

$p \geq 3$ — SUGRA KK modes

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Half-BPS operators

So far, intense study of $\frac{1}{2}$-BPS operators

- protected dimension and three-point functions
- maximal amount of supersymmetry
- Schur ops in the chiral algebra construction [Beem, Rastelli, Rees (2013)]
  Chiral algebra idea: correlators of protected operators reduce to meromorphic functions when after an R-symmetry twist are restricted to lay on a plane.

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- Superconformal WI are solved [Nirschl, Osborn (2005); Beem et al. (2017)]
- Completely fixed up to N^{-4} [Alday, Bissi (2017)]
- **However** as we go to higher orders at large N we run into problems pretty fast
  - **mixing** among degenerate ops
  - appearance of **higher trace** ops

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Let us have a closer look to $\frac{1}{4}$-BPS operators

$$\mathcal{O}_{pq} \sim \mathrm{tr}(\varphi^{M_1}\cdots)\mathrm{tr}(\cdots\varphi^{M_\Delta})P_{M_1\cdots M_\Delta} + \frac{1}{N}(\text{single trace})$$

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  Ex: 10 structures in $\mathcal{O}_{02} \times \mathcal{O}_2$, 42 in $\mathcal{O}_{02} \times \mathcal{O}_{02}$ vs 6 in $\mathcal{O}_2 \times \mathcal{O}_2$.
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  $\Rightarrow$ We introduce null polarization vectors and we build invariants out of them similar to [Cuomo, Karateev, Kravchuk (2018)]
- Contain $\frac{1}{2}$-BPS Schur operators when expanded in $\mathcal{N} = 2$ supermultiplet $\Rightarrow$ Ward Identities

Quarter-BPS operators: four-point function

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$$\langle \mathcal{O}_{p_1 q_1} \mathcal{O}_{p_2 q_2} \mathcal{O}_{p_3 q_3} \mathcal{O}_{p_4 q_4} \rangle \sim \sum_k \mathbb{T}_k \, \mathcal{G}_k(z, \bar{z}) \,,$$

where we consider $\mathcal{O}_{p_1 q_1}$ to be $\mathcal{O}_2$ or $\mathcal{O}_{02}$.

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$$\mathcal{G}_k(z, \bar{z}) = w_k(z, \bar{z}) + \sum_{m=1}^{\dim(\ker \chi)} \mathcal{H}_k(z, \bar{z}) v_k^{(m)}(z, \bar{z})$$

w_k ↓ protected χ algebra ⟸ Extract new OPE data [Alday, Caron-Huot (2018)]

$\mathcal{H}_k v_k^{(m)}$ ↓ non protected

$$\chi\left[\sum \mathbb{T}_k w_k\right] = \mathfrak{f}(z)$$

$$\chi\left[\sum \mathbb{T}_k v_k^{(m)}\right] = 0$$

OPE data

In the regime under analysis, the OPE data of a non protected exchanged operator get expanded as

$$a_{\Delta,\ell} = a^{(0)}_{\Delta,\ell} + \sum_{\kappa} \frac{a^{(\kappa)}_{\Delta,\ell}}{N^{2\kappa}}\,, \qquad \Delta = \Delta^{(0)} + \sum_{\kappa} \frac{\gamma^{(\kappa)}_{\Delta,\ell}}{N^{2\kappa}}\,.$$

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We have studied

- $\langle \mathcal{O}_{02}\mathcal{O}_{2}\mathcal{O}_{2}\mathcal{O}_{2}\rangle \to$ protected (see next)
- $\langle \mathcal{O}_{2}\mathcal{O}_{02}\mathcal{O}_{02}\mathcal{O}_{2}\rangle \to$ OPE data in $\mathcal{O}_{02}\times\mathcal{O}_{2}$
- $\langle \mathcal{O}_{02}\mathcal{O}_{02}\mathcal{O}_{02}\mathcal{O}_{02}\rangle \to$ OPE data in $\mathcal{O}_{02}\times\mathcal{O}_{02}$

Higher-point functions

Why higher-points? Quarter-BPS operators appear as leading singularities in the OPE of $\mathcal{O}_2 \times \mathcal{O}_2$. For instance

$$\lim_{x_1 \to x_2} \langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\cdots\rangle\big|_{(2,0,2)} \supset \lambda_{\mathcal{O}_{02}} \langle \mathcal{O}_{02}\cdots\rangle$$

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So to extract the contribution of a specific operator, we just have to take limits of higher-point functions. Consider the 5-point function of [Gonçalves, Pereira, Zhou (2019)] in supergravity

$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle = \frac{1}{N} \mathcal{G}^{(5)}_{\text{disc}} + \frac{1}{N^3} \mathcal{G}^{(5)}_{\substack{\text{sugra} \\ \text{tree}}}$$

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protected!

Conclusions

Summary

We have started a systematic study of quarter-BPS operators in $\mathcal{N} = 4$ SYM. In particular

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Outlook

Several directions can be pursued

- Find the explicit form of **superconformal blocks**, similarly to what has been done for $\frac{1}{2}$-BPS
- Understand the **basis of function** of the SUGRA correlator (not only $\bar{D}$?)
- **Higher-point functions**
- Determine the OPE data of **triple trace** operators appearing in $\mathcal{O}_2 \times \mathcal{O}_2$