Risking your NEC

José Manuel Penín Ascariz

November 24, 2021

University of Helsinki

• 1st version of the gauge/gravity correspondence (Maldacena '97):

$$\mathcal{N} = 4 \text{ SYM}, SU(N) \Leftrightarrow \text{type IIB strings on } AdS_5 \times S^5$$

• Color branes → p-brane SUGRA solutions
• Only adjoint matter? Fundamentals? → Add N_f flavor D-branes (Karch '01)

- N_f small → Flavors added as probe branes. (non-dynamical, infinitely massive quarks)
- N_f large → Backreaction of flavor branes important! → $S_{SUGRA} + S_{branes}$ (more difficult problem)

• flavor branes = sources to supergravity (SUGRA) eoms → violation of Bianchi id. for fluxes $dF \neq 0$

• D-branes are localized: (Dp brane = codimension q-p defect)
• Localized sources → $dF \sim \delta(x)$ Challenging!

⇒ Use smeared sources: "a continuous distribution of branes" → avoids $\delta(x)$ (easier problem...)

Consider a continuous distribution of branes:

- Flavor group: $U(N_f) \to U(1)^{N_f}$
- Brane action:
  - probe brane: DBI+WZ defined in $q+1$ worldvolume
  - smeared branes: probe action $\to$ 10 d action
- smeared action $\sim$ (probe action) $\wedge$ (distribution)
  10 - form $\quad$ $(q+1)$ - form $\quad$ $(9-q)$ - form
- this smearing preserves supersymmetry

$$S_{brane} = -T_q \int_{\mathcal{M}_{10}} (e^{-\phi}\mathcal{K}_{(q)} - C_{q+1}) \wedge \Xi \tag{1}$$

- modified Bianchi identity:

$$dF_{8-q} = 2\kappa_{10}^2 T_q \Xi \tag{2}$$

Smearing technique

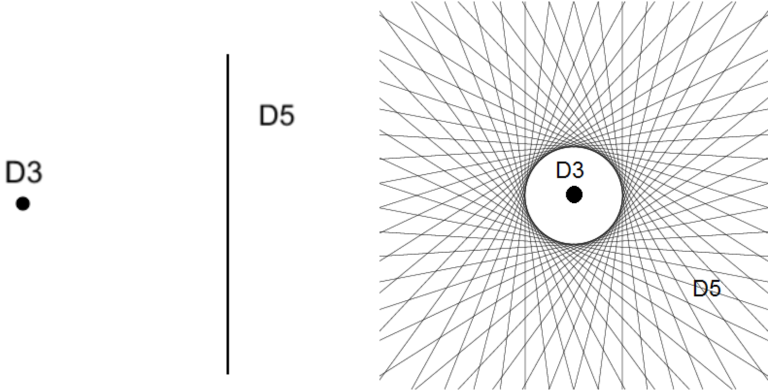


Figure: Localized vs smeared branes

- Massless embeddings:
  - Probe branes: no dependence on holographic radial direction:
  - Smeared branes: no dependence on radial direction, legs transverse to massless embedding
- Massive embeddings:
  - Probe branes: dependence on holographic direction
  - Smeared branes: dependence on holographic direction

$$Q_f \rightarrow Q_f \, p(r) \tag{3}$$

$p(r)$ 'profile function' $\Rightarrow$ comparison from microscopic DBI with macroscopic DBI fixes the profile.
But no constraints coming from SUSY!
• (But SUSY constrains structure e.g.: internal space Sasaki-Einstein (5D), Nearly Kahler (6D))
• Susy variations $\Rightarrow$ BPS equations for dilaton and metric

- An arbitrary $p(r)$ is not related to usual embeddings in any obvious way
- Flavor-like profiles: $p(r \to \infty) = 1, p(r = r_q) = 1$

Examples of application:

- D3-D7: Benini, Canoura, Cremonesi, Nunez, Ramallo 0612118/ Bigazzi, Cotrone, Paredes, Ramallo 0810.5220
- D2-D6: Faedo, Mateos, Tarrio 1505.00210
- ABJMf: Conde, Ramallo 1105.6045, Bea, Conde, Jokela, Ramallo 1309.4453
- D3-D5 Conde, Lin, P., Ramallo, Zoakos 1607.04998

What about using more general profiles? coming from:

- antibranes (susy)
- orientifolds
- not in string theory

⇒ Let us explore other profiles giving new supergravity duals! (2104.11749 C. Hoyos, N. Jokela, J. M. P., A. Ramallo, J. Tarrio)

Duals to $2+1$ QFTs:

- ABJMf: $AdS_4 \times M_6$ + D6 flavor branes
- D2-D6: D2 geometry+ D6 flavor branes

Duals to $3+1$ QFTs:

- D3-D7: $AdS_5 \times M_5$ + D7 flavor branes
- D3-D5: $AdS_5 \times M_5$ + D5 defect branes

Consider distributions satisyfing:

$$\lim_{r\to 0} p(r) < \infty, \quad \lim_{r\to\infty} = 0, \quad p(r) \geq 0 \tag{4}$$

If $p(0)=0$, non-monotonic! Requirement of positive tension:

$$T_{00}^{BRANE} \sim N_f(d\ p(r) + rp'(r)) \geq 0 \tag{5}$$

Where d= {2-ABJMf, 3-D3-D5, 4-D2-D6 & D3-D7}

$$\Rightarrow p(r) = \frac{r^n}{(1+r^m)^{\frac{n+d}{m}}}, \quad n \geq 0, m \geq 1 \tag{6}$$

Our findings?

- We find analytic solutions (up to a warp factor given by an integral)
- Series expansion for $r \to 0$ (IR) & $r \to \infty$ (UV).
- No curvature singularities
- Do reductions to 4D or 5D
- Study effects of non-monotonic profiles on c-theorems

Brane distribution $\sim p(r) \Rightarrow$ not monotonic with r (energy scale in the holographic dual)

•c-theorems: 'there should be a monotonic quantity along the RG flow related to the number of degrees of freedom'

Question: does a c-theorem hold?

• c-theorems rely on Lorentz invariance $\rightarrow$ do not apply to D3-D5.

• there are proposals for holographic c-functions in anisotropic geometries In Poincare invariant geometries, monotonicity requires the null-energy condition (NEC):

$$T_{MN} n^M n^N = 0 \ \Rightarrow \ R_{MN} n^M n^N \geq 0 \tag{7}$$

n null vector in the bulk $n^2 = 0$

• Holographic c-theorems aere formulated in 4D, 5D $\Rightarrow$ use the reduced actions.

Relevant NEC is in 4D or 5D (different from 10D NEC)

Consider first the isotropic geometries (ABJMf, D2-D6, D3-D7):
Holographic c-theorems that DO NOT hold if the NEC is satisfied VIOLATED :

- (NO) AdS radius $L_{UV} > L_{IR}$. (Freedman, Gubser, Pilch, Warner 9904017)
- (NO) Radial expansion of null congruences (Sahakian 9910099)
- (?)Entanglement entropy of a strip: $C_d = l^d \frac{\partial S_{EE}}{\partial l}$ (Ryu-Takayanagi 0605073), (Myers-Singh 1202.2068). Only if the NEC is violated in a finite region

• We check the NEC in the reductions to 4d and 5d and also in the 10d case.
• We plot the NEC for $0 \leq r \leq \infty$

	UV	IR	Intermediate energy
ABJMf ($N_f/N_c \ll 1$)	always	$n < 2(\sqrt{2}-1)$	always
D2-D6	always	always	$N_f < N_f^{\text{crit}}$
D3-D7	never	always	$r < r_{\text{crit}}$

D2-D6

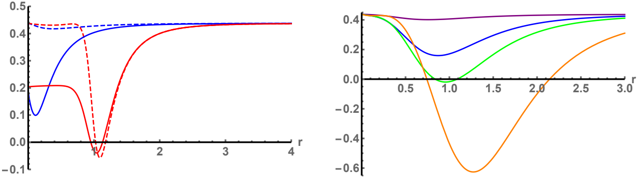


Figure: **Left**: varying (n, m). **Right**: varying Q_f

ABJMf

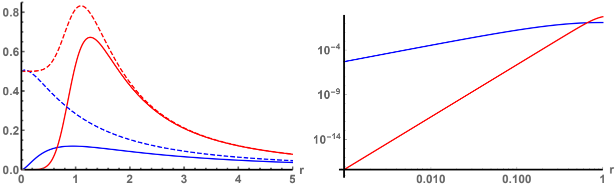


Figure: $n = 0$ (dashed), $n = 2(\sqrt{2} - 1)$ (solid) and $m = 1$ (blue), $m = 5$ (red). If the curves are positive then it means that the four-dimensional NEC holds. We observe that the NEC is satisfied for the full range of r.

Let's study the D3-D5 case:

• For n>1, we have boomerang solutions (Donos, Gauntlett, Rosen / Sosa-Rodriguez 1705.03000,1712.08017)

• $\frac{1}{3} \leq n < 1$ Lifshitz solutions

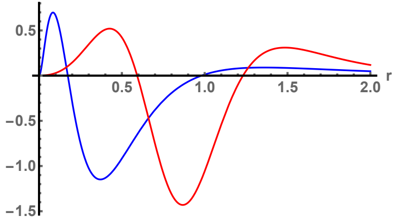
$$ds^2 = \frac{dr^2}{r^2} + r^2 \eta_{mn} dx^m dx^n + r^{\frac{2}{z}} dx_3^2, \quad z = \frac{1}{n} \tag{8}$$

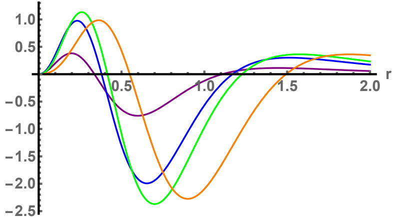
For $n = 1/3$ we have massless D5

NEC for D3-D5:

UV: always, IR: boomerang → always, Lifshitz → never.

$0 < r < \infty$ boomerang: violated in a finite region





We focus on 2 types of anisotropic c-functions:
1-based on entanglement entropy: Chu, Giataganas 1906.0962.
Having scaling symmetry:

$$t \to \Lambda t, \quad x_i \Lambda^{n_i} x_i, \quad y_j \to \Lambda^{n_2} y_j, \quad i = 1, ..., d_1, \quad j = 1, ..., d_2 \tag{9}$$

The entanglement entropy with these scaling geometries goes like:

$$S_{EE}^{(x)} \sim -\frac{1}{l^{d_x}}, \quad d_x = d_1 - 1 + d_2 \frac{n_2}{n_1} \tag{10}$$

$$S_{EE}^{(y)} \sim -\frac{1}{l^{d_y}}, \quad d_y = d_2 - 1 + d_1 \frac{n_1}{n_2} \tag{11}$$

For boomerang flows we have: $C_{||} = C_{\perp} \equiv \beta_4 l^3$. We can define an average c-function $\bar{c} = (c_{||} c_{\perp}^2)^{\frac{1}{3}}$.
For Lifshitz flows, the behaviour for IR and UV is:

$$UV : l \to 0, \quad C_{||}(l) \approx C_{\perp}(l) \approx \beta_4 l^3$$

$$IR : l \to \infty, \quad C_{||}(l) \approx \beta_{d_{||}+2} l_0^3 \left(\frac{l}{l_0}\right)^{1+\frac{2}{n}}, \quad C_{\perp}(l) \approx \beta_{d_{\perp}+2} l_0^3 \left(\frac{l}{l_0}\right)^{n+2} \tag{12}$$

$$\frac{2}{n} = d_{||} > d_{UV} = 2 > d_{\perp} = n + 1 \Rightarrow \text{Define} \quad \bar{c} = (c_{||}^n c_{\perp}^2)^{\frac{1}{n+2}} \tag{13}$$

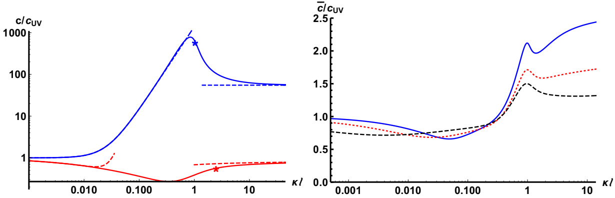


Figure: Left: The c-functions for the in-plane ($\|$,blue) and off-plane ($\perp$,red) directions at $Q_f = 1$ and $n = 1/2$, $m = 2$ as functions of $\ell_{\|}$ and $\ell_{\perp}$, respectively. The dashed curves are the analytic UV and IR expansions. Right: We depict the average $\bar{c}$-functions

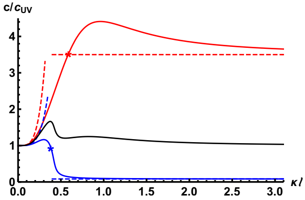


Figure: The c-functions for the boomerang flows. The solid curves are produced numerically, while the dashed curves follow from the asymptotic UV and IR analytics The black curve is the average c-function

2-Other candidates of c-functions are based on the generalization of the AdS radius: Giataganas, Gursoy, Pedraza 1708.05691 and Cremonini, Li, Ritchie, Tang 2006.10780

$\partial_r c$?	UV	IR Boomerang	IR Lifshitz
GGP	$\partial_r c < 0$	$\partial_r c > 0$	$\partial_r c > 0$
CLRT	$\partial_r c < 0$	$\partial_r c > 0$	$\partial_r c < 0$

So we have decreasing c-functions in the UV, and is non-monotonic in most cases.

- NEC can be satisfied for ABJMf and D2-D6 (2+1)
- NEC violated for D3-D7 and D3-D5
- The anistropic c-theorems do not hold for D3-D5
- All the gravity backgrounds constructed are susy, causal and non singular

So

- non-monotonic brane distribution does not imply violation of NEC
- NEC violated usually means violation of c-theorem. Not correct c-function?
- Quantum corrections for classical bckg saturating NECs?

Thanks for your attention!