A giant artificial atom: Nonexponential decay


A driven resonator in the quantum regime: Period multiplication
A giant artificial atom: Nonexponential decay

Outline
- Atoms coupling to sound
- The size of an atom
  - Small, Large, or Giant atoms
- Experiments on a giant atom
  - Atom emission
  - Nonexponential decay
- Conclusions

Qubit coupling to propagating surface acoustic waves

Nonlinear reflection
Qubit decay by emitting phonons

Waveguide Quantum AcoustoDynamics
w-QAD

M. V. Gustafsson et al. Nature Physics 8, 338 (2012)
M. V. Gustafsson et al. Science 346, 207 (2014)
Coupling qubits to mechanical resonators

UChicago Qubit+SAW resonator

Yale: Qubit+Bulk acoustic wave resonator


Phonon Fock states
Surface Acoustic Waves (SAW)

SAW exist at different length scales, from earthquakes to filters in cell phones. They can be exited either mechanically or electrically using the piezoelectric effect. Confined to the surface within approximately $\lambda$

GHz frequencies
mK temperatures

$$\hbar \omega \gg k_B T$$

Very low powers
~ -130 dBm

Animation by L. Braile
Generating and detecting SAW with an IDT

- Piezoelectric substrate (GaAs, quartz, LiNbO₃…)
- Propagation speed: \( v \approx 2900 \text{ m/s} \)
- \( f \approx 2.3 \text{ GHz}, \lambda \approx 1.25 \mu\text{m} \)

- Generator and receiver:
  **The Interdigital Transducer (IDT)**

  \[
  f = \frac{v}{\lambda_0} \\
  \text{Finger distance, } p = \lambda \\
  \]

  Photon to phonon converter
  \[
  b^\dagger a \quad \text{anihilates a photon and creates a phonon} \\
  a^\dagger b \quad \text{anihilates a phonon and creates a photon} \\
  \]

  Datta, *Surface Acoustic Wave devices*, 1986
  Morgan, *Surface acoustic wave filters*, 2007
Superconducting qubits

- Quantized electrical circuit
- Harmonic oscillator is not an atom
- Nonlinearity makes the circuit anharmonic and addressable
- Small JJ is a good nonlinear inductor

Koch et al. PRA (2007)
The transmon qubit as an artificial atom

A capacitively shunted Cooper-pair box

\[ H = 4E_C(n - n_g)^2 - E_J \cos(\pi \frac{\Phi}{\Phi_0}) \cos \theta \]

\[ E_C = \frac{e^2}{2C}, \quad n = \frac{Q}{2e}, \quad n_g = \frac{C_g V_g}{2e} \]

\[ f_{01} \approx 4-8 \text{ GHz} \]
\[ \text{Anharmonicity} \approx E_C \approx 0.1-0.5 \text{ GHz} \]

Atom frequency is flux tunable

\[ f_{12} = \frac{\sqrt{8E_J E_C - 2E_C}}{\hbar} \]
\[ f_{01} = \frac{\sqrt{8E_J E_C - E_C}}{\hbar} \]

\[ \alpha = \hbar(f_{12} - f_{01}) \approx E_C \]

Jens Koch et al. PRA (2007)
A large SAW-coupled transmon qubit

The qubit has two important frequencies

- $f_{01}$ where it stores its energy
- $f_{QDT}$ where it couples to SAW

Qubit levels

$|0\rangle$

$|1\rangle$

$|2\rangle$

$\Gamma = \frac{1}{2\pi} \frac{K^2}{N_p} \frac{f_{01}(\Phi) - f_{QDT}}{f_{QDT}}$

$N_p = $ Number of finger pairs

$K^2 = 0.07\%$, the electromechanical coupling constant for GaAs
Atoms are normally small compared to the wavelength, $d \ll \lambda$

**Atomic physics**

$\lambda \sim 10^{-6} \text{ m}$
$d \sim 10^{-10} \text{ m}$

**Cavity QED**

$\lambda \sim 10^{-3} \text{ m}$
$d \sim 10^{-7} \text{ m}$

**Circuit QED**

$\lambda \sim 10^{-2} \text{ m}$
$d \sim 10^{-4} \text{ m}$

SAW coupled atoms are larger than the wavelength of SAWs

$\lambda_{\text{SAW}} = 1.25 \mu\text{m}$
Atoms coupled to sound are big

Normally, the size of an atom is much smaller than the wavelength of the bosonic field that the atom interacts with. \( d \ll \lambda \)

**Large atoms**

Atom larger than the wavelength

\[
d = N_p \lambda > \lambda
\]

\[
N_p > 1
\]

Dipolar approximation breaks down

This allows to put an “antenna” on the atom

Coupling can be tailored both in space and frequency

**Giant atoms**

Atom is long compared to the distance that the wave propagates during one lifetime

\[
d = N_p \lambda > v \tau
\]

\[
N_p > \frac{1}{\sqrt{\pi K^2}}
\]

a stronger requirement than \( N_p > 1 \) since \( K^2 \) is small

Emitted phonons can be reabsorbed.

Non-Markovian behavior, nonexponential decay
**Different regimes**

- **Small**
  - $d = \lambda$
  - Dipolar approximation holds

- **Large**
  - $d = \nu \tau$
  - Tailor emission in Frequency and space

- **Giant**
  - Non-Markovian dynamics

**Formulas**:
- $d = \text{atom size}$
- $\lambda = \text{wavelength of interacting field}$
- $\nu = \text{velocity of field}$
- $\tau = \text{relaxation time of atom}$

**References**
- Large atom in circuit QED setting without sound: B. Kannan et al, Nature **583**, 775 (2020)
The giant artificial atom

Aluminium in GaAs

$L = 450 \mu m$
$v_{SAW} = 2900 \text{ m/s}$

$T = L/v_{SAW} \approx 170 \text{ ns} > 1/2\gamma$

Predictions:
- Non-Markovian dynamics
- Nonexponential decay

Measured samples

Qubit, IDT and resonators made from Aluminum on GaAs

Type A samples

Sample parameters

<table>
<thead>
<tr>
<th>sample</th>
<th>$N_p$</th>
<th>$\gamma/2\pi$</th>
<th>$T$</th>
<th>$\gamma T$</th>
<th>$L$</th>
<th>$\gamma_{gate}/2\pi$</th>
<th>$2\gamma/\gamma_{ext}$</th>
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<td>A1</td>
<td>14</td>
<td>6.1 MHz</td>
<td>19 ns</td>
<td>0.8</td>
<td>55 $\mu$m</td>
<td>1.25 MHz</td>
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<tr>
<td>A2</td>
<td>14</td>
<td>4.4 MHz</td>
<td>46 ns</td>
<td>1.4</td>
<td>125 $\mu$m</td>
<td>1.5 MHz</td>
<td>1.8</td>
</tr>
<tr>
<td>A3</td>
<td>18</td>
<td>5.8 MHz</td>
<td>190 ns</td>
<td>7.0</td>
<td>550 $\mu$m</td>
<td>2.2 MHz</td>
<td>1.9</td>
</tr>
<tr>
<td>A4</td>
<td>18</td>
<td>5.3 MHz</td>
<td>190 ns</td>
<td>6.3</td>
<td>550 $\mu$m</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B1</td>
<td>14</td>
<td>4.8 MHz</td>
<td>160 ns</td>
<td>4.8</td>
<td>450 $\mu$m</td>
<td>-</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Giant atom if $\gamma T > 1$
Total coupling $\Gamma = 2\gamma$

Qubits designed with $f_{01}$ higher than $f_{QDT}$ and then tuned down by flux

$f_{IDT} = f_{QDT} = 2.29$ GHz
$Bw_{IDT} = 14$ MHz
$N_{p,IDT} = 150$

Type B samples

$f_{cavity} = 2.77$ GHz
$\Delta = 480$ MHz
$g = 15$ MHz
Acoustic transmission is probed at low power

\[ P < \hbar \omega \Gamma \]

A small atom would reflect on resonance, resulting in a Lorentzian line shape

We observe interference fringes in the acoustic transmission with a period of \(~5\text{MHz}\) agreeing well with \(1/190\text{ ns}\).

Limited bandwidth due to IDTs, 14 MHz
SAW emission from the giant atom

SAW emission probed by measuring the reflection from the gate (larger bandwidth)

\[ L = 550 \, \mu m \]

Reflectance from sample A3

Maximal emission (minimal reflection) when

\[ \omega_0 = \omega_d - \gamma \sin \omega_d T \]

\[ T = L / v_{\text{SAW}} \approx 190 \, \text{ns} \]
Reflectance depending on atom size

**Experiment**

Sample A1, $L = 55 \, \mu m$, $T = 19 \, ns$

ample A2, $L = 125 \, \mu m$, $T = 46 \, ns$

Sample A3, $L = 550 \, \mu m$, $T = 190 \, ns$

**Theory**
Measuring the giant atom via the mw-resonator

A superconducting cavity is dispersively coupled to the atom

\[ f_{01} = 2.29 \text{ GHz} \]
\[ f_{\text{cavity}} = 2.77 \text{ GHz} \]
\[ \Delta = 480 \text{ MHz} \]
\[ g = 15 \text{ MHz} \]

Dispersive regime

\[ |\Delta| = |\omega_{01} - \omega_r| \gg g \]
\[ \omega'_r = \omega_r + \frac{g^2}{\Delta} \sigma_z \]
**Spectroscopy of the giant atom**

**Two tone spectroscopy**
- Fixed readout at $f_{\text{cavity}}$
- Sweep drive frequency close to $f_{01}$

Small atom gives Lorentzian

Giant atom gives multi peaked structure with Lorentzian envelope

Peak distance depends on $1/T$
Lorentzian width depends on $\gamma$
# of peaks $\approx \gamma T$

Red curve is theory
L. Guo et al. PRA 95, 053821 (2017)
We apply a $\pi$-pulse and do a weak continuous measurement of the population in the qubit.

Averaging a large number ($10^7$) of weak measurements, we observe revivals in the population at times that agree well with the distance between the coupling points. $T = 160$ ns

Summary I: Giant atoms

- Artificial atoms in a new regime: Large atoms and Giant atoms
- Non-Markovian dynamics
- Nonexponential decay with revivals

Period-tripling in a superconducting resonator

- **Nonlinear resonators**
  - Parametric pumping
  - Subharmonic oscillations, period tripling
  - Further multiples

Generating three photons from one, $a_1^+ a_1 a_2$

A flux tunable resonator

$\lambda/4$ Coplanar Waveguide
terminated by a Josephson inductance of a SQUID

Resonance frequency, $f$

$$f(\Phi) = \frac{f_0}{1 + \frac{L_{SQ}(\Phi)}{L_{cav}}}$$

$$L_{SQ}(\Phi) = \frac{\hbar}{2eI_c \cos(\pi \frac{\Phi}{\Phi_0})}$$

$$\Phi = \Phi_{dc} + \Phi_A \sin\left(2\pi f_D t\right)$$

Time varying flux gives time varying resonance frequency

Resonance frequency can be varied faster than the oscillation period

M. Sandberg et al. APL 92, 203501 2008
Parametric oscillations: Flux pumping at twice the resonance frequency

Pumping at $f_D \approx 2f_0$, vary amplitude and frequency we observe photons coming out at $f_0$
Note $\lambda/4$: no mode at $2f_0$

Theory:

Experiment:

Flux pumping at $3f_0$, nothing should happen
Sample and Measurement configuration

$$L_{SQ}(t) = \frac{L_{SQ,0}}{\cos(\pi \frac{\Phi(t)}{\Phi_0}) \sqrt{1 - \frac{I(t)^2}{I_C^2}}}$$

- Flux pump
- Current drive

Aluminum SQUID
Niobium resonator
Sapphire substrate

External drive
AC flux pump
Down-converter
DC flux bias

12 mK
Chip
Coil
50 Ω
The current pumped resonator

\[ \lambda/4 \text{-cavity with a SQUID at the end} \]

**Resonator properties at zero flux**

- Resonance frequency, \[ \omega_1/2\pi = 5.504 \text{ GHz} \]
- Mode linewidth first mode, \[ 2\Gamma_1/2\pi = 0.38 \text{ MHz} \]
- First mode quality factors, \( Q_{int} = 61000, Q_c = 19000 \)
- Spectrum anharmonicity, \( (3\omega_1 - \omega_2)/2\pi = 136 \text{ MHz} \)
Current driving at $\Phi=0$: Observation of period-tripling

Model with two coupled modes, Vitaly Shumeiko

\[
\delta_1 = \omega - \omega_1 \\
\delta_2 = 3\omega - \omega_2
\]

The second mode acts as a parametric pump of the first mode

\[
i\dot{a}_1 + (\delta_1 + i\Gamma_1 + \alpha_1 |a_1|^2 + 2\alpha |a_2|^2)a_1 + \tilde{\alpha}a_1^*a_2 = 0
\]

\[
i\dot{a}_2 + (\delta_2 + i\Gamma_2 + \alpha_2 |a_2|^2 + 2\alpha |a_1|^2)a_2 + \frac{\tilde{\alpha}}{3}a_1^3 = \sqrt{2\Gamma_{2,\text{ext}}}B_2
\]

Quantum description

Experiment and theory:
Subharmonic oscillation region

Regions:

I  Silent state
II  Oscillating state
III Oscilating state exists but not energetically favorable

More properties of period-tripling subharmonic oscillations

Increasing intensity with $\delta_1$
Decreasing intensity with flux

$\delta_1 = \omega - \omega_1$
$\Gamma_1$, damping rate

Threshold frequency
$\delta_1 \leq -\sqrt{7}\Gamma_1 = \delta_{th}$
What about higher order pumping

- \( f_{\text{pump}} = n f_0, \)
- \( n=2 \) flux pumping, \( n=3 \) Current pumping
- Good agreement with theory

- Pumping at higher order multiples can also generate subharmonic states
- \( n = 4 \) and \( 5 \) also observed (no theory yet)

- Phase space crystals
- Similar to time crystals

Summary II: Period tripling

• Current pumping a multimode nonlinear oscillator close to a higher mode can generate oscillations at the fundamental mode which have an n-fold phase symmetry.
• The oscillations in the higher mode act as a parametric pump for the fundamental mode.
• Observation of period tripling, n=3, good agreement with theory. n = 4, 5 also observed.
• Hamiltonians of the type $a_1^n a_2$ implemented.

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