PHASE TRANSITIONS IN THE SMEFT:
A CAREFUL APPROACH.

Eliel Camargo-Molina, Rikard Enberg, Johan Löfgren

GAUGE INVARIANCE AND THE EFF. POTENTIAL

IS IT A PROBLEM?

It is well known that the effective potential approach is gauge dependent to some extent.

- The shape of the potential might depend on the gauge choice
- The location of minima might also depend on gauge choice
- The depth of minima does not depend on gauge choice

$$\xi \partial_\xi V(\phi,\xi) + \mathcal{C}(\phi,\xi)\partial_\phi V(\phi,\xi) = 0;$$

GAUGE INVARIANCE AND THE EFF. POTENTIAL

IS IT A PROBLEM?

Things are (as usual) much simpler at lower orders, and one can say e.g. that:

- At one-loop all gauge dependence manifests itself in the Goldstone boson masses!
- If one works at $T = 0$, then as long as you are minimally careful, all is well. Just be sure Goldstone masses vanish.
- At $T \neq 0$, resummations have to be included to improve the perturbative expansion. Then more care has to be taken with power counting to get the Goldstone masses to be 0.

POWER COUNTING CAN REALLY CHANGE THE PICTURE

PHASE TRANSITIONS

POWER COUNTING IS IMPORTANT!

Without careful power counting, phase transitions can even disappear for some gauge choices!

Power counting depends on the parameters of your theory! It is not a unique recipe

Incredibly relevant for > 1 loop

Baryon washout, electroweak phase transition, and perturbation theory

Hiren H. Patel[a] and Michael J. Ramsey-Musolf[a,b]

$$V_{\rm eff}(\phi,T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\bar\lambda}{4}\phi^4 + \ldots .$$

The coefficients D, T_0^2, E and $\bar\lambda$ depend on the parameters of the underlying model. In the SM, the coefficients are [37]

$$\begin{aligned} D &= \frac{1}{32}(g_1^2 + 3g_2^2 + 4y_t^2 + 8\lambda)\,, \\ T_0^2 &= \mu^2/2D\,, \\ E &= \frac{3-\xi^{3/2}}{96\pi}\left(2g_2^3 + (g_1^2+g_2^2)^{3/2}\right), \\ \text{and }\ \bar\lambda &= \lambda + (\xi\text{-dep. log})\,, \end{aligned} \tag{2.23}$$

where y_t is the top yukawa coupling; g_1 and g_2 are the U(1) and SU(2)$_L$ gauge coupling constants; and the scalar quartic self coupling $\bar\lambda$ picks up a logarithmic ξ-dependence.

We observe that the coefficient of the quadratic term is gauge-independent, as one expects based on the gauge-independence of thermal masses (see e.g., ref. [38]). In appendix C, we explicitly demonstrate this property for the general model. As we will discuss below, we take advantage of this property to define the high-temperature effective theory used to obtain a gauge-independent sphaleron scale.

Unfortunately, the coefficient E is not only gauge-dependent but strongly so. For example, by choosing $\xi = 3^{2/3}$ the E-coefficient can be made to vanish, and the barrier necessary for a first order phase transition is permanently absent. One might hope that

PHASE TRANSITIONS

The order depends on how the potential looks/ develops for T>0.

The proper contributions at the leading-order have to be properly calculated.

This depends on the specific theory, and the region of parameter space of interest.

A feedback loop between valid expansion and the possible order for transition.

For SM (or toy-SM) this has been done ->

Needs to be combined with proper gauge invariant methods!!

The situation is alleviated with a proper power-counting. Consider the first-order transition scaling $\lambda \sim e^3$, $m^2_{\text{eff}}(T) \sim e^3 T^2$, $T \sim \frac{1}{e}$. A new minimum develops when the quartic term competes with the mass term: $\phi \sim T$. Now, the Goldstone mass is of order $\overline{G} \sim e^3 T^2$, while the photon mass is of order $e^2\phi^2 \sim e^2 T^2$. This means that the gauge dependent terms (to leading order) cancel, leaving

$$(\overline{G} + \xi e^2\phi^2)^{3/2} T - \xi^{3/2} e^3 \phi^3 T = \frac{3}{2} T \sqrt{\xi} e \phi \overline{G} \sim e^4 T^4. \tag{3.8}$$

3.1 Second-order transition

Consider first a second-order transition. With the scaling $T \sim 1/\sqrt{\hbar}$ the energy is

$$V_{\text{min}} = \Big\{ \left(V_0 + T^2 V_1^2\right) + \sqrt{\hbar} T \overline{V}_1^1 + \hbar \left(T^2 \overline{V}_2^2 + \overline{V}_1^0 - \sum_X \Pi_X \partial_X V_1^0 - T^2 \frac{(\phi_{1/2}(T))^2}{2} \left(\partial^2 V_0 + T^2 \partial^2 V_1^2\right) \right) + \dots \Big\} \Big|_{\phi_0(T)}. \tag{3.2}$$

The leading-order term $\left(V_0 + T^2 V_1^2\right)$ determines the temperature dependent VeV $\phi_0(T)$. Terms in $T^2 V_1^2$ are gauge invariant and are of the form $\sim e^2 \phi^2 T^2$ for some coupling e [5]. So all that changes for finite T is $m^2 \to m^2_{\text{eff}}(T)$. The transition occurs at the temperature where $m^2_{\text{eff}}(T)$ changes sign: $m^2_{\text{eff}}(T_{2\text{nd}}) = 0$. This is a second-order transition.

3.2 First-order transition

To be concrete, consider a high temperature expansion in the Abelian Higgs model. For high temperatures the potential is approximately

$$V(\phi) \sim -m^2\phi^2 + T^2\phi^2(e^2 + \lambda) - e^3 T \phi^3 + \lambda \phi^4. \tag{3.3}$$

Following [8], these various terms have to balance each other for a barrier to develop. The balance occurs if $\lambda\phi^2 \sim e^3 T \phi \sim (-m^2 + T^2 e^2 + \lambda T^2) \equiv m^2_{\text{eff}}(T)$, or

$$\phi \sim \frac{e^3}{\lambda} T \quad \& \quad m^2_{\text{eff}}(T) \sim \frac{e^6}{\lambda} T^2. \tag{3.4}$$

This leaves only one option [8],

$$\lambda \sim e^3 : \qquad \phi \sim T \quad \& \quad m^2_{\text{eff}}(T) \sim e^3 T^2 \quad \& \quad T \sim \frac{1}{e}. \tag{3.5}$$

Andreas Ekstedt[*,a] and Johan Löfgren[†,b]

SMEFT

FIRST OR SECOND ORDER?

In the SM, we need the quartic coupling to be large to get the right Higgs mass.

$$\lambda \sim e^3 : \qquad \phi \sim T \quad \& \quad m^2_{\text{eff}}(T) \sim e^3 T^2 \quad \& \quad T \sim \frac{1}{e}.$$

With nothing extra, it is really hard to get a first order phase transition with m_H = 125 GeV.

How about the SMEFT?
The quartic coupling could be smaller
The potential looks different
Power counting has to be done properly

$$\mathcal{L}_{\text{H}} = (D_\mu \phi)^\dagger (D^\mu \phi) + m^2 (\phi^\dagger \phi) - \frac{\lambda}{2} (\phi^\dagger \phi)^2 + C^{\phi} (\phi^\dagger \phi)^3 \; + \; C^{\phi \Box} (\phi^\dagger \phi) \Box (\phi^\dagger \phi) \; + \; C^{\phi D} (\phi^\dagger D_\mu \phi)^* (\phi^\dagger D^\mu \phi) \; .$$

$$m_H^2(\phi) = -m^2 + \frac{1}{2}(m^2(C^{\phi D} - 4C^{\phi\Box}) + 3\lambda)\phi^2 - \frac{3}{4}(5C^{\phi} + (C^{\phi D} - 4C^{\phi\Box})\lambda)\phi^4$$

What are the allowed regions for the WCs?
Are they consistent with a 1st order phase transition?
What about other observables?

PHASE TRANSITIONS IN THE SMEFT

- Really early stages, but looking promising
- We will establish a power counting scheme for the SMEFT at T > 0, first at one-loop
- We want to carefully map regions of allowed parameter space consistent with 1st order PT
- We want to compare those regions with future interesting regions in collider experiments and consider contributions to di-Higgs production