# Adiabatic construction of (hierarchical) quantum Hall states

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material contained in:

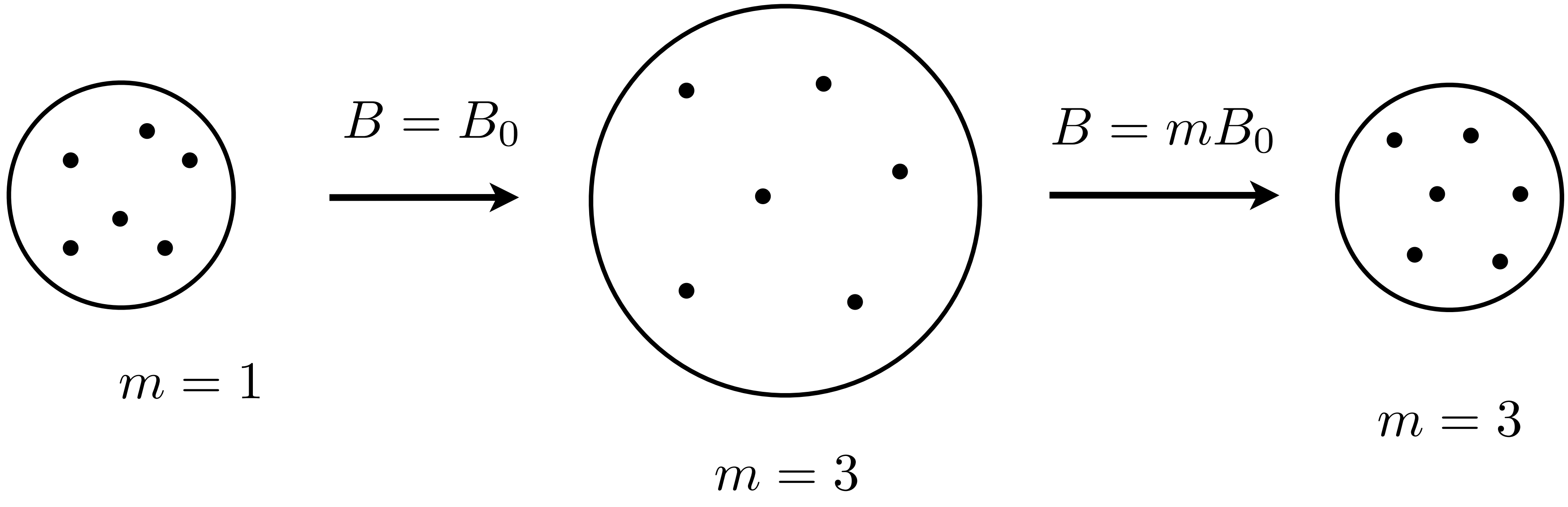
Heuristic principle for quantized Hall states, Mod. Phys. Lett. B4, 1063 (1990)
Exact solutions and the adiabatic heuristic for quantum Hall states, Nucl. Phys. B370, 577 (1992)
Paired Hall states (with XG Wen), Nucl. Phys. B 374, 567 (1992)

Adiabatic Construction of Hierarchical Quantum Hall States, arXiv:2105.05625 (2021)

States in the lowest Landau level: $\psi[z] = f[z] \prod_{i=1}^{N} e^{-\frac{B}{4}|z_i|^2} \qquad z = x + \mathrm{i}y$

Laughlin state for the (F)QH state: $\psi[z] = \prod_{i<j}^{N} (z_i - z_j)^m \prod_{i=1}^{N} e^{-\frac{B}{4}|z_i|^2}$

$$\nu = \frac{\#\text{ of electrons}}{\#\text{ of states in the LLL}} = \frac{1}{m}$$



## adiabatic evolution form filled LL $m=1$ to Laughlin state $m=3$ :

$$\psi_\theta^{\text{anyonic}}[z] = \boxed{\prod_{i<j}^{N}(z_i - z_j)^{1+\frac{\theta}{\pi}}} \prod_{i=1}^{N} e^{-\frac{B}{4}|z_i|^2} \qquad B = \left(1+\frac{\theta}{\pi}\right)$$

anyons

$$\text{as } \theta : \quad 0 \longrightarrow 2\pi : \quad \text{fermions} \quad \longrightarrow \quad \text{"superfermions"}$$

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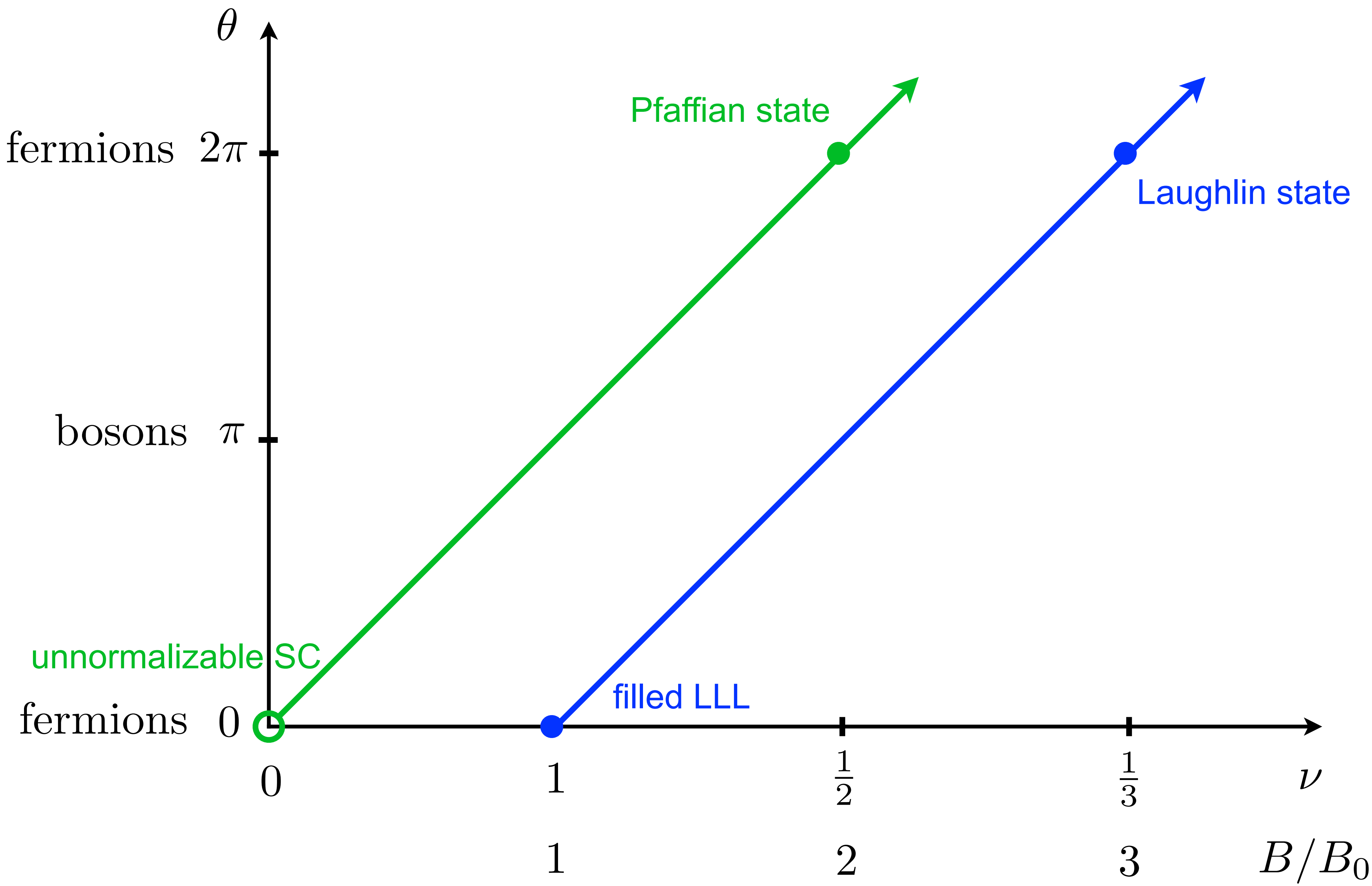
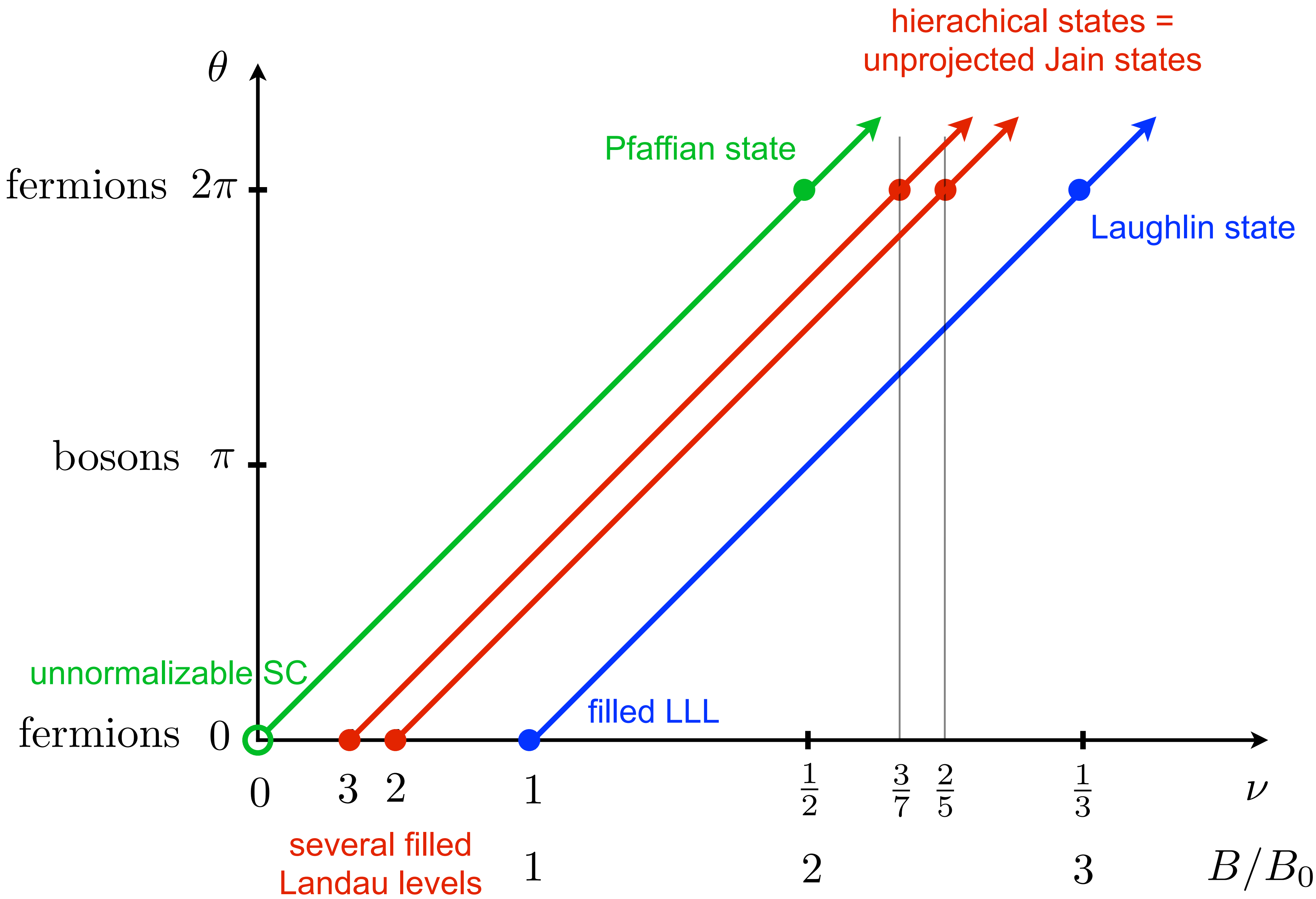
exact model: $$\psi_\theta^{\text{fermionic}}[z] = \prod_{i<j}^{N} (z_i - z_j) \prod_{i<j}^{N} |z_i - z_j|^{\frac{\theta}{\pi}} \prod_{i=1}^{N} e^{-\frac{B}{4}|z_i|^2}$$

is the unique zero energy ground state of $H_\theta$ : $\quad H_\theta \left|\psi_\theta^{\text{fermionic}}\right\rangle = 0$

→ existence and energy gap of FQH state derived from IQH state!


$\theta$
fermions $2\pi$
bosons $\pi$
fermions $0$
Pfaffian state
Laughlin state
unnormalizable SC
filled LLL
0
1
$\frac{1}{2}$
$\frac{1}{3}$
$\nu$
1
2
3
$B/B_0$



hierachical states =
unprojected Jain states
$\theta$
Pfaffian state
fermions $2\pi$
Laughlin state
bosons $\pi$
unnormalizable SC
filled LLL
fermions $0$
$0$
$3$
$2$
$1$
$\frac{1}{2}$
$\frac{3}{7}$
$\frac{2}{5}$
$\frac{1}{3}$
$\nu$
several filled
Landau levels
$1$
$2$
$3$
$B/B_0$

# Technical details of the model(s):

in the plane spanned by unit vectors $\hat{\boldsymbol{e}}_{\mathrm{x}}$ and $\hat{\boldsymbol{e}}_{\mathrm{y}}$

$$H = \frac{1}{2m} \sum_{i=1}^{N} \left[ (-\mathrm{i}\nabla_i + e\boldsymbol{A}_i)^2 - eB_i \right] \qquad \boldsymbol{B}_i = \nabla_i \times \boldsymbol{A}_i =: -B_i \hat{\boldsymbol{e}}_{\mathrm{z}}$$

$$z \equiv x + \mathrm{i}y, \; \partial = \tfrac{1}{2}(\partial_{\mathrm{x}} - \mathrm{i}\partial_{\mathrm{y}}), \; A = A_{\mathrm{x}} + \mathrm{i}A_{\mathrm{y}}$$

$$a_i \equiv \sqrt{2}\left(\bar{\partial}_i + \frac{\mathrm{i}e}{2} A_i\right), \quad a_i^\dagger = \sqrt{2}\left(-\partial_i - \frac{\mathrm{i}e}{2} \bar{A}_i\right)$$

$$\{a_i, a_i^\dagger\} = (-\mathrm{i}\nabla_i + e\boldsymbol{A}_i)^2$$

$$[a_i, a_i^\dagger] = \mathrm{i}e\left(\partial_i A_i - \bar{\partial}_i \bar{A}_i\right) = eB_i$$

and hence $H = \frac{1}{m} \sum_{i=1}^{N} a_i^\dagger a_i$

Girvin et.al PRL1990, Jackiw and Pi, PRL 1990

For the present analysis, we take

$$B_i = -\frac{2\theta}{e}\sum_{j(\neq i)} \delta^2(\boldsymbol{r}_i - \boldsymbol{r}_j) + B \qquad \nabla_i \boldsymbol{A}_i = \partial_i A_i + \bar{\partial}_i \bar{A}_i = 0$$

write $\quad A_i = -\frac{2\mathrm{i}}{e}\bar{\partial}_i S, \quad \bar{A}_i = \frac{2\mathrm{i}}{e}\partial_i S, \quad B_i = \frac{4}{e}\partial_i\bar{\partial}_i S$

$$S \equiv -\frac{\theta}{\pi}\sum_{i<j} \ln|z_i - z_j| + \frac{1}{4}eB\sum_i |z_i|^2$$

$$a_i = +\sqrt{2}\, e^{-S}\, \bar{\partial}_i\, e^{+S},$$

$$a_i^\dagger = -\sqrt{2}\, e^{+S}\, \partial_i\, e^{-S} = \sqrt{2}\, e^{-S}\big(-\partial_i + 2(\partial_i S)\big)e^{+S}$$

zero energy ground states: $\psi_0[z] = f[z]\, e^{-S}$

filled Landau level as initial state: $f[z] = \prod_{i<j} (z_i - z_j)$

$$\psi^{(\theta)}[z] = \prod_{i<j} (z_i - z_j) \prod_{i<j} |z_i - z_j|^{\theta/\pi} \prod_i e^{-\frac{1}{4} eB|z_i|^2}$$

supplement $H = \frac{1}{m} \sum_{i=1}^{N} a_i^\dagger a_i$ with a repulsive interaction:

$$V^{(1+\theta/\pi)} \propto \sum_{i \neq j} |z_i - z_j|^{2n-\theta/\pi} \left(\nabla_i^2\right)^n \delta^2(z_i - z_j)$$

where n is an integer such that $2n - \theta/\pi \geq 0$

Trugman and Kivelson PRL 1985

To fully recover the Laughlin we need to remove the flux attached to the particles via a "singular gauge transformation"

## Generalization to higher Landau levels:

$p$ filled levels: $\psi_p^{(0)}[z] = f_p[z,\bar{z}]\, e^{-S_0}$

during the evolution:

$$\psi_p^{(\theta)}[z] =: f_p[z,\bar{z}]\, e^{-S} = \prod_{i<j} |z_i - z_j|^{\theta/\pi}\, f_p[z,\bar{z}] \prod_i e^{-\frac{1}{4}eB|z_i|^2}$$

kinetic part of the parent Hamiltonian: $H_p^{(\theta)} = \dfrac{1}{m(eB)^{p-1}} \displaystyle\sum_{i=1}^{N} (a_i^\dagger)^p (a_i)^p$

states are still annihilated by

$$V^{(1+\theta/\pi)} \propto \sum_{i\neq j} |z_i - z_j|^{2n-\theta/\pi}\, \left(\nabla_i^2\right)^n\, \delta^2(z_i - z_j)$$