FRACTIONALIZATION OF CHA STATISTICS IN TWO DIMENS

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Topics to cover…

- Anyons from an experimentalist's view

- Short introduction to the 2DEG in semiconductors and FQHE

- How can we "measure" anyons?
  Challenges of operation of electronic Fabry-Perot interferometers in the quantum Hall regime

- Determination of anyonic braiding statistics at ν=1/3

- Beyond ν=1/3 in the lowest Landau level; ν=2/3 and ν=2/5

- Routes to ν=5/2 interferometry

Elementary particles

Emergent particles in 2D

FERMIONS

Pauli Exclusion Principle
Fermions switch places:
(-1) * Wavefunction

electrons
quarks
muons
neutrons
protons

BOSONS

Flock Together
Bosons switch places:
(1) * Wavefunction

photons
gluons
Higgs boson
gravitons

ANYONS

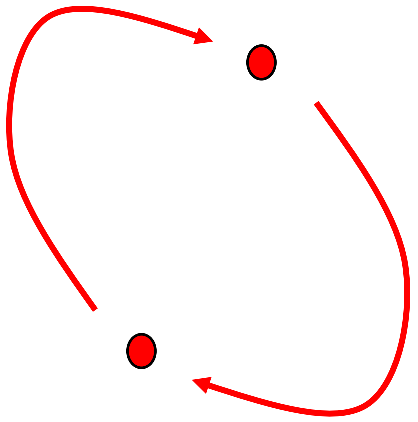
Novel Quantum Statistics
- Fractional charge
- Anyonic braiding statistics

($e^{i\boldsymbol{\theta}}$) * Wavefunction

Quasiparticles in the fractional quantum Hall state at ν=1/3

$\boldsymbol{\theta}=2\pi/3$

What is an Anyon? Quantum Statistics in 2 dimensions



Quantum Statistics:

What happens to a many-particle wave function under exchange or “braiding” of identical particles

3 Dimensions: Bosons or Fermions

Bosons: $\psi(x_1, x_2, x_3,\ldots) \longrightarrow \psi(x_2, x_1, x_3,\ldots)$ same wave function: phase factor 2π

Fermions: $\psi(x_1, x_2, x_3,\ldots) \longrightarrow -1^*\psi(x_2, x_1, x_3,\ldots)$ same wave function multiplied by -1: phase factor π

In 3D: : Two permutations equivalent to identity operation

In 2D: Can have fractional statistics (Abelian anyons) :$e^{i2\pi\alpha}$ (e.g., ν=1/3 FQHE)
Why? Because world lines can form knots in 2D

Non-Abelian anyons possible as well (e.g., ν=5/2 FQHE, topological superconductors)
Braiding corresponds to a unitary rotation within a degenerate manifold of states

Leinaas and Myrheim, *Nuovo Cimento* B **37**, 1-23 (1977)

F. Wilczek, PRL **49**, 957-959 (1982) “anyons” defined for the first time

Early analysis of fractional statistics and the FQHE

Quantum Mechanics of Fractional-Spin Particles

Frank Wilczek
Institute for Theoretical Physics, University of California, Santa Barbara, California 9310
(Received 22 June 1982)

Composites formed from charged particles and vortices in (2+1)-dimensional models, or flux tubes in three-dimensional models, can have any (fractional) angular momentum. The statistics of these objects, like their spin, interpolates continuously between the usual boson and fermion cases. How this works for two-particle quantum mechanics is discussed here.

Statistics of Quasiparticles and the Hierarchy of Fractional Quantized Hall States

B. I. Halperin
Physics Department, Harvard University, Cambridge, Massachusetts 02138
(Received 9 November 1983)

Quasiparticles at the fractional quantized Hall states obey quantization rules appropriate to particles of fractional statistics. Stable states at various rational filling factors may be constructed iteratively by adding quasiparticles or holes to lower-order states, and the corresponding energies have been estimated.

Although practical applications of these phenomena seem remote, I think they have considerable methodological interest and do shed light on the fundamental spin-statistics connection.

The appearance of fractional statistics in the present context is strongly reminiscent of the fractional statistics introduced by Wilczek to describe charged particles tied to "magnetic flux tubes" in two dimensions.[6]

Fractional Statistics and the Quantum Hall Effect

Daniel Arovas
Department of Physics, University of California, Santa Barbara, California 93106

and

J. R. Schrieffer and Frank Wilczek
Department of Physics and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 18 May 1984)

The statistics of quasiparticles entering the quantum Hall effect are deduced from the adiabatic theorem. These excitations are found to obey fractional statistics, a result closely related to their fractional charge.

PACS numbers: 73.40.Lq, 05.30.−d, 72.20.My

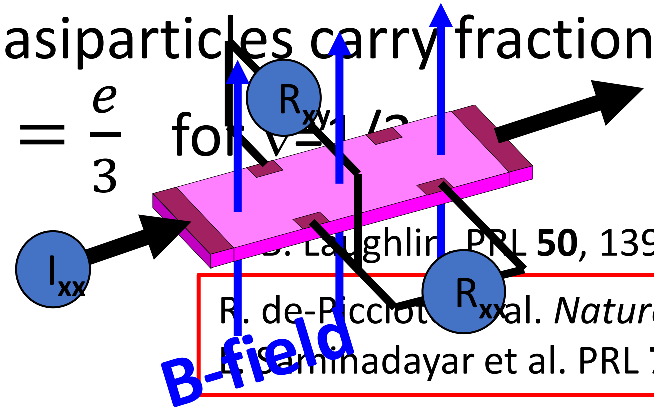
Excitations of the FQHE are anyons: fractional c

- Quasiparticles carry fractional charge

$$e^* = \frac{e}{3} \quad \text{for } \nu=1/3$$

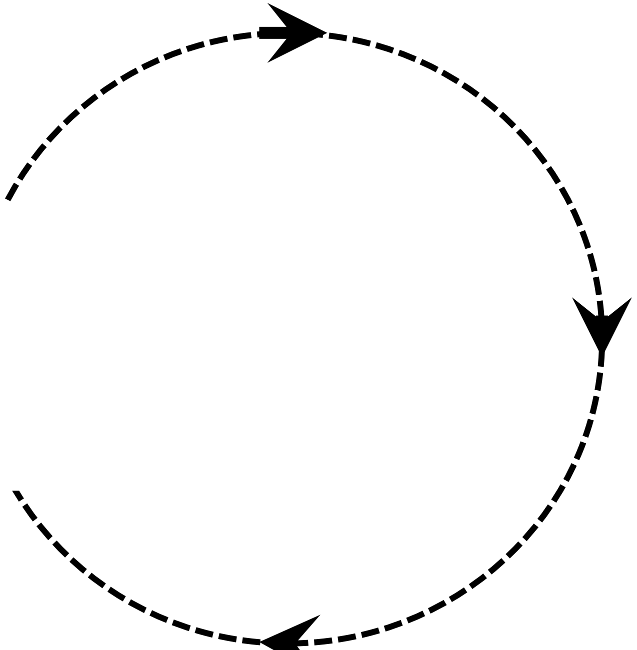
R. B. Laughlin, PRL **50**, 1395 (1983)

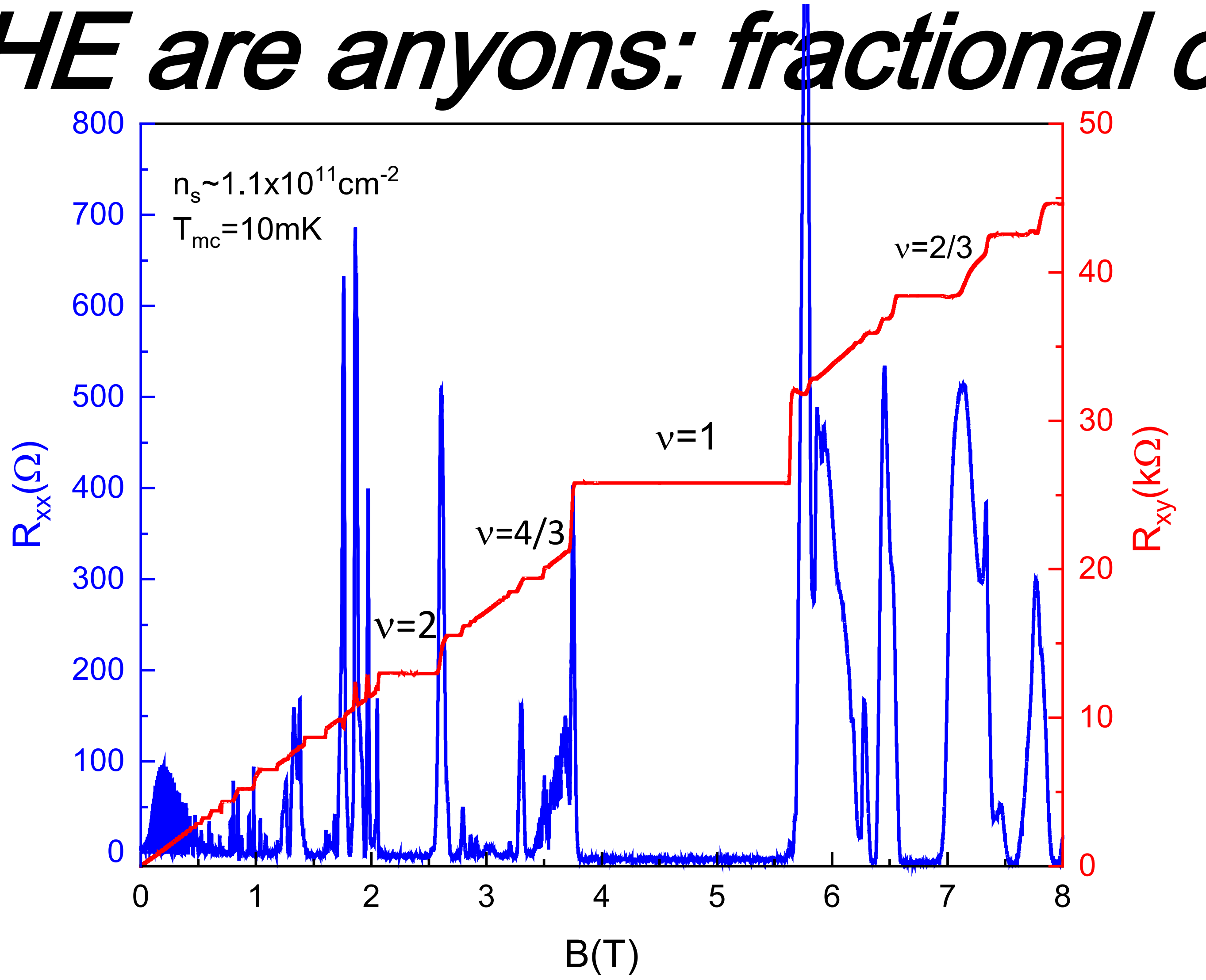
R. de-Picciotto et al. *Nature* **389**, 162 (1997)
L. Saminadayar et al. PRL **79,** 2526 (1997)



- Anyonic braiding statistics: $\theta_{anyon} = 2\pi \frac{e^*}{e}$

B. I. Halperin, PRL **52**, 1583 (1984)
Arovas, Schrieffer, and Wilczek, PRL **53**, 722 (1984)



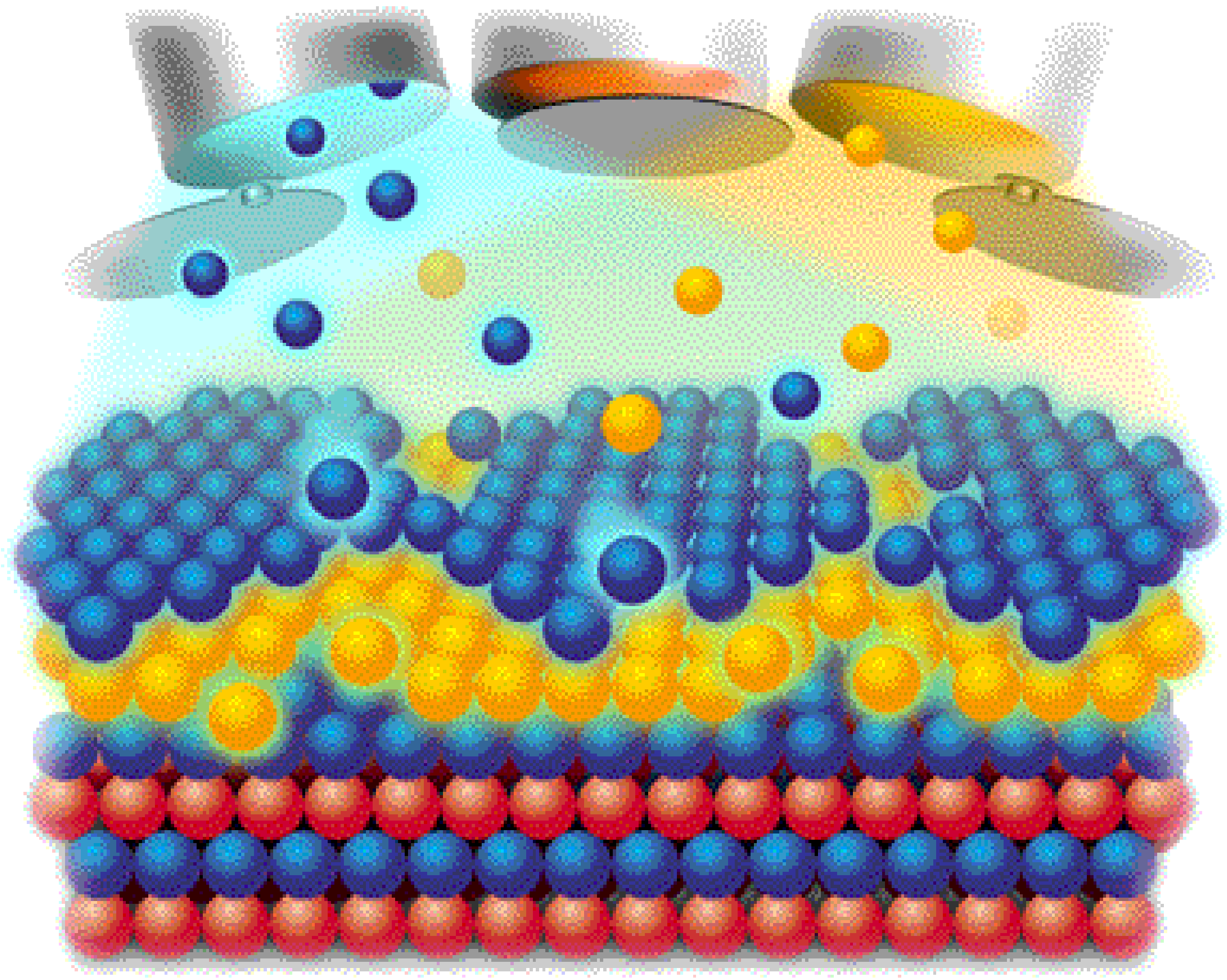


$$\boxed{\nu=1/3:\ \theta_{anyon} = \frac{2\pi}{3}}$$

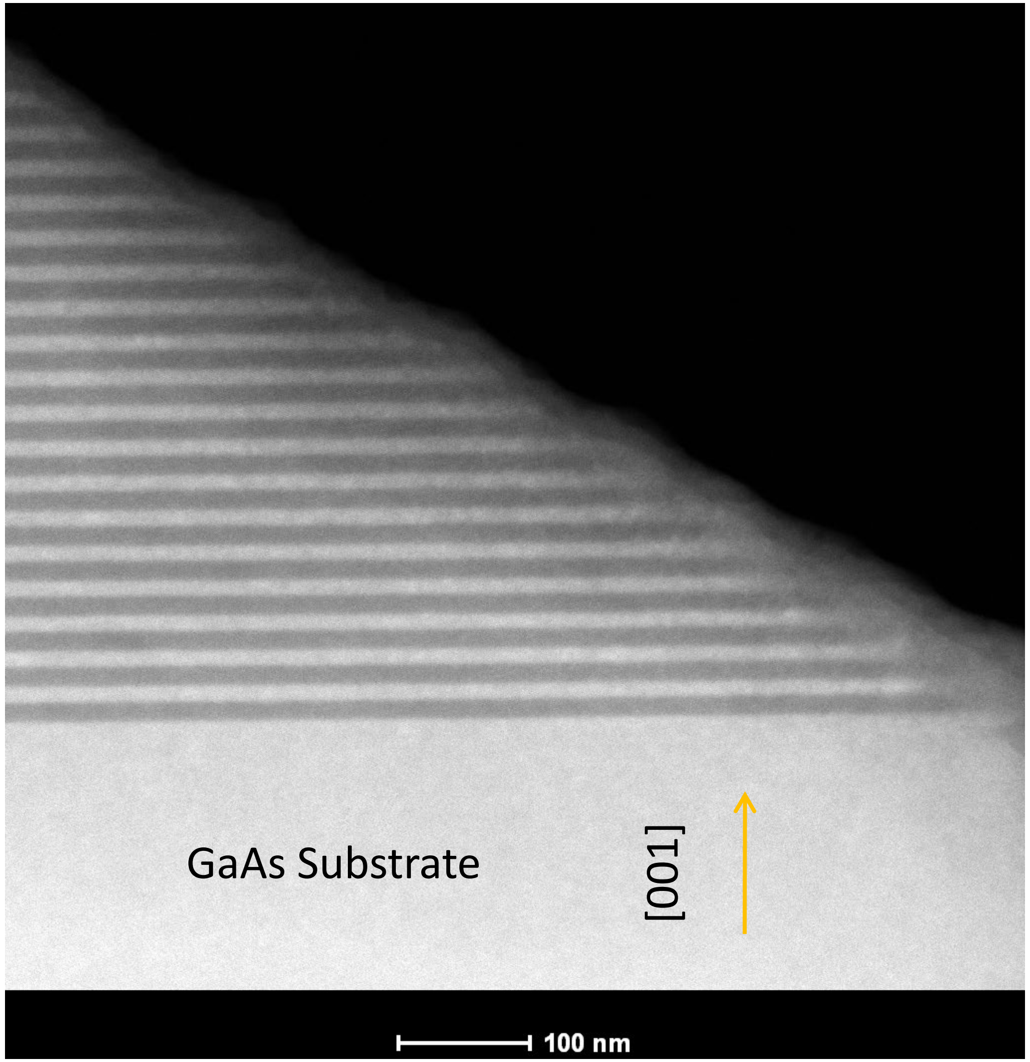
How does one make a two-dimensional electron gas and what’s the quantum Hall effect?

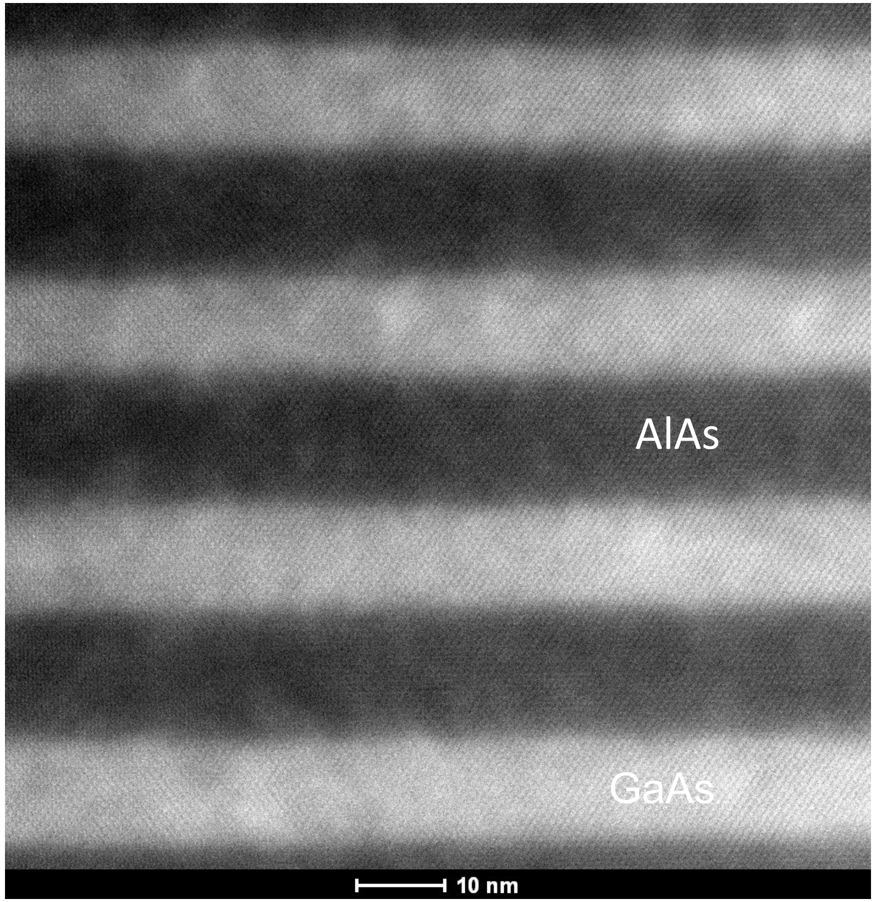
How does one make a two-dimensional electron gas?

Molecular Beam Epitaxy: (Al Cho, Bell Labs, 1970's)

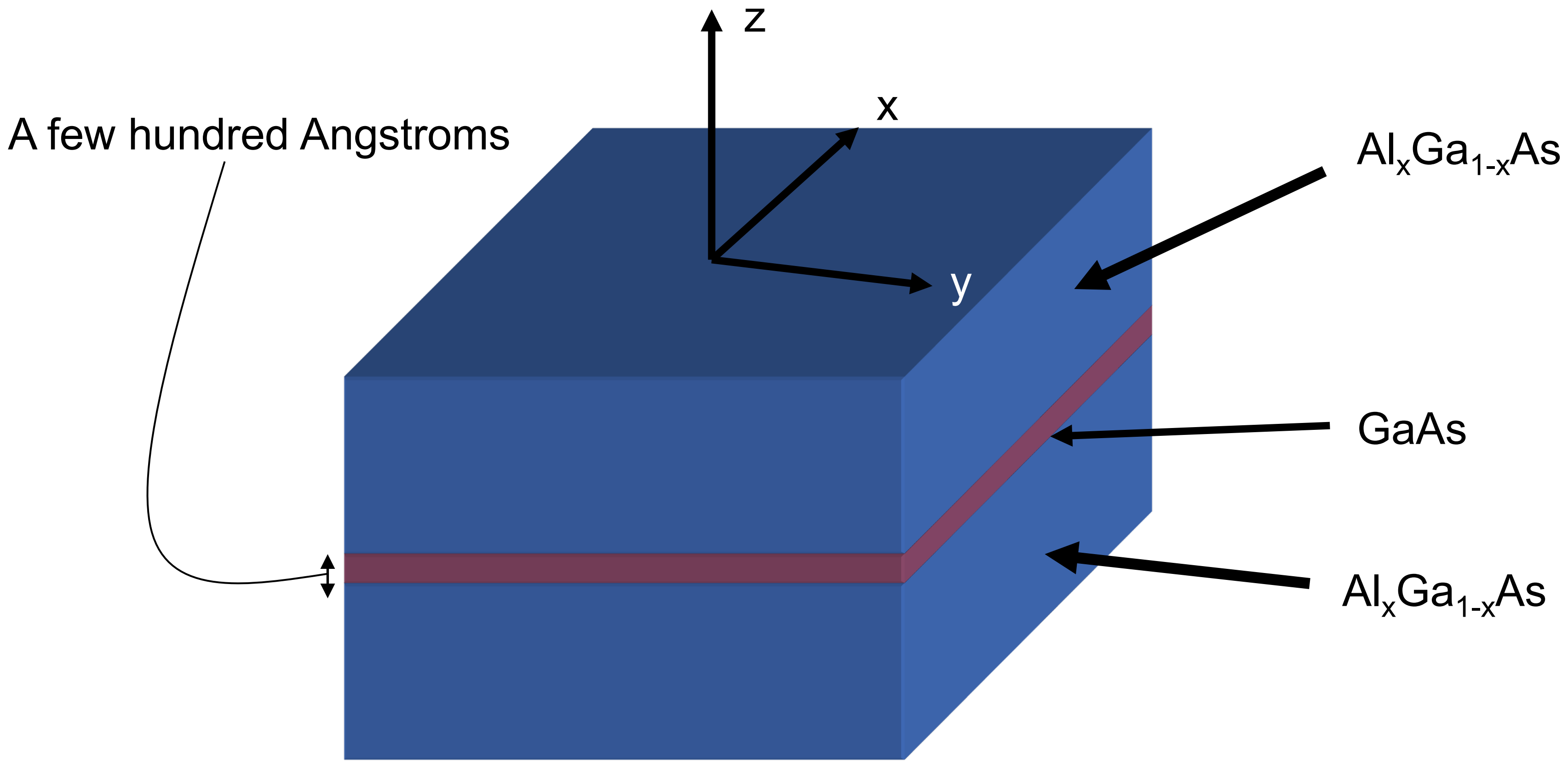


TEM images of AlAs/GaAs superlattice

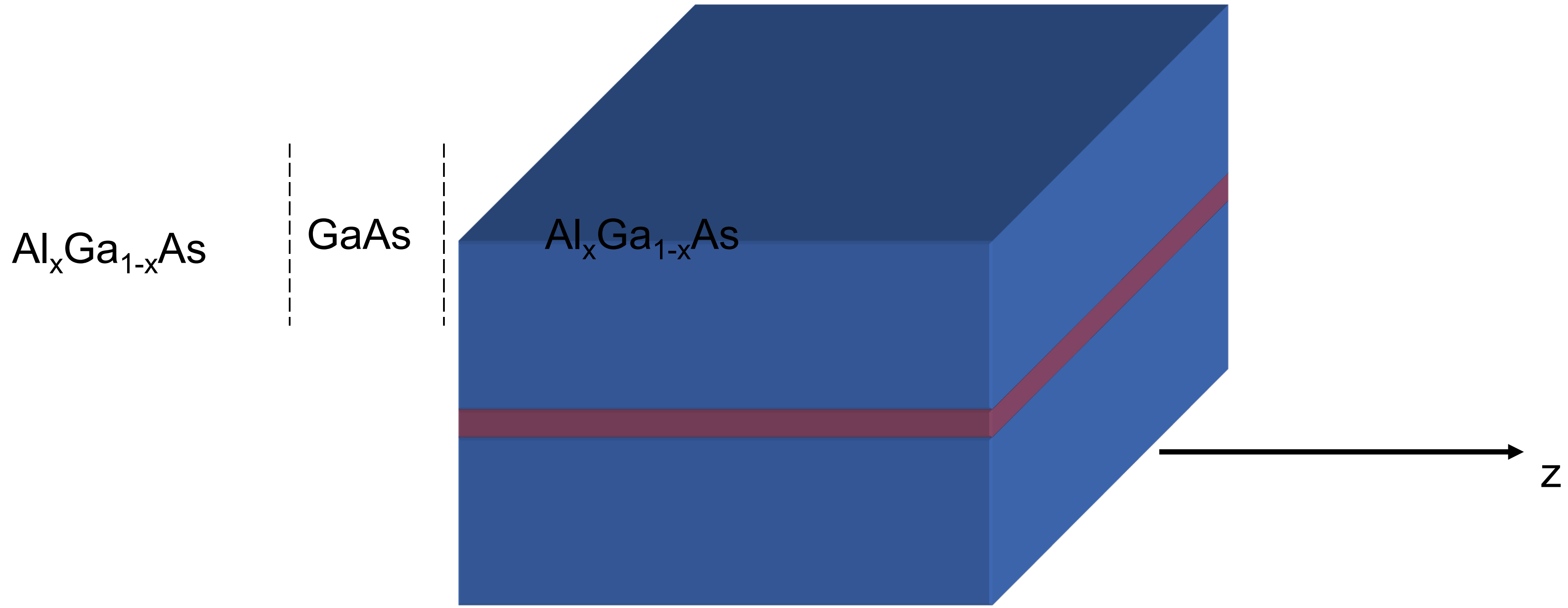


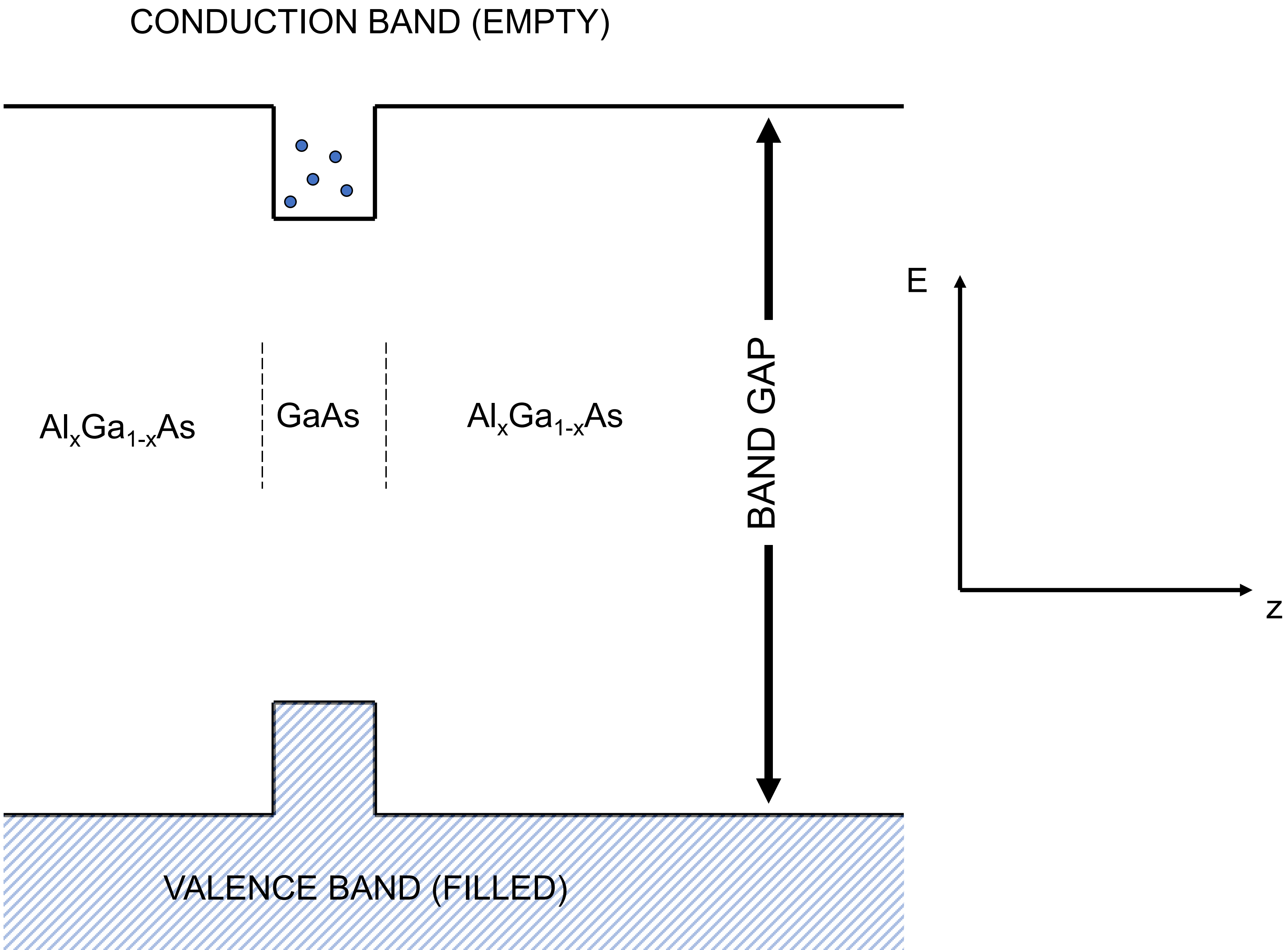


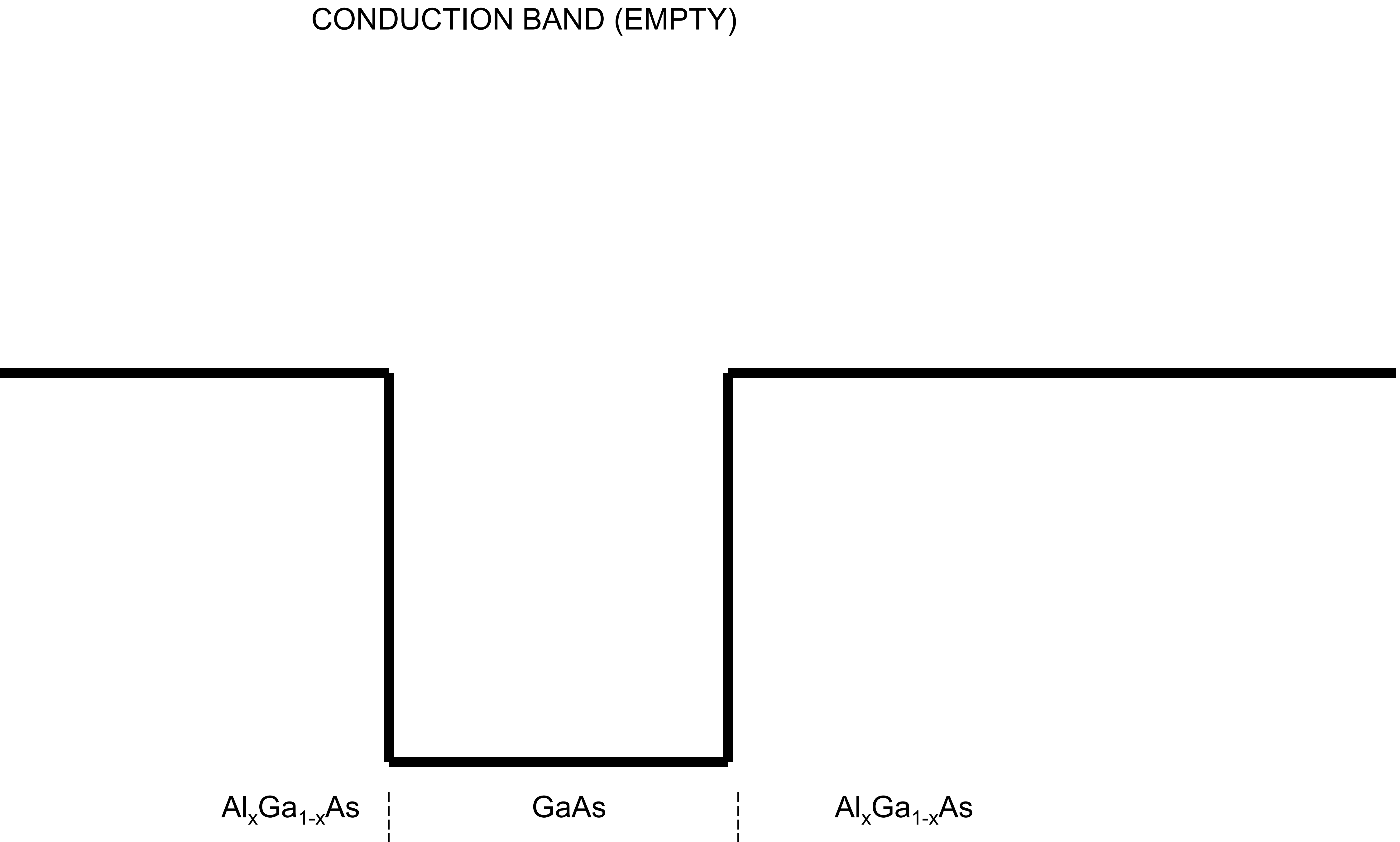
The AlAs/GaAs interface is among the most perfect in all of nature.



Translational Invariance in x-y plane

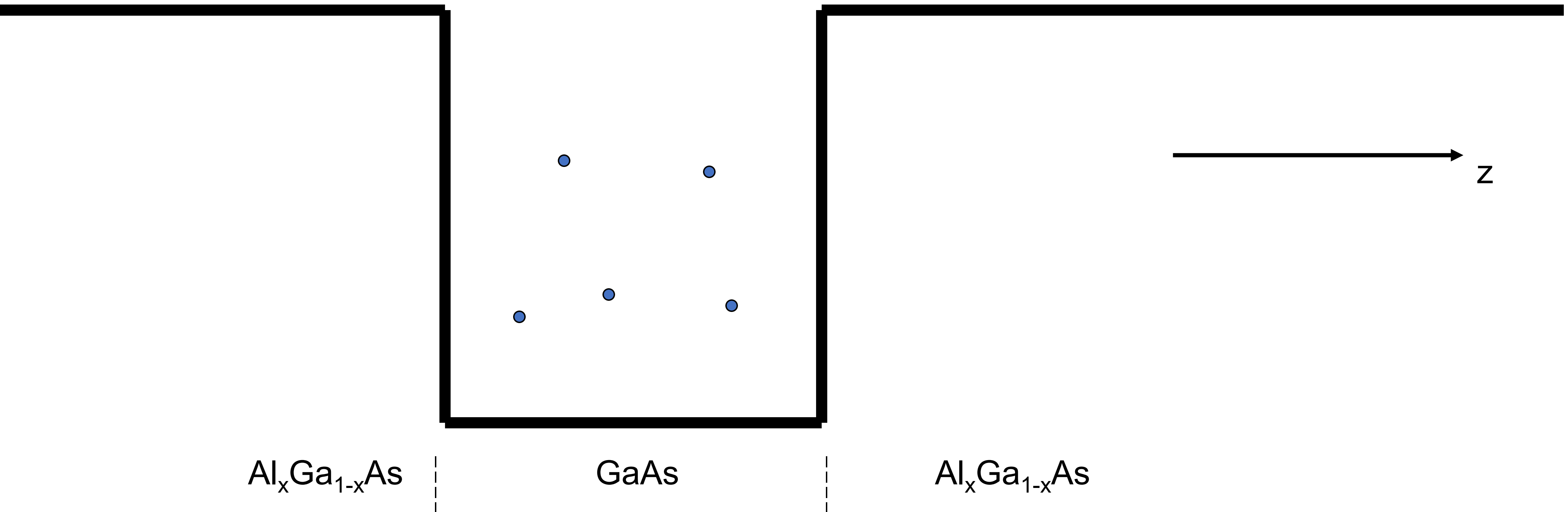

$Al_xGa_{1-x}As$
GaAs
$Al_xGa_{1-x}As$
z


CONDUCTION BAND (EMPTY)
$Al_xGa_{1-x}As$
GaAs
$Al_xGa_{1-x}As$
BAND GAP
E
z
VALENCE BAND (FILLED)


CONDUCTION BAND (EMPTY)
$Al_xGa_{1-x}As$
GaAs
$Al_xGa_{1-x}As$

$$H = H(x,y) + H(z)$$

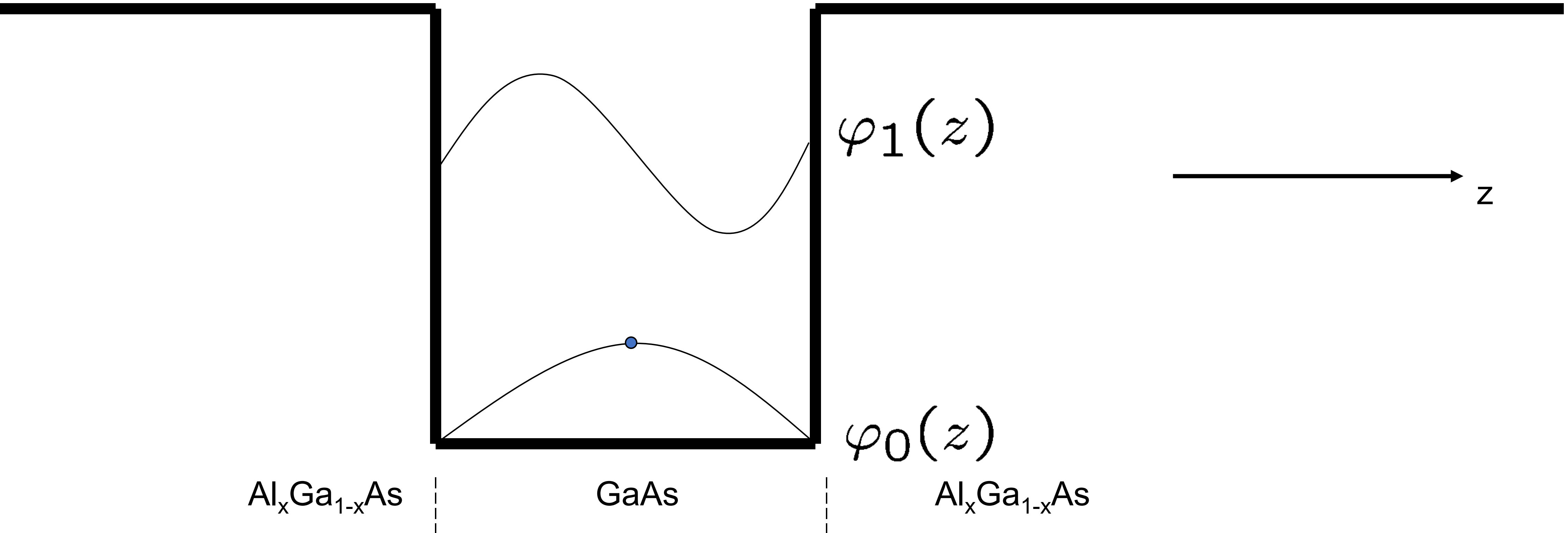
$$\Psi(x,y,z) = \psi(x,y)\varphi(z)$$



$$H = H(x,y) + H(z)$$
$$\Psi(x,y,z) = \psi(x,y)\varphi(z)$$

LOW!! T

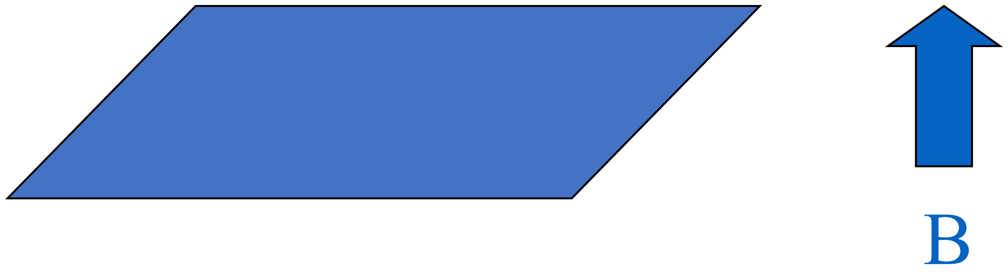
Z-motion is FROZEN OUT
Strictly 2-D Motion



Simple theory of the QHE

(ignore spin)

$$\nabla \times \vec{A}(\vec{r}) = B\hat{z}$$



$$H = \sum_j \frac{\left[\vec{p}_j - \frac{e}{c}\vec{A}(\vec{r}_j)\right]^2}{2m} + \sum_j U(\vec{r}_j) + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

Kinetic Energy in Magnetic Field

Disorder Potential

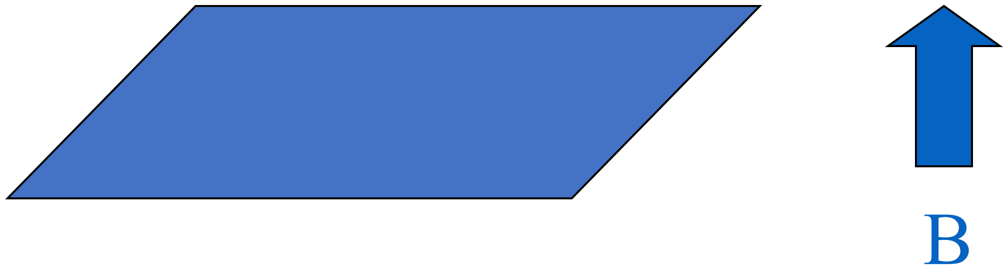
Electron-Electron Interactions

(necessary for FQHE, But not for IQHE)

Simple theory of the QHE

(ignore spin)

$$\nabla \times \vec{A}(\vec{r}) = B\hat{z}$$



$$H_0 = \sum_j \frac{\left[\vec{p}_j - \frac{e}{c}\vec{A}(\vec{r}_j)\right]^2}{2m} + \sum_j U(\vec{r}_j) + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

Kinetic Energy in Magnetic Field

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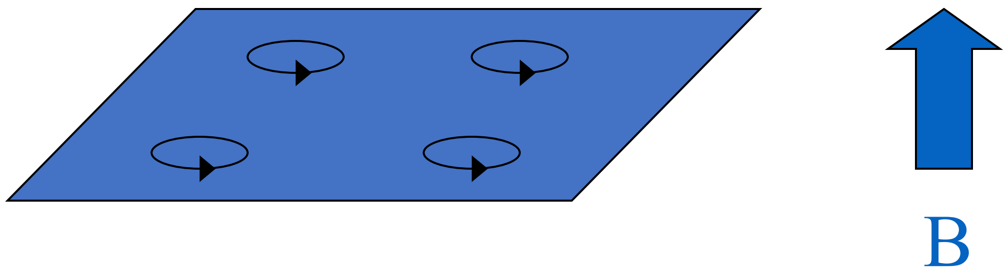
Simple theory of the QHE

(ignore spin)

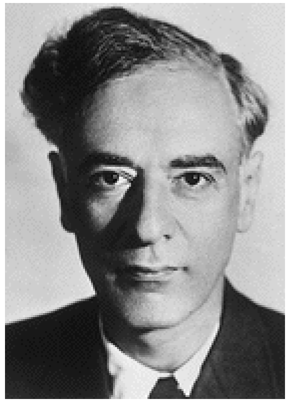
$$\omega_c = \frac{eB}{mc}$$

Cyclotron frequency



$$H_0 = \frac{\left[\vec{p} - \frac{e}{c}\vec{A}(\vec{r})\right]^2}{2m}$$

Free Electron in 2D
in a Magnetic Field


Lev Landau

Eigenstates: $\psi_{n,k}(\vec{r})$

Eigenenergies: $\epsilon_{n,k} = \hbar\omega_c(n + \frac{1}{2})$

"Landau Level" (orbital quantization)

Positional Degeneracy = (Area) B / ϕ_0

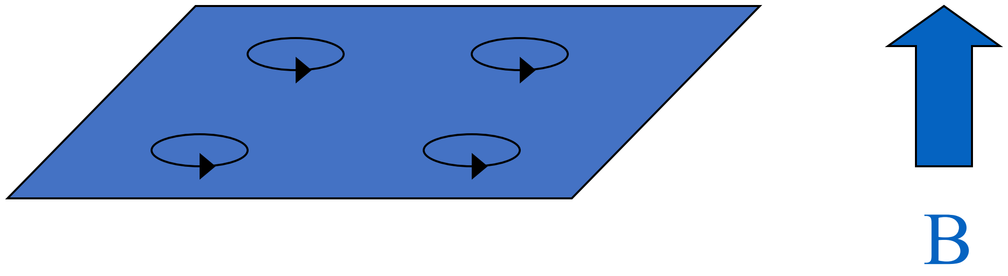
$\phi_0 = 2\pi\hbar c/e$ = flux quantum

Simple theory of the QHE

(ignore spin)

$$\omega_c = \frac{eB}{mc}$$

Cyclotron frequency



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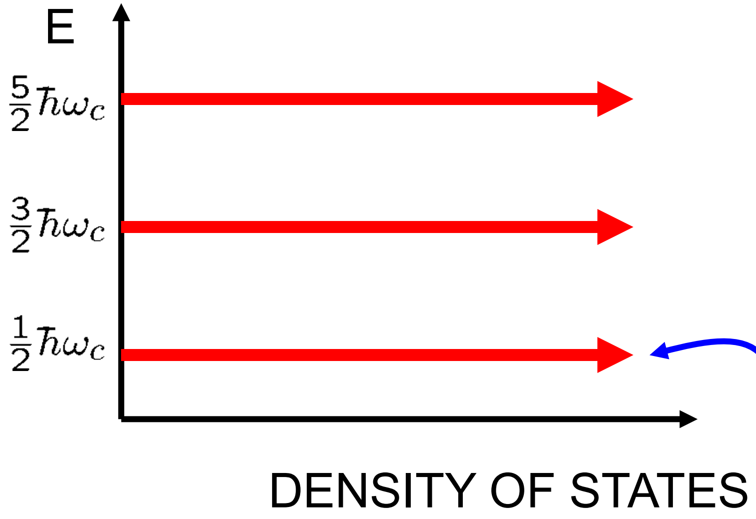
Free Electron in 2D
in a Magnetic Field



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"Landau Level" (orbital quantization)

Positional Degeneracy = (Area) B / ϕ_0

$\phi_0 = 2\pi\hbar c/e$ = flux quantum

$$\nu = \frac{N\phi_0}{(Area)B} = \frac{n\phi_0}{B}$$ = # Landau Levels Filled With Electrons

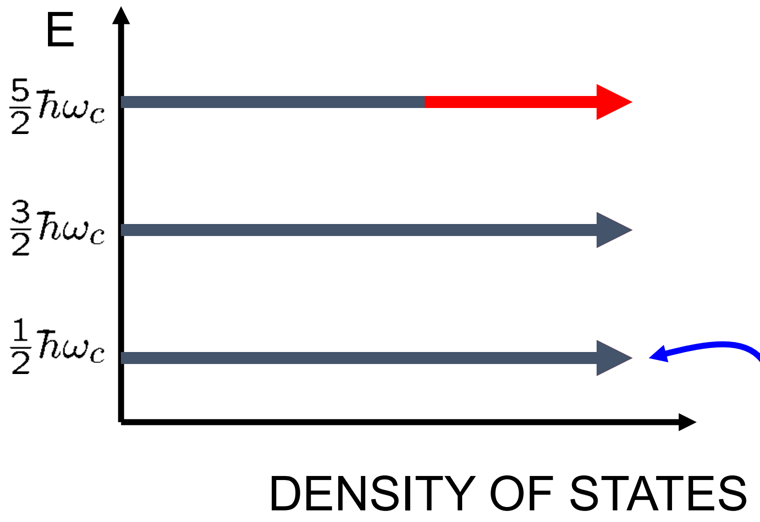
= Filling Fraction

$$H_0 = \frac{\left[\vec{p} - \frac{e}{c}\vec{A}(\vec{r})\right]^2}{2m}$$

Free Electron in 2D
in a Magnetic Field

Eigenstates: $\psi_{n,k}(\vec{r})$

Eigenenergies: $\epsilon_{n,k} = \hbar\omega_c(n + \frac{1}{2})$



"Landau
Level"
(orbital
quantization)

Positional Degeneracy = (Area) B / ϕ_0

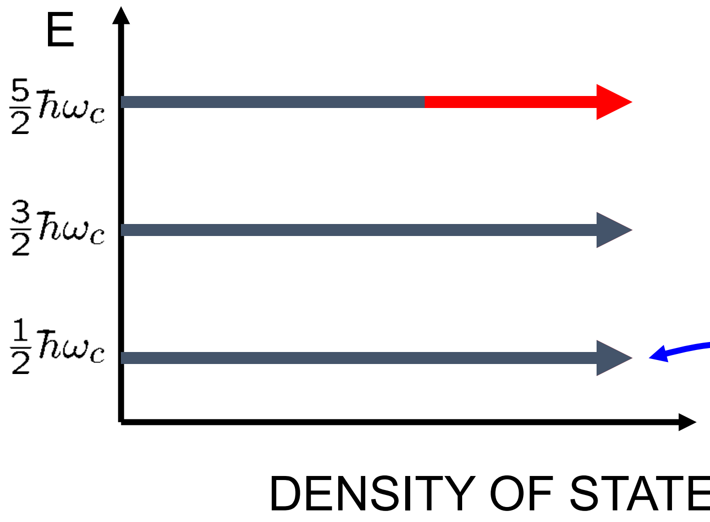
$\phi_0 = 2\pi\hbar c/e$ = flux quantum

$$\nu = \frac{N\phi_0}{(Area)B} = \frac{n\phi_0}{B}$$ = # Landau Levels Filled With Electrons

= Filling Fraction

Incompressible when
An integer number of
Landau Levels are full

Compressible when
A fractional number of
Landau Levels are full
(in noninteracting picture)

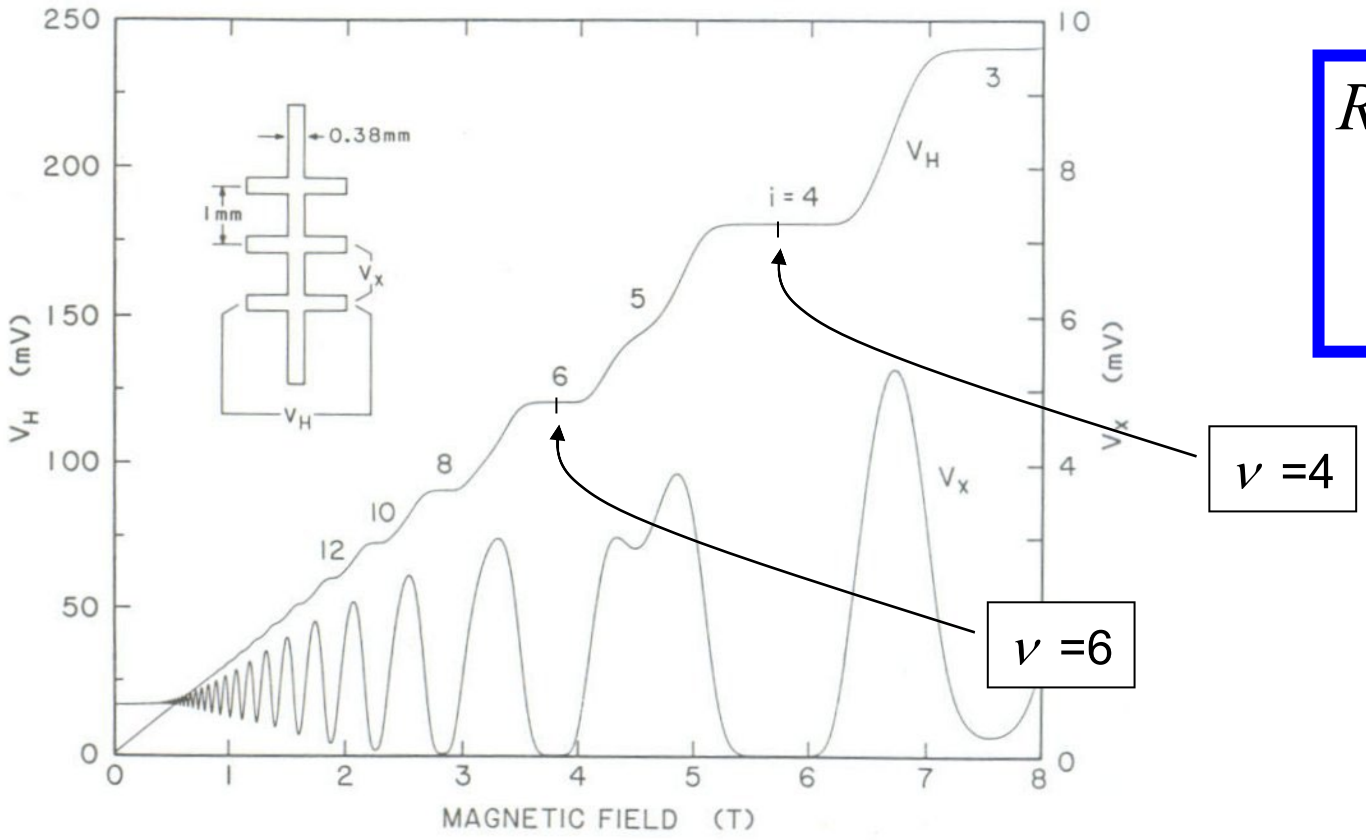


Positional Degeneracy = (Area) B / ϕ_0

But why the Plateau?

$$\nu = \frac{N\phi_0}{(Area)B} = \frac{n\phi_0}{B}$$ = # Landau Levels Filled With Electrons

= Filling Fraction



$$R_H = V_H / I = \frac{2\pi\hbar}{e^2}\frac{1}{\nu}$$

Simple theory of the QHE

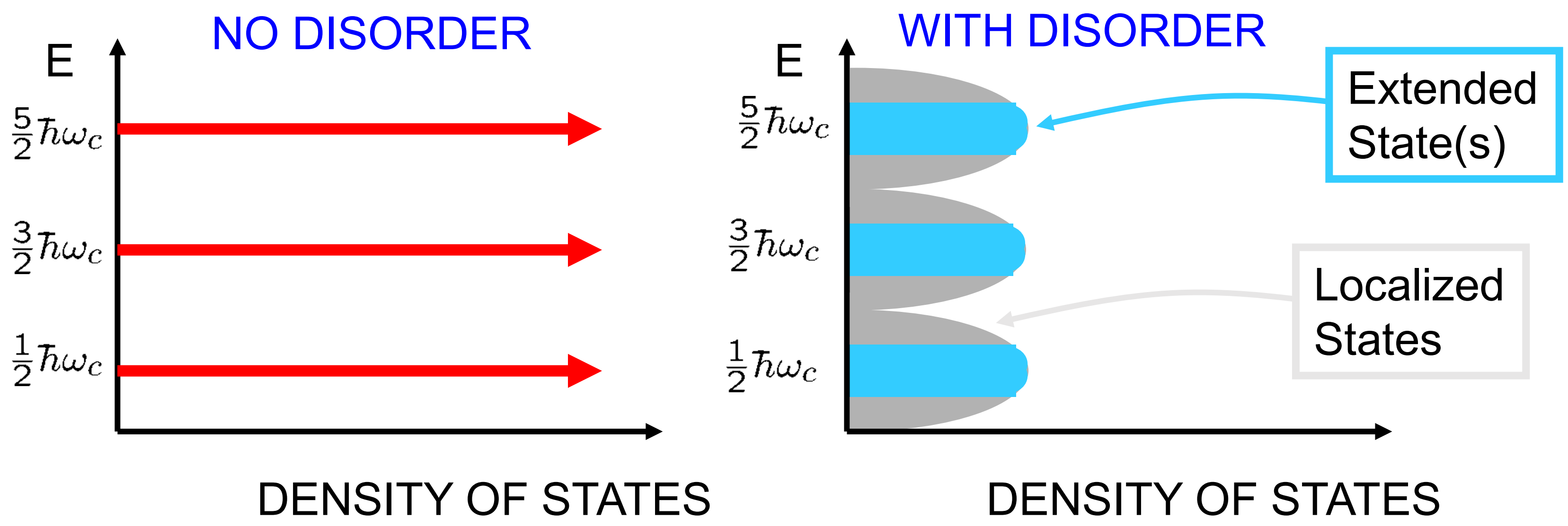
NOT ENOUGH IN THIS SIMPLE THEORY!

RESTORE DISORDER

$$H=\sum_j \frac{\left[\vec{p}_j-\frac{e}{c}\vec{A}(\vec{r}_j)\right]^2}{2m} + \sum_j U(\vec{r}_j)$$

Free Electrons in 2D
in a Magnetic Field

+ Disorder Potential



Why Hall Quantization?
Why So Precise?

Laughlin 1981 :
Must be some “conservation” law…

Gauge Invariance

Bob Laughlin

Bert Halperin

Detailed arguments, importance of edge states (1982)

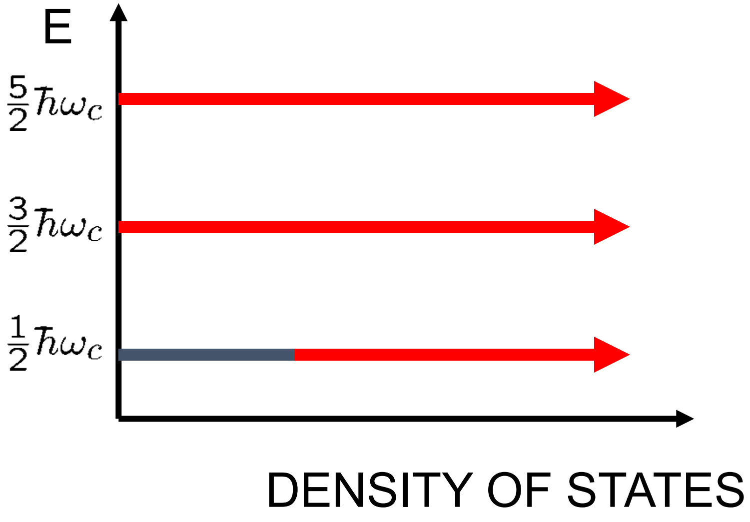
FQHE: What’s is missing?

$$H = \sum_j \frac{\left[\vec{p}_j - \frac{e}{c}\vec{A}(\vec{r}_j)\right]^2}{2m} + \sum_j U(\vec{r}_j) + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

Kinetic Energy in Magnetic Field

Disorder Potential

Electron-Electron Interactions



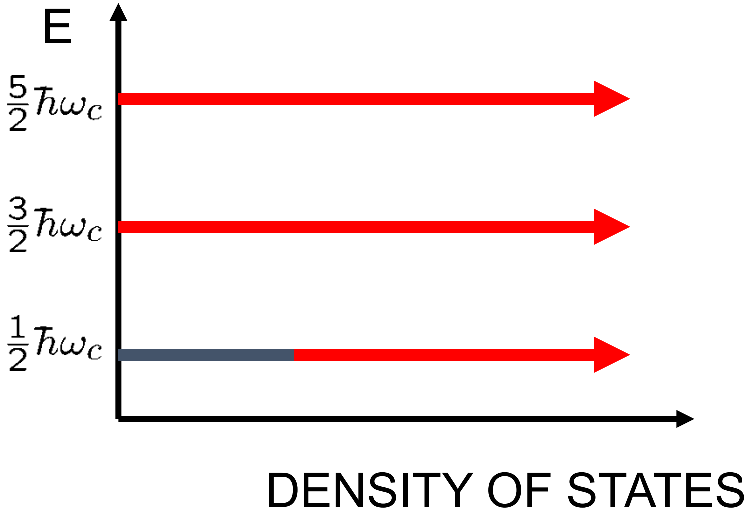
HUGE DEGENERACY

VERY COMPRESSIBLE

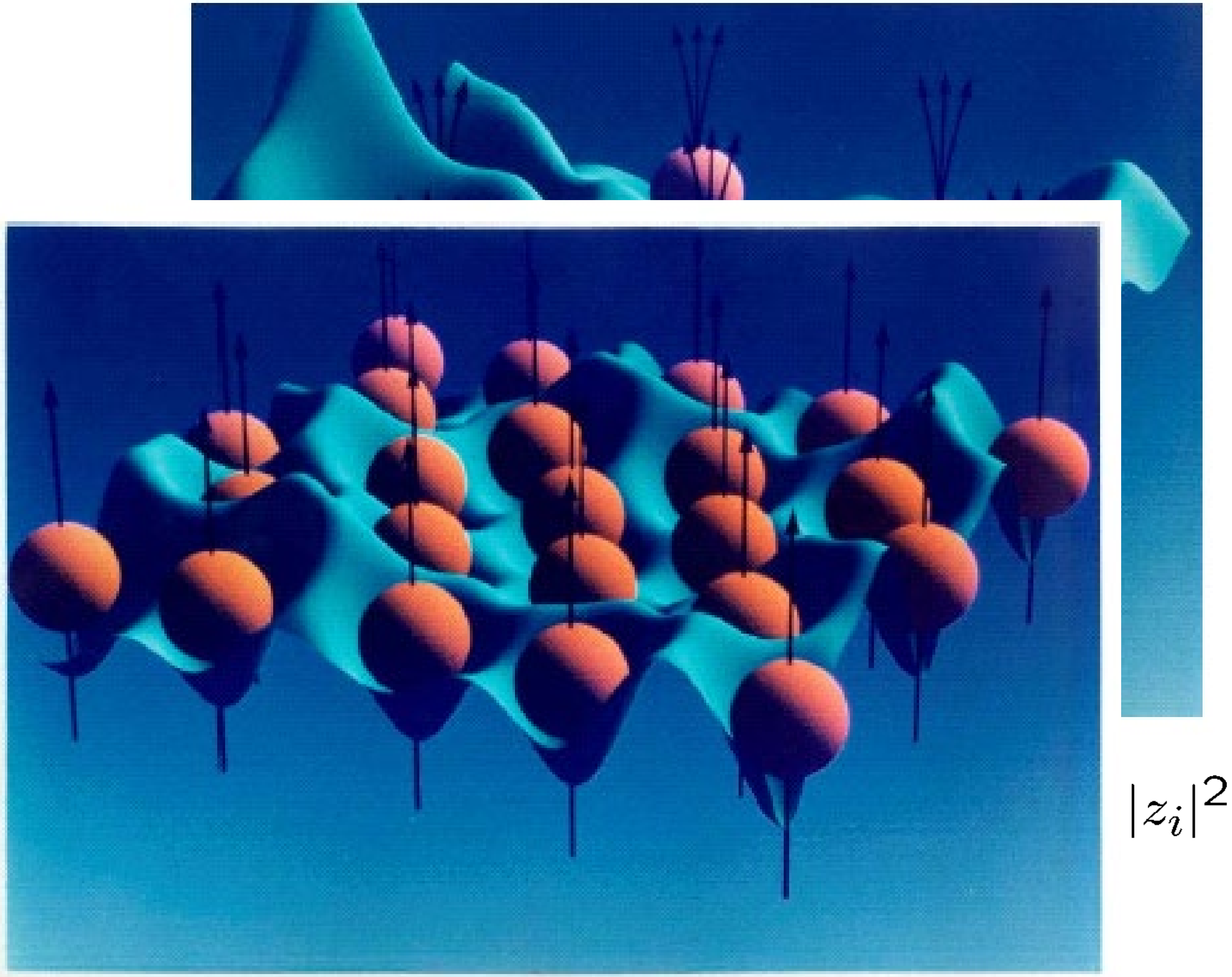
$$H = \sum_j \frac{\left[\vec{p}_j - \frac{e}{c}\vec{A}(\vec{r}_j)\right]^2}{2m} + \sum_{i<j} V(\vec{r}_i - \vec{r}_j)$$

Kinetic Energy in Magnetic Field

Electron-Electron Interactions



INCOMPRESSIBLE?



Bob Laughlin

PRL **50**, 1395 (1983)

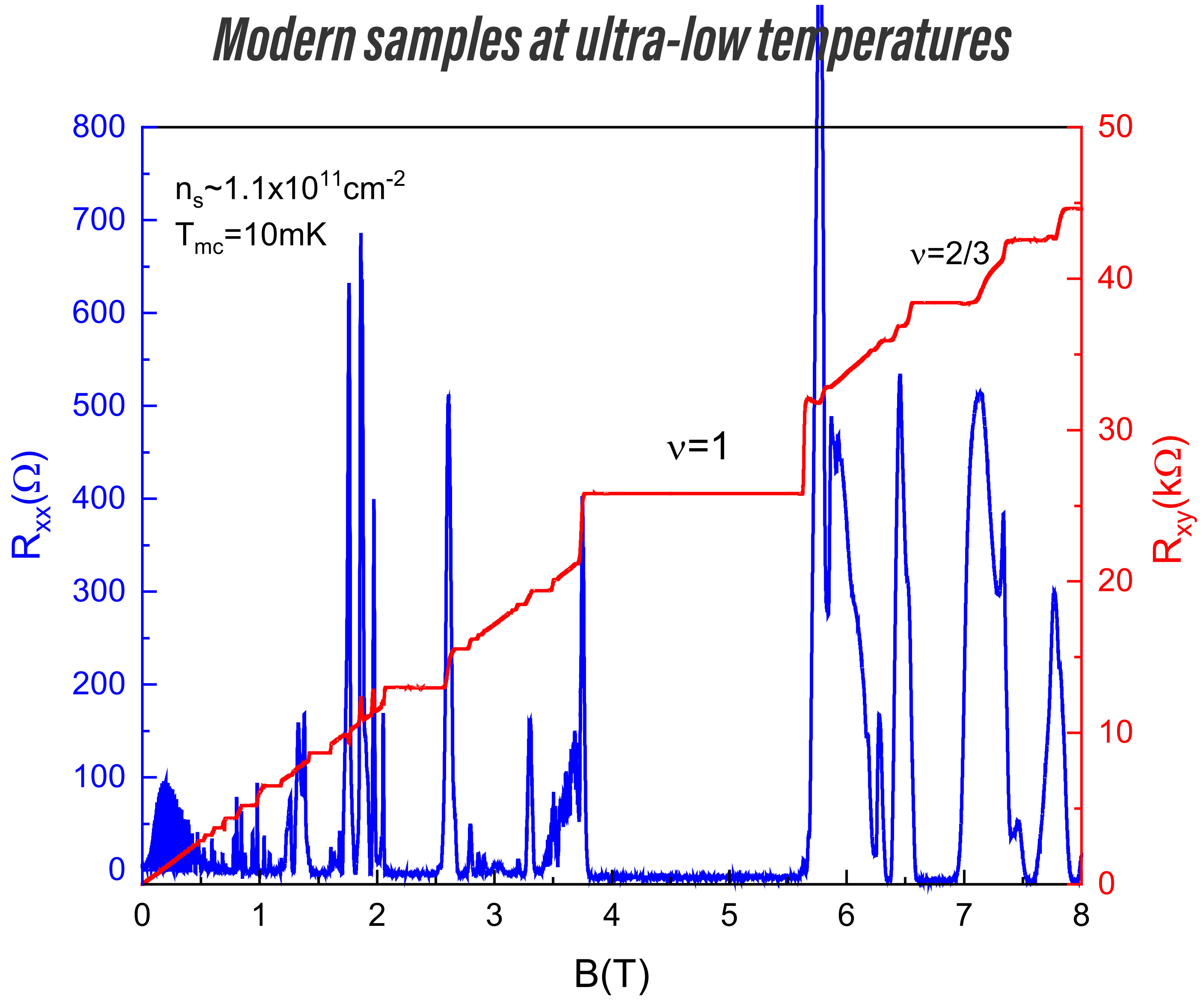
$|z_i|^2/4l_B^2$

An Inspired and Highly Accurate Guess (.999…)

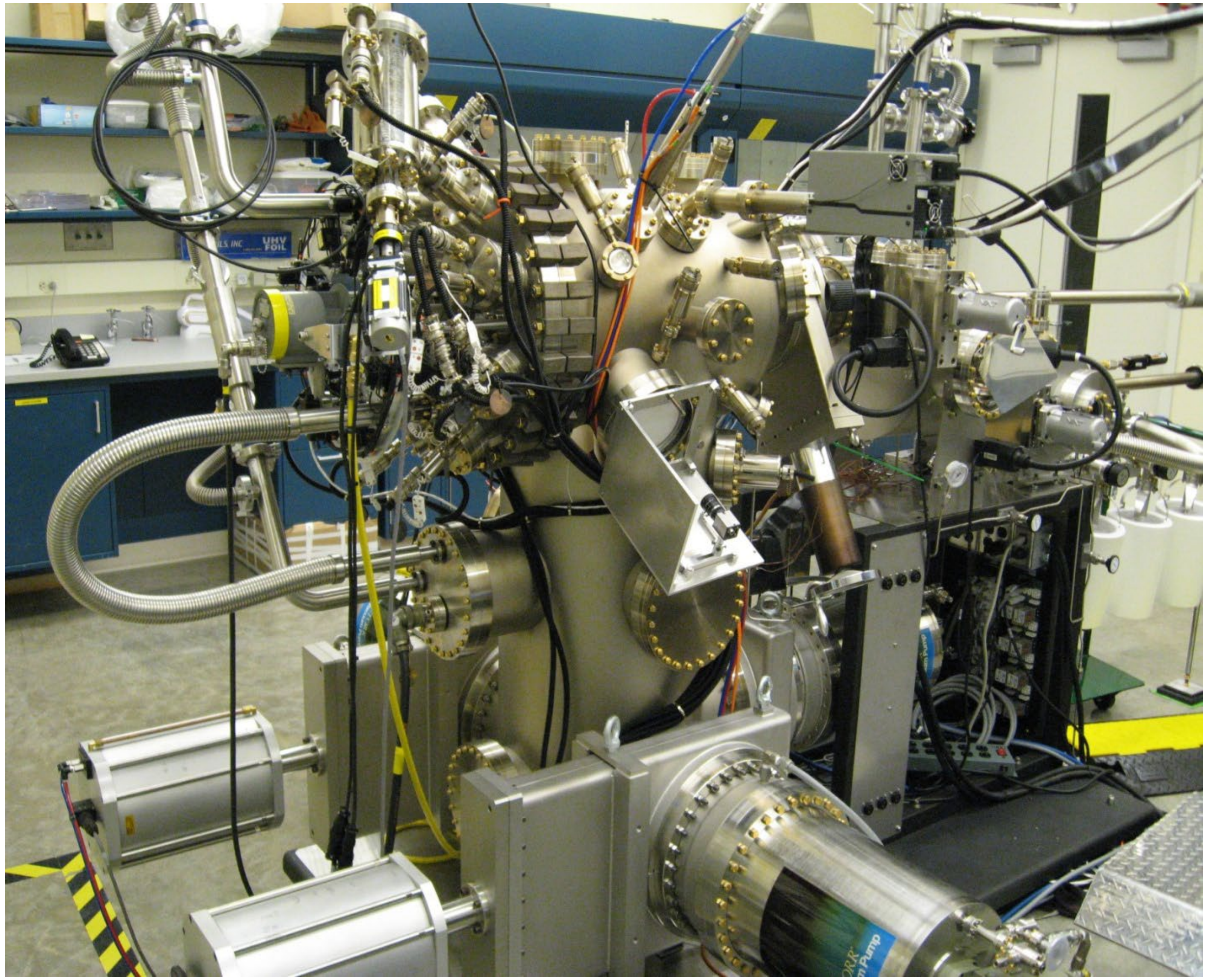
- **Angular Momentum Eigenstate (Homogeneous in z's)**
- **Antisymmetric (Fermions)**
- **Minimize Coulomb Interaction**
  (Keep Electrons Far Away From Each Other)

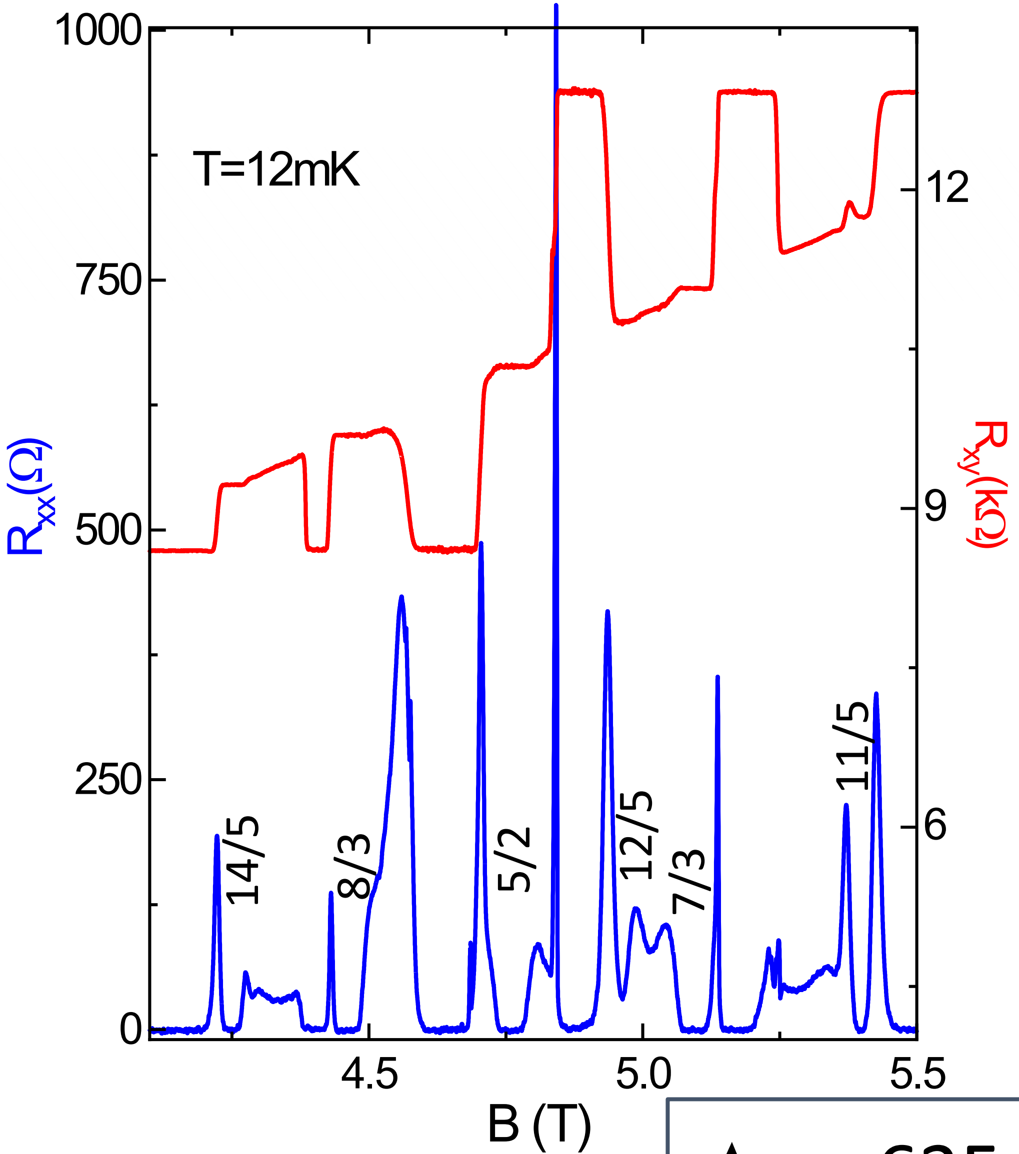
$$\Psi_{\text{Filled LLL}} = \left[\prod_{i<j}(z_i - z_j)\right] e^{-\sum_i |z_i|^2/4l_B^2}$$

Modern samples at ultra-low temperatures



Fractional states in the N=1 Landau level

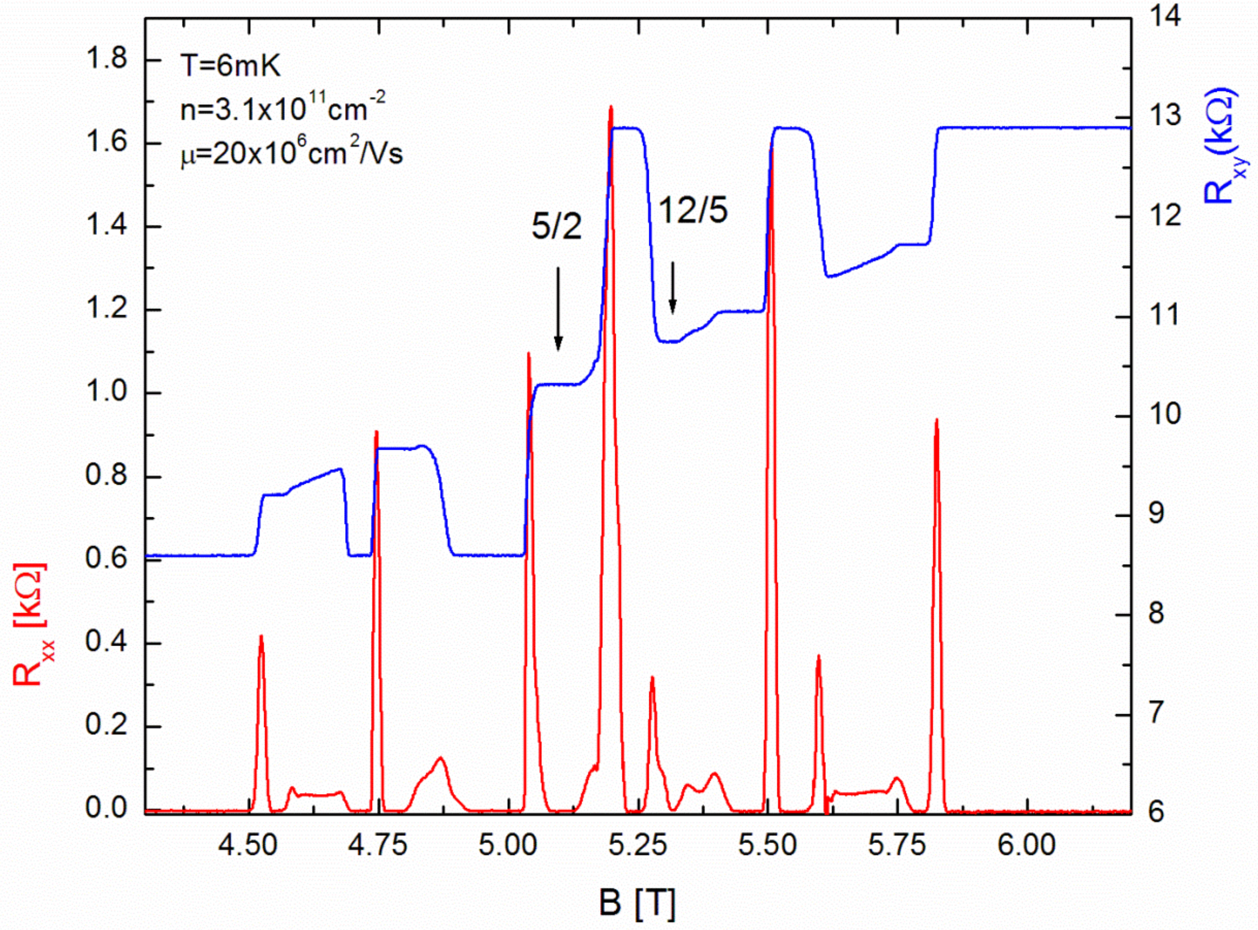


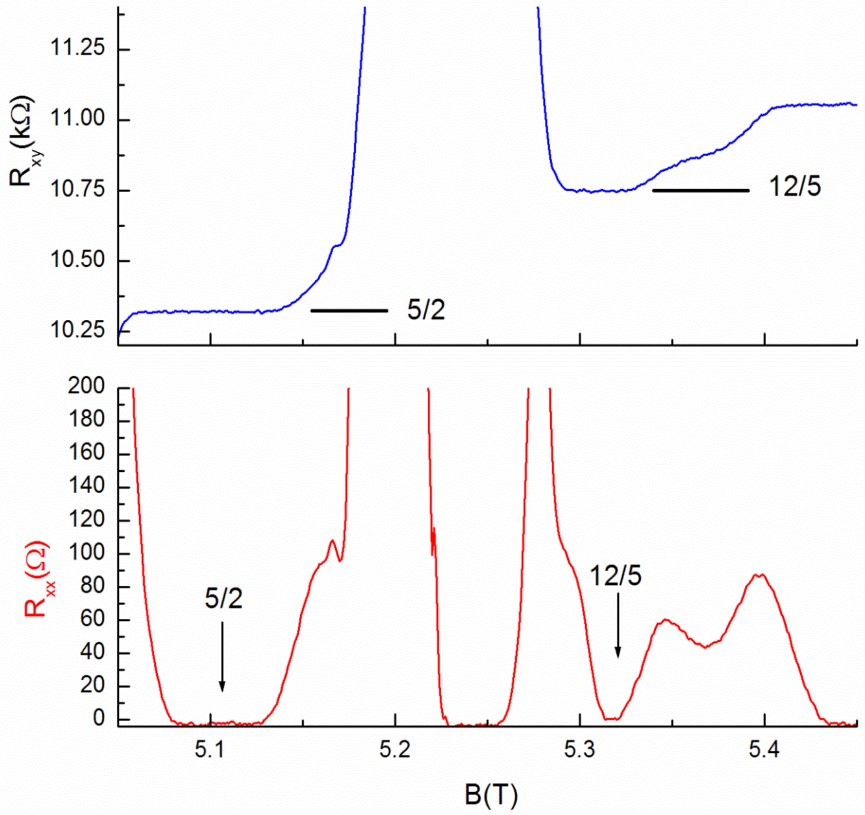


Electron mobility: >35x10^{6}cm^2/Vs

$\Delta_{5/2}$=625mK

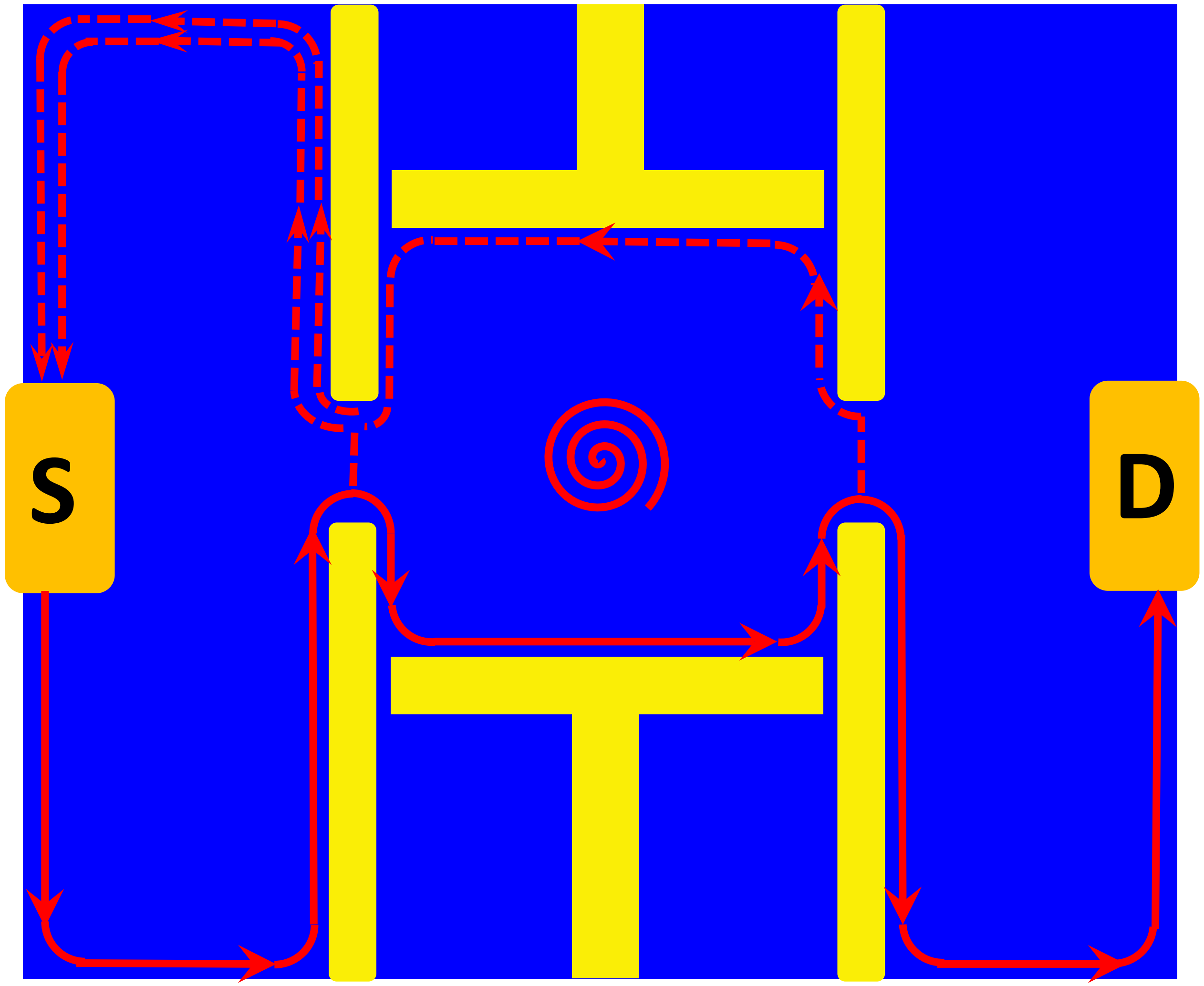
Putative non-Abelian states in 2^{nd} Landau level





Moore and Read, Nucl. Phys. B, 362-396 (1991)
Nayak and Wilczek, Nucl. Phys. B, 529-553 (1996)
Read and Green, Phys. Rev. B **61**, 10267-10297 (2000)

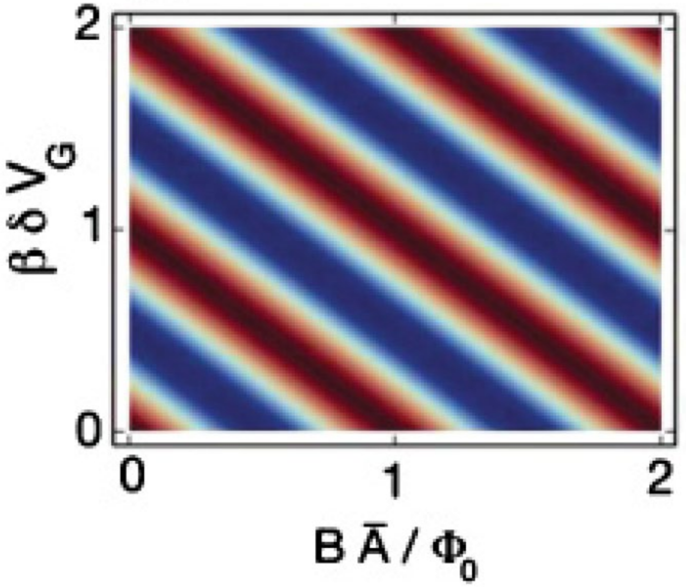
Electronic Fabry Perot interferometer in the IQHE and FQHE reg



- Surface gates define electron interference path
- Quantum point contacts (QPCs) act as beam splitters

$$I_{backscattered} \sim |r_1||r_2|\cos(\theta)$$

$$\theta = \underbrace{2\pi\left(\frac{AB}{\Phi_0}\right)\frac{e^*}{e}}_{\text{Aharonov-Bohm phase}} + \underbrace{N_L\theta_{anyon}}_{\text{Braiding phase}} \qquad \Phi_0 \equiv \frac{h}{e}$$

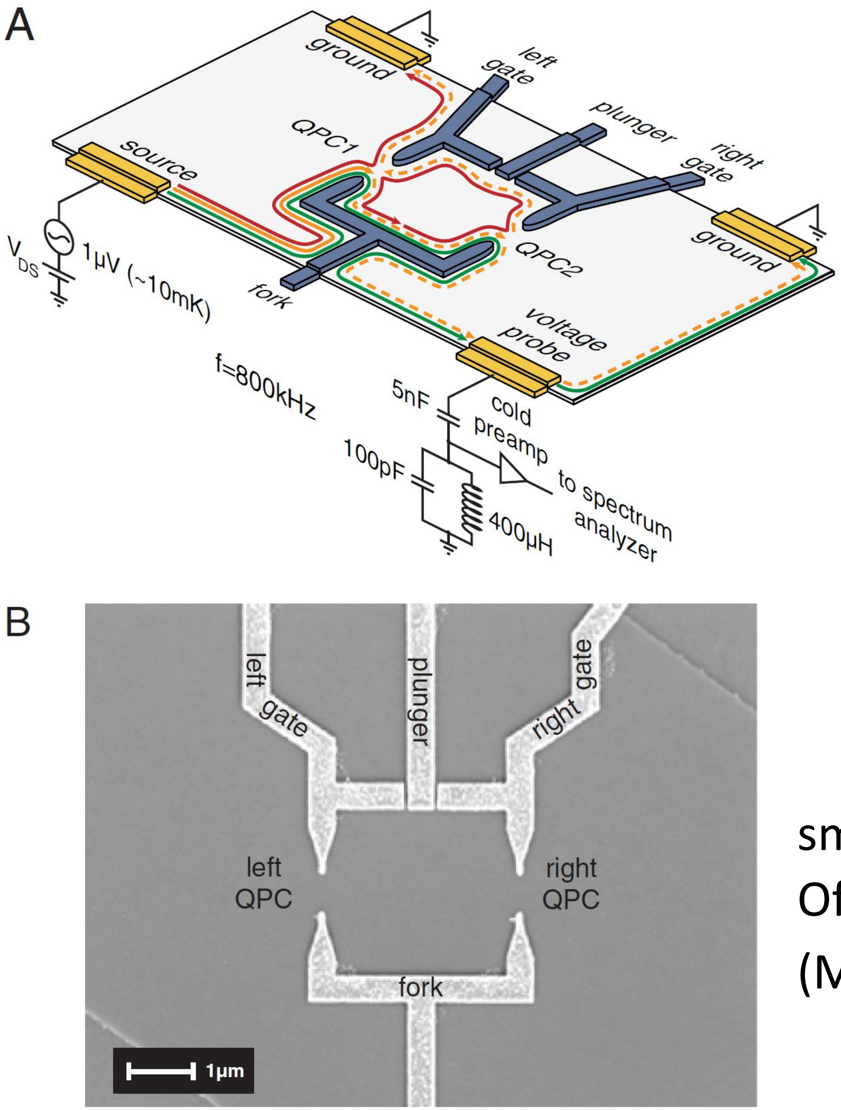


$B \otimes$

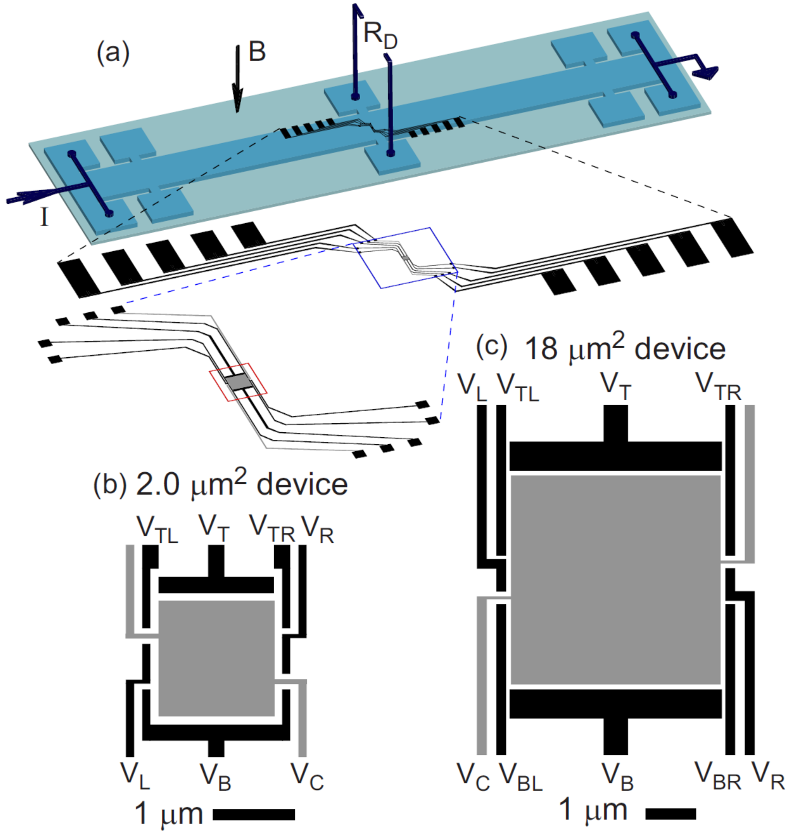
B.I. Halperin, A. Stern, I. Neder, and B. Rosenow PRB **83**, 155440 (2011)

C. de C. Chamon, D. Freed, S. Kivelson, S. Sondhi, X. Wen Phys. Rev. B **55**, 2331 (1997)

Early pathfinding experiments: challenges and clues



small devices ~4μm^2
Ofek, PNAS 2010
(M. Heiblum group)

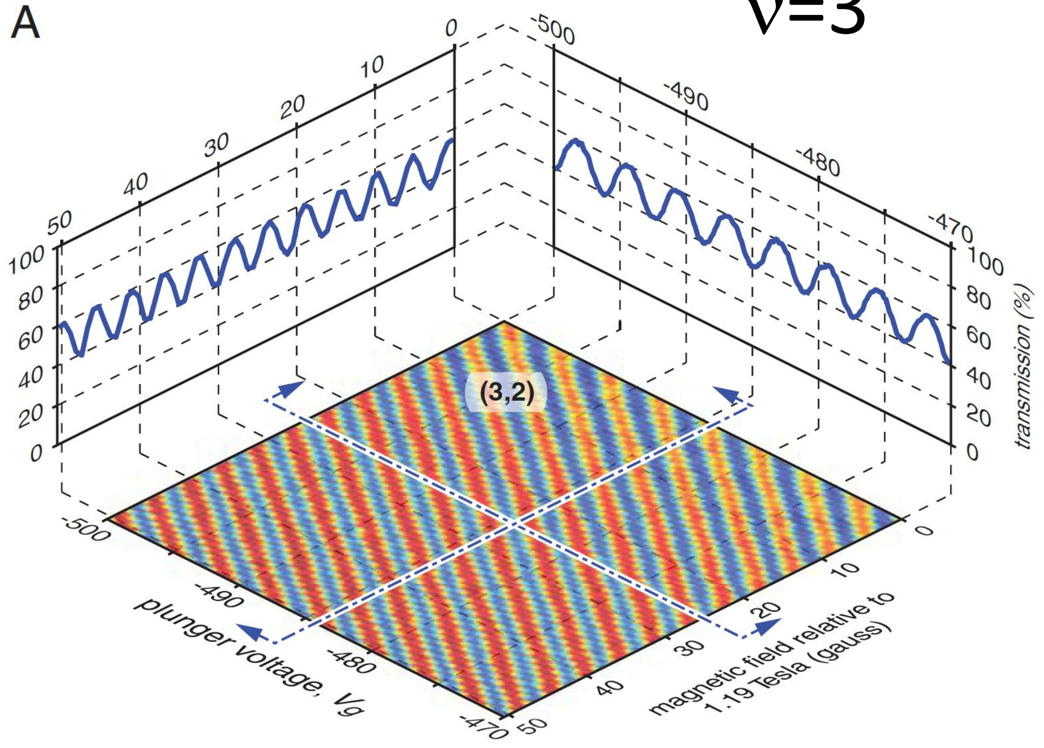


large devices ~20μm^2
Zhang, PRB **79**, 241304 (R) (2009)
(C. Marcus group)

small interferometers

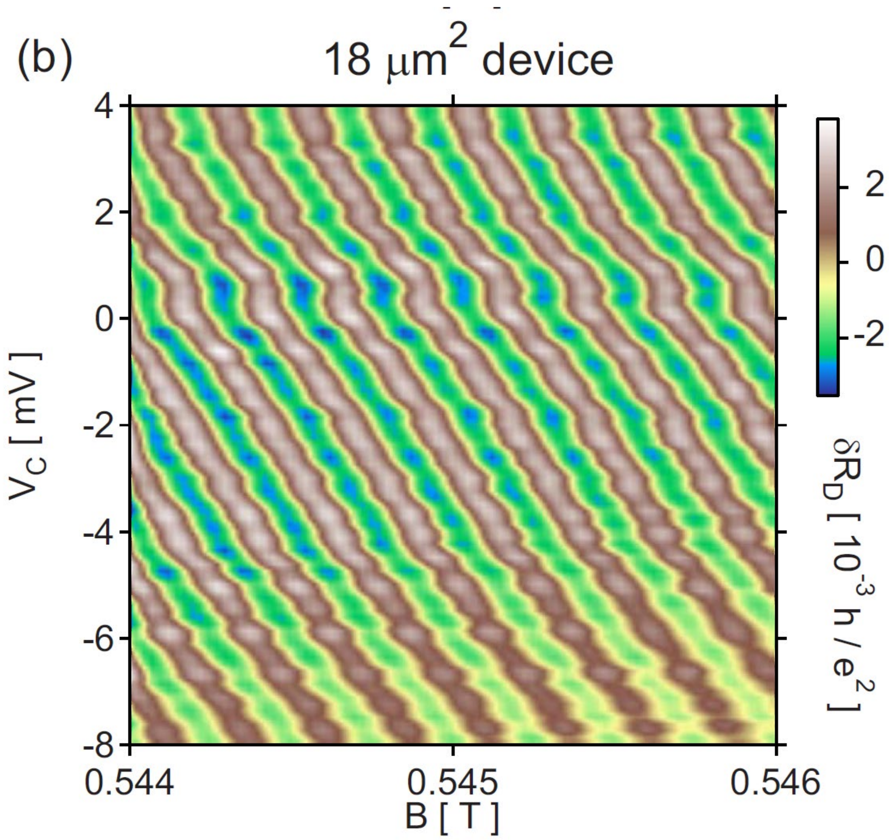
big interferometers

$\nu=3$



Not AB oscillations, positive slope

Ofek, PNAS 2010



AB oscillations, but only
at low magnetic fields

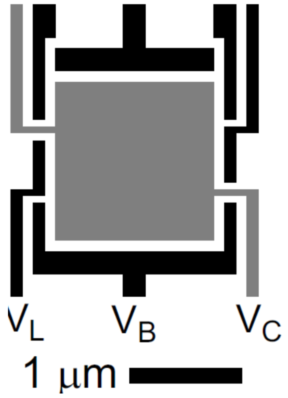
Zhang, PRB **79**, 241304 (R) (2009)

C. Marcus' group

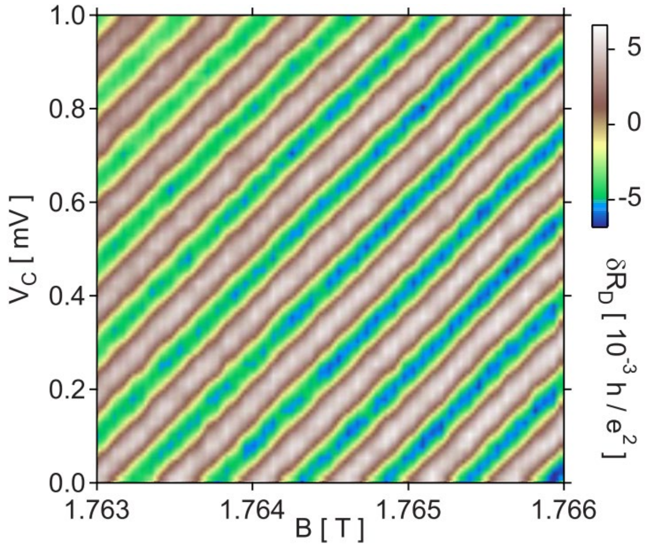
AB vs. CD in early experiments: a valuable lesson

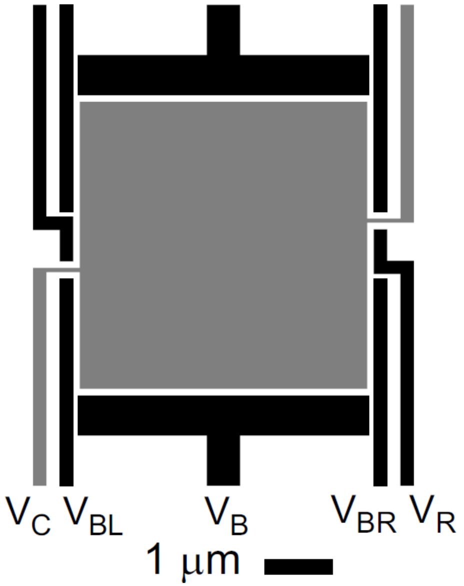
- Many early experiments observed Coulomb dominated behavior
- C. Marcus group observed AB behavior (negative slope) in devices with large area which included metal screening gates
- Coherence was poor due to large path length
- Need better way to screen to observe AB interference in smaller devices

Zhang et al. PRB **79**, 241304 (2009)

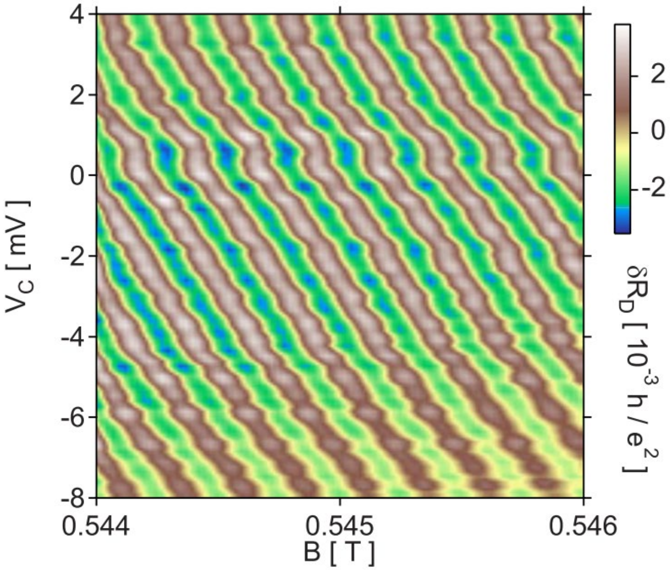


$2\mu m^2$
Device





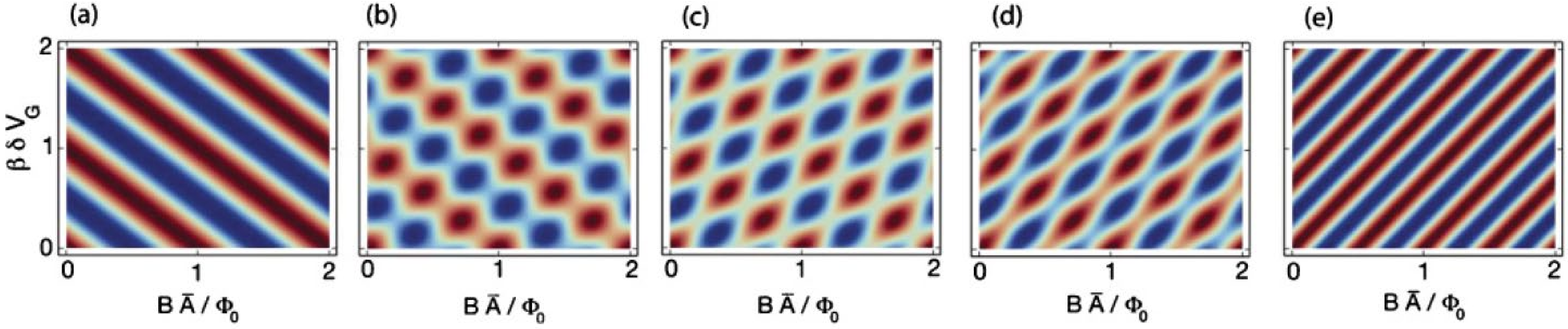
$18\mu m^2$
Device



Aharonov-Bohm vs. Coulomb dominated regime

Aharonov-Bohm

Coulomb dominated



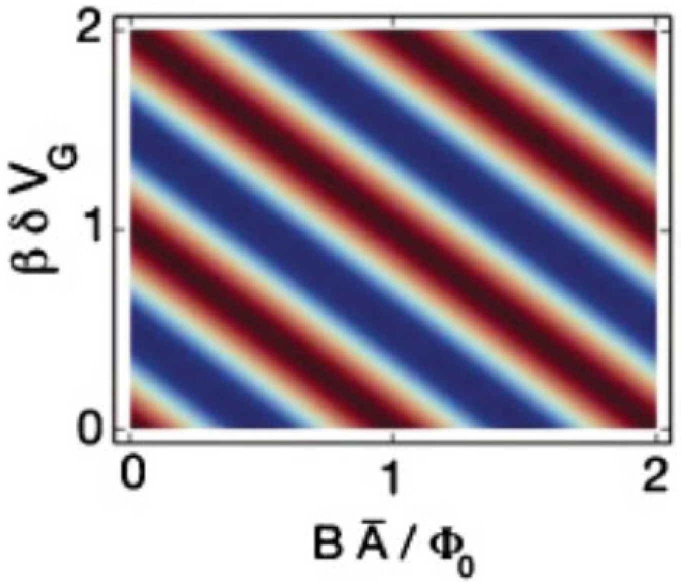
- **Regime of operation depends on the ratio of K_{IL}/K_I, where K_{IL} parameterizes bulk-edge interaction and K_I parameterizes the energy cost to add charge to the edge**

- **Critically, θ_{anyon} is unobservable in the Coulomb dominated regime: phase change is multiple of 2π.**

B. I. Halperin, A. Stern, I. Neder, and B. Rosenow. PRB **83**, 155440 (2011)
D. Feldman and B. Halperin, arXiv.org/abs/2102.08998 (2021)
C. W. von Keyserlingk, S. H. Simon, B. Rosenow, ***PRL*** 115, 126807 (2015)

Problem: strong bulk-edge interaction

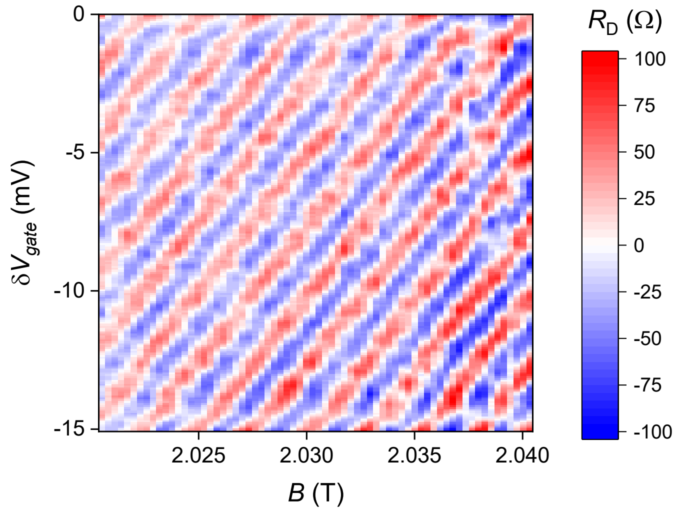
Expectation: Aharonov-Bohm Interference



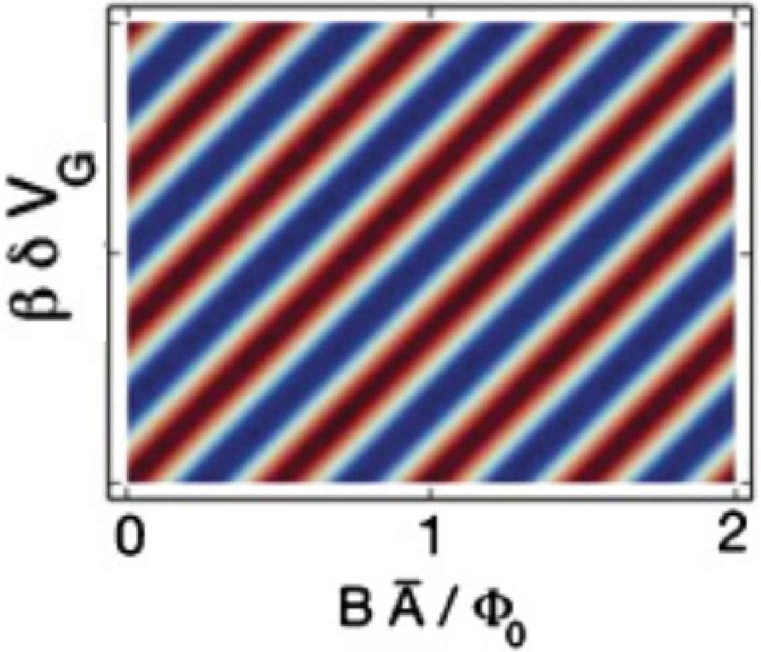
$$\theta_I = 2\pi \frac{AB}{\Phi_0}$$

- Bulk-edge interactions cause area to change with magnetic field
- Cannot change A and B independently – flux *decreases* when increase B!
- Makes braiding unobservable

Reality: Coulomb-dominated oscillations



Wrong slope!



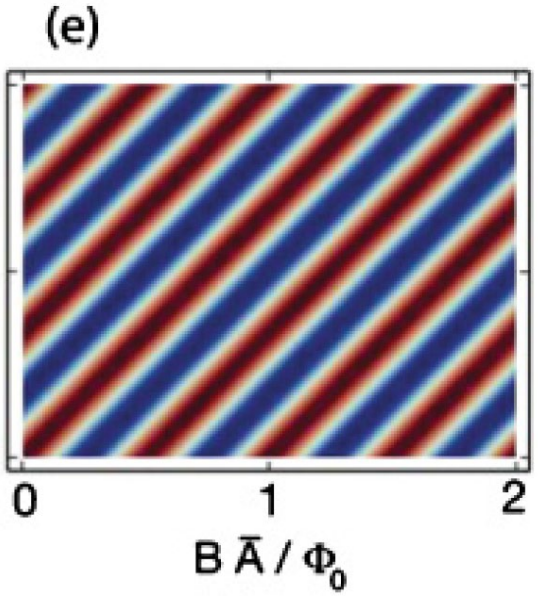
$$\delta A = -\nu \bar{A} \delta B$$

B.I. Halperin, A. Stern, I. Neder, and B. Rosenow. PRB 83, 155440 (2011)

Challenges for QHE interferometry

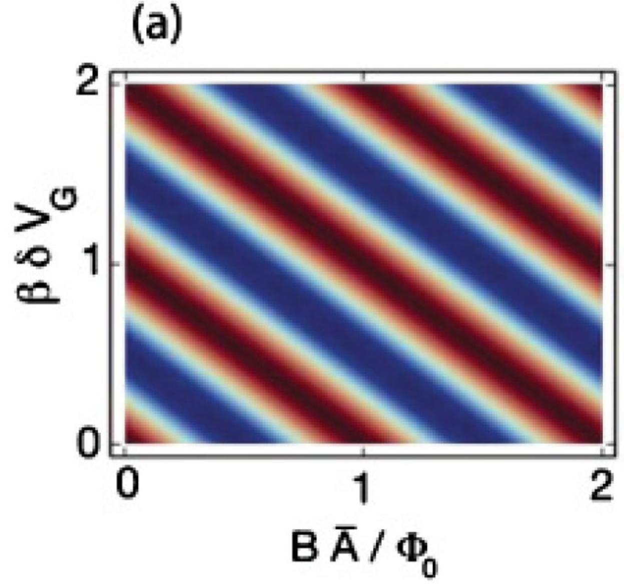
Small interferometers

- good coherence (short path length)
- Coulomb-dominated transport – (large charging energy)

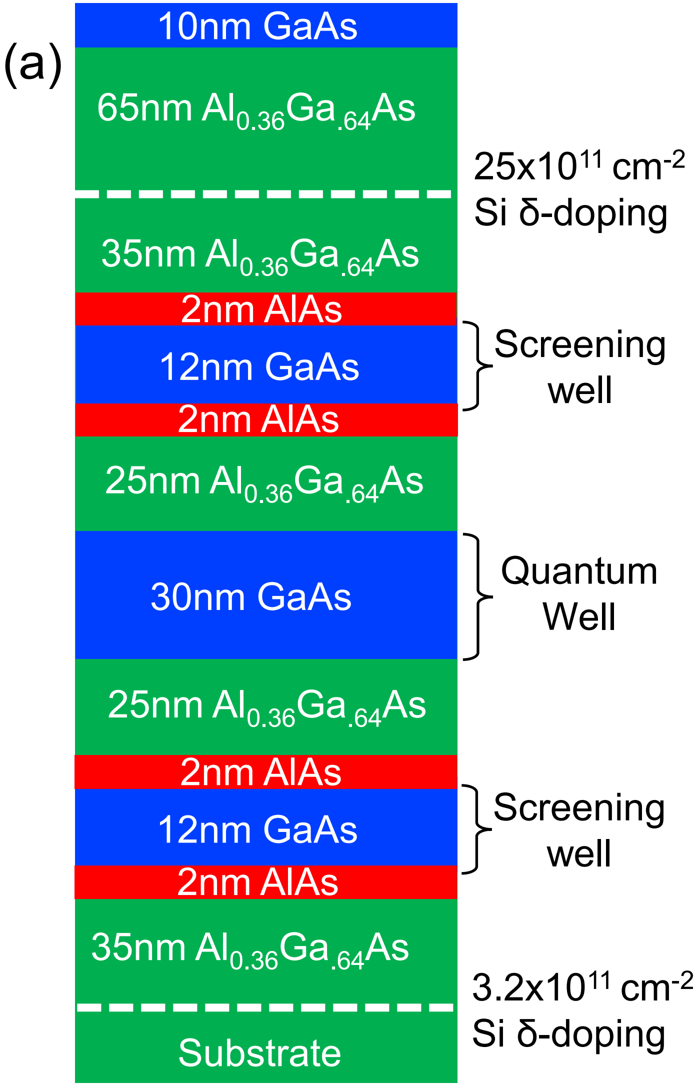


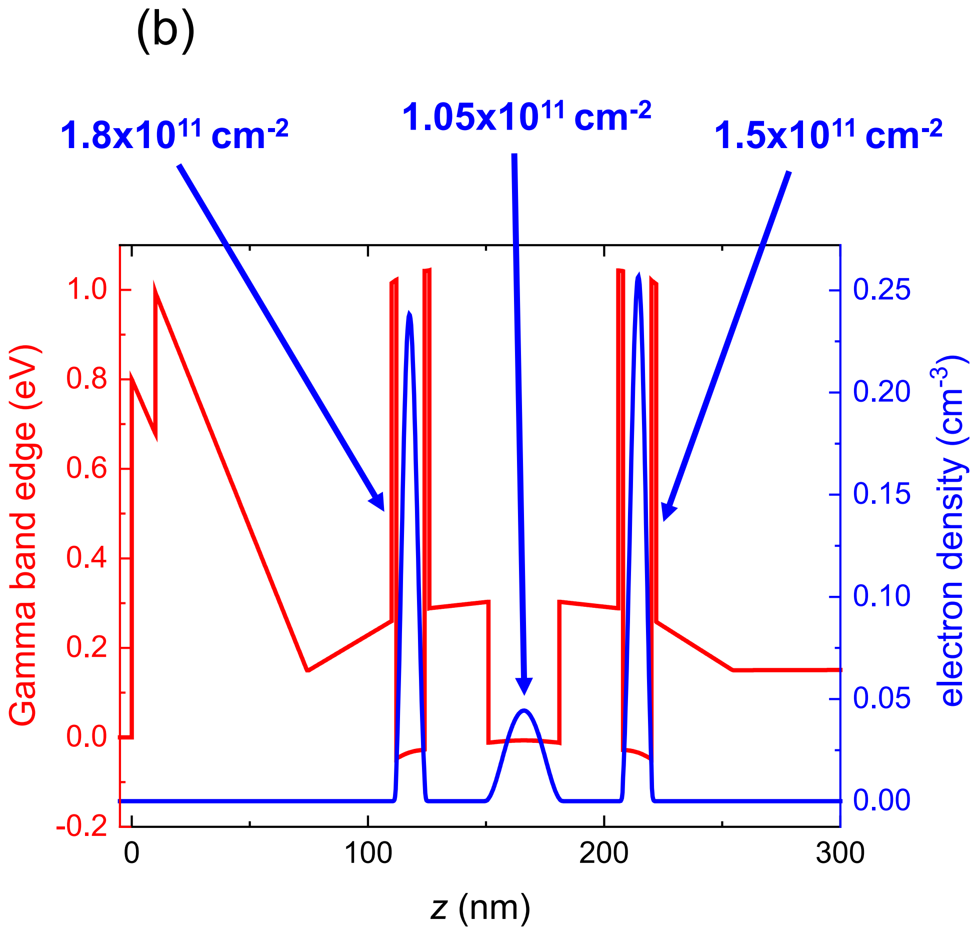
Large interferometers

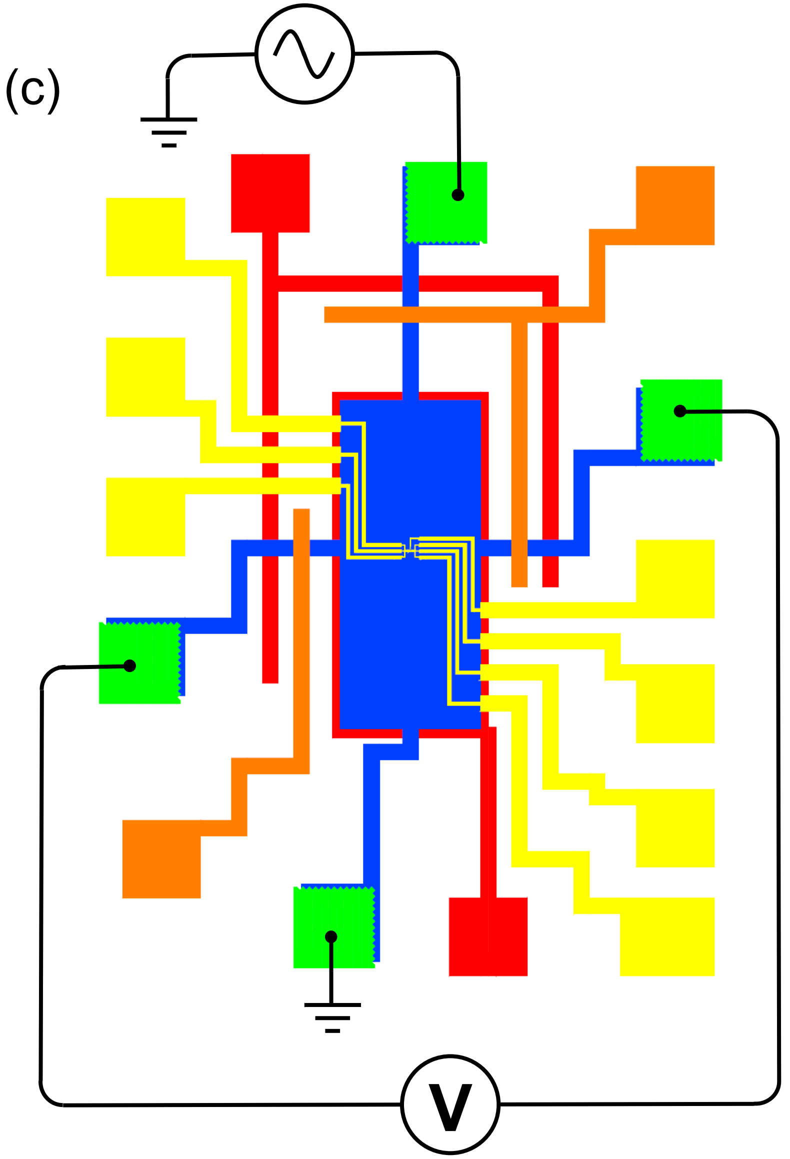
- low coherence (long path length)
- Aharonov-Bohm interference – (low charging energy)
- Limited to large filling factor (low B-field)



Screening well heterostructure and device desi

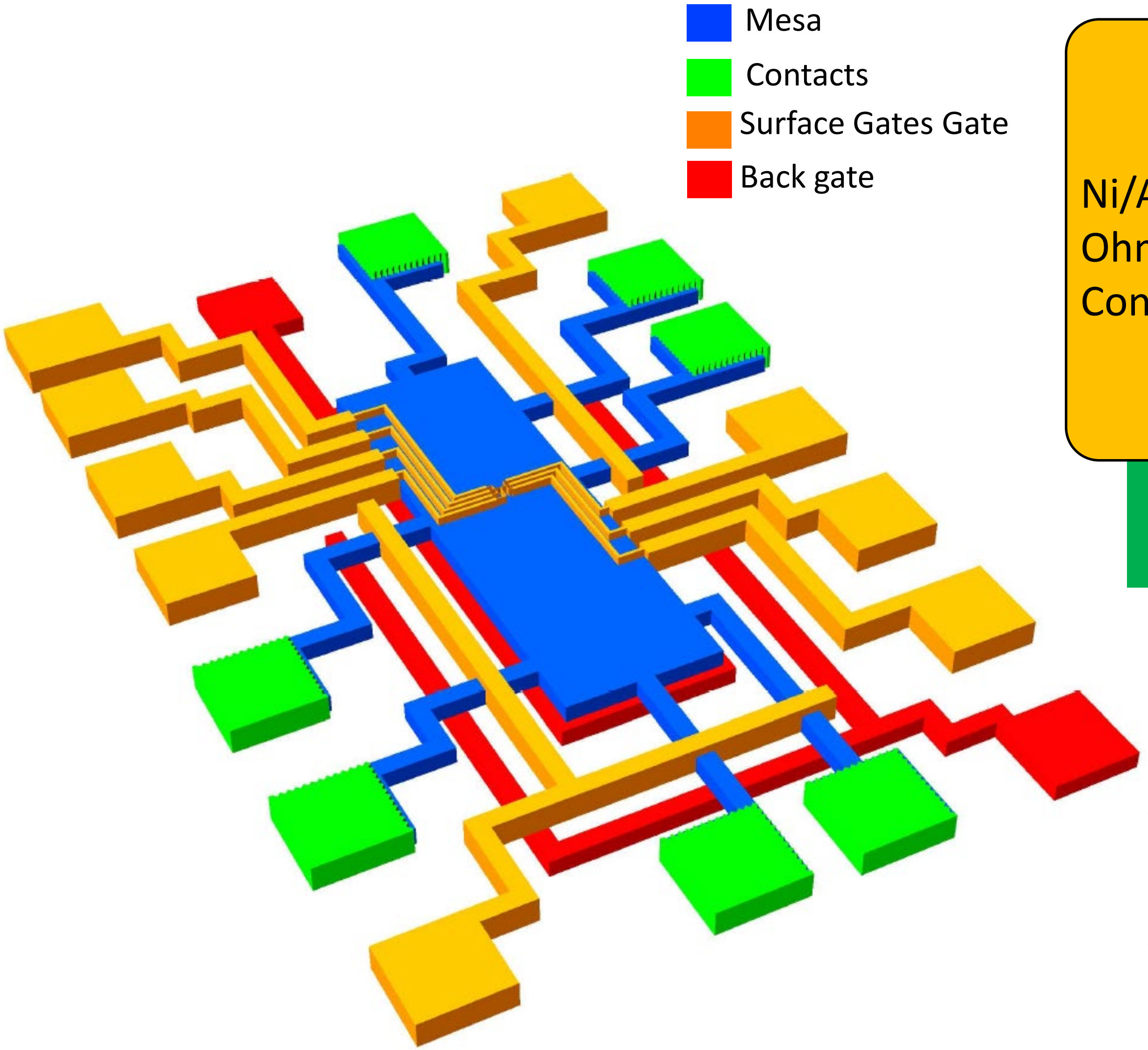


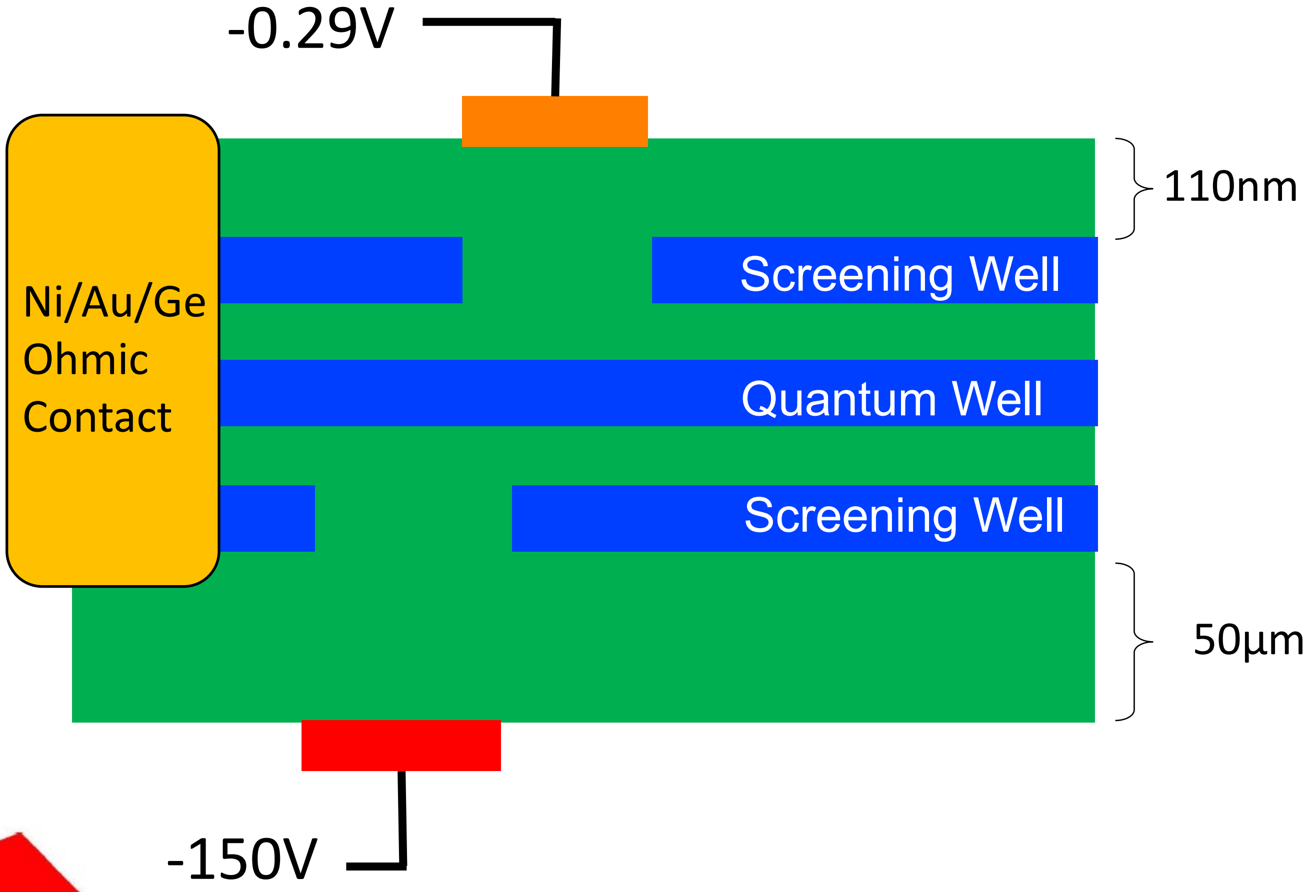




Work from 2018 - 2019

Top and back-gated interferometer

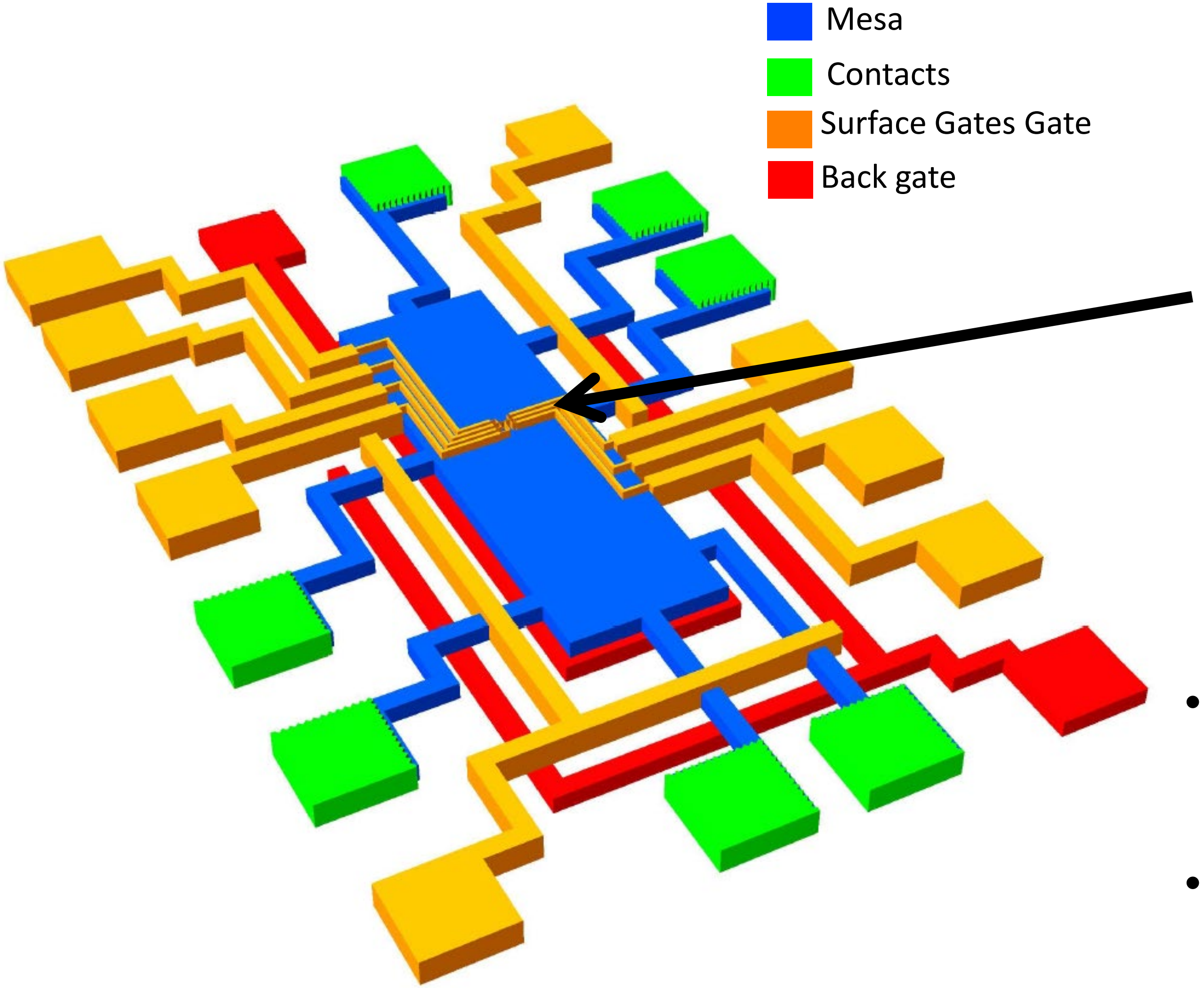


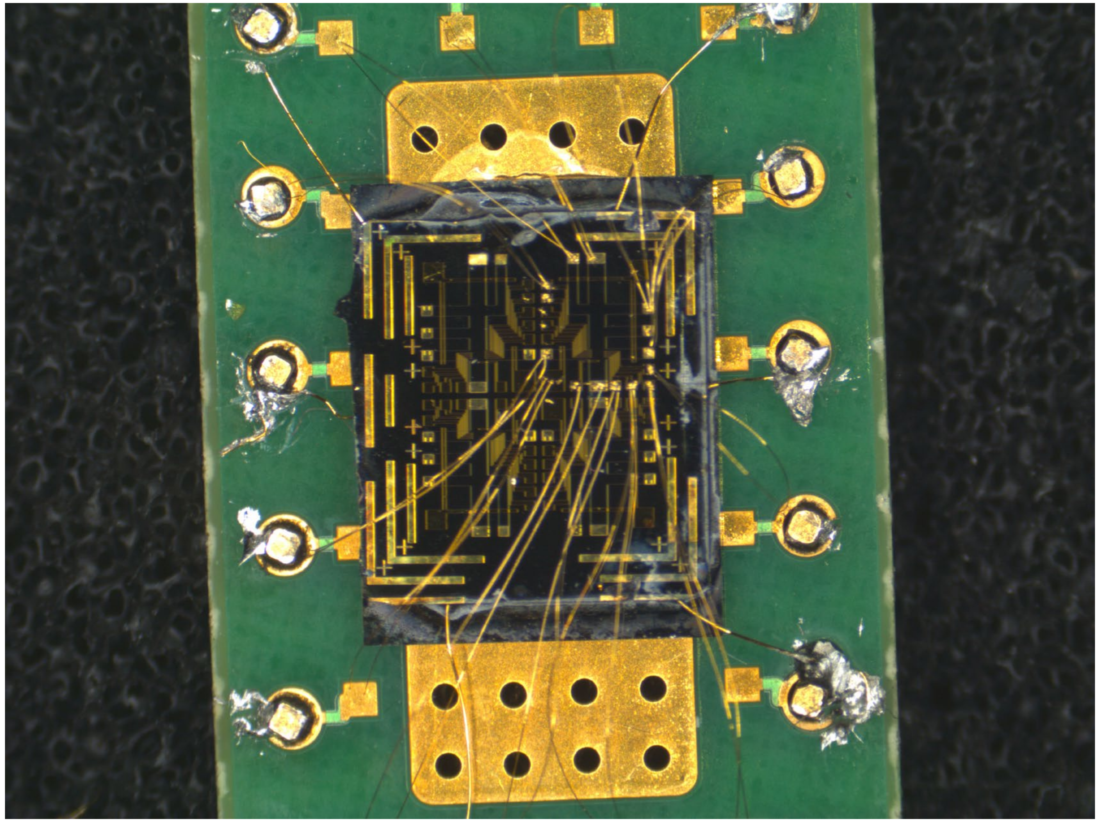


- Need to eliminate parallel conduction through screening wells
- Adapt technique used in bilayer systems – use gates around Ohmics to disconnect SWs from contacts

J. P. Eisenstein, L. N. Pfeiffer, & K. W. West. APL 57, 2324 (1990)

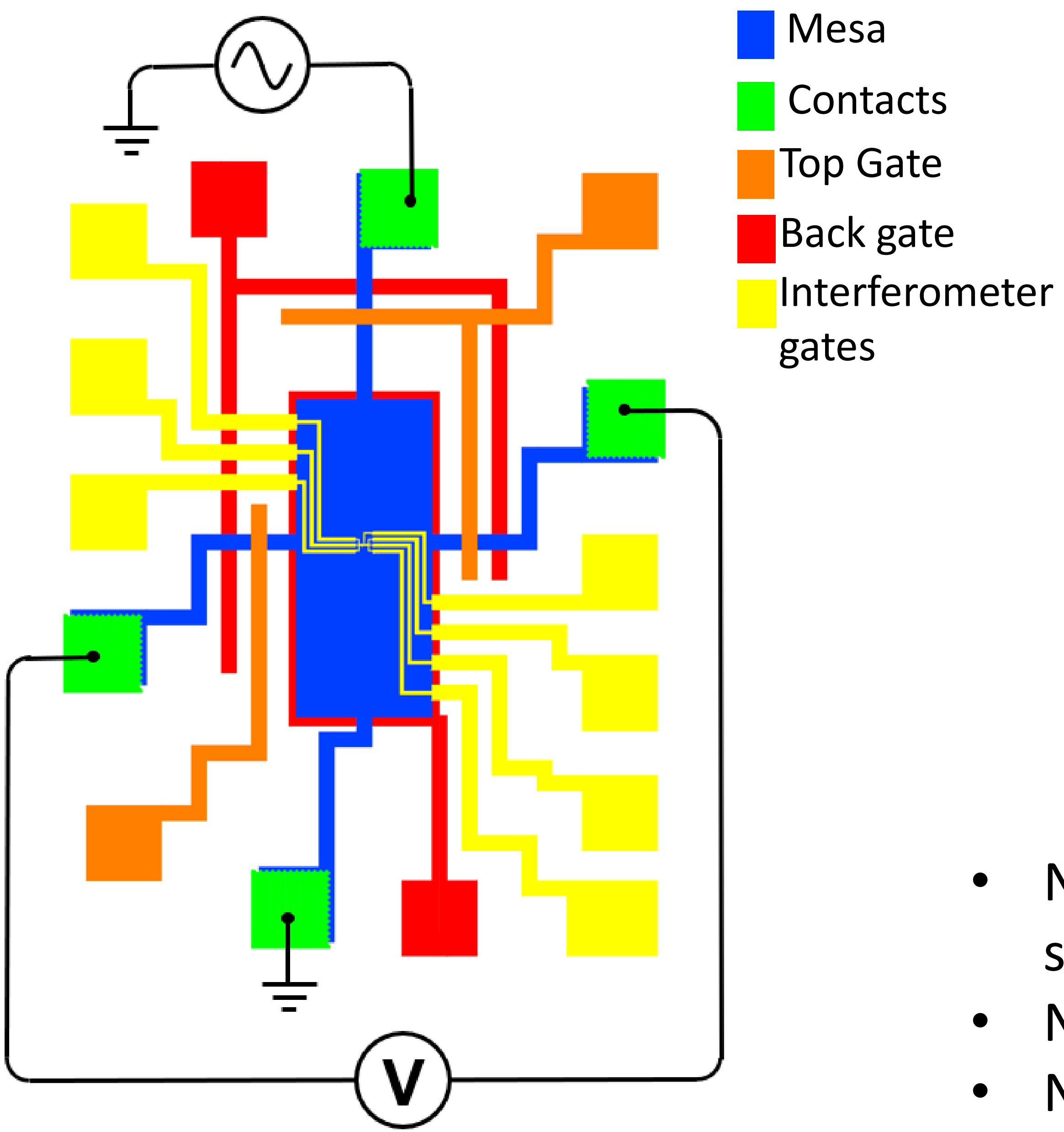
Interferometer with screening layers

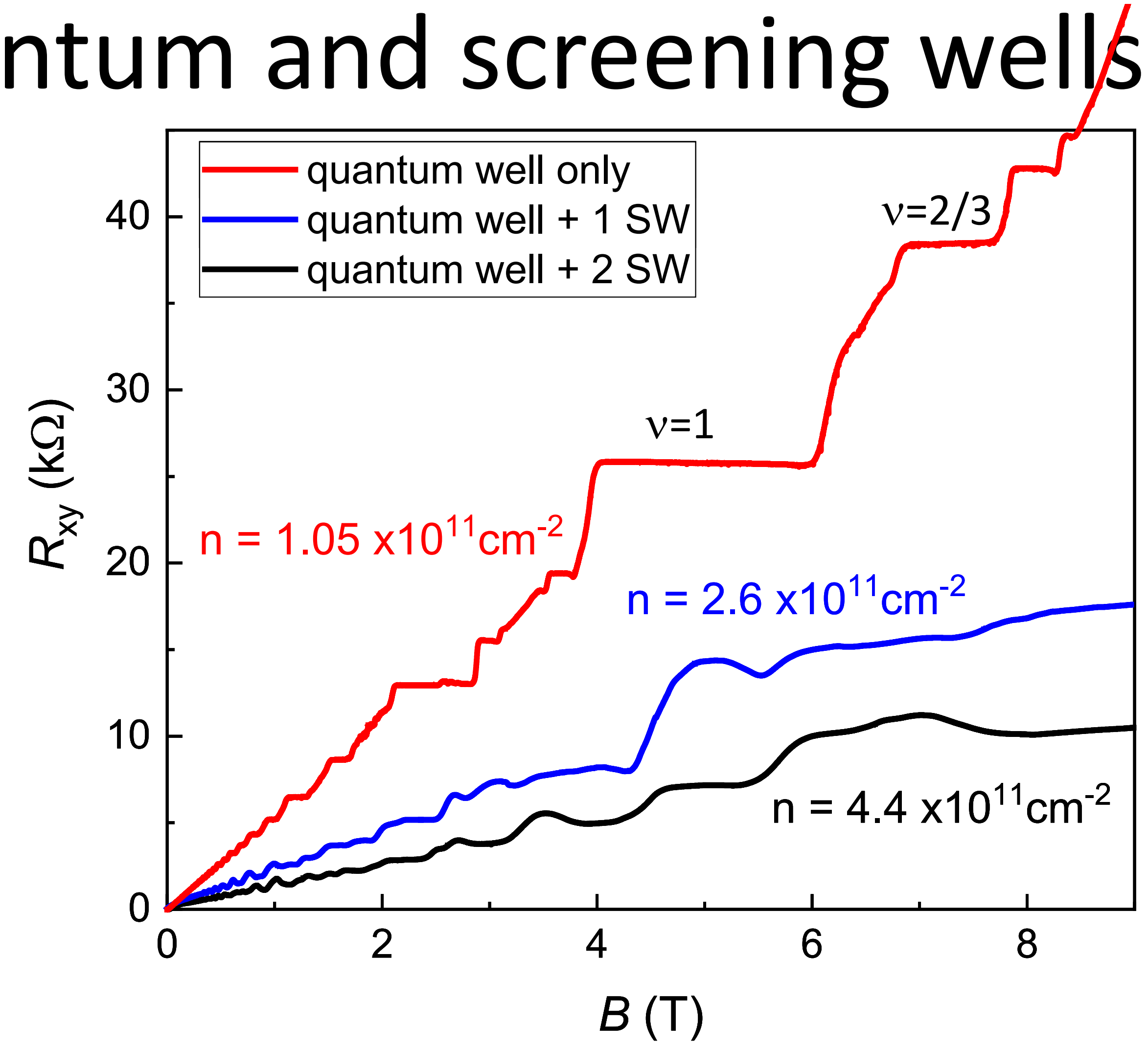




- Presence of screening wells enables AB oscillations in small device size (~$0.7 \mu m^2$) and thus improved coherence
- Simulations indicate SWs promote steeper confining potential

Transport in primary quantum and screening wells

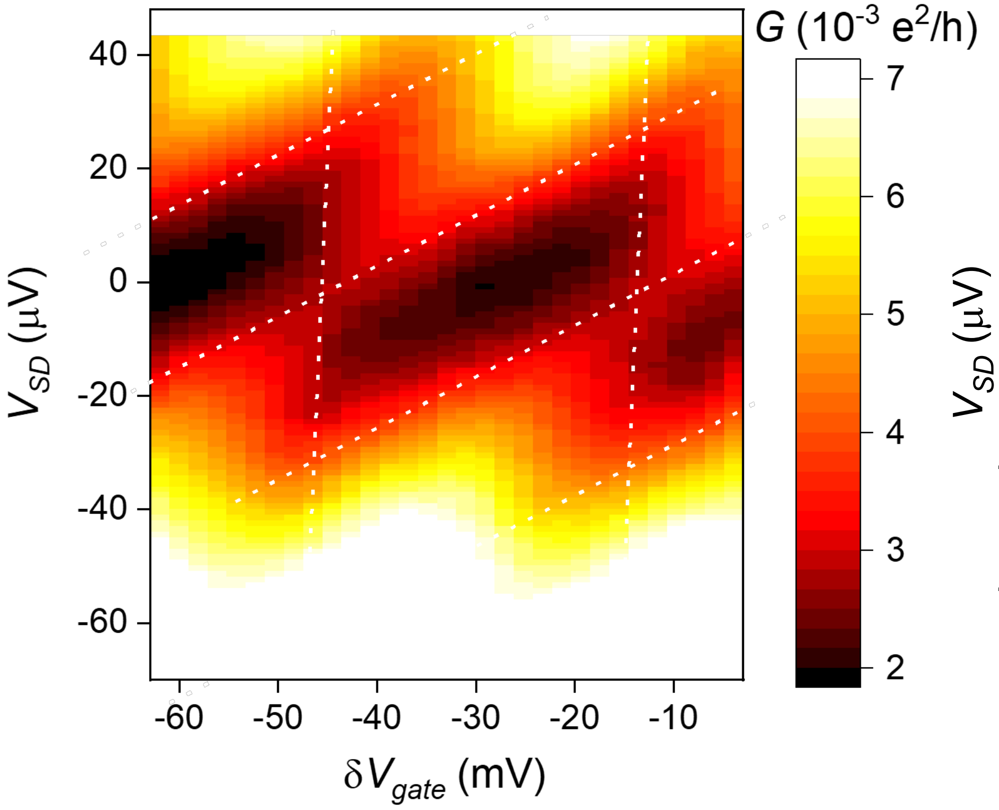




- Need to eliminate parallel conduction in screening wells
- Negatively bias top gate to isolate top SW
- Negatively bias back gate to isolate bottom SW

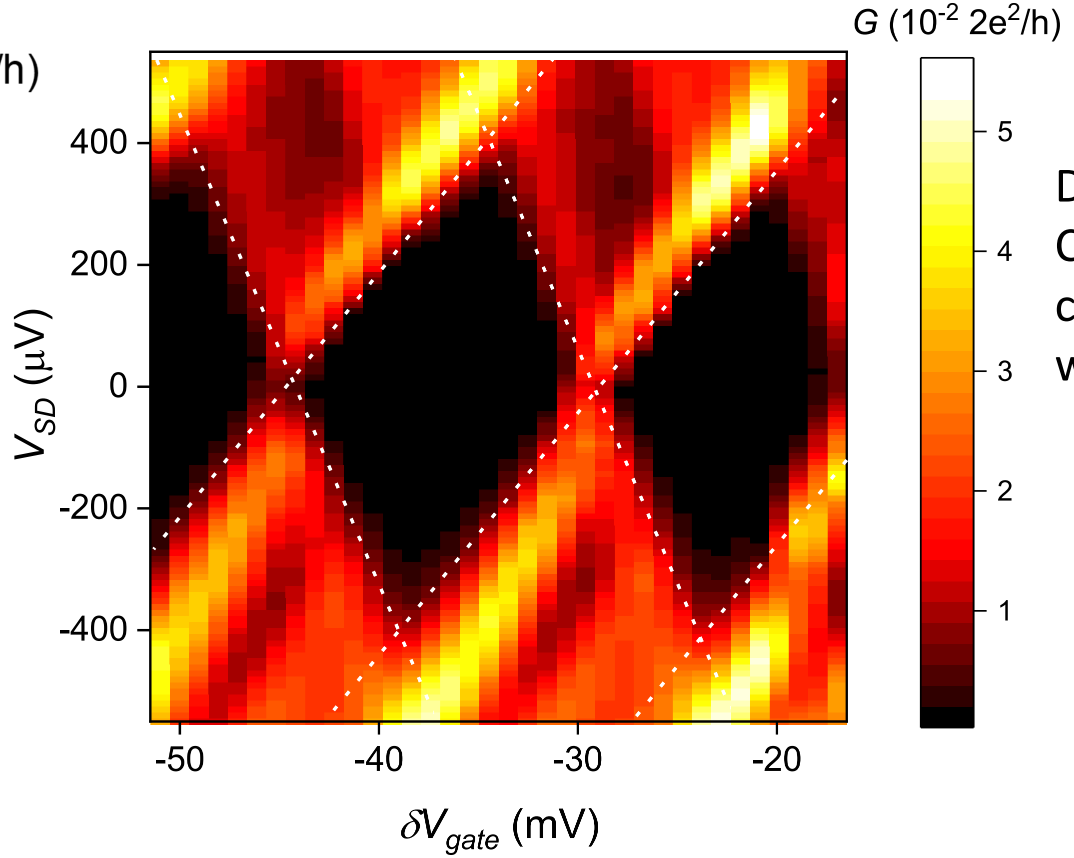
Suppression of Coulomb charging effects

Device with SWs



$$\frac{e^2}{2C} \sim 17\mu eV$$

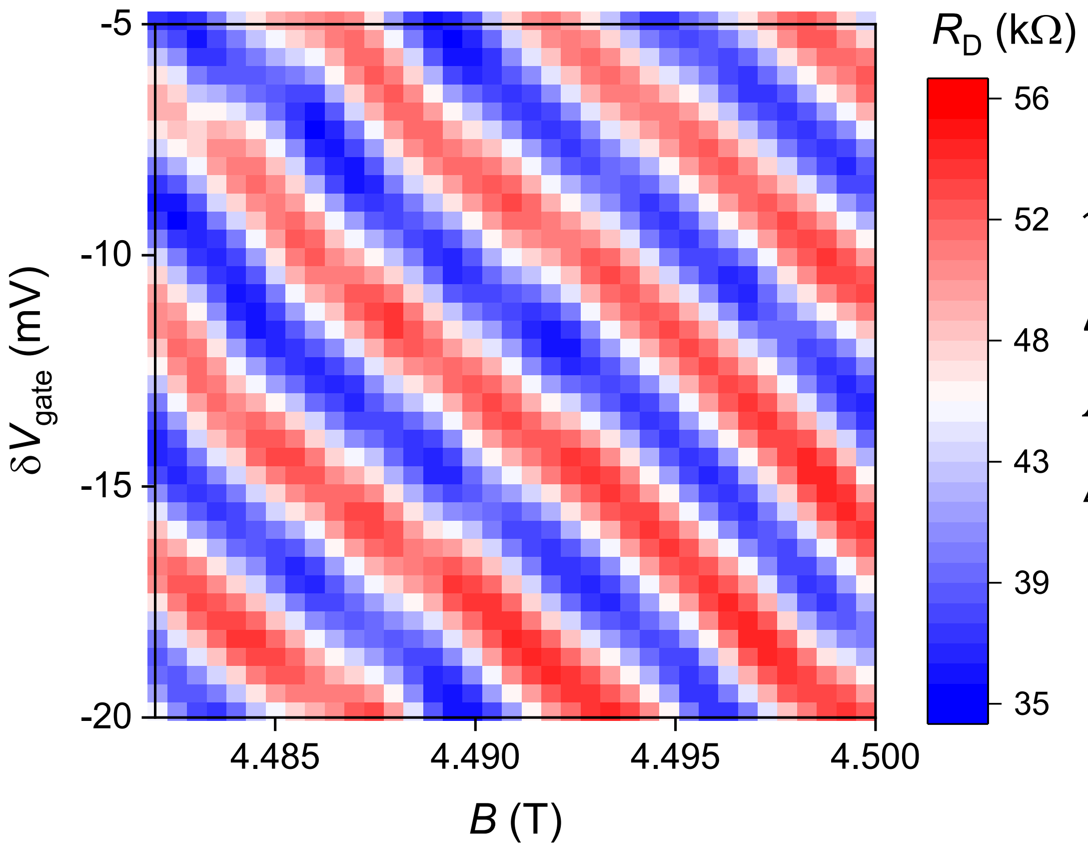
Device without SWs



$$\frac{e^2}{2C} \sim 200\mu eV$$

Dramatic reduction in Coulomb charging energy compared to device without screening wells

Aharonov-Bohm interference at $\nu = 1$ using screening wells



$\nu = 1$

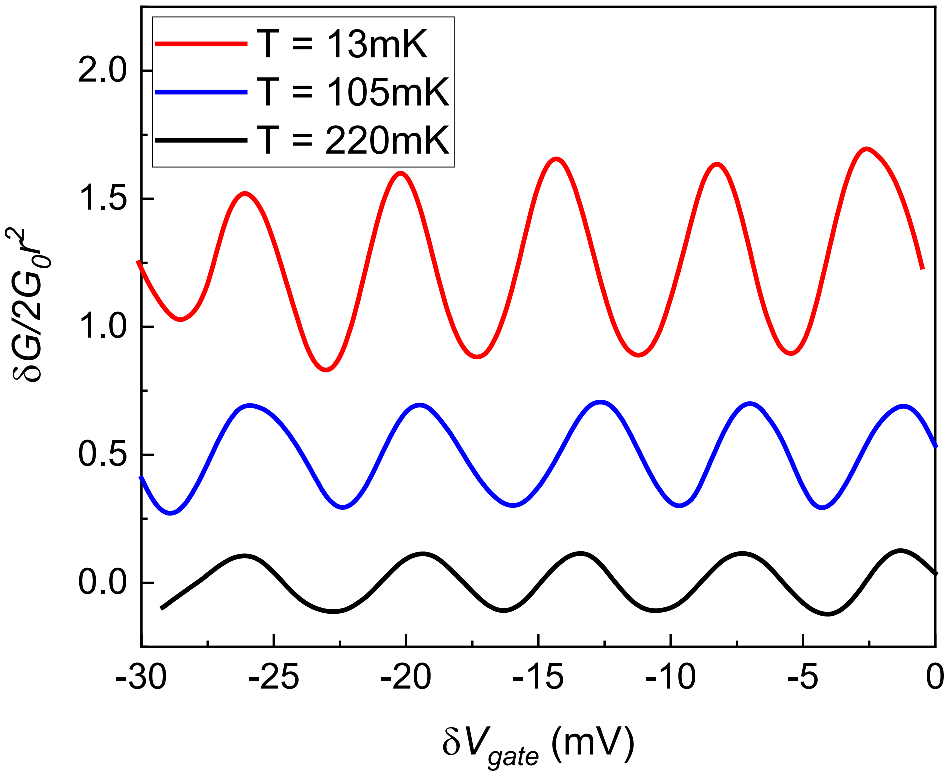
$\Delta B = 5.7\ \mathrm{mT}$

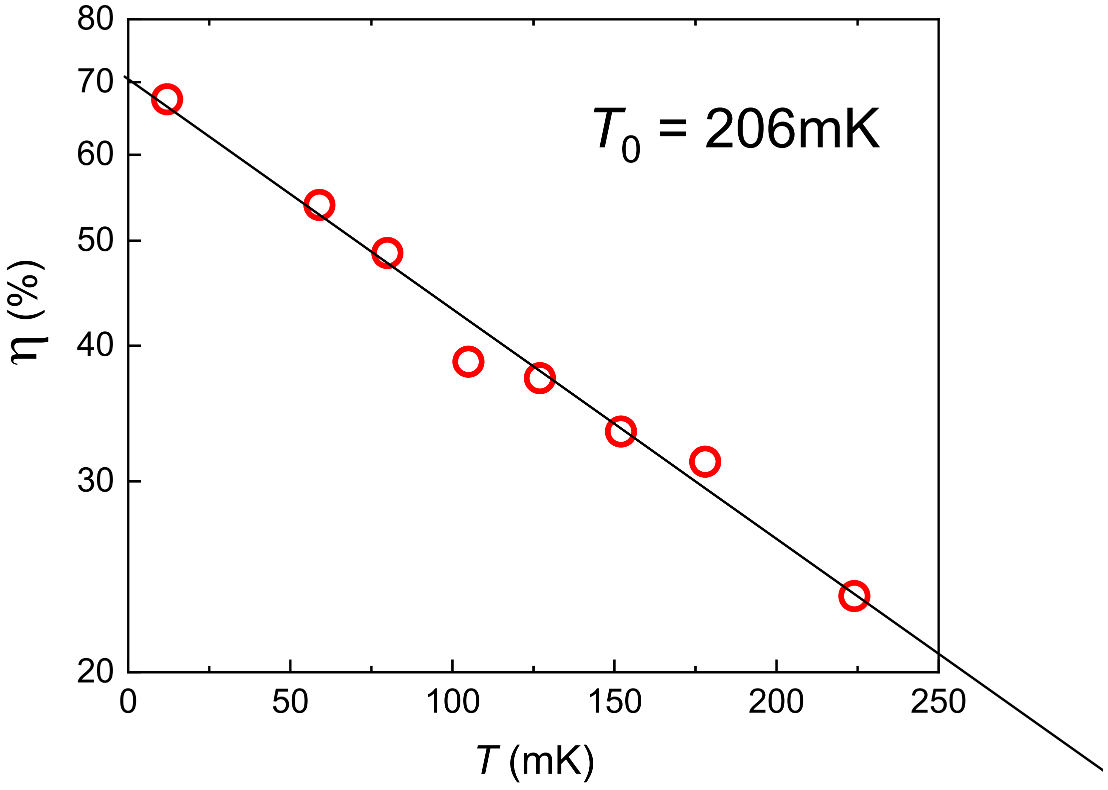
$A \approx 0.73\mu m^2$

$\Delta V_{gate} \approx 5\mathrm{mV}$

- Aharonov-Bohm interference in device ~20x smaller
- Interference is large amplitude and robust (survives up to hundreds of mK)

Coherence: temperature dependence at $\nu = 1$

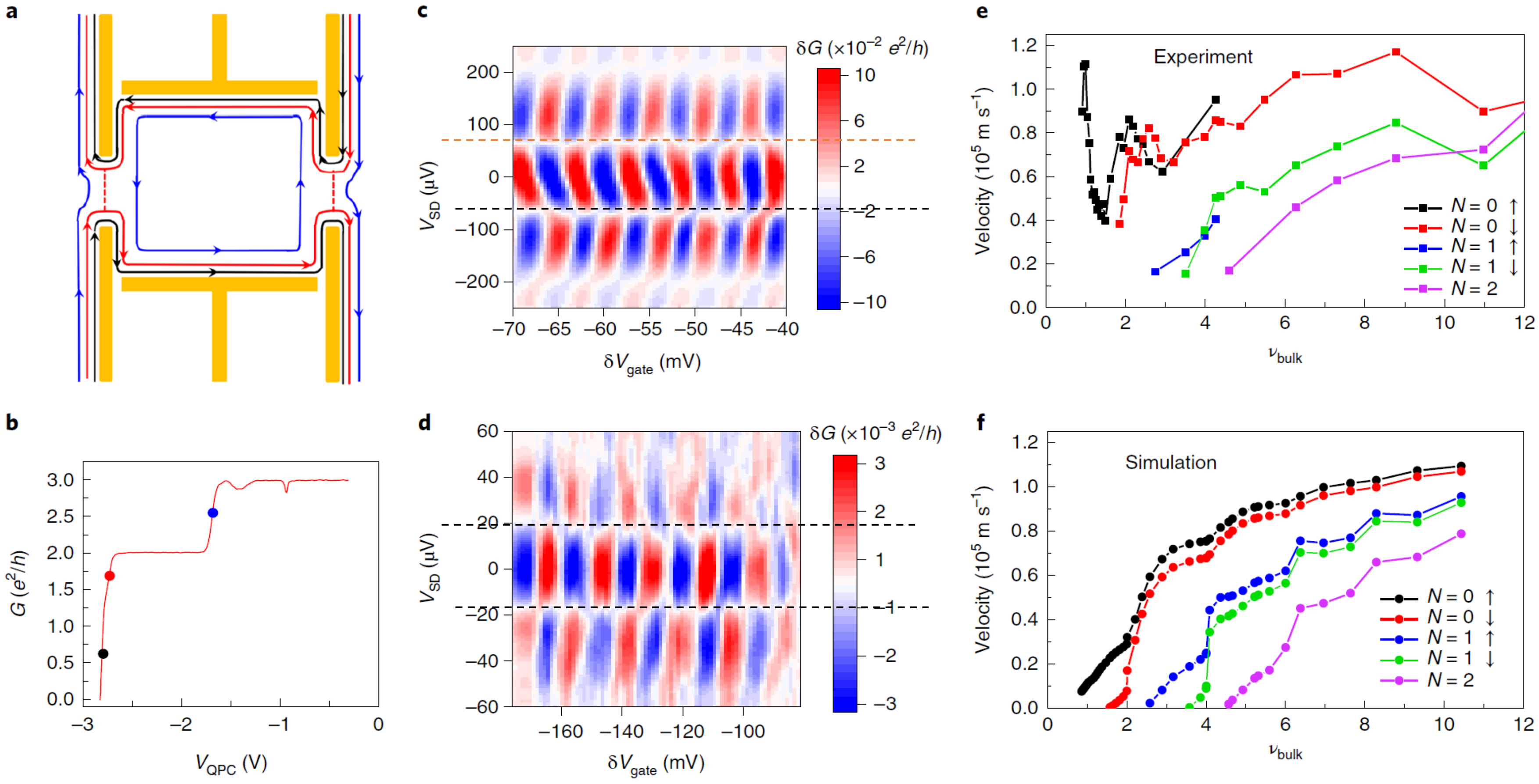




$$G/_{G_0} = 1 - 2r^2\left[1 + \eta\cos\left(2\pi\frac{AB}{\phi_0}\right)\right]$$

Selective interference of inner modes and edge mode velocity determination

ν=1, ν=1 edge mode



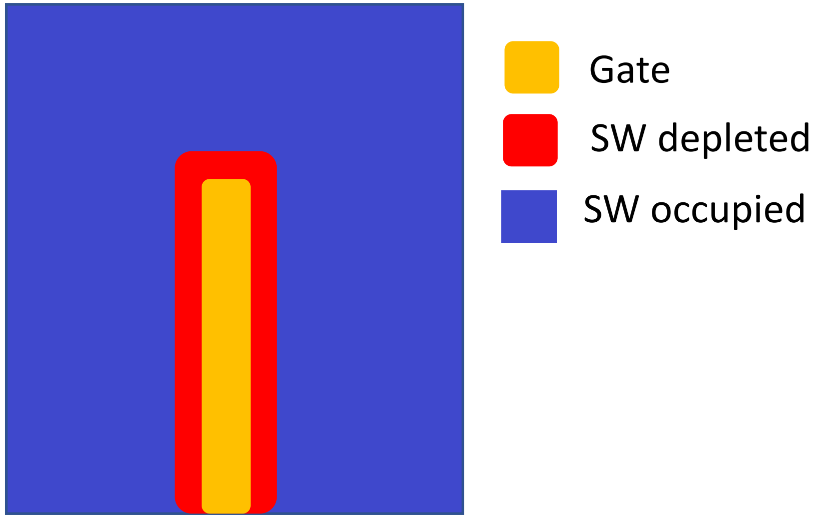
ν=3, ν=3 inner edge mode

$$\delta G \propto \cos\left(2\pi \frac{AB}{\phi_0}\right)\cos\left(\frac{eV_{SD}L}{2\hbar v_{edge}}\right)$$

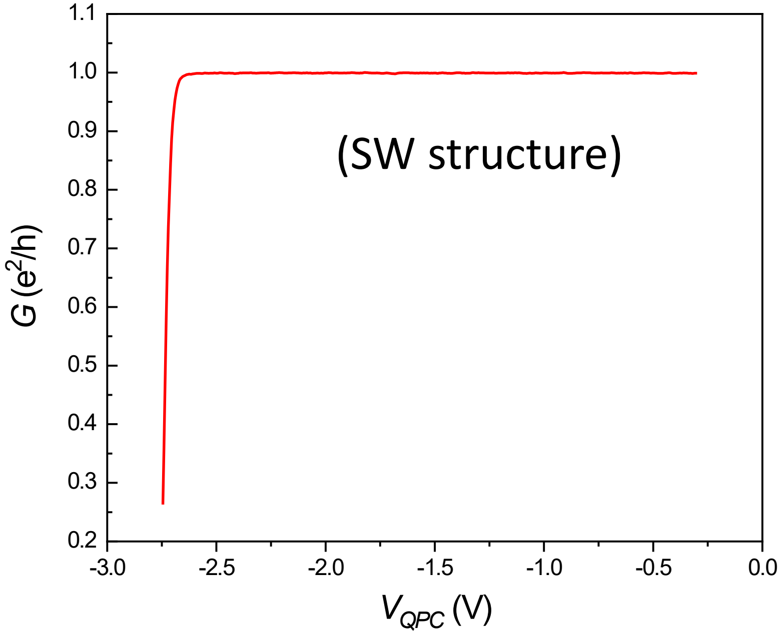
$$v_{edge} = \frac{eL\Delta V_{SD}}{2\pi\hbar}$$

H. Sahasrabudhe et al. “Optimization of edge state velocity in the integer quantum Hall regime”, *Phys. Rev. B.* **97**, 0853202 (2018)

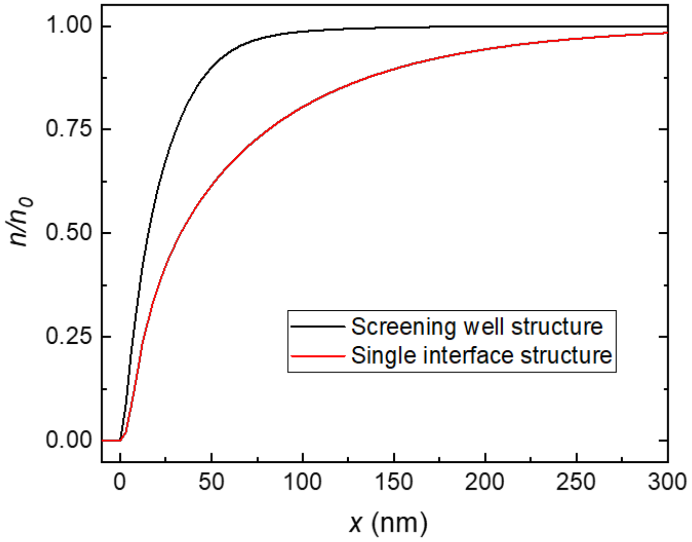
Sharper Confining Potential due to SW structure

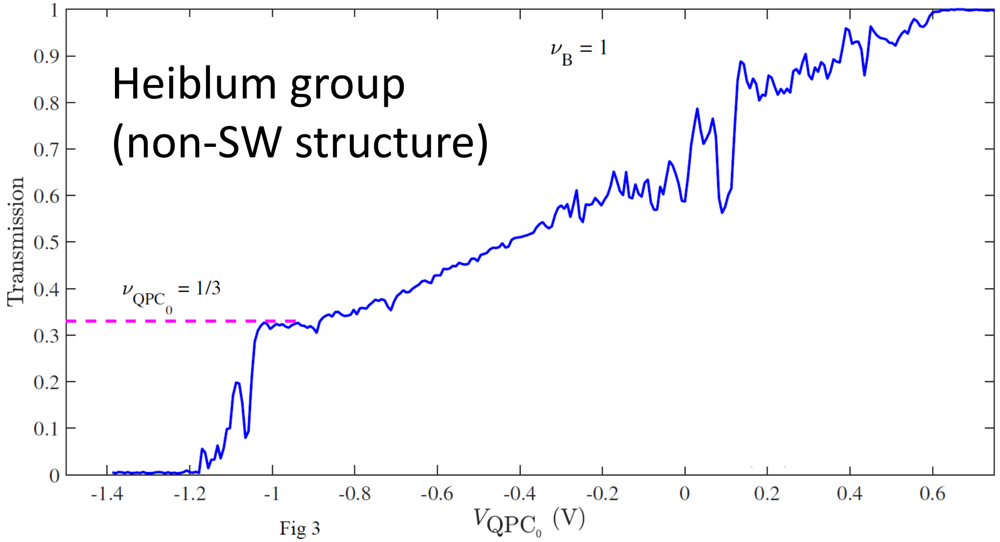


Single QPC sweeps at $\nu = 1$



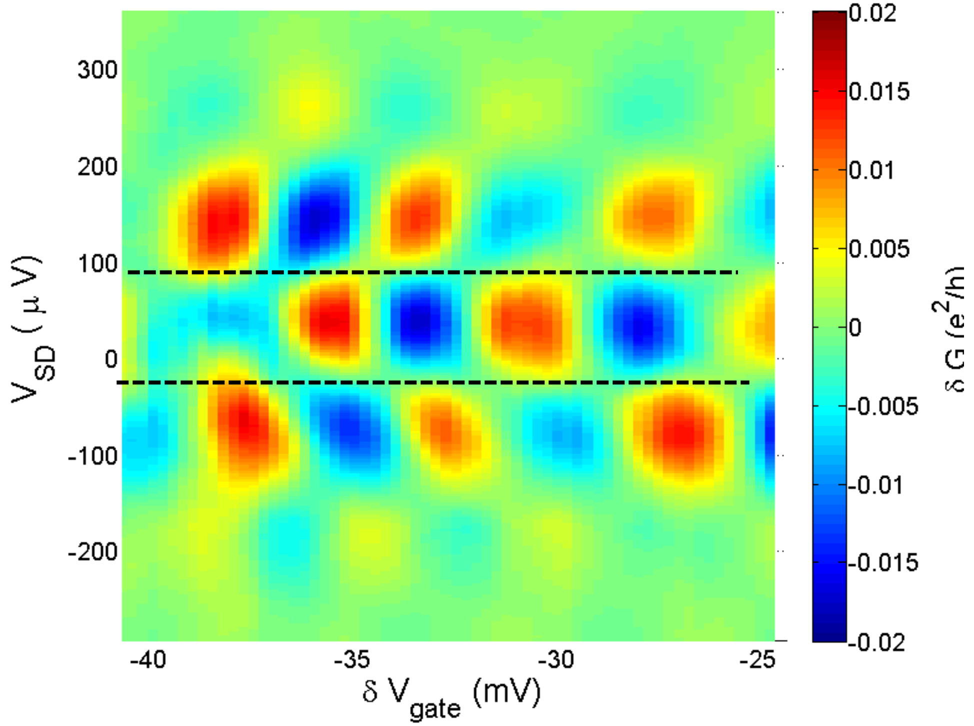
- Simulations indicate that SW structure results in a sharper confining potential at the edge of the gates
- Qualitatively, SW creates a "mask" so QW feels gate potential only in a sharply defined area
- QPCs exhibit much sharper conductance curves compared to standard structures





Edge mode velocity at ν=1/3

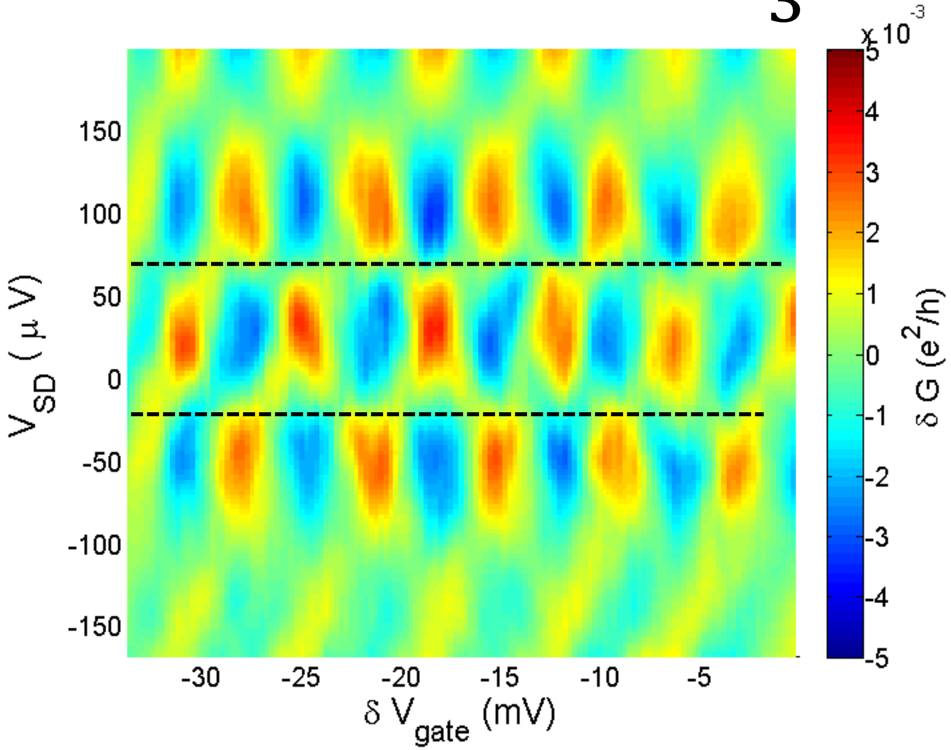
velocity at $\nu = 1$



$\Delta V_{SD} = 108\mu V$

$\boldsymbol{v_{edge} = 8.9 \times 10^4 m/s}$

velocity at $\nu = \frac{1}{3}$

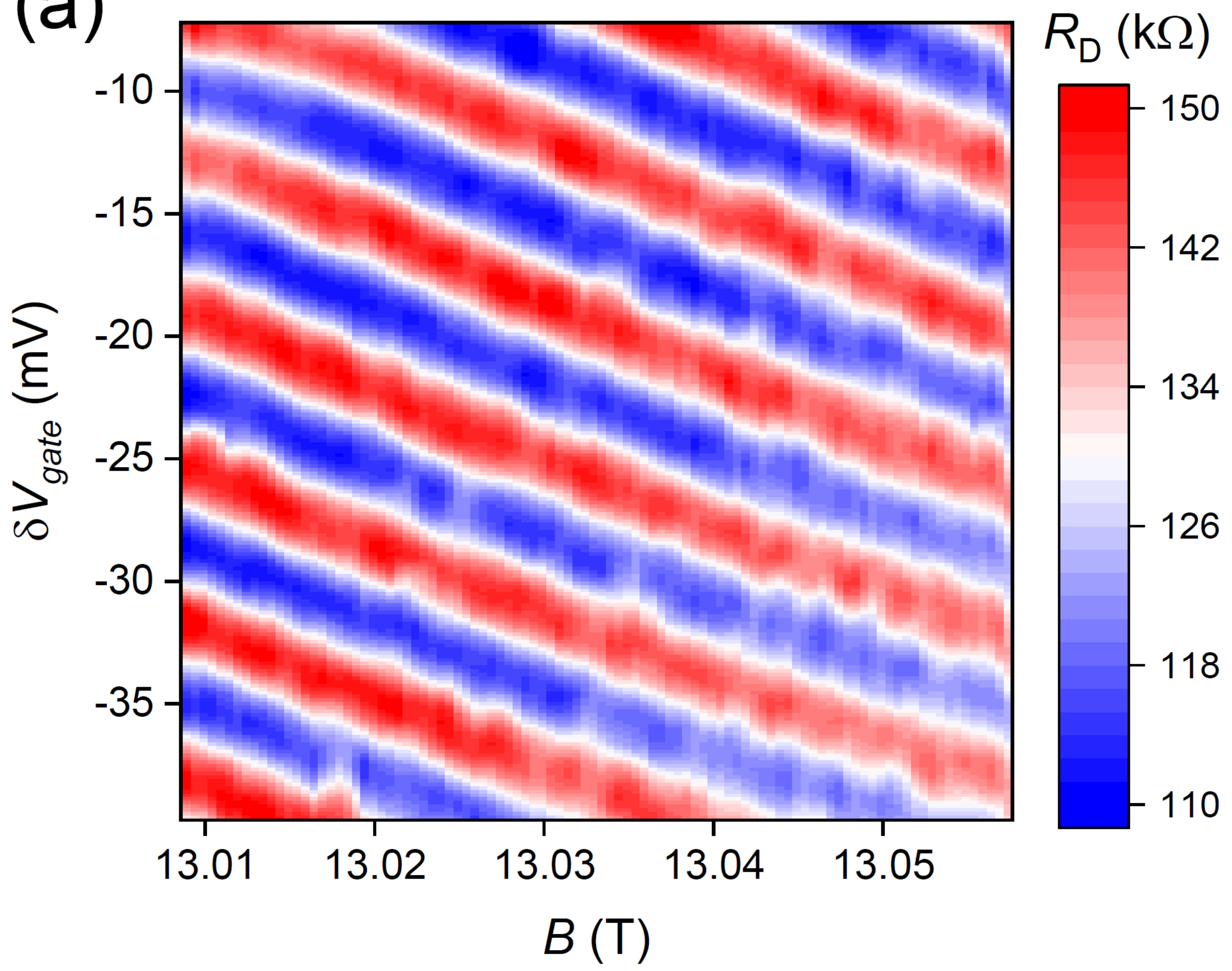


$\Delta V_{SD} = 74\mu V$

$\boldsymbol{v_{edge} = 6.1 \times 10^4 m/s}$

Aharonov-Bohm interference in the FQH regime

(a)



$$\nu_{bulk} = \frac{1}{3}$$

$$\Delta B = 22.2 \text{ mT}$$

$$\Delta V_{gate} = 6.1 \text{ mV}$$

$$\frac{e^*}{e} = 0.29$$

ν=1/3

$$\frac{e^*}{e} = \frac{\Phi_0}{B \Delta V_g \frac{\partial A}{\partial V_g}}$$

nature physics

ARTICLES

https://doi.org/10.1038/s41567-019-0441-8

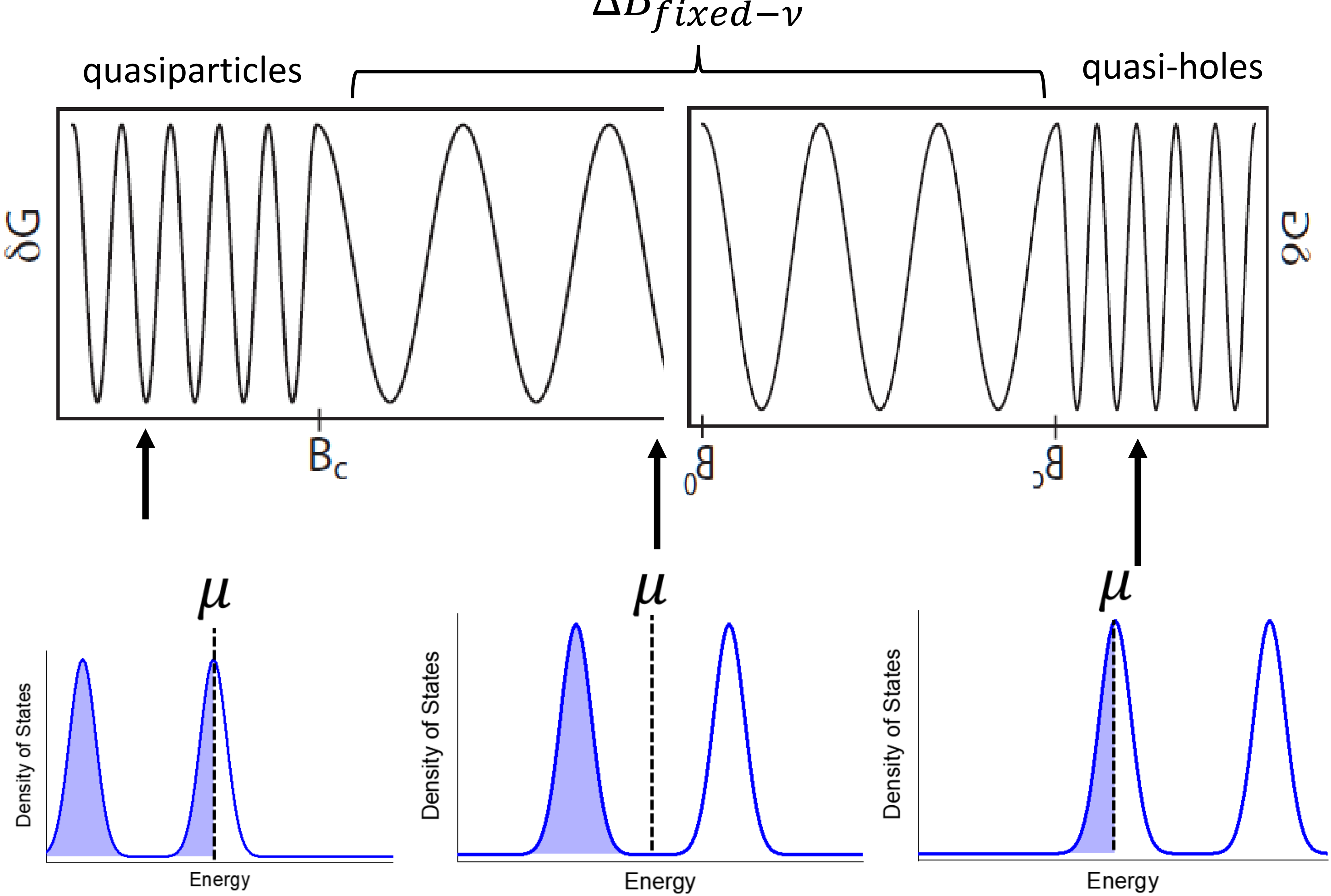
Aharonov–Bohm interference of fractional quantum Hall edge modes

J. Nakamura[1,2], S. Fallahi[1,2], H. Sahasrabudhe[1], R. Rahman[3], S. Liang[1,2], G. C. Gardner[2,4] and M. J. Manfra[1,2,3,4,5]*

Theoretical analysis: fixed ν vs. fixed density

B. Rosenow and A. Stern. *PRL* **124**, 106805 (2020)

- Competition between energy cost to create quasiparticles Δ and electrostatic energy cost to keep ν fixed
- Predicted transition from AB with $3\Phi_0$ period regime of no magnetic field dependence with higher order Φ_0 modulations.



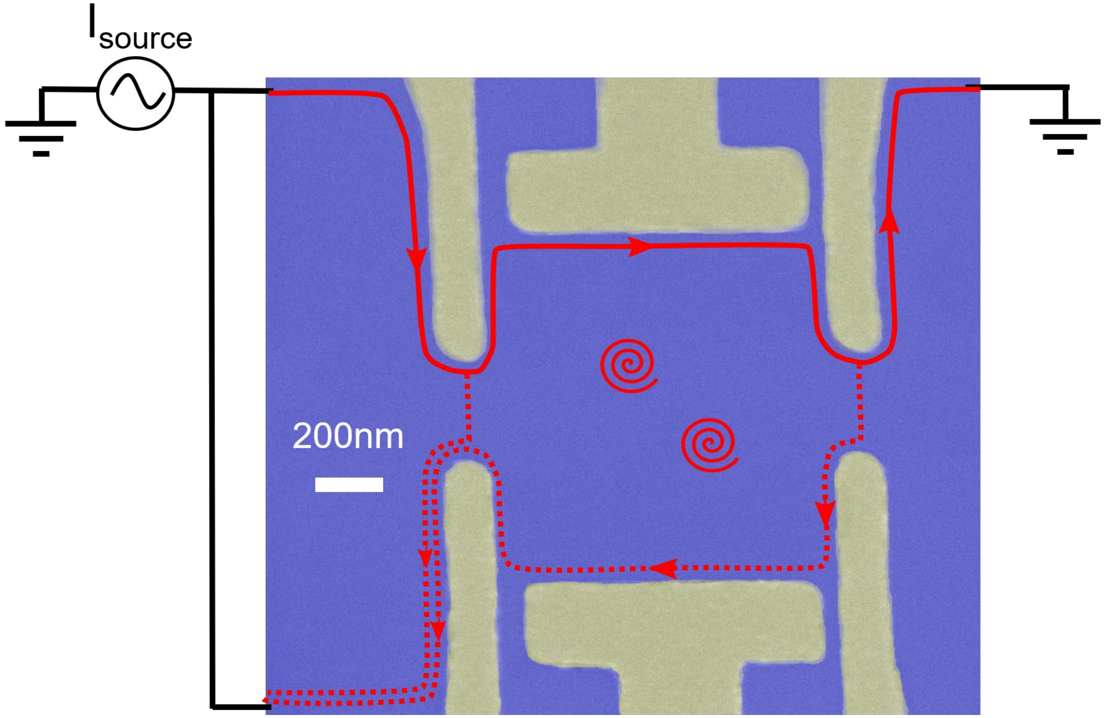
Equation for width of B with fixed ν and $3\Phi_0$ oscillations:

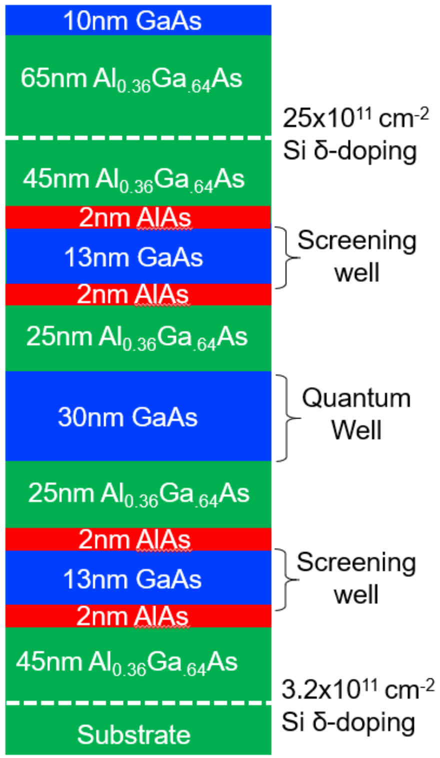
$$\Delta B_{fixed-\nu} = \frac{\Delta \times \Phi_0}{\nu e^* \times \frac{e^2}{C}}$$

Δ = Energy gap of quantum Hall state

C = capacitance to screening layers (per unit area)

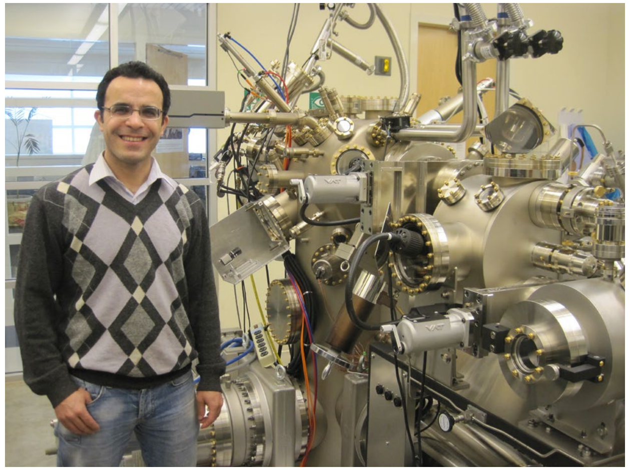
Braiding experiments in lower density and smaller device

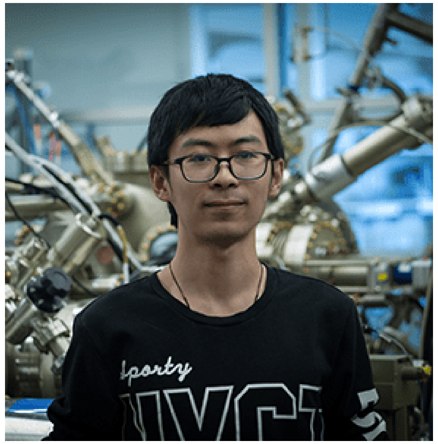


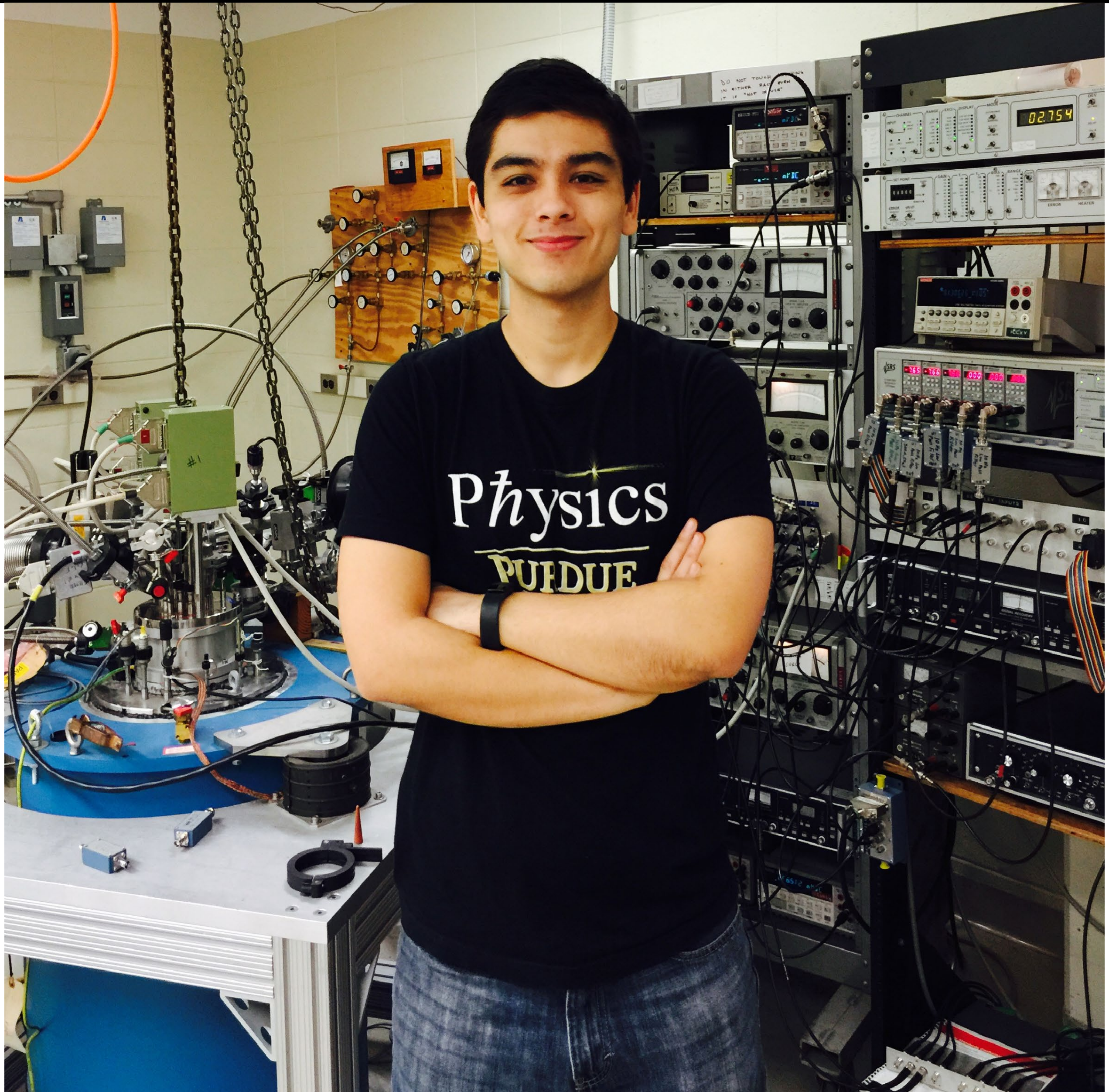


- **Smaller interferometer 1micron by 1micron**
- **Lower density 2DEG: $n \sim 7\times10^{10}cm^{-2}$: lower gate potential enhances stability**
- **Examine interference over broad range of magnetic field around $\nu=1/3$**

Experimental lead: Dr. James Nakamura







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Interference at $\nu = 1/3$: discrete phase jumps

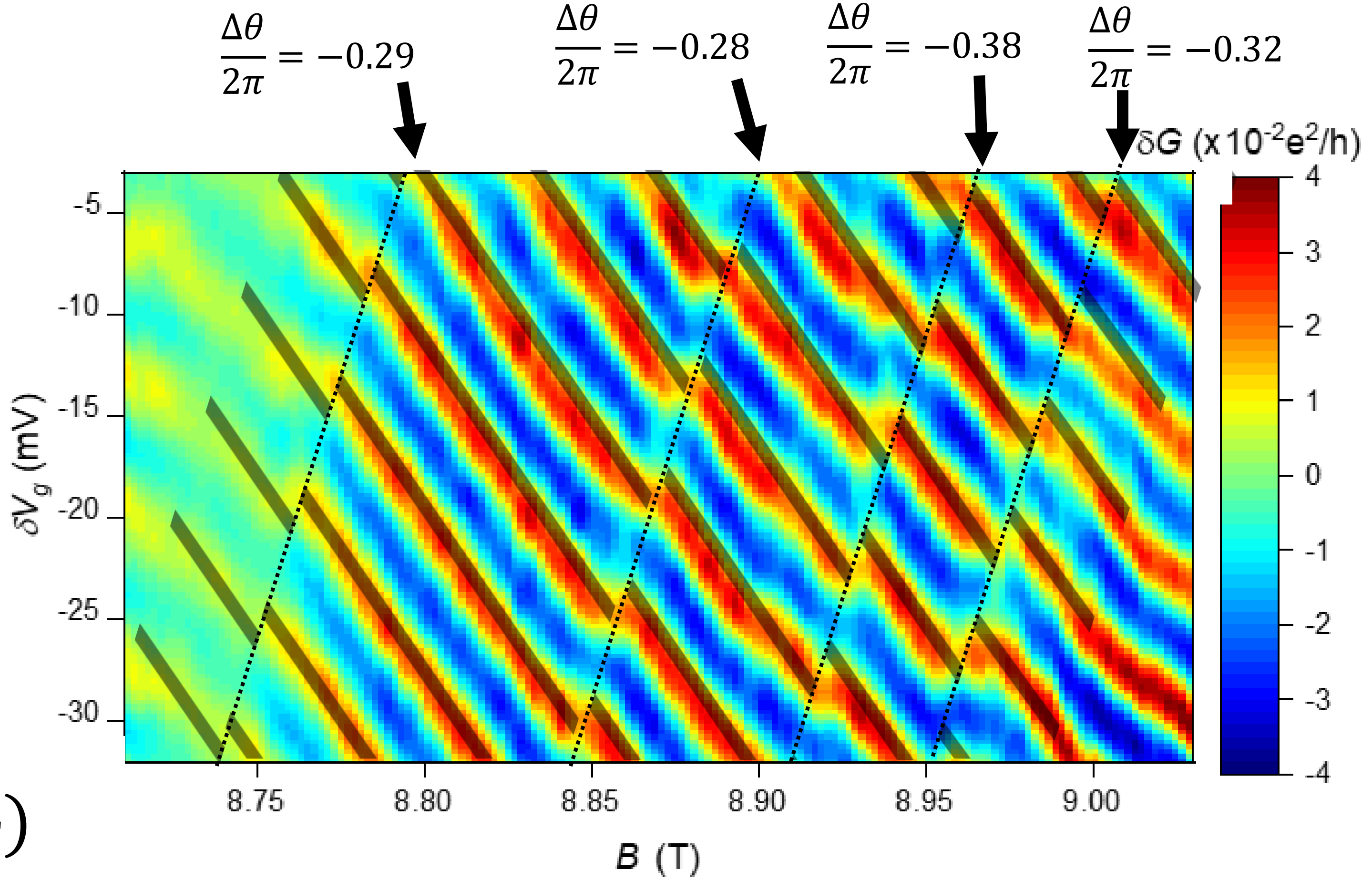
- Observe discrete jumps in interference pattern:

$$\theta = 2\pi\left(\frac{AB}{\Phi_0}\right)\frac{e^*}{e} + N_L\theta_{anyon}$$

- Fit conductance *between* jumps to

$$\delta G = \delta G_0 \cos\left(2\pi\frac{e^*}{e}\frac{AB}{\Phi_0} + \theta_0\right)$$

to get phase jumps $\Delta\theta$



Take average:

$$\Delta\theta = -2\pi \times (0.31 \pm 0.04)$$

$$\boxed{\text{Theory: } \theta_{anyon} = \frac{2\pi}{3}}$$

Demonstration Experiments

nature physics 2019 ARTICLES
https://doi.org/10.1038/s41567-019-0441-8

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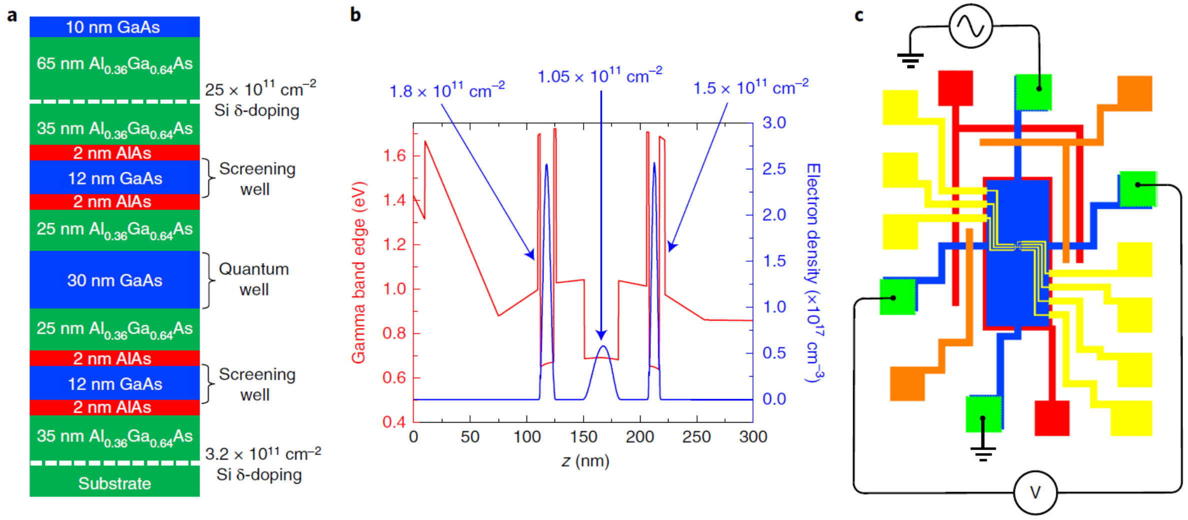
nature physics 2020 ARTICLES
https://doi.org/10.1038/s41567-020-1019-1
Check for updates

Direct observation of anyonic braiding statistics

J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]

Aharonov-Bohm Interference in the FQHE via novel heterostructure and device



Measured $\theta_{anyon} = \frac{2\pi}{3}$ at ν=1/ 3