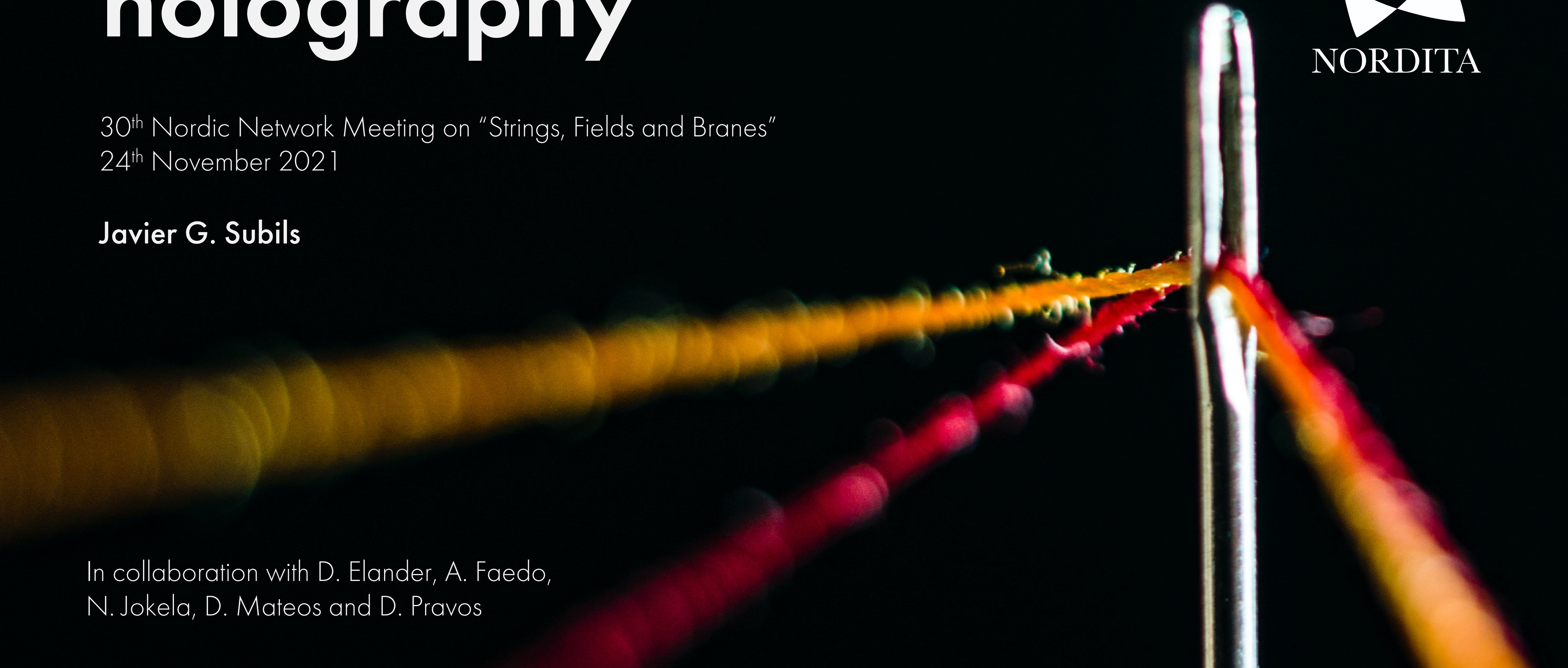
Probing confinement in holography

30th Nordic Network Meeting on "Strings, Fields and Branes"
24th November 2021

Javier G. Subils



In collaboration with D. Elander, A. Faedo,
N. Jokela, D. Mateos and D. Pravos

NORDITA

Confinement and mass gap

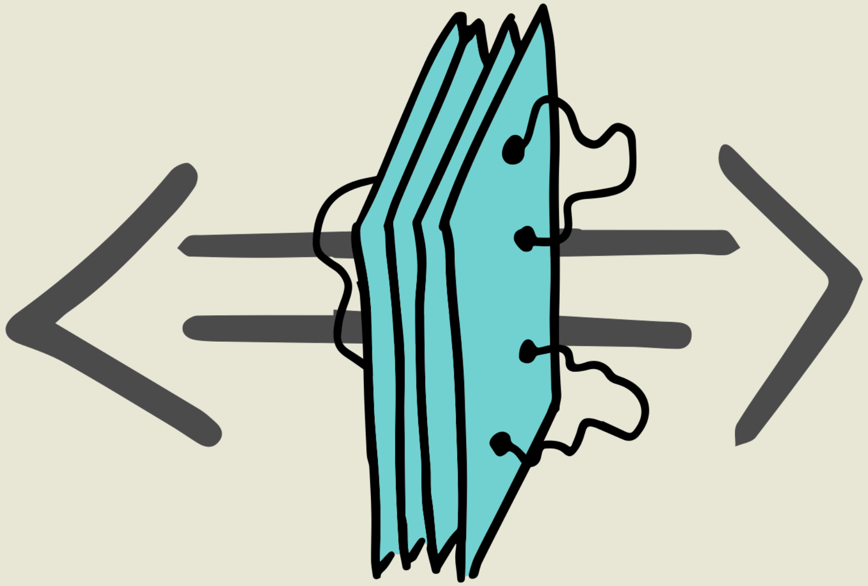
Mass gap: every excitation of the vacuum has energy al least $\Delta > 0$

Confinement: there are different notions of confinement:

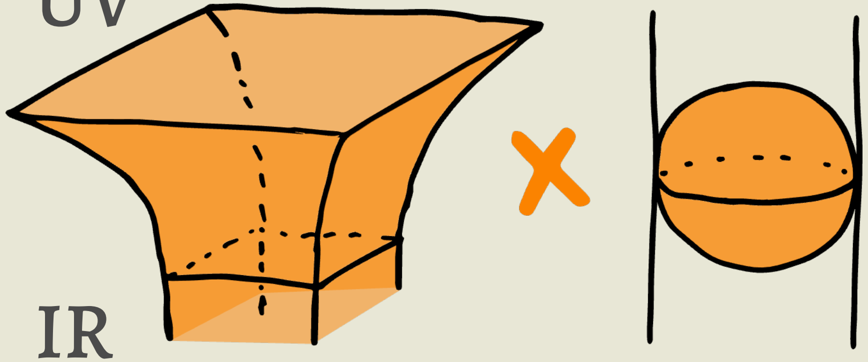
- The physical particle states are “colourless”.
- At large N, degrees of freedom scale as $\mathcal{O}(N_c^2)$ at large temperatures and $\mathcal{O}(N_c^0)$ at low.
- Linear quark-antiquark potential.

N D3 - branes

type IIB
$AdS_5 \times S^5$



$\mathcal{N}=4$ SYM
$SU(N)$



Conformal Theory

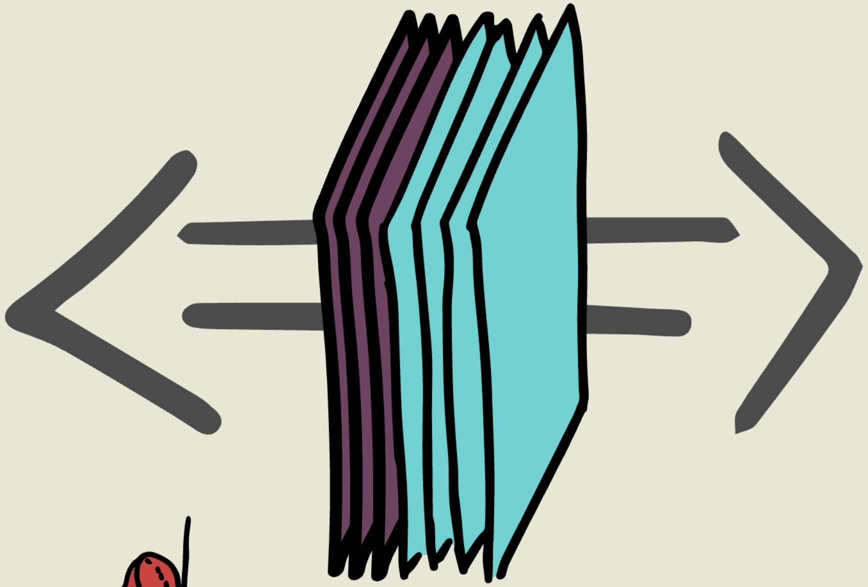
[Maldacena, '97]

N D3 - branes

M fractional D3 - branes

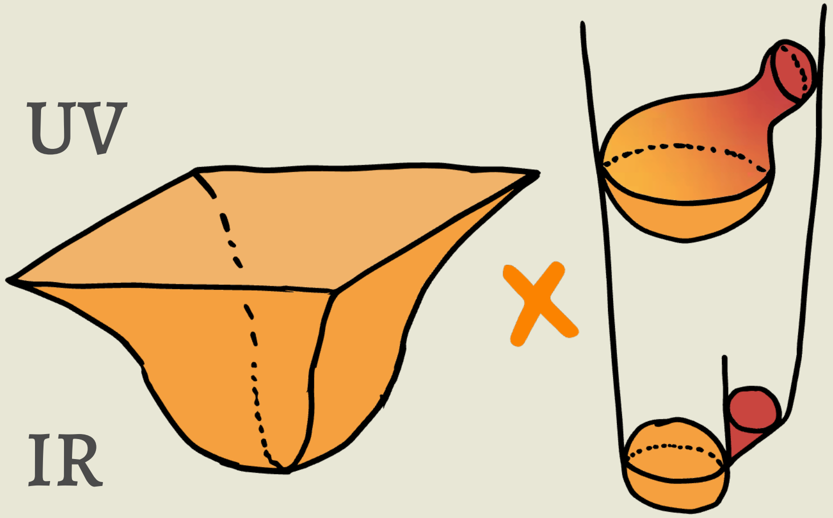
type IIB

$\cancel{AdS_5} \times X^5$



$\mathcal{N} = 1$ SYM

$SU(M+N) \times SU(N)$

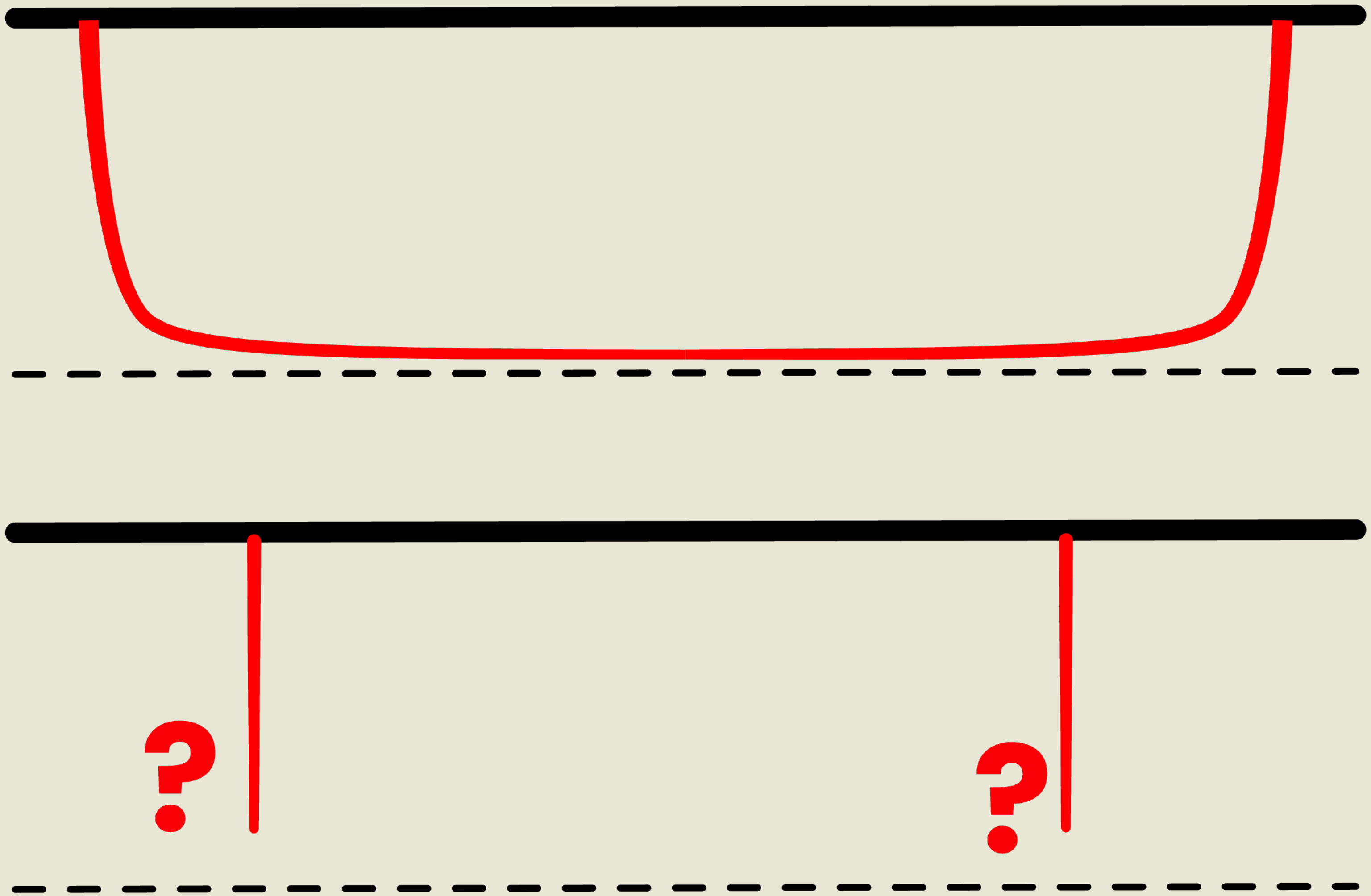


Mass Gap

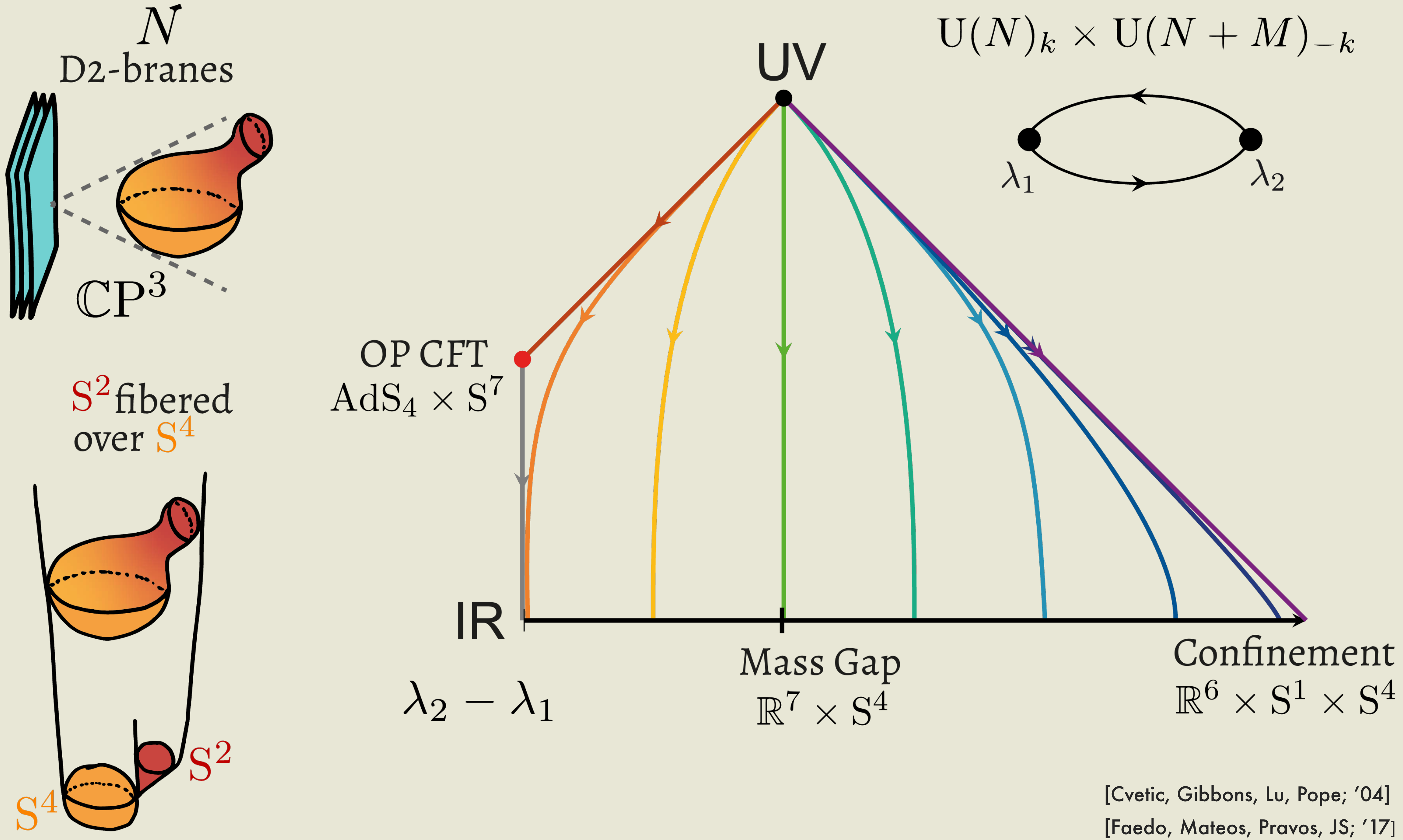
Confining theory

[Klebanov, Witten, '98]
[Klebanov, Tseylin, '99]
[Klebanov, Strassler, '00]

$$V_{q\bar{q}} \sim \text{Length of the string}$$

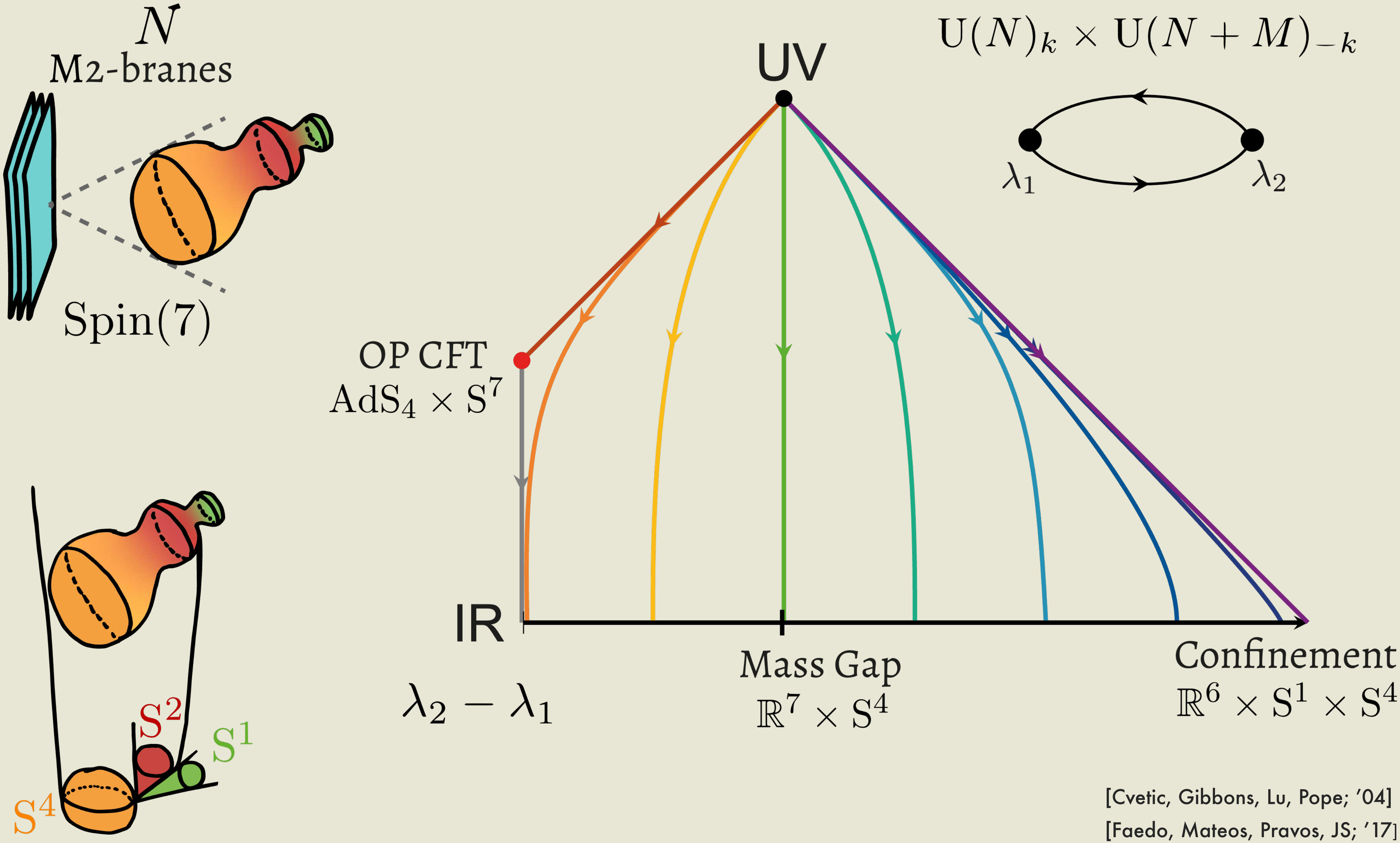


Super-Yang-Mills
Chern-Simons Matter theories

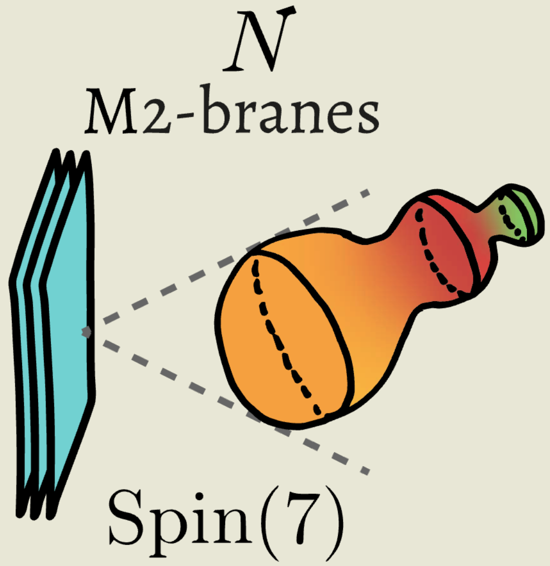


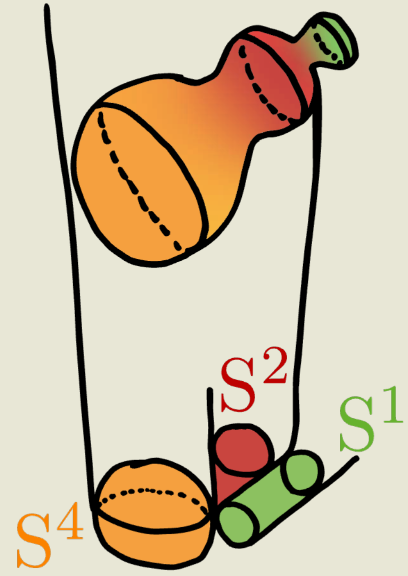
[Cvetic, Gibbons, Lu, Pope; '04]

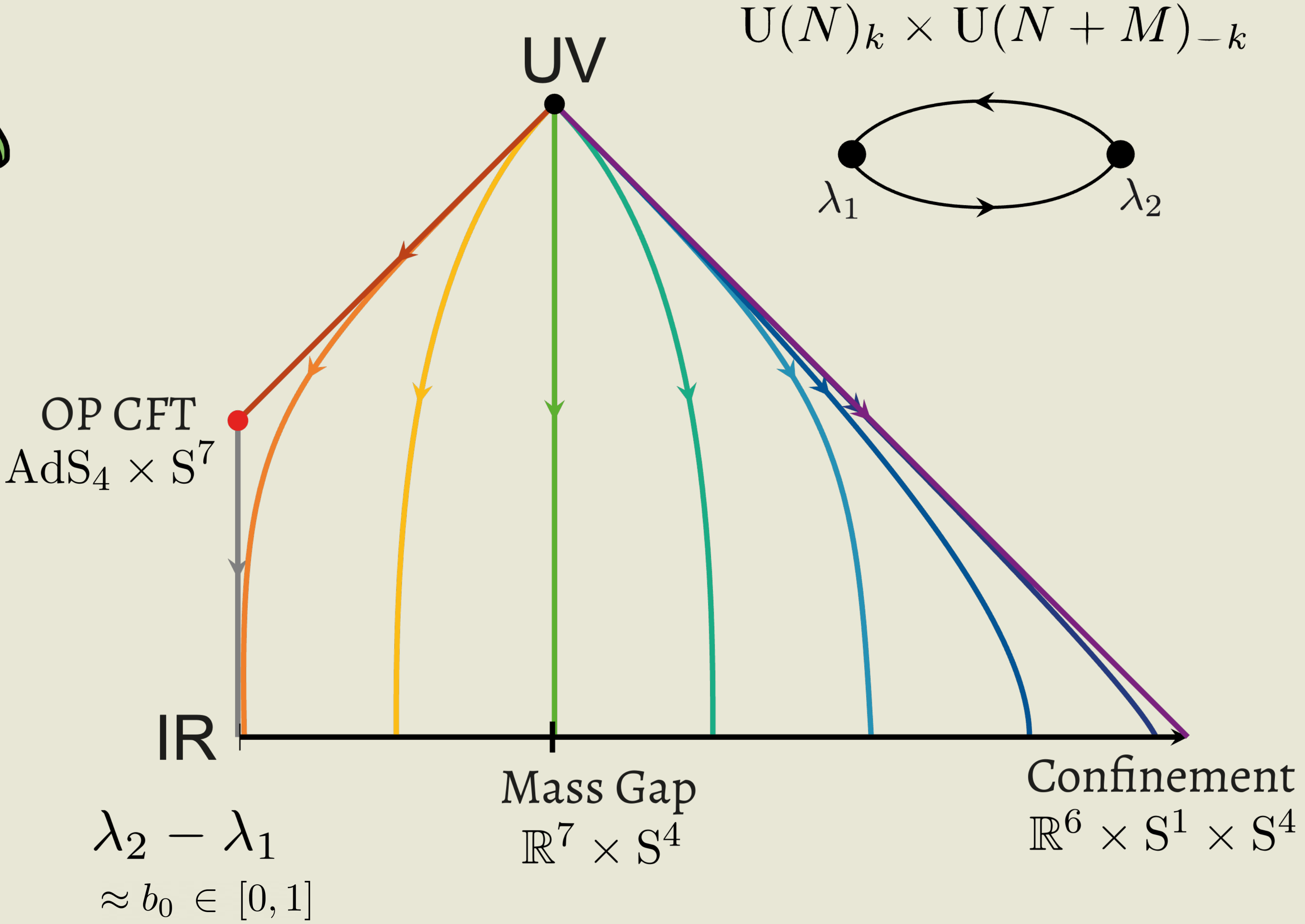
[Faedo, Mateos, Pravos, JS; '17]



[Cvetic, Gibbons, Lu, Pope; '04]
[Faedo, Mateos, Pravos, JS; '17]

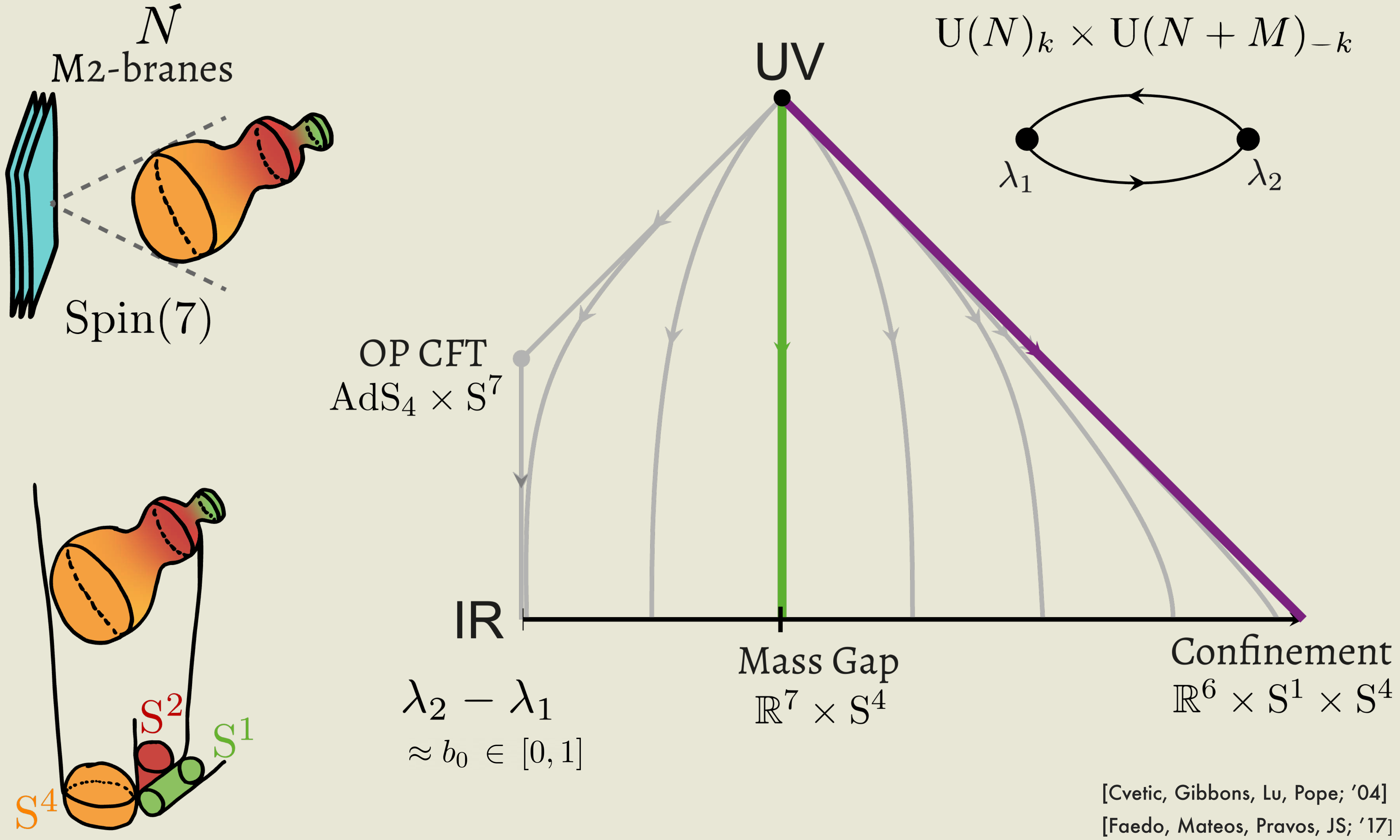




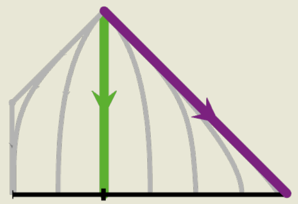


[Cvetic, Gibbons, Lu, Pope; '04]

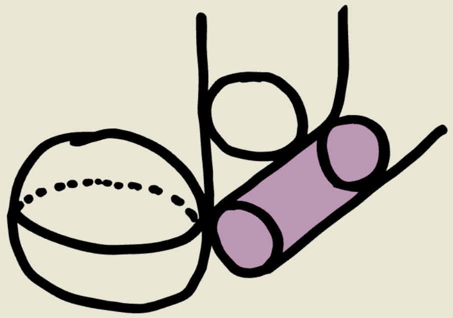
[Faedo, Mateos, Pravos, JS; '17]

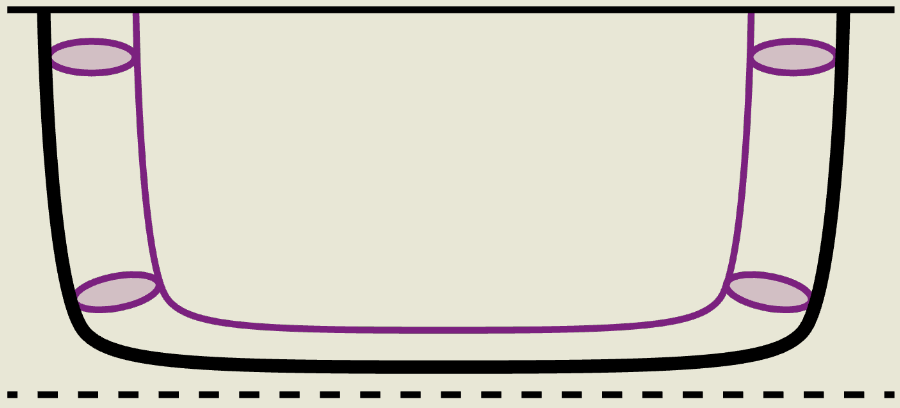


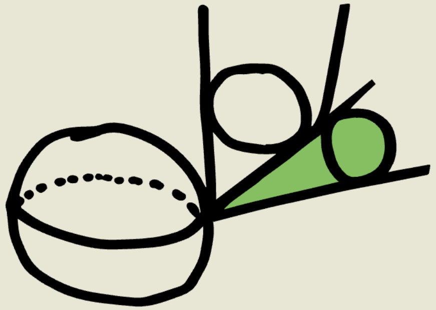
N
M2-branes
Spin(7)
S^2
S^1
S^4
UV
$\mathrm{U}(N)_k \times \mathrm{U}(N+M)_{-k}$
λ_1
λ_2
OP CFT
$\mathrm{AdS}_4 \times \mathrm{S}^7$
IR
$\lambda_2 - \lambda_1$
$\approx b_0 \in [0,1]$
Mass Gap
$\mathbb{R}^7 \times \mathrm{S}^4$
Confinement
$\mathbb{R}^6 \times \mathrm{S}^1 \times \mathrm{S}^4$
[Cvetic, Gibbons, Lu, Pope; '04]
[Faedo, Mateos, Pravos, JS; '17]

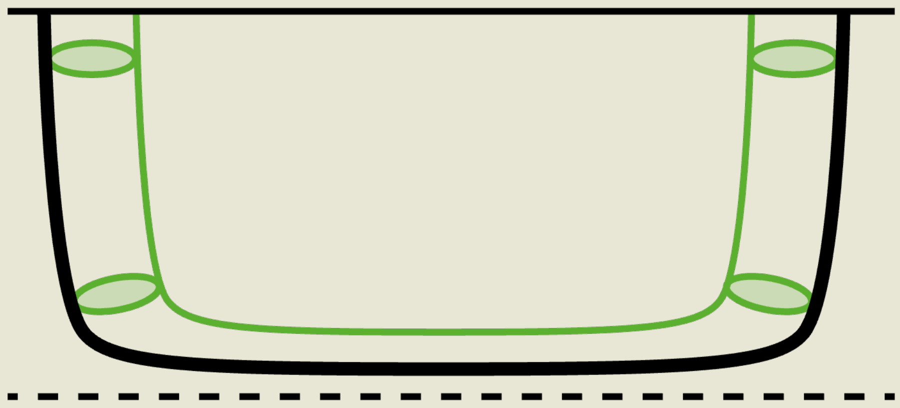


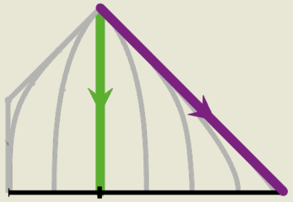
$$V_{q\bar{q}} \sim \text{Area of the membrane}$$



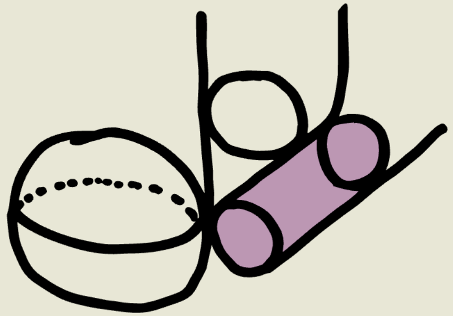


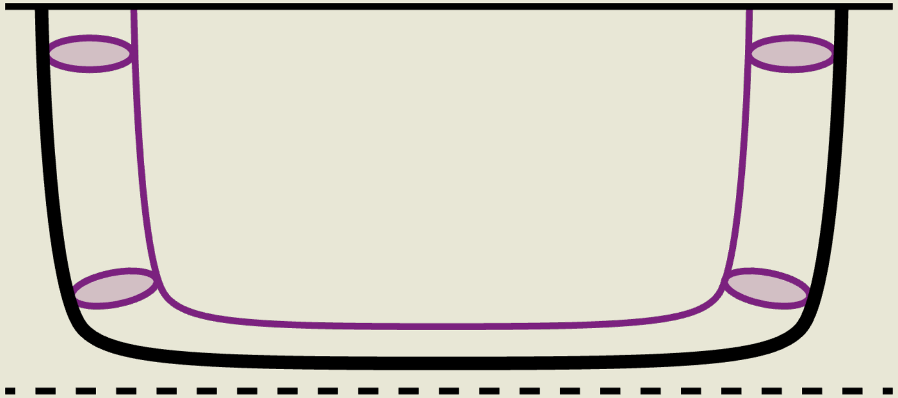


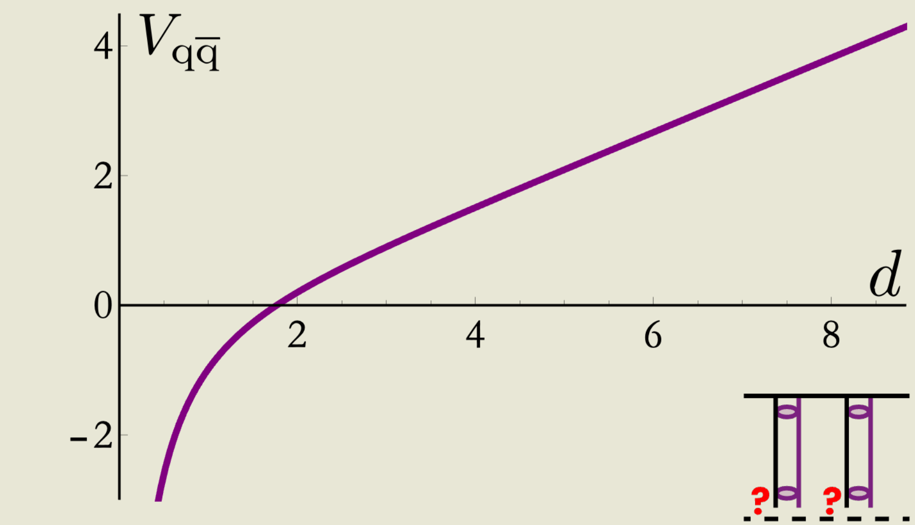


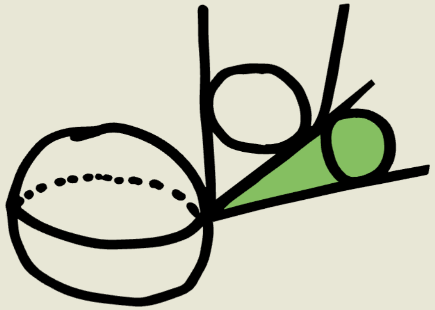


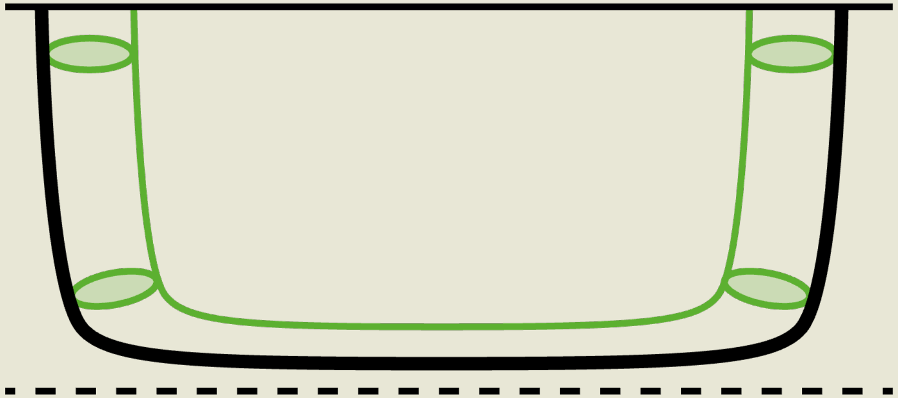
$V_{q\bar{q}} \sim$ Area of the membrane

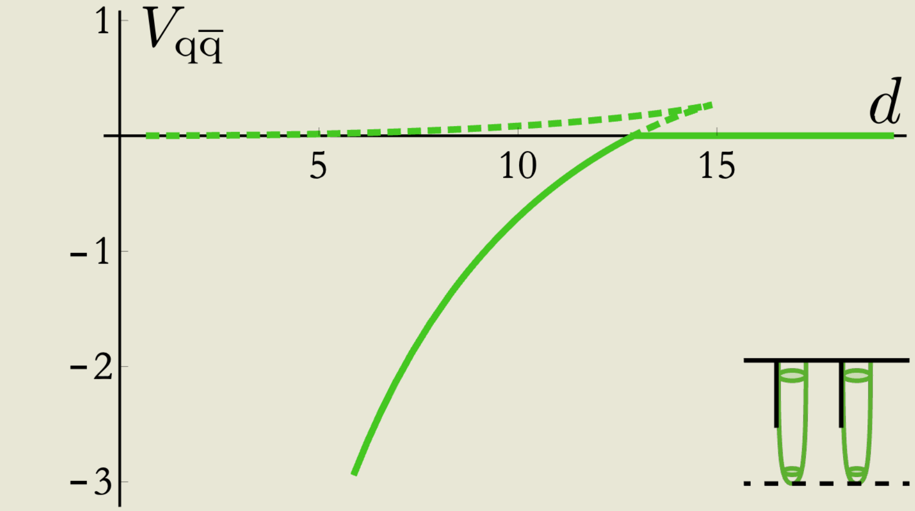






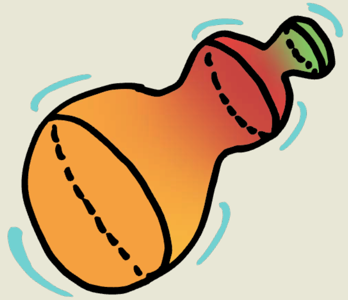




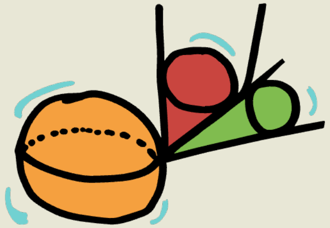


Spectrum

UV



IR

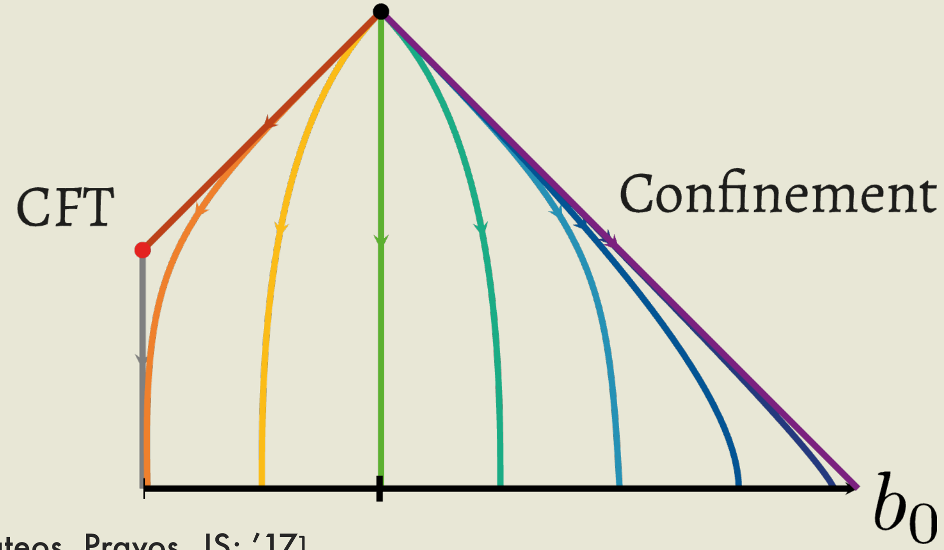


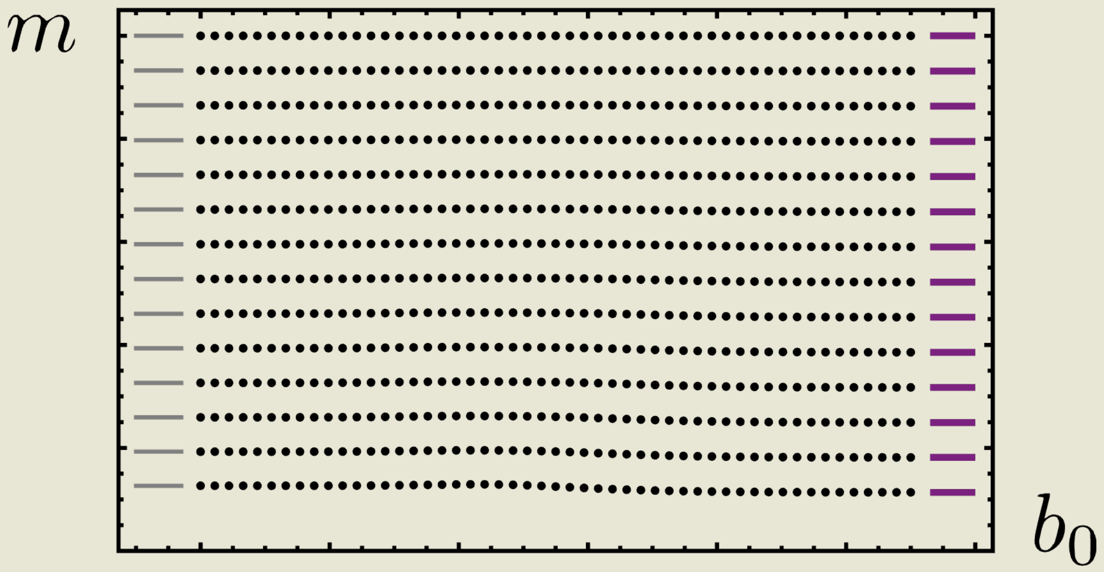
We need to consider fluctuations of the fields

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$$

$$\Phi \rightarrow \Phi + \delta\Phi$$

Spin-2 corresponds to finding eigenvalues and eigenstates with a simple potential





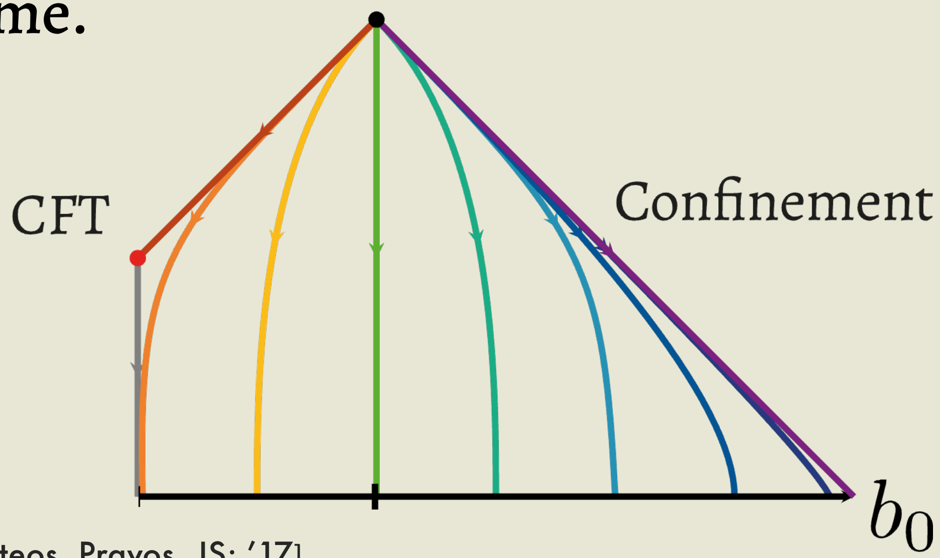
[Elander, Faedo, Mateos, Pravos, JS; '17]

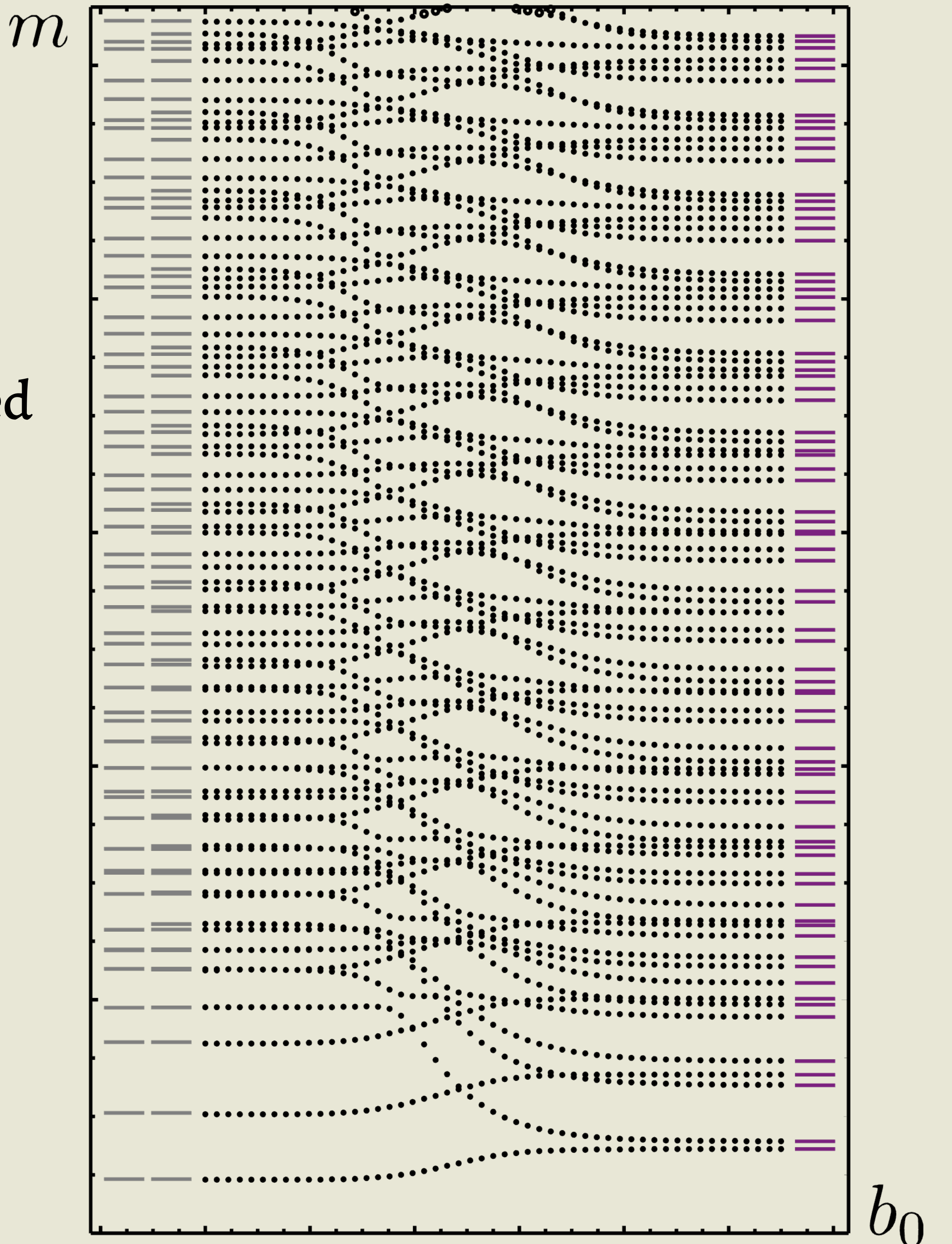
Spin-0 states are more sensitive to IR physics.

No light dilaton is found in the spectrum.

- The **RG flow whose UV is the CFT** is triggered by a VEV and a source of comparable size

The spectrum nicely interpolates between the quasi-conformal and the quasi-confining regime.





[Elander, Faedo, Mateos, Pravos, JS; '17]

Is entanglement a probe of confinement?

Entanglement entropy

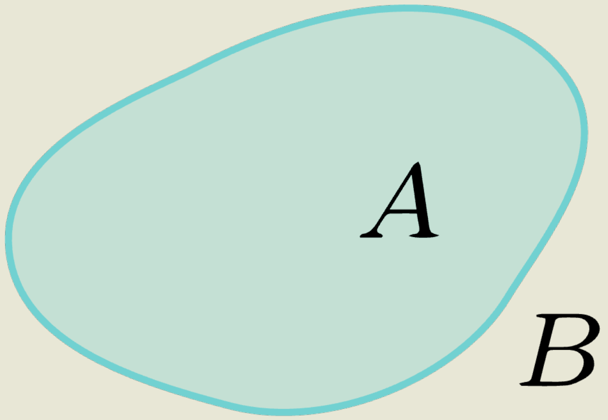
We can think of it as the entropy for an observer who is only accessible to the subsystem A and cannot receive any signals from B

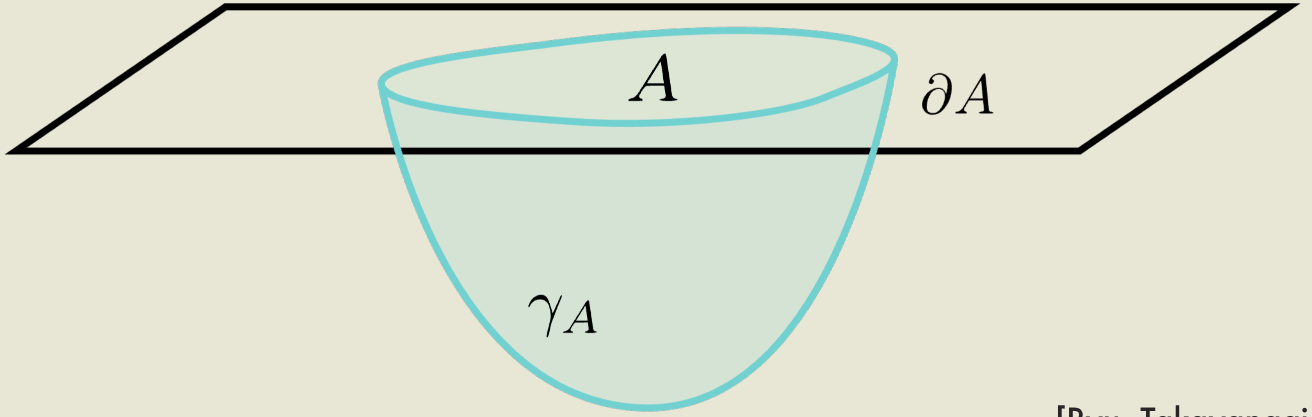
$|\psi\rangle$ vacuum state

$\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$ reduced density matrix

$$S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A$$

$$S_A = \frac{\text{Area of } \gamma_A}{4G_N}$$



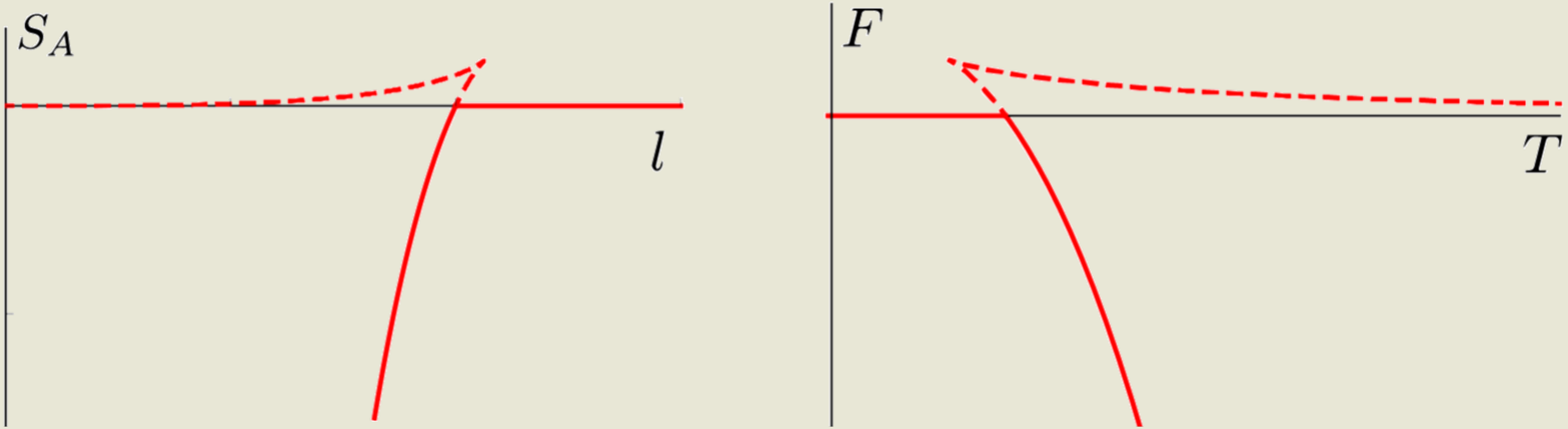


[Ryu, Takayanagi; '20]

Entanglement entropy

Entanglement entropy has been proposed as a probe of confinement.

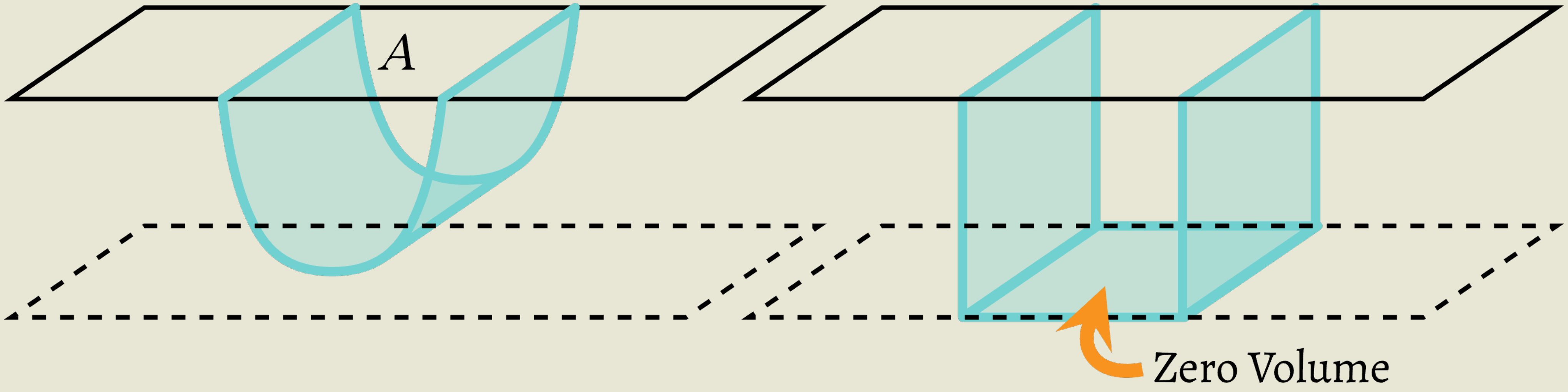
Similar phase transitions are found in entanglement entropy measures and thermodynamical properties in confining backgrounds.

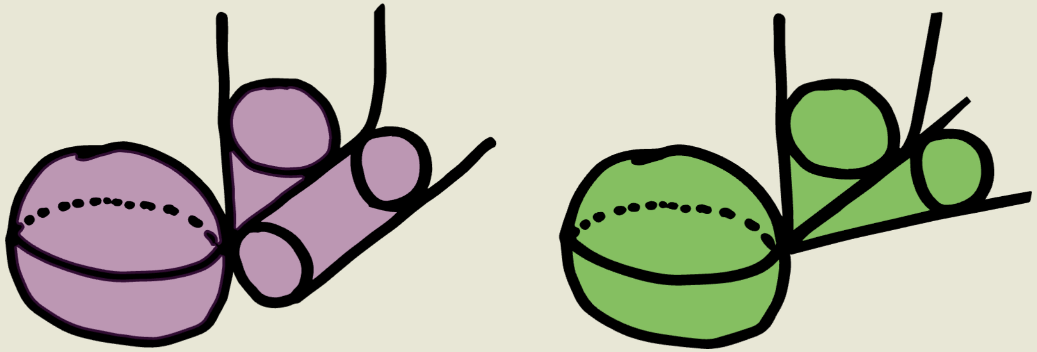


We examined if this was the case in our theories.

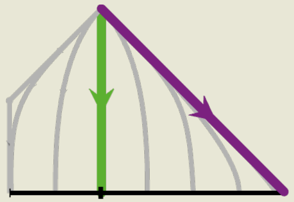
[Klebanov, Kutasov, Takayanagi; '09]
[Nishioka, Takayanagi; '08]

$S_A \sim$ Volume of a codimendion 2 subspace



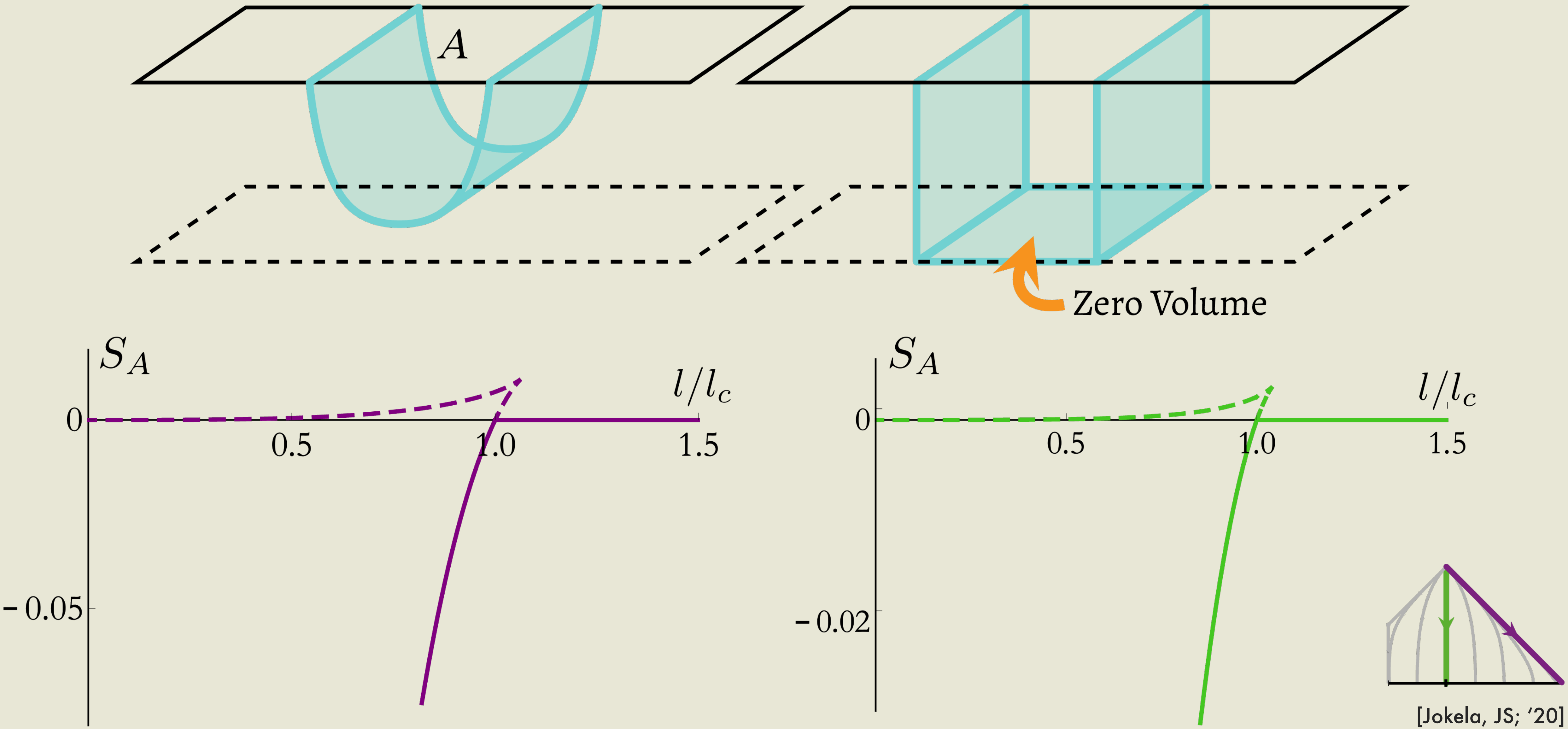


The shrinking of the M-Theory circle has no physical consequence in this computation

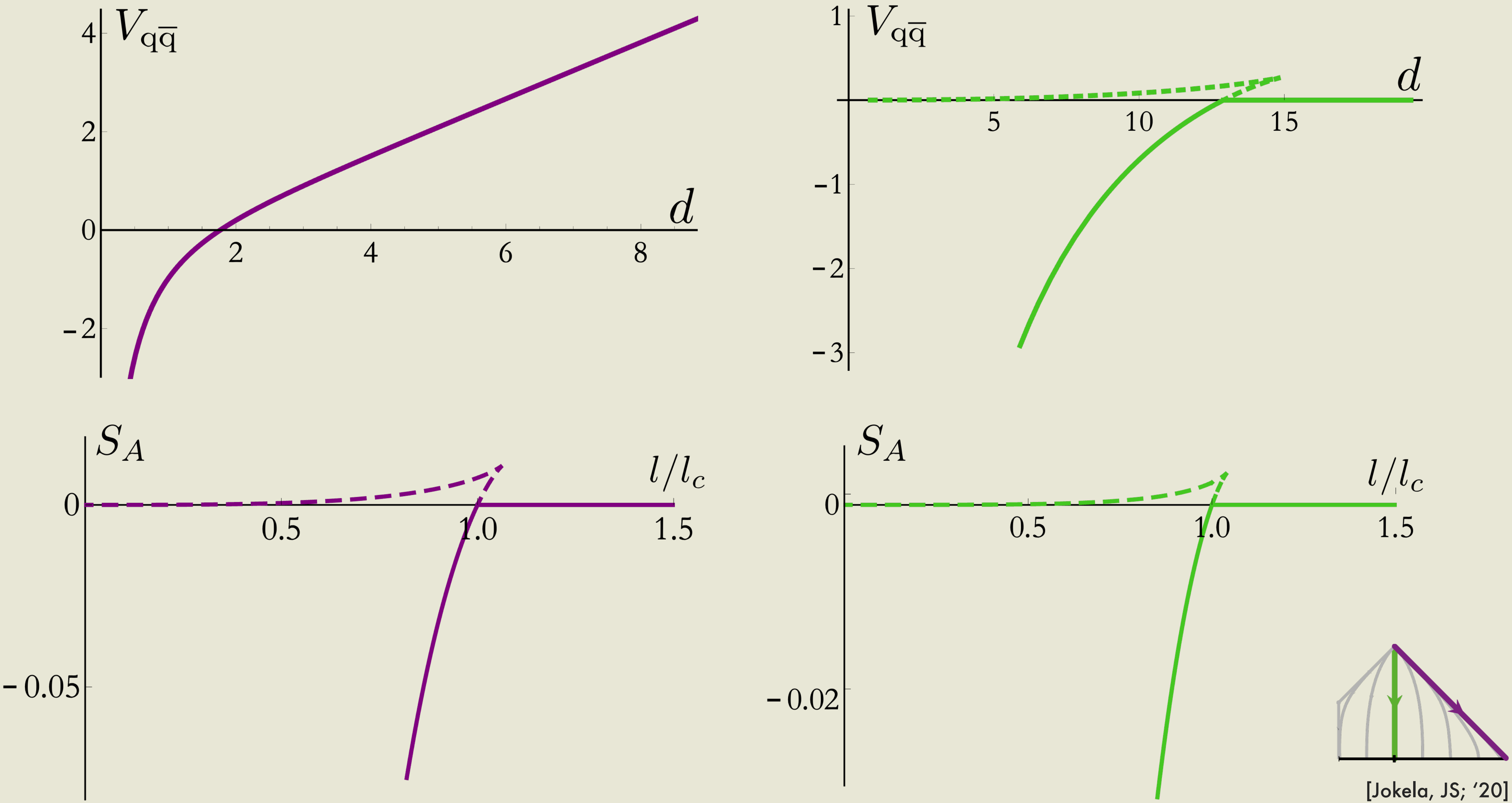


[Jokela, JS; ’20]

$S_A \sim$ Volume of a codimendion 2 subspace



[Jokela, JS; '20]



$V_{q\bar{q}}$
d
4
2
0
-2
2
4
6
8
$V_{q\bar{q}}$
d
1
-1
-2
-3
5
10
15
S_A
l/l_c
0
-0.05
0.5
1.0
1.5
S_A
l/l_c
0
-0.02
0.5
1.0
1.5

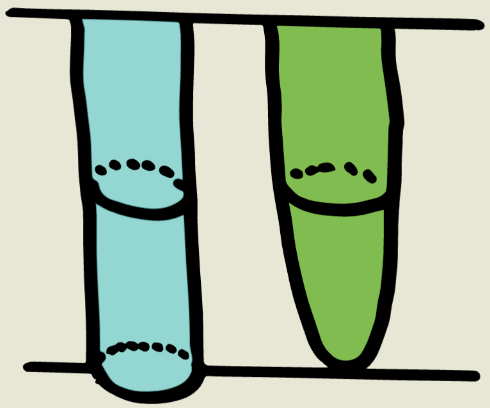
[Jokela, JS; '20]

Thermodynamical phase diagram

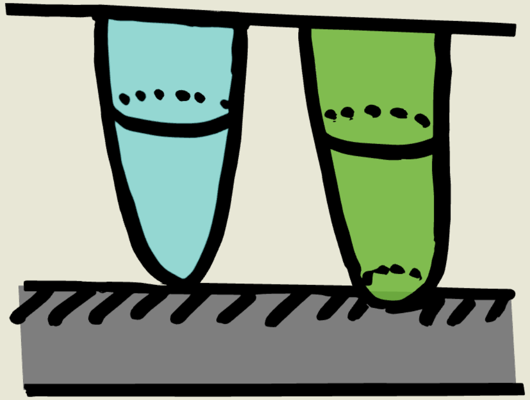
Finite temperature

Low – temperature phases (gapped):

$$\tau \sim \tau + \beta$$

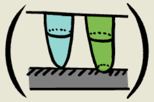
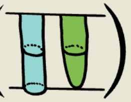


High – temperature phases (degapped):
β given by surface gravity at the Horizon of the black brane



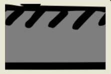
Finite temperature

Canonical ensemble: The dominant phase will be that of the smallest free energy

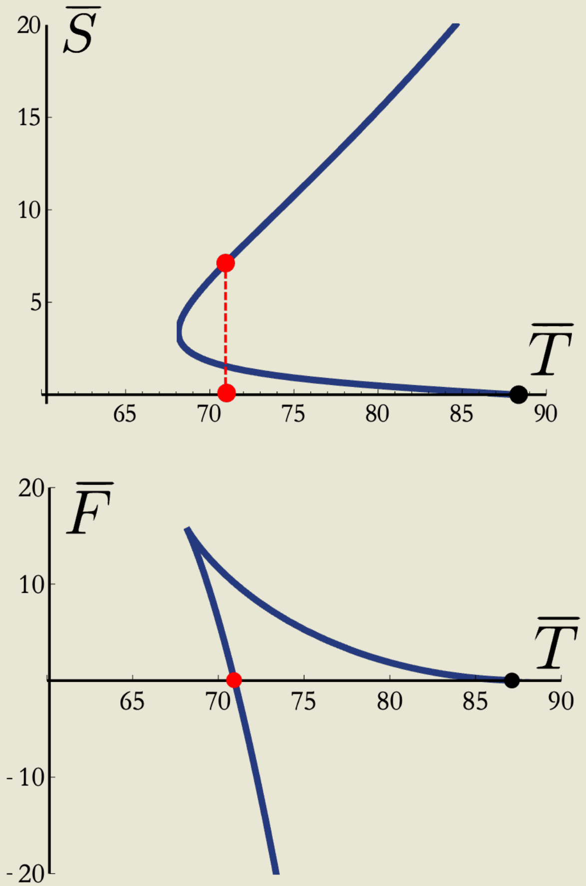
$\bar{F} = F($$) - F($$)$

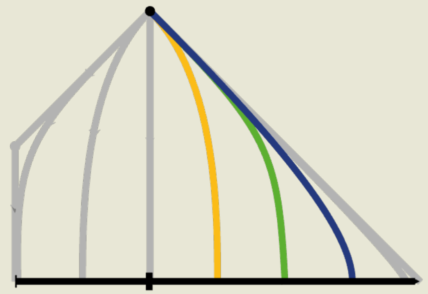
- If it is positive, gapped phase
- If it is negative, black brane solution is preferred

The entropy is given by the are of the horizon:

$\bar{S} =$ Area()

Case A

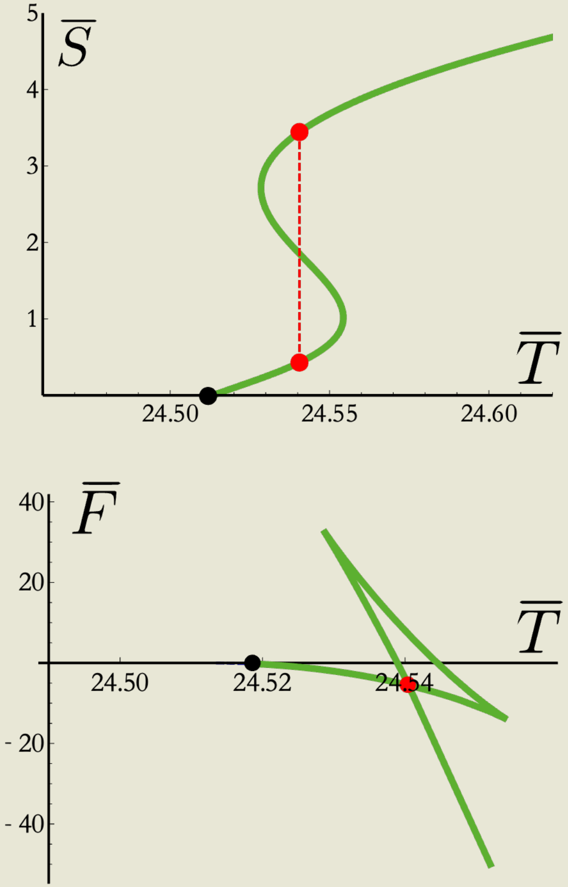


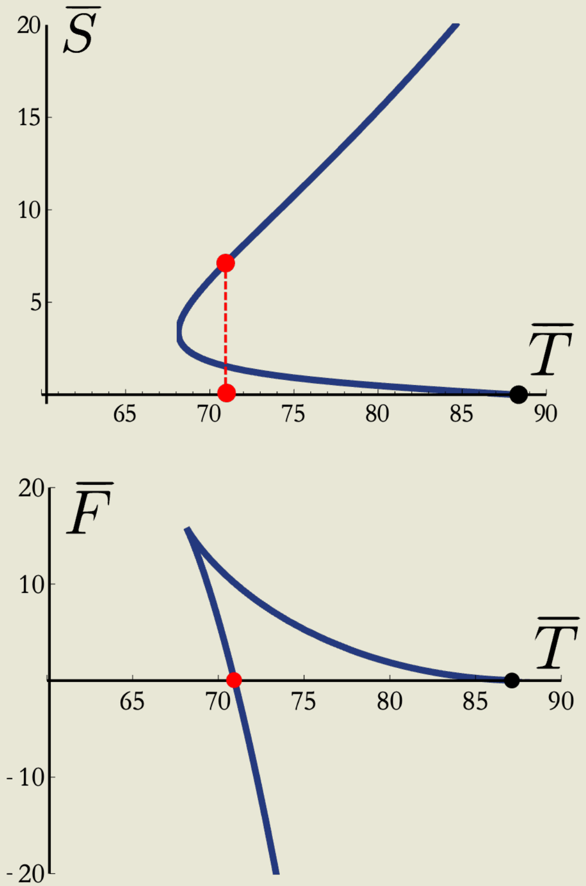


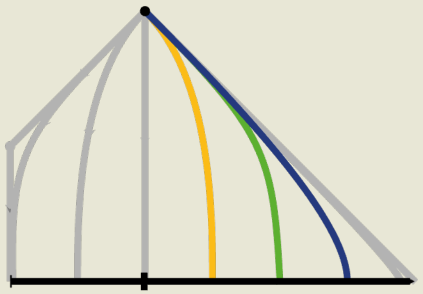
[Elander, Faedo, Mateos, JS; '20]

Case B

Case A





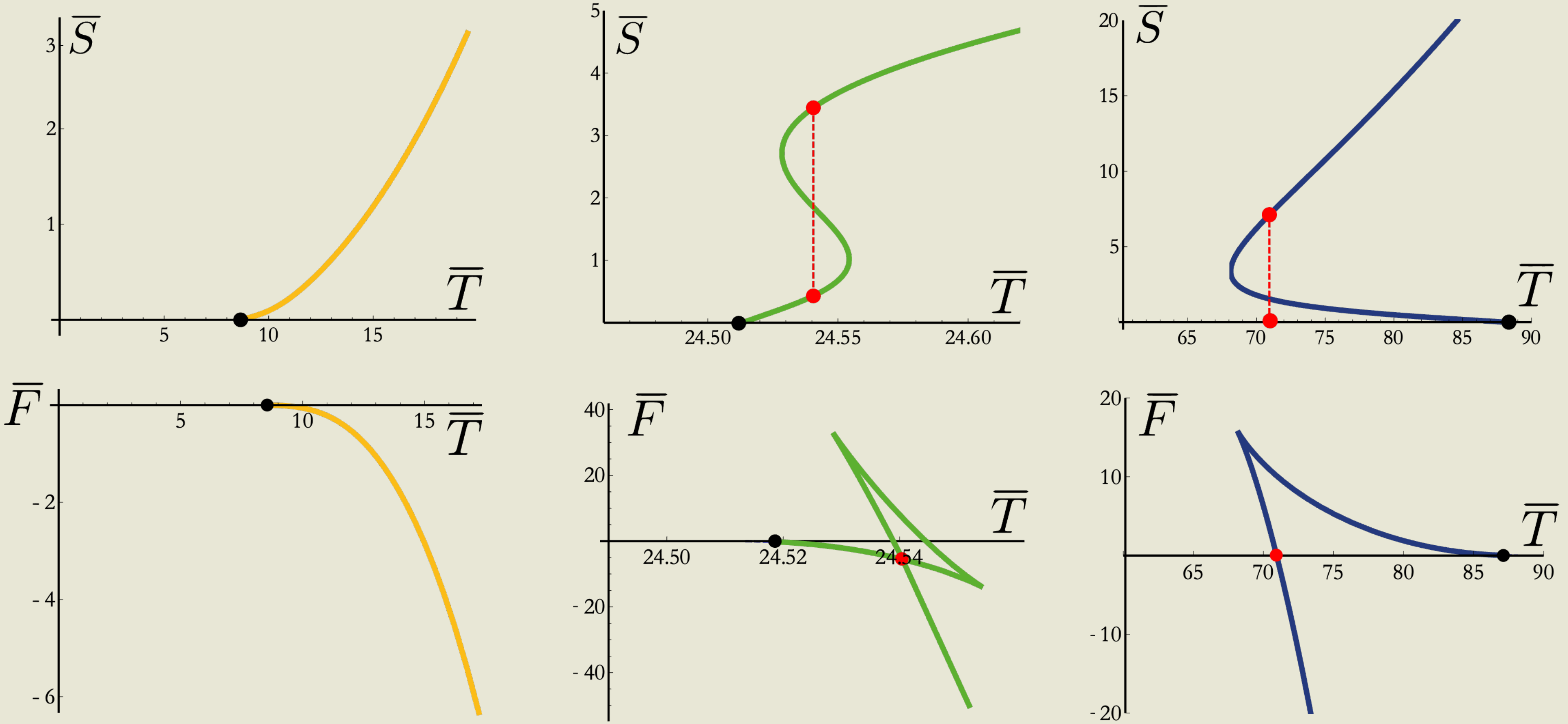


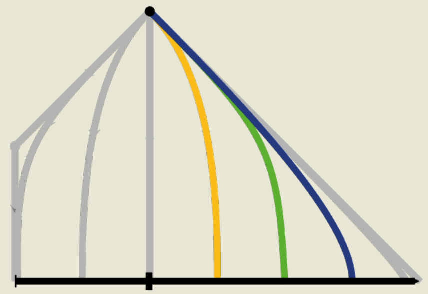
[Elander, Faedo, Mateos, JS; '20]

Case C

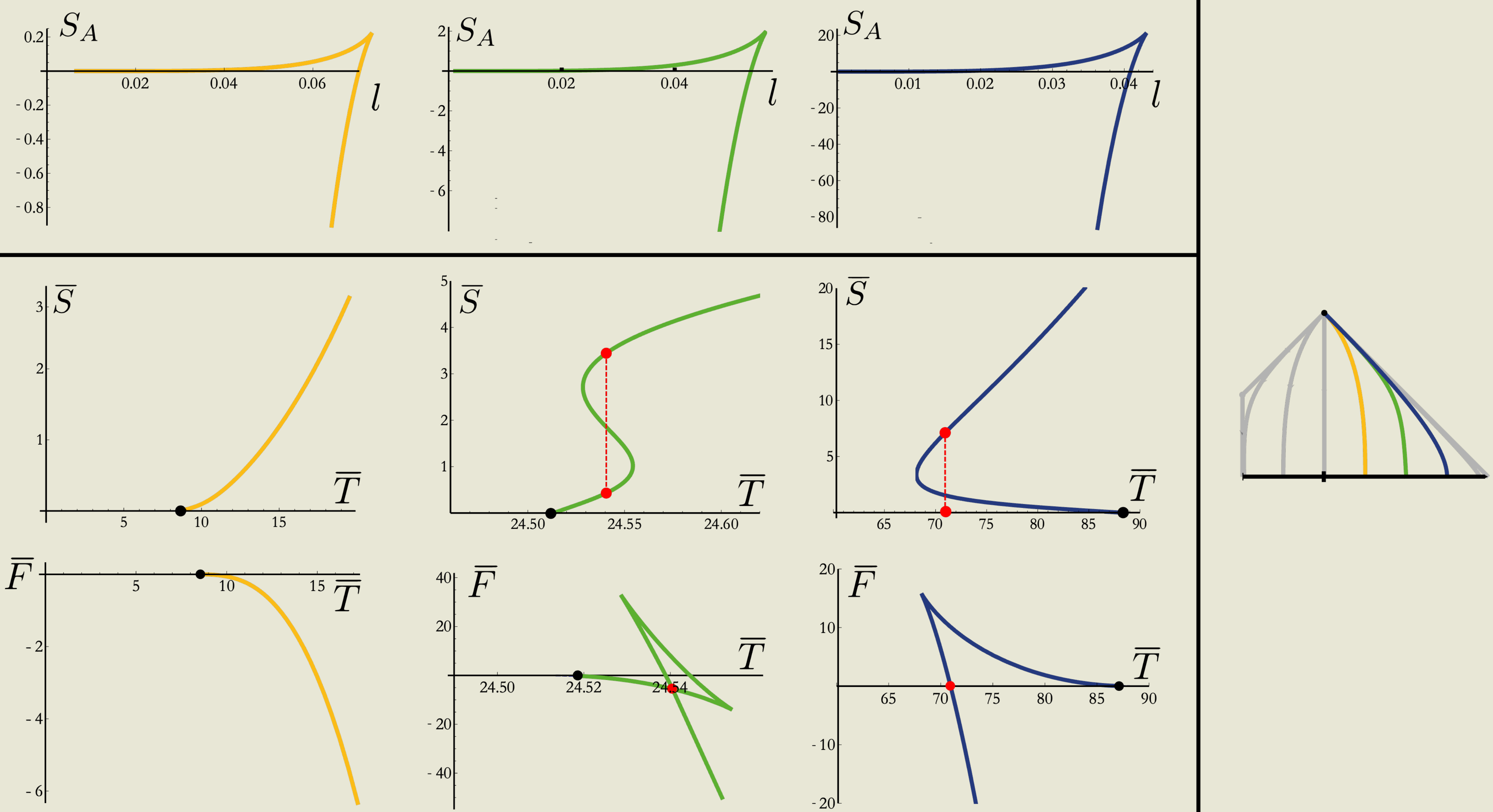
Case B

Case A



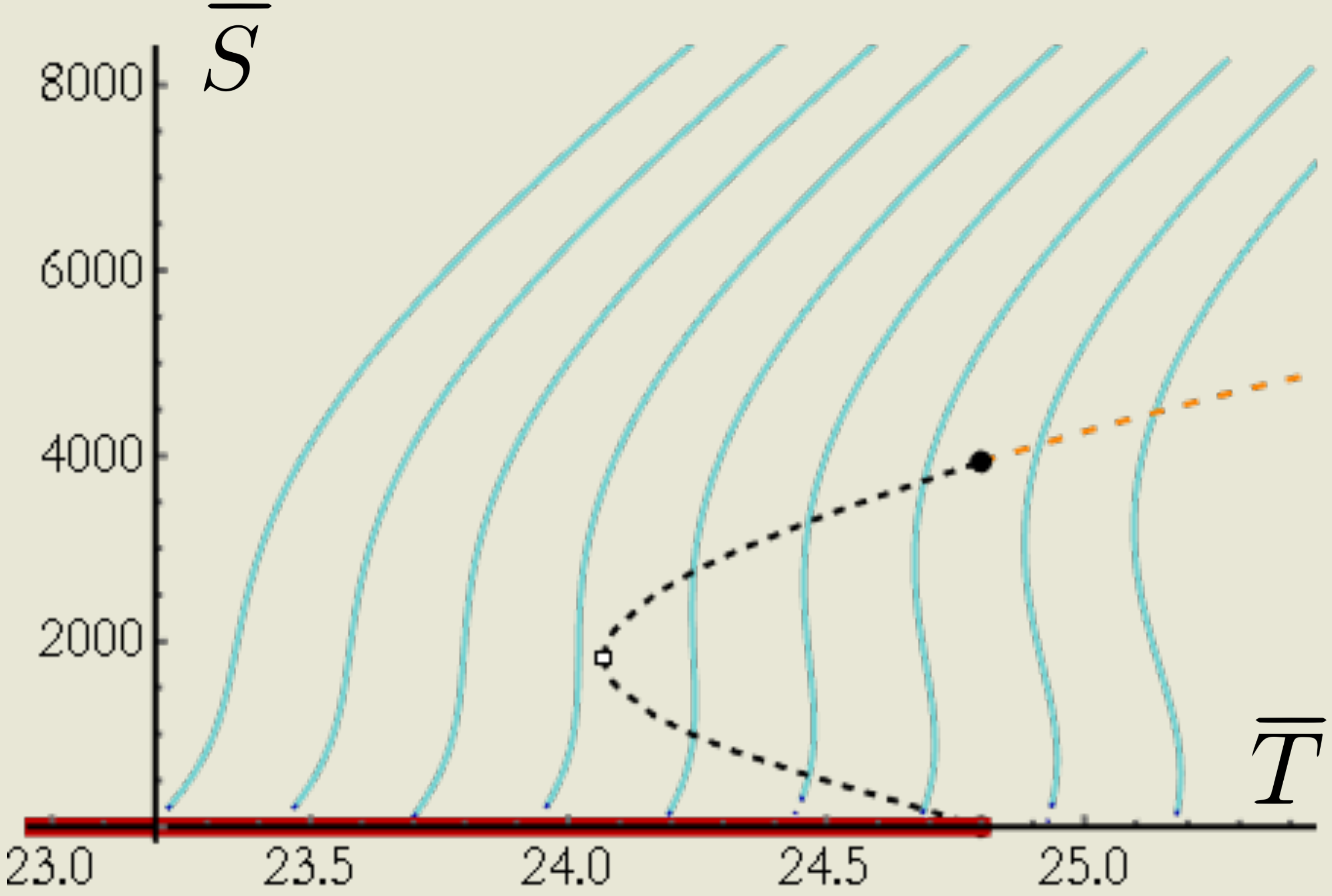


[Elander, Faedo, Mateos, JS; '20]



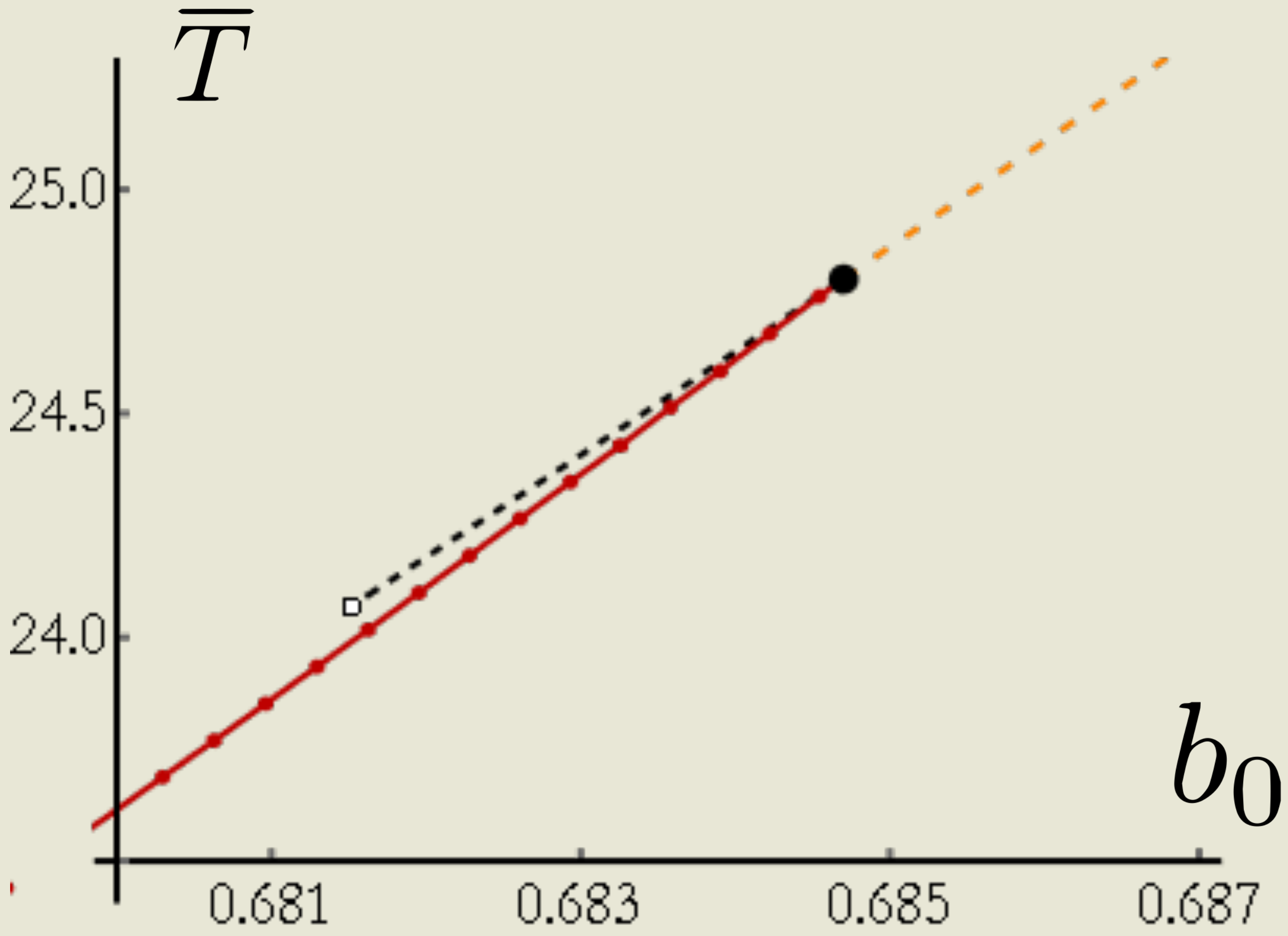
[Elander, Faedo, Mateos, JS; '20]

Phase diagram



[Elander, Faedo, Mateos, JS; '20]

Phase diagram



[Elander, Faedo, Mateos, JS; '20]

Take home messages

We studied a (2+1) dimensional family of theories with **mass gap and without confinement**.

For this set of theories, **entanglement entropy** is not a good probe of confinement.

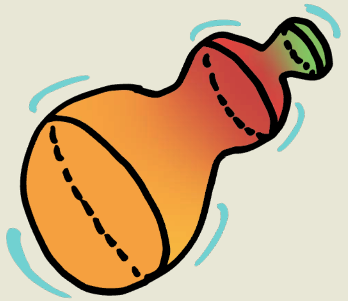
Their rich **thermodynamical phase structure**, possesses a critical point where a second order phase transition occurs and a branch of transitions at zero entropy but finite temperature.

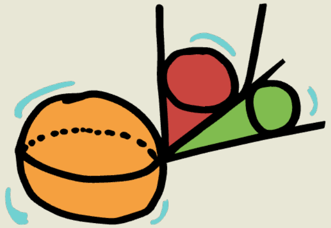

Thanks.

(Back-up slides)

$$g_{\mu\nu} \;\to\; g_{\mu\nu} + \delta g_{\mu\nu}$$

$$\Phi \;\to\; \Phi + \delta\Phi$$

UV 

IR 

$$\Phi^a = \bar{\Phi}^a + \varphi^a,$$

$$\mathrm{d}s_4^2 = (1 + 2\nu + \nu_\sigma \nu^\sigma)\mathrm{d}\rho^2 + 2\nu_\mu \mathrm{d}x^\mu \mathrm{d}\rho + e^{2A}(\eta_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu,$$

$$h^\mu{}_\nu = \mathfrak{e}^\mu{}_\nu + iq^\mu \epsilon_\nu + iq_\nu \epsilon^\mu + \frac{q^\mu q_\nu}{q^2} H + \frac{1}{2}\delta^\mu{}_\nu h,$$

[Berg, Haack, Mueck; '05]

[Elander, Piai; '10]

Spin-2

$$\left[\partial_\rho^2 + 3A'\partial_\rho + e^{-2A}m^2\right]\mathfrak{e}^\mu{}_\nu = 0$$

Spin-0

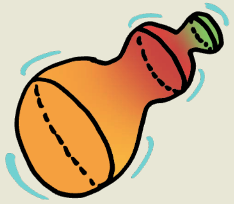
$$\mathfrak{a}^a = \varphi^a - \frac{\bar{\Phi}'^a}{4A'}h,$$

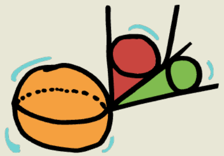
$$\left[\mathcal{D}_\rho^2 + 3A'\mathcal{D}_\rho + e^{-2A}m^2\right]\mathfrak{a}^a$$
$$- \left[\mathcal{V}^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d + \frac{2(\bar{\Phi}'^a\mathcal{V}_c + \mathcal{V}^a\bar{\Phi}'_c)}{A'} + \frac{4\mathcal{V}\bar{\Phi}'^a\bar{\Phi}'_c}{A'^2}\right]\mathfrak{a}^c = 0$$

[Berg, Haack, Mueck; '05]
[Elander, Piai; '10]

The boundary conditions are obtained by requiring that the variational problem is well defined.

- Localised boundary actions at IR and UV cutoffs,
- Remove the cutoff and check convergence.

UV 

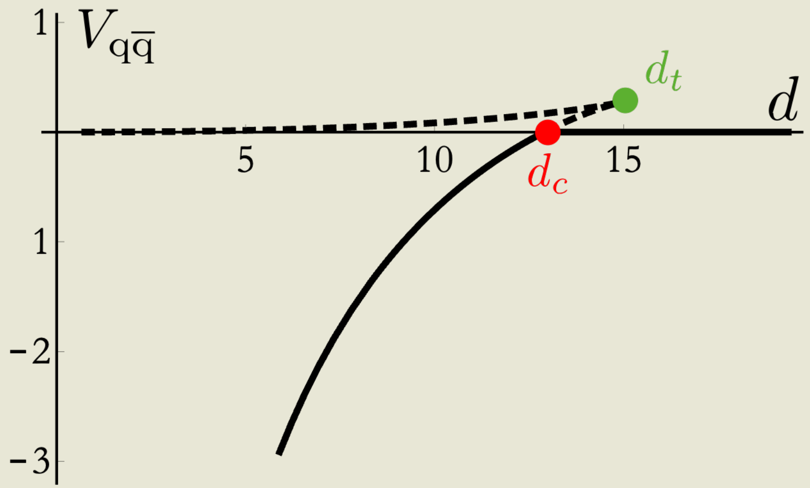
IR 

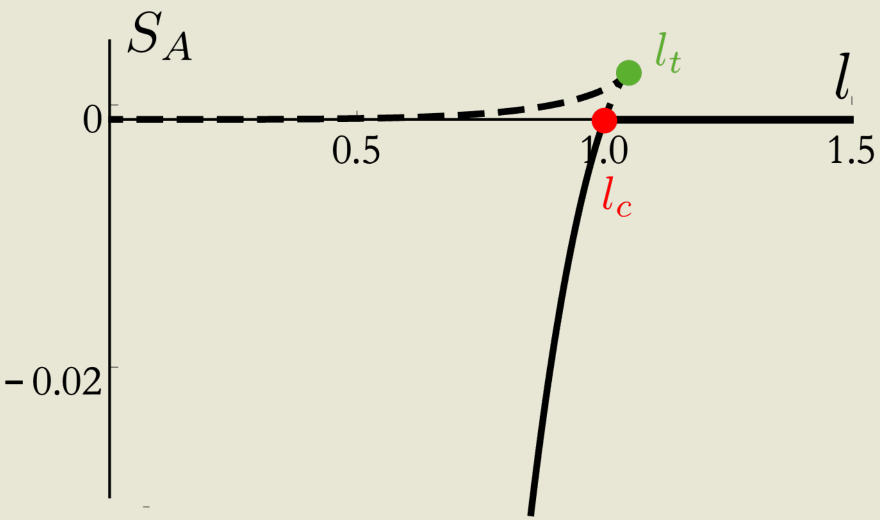
$$\varphi^a\big|_{\rho_{I,U}} = 0 \quad \Rightarrow \quad -\frac{e^{2A}}{m^2}\frac{\bar{\Phi}'^a}{A'}\left[\bar{\Phi}'_b\mathcal{D}_\rho - \frac{2\mathcal{V}\bar{\Phi}'_b}{A'} - \mathcal{V}_b\right]\mathfrak{a}^b\Big|_{\rho_{I,U}} = \mathfrak{a}^a\Big|_{\rho_{I,U}}$$

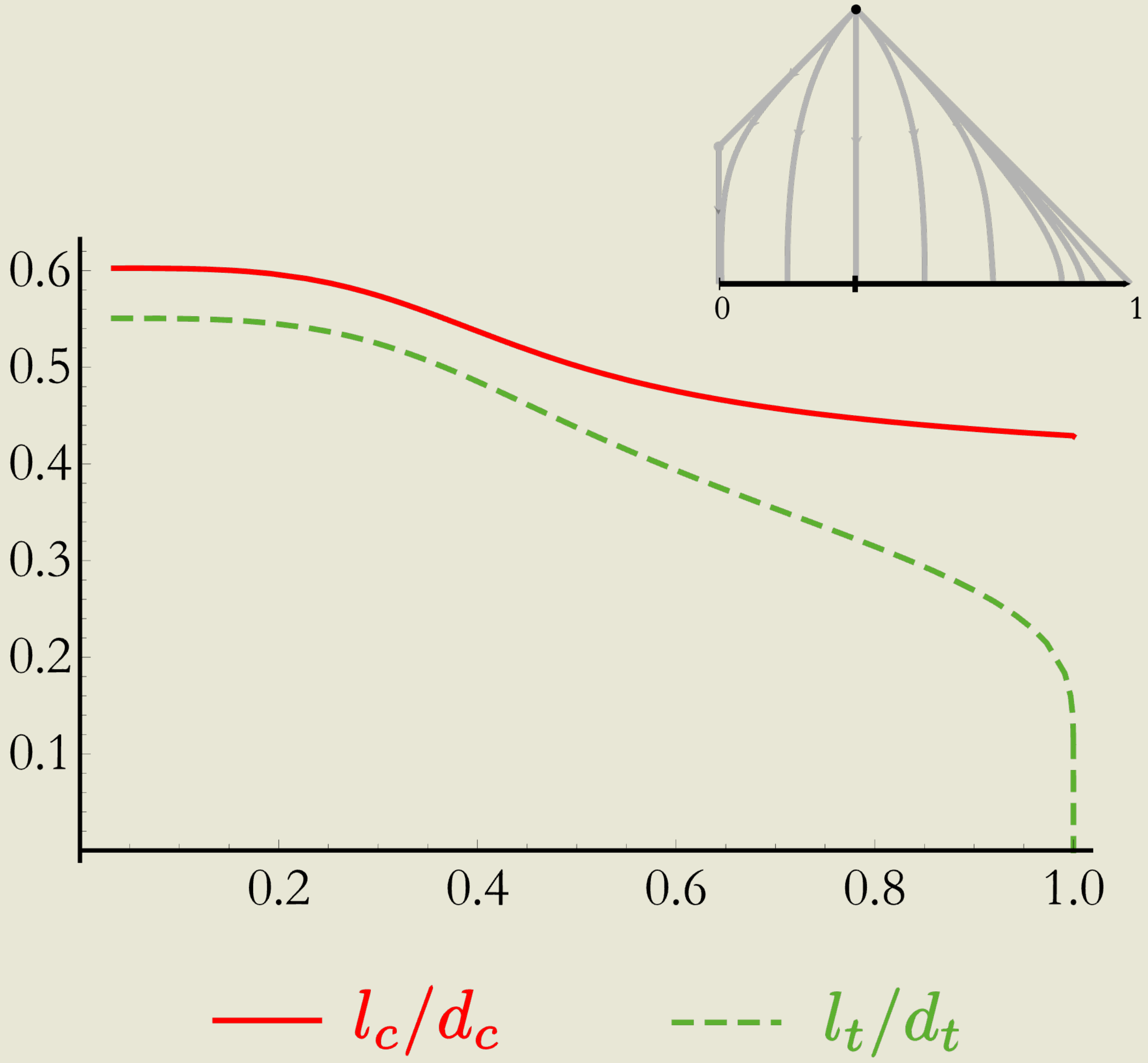
$$\partial_\rho \mathfrak{e}^\mu{}_\nu\big|_{\rho_{I,U}} = 0$$

At the UV, we took advantage of knowing the UV expansion.

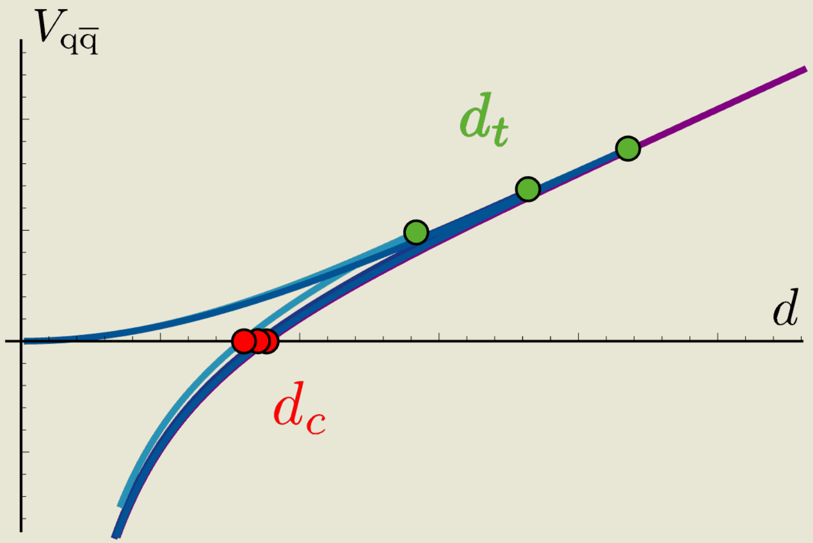
Comparison between scales

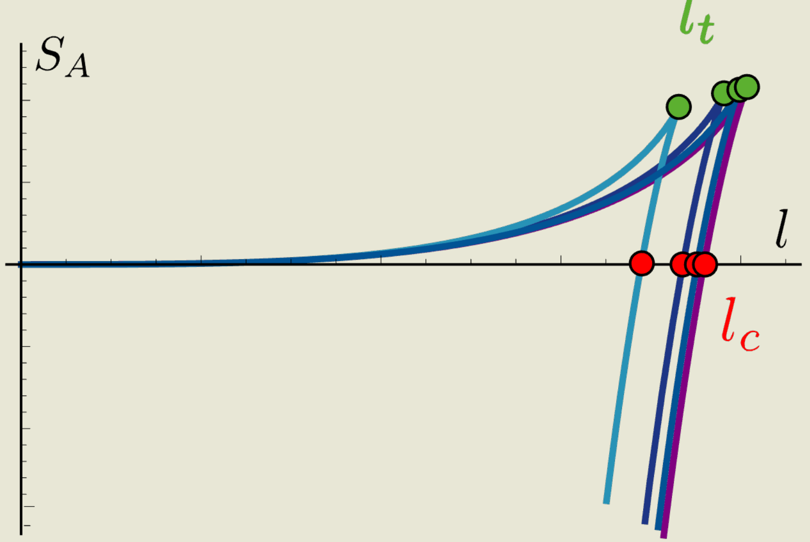


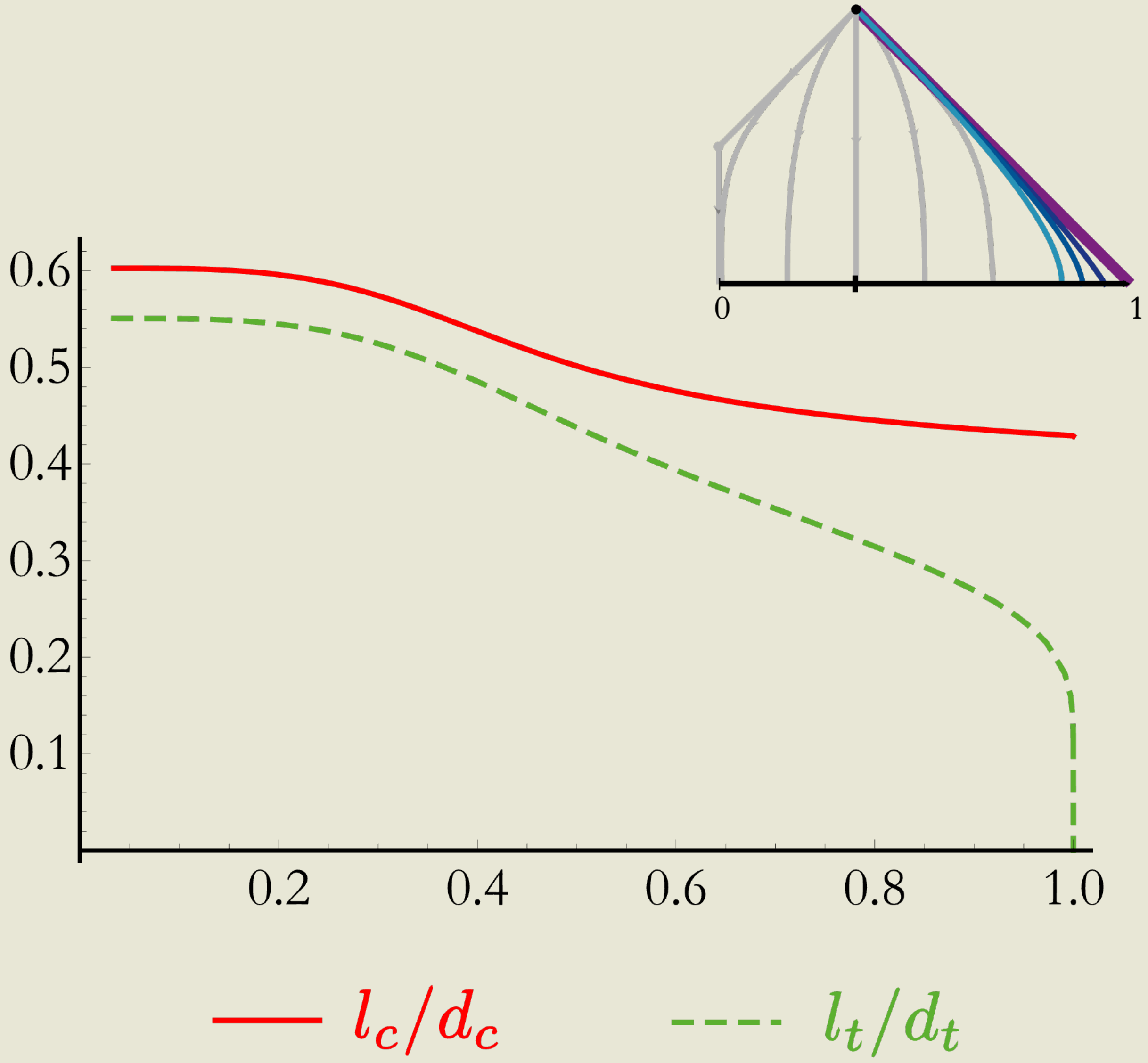


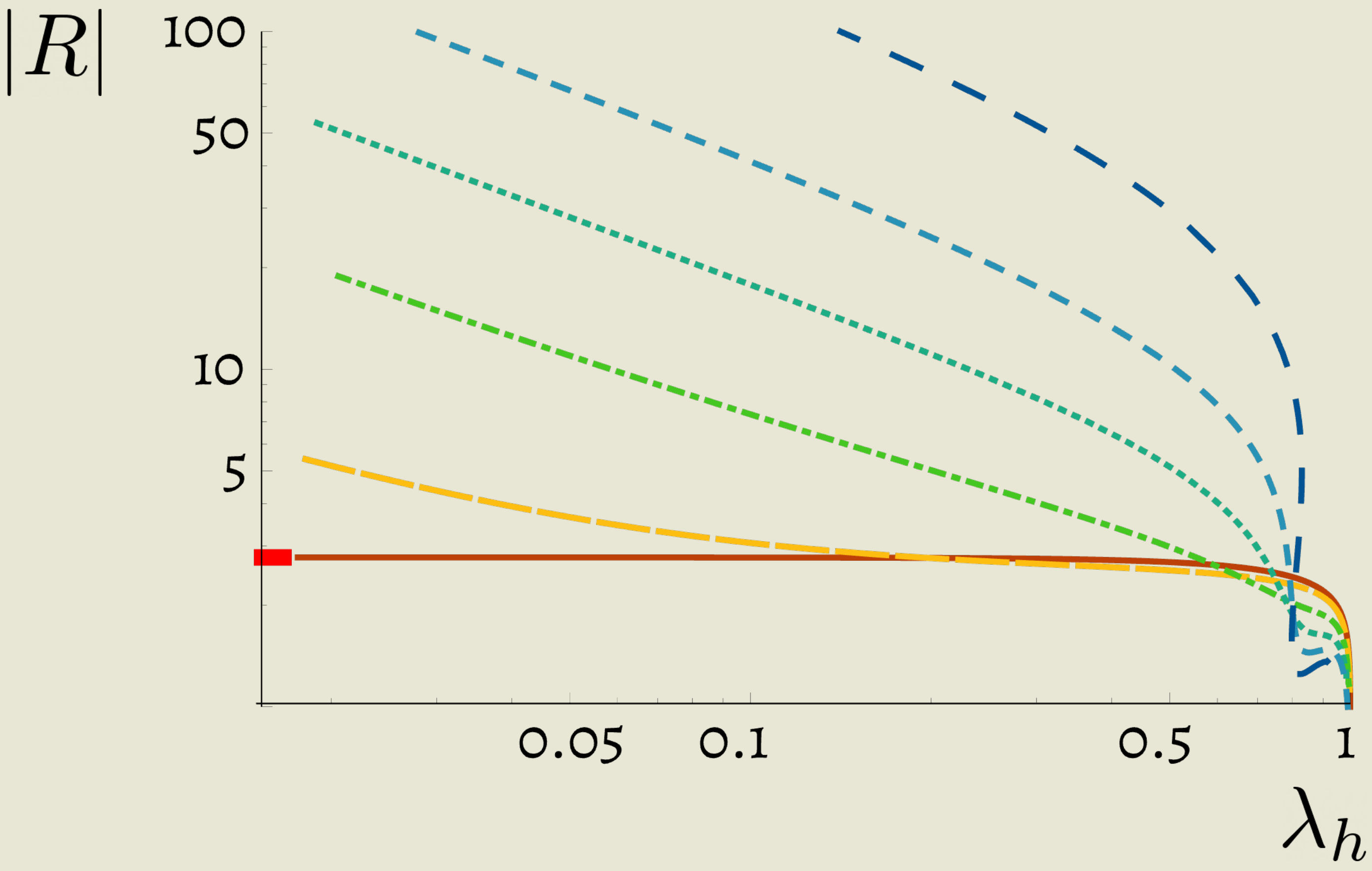


Comparison between scales









The zero entropy limit is supersymmetric but singular