Non-Boost Invariant Fluid Dynamics

Nordita (Astrophysics seminar), April 29, 2020

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based on:

2004.10759 (de Boer, Have, Hartong, NO, Sybesma)

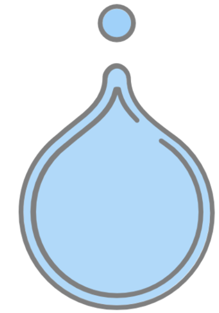
&

1710.04708 (SciPost); 1710.06885 (SciPost)

(de Boer, Hartong, NO, Sybesma, Vandoren)

Introduction

- Hydrodynamics is widely applicable effective description for many physical systems at long length/time scales -> system can relax to approximate thermal equilibrium



→ powerful: universal description at finite T, symmetry principles

symmetries that underlie Navier-Stokes equations:

- time and space translations
- spatial rotations
- ~~boosts~~

$$\text{Galilei boost}: \quad \vec{x}\,' = \vec{x} - \vec{v}\,t\,, \qquad t' = t\,,$$

$$\text{Lorentz boost}: \quad \vec{x}\,' = \gamma(\vec{x} - \vec{v}\,t)\,, \qquad t' = \gamma\left(t - \frac{\vec{v}\cdot\vec{x}}{c^2}\right)\,,$$

- *often*: extra U(1) symmetry (e.g. particle number)

- **topic of this talk:**

perfect fluid description & example (part I)
& 1^{st} order hydrodynamics (part II)
of systems that are not necessarily boost-invariant

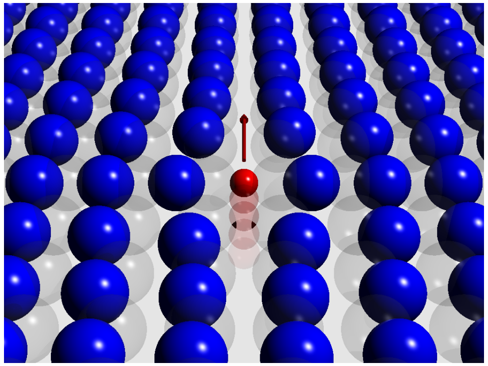
Motivation: why fluids without boost symmetry ?

- many systems in nature in which boost symmetry is broken

bird flocks in air

[e.g. J. Toner, Y. Tu, and S. Ramaswamy 2005]

electron gas moving in lattice of atoms



- existence of medium defines preferred frame:

→ important when interactions between fluid particles and medium cannot be ignored

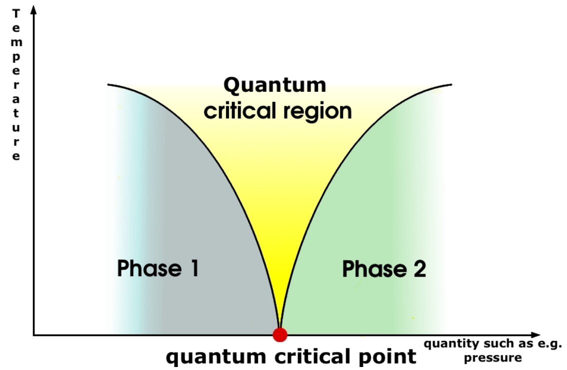
- integrating out dof of medium: can loose symmetries (e.g. Lorentz/Galilean boost) of the fluid particles

Motivation (cont'd)

- Lifshitz fluids (and their dual holographic black brane description)

- in CM: IR effective theories can have non-CFT scaling exponents
  typically such theories have no boost symmetries
  (cf. no-go theorem larer this talk)

$$t \to \lambda^z t \qquad , \qquad x \to \lambda x$$



- near quantum critical points electrons may be strongly coupled → form a fluid

will see already at perfect fluid level: novel expression for speed of sound
also: new transport coefficients that signal boost breaking
→ new observable quantities

- to describe hydro phase of any field theory with scaling z>1 (z not 2) at finite T
  we need to understand non-boost invariant hydro !

Further examples

- non-analytic dispersion relations of:

  - capillary waves — Watanabe,Murayama(2014)
  - domain wall fluctuations in superfuid interfaces (ripplons)

requirements

- EM conservation: weak coupling of excitations to the medium
- hydro regime:

  interaction times/length scales of excitations with themselves << exc. with medium

  Lucas,Fong(2017) **(electrons in graphene)**

Nicolis,,Penco,Piazza, Rattazzi (2015)

- EFT pov: integrate out dofs of the medium in state that breaks boost symmetry (simplest possibility: type I framid (requires E+P=0))

- superfluid with spontaneously broken U(1) symmetry

$$\int d^{d+1}x A_\mu J^\mu$$

$$A_\mu = \lambda \delta^0_\mu.$$

$$T^{\rm new}_{\mu\nu} \sim T^{\rm old}_{\mu\nu} + J_\mu A_\nu$$ not symmetric

Main results

- crucial ingredient in thermodynamics formulation:
  extra term in 1st law of thermodynamics (kinetic mass density – velocity)

- derivation of novel expressions for speed of sound (and attenuation)

- new 1st order transport coefficients (as compared to Lorentz/Galilean case):

  -10 dissipative, 2 hydrostatic non-dissipative, 4 nonhydrostatic non-dissipative

  -for Lifshitz scaling: 7 – 1 – 2

- powerful technical tool:
  use appropriate curved space for non-boost invariant systems
  - absolute spacetime (aka Aristotelian geometry)

Outline

- Perfect fluids
  - **extra thermodynamic quantity: kinetic mass density**
  - **most general stress tensor**
  - **corrections to Euler equation**
  - **new expressions for speed of sound**
  - brief illustration: Ideal gas of Lifshitz particles

- 1^{st} order hydro
- - **curved space formulation**
  - outline of the method (entropy current, hydrostatic PF)
  - main results (**constitutive relations and positivity of entropy current**)
  - effects on **hydrodynamic modes** (new (non)-dissipative effects)
  - examples of **effect on sound, shear, diffusion**

- Outlook

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Thermodynamics

- consider grand canonical ensemble with partition function

$$\mathcal{Z}(T, V, \mu, v_i) = \mathrm{Tr}\left[e^{-\beta\left(\hat{H}-\mu\hat{N}-v_i\hat{P}_i\right)}\right]$$

temperature T, volume V, chemical potentials: mu **and velocity v**

- grand canonical potential $\Omega(T, V, \mu, v_i) = -\frac{1}{\beta}\log\mathcal{Z}$

$\Omega = -PV.$ $$d\Omega = -SdT - PdV - P_i dv_i - Nd\mu$$

P pressure, s entropy, P_i momentum, N charge/# particles

- express in terms of densities

→ thermodynamic identities

momentum density

- total energy density
- 1st law

$$\mathcal{E} = Ts - P + v^i\mathcal{P}_i + \mu n\,,$$
$$d\mathcal{E} = Tds + v^i d\mathcal{P}_i + \mu dn\,.$$

internal energy: $\tilde{\mathcal{E}} = \mathcal{E} - \rho v^2$

Kinetic mass density

There is only one vector v^i so momentum density:

$$\mathcal{P}_i = \rho v^i$$

ρ is (in general) new thermodynamic quantity:
``kinetic mass density”
(expresses relation between momentum and velocity)

1st law:

$$\mathrm{d}P = s\mathrm{d}T + n\mathrm{d}\mu + \frac{1}{2}\rho\mathrm{d}v^2 \qquad P(T,\mu,v^2),$$

ρ can be computed e.g. as: $\rho(T,\mu,v^2) = 2\left(\frac{\partial P}{\partial v^2}\right)_{T,\mu}$

- reduces to known quantities when system has boost symmetry:

Lorentz (relativistic) $\rho = \mathcal{E} + P$ (enthalpy)

Bargmann (non-relativistic) $\rho = mn$ (particle mass density)

Energy-Momentum tensor and charge current

underlying microscopic theory is assumed to have at least symmetries:

$$H, P_i, J_{ij}, Q$$

→ there is conserved energy-momentum tensor and conserved current with associated conserved charges (enough for the effective fluid theory !)

$T^\mu{}_\nu, J^\mu$ spacetime tensors/transform in rep of symmetry algebra (if more symmetries, e.g. boosts, then larger algebra)

- perfect fluid in LAB (or rest) frame

energy density — momentum density — charge density

$$T^\mu{}_\nu = \begin{pmatrix} -\mathcal{E} & \rho v_j \\ -(\mathcal{E}+P)v^i & P\delta^i{}_j + \rho v^i v_j \end{pmatrix} \qquad J^\mu = (n, nv^i)$$

energy flux — pressure + momentum flow — charge flux

Lorentz (relativistic) $\quad T^i{}_0 = -T^0{}_i$

Bargmann (non-relativistic) $\quad T^0{}_j = mJ_j$

Entropy current and (modified) Euler equation

Conservation of energy-momentum/particle current

- particle number conservation $\partial_t n + \partial_i \left(n v^i\right) = 0$

- entropy current $\partial_t s + \partial_i \left(s v^i\right) = 0$

- Euler equation of homogeneous and isotropic fluids gets an extra term

$$\partial_0 \vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} P - \frac{\vec{v}}{\rho}\left[\partial_0 \rho + \partial_i(\rho v^i)\right]$$

Galilei fluid: extra term vanishes due to particle number conservation

relativistic fluid: correct extra term $-\frac{1-v^2}{\tilde{\mathcal{E}}+P}\left(\vec{\nabla} P + \vec{v}\,\partial_t P\right)$

Speed of sound

standard LL formula for speed of sound does not hold anymore

- fluctuation analysis of conservation equations

(around background with zero velocity for simplicity): novel sound speed

$$v_s^2 = \frac{n_0}{\rho_0}\left(\frac{\partial P_0}{\partial n_0}\right)_{\frac{s_0}{n_0}}$$

generalizes non-relativstic $v_s^2 = \left(\frac{\partial P_0}{\partial \rho_0}\right)_{\frac{s_0}{n_0}}$ when $\rho = mn$

without U(1) current $v_s^2 = \frac{\tilde{\mathcal{E}}_0 + P_0}{\rho_0}\left(\frac{\partial P_0}{\partial \tilde{\mathcal{E}}_0}\right)$ generalizes relativistic $\left(\frac{\partial P_0}{\partial \tilde{\mathcal{E}}_0}\right)$

→ new formula for Lifshitz perfect fluid:

$$v_s^2 = \frac{z}{d}\frac{\tilde{\mathcal{E}}_0 + P_0}{\rho_0}$$

scale Ward identity $t \to \lambda^z t$ and $x^i \to \lambda x^i$

- more complicated expressions around background with non-zero v (reproducing correct transformations for boost-inv. cases)

Ideal gas of Lifshitz particles

gas of N identical free Lifshitz particles with single-particle Hamiltoniana;

$$H_1 = \lambda \left(\vec{p}^2\right)^{\frac{z}{2}}$$

$$z = 1: \quad \lambda = c$$

$$z = 2: \quad \lambda = \frac{1}{2m}$$

- no boost invariance for z not equal to 1,2

- momentum as function of velocity $$\vec{k} = \left(\frac{1}{\lambda z}\right)^{\frac{1}{z-1}} \frac{1}{(\vec{v}^2)^{\frac{z-2}{2(z-1)}}} \vec{v}$$

- sound modes $\omega = v_s k$. $$v_s = \#\,(k_B T)^{\frac{z-1}{z}} \lambda^{\frac{1}{z}}$$ (from scaling analysis)

-contrast with dispersion relation of Lif particle: $$\omega = c_z\, k^z ,$$

$$k \to \alpha k\,, \quad \omega \to \alpha^z \omega$$

Boltzmann gas

partition function:

$$Z(N,T,V,\vec{v}\,) = \frac{1}{N!}\left[Z_1(T,V,\vec{v}\,)\right]^N$$

$$Z_1(T,V,\vec{v}\,) = \frac{V}{h^d}\int d^d\vec{p}\, e^{-\beta H_1 - \beta\vec{v}\cdot\vec{p}} \qquad \beta = \frac{1}{k_B T}$$

$$Z_1(T,V,\vec{v}\,) = \frac{2V}{z}\left(\frac{\sqrt{\pi}}{h}\right)^d (\lambda\beta)^{-\frac{d}{z}} \sum_{n=0}^{\infty} \frac{\left(-\frac{\beta v}{2}\right)^{2n}}{n!} \frac{\Gamma\left[\frac{d+2n}{z}\right]}{\Gamma\left[\frac{d+2n}{2}\right]} (\lambda\beta)^{-\frac{2n}{z}}$$

- approximation valid when: $\lambda_{th} \ll \left(\frac{V}{N}\right)^{\frac{1}{d}}$, $\quad \lambda_{th}^{-d} \equiv \frac{Z_1}{V} = \frac{2}{z}\left(\frac{\sqrt{\pi}}{h}\right)^d \frac{\Gamma\left[\frac{d}{z}\right]}{\Gamma\left[\frac{d}{2}\right]}\left(\frac{k_B T}{\lambda}\right)^{\frac{d}{z}}$

- grand canonical partition function:

$$\mathcal{Z}(\mu,T,V,\vec{v}\,) = \sum_{N=0}^{\infty} e^{\beta\mu N} Z(N,T,V,\vec{v}\,) = \sum_{N=0}^{\infty} \frac{1}{N!}\left(e^{\beta\mu} Z_1(T,V,\vec{v}\,)\right)^N$$

Thermodynamics

at zero velocity (see also Yan(2000)):

- ideal gas law $$PV = Nk_BT$$
- equipartition: $$U_0 \equiv \langle \tilde{E}_0 \rangle = -\frac{\partial \ln Z}{\partial \beta} = \frac{d}{z} N k_B T$$
- heat capacities $$\hat{C}_V \equiv \frac{C_V}{Nk_B} = \frac{d}{z} \qquad \hat{C}_P \equiv \frac{C_P}{Nk_B} = \frac{d}{z} + 1$$
- adiabatic expansion $$PV^{\gamma} = constant \qquad \gamma \equiv \frac{C_P}{C_V} = 1 + \frac{z}{d}$$
- mass/particle $$\rho = \rho_0 \left(1 + \frac{1}{2} \frac{(d+z)}{(d+2)} \frac{\Gamma\left[\frac{d}{z}\right] \Gamma\left[\frac{d+4}{z}\right]}{\Gamma\left[\frac{d+2}{z}\right]^2} \frac{v^2}{v_s^2} + \cdots \right)$$
- speed of sound $$\boxed{c_s^2 = (d+z) \frac{\Gamma\left[\frac{d}{z}\right]}{\Gamma\left[\frac{d+2}{z}\right]} (k_B T)^{2\left(\frac{z-1}{z}\right)} \lambda^{\frac{2}{z}}} \qquad c_s^2 = \gamma \frac{P}{\rho_0}$$

$$z = 2: \quad c_s^2 = \frac{d+2}{d} \frac{k_B T}{m} \qquad\qquad z = 1: \quad c_s^2 = \frac{c^2}{d^2}$$

1st order Hydro: prescription

derivative expansion around local thermal equilibrium
- focus on small fluctuations: 1st order in derivatives

- hydrodynamic frame choice: specify choice local fluid variables: temperature, velocity
- general constitutive relations for conserved currents and entropy current
- positivity of entropy production (restrictions on free functions in const. rel.)

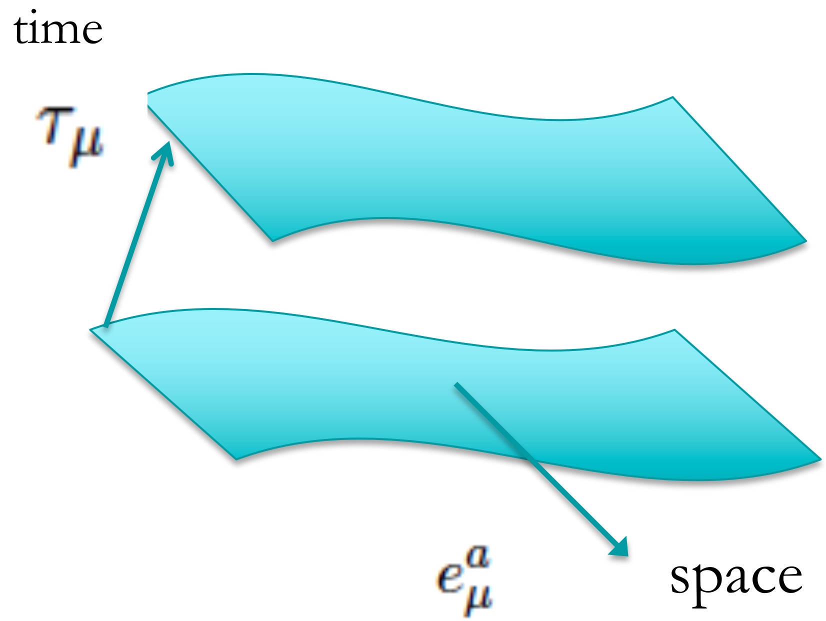
→ allowed transport coefficients

- subsequently examine: effect on dispersion relations of hydrodynamic modes

highly beneficial tools: curved space
& hydrostatic partition function/Lagrangian formulation: non-dissipative transport

Curved geometry for non-boost invariant fluids

non-boost invariant systems live on the geometry of absolute spacetime (aka Aristotelian spacetime)



- clock form τ_μ

- spatial metric $h_{\mu\nu} = \delta_{ab} e^a_\mu e^b_\nu$

$$\nabla_\mu \tau_\nu = 0 \,, \qquad \nabla_\mu h_{\nu\rho} = 0 \,.$$

- useful quantities:

- torsion tensor: $\tau_{\mu\nu} = \partial_\mu \tau_\nu - \partial_\nu \tau_\mu$

- extrinsic curvature $K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_v h_{\mu\nu}$

Geometry and hydrostatic partition function

stationary curved background $\mathcal{M}_S$
time-translations symmetry generated by H

$$\mathcal{Z} = \mathrm{Tr}\left[\mathrm{e}^{-H/T}\right]$$

for weakly curved background → hydrostatic partition funtion
(or equilibrium partition function)

time-translation of background
generated by Killing vector β^μ

$$\mathcal{L}_\beta \tau_\mu = 0\,,$$
$$\mathcal{L}_\beta h_{\mu\nu} = 0\,.$$

→ gives local temperature and local velocity:

$$T = 1/(\tau_\mu \beta^\mu)$$
$$u^\mu = T\beta^\mu\,, \qquad u^\mu \tau_\mu = 1$$

(analytically continue time)

$$S_{\mathrm{HPF}} = -i \log \mathcal{Z}$$

derivative expansion:

$$S_{\mathrm{HPF}} = \sum_n S_{\mathrm{HPF}}^{(n)}\,,$$

$$\delta_\xi S_{\mathrm{HS}} = \int_{\mathcal{M}} d^{d+1}x\; e\left(-T^\mu \delta_\xi \tau_\mu + \frac{1}{2} T^{\mu\nu} \delta_\xi h_{\mu\nu} + F_\mu \delta_\xi \beta^\mu\right)$$

Geometry and equilibrium partition function

- for background with time symmetry: Killing vector β^μ

$$T = 1/(\tau_\mu \beta^\mu)$$
$$u^\mu = T\beta^\mu \,,$$

- on flat spacetime: $u^\mu = (1, v^i)$;

can build two scalars (at 0th order): T $\quad u^2 = h_{\nu\rho} u^\nu u^\rho.$ $\quad (= v^2$ on flat spacetime)

→ hydrostatic partition function:

$$S_{(0)} = \int_{\mathcal{M}} d^{d+1}x \, eP(T, u^2) \,,$$

$$\delta S = \int \mathrm{d}^{d+1}x e \left(-T^\mu \delta\tau_\mu + \frac{1}{2} T^{\mu\nu} \delta h_{\mu\nu} \right) \qquad T^\mu{}_\nu = -T^\mu \tau_\nu + T^{\mu\rho} h_{\rho\nu} \,,$$

-gives covariant EM tensor:

$$T^\mu{}_\nu = -(\mathcal{E} + P) u^\mu \tau_\nu + P\delta^\mu_\nu + \rho u^\mu u^\rho h_{\rho\nu}$$

-EM conservation from diffeomorphism invariance

$$e^{-1}\partial_\mu \left(eT^\mu{}_\rho \right) + T^\mu \partial_\rho \tau_\mu - \frac{1}{2} T^{\mu\nu} \partial_\rho h_{\mu\nu} = 0 \,.$$

Entropy current

- 2nd law of thermo: $e^{-1}\partial_\mu\left(eS^\mu\right) \geq 0$.

entropy current has canonical and non-canonical part: $S^\mu = S^\mu_{\text{can}} + S^\mu_{\text{non}}$

$$s = \frac{1}{T}\tilde{\mathcal{E}} + \frac{1}{T}P \quad \rightarrow \quad S^\mu_{\text{can}} = -T^\mu{}_\nu\beta^\nu + P\beta^\mu$$

divergence takes form:

$$e^{-1}\partial_\mu\left(eS^\mu\right) = \left(T^\mu - T^\mu_{(0)}\right)\mathcal{L}_\beta\tau_\mu - \frac{1}{2}\left(T^{\mu\nu} - T^{\mu\nu}_{(0)}\right)\mathcal{L}_\beta h_{\mu\nu} + e^{-1}\partial_\mu\left(eS^\mu_{\text{non}}\right)$$

$$T^\mu - T^\mu_{(0)} = T^\mu_{\text{D}} + T^\mu_{\text{HS}} + T^\mu_{\text{NHS}} \,,$$
$$T^{\mu\nu} - T^{\mu\nu}_{(0)} = T^{\mu\nu}_{\text{D}} + T^{\mu\nu}_{\text{HS}} + T^{\mu\nu}_{\text{NHS}} \,.$$

split corrections to perfect fluid:

- dissipative
- hydrostyatic non-dissipative
- non-hydrostatic non-dissipative

Properties of the 3 parts

- dissipative produces entropy:

$$e^{-1}\partial_\mu\left(eS^\mu\right) = T^\mu_{\rm D}\mathcal{L}_\beta\tau_\mu - \frac{1}{2}T^{\mu\nu}_{\rm D}\mathcal{L}_\beta h_{\mu\nu} \geq 0 \ ,$$

- non-hydrostatic non-dissipative does not contribute to the divergence:

$$T^\mu_{\rm NHS}\mathcal{L}_\beta\tau_\mu - \frac{1}{2}T^{\mu\nu}_{\rm NHS}\mathcal{L}_\beta h_{\mu\nu} = 0 \ .$$

- hydrostatic non-dissipative cancels divergence of non-canonical part

$$e^{-1}\partial_\mu\left(eS^\mu_{\rm non}\right) = -T^\mu_{\rm HS}\mathcal{L}_\beta\tau_\mu + \frac{1}{2}T^{\mu\nu}_{\rm HS}\mathcal{L}_\beta h_{\mu\nu}$$

Hydrostatic non-dissipative contributions

1st order terms in HPF: 2 possible terms

$$S_{(1)} = \int d^{d+1}xe\left(F_1(T,u^2)v^\mu\partial_\mu T + F_2(T,u^2)v^\mu\partial_\mu u^2\right)$$

need to convert to Landau frame $T^\mu{}_\nu u^\nu = -\tilde{\mathcal{E}}u^\mu\,,$

resulting EM tensor takes form:

$$T^{\mu\nu}_{(1)\mathrm{HS}} = \frac{1}{2}\eta^{\mu\nu\alpha\beta}_{\mathrm{HS}}\left(\mathcal{L}_\beta h_{\alpha\beta} - u_\alpha\mathcal{L}_\beta\tau_\beta - u_\beta\mathcal{L}_\beta\tau_\alpha\right) + \frac{1}{2}\eta^{\mu\nu\alpha\beta}_{\mathrm{tor}}\tau_{\alpha\beta} + \frac{1}{2}\eta^{\mu\nu\alpha\beta}_{\mathrm{rot}}\omega_{\alpha\beta} + \frac{1}{2}\eta^{\mu\nu\alpha\beta}_{\mathrm{ext}}K_{\alpha\beta}\,,$$

$$\omega_{\rho\mu} = \partial_\rho u_\mu - \partial_\mu u_\rho$$

→ 2 hydrostatic non-dissipative transport coefficients

- can solve for the non-canonical part of entropy current

Non-hydrostatic non-dissipative

β^μ not necessarily Killing vector anymore:

- more possible terms in Lagrangian:

$$S_{\mathrm{NHS}} = \int \mathrm{d}^{d+1}xe \left(F_3 u^\mu \partial_\mu T + F_4 u^\mu \partial_\mu u^2 - T F_5 v^\mu \mathcal{L}_\beta \tau_\mu - 2T F_6 u^\mu v^\nu \mathcal{L}_\beta h_{\mu\nu} \right)$$

→ 4 non-hydrostatic non-dissipative transport coefficients

Non-hydrostatic part (dissipation)

contributing to divergence of entropy current can be written as

$$e^{-1}\partial_\mu\left(eS^\mu\right) = -\frac{1}{2T}\left(T^{\mu\nu}_{(1)} - T^{\mu\nu}_{(1)\mathrm{HS}}\right)\left(\mathcal{L}_u h_{\mu\nu} - h_{\rho\nu}u^\rho\mathcal{L}_u\tau_\mu - h_{\mu\rho}u^\rho\mathcal{L}_u\tau_\nu\right),$$

fluid variables

constitutive relations for non-hydrostatic part of EM tensor:

$$T^{\mu\nu}_{(1)} - T^{\mu\nu}_{(1)\mathrm{HS}} = \frac{1}{2}\eta^{\mu\nu\rho\sigma}\left(\mathcal{L}_u h_{\rho\sigma} - h_{\kappa\sigma}u^\kappa\mathcal{L}_u\tau_\rho - h_{\rho\kappa}u^\kappa\mathcal{L}_u\tau_\sigma\right) + \zeta^{\mu\nu\rho}\mathcal{L}_u\tau_\rho\,.$$

positivity of entropy production: $\zeta^{\mu\nu\rho}$ is zero

= eta-tensor can be obtainted by decomposing into SO(d-1) invariant tensors

- symmetric part is dissipative
- antisymmetric part is (non-hydrostatic) non-dissipative

Results in flat space

$$(T_{(1)\mathrm{D,NHS}})^0{}_j = \frac{1}{2}\eta_{jkl}\left(\partial_k v^l + \partial_l v^k\right) + \kappa_{jk}\partial_t v^k\,,$$

$$(T_{(1)\mathrm{D,NHS}})^i{}_j = \frac{1}{2}\eta^{ijkl}\left(\partial_k v^l + \partial_l v^k\right) + \kappa^{ijk}\partial_t v^k\,,$$

$$\kappa_{jk} = \frac{f_1}{v^2}P_{jk} + \frac{s_1}{v^2}n_j n_k\,,$$

$$\kappa^{ijk} = \frac{f_3}{\sqrt{v^2}}\left(P^{jk}n^i + P^{ik}n^j\right) + \frac{1}{\sqrt{v^2}}s_5 P^{ij}n^k + \frac{1}{\sqrt{v^2}}s_4 n^i n^j n^k\,,$$

$$\eta^{ijkl} = \mathfrak{t}\left(P^{ik}P^{jl} + P^{il}P^{jk} - \frac{2}{d-1}P^{ij}P^{kl}\right) + f_2\left(P^{ik}n^j n^l + P^{jk}n^i n^l + P^{il}n^j n^k + P^{jl}n^i n^k\right) + s_3 P^{ij}P^{kl} + s_6\left(P^{kl}n^i n^j + P^{ij}n^k n^l\right) + s_2 n^i n^j n^k n^l\,,$$

$$n^i = \frac{v^i}{\sqrt{v^2}}, \qquad P^{ij} = \delta^i_j - \frac{v^i v^j}{v^2} = \delta^i_j - n^i n^j$$

10 dissipative transport coefficients.

& certain conditions to make divergence of S quadratic form

(f1 identified also in: Hoyos,Kim,Oz(2013))

Hydrodynamic modes (linearized around v=0)

new

$$T^{0}_{(1)j} = -\varpi\partial_t v^j + \dots\,,$$

$$T^{i}_{(1)j} = -\zeta\delta_{ij}\partial_k v^k - \eta(\partial_i v_j + \partial_j v_i - \frac{2}{d}\delta^i_j\partial_k v^k) + \dots\,,$$

compute (generalized) Navier-Stokes equations and consider linearized perturbations

$$\omega_{\text{shear}} = -i\frac{\eta_0}{\rho_0}k^2\,, \qquad \text{with multiplicity } d-1\,,$$

$$\omega_{\text{sound}} = \pm v_s k - i\Gamma k^2\,, \qquad \text{with multiplicity } 2\,,$$

sound attenuation

$$\Gamma = \frac{1}{2\rho_0 v_s^2}\left[\left[\bar{\zeta}_0 + \frac{2}{d}(d-1)\eta_0\right]v_s^2 + \bar{\pi}_0 v_s^4\right]$$

Lifshitz fluid

$$zT^0{}_0 + T^i{}_i = 0\,.$$

$$= \frac{1}{d}(d-1)\frac{\eta_0}{\rho_0} + \frac{z}{2d}\frac{\tilde{\mathcal{E}}_0 + P_0}{\rho_0^2}\bar{\pi}_0$$

Outlook

- Kubo formulae:
  relate individual transport coefficients to particular linear respons

$$\eta \sim \lim_{\omega \to 0} \frac{1}{\omega}(\langle TT \rangle - \langle TT \rangle_{\text{leading}}).$$

- hydrodynamic modes around non-zero velocity configurations
- stability of hydrodynamic spectrum at 1st order in curved spacetime
- include U(1) charge current (see also Novak, Sonner, Withers(2019))

- fluid/gravity correspondence/ holographic computation of transport /universal behaviour

- other aspects for non-boost hydro:
  momentum dissipation, turbulence, shock waves, surface phenomena, non-boost inv. fluids on surfaces

- experimental consequences/inventory of type of systems
- corrections to Euler/Navier-Stokes involve kinetic mass density: determine velocity profile using measurements ?
- applications to astrophysics and cosmology

The end