

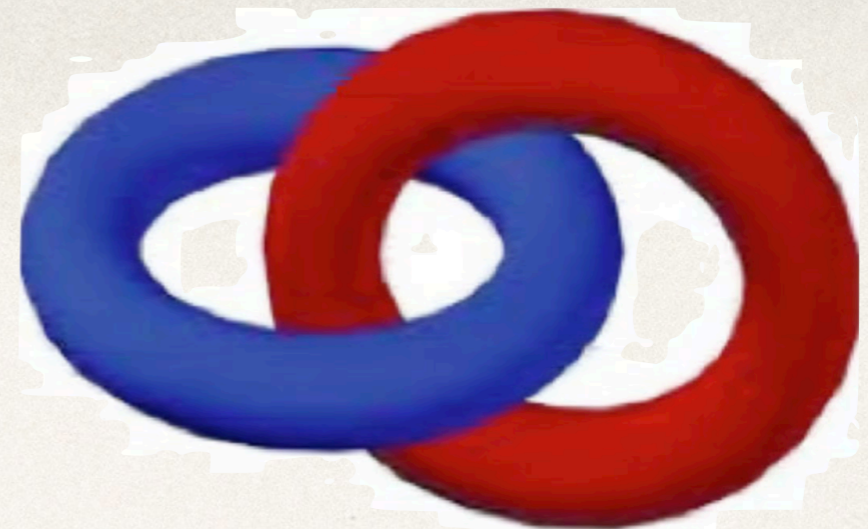
Helicity proxies from linear polarisation for solar active regions

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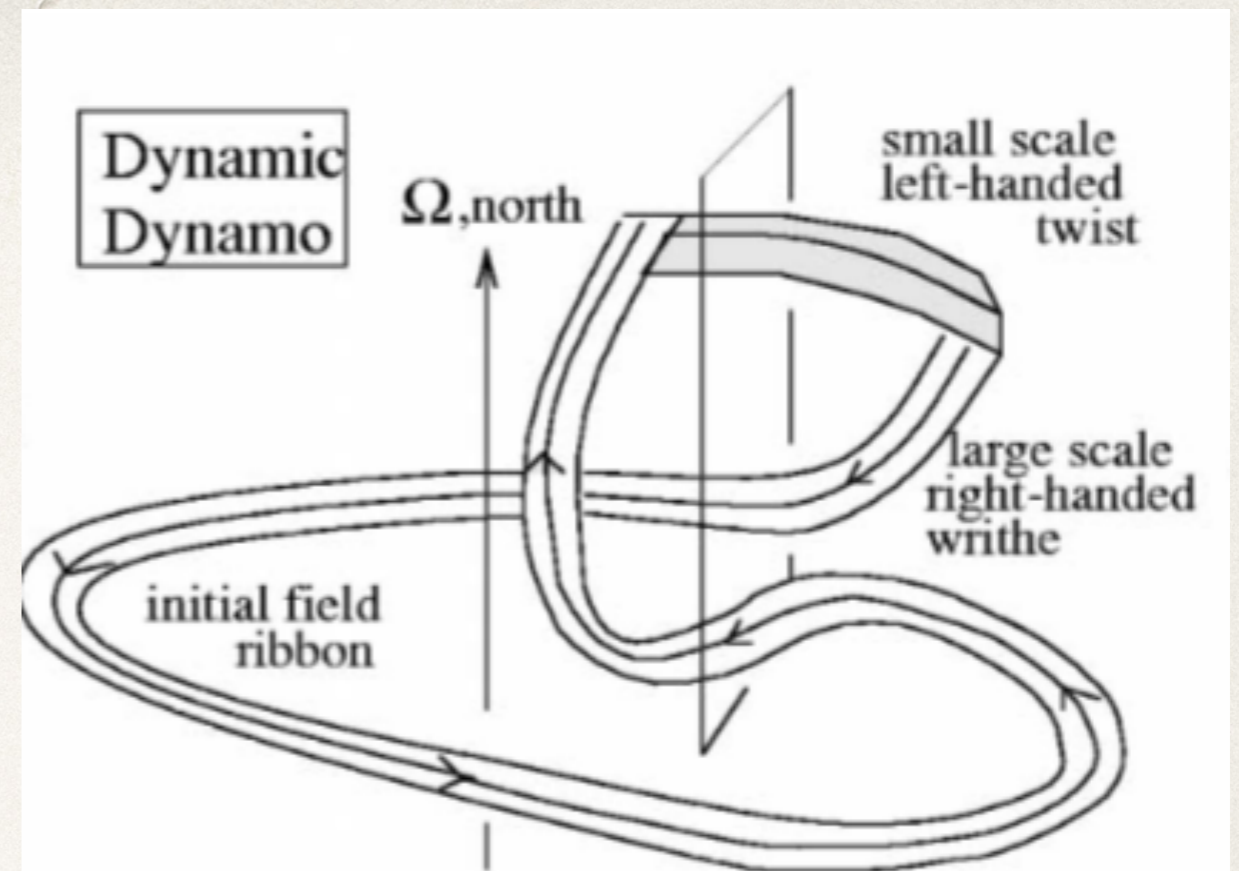
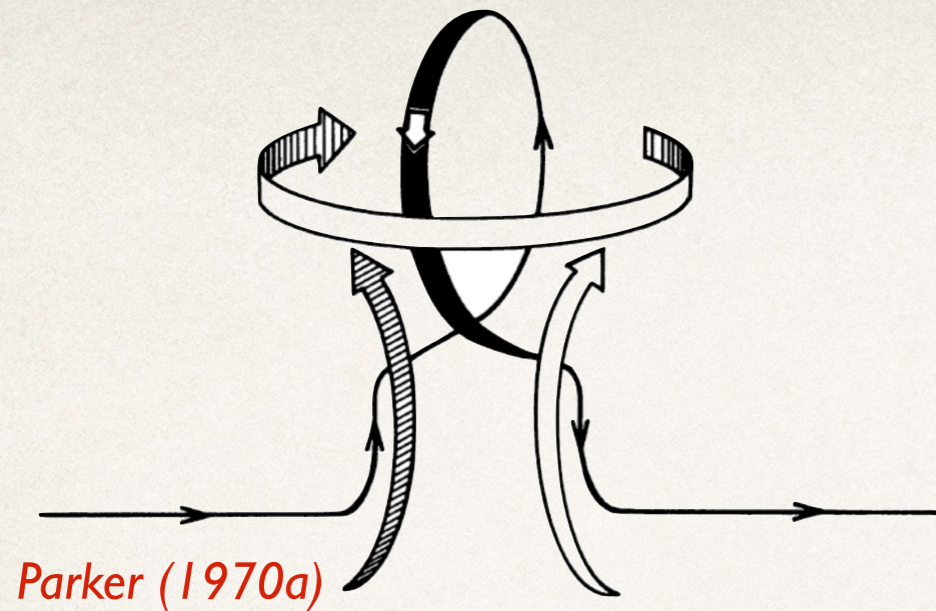


Why magnetic helicity?

$$H_m = \int_V \vec{A} \cdot \vec{B} \, dV$$



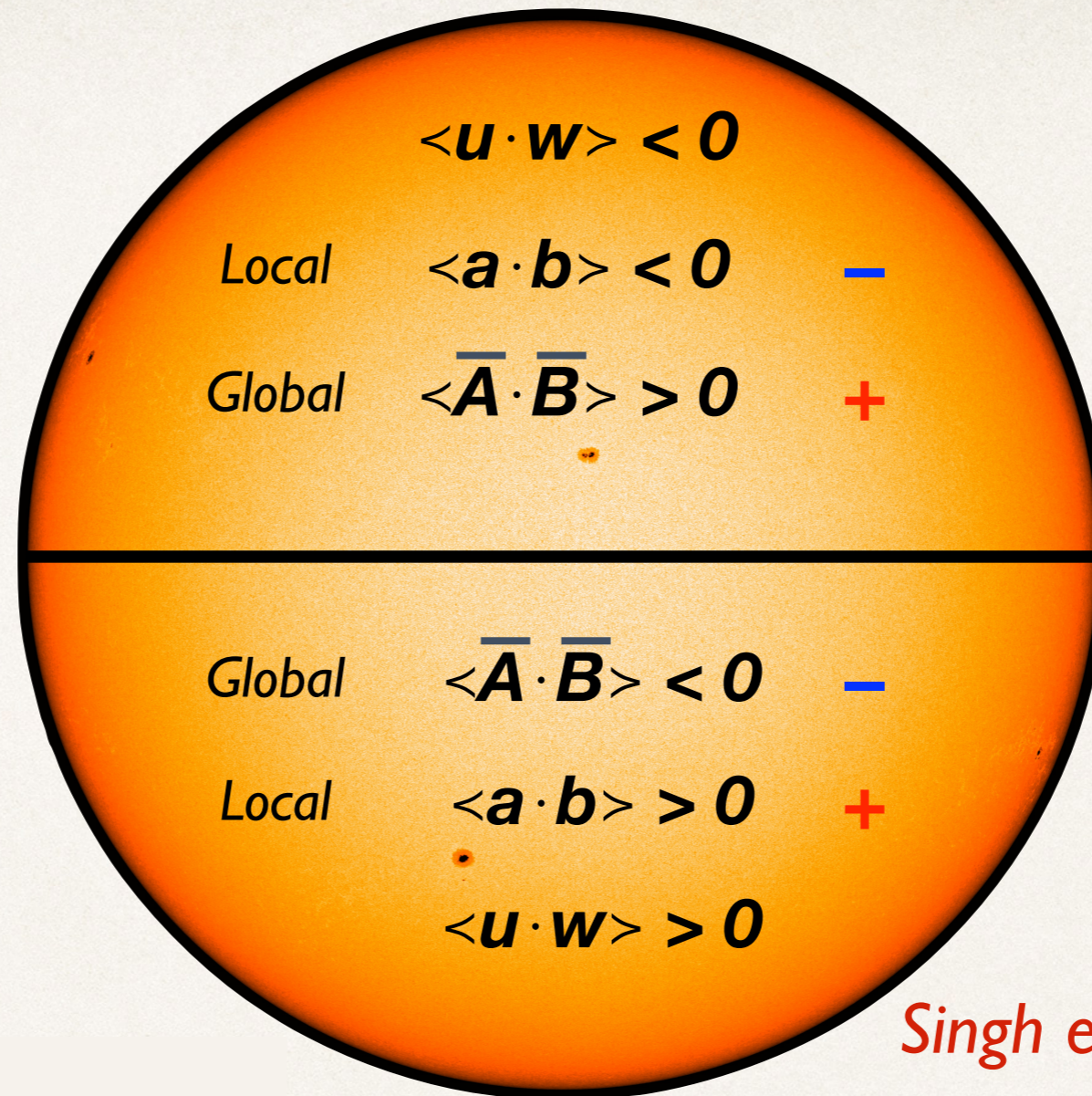
- It's a measure of complexity or internal twist of magnetic fields. Can be understood in terms of twists or linkage flux tubes (*Berger & Field 1984*)
- H_m is a topological invariant of ideal MHD, *almost* perfectly conserved even in non-ideal MHD (*Berger 1984, Pariat et al. 2015*)
- Imposes crucial constraint on the evolution of magnetic fields via a dynamo mechanism (*Brandenburg and Subramanian 2005*)
- Solar magnetic field and its spatio-temporal features \longrightarrow dynamo process acting within the Sun's convection zone
- One scenario : turbulent dynamo \longrightarrow α effect i.e. a measure of *helicity* of turbulence in the convection zone



A sketch from *Blackman and Brandenburg (2002)*

- Convective turbulence under the effect of stratification and rotation possesses small-scale kinetic helicity \rightarrow results in magnetic helicity of same handedness at small scales
- Helicity is conserved; helicity of equal magnitude but opposite sign is generated at larger scales (see for eg. *Seehafer 1996*). Thus theory predicts a helicity spectrum with **opposite signs on small and large scales!**
- Coriolis force breaks reflectional symmetry across the equator
- Another key consequence: **magnetic helicity changes sign across the equator**

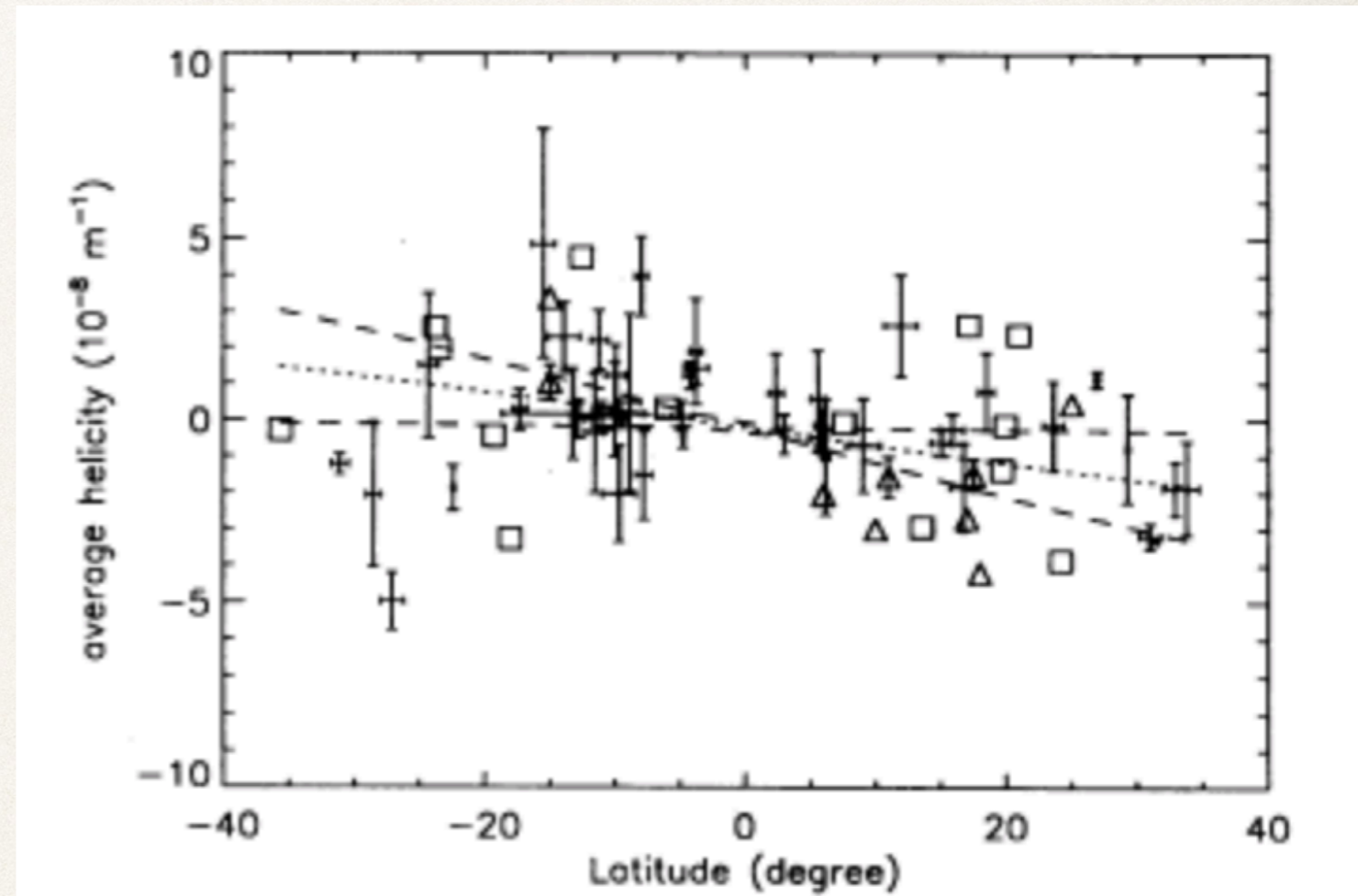
IF an α effect is a key ingredient of the solar dynamo then...



- A prediction is the hemispheric sign rule for magnetic helicity
- **Goal** : To test if we indeed see this sign rule from actual solar observations and thus verify the significance of the α effect for the dynamo

Observations of helicity on the Sun

- Seehafer (1990), Pevtsov et al. (1995) looked at current helicity as a proxy
- $\nabla \times \vec{B} = \alpha \vec{B}$
- This does not however, elucidate the scale dependence of helicity, thus **spectra are desirable**



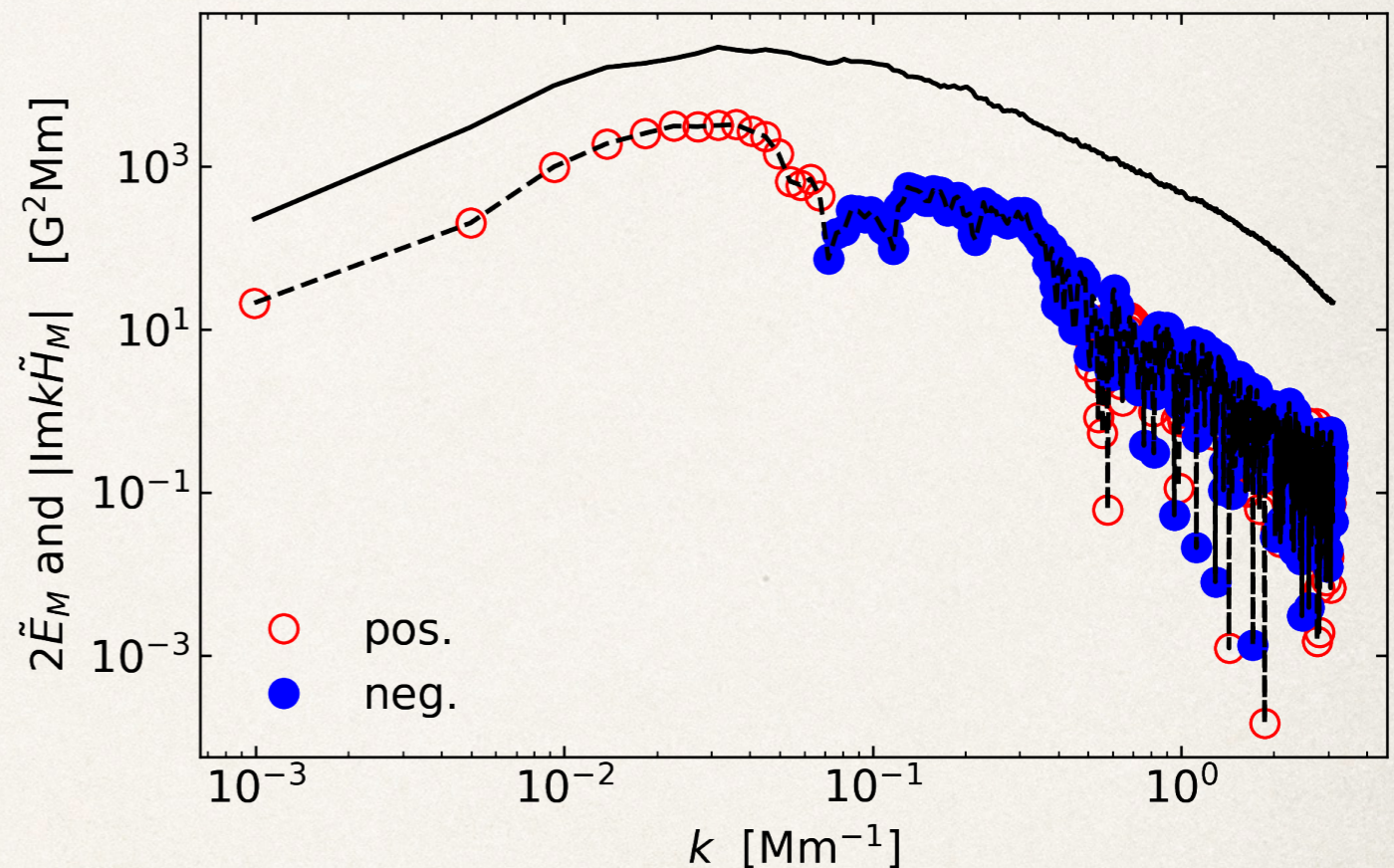
Pevtsov et al. (1995)

Observations of helicity on the Sun

- Zhang et al. 2014, 2016 computed “local” spectra from active regions
- Brandenburg et al. 2017, Singh et al. 2018 computed “global” spectra from synoptic maps

$$M_{ij}(\vec{x}) = \int \langle B_i(\vec{X}) B_j(\vec{X} + \vec{x}) \rangle dX$$

$$M_{ij}(\vec{k}) = \frac{2E_M(k)}{4\pi k} (\delta_{ij} - \hat{k}_i \hat{k}_j) + \frac{iH_m(k)}{4\pi k} \epsilon_{ijk} k_k$$



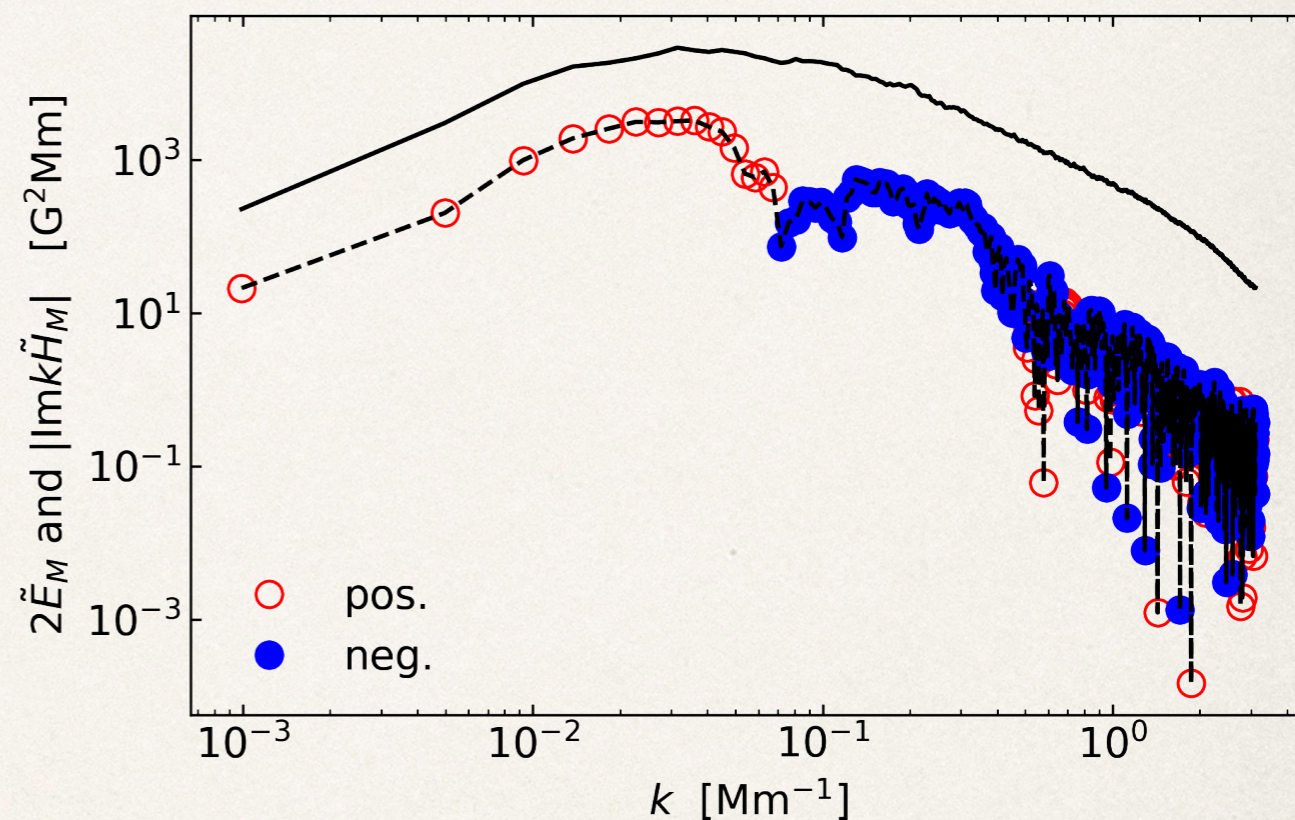
HMI, CR 2156-2158

Observations of helicity on the Sun

- All these studies are based on (directly or indirectly) on the magnetic field
- Photospheric magnetic fields are inferred using inversions of solar spectra via Zeeman effect $\rightarrow \pi$ ambiguity
- One can only measure the transverse field without arrow heads

$$M_{ij}(\vec{x}) = \int \langle B_i(\vec{X}) B_j(\vec{X} + \vec{x}) \rangle dX$$

$$M_{ij}(\vec{k}) = \frac{2E_M(k)}{4\pi k} (\delta_{ij} - \hat{k}_i \hat{k}_j) + \frac{iH_m(k)}{4\pi k} \epsilon_{ijk} k_k$$

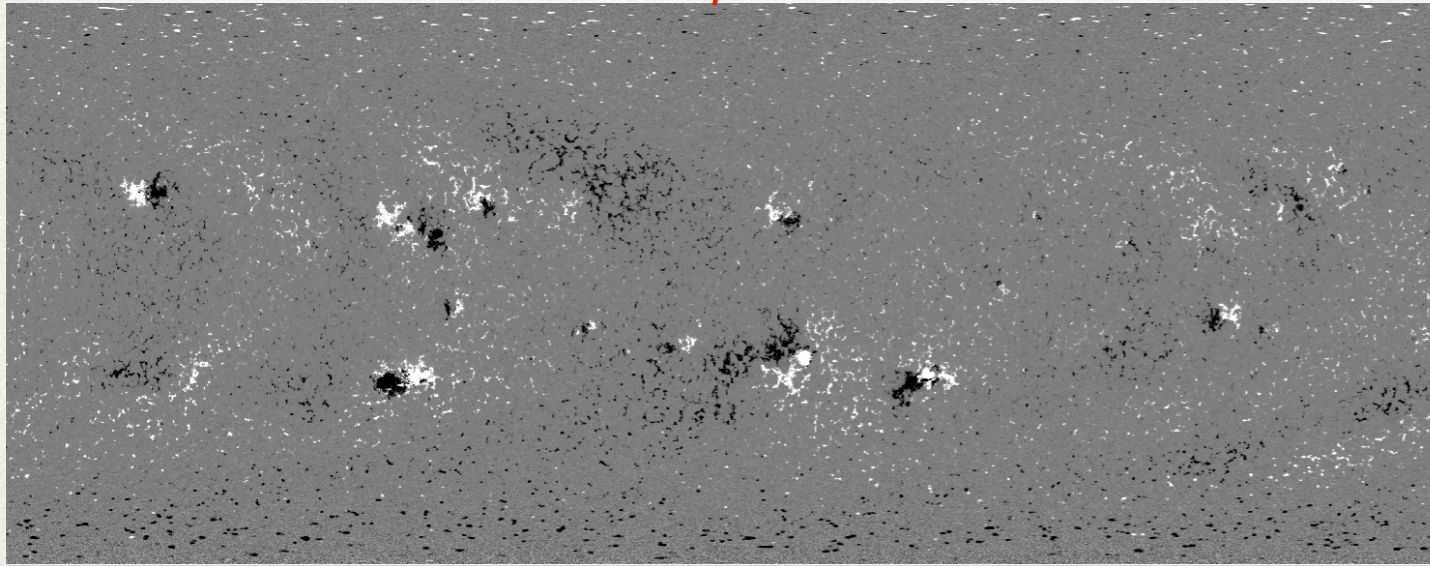


HMI, CR 2156-2158

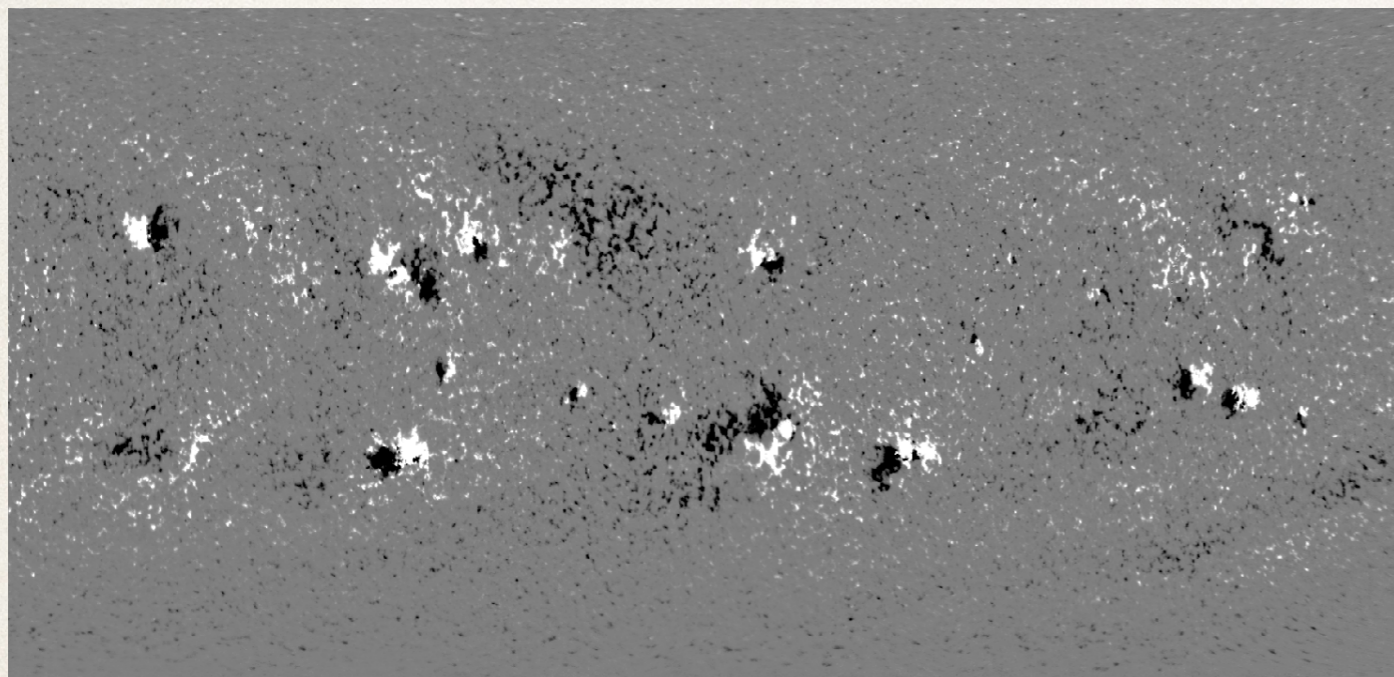
- To disambiguate the transverse component of the magnetic field and transform from a line-of-sight coordinate system to a solar coordinate system several methods exist (see *Metcalf et al. 2006*)

HMI

B_r



Carrington map 2168,
Sep-Oct 2015.

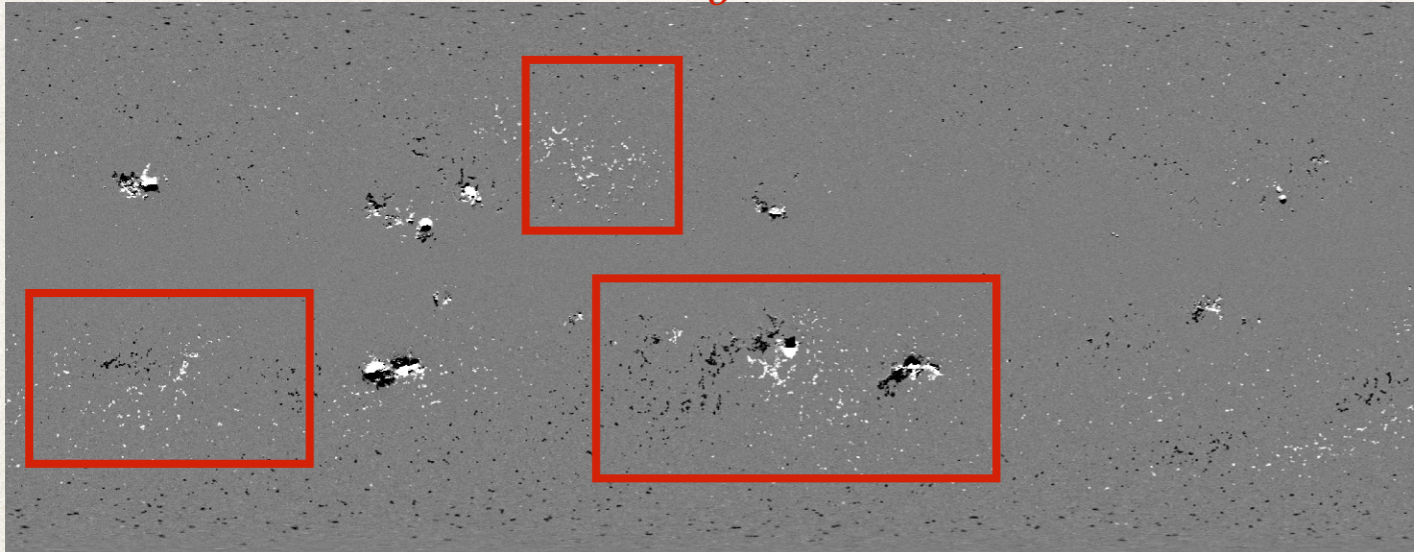


SOLIS

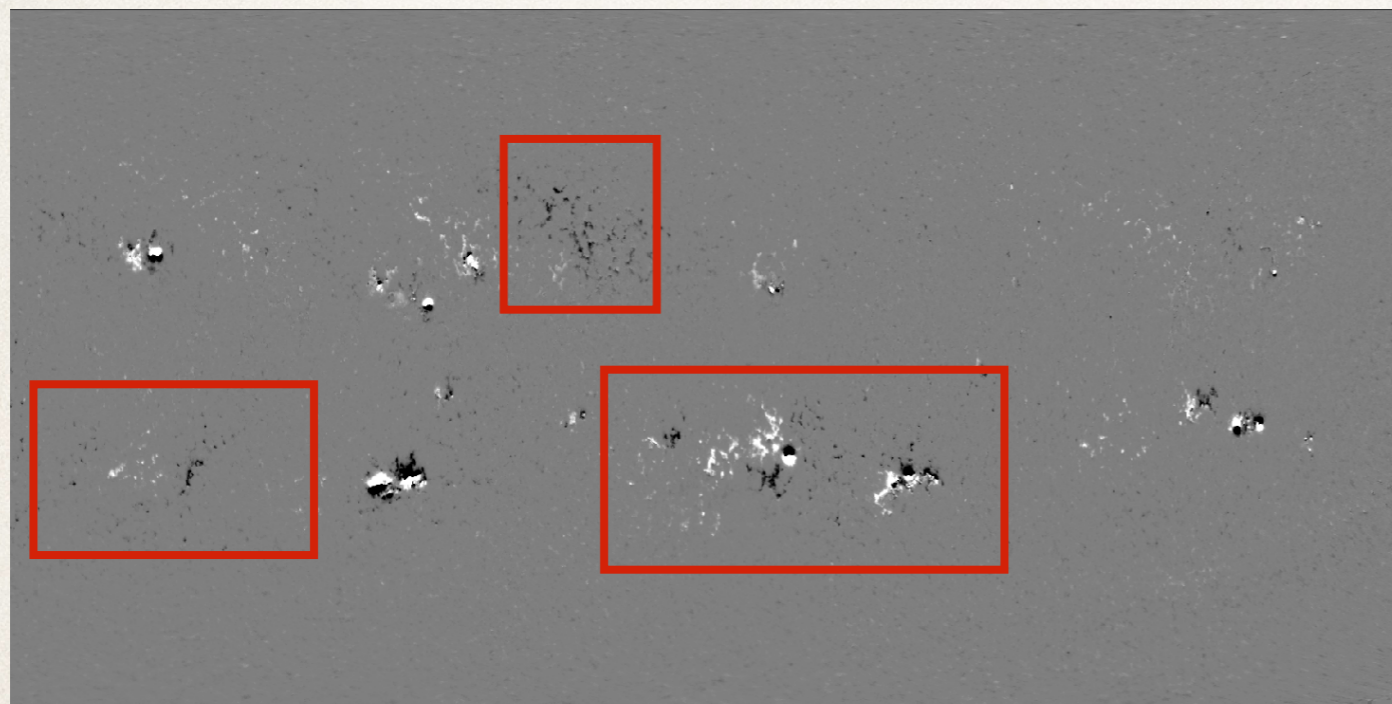
- These disambiguation methods face difficulties in regions of complex magnetic field geometries or where there is a strong influence of noise on the measurement

HMI

B_{θ}



Carrington map 2168,
Sep-Oct 2015.

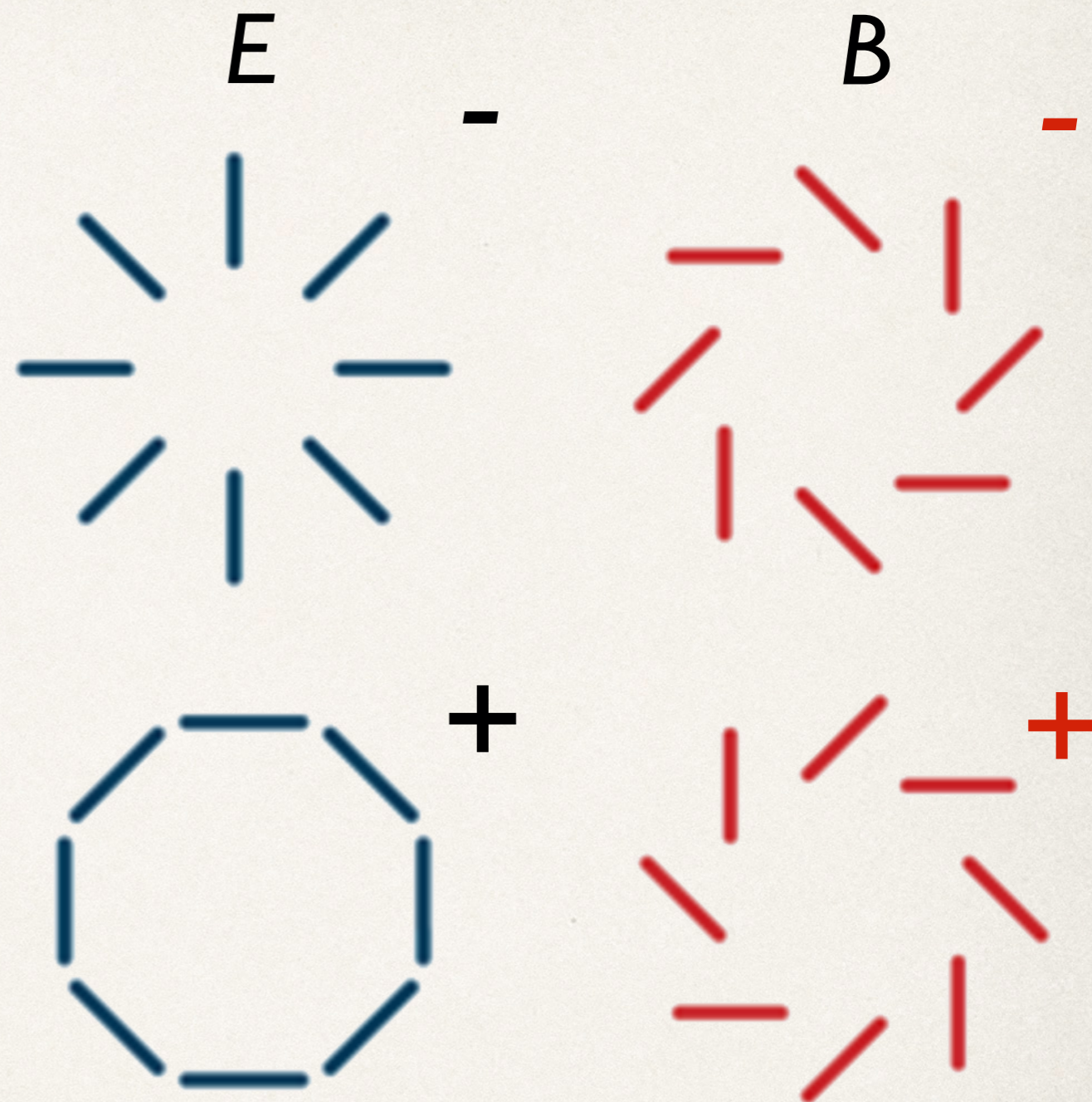


- This can lead to disagreement in the helicity spectra retrieved from different instruments.
- It highlights the need for better synoptic maps

SOLIS Another proxy of magnetic helicity independent of the π ambiguity could be useful!

E and B polarisation

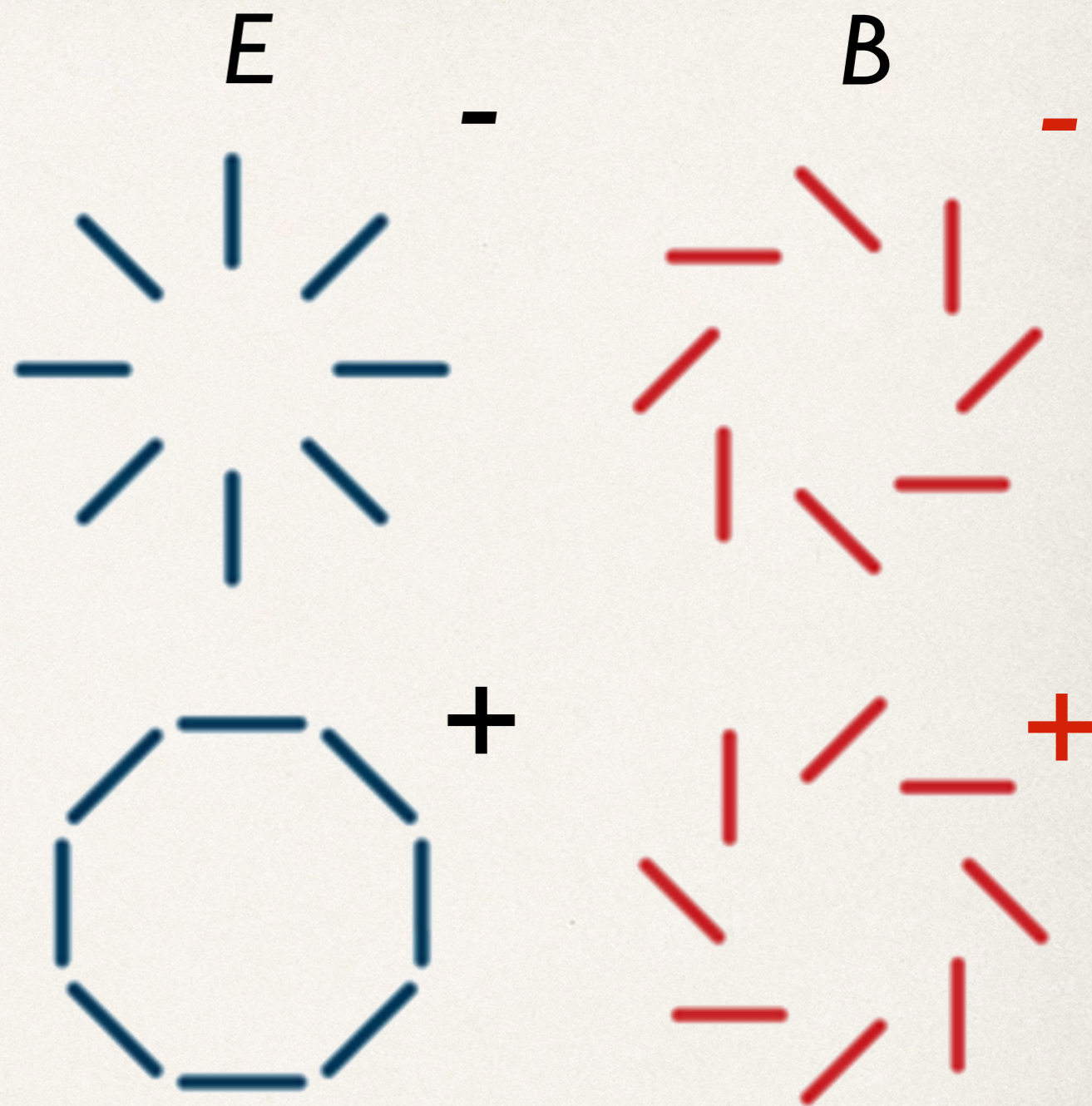
- Linear polarisation (Stokes Q and U) is decomposed into E and B .
- E and B are parity even and parity odd respectively see for eg. [Zaldariaga & Seljak 1997](#)
- $(\tilde{E} + i\tilde{B}) = (\hat{k}_x - i\hat{k}_y)^2(\tilde{Q} + i\tilde{U})$
- EB correlations are indicative of helicity of underlying magnetic field (for eg: [Pogosian et al. 2002](#), [Kahniashvili et al. 2014](#))
- First tested in the solar context by [Brandenburg et al. 2019](#)



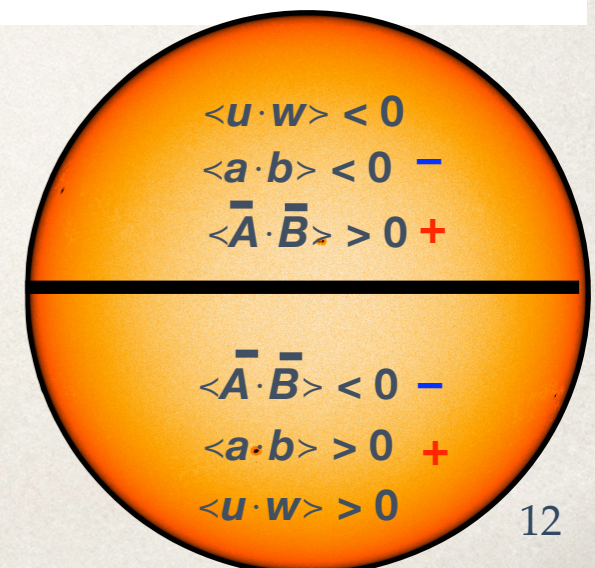
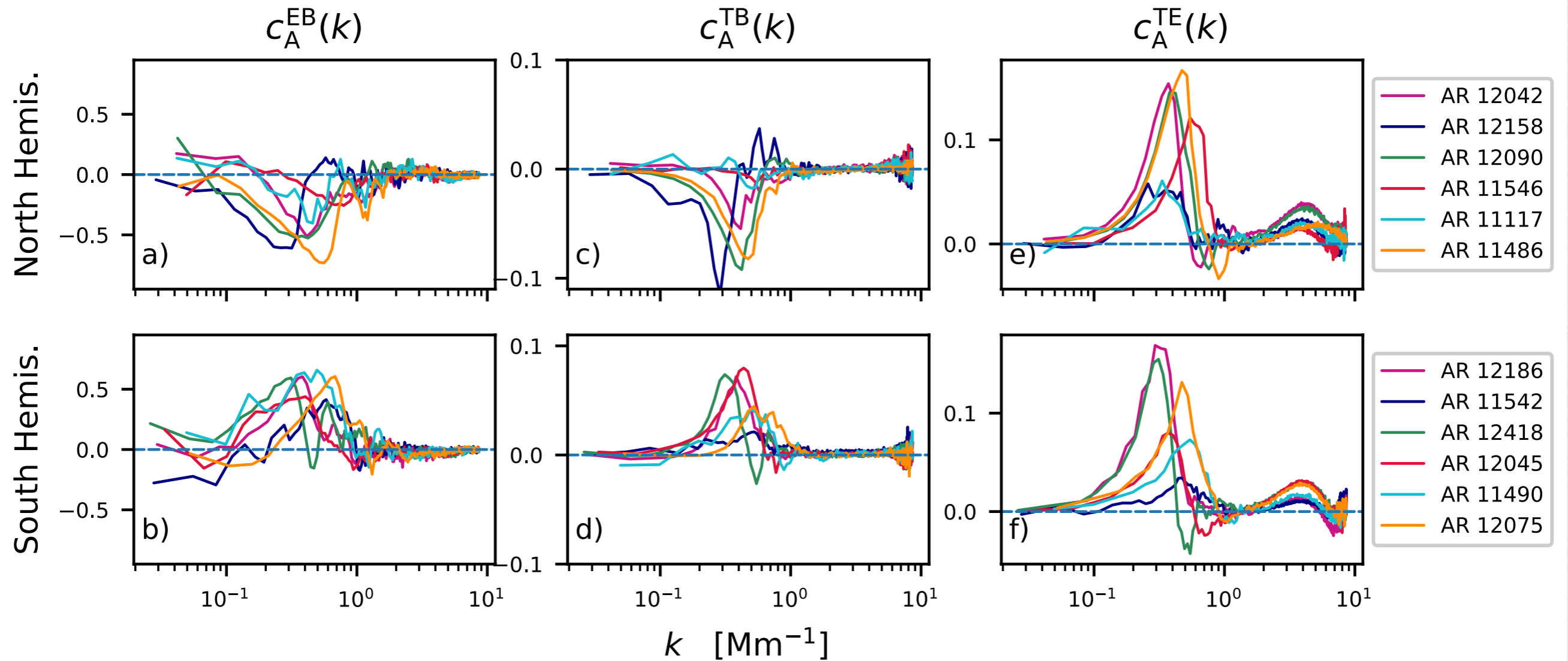
$$a_{lm}^E = - (a_{lm}^2 + a_{lm}^{-2})/2, \quad a_{lm}^B = i(a_{lm}^2 - a_{lm}^{-2})/2$$

E and B polarisation

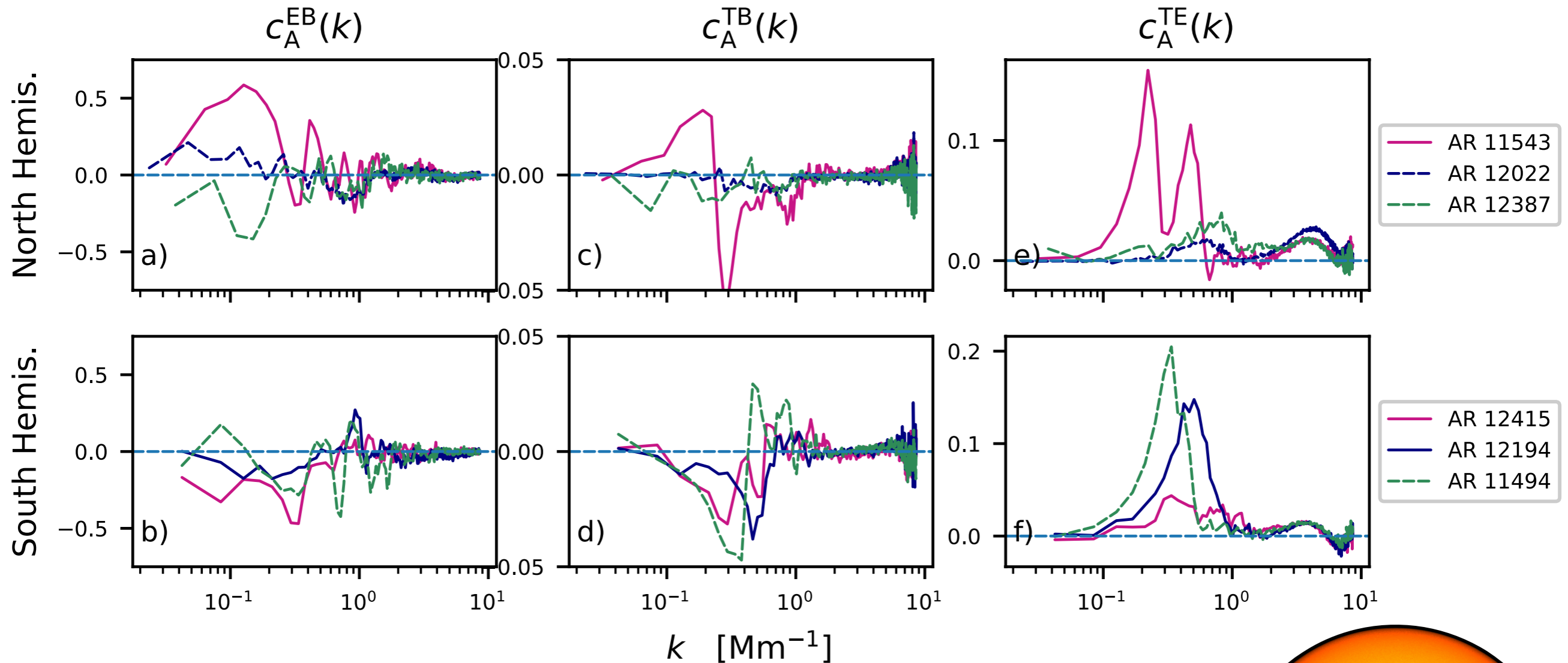
- ❖ One would expect that **helicity** and thus **parity-odd** correlations to ideally show different signs in different hemispheres!
- ❖ We looked at active regions in different hemispheres with SDO/HMI
- ❖ Computed EB from Q and U and checked for systematic preference for a sign based on hemisphere



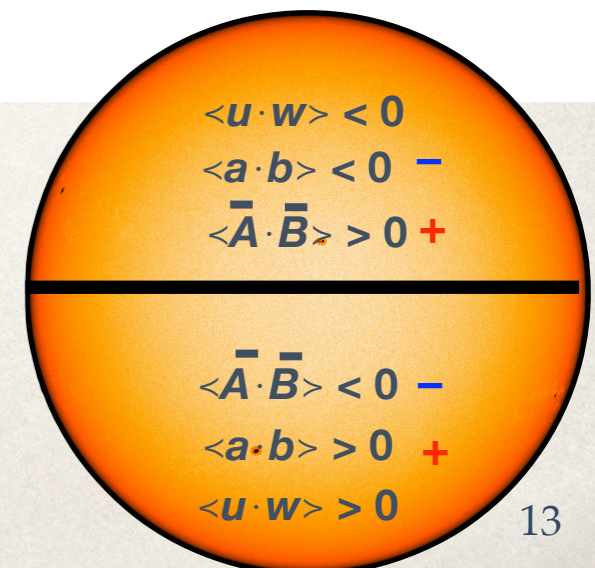
Computed EB correlations from Stokes Q&U at 4 filter positions of HMI and then averaged!



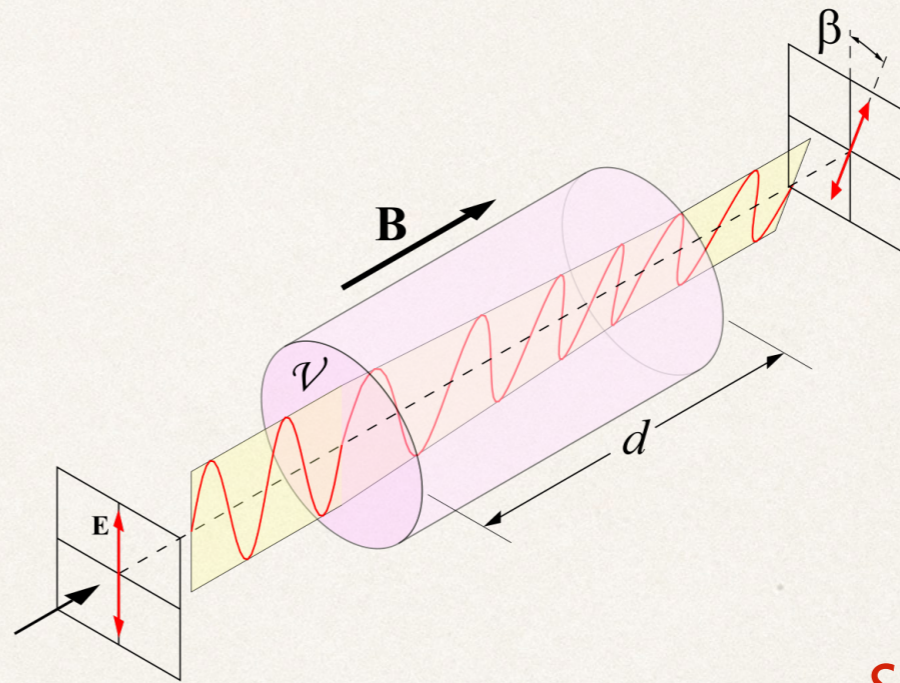
Active regions with reversed (solid) and no preference for sign (dashed)



- Singh et al. (2018) found evidence for the hemispheric sign rule in nearly 75% of the synoptic maps over solar cycle 24

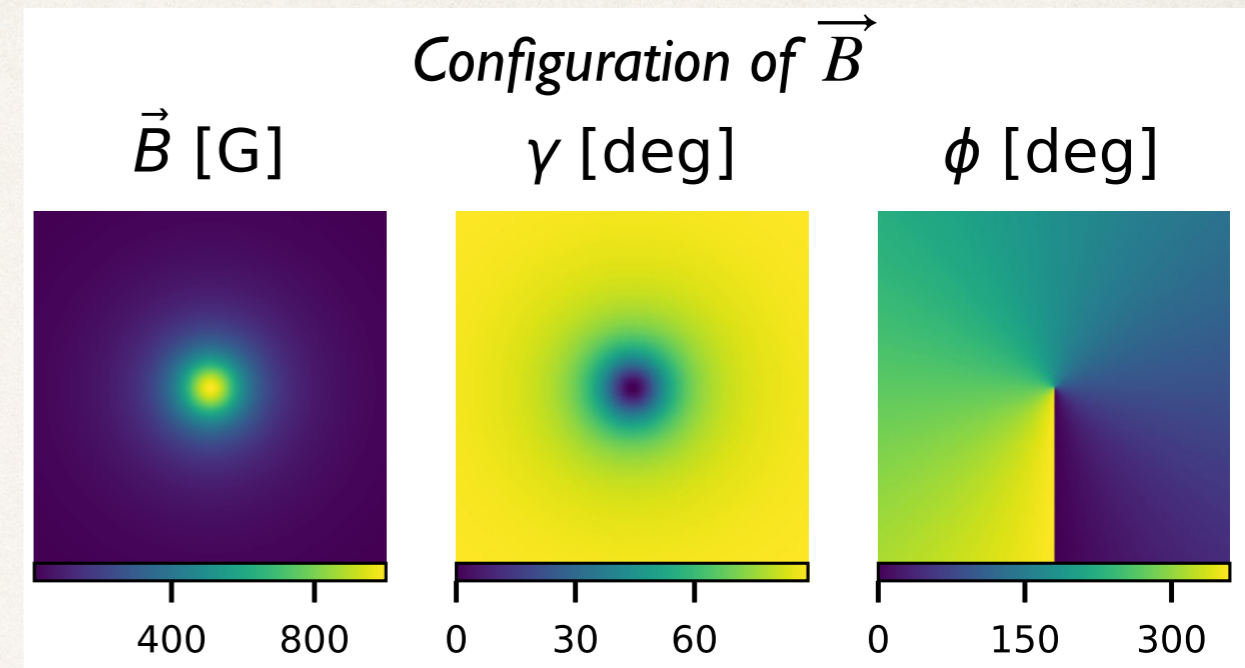


Faraday rotation could play a role : a homogeneous non-helical field could contribute to parity-odd correlations (*Scannapieco & Ferreira 1997*)



Source: wikipedia

Aim: To test if a non-helical magnetic field on the Sun's surface can contribute to significant parity odd C_A^{EB} or C_A^{TB} correlations purely due to Faraday rotation

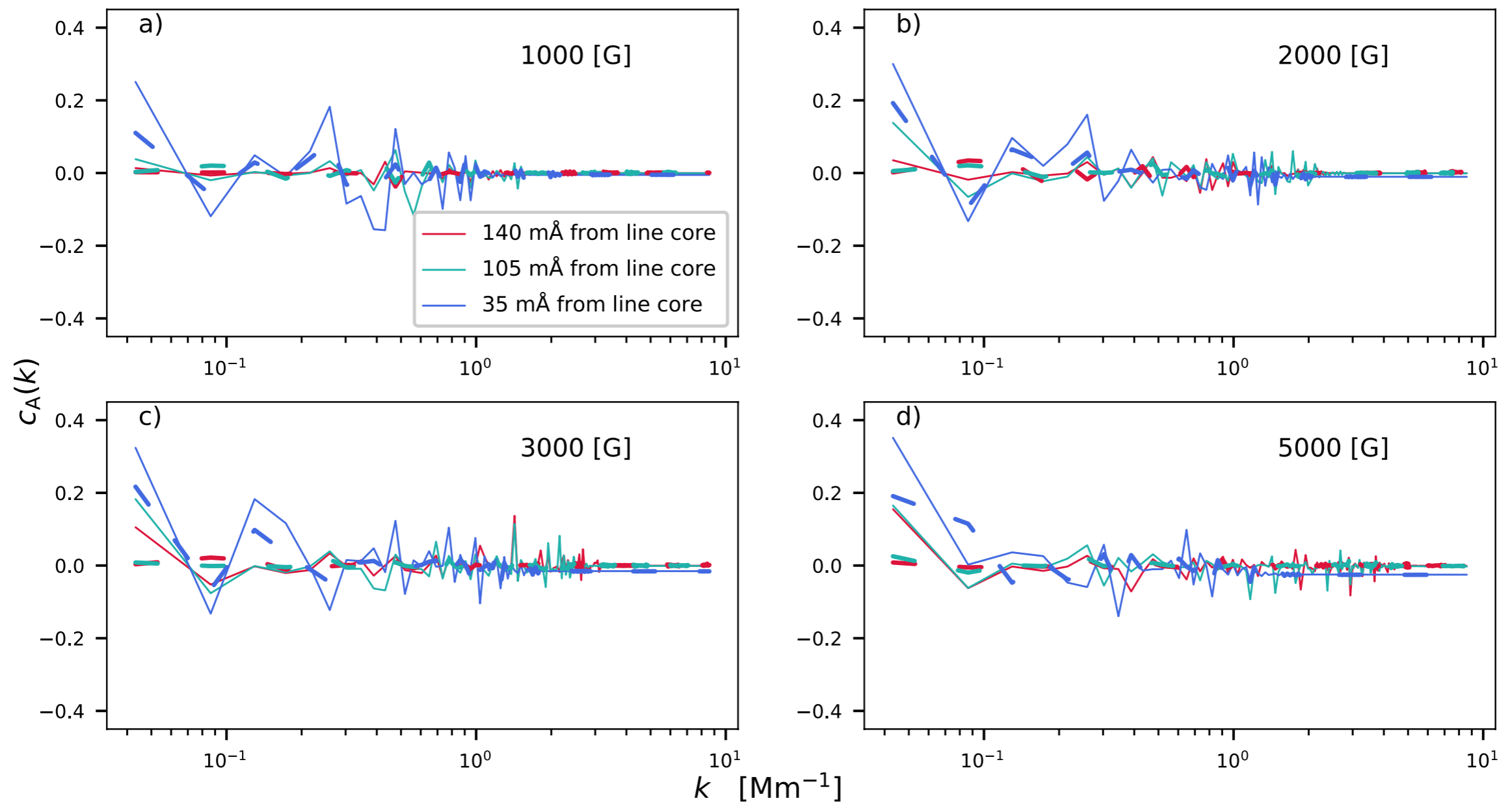


1. Simple model of solar atmosphere
2. Solve the radiative transfer equation for polarised light i.e.

$$\frac{d\mathbf{I}}{d\tau_c} = \mathbf{K}(\mathbf{I} - \mathbf{S})$$

3. Synthesise spectra, compute E and B from Stokes Q and U and check for any significant parity odd correlations arising purely due to Faraday rotation

Contributions of Faraday effect from a non-helical field seem negligible!



Conclusions

- EB decomposition of linear polarisation is a promising **proxy** for magnetic helicity for solar observations
- It can be used to infer the sign of H_m directly from polarisation without having to reconstruct \vec{B}
- A next step is to include the information Stokes V in this approach (if it's even feasible)

Appendix

See Zaldarriaga & Seljak 1997

$$(Q \pm iU)' = e^{\mp 2i\phi} (Q \pm iU)(\hat{n}),$$

$$(Q \pm iU)(\hat{n}) = \sum_{lm} a_{lm}^{\pm 2} {}_{\pm 2}Y_{lm}(\hat{n}),$$

With $a_{lm}^E = -(a_{lm}^2 + a_{lm}^{-2})/2$, $a_{lm}^B = i(a_{lm}^2 - a_{lm}^{-2})/2$, redefine above equation

We work within the confines of the small scale limit i.e.

$$\begin{aligned} {}_2Y_{lm} &= \left[\frac{(l-2)!}{(l+2)!} \right]^{1/2} \delta^2 Y_{lm} \longrightarrow \frac{1}{2\pi} \frac{1}{l^2} \delta^2 e^{ik \cdot x}, \\ {}_{-2}Y_{lm} &= \left[\frac{(l-2)!}{(l+2)!} \right]^{1/2} \bar{\delta}^2 Y_{lm} \longrightarrow \frac{1}{2\pi} \frac{1}{l^2} \bar{\delta}^2 e^{ik \cdot x}, \end{aligned}$$

And invoking the definition of the spin-up and down operators:

$$(\tilde{E} + i\tilde{B}) = (\hat{k}_x - i\hat{k}_y)^2 (\tilde{Q} + i\tilde{U})$$