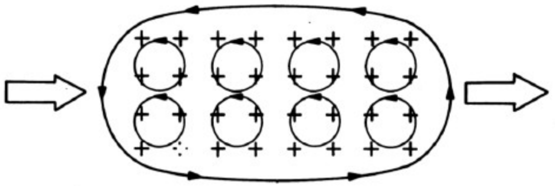
Lúcia D. V. Duarte, Johannes Wicht, Matthew K. Browning, Thomas Gastine

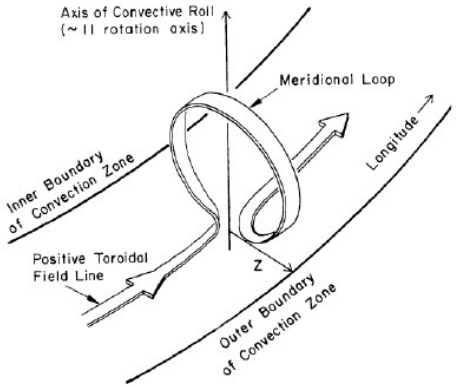
Motivation – the Parker cycle

According to the Parker cycle model, the direction of propagation of the dynamo wave may be controlled by one of these two properties of the flow:

- sign of the differential rotation, which determines the sign of the toroidal field generated by the shear from the poloidal field (Ω-effect)



- sign of kinetic helicity, which determines the sign of the poloidal field generated from the toroidal field (α-effect)



MHD Days 2016 – *Helicity inversion in spherical convection as a means for equatorward dynamo wave propagation*

Lúcia D. V. Duarte, Johannes Wicht, Matthew K. Browning, Thomas Gastine

MHD equations

The dimensionless equations are:

$$E\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla\frac{p}{\tilde\rho} - 2\mathbf{e}_z\times\mathbf{u} + \frac{Ra\,E}{Pr}\frac{r}{r_o}s\,\mathbf{e}_r + \frac{1}{Pm_i\,\tilde\rho}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{E}{\tilde\rho}\nabla\cdot\mathsf{S}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) - \frac{1}{Pm_i}\nabla\times(\tilde\lambda\nabla\times\mathbf{B})$$

$$\tilde\rho\,\tilde T\left(\frac{\partial s}{\partial t} + \mathbf{u}\cdot\nabla s\right) = \frac{1}{Pr}\nabla\cdot(\tilde\rho\tilde T\nabla s) + \epsilon\tilde\rho + \frac{Pr\,Di}{Ra}\left[Q_\nu + \frac{1}{Pm_i^2\,E}Q_j\right]$$

$$\nabla\cdot(\tilde\rho\mathbf{u}) = 0$$

$$\nabla\cdot\mathbf{B} = 0$$

traceless rate-of-strain tensor:

$$\mathsf{S} = 2\tilde\rho\left[\mathsf{e}_{ij} - \frac{1}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right], \quad \mathsf{e}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

viscous and ohmic heating contributions:

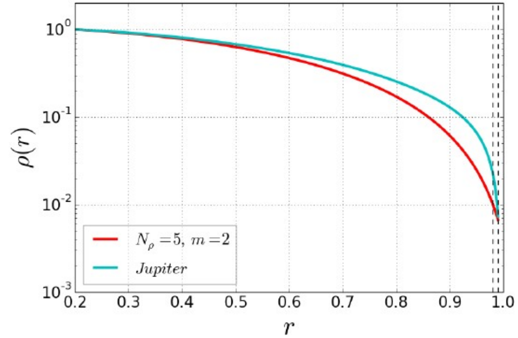
$$Q_\nu = 2\tilde\rho\left[\mathsf{e}_{ij}\mathsf{e}_{ji} - \frac{1}{3}(\nabla\cdot\mathbf{u})^2\right]$$

$$Q_j = \tilde\lambda(\nabla\times\mathbf{B})^2$$

Set-up

Thick shell models with:

- aspect ratio $r_{bot}/r_{top} = 0.2$
- density contrast $N_\rho = \rho_{bot}/\rho_{top} \approx 5$



MagIC code: https://github.com/magic-sph/magic

Jones, C. A., 2014, A dynamo model of Jupiter's magnetic field. Icarus. 241, 148-159.

Main parameter changes between models:

- Prandtl number: 0.1 or 1
- Heating mode: bottom heating with fixed entropy at the boundaries
  or
  volumetric internal heating with fixed entropy flux at the boundaries

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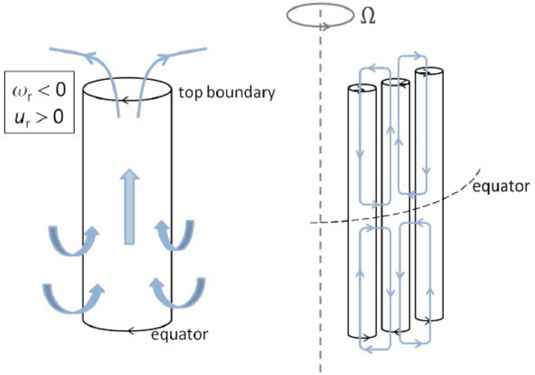
Standard kinetic helicity pattern – Two possible scenarios I

- Columnar convection: dominated by geostrophic convection with an additional secondary axial flow along the columns



- Example: the Geodynamo

Olson, P., Christensen, U., Glatzmaier, G. A., 1999, Numerical modeling of the geodynamo: Mechanisms of field generation and equilibration. J. Geophys. Res. 104, 10383-10404.

Northern Hemisphere
– **negative** kinetic helicity

Southern hemiphere
– **positive** kinetic helicity

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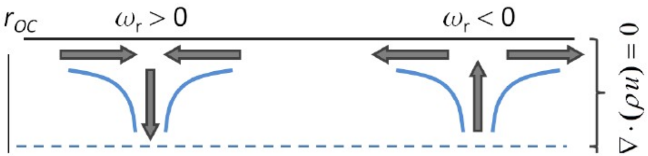
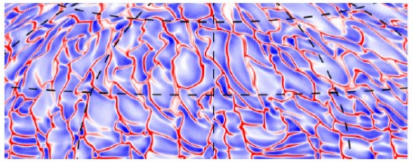
Standard kinetic helicity pattern – Two possible scenarios II

- Plume-like convection: strong density gradient → convecting cells expand as they rise (compress as they sink)



- Example: stellar surfaces



radial velocity (blue outward, red inward)
Yadav et al. (2014)

Gilman, P. A., Glatzmaier, G. A., 1984, Compressible convection in a rotating spherical shell. I - Anelastic equations. II - A linear anelastic model. III - Analytic model for compressible vorticity waves. ApJS. 55, 461-484.

Northern Hemisphere
– **negative** kinetic helicity

Southern hemiphere
– **positive** kinetic helicity

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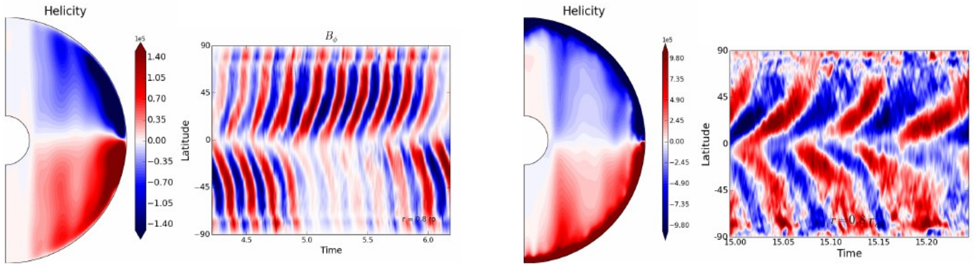
Standard kinetic helicity pattern in numerical models

a) Standard helicity pattern



Northern Hemisphere
– **negative** kinetic helicity

Southern hemiphere
– **positive** kinetic helicity

Typical result:

wave propagating
in **poleward** direction

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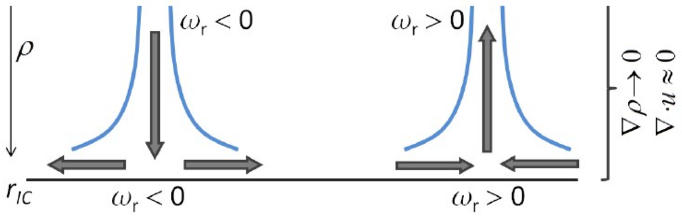
Inverted helicity pattern



- weak or no density gradient → rising+accelerating convecting cells contract (mass conservation)

Northern Hemisphere
– **positive** kinetic helicity

Southern hemiphere
– **negative** kinetic helicity

MHD Days 2016 – *Helicity inversion in spherical convection as a means for equatorward dynamo wave propagation*

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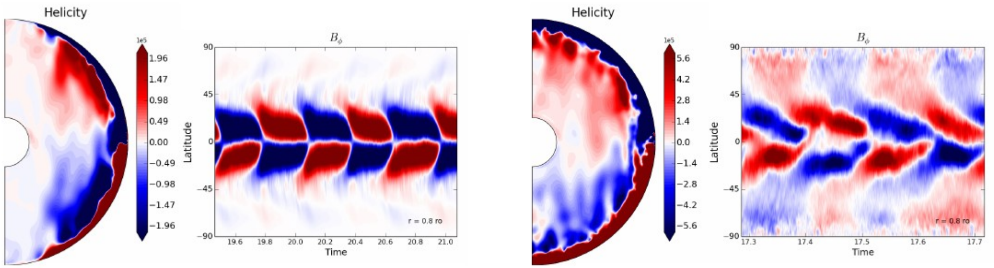
Helicity in numerical models - invered helicity pattern

a) Inverted helicity pattern



Northern Hemisphere
– **positive** kinetic helicity

Southern hemiphere
– **negative** kinetic helicity

Result:

wave propagating
in **equatorward** direction

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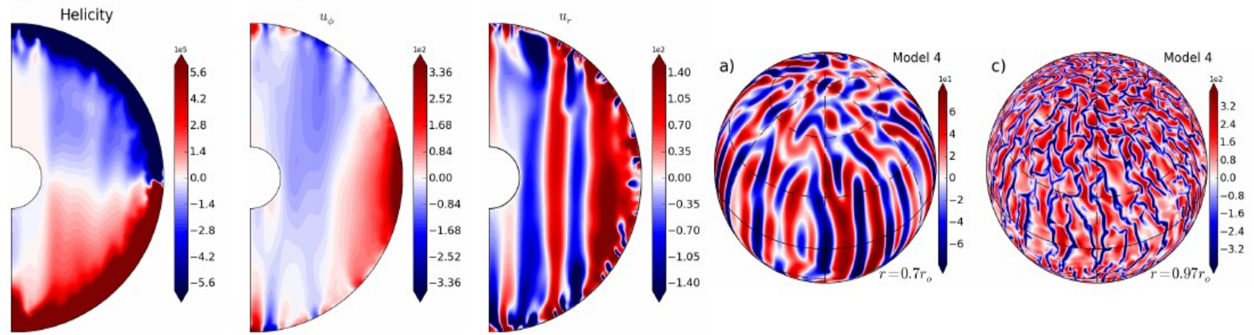
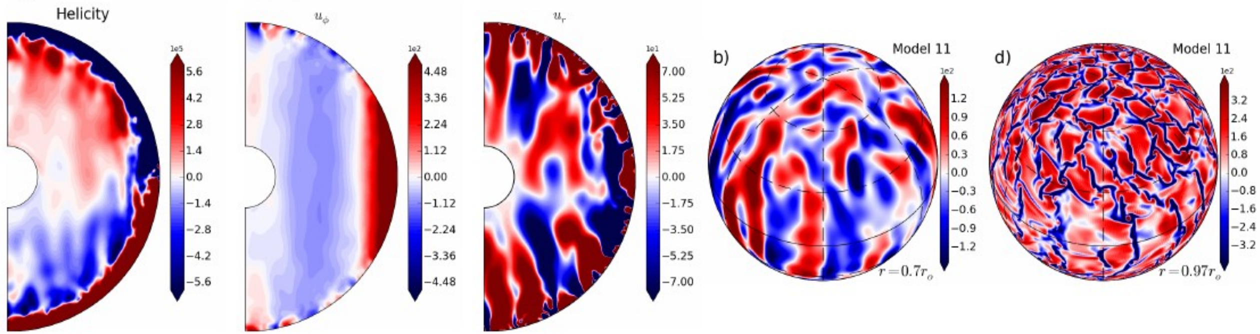
Flow components

a) Standard helicity pattern $r = 0.7r_o$ $r = 0.97r_o$



b) Inverted helicity pattern $r = 0.7r_o$ $r = 0.97r_o$



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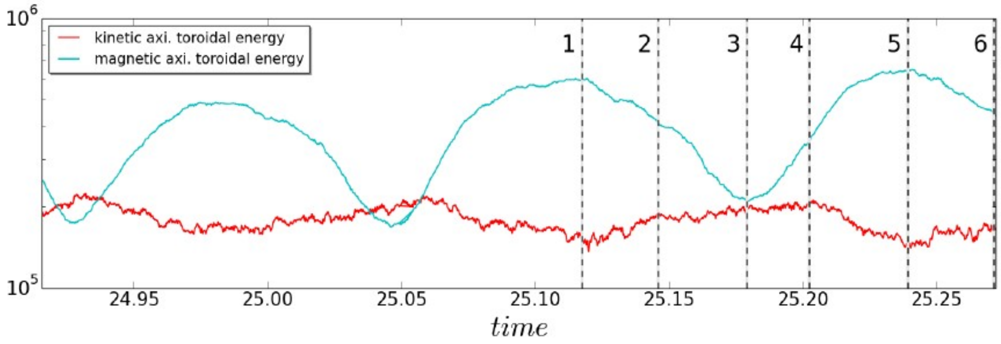
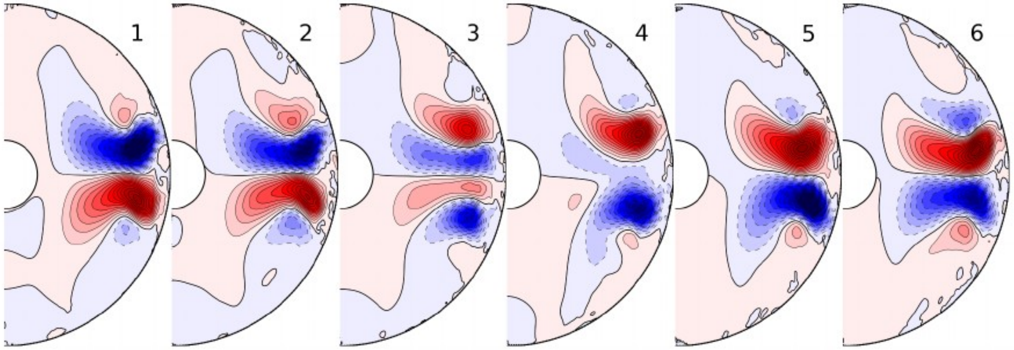
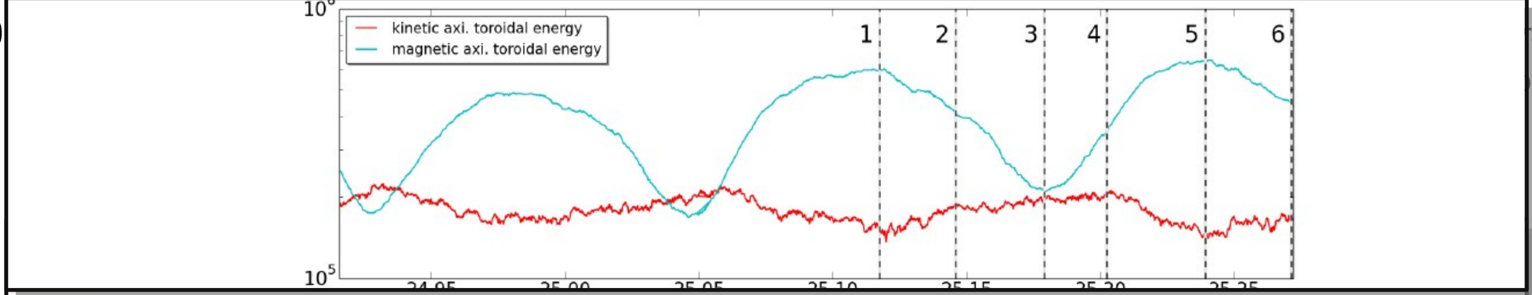
Cycles

- Axisymmetric toroidal energies

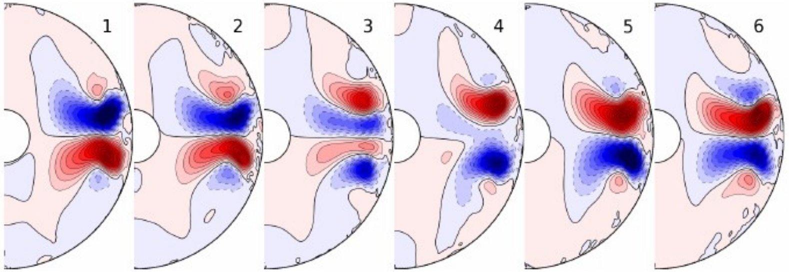


- Toroidal field $\overline{\mathbf{B}}_\phi$

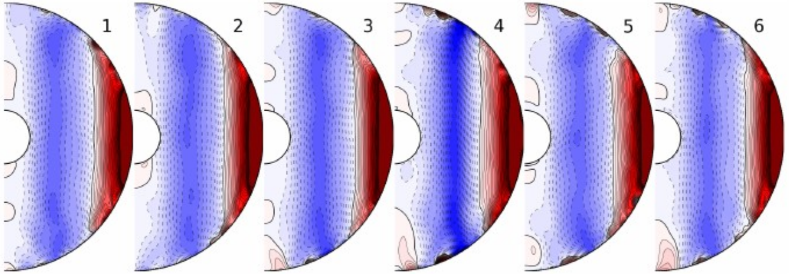




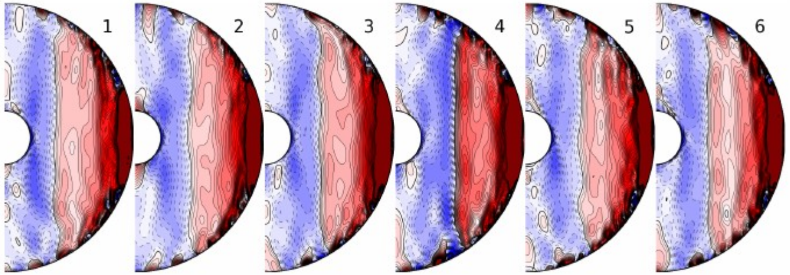
- $\overline{\mathbf{B}}_\phi$



- $\overline{\mathbf{v}}_\phi$



- $DR = \frac{\partial \overline{\mathbf{v}}_\phi}{\partial s}$



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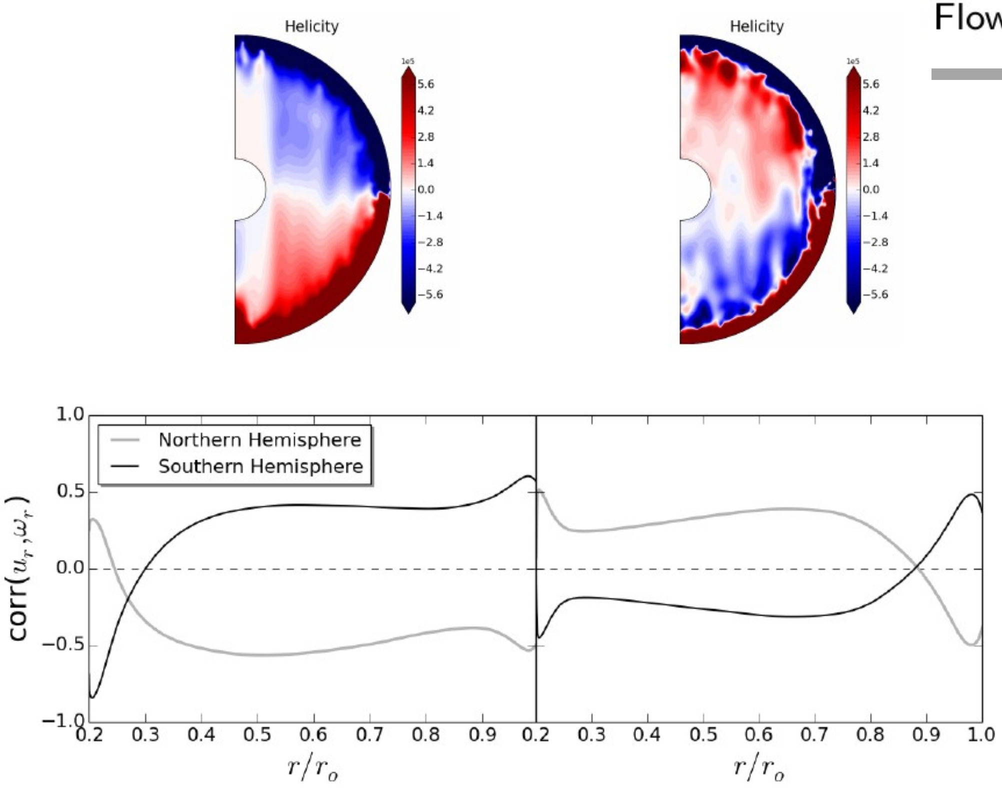
Correlations: rising/sinking velocity *vs.* flow vorticity

Rising (positive u_r) / sinking (negative u_r) velocity

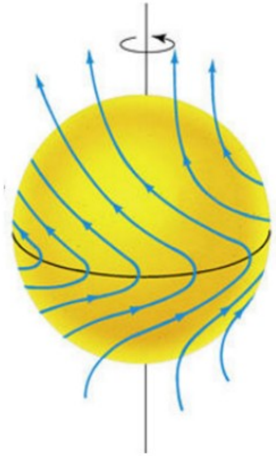
Flow vorticity: $\omega_r = (\nabla \times u) \cdot \hat{\mathbf{r}}$

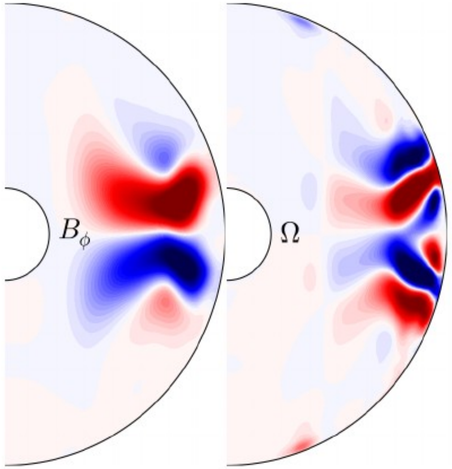


The omega effect

- Induction equation for the mean field expanded:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \overline{\nabla \times (\mathbf{u} \times \mathbf{B} - \lambda \nabla \times \mathbf{B})}$$
$$= \nabla \times (\overline{\mathbf{u}} \times \overline{\mathbf{B}} - \lambda \nabla \times \overline{\mathbf{B}}) + \overline{\nabla \times (\mathbf{u}' \times \mathbf{B}')}$$
$$= \underbrace{\overline{\mathbf{B}} \cdot \nabla \overline{\mathbf{u}}}_{\text{mean shear term} \equiv \Omega} + \overline{\mathbf{B}' \cdot \nabla \mathbf{u}'} + \overline{\mathbf{u}} \cdot \nabla \overline{\mathbf{B}} + \overline{\mathbf{u}' \cdot \nabla \mathbf{B}'} - \nabla \times (\lambda \nabla \times \overline{\mathbf{B}})$$





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The alpha effect

- Induction equation – mean field simplest approximation

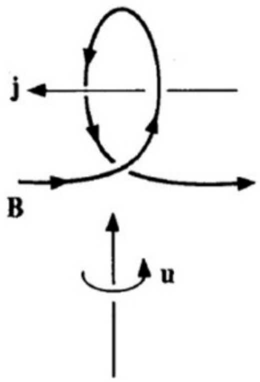
$$\mathrm{emf}' = \overline{\mathbf{u}' \times \mathbf{B}'} \sim \alpha\, \overline{\mathbf{B}}$$

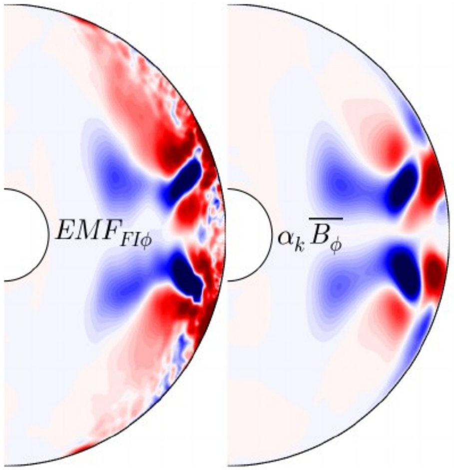
  - Simplified α:

$$\alpha \sim \frac{\tau_c}{3}\, Hel_{kin} = \frac{\tau_c}{3}\, \mathbf{u}' \cdot (\nabla \times \mathbf{u}')$$

  - where the correlation time is a function of the density scale height H_ρ:

$$\tau_c = \frac{H_\rho}{\mathbf{u}'}$$





Outlook and future work

1. The direction of propagation of the dynamo wave may be controlled by the inversion of the sign of the flow helicity
   - assuming that kinetic helicity is a good proxy for the α-effect (Parker cycle)
2. Equatorial asymmetry in radial correlations for the standard model shows that helicity of the flow is determined by local effects (columnar convection)

   $\rightarrow$ Plume-like convection is essential to control the sign of kinetic helicity
3. The combination of three aspects of the model set-up seems to be key for the inversion of the helicity pattern in our thick shell models:
   - strong density gradient
   - low Prandtl number (high Rossby number)
   - ~~fixed flux boundary conditions + internal heat sources (?)~~

~~Future work...~~

1. ~~Analysis of dynamo models~~
2. ~~Dependence of the helicity pattern on control parameters~~
3. ~~Dependence of dynamo wave properties on control parameters~~

Thin shells

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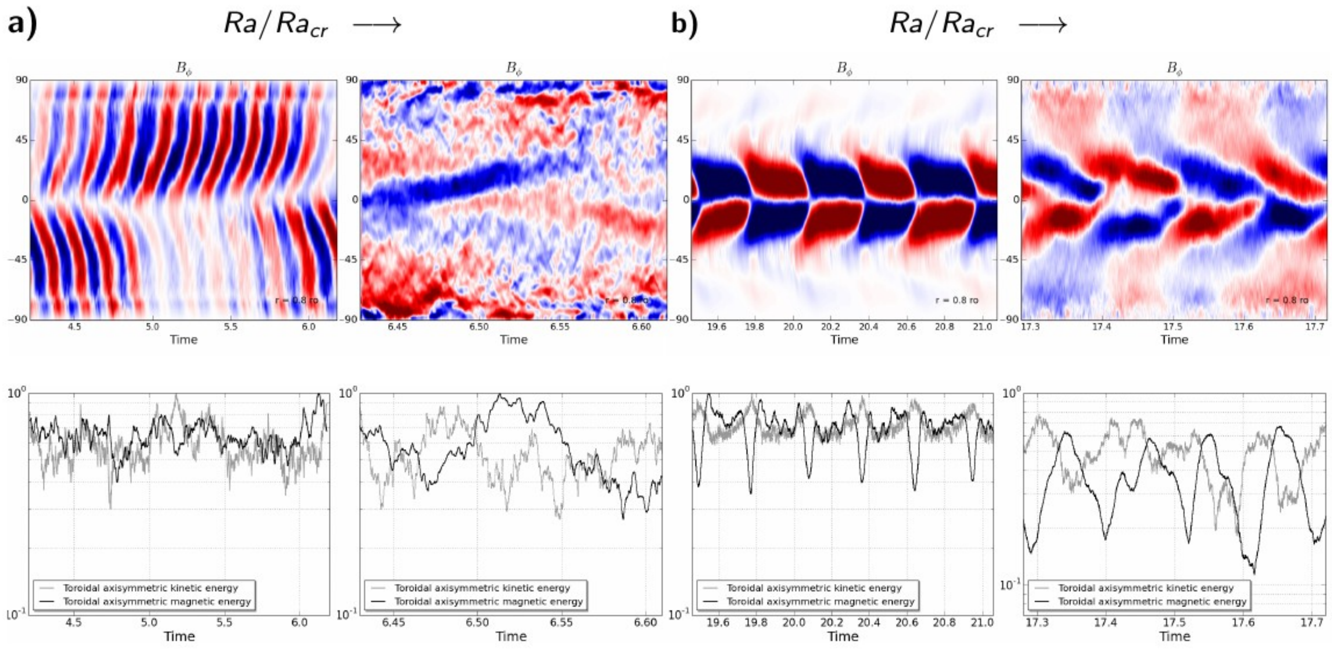
Additional dependence on the supercriticality...



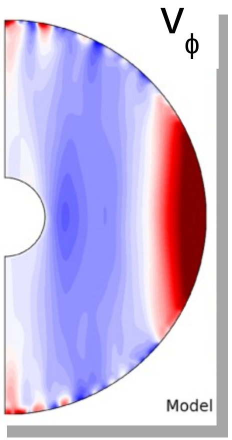
COFFIES

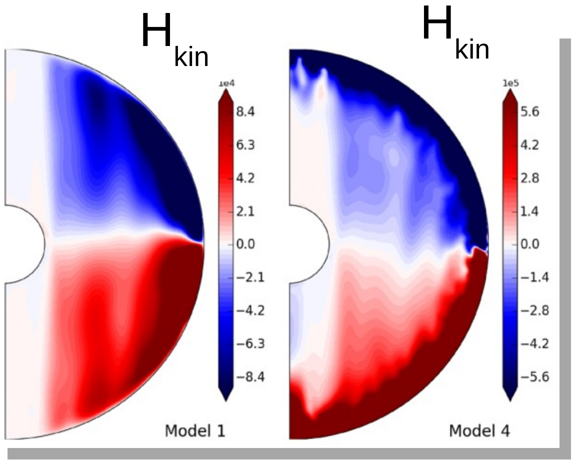
Kinetic helicity sign reversal

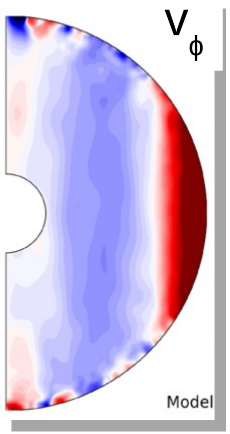
Duarte et al., MNRAS, 2015

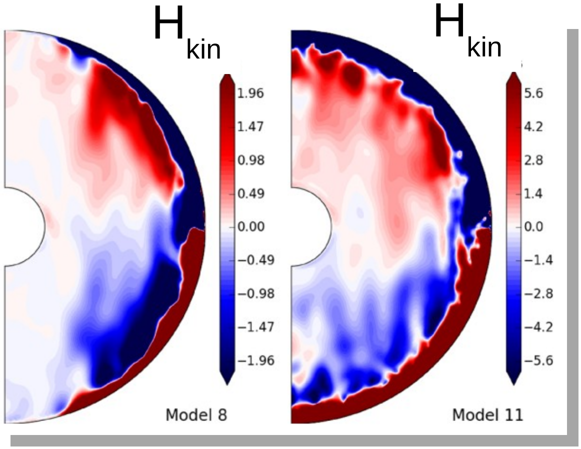
negative (NH) / positive (SH)

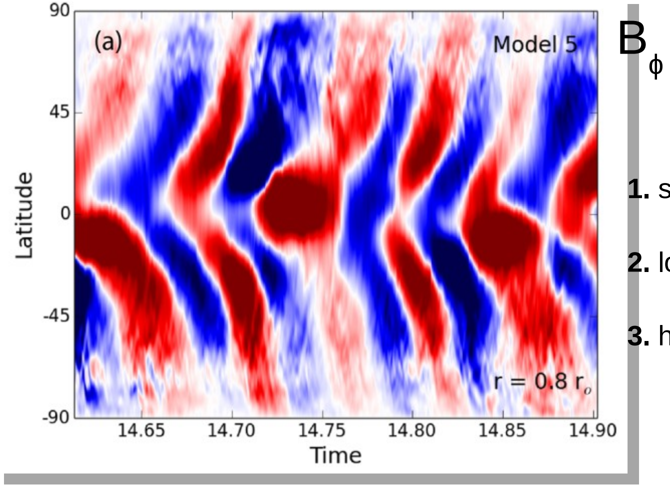
positive (NH) / negative (SH)

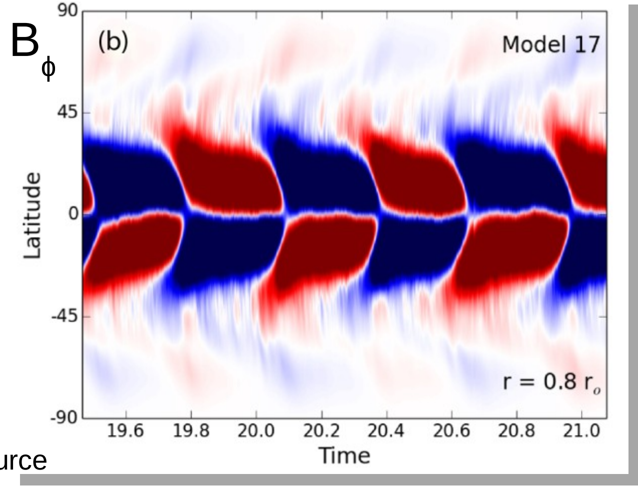












<u>Suggested list of requirements for inversion:</u>

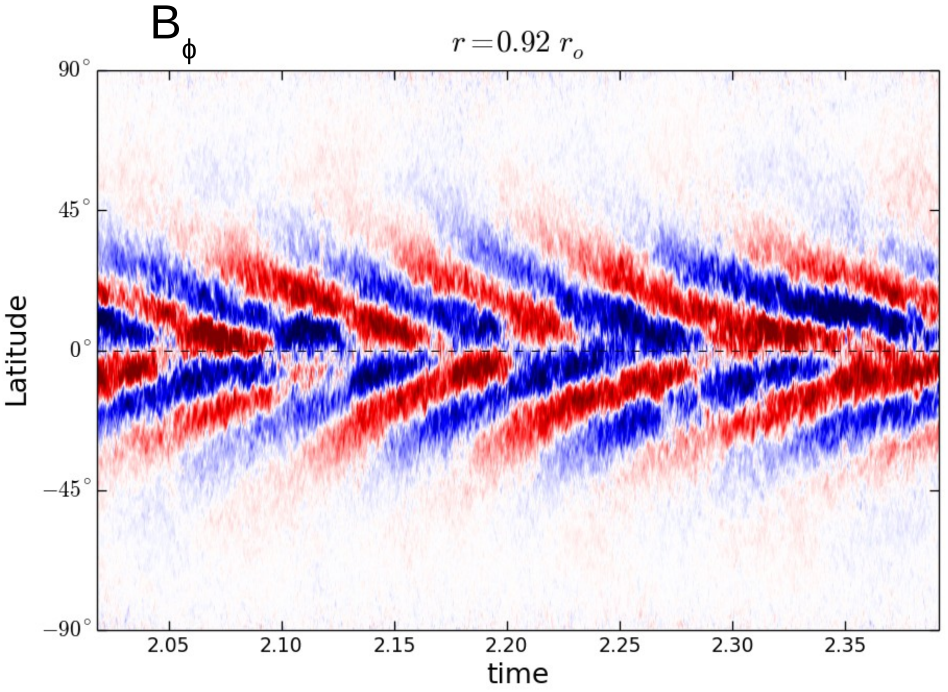
1. strong density stratification
(~5 density scale heights used)

2. lowering the fluid Prandtl number:
Pr=1 *vs.* Pr=0.1

3. heating mode:

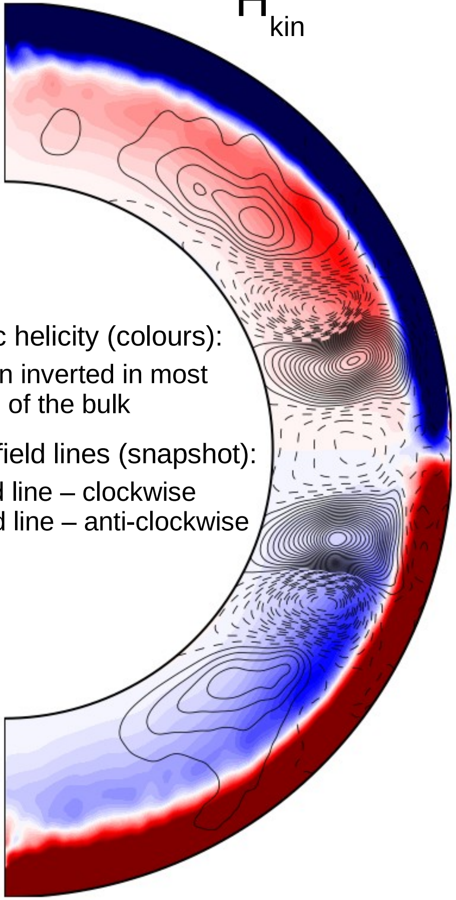
fixed entropy at the boundaries and no internal heat sources		fixed entropy flux at the boundaries and volumetric heating source

Kinetic helicity sign reversal
Thinner shell



averages in time and longitude

H_{kin}



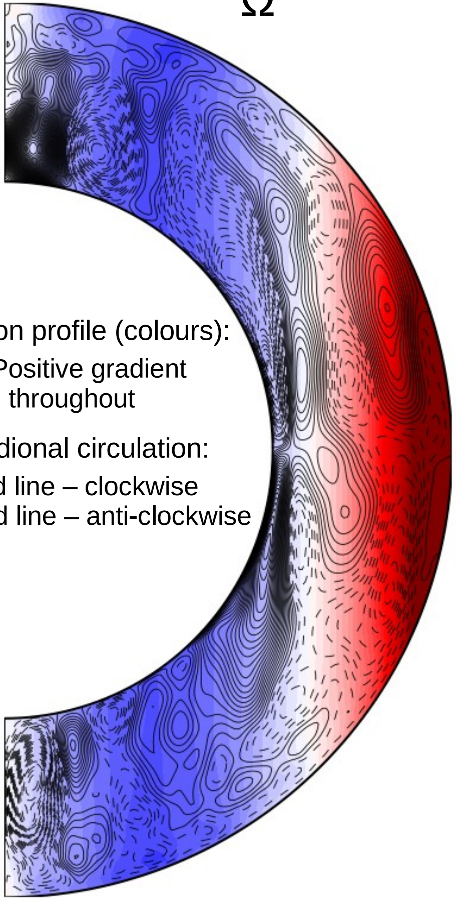
Kinetic helicity (colours):

► sign inverted in most of the bulk

Poloidal field lines (snapshot):

solid line – clockwise
dashed line – anti-clockwise

Ω



Rotation profile (colours):

► Positive gradient throughout

Meridional circulation:

solid line – clockwise
dashed line – anti-clockwise

Reviewed list of requirements for inversion:

1. strong density stratification
(~5 density scale heights used)

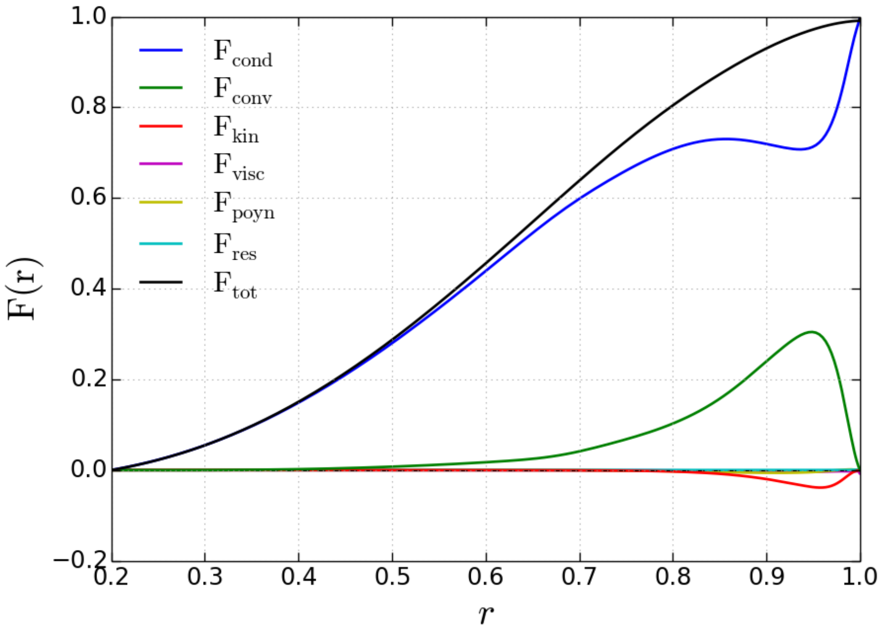
2. lowering the fluid Prandtl number: Pr=0.1

~~**3.** heating mode:~~
fixed entropy at the boundaries
and no internal heat sources

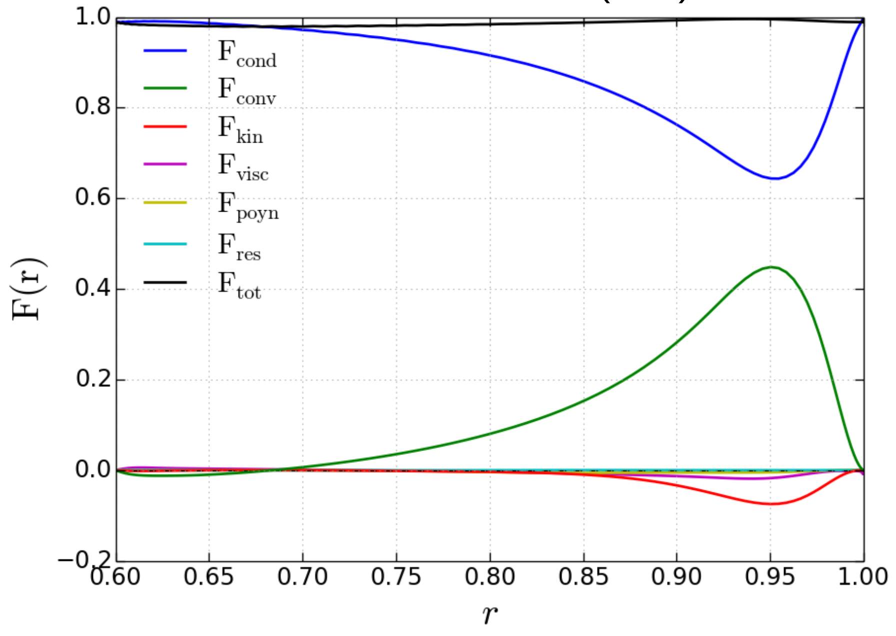
Kinetic helicity sign reversal

Flux components

Thick shell model (Duarte et el. 2015)



Thin shell model (new)



Reviewed list of requirements for inversion:

1. strong density stratification
(~5 density scale heights used)

2. lowering the fluid Prandtl number: Pr=0.1

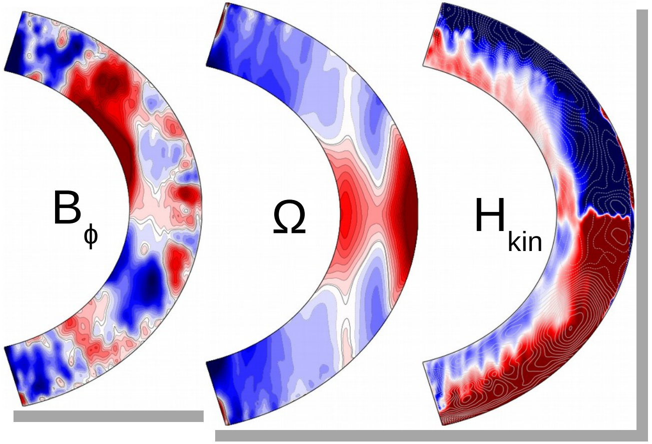
~~**3.** heating mode:~~
fixed entropy at the boundaries
and no internal heat sources

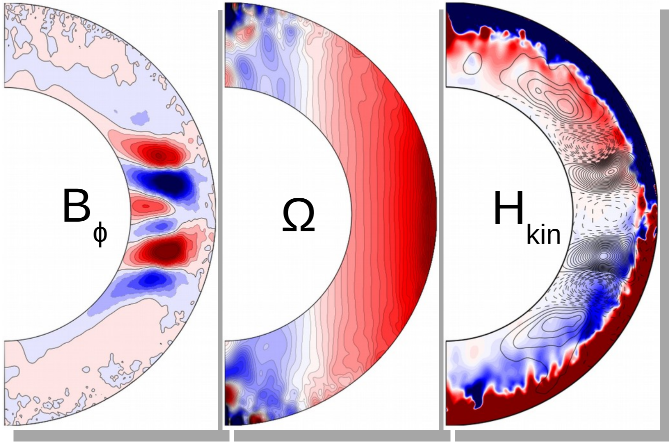
- ► Internal heating is not a requirement
- ► Conduction clearly dominates over convection
(natural result of low fluid Prandtl number?)

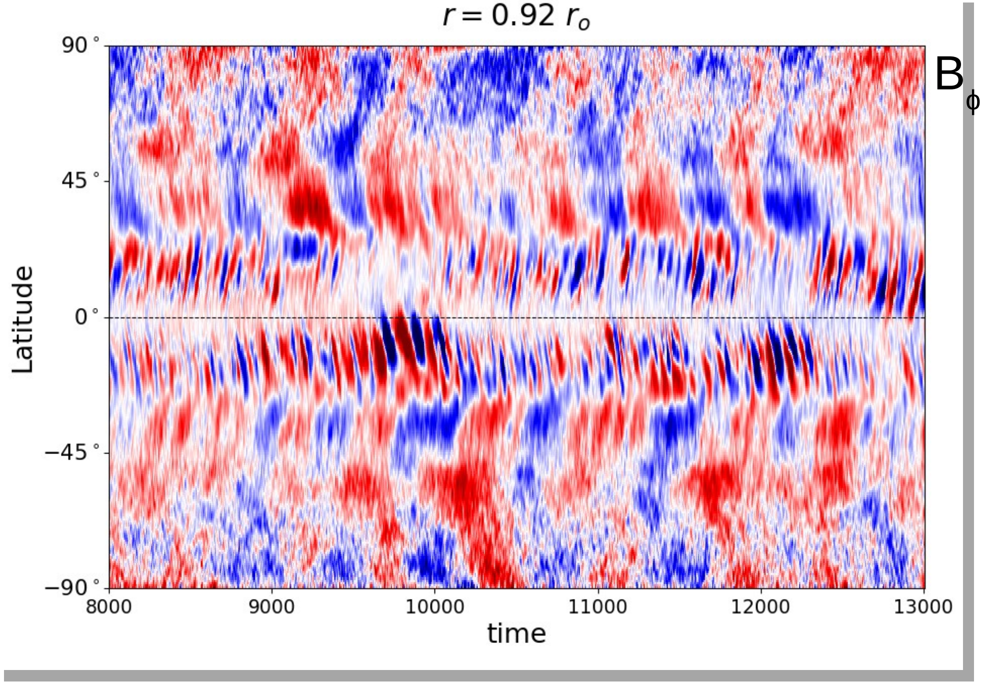
Kinetic helicity sign reversal

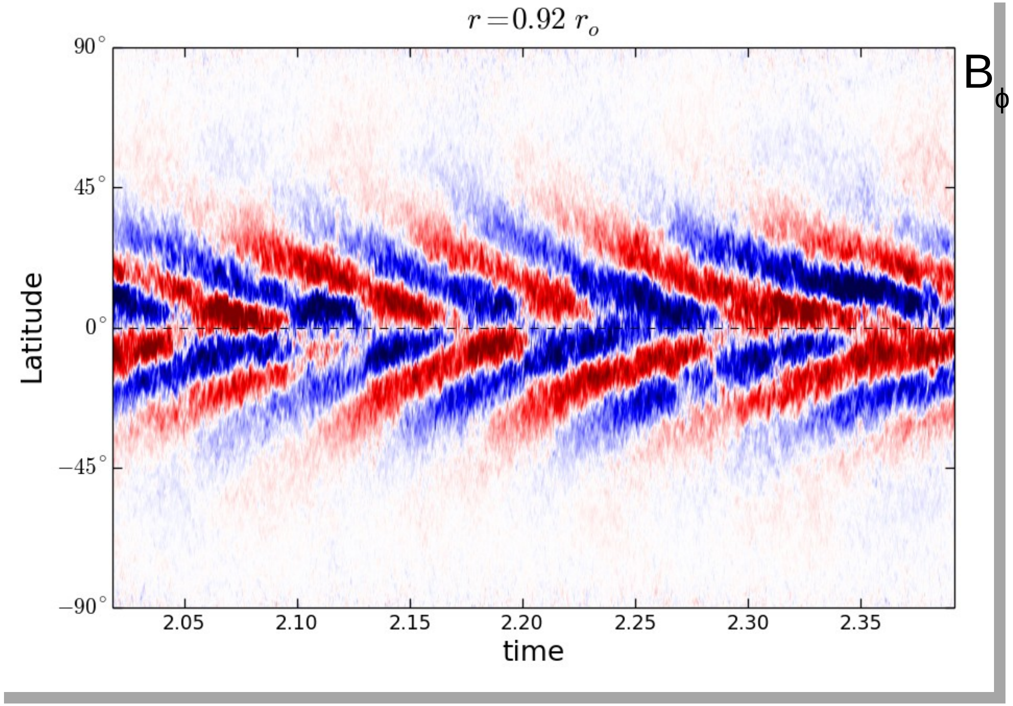
vs.

Differential rotation sign reversal









Thank you.

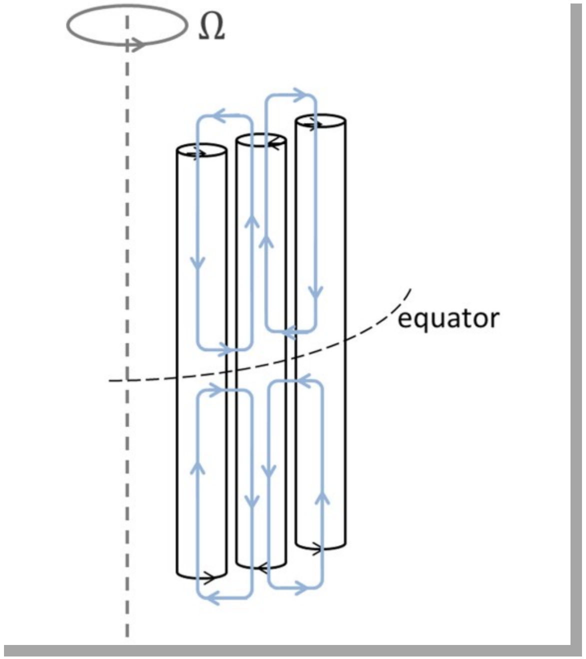
(movies?)

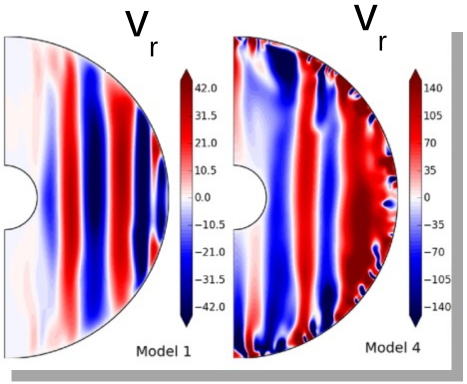
additional slides...

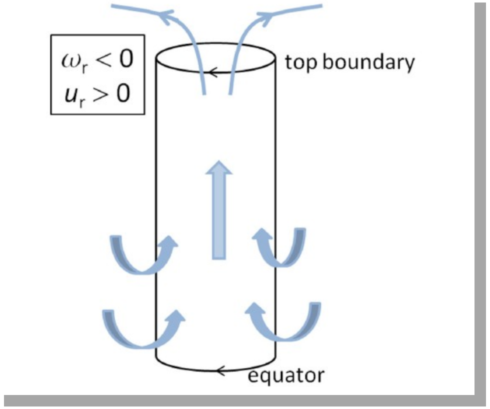
Kinetic helicity sign reversal

Duarte et al., MNRAS, 2015

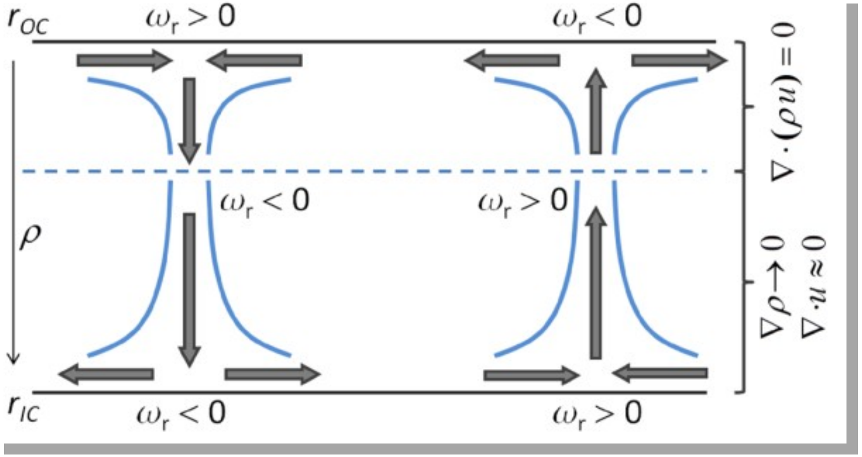
negative (NH) / positive (SH)

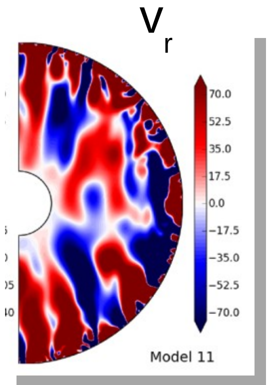






positive (NH) / negative (SH)





Kinetic helicity sign reversal

Duarte et al., MNRAS, 2015