Josephson, Canonically

A Hamiltonian Time Crystal

The Canonical Difficulty

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

$\overset{?}{\Rightarrow}$ at minimum of H, nothing moves

$$L = n\dot{\phi} - g\cos\phi - 2eVn$$

$$\pi_\phi = n$$

$$H = g\cos\phi + 2eV\pi_\phi$$

$$\dot{\phi} = \frac{\partial H}{\partial \pi_\phi} = 2eV$$

$$\dot{\pi}_\phi = -\frac{\partial H}{\partial \phi} = g \sin \phi$$

$$\phi = 2eVt + \alpha$$

$$\pi_\phi = \frac{-g}{2eV} \cos(2eVt + \alpha) + \kappa$$

$$j = \dot{n} = g \sin(2eVt + \alpha)$$

The Hamiltonian, being linear in π, has no bottom.

Is that really so bad?

The Schrödinger equation:

$$i\frac{\partial\psi(t,\phi)}{\partial t} \;=\; -\, i2eV\frac{\partial\psi}{\partial\phi} + g\cos\phi\,\psi$$

Its solution:

$$\psi(t,\phi) \;=\; e^{-i\frac{g}{2eV}\sin\phi}\eta(t - \frac{\phi}{2eV})$$

This is not singular in any way.

$$[\,\psi(t,\phi) \;=\; e^{-i\frac{g}{2eV}\sin\phi}\eta(t-\frac{\phi}{2eV})\,]$$

- The "classic" Josephson effect corresponds to taking a δ function for η

- To get energy eigenfunctions we make η a traveling wave: $\eta_E \;=\; e^{-iE(t-\frac{\phi}{2eV})}$

- The stationary states are: $\psi_E(\phi) \;=\; e^{i(-\frac{g\sin\phi + E\phi}{2eV})}$

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- Imposing 2π periodicity on ϕ, we get the eigenvalues $E_l = (2eV)\,l$, for integers l.

Which is more natural?

Which is more appropriate?

t as a quantum variable