



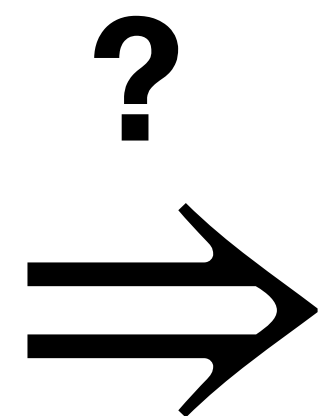
# Josephson, Canonically

A Hamiltonian Time Crystal

# The Canonical Difficulty

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$



at minimum of  $H$ , nothing moves

$$L = n\dot{\phi} - g \cos \phi - 2eVn$$

$$\pi_{\phi} = n$$

$$H = g \cos \phi + 2eV\pi_{\phi}$$

$$\dot{\phi} = \frac{\partial H}{\partial \pi_{\phi}} = 2eV$$

$$\dot{\pi}_{\phi} = -\frac{\partial H}{\partial \phi} = g \sin \phi$$

$$\phi = 2eVt + \alpha$$

$$\pi_{\phi} = \frac{-g}{2eV} \cos(2eVt + \alpha) + \kappa$$

$$j = \dot{n} = g \sin(2eVt + \alpha)$$

The Hamiltonian, being linear in  $\pi$ , has no bottom.

*Is that really so bad?*

The Schrödinger equation:

$$i \frac{\partial \psi(t, \phi)}{\partial t} = -i2eV \frac{\partial \psi}{\partial \phi} + g \cos \phi \psi$$

Its solution:

$$\psi(t, \phi) = e^{-i \frac{g}{2eV} \sin \phi} \eta\left(t - \frac{\phi}{2eV}\right)$$

This is not singular in any way.

$$\left[ \psi(t, \phi) = e^{-i\frac{g}{2eV} \sin \phi} \eta\left(t - \frac{\phi}{2eV}\right) \right]$$

- The “classic” Josephson effect corresponds to taking a  $\delta$  function for  $\eta$
- To get energy eigenfunctions we make  $\eta$  a traveling wave:  $\eta_E = e^{-iE\left(t - \frac{\phi}{2eV}\right)}$
- The stationary states are:  $\psi_E(\phi) = e^{i\left(-\frac{g \sin \phi + E\phi}{2eV}\right)}$



- [ The stationary states are:  $\psi_E(\phi) = e^{i\left(-\frac{g \sin \phi + E\phi}{2eV}\right)}$  ]
- Imposing  $2\pi$  periodicity on  $\phi$ , we get the eigenvalues  
 $E_l = (2eV) l$ , for integers  $l$ .

Which is more natural?

Which is more appropriate?



**t as a quantum variable**