Simple Models of Hidden Purity

Griffiths decay model

unit time step

$$|x\rangle \;\to\; |x+1\rangle \qquad \text{for } x < -1 \text{ or } x \geq 1$$

$$|0\rangle \;\to\; \alpha|0\rangle + \beta|1\rangle$$

$$|-1\rangle \;\to -\beta^*|0\rangle + \alpha^*|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

manifestly unitary

model of right-mover with an impurity or radial “scattering center” / “decaying object” / “lump of coal” / “liquid drop” /“black hole”

$$V_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta^* & \alpha & 0 & 0 \\ 0 & 0 & \alpha^* & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

… (extended)

$$V_0^n|0\rangle \ = \ \alpha^n|0\rangle + \beta|n\rangle + \beta\alpha|n-1\rangle + \beta\alpha^2|n-2\rangle + \ldots + \beta\alpha^{n-1}|1\rangle$$

Introduce “detector” or “internal state” at $|1\rangle$

$$W = (\mathbb{1} - |1\rangle\langle 1|) \otimes \mathbb{1} + |1\rangle\langle 1| \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Complete model

$$V \equiv V_0 \otimes \mathbb{1}$$

$$U = WV$$

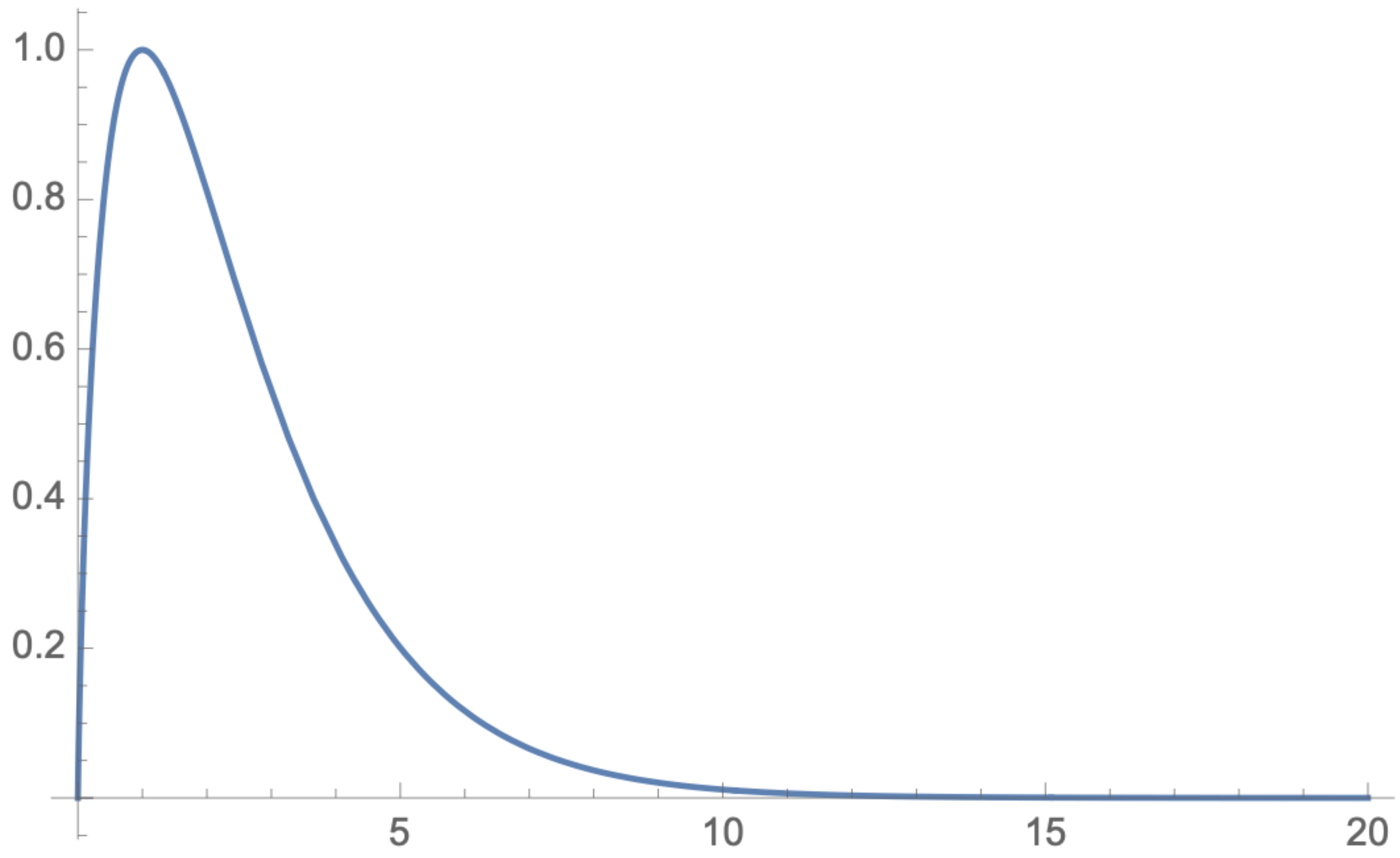
$$\psi_0(0) = |0\rangle \otimes |e\rangle$$

$$\begin{aligned} \psi_0(n) &\equiv U^n \psi_0(0) \\ &= \alpha^n |0\rangle \otimes |e\rangle + (\beta|n\rangle + \beta\alpha|n-1\rangle + \ldots) \otimes |g\rangle \end{aligned}$$

$$\rho_0(n) = \begin{pmatrix} |\alpha|^{2n} & 0 \\ 0 & 1 - |\alpha|^{2n} \end{pmatrix}$$

$$\mathrm{Ent}_0(n) \; = \; -\left(|\alpha|^{2n}\log_2|\alpha|^{2n} \, + \, (1-|\alpha|^{2n})\log_2(1-|\alpha|^{2n})\right)$$

$$\mathrm{CEnt}(t) \; = \; t\,2^{-t} - (1-2^{-t})\log_2(1-2^{-t})$$



1.0
0.8
0.6
0.4
0.2
5
10
15
20

One can enhance this model in many ways that keep it tractable, e.g.:

more elaborate centers, with richer internal structure and long delays

many-particle states

and in principle:

different particle species (modes)

number-changing processes (field theory)

more dimensions

Entropy and quantum statistics

start with two particles at $|-1\rangle$, $|0\rangle$, introduce two detectors (or one counter) …

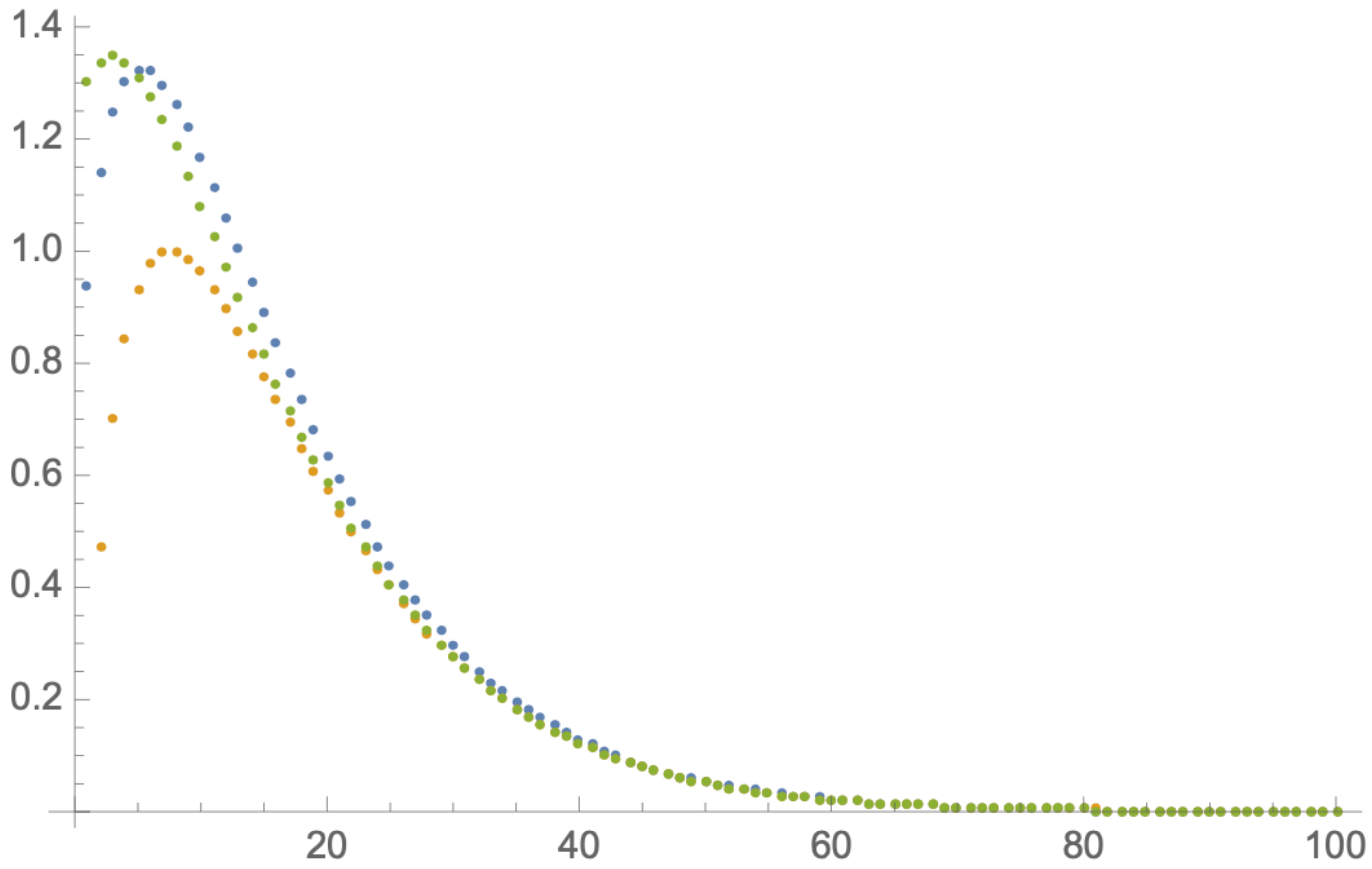


Figure 3: Entropy of the radiation field evolving from initial fermionic (orange), bosonic (green), and distinguishable (blue) initial states based on $\psi_0(0) \otimes \psi_{-1}(0)$.

We can access physical systems that embody these models quite closely.

A most interesting possibility is to make different “temporally separated” parts of quantum radiation fields interfere.