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Dynamo in weakly collisional non-magnetized plasmas impeded by Landau damping of magnetic fields

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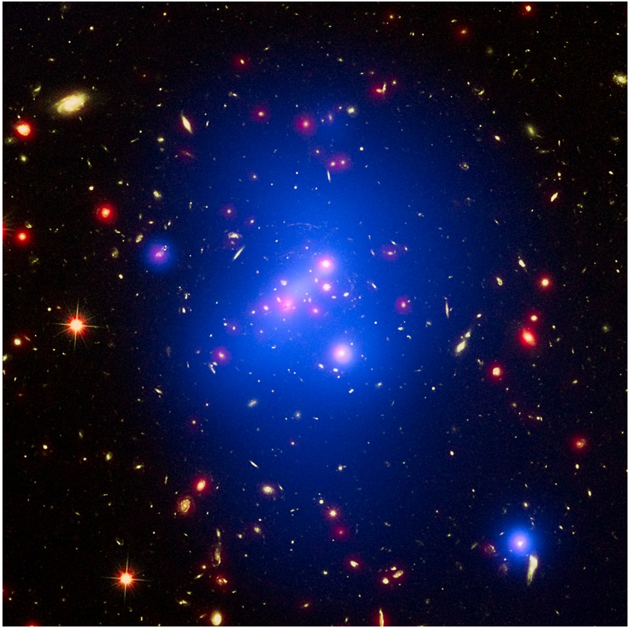
In turbulent non-magnetized weakly collisional plasmas, governed by kinetic physics, the smallest scale that supports magnetic fluctuations is not the resistive scale, as in MHD, but a larger scale, where magnetic perturbations are Landau damped on electrons.

This has a relevance in the early stages of the magnetic field generation in the universe, and it makes the fully kinetic simulation of non-magnetized dynamo difficult.

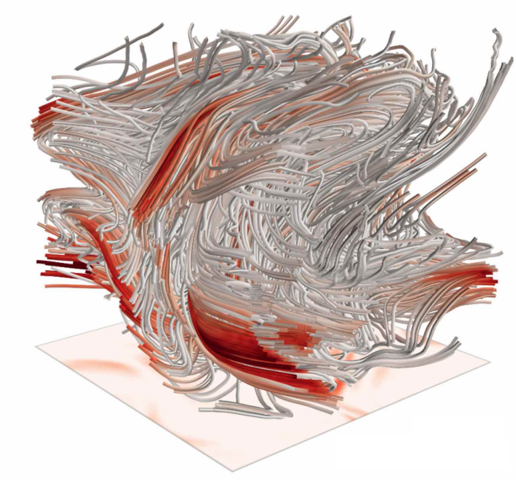
- Background
  - Turbulent dynamo in galaxy clusters
  - Previous kinetic dynamo studies
- Modeling
  - The collisionless, non-magnetized setup
  - `Gkeyll`
- Landau damping effects
  - No dynamo in simulations
  - Landau damping of magnetic fluctuations
  - Consequences on asymptotically scale-separated systems
- Conclusions

- Mass $10^{14-15} M_\odot$
  $\sim 10\%$ visible matter,
  $\sim 10\%$ of which is galaxies.
- Intra-cluster medium:
  $T \sim 1 - 10\,\mathrm{keV}$
  $n \sim 10^{2-5}\,\mathrm{m}^{-3}$
  $L \sim \mathrm{Mpc}\,(= 3 \cdot 10^{22}\,\mathrm{m})$
  velocity disp. $10^{2-3}\,\mathrm{km/s}$
- $B \sim \mu\mathrm{G}\,(= 10^{-10}\,\mathrm{T})$
- Taking $10\,\mathrm{keV}$ and $10^3\,\mathrm{m}^{-3}$
  $\lambda_{\mathrm{mfp}} \sim 10^{21}\,\mathrm{m}$
  $\rho_{\mathrm{e}} \sim 3 \cdot 10^6\,\mathrm{m}$



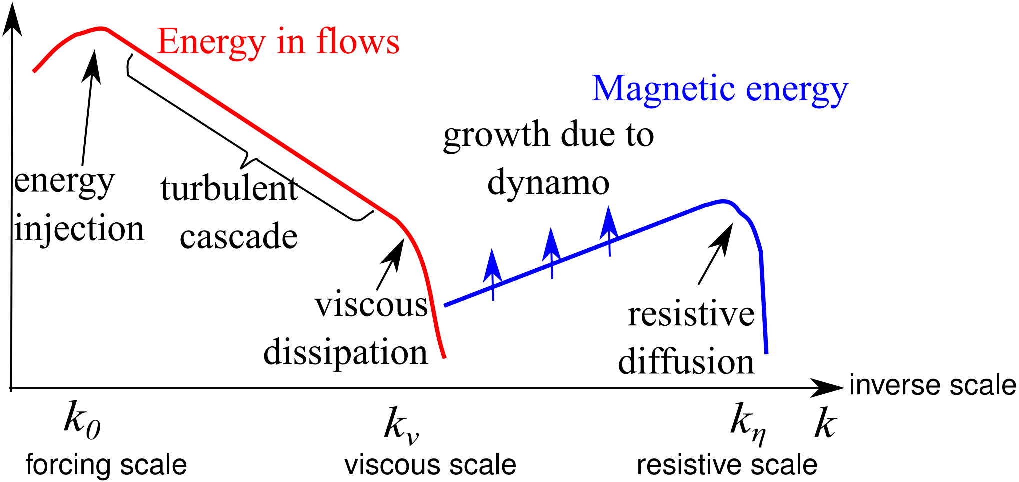
Composite image of cluster IDCS J1426, blue: X-ray

- Now: $B \sim 10^{-10}\,\mathrm{T}$ ($\rho_{\mathrm{e}} \sim 3 \cdot 10^{6}\,\mathrm{m}$)
- Biermann battery at ionization fronts in the early universe (induction due to misaligned n and T gradients)
  Seed field: $\sim 10^{-23}\mathrm{T}$
- Energy content in turbulent flows ($\sim$ observed velocity dispersion) comparable magnetic field energy.
- Turbulent dynamo

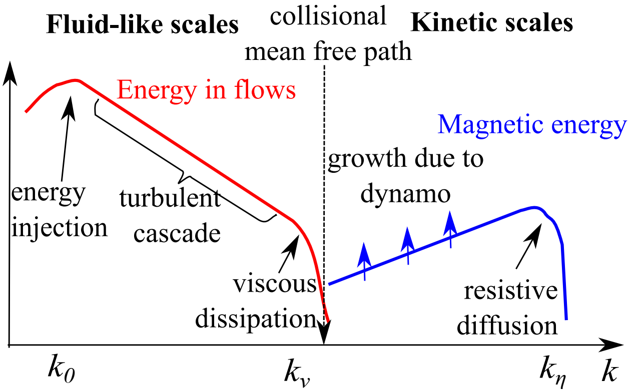


Folded structure of magnetic field in turbulent dynamo; hybrid simulation [F. Rincon 2019 JPP]

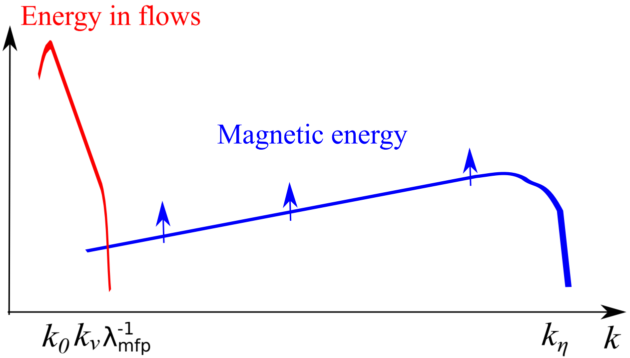
Magnetic Prandtl number $\mathrm{Pm} = \nu/\eta \gg 1$, ν viscosity, η magnetic diffusivity



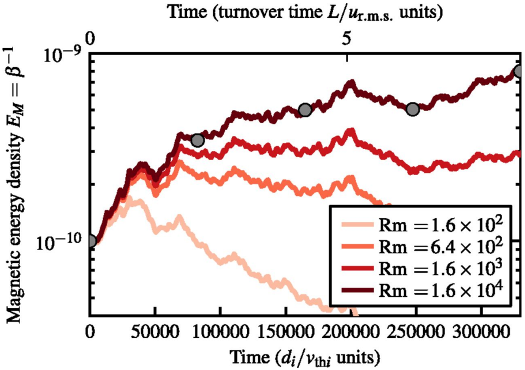
- Assume non-magnetized collisional viscosity, Spitzer resistivity, and $u_0 \sim 100\,\mathrm{km/s}$, $l_0 \sim 1\,\mathrm{Mpc}$, $T \sim 10\,\mathrm{keV}$, $n \sim 10^3/\mathrm{m}^3$
- $\mathrm{Re} = \frac{u_0 l_0}{\nu} \gtrsim 1$
  $\mathrm{Pm} = \nu/\eta \sim 10^{28}$
  $k_\nu \sim \mathrm{Re}^{3/4}/l_0 \sim k_0$
  $l_0/\lambda_{\mathrm{mfp}} \sim 10$
  $k_\eta/k_0 \sim \mathrm{Re}^{3/4}\mathrm{Pm}^{1/2} \sim 10^{14}$
- Not much fluid cascade
- k_ν and k_η well separated



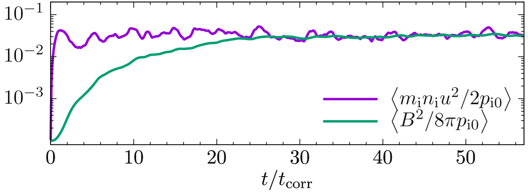
- Most of the scales of interest are collisionless, kinetic
- Smallest flow structures are at k_ν, for $k \gg k_\nu$ flow is smooth
- Could mock up effect of "fluid cascade" by smooth, large scale forcing
- Caveat: Today $B \sim \mu\mathrm{G}$, $\Rightarrow k_\eta \rho_e \sim 1$ Non-magnetized fluid viscosity does not apply



- Focus on high Prandtl numbers, random forcing on box-size scale
- **F. Rincon *et al* 2016**:
  Ions: Continuum kinetic.
  Electrons: simple Ohm's law.
  Demonstrates magnetic field growth in this hybrid system, and shows that temperature anisotropy-driven instabilities are active.

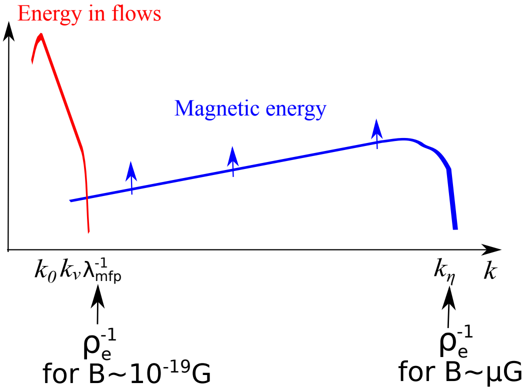


- **D.A. St-Onge *et al* 2018**:
  Ions: Particle-in-cell.
  Electrons: isothermal massless fluid.
  Clarifies essential role of instabilities. Simulations proceed to the saturated state.

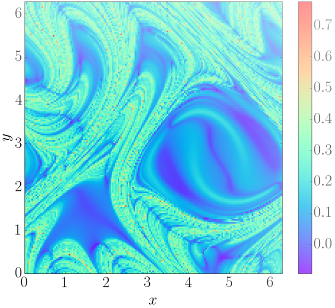


The weakly collisional non-magnetized case

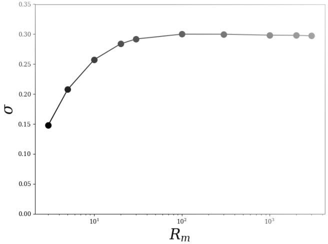
- At typical Biermann seed fields, kinetic scales are non-magnetized.
- No problem resolving ρ scale
- How do electrons affect dynamo? $\rightarrow$ Fully kinetic



- Consider sub-viscous scales,
  $L_0 \sim l_0 \sim l_\nu \sim \lambda_{\mathrm{mfp}}$.
- Physics constants not modified
- To get $\mathrm{Pm} \approx 20$
  ($\mathrm{Rm} \approx 13$ and $\mathrm{Re} \approx 0.64$)
  Use $T = 1\,\mathrm{keV}$, $n = 2.3 \cdot 10^{28}\,\mathrm{m}^{-3}$,
  $u_0 = 0.35\sqrt{T/m_i}$, $L_0 = 9.73\,\mu\mathrm{m}$,
  $\ln\Lambda = 10$, $\lambda_{\mathrm{mfp}} = 1.25\,\mu\mathrm{m}$
- Galloway-Proctor model flow
  $u_0\{\sin(kz + \sin\omega t) + \cos(ky + \cos\omega t),$
  $\cos(kz + \sin\omega t), \sin(ky + \cos\omega t),$
  with $k = 2\pi/L_0$.
  Chaotic flow, produces *fast dynamo*
  Low critical $\mathrm{Rm} = \frac{u_0 l_0}{\eta}$
- Do it fully kinetically!



Lyapunov exp. of G-P flow. $z = 0$ plane



Dynamo growth rate of G-P flow, vs. Rm

[figures from S. M. Tobias (2020) subm. *J. Fluid Mech.*]

- The `Gkeyll` framework: Freely available plasma physics solver package, using a discontinuous Galerkin scheme. https://gkyl.readthedocs.io
- Fully kinetic, gyrokinetic and advanced multi-fluid solvers.
- **Kinetic-Maxwell** solver: High order accurate. Discretization conserves particles and energy exactly. Scales well on distributed memory clusters. Permits noise free calculations of the distribution function. Serendipity basis set provides efficiency at high dimensionality.

$$\partial_t f_a + \mathbf{v} \cdot \nabla f_a + (e_a/m_a)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_a = C[f_a]$$

- Inter- and intra-species Coulomb **collisions**, $C[f_a]$, modeled by a conservative Dougherty operator (drag and diffusion in velocity space; reproduces Spitzer resistivity).
- Has full Maxwell equations, but kinetics is non-relativistic.
- We need 3X3V, time dependent forcing.

- Exerting force on ions (added to $e\mathbf{E}$)

$$\mathbf{f}(\mathbf{x},t) = C_f m_i \mathbf{u}(t)/t_i$$

  $\mathbf{u}$ is G-P flow, $t_i = L_0/\sqrt{2T/m_i}$; we set $C_f = 1$.
- Initial (seed) magnetic field

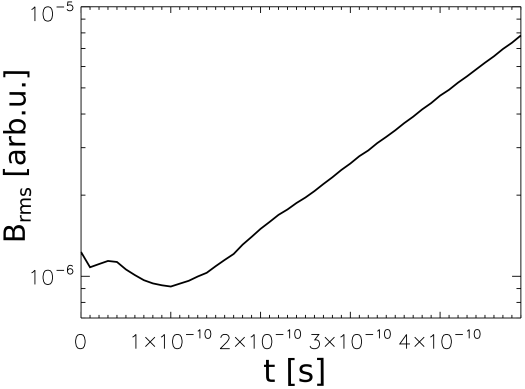
$$B_i = B_0 \sum_{j\neq i,n} b_{ij,n} \cos[nk(x_i + \varphi_{ij,n})]$$

  $b_{ij,n}$ and $\varphi_{ij,n}$ random on $[0,1]$, $n = 1, 2, ..., N$, ($N = 4$)
  $B_0 = 40\,\mathrm{T}$ ($\rho_e \sim 3\,\mu\mathrm{m}$).
- Both species initialized with Maxwellian with flow $\mathbf{u}(t=0)$, current density corresponding to $\mathbf{B}(t=0)$, deposited into electron flows.

- Solve the MHD induction equation using the PENCIL CODE

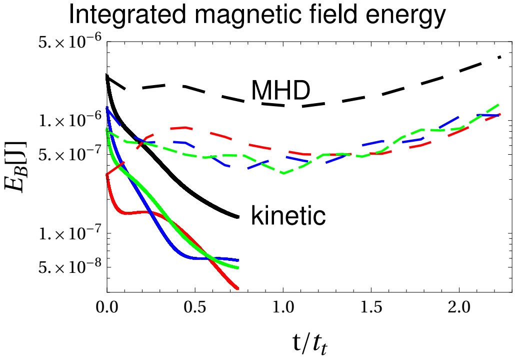
$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Use same initial condition
- After ≈ 1 turnover time ($t_\mathrm{t} = l_0/u_0 = 8.9 \cdot 10^{-11}\,\mathrm{s}$) exponential growth begins
- Changing spatial resolution from 12^3 to 32^3, leads to no appreciable difference in results



No magnetic field amplification in kinetic simulation

- Magnetic energy in kinetic simulations strongly decays



Red/blue/green: Contribution from $x/y/z$ field components. Black: total

Thermal energy: $(3/2)n_iT_iL_0^3 = 5.1\cdot10^{-3}$ J.

- In a collisionless plasma $B_z(x, t = 0) = B_0 \cos(kx)$ decays with damping rate

$$\gamma = \frac{|k|^3 c^2 v_e}{\sqrt{\pi}\omega_{pe}^2} = \frac{|k|^3 v_e m_e}{\sqrt{\pi}\mu_0 n_e e^2}$$

$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$, $v_e = \sqrt{\frac{2T_e}{m_e}}$ [A. B. Mikhailovskii 1980 Plasma Phys. **22** 133]

- Without effect of fields, current would decay on time scale $(v_e|k|)^{-1}$, due to free streaming of electrons. $\mathbf{E}$ is induced to keep current running. $|\mathbf{E}| \propto |\mathbf{j}|$
- Compare to damping due to resistive diffusion

$$\gamma = \frac{k^2}{\sigma\mu_0} \approx \frac{k^2 m_e \nu_{ei}}{2\mu_0 n_e e^2}$$

- Landau damping similar to the effect of scale dependent resistivity

$$\frac{1}{\sigma_{\text{eff}}} = \frac{|k| v_e m_e}{\sqrt{\pi} n_e e^2}$$

- Effective magnetic diffusivity $\eta_{\text{eff}} \sim \eta\lambda_{\text{mfp}}|k|$

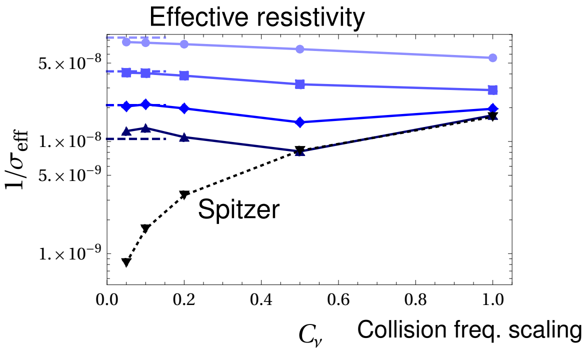
Landau damping of magnetic fluctuations

- $B_z(x, t = 0) = B_0 \cos(kx)$ decays as

$$B_z \propto \exp(-\gamma t) = \exp(-k^2 \eta t)$$

when $\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B}$, $\eta = (\sigma \mu_0)^{-1}$

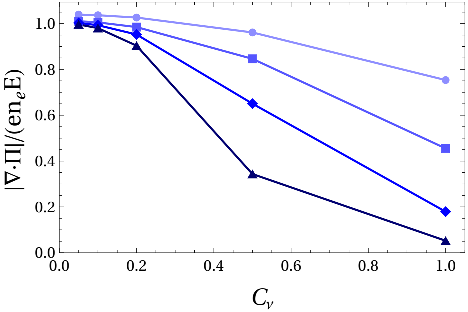
- in 1X2V simulations exponential decay consistent with instantaneous $\sigma_{\text{eff}} = j_y / E_y$
- When $\lambda_{\text{mfp}} k \ll 1$ Spitzer result reproduced
- For low collisionality $C_\nu \ll 1$ $\Rightarrow \lambda_{\text{mfp}} k \gg 1$, Landau damping of magnetic field on electrons
- A stronger, wave number dependent "diffusion"



Wavelength (and box size) increases as $\{1/8, 1/4, 1/2, 1\} \cdot 9.73\mu\text{m}$ with darkening curves.
$\lambda_{\text{mfp}} = 1.25\,\mu\text{m}$ at $C_\nu = 1$
Dashed: Collisionless Landau damping

$$\frac{\partial(mn\mathbf{V}_e)}{\partial t}+\nabla\cdot\mathbf{\Pi}_e = -ne(\mathbf{E}+\mathbf{V}_e\times\mathbf{B})+\mathbf{R}_{ei}$$

Importance of electron viscous stress

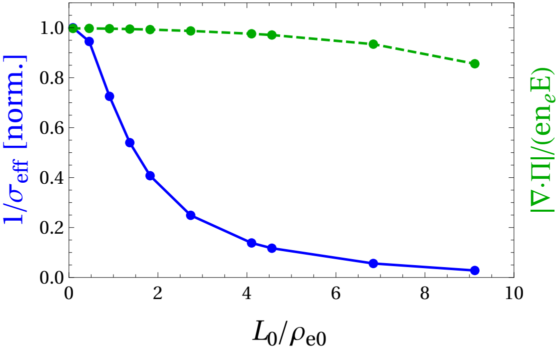


- In a collisional plasma, electric force is balanced by friction on ions (resistivity)
- In collisionless plasma it is balanced by viscous stress $\nabla\cdot\mathbf{\Pi}_e$, involving off-diagonal terms in $\mathbf{\Pi}_e$

Wavelength (and box size) increases as
$\{1/8,\ 1/4,\ 1/2,\ 1\}\cdot 9.73\mu\mathrm{m}$
with darkening curves.
$\lambda_{\mathrm{mfp}} = 1.25\,\mu\mathrm{m}$ at $C_\nu = 1$

- Increase $B(t=0)$, and so L_0/ρ_e
- Perpendicular free streaming of electrons inhibited
- Distribution function becomes gyrotropic
- Landau damping of magnetic perturbations is not relevant in magnetized plasmas

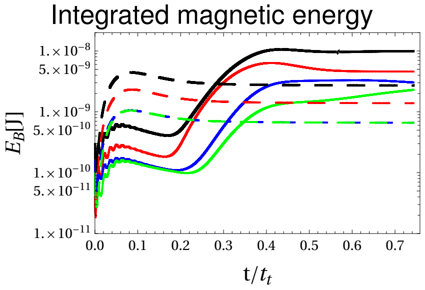
Scaling with magnetization

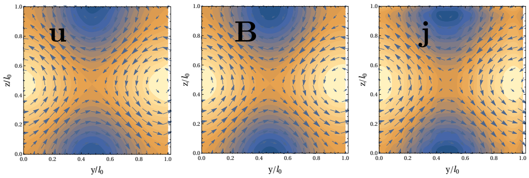


- Consider Roberts flow

  $u_0\{\cos(ky)-\cos(kz), \sin(kz), \sin(ky)\}$

  $L_0 = 1.22\,\mu\mathrm{m}$, $B_0 = 10\,\mathrm{T}$, $C_f = 3$, $C_\nu = 0$ (solid), $C_\nu = 0.3$ (dashed)
- Forcing introduces a current. **Unavoidable** in a fully kinetic setup, due to different transport properties of electrons and ions.
- $B(t \to \infty) \sim e u_0 n_i \mu_0 L_0$ force free field
- Perturbations decayed away





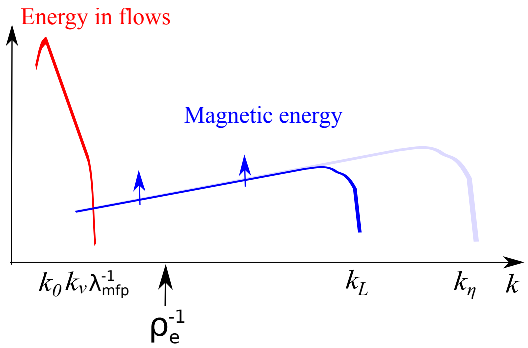
- In resistive MHD, balance rate of stretching of magnetic fluctuations at viscous scale u_ν/l_ν with dissipation rate at resistive scale η/l_η^2:

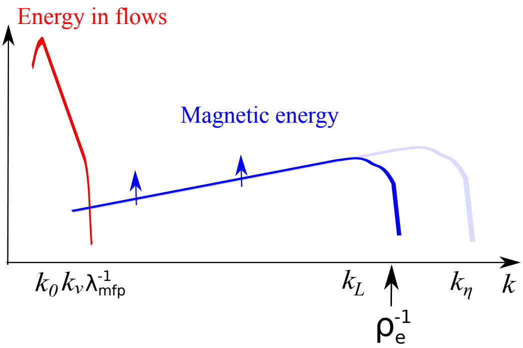
$$l_\eta \sim l_0 \mathrm{Re}^{-3/4}\mathrm{Pm}^{-1/2} \sim l_\nu \mathrm{Pm}^{-1/2}$$

- In collisionless non-magnetized system balance u_ν/l_ν and $\eta\lambda/l_L^3$, and use $\lambda_{\mathrm{mfp}} \sim l_0 M_0/\mathrm{Re}$

$$l_L \sim l_0 \frac{M_0^{1/3}}{\mathrm{Re}^{5/6}\mathrm{Pm}^{1/3}} \sim l_\nu \frac{M_0^{1/3}}{\mathrm{Re}^{1/12}\mathrm{Pm}^{1/3}}$$

When $\mathrm{Re}^{1/2}/M_0^2 \ll \mathrm{Pm}$, as for instance in galaxy clusters, $l_\eta \ll l_L$.





- The magnetic spectrum cutoff is initially at inverse scale k_L.
- As the magnetic field grows, at some point ρ_e^{-1} passes k_L, then cutoff is at ρ_e^{-1}.

- Using the kinetic-Maxwell solver `Gkeyll`, we have performed fully kinetic simulations of forced model flows known to produce dynamo, focusing on the weakly collisional, non-magnetized limit, and $\mathrm{Pm} \gg 1$, with relevance to earlier stages of dynamo in galaxy clusters.
- Landau damping of magnetic perturbations on electrons leads to a decaying magnetic field energy.
  We have not yet found dynamo growth in these kinetic simulations.
- Landau damping acts like a scale-dependent magnetic diffusion.
- In astrophysical systems with asymptotically large Pm, the cutoff of the magnetic spectrum is at the "Landau damping scale", l_L, or at ρ_e, whichever is smallest (as long as $\rho_e > l_\eta$).
- Beware of current drive due to forcing of flows in fully kinetic simulations.

- GITHUB
  `https://github.com/ammarhakim/gkyl`
- The kinetic-Maxwell solver
  J. Juno et al 2018, *Discontinuous Galerkin algorithms for fully kinetic plasmas*, Journal of Computational Physics **353** 110.
- The fluid solvers: L. Wang et al 2015, *Comparison of multi-fluid moment models with particle-in-cell simulations of collisionless magnetic reconnection*, Physics of Plasmas **22** 012108.