Thoughts on time crystals

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U. Kentucky

A brief history of time crystals

- A brief look back at the origins of the idea.
- As far as I know, the term “time crystal” originated in the 60’s with Dr. Who.
- The term was first used scientifically in the 70’s by biologist Arthur Winfree to describe self-organizing oscillations and rhythms in biological systems.
  - Circadian rhythms, cardiac arrhythmias
  - Nonequilibrium, driven systems

A brief history of time crystals

- In physics, the concept of a time crystal was born in 2010. Frank had been thinking about spontaneous synchronization of oscillators. I came to visit him in Cambridge and we spent a long June afternoon trying to sharpen questions and come up with examples. The key question became: “Can time translation symmetry be broken spontaneously?”
- In other words, can the ground state of a system (classical or quantum) be time-dependent?

Selected for a Viewpoint in *Physics*
PRL **109,** 160401 (2012) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2012

Quantum Time Crystals

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Some subtleties and apparent difficulties associated with the notion of spontaneous breaking of time-translation symmetry in quantum mechanics are identified and resolved. A model exhibiting that phenomenon is displayed. The possibility and significance of breaking of imaginary time-translation symmetry is discussed.

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Classical Time Crystals

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(Received 3 April 2012; published 15 October 2012)

We consider the possibility that classical dynamical systems display motion in their lowest-energy state, forming a time analogue of crystalline spatial order. Challenges facing that idea are identified and overcome. We display arbitrary orbits of an angular variable as lowest-energy trajectories for nonsingular Lagrangian systems. Dynamics within orbits of broken symmetry provide a natural arena for formation of time crystals. We exhibit models of that kind, including a model with traveling density waves.

DOI: 10.1103/PhysRevLett.109.160402 PACS numbers: 45.50.−j, 03.50.Kk, 05.45.−a, 11.30.Qc

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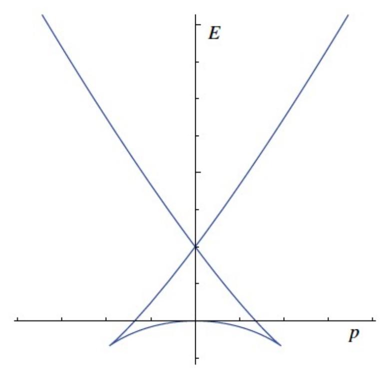
$$L = -\frac{\kappa}{2}\dot{\phi}^2 + \frac{\lambda}{4}\dot{\phi}^4$$



Double-well kinetic terms

Branched energy function

Branched Quantization

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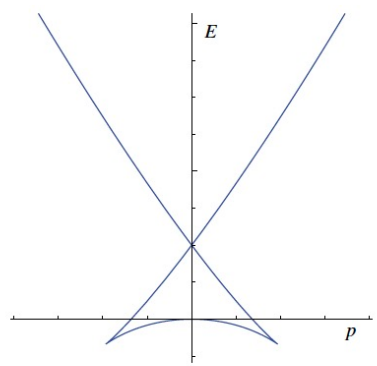
We propose a method for quantization of Lagrangians for which the Hamiltonian, as a function of momentum, is a branched function, possibly with cusps. Appropriate boundary conditions, which we identify, ensure unitary time evolution. In special cases a dual (canonical) transformation maps the problem into a problem of quantum mechanics on singular spaces, which we also develop. Several possible applications are indicated.

DOI: 10.1103/PhysRevLett.109.200402 PACS numbers: 03.65.Ca, 03.65.Ta, 84.30.Bv



Models of Topology Change

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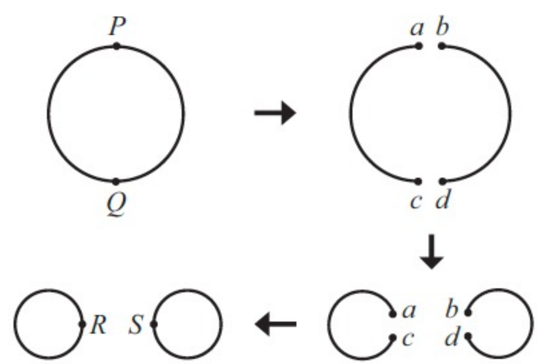
We show how changes in unitarity-preserving boundary conditions allow continuous interpolation among the Hilbert spaces of quantum mechanics on topologically distinct manifolds. We present several examples, including a computation of entanglement entropy production. We discuss approximate realization of boundary conditions through appropriate interactions, thus suggesting a route to possible experimental realization. We give a theoretical application to quantization of singular Hamiltonians, and give tangible form to the "many worlds" interpretation of wave functions.



PNAS PNAS

Regularizations of time-crystal dynamics

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Contributed by Frank Wilczek, July 19, 2019 (sent for review May 21, 2019; reviewed by Robert V. Kohn and Krzysztof Sacha)

We demonstrate that nonconvex Lagrangians, as contemplated in the theory of time crystals, can arise in the effective description of conventional, physically realizable systems. Such embeddings resolve dynamical singularities which arise in the reduced description. Microstructure featuring intervals of fixed velocity interrupted by quick resets—"Sisyphus dynamics"—is a generic consequence. In quantum mechanics, this microstructure can be blurred, leaving entirely regular behavior.

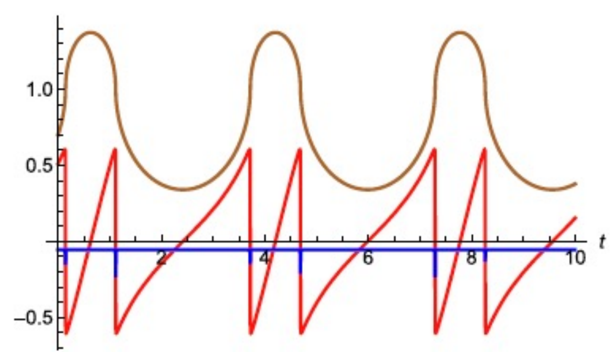
time crystal | microstructure | Lagrangian

rials (18, 19). Our treatment of the time-crystal problem suggests opportunities in those areas, as we shall discuss further below.

We can gain a more general perspective by considering not only the (problematic) ground state, but solutions of the equations of motion more generally. In the equation of motion

$$(\dot{y}^2 - 1)\ddot{y} = -V'(y), \qquad [4]$$

we see that the effective mass, $\dot{y}^2 - 1$, can vanish and change sign. Negative effective mass is unusual, though perhaps not problematic in itself, at the level of differential equations. But vanish-

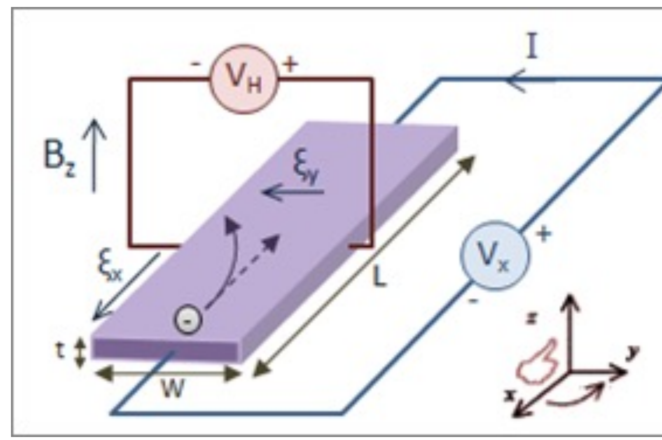


How to realize a mechanical time crystal?

- Planar particle in a nonuniform electromagnetic field

$$L = \frac{\mu}{2}\dot{x}^2 + f(x)\dot{y} - g(x) - V(y)$$

  - magnetic field $B_z = f'(x)$
  - electric potential $\Phi = g(x) + V(y)$
  - B_z constant in y-direction



- Equations of motion

$$\mu\ddot{x} = f'(x)\dot{y} - g'(x)$$
$$\dot{x}f'(x) = -V'(y).$$

- In limit $\mu \to 0$, get

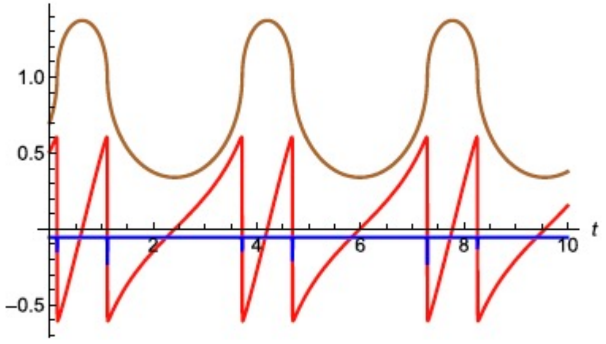
$$\dot{y} = g'/f'$$

How to realize a mechanical time crystal?

- Choose $f(x) = \frac{1}{3}x^3 - x,\; g(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$
- Then $\dot{y} = g'/f'$ gives $\dot{y} = x$
- Plug into *L* to get

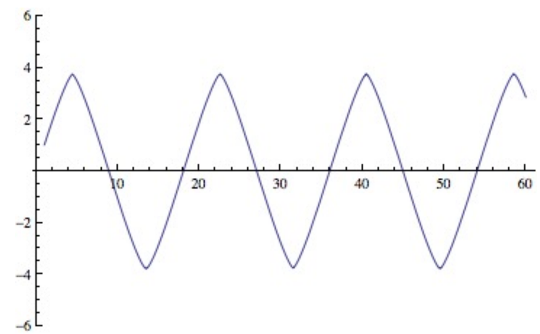
$$L \;=\; \frac{1}{12}\dot{y}^4 - \frac{1}{2}\dot{y}^2 - V(y).$$

  - our favorite Lagrangian.
- Turn μ back on: a regulator, important near turning points…



Other regularizations

- Near turning points, the effective Lagrangian breaks down and addition info is required.
- If instead we add a term $\mu\ddot{y}^2$ as regulator to suppress sudden acceleration, we get very different behavior:



- Which regulator is preferred depends on microscopic properties of system, determines very different macroscopic behavior.

Cold atoms

- Nonlinear Schrodinger equation (Gross-Pitaevskii)

$$-\frac{1}{2}\Psi_{xx} + g|\Psi|^2\Psi + V(x)\Psi = \mu\Psi$$

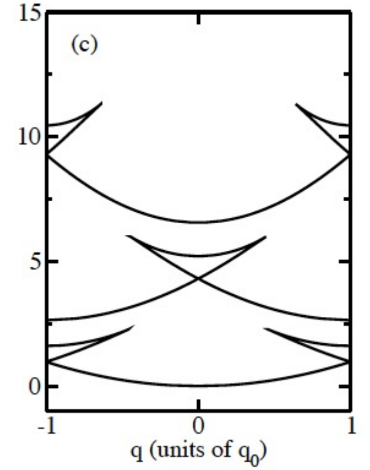
$g > 0$: repulsive

- With periodic (repulsive) KP potential

$$V(x) = V_0 \sum_{j=-\infty}^{+\infty} \delta(x-j)$$

has swallowtail–shaped bands

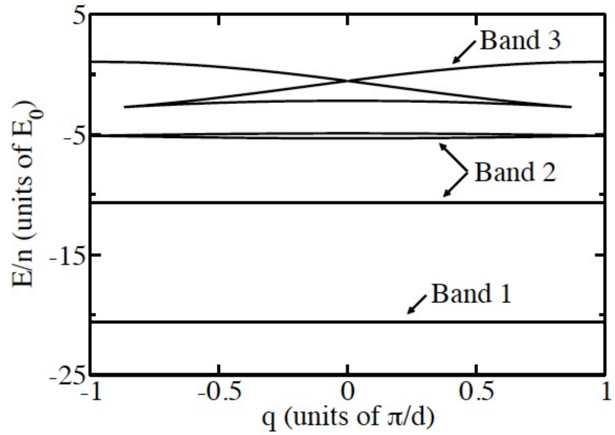
  - But these swallowtails are “upside-down”….



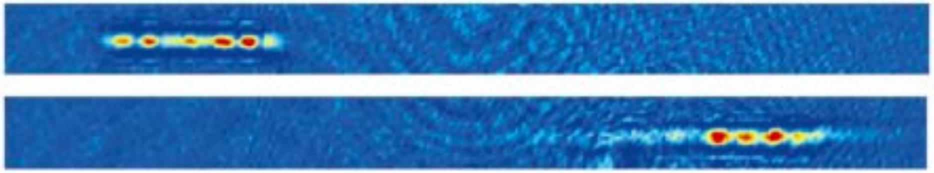
[Seaman, Carr, Holland 2005]

Cold atoms

- For an attractive potential, the swallowtail turns right-side up:



- Swallowtail appears in third band.
- Along lower branch of swallowtail, atoms clump into solitons. “Soliton train” states – have been observed in cold atom experiments:



[Strecker et al 2002]

- Do these deserve to be called time crystals?

Cosmology

- Lagrangians similar to ours have been proposed as a source of inflationary vacuum energy:
  - k-inflation [Armendariz-Picon, Damour, Mukhanov 99]
  - ghost condensation [Arkani-Hamed, Cheng, Luty, Mukohyama 04]

- Funny kinetic terms for scalar field: "wrong" sign quadratic term.
  - non-equilibrium, external *t*-dependent background
  - typically a potential for ϕ is not considered,
    - but could be added to produce interesting cosmological bounce.
- A similar mechanism appears in Starobinsky's original model…

Inflation

- Starobinsky's inflation mechanism also involved a time-crystal-like Lagrangian. Highly constrained; consistent with PLANCK data (but not BICEP2)
- A model of inflation involving only gravity

$$\mathcal{L} = \sqrt{-g}\left(R + \beta R^2 + \gamma R^3 + \cdots - 2\Lambda\right)$$

- Consider dynamics of scale factor

$$g_{\mu\nu} = a^2(x)\eta_{\mu\nu}$$

- Substitute in $\mathcal{L}$ …

Inflation

- Substitute $g_{\mu\nu} = a^2(x)\eta_{\mu\nu}$ into

$$\mathcal{L} = \sqrt{-g}\left(R + \beta R^2 + \gamma R^3 + \cdots - 2\Lambda\right)$$

  – For flat homogeneous, isotropic metric get (in –+++ signature)

$$\mathcal{L} = -6\dot{a}^2 + 108\beta \left(\frac{\dot{a}}{a}\right)^4 + \cdots - 2\Lambda a^4$$

  – First coefficient is negative.

- For positive β get time crystals
  – Exponential inflation
  – Cosmological term → a natural bounce scenario…
- Plausible that the Universe itself is a time crystal!

Who’s afraid of no-go theorems?

- Driven, nonequilibrium systems, etc., but
- Even in zero-temp, closed systems can have time-crystal-like behavior
  - Effective Lagrangians (higher derivatives)
  - Metastable vacua (cold atoms)
  - Weird dynamical systems, e.g. with multivalued/singular H
  - Spatially localized (Q-balls…)
  - …..