Superconducting Qubits

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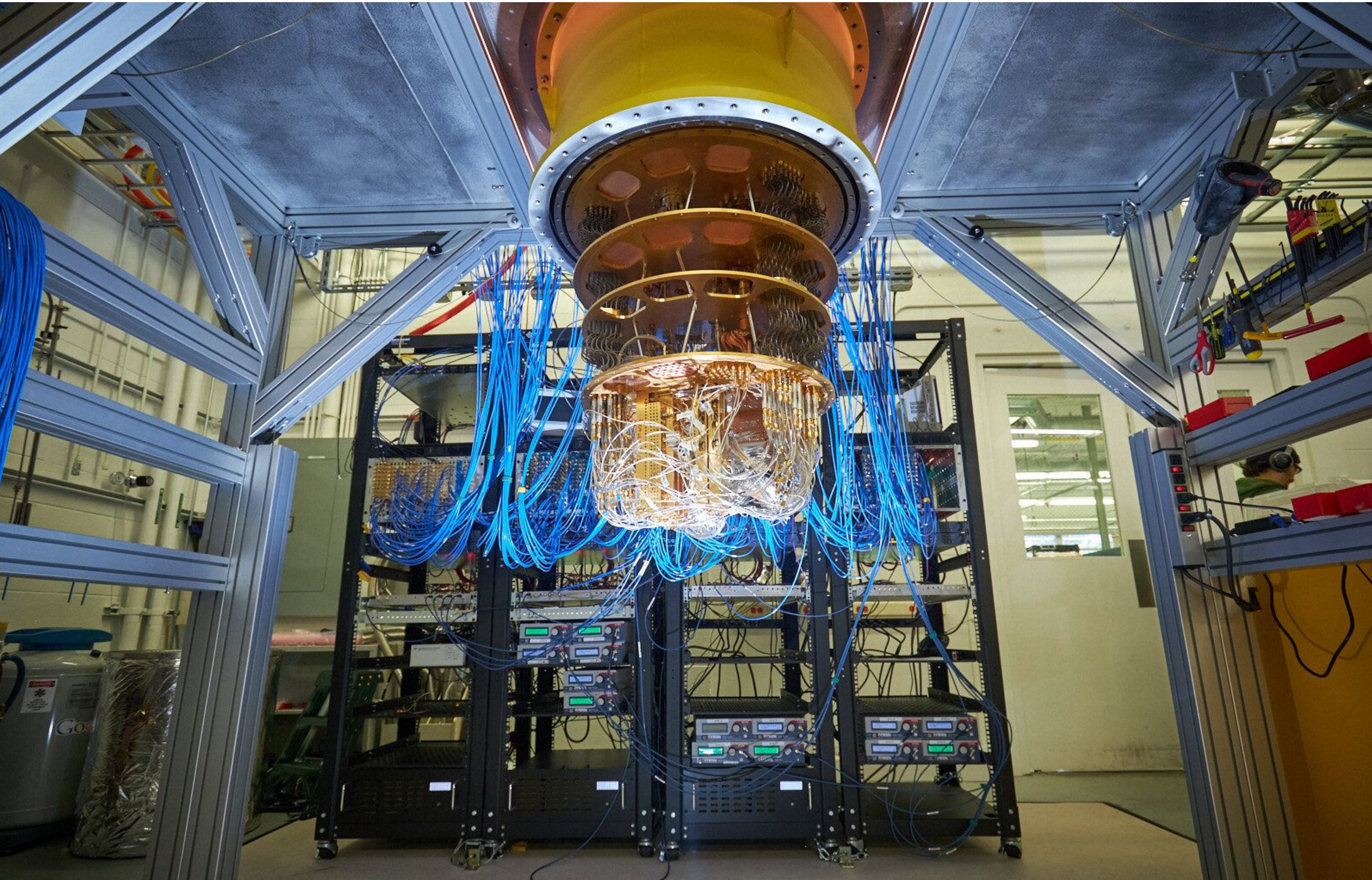
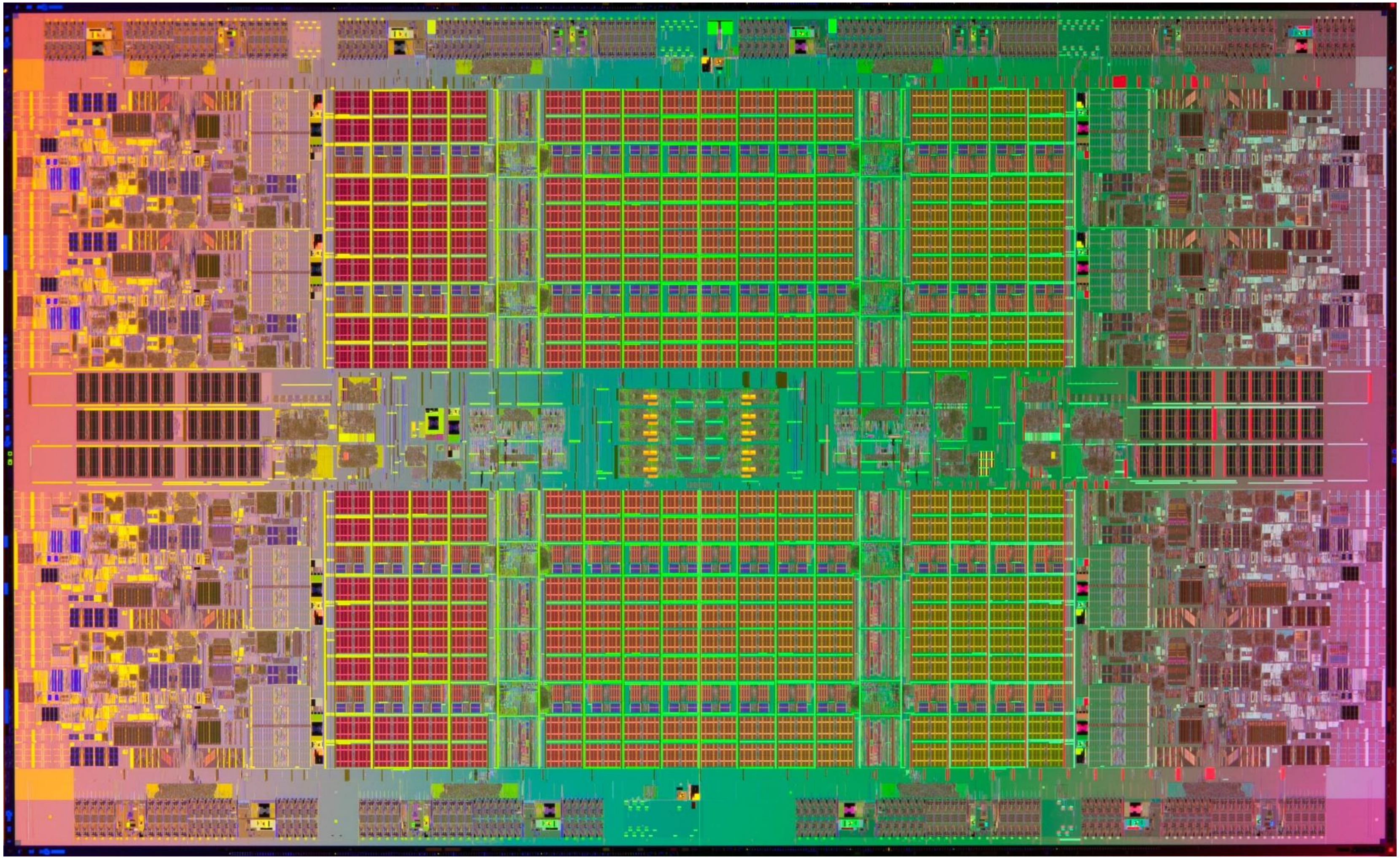


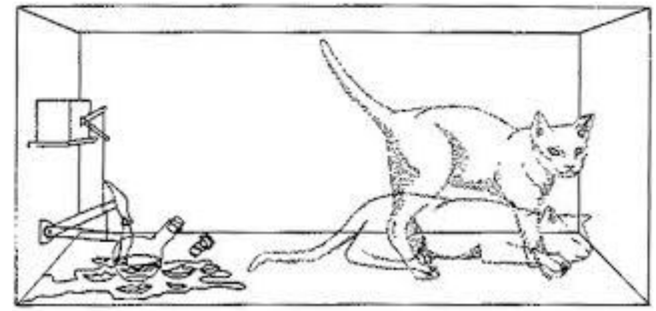
Hamiltonians and Systems Physics



Need simple and scalable concepts for complex systems:
(Combine circuit theory (linear) + quantum physics)

A. Leggett (1980):
Do Macroscopic Variables Obey Quantum Mechanics?

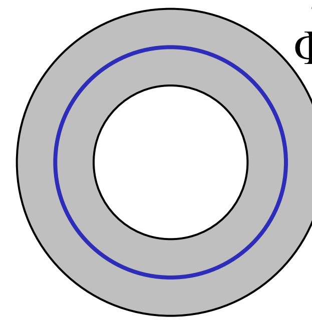
Hypothesis: Schrodinger's cat collapses to classical states because QM not allowed on macroscopic object



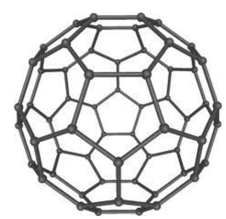
Note: Microscopic QM observable on macroscopic scale;
e.g. crystals, superconductors

$$\Delta\phi = n \cdot 2\pi$$

$$\Phi = n \cdot \Phi_0$$



Will macroscopic object show QM?
(normally tiny effect)



diffraction effects
(Zeilinger group)

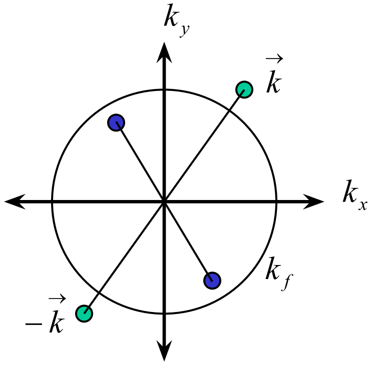
Macroscopic Variable: Current in SC wire

Center of mass of ball
(single variable describes position)

X_{cm}

Would ball tunnel through wall?

Phase of superconductor
(Single phase for all Cooper pairs)



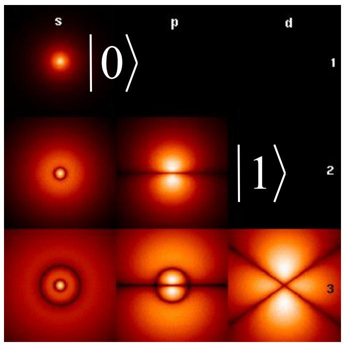
$$\Psi = \prod_k \left(u_k + v_k e^{i\varphi} c_k^+ c_{-k}^+\right) |\Psi_0\rangle$$

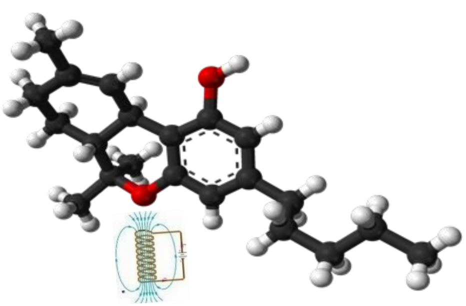
$$I \propto \nabla \phi$$

Macroscopic QM Enables New Physics
Control of single quantum systems, to quantum computers

<u>1 nm</u>

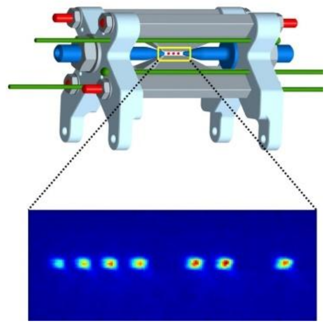
H atom wavefunctions:

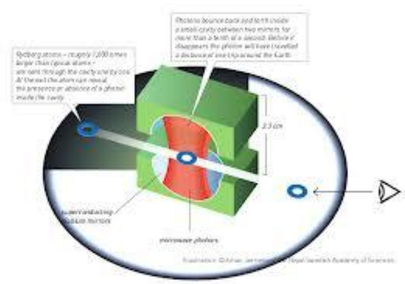


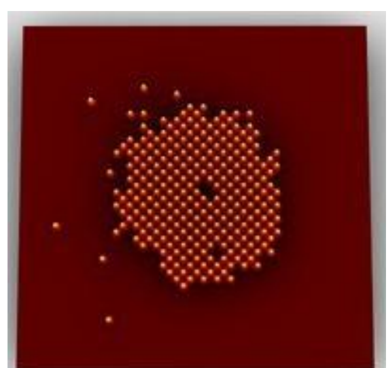


Need
large "Molecules"
for Control Signals

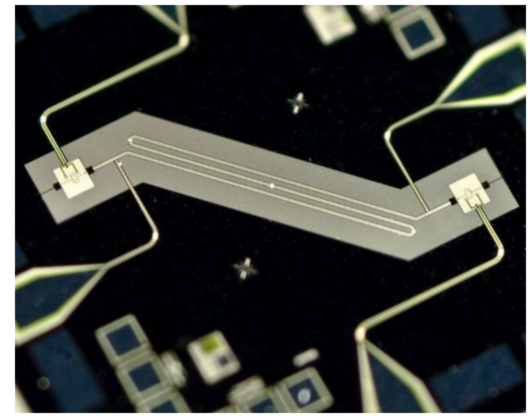
<u>1 μm</u>

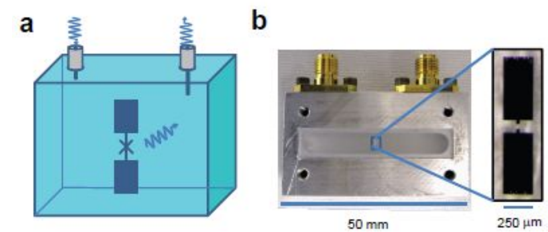






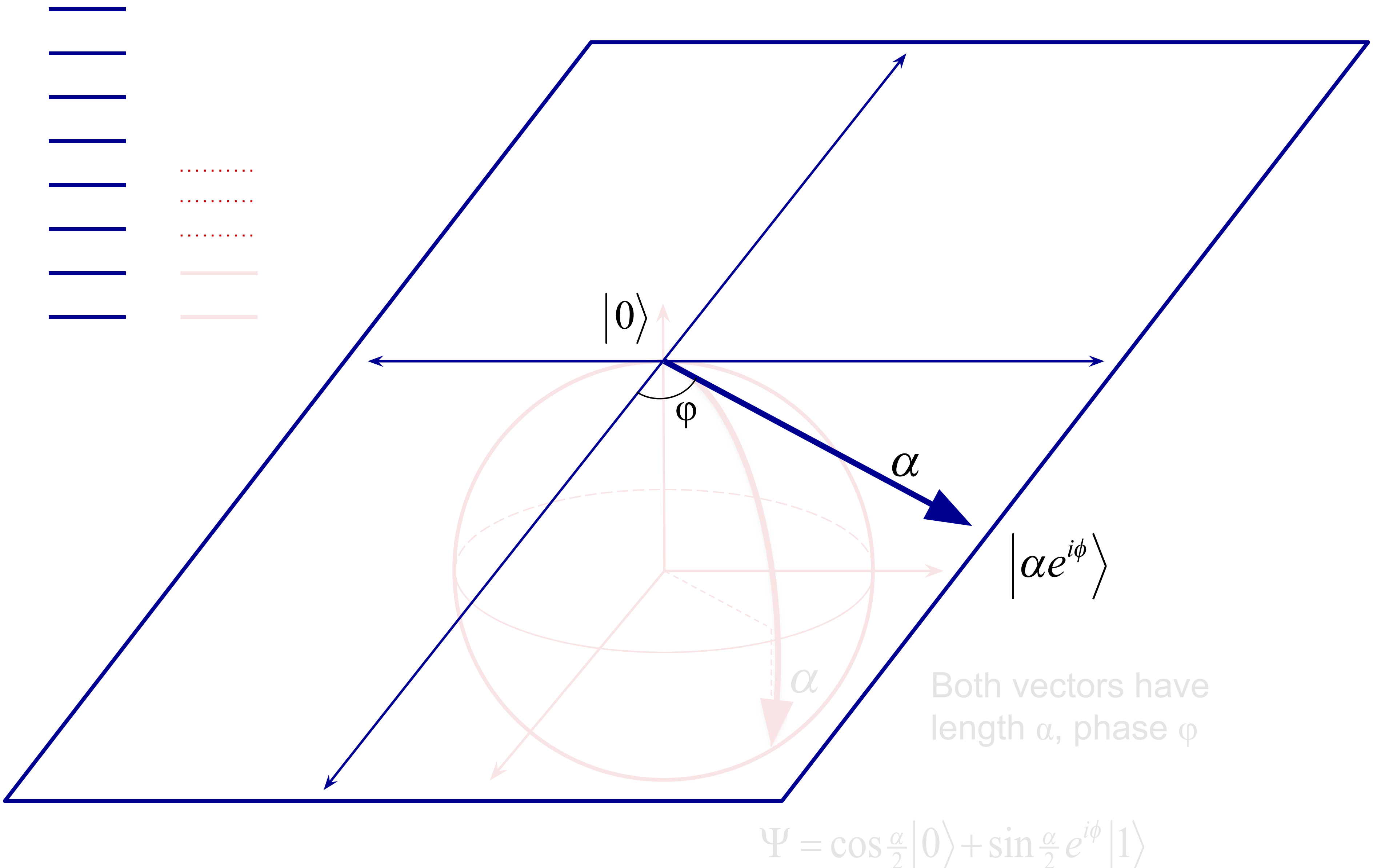
<u>1 cm</u>





- New quantum systems
  e.g. nanomechanics
- Strategy: Large "atom" has
  room for complex control

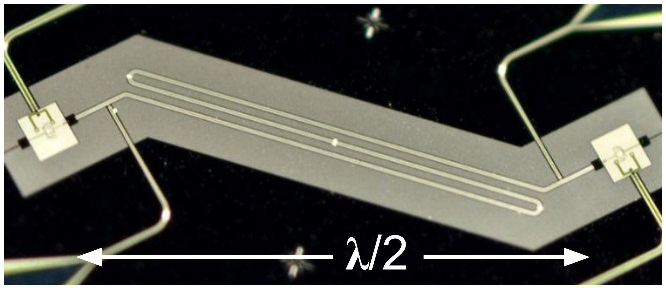
Geometry of Harmonic Oscillators and Qubits

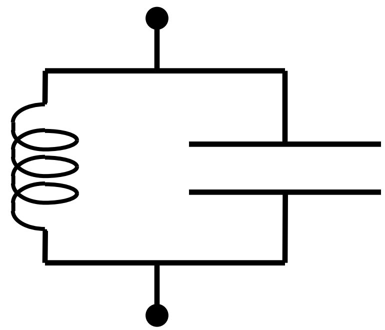


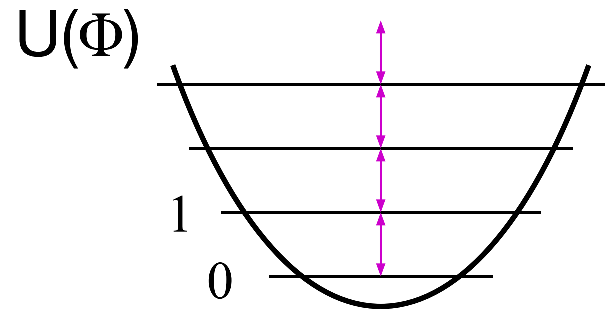
Superconducting Qubits

- Macroscopic "atom": quantize I and V,   5 GHz >> 20 mK

- LC oscillator (linear): memory and communication

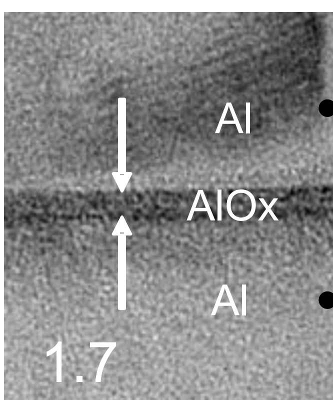


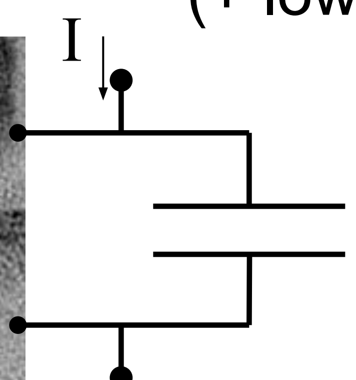


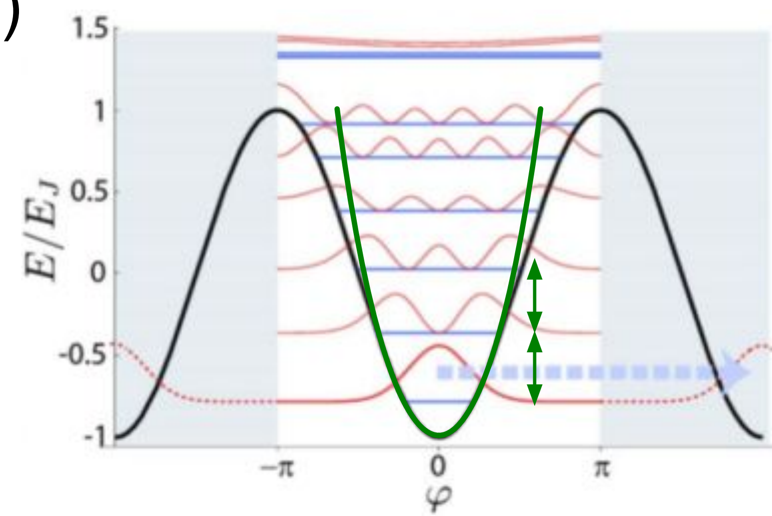


- Josephson junction: <u>non-linear</u> inductance with <u>1 photon</u>

(+ low loss)

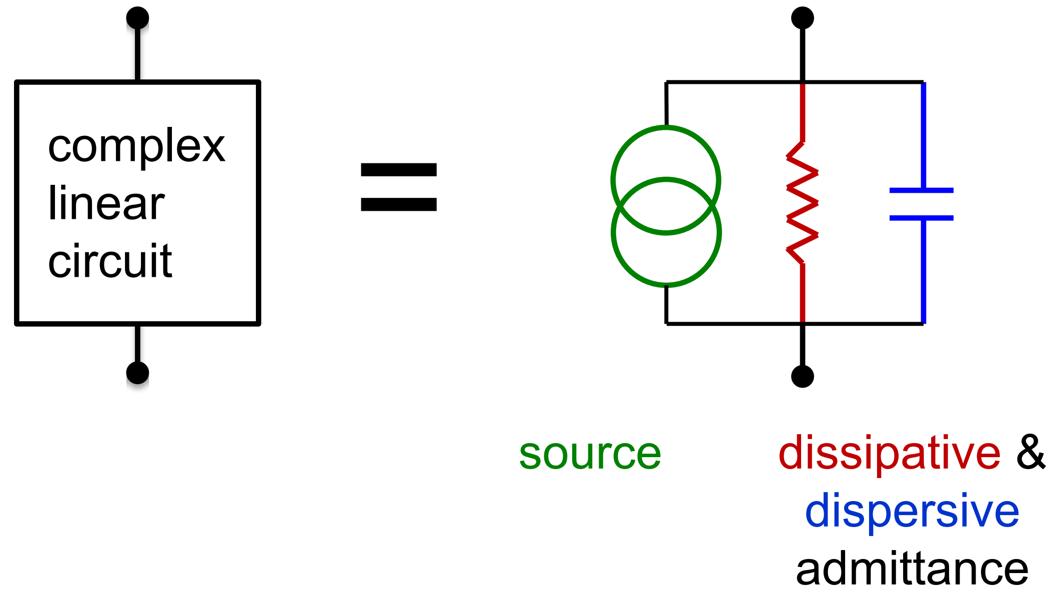






Linear Circuits & Universality / Equivalence

describe with
simple circuits

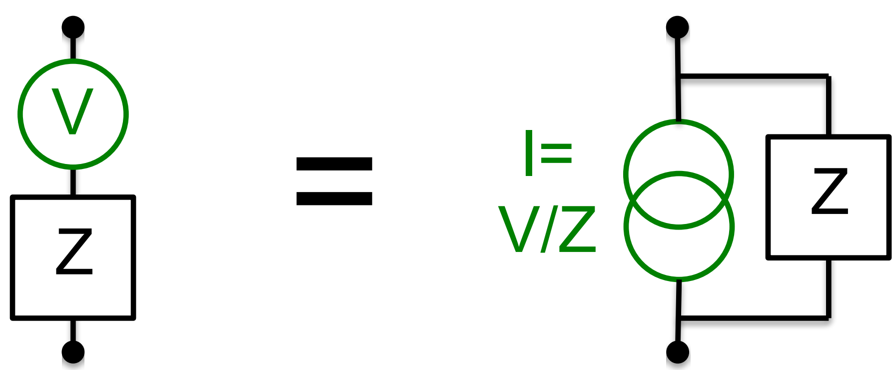


1: $Z_{series} = Z_1 + Z_2$

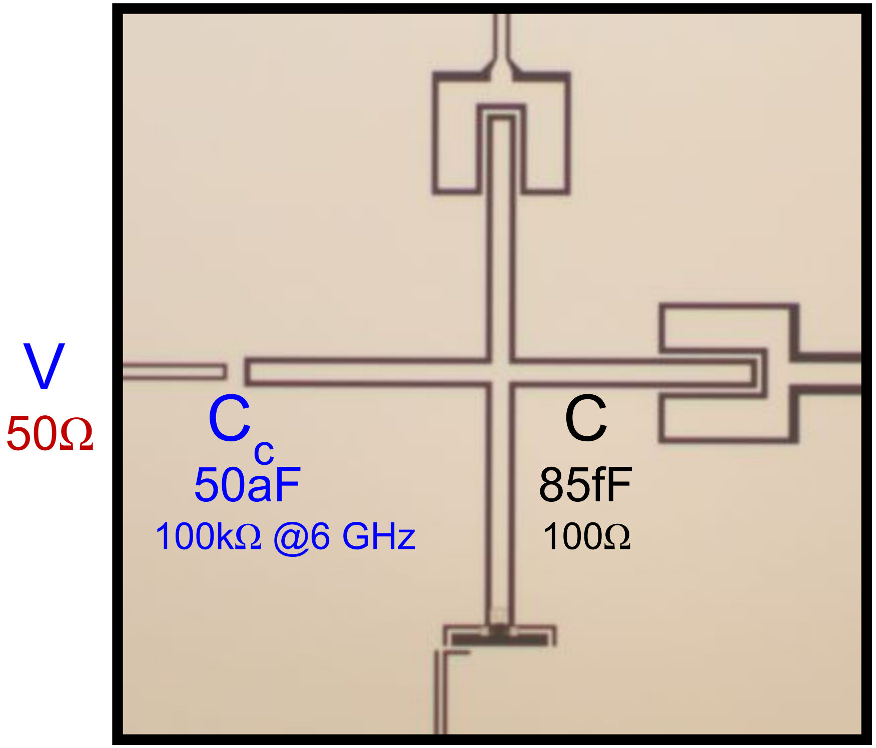
2: $1/Z_{parallel} = 1/Z_1 + 1/Z_2$

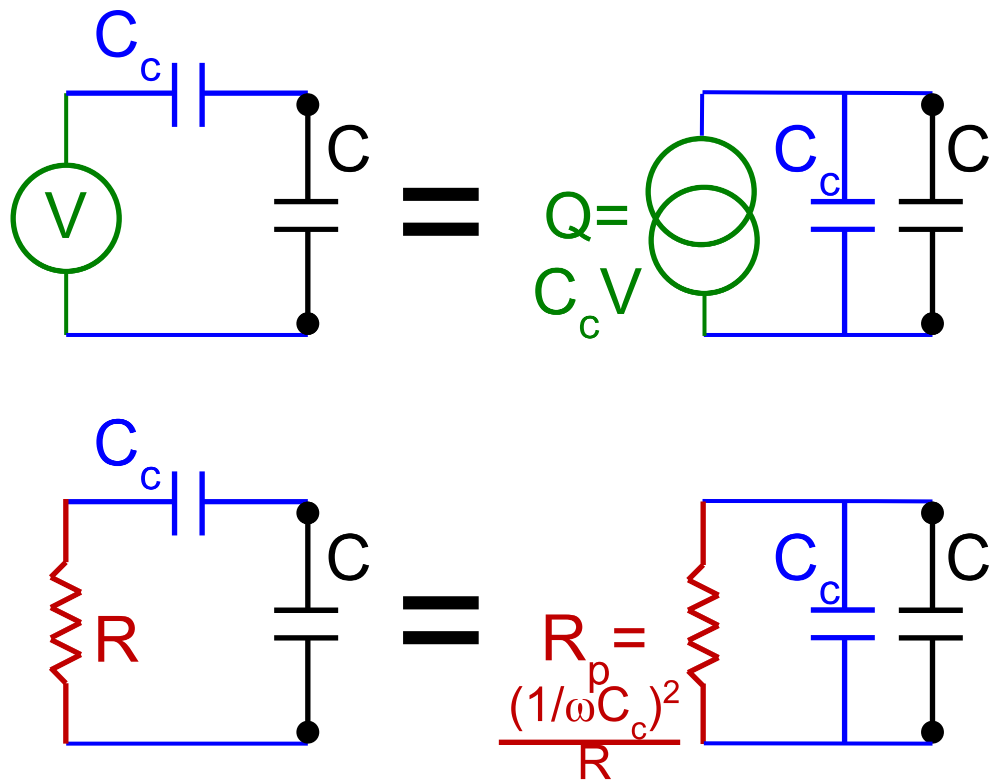
3: Thevanin-Norton
(series – parallel)

proof:
linear, so match 2 cases –
short & open



Example: Capacitor Coupling





Easier, match loss tangents:

$$\frac{R}{1/i\omega C_c} = \frac{1/i\omega C_c}{R_p}$$

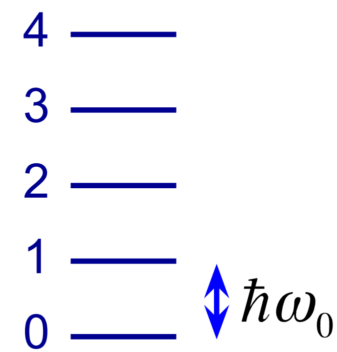
$$\frac{1}{R+1/i\omega C_c} = \frac{R-1/i\omega C_c}{\cancel{R^2}+(1/\omega C_c)^2}$$

$$\cong \frac{R}{(1/\omega C_c)^2} + i\omega C_c$$

Quantum Harmonic Oscillator

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \qquad [\hat{\Phi},\hat{Q}] = i\hbar$$

Frequency: $\omega_0 = 1/(LC)^{1/2}$



$$\hat{\Phi} = \Phi_{zp}(\hat{a} + \hat{a}^+)$$

$$\hat{Q} = iQ_{zp}(\hat{a} - \hat{a}^+)$$

$$\frac{\Phi_{zp}^2}{2L} = \frac{Q_{zp}^2}{2C} = \frac{1}{4}\hbar\omega_0$$

$$\frac{\Phi_{zp}}{\Phi_0} = \sqrt{\frac{1}{\pi}\frac{Z_0}{R_K}}$$

$$\frac{Q_{zp}}{2e} = \frac{1}{4}\sqrt{\frac{1}{\pi}\frac{R_K}{Z_0}}$$

$$R_K = \frac{h}{e^2} = 25.8\ \mathrm{k\Omega}$$

Impedance: $Z_0 = 1/\omega_0 C$

Nature typically gives
$Z_0 \sim 377\Omega \ll R_K$

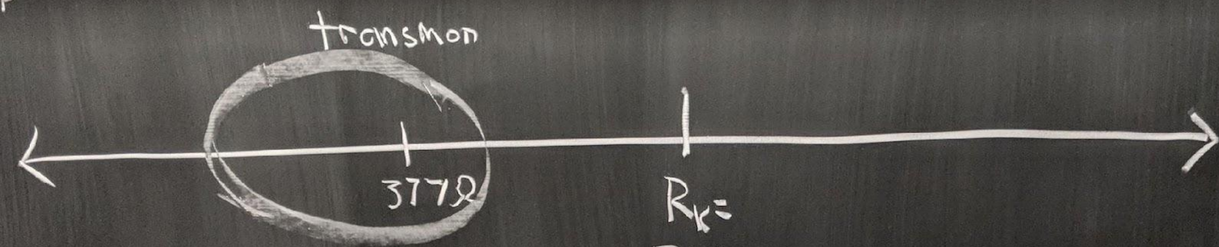
$\Phi_{zp} \ll \Phi_0$
$Q_{zp} \gg 2e$

Flux (phase) basis

$\Phi_{zp} \ll \Phi_0$

Charge basis

$Q_{zp} \ll e$



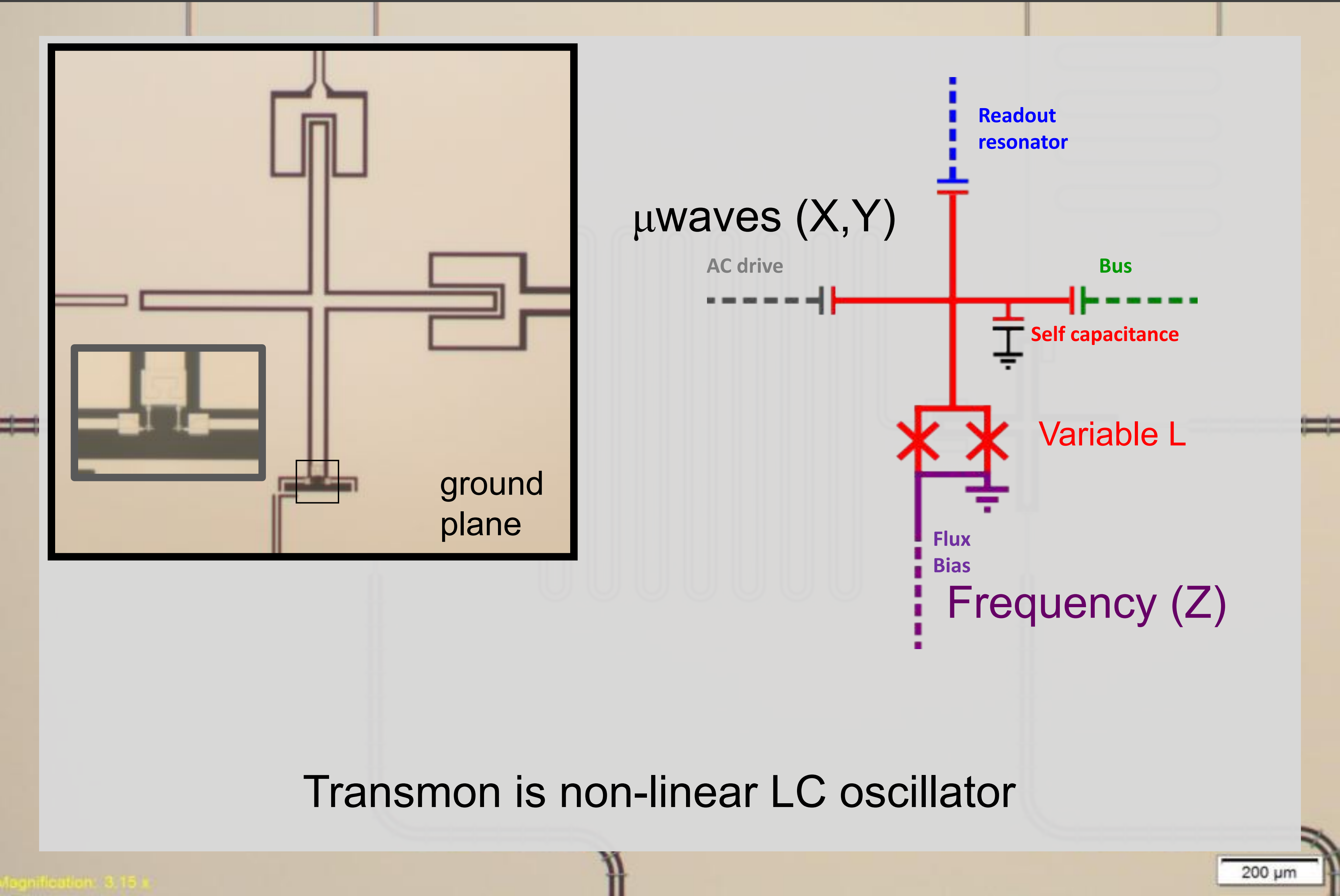
$Z_0 = \frac{1}{\omega_0 C}$

$R_K =$
$25\,k\Omega$
Quantum impedance

$\xrightarrow{\text{non-lin. gets bigger}}$

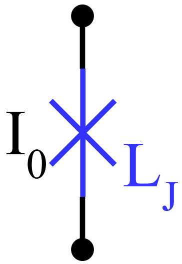
$\frac{Z_0}{R_K} \propto \left(\frac{E_C}{E_J}\right)^2$

Xmon Circuit



Transmon is non-linear LC oscillator

Junction is Nonlinear Inductor



AC: $V = (\Phi_0/2\pi)\dot{\delta}$

Looks like $V = \dot{\Phi}$

δ is dimensionless flux

DC: $I = I_0 \sin\delta$

$= I_0 \sin(2\pi\Phi / \Phi_0)$

Non-linear I-Φ

Small signal: $\dot{I} = I_0 \cos\delta \, \dot{\delta}$

$\equiv (1/L_J)V$

$L_J = \Phi_0/2\pi I_0 \cos\delta$
nonlinear inductor

Junction energy: $U(\delta) = \int dt \, IV$

$= (I_0\Phi_0 / 2\pi)\int dt \sin\delta \; d\delta / dt$

$= -(I_0\Phi_0 / 2\pi)\cos\delta$

Nonlinear Resonator

$$E = \frac{Q^2}{2C} - \frac{I_0\Phi_0}{2\pi}\cos\left(2\pi\Phi/\Phi_0\right)$$

dim'less: $\{Q,\Phi\} \Rightarrow \{2en, (\Phi_0/2\pi)\delta\}$

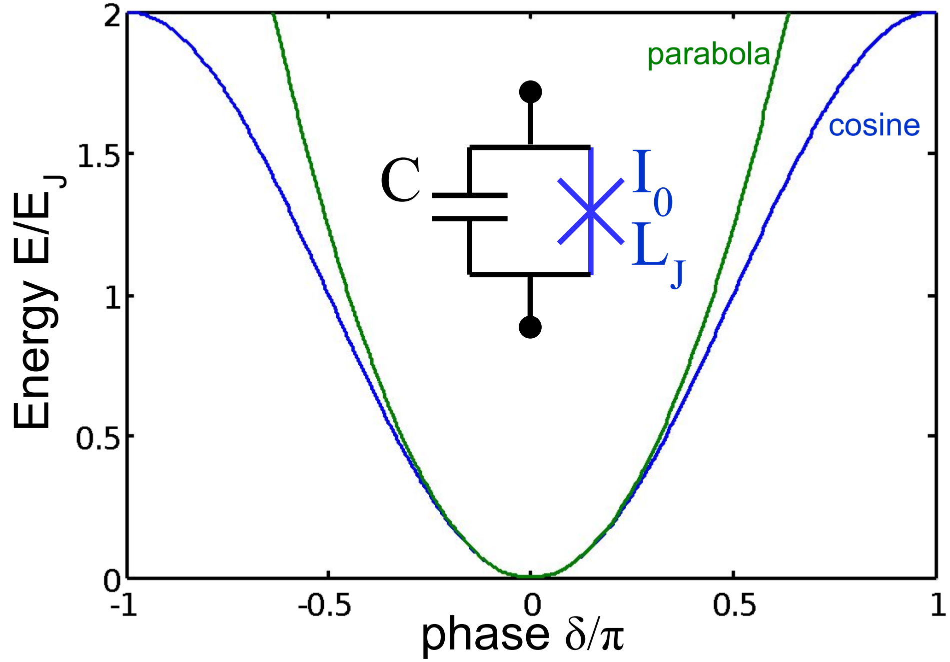
$$E = 4E_C n^2 - E_J \cos\delta$$

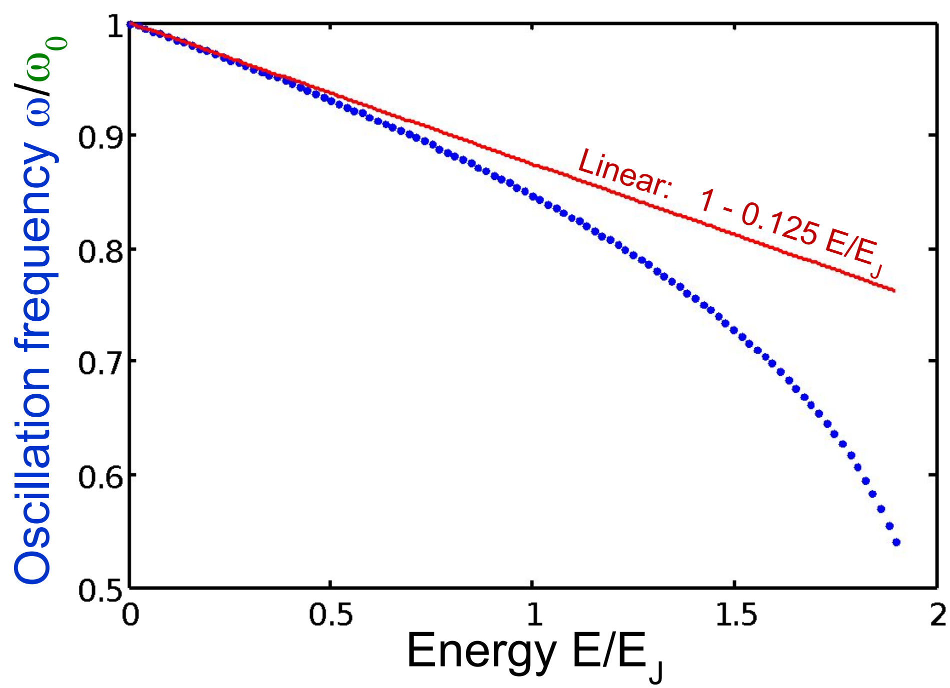
energies: $E_C = e^2/2C, \quad E_J = I_0\Phi_0/2\pi$

charging Josephson

At low energy E,
classical oscillation frequency:

$$\omega_0 = 1/\sqrt{L_{J0}C} = \sqrt{8E_J E_C}\,/\,\hbar$$





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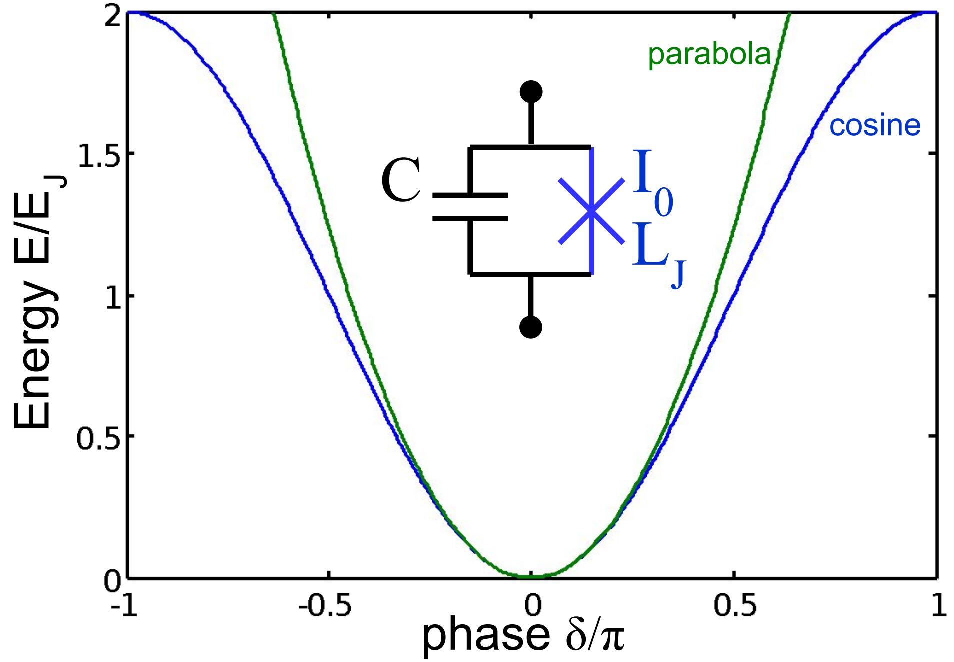

At low energy E,
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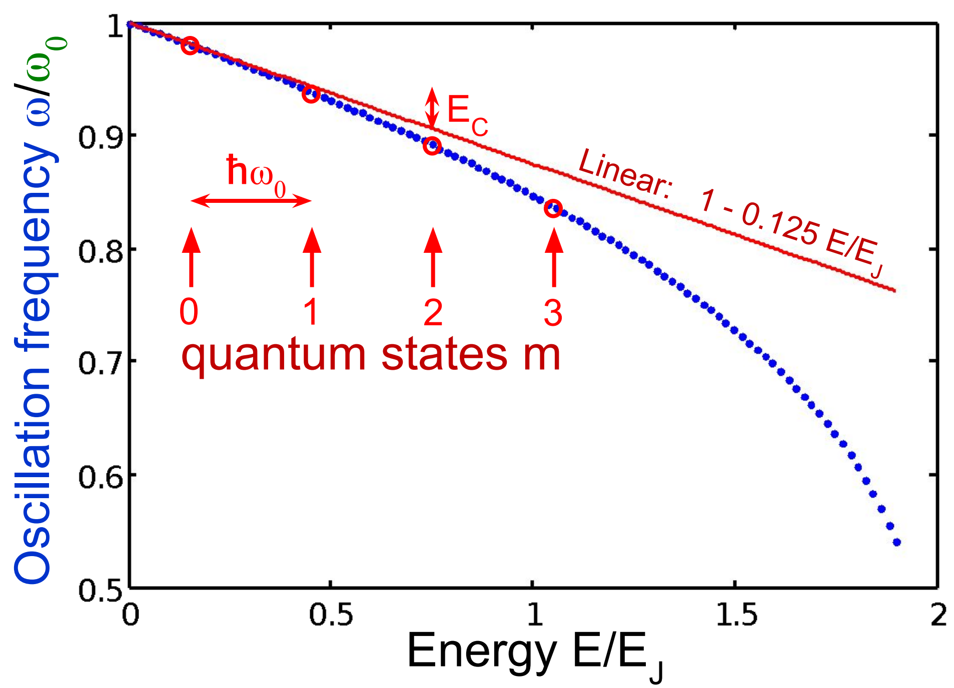
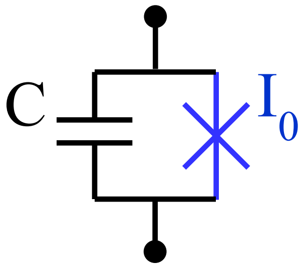
Semiclassical perturbation:

quantized E level

$$\omega_m = \omega_0\left[1 - 0.125(m+1/2)\hbar\omega_0/E_J\right]$$
$$= \omega_0 - (m+1/2)E_C/\hbar$$



Qubit: Weakly Anharmonic LC Resonator



$$\hat{H} = 4E_C\hat{n}^2 - E_J\cos\hat{\delta}$$

$$\cong 4E_C\hat{n}^2 + E_J\left(\frac{\hat{\delta}^2}{2} - \frac{\hat{\delta}^4}{24} + \ldots\right)$$

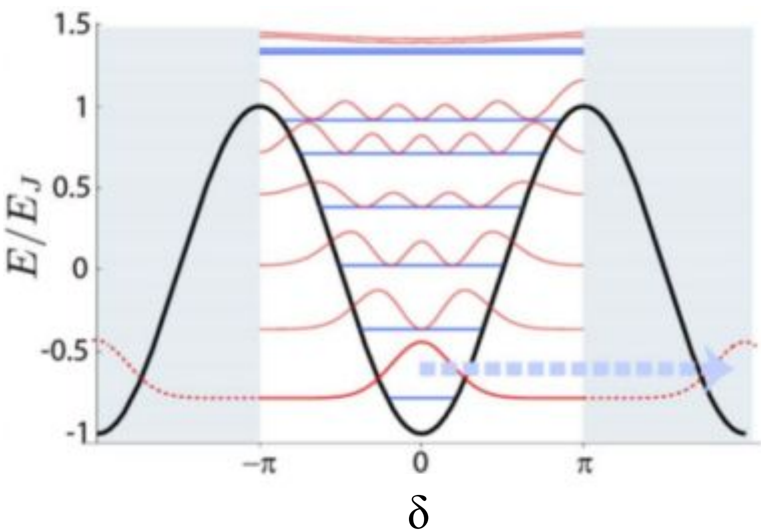
commuting operators:

$$[\hat{\Phi},\hat{Q}] = i\hbar \qquad [\hat{\delta},\hat{n}] = i$$

Harmonic oscillator: $\omega_0 = 1/\sqrt{L_{J0}C}$

$$\hat{\Phi} = (\Phi_0/2\pi)\hat{\delta} = \sqrt{\hbar/2\omega_0 C}\left[\hat{a}+\hat{a}^+\right]$$

$$\hat{Q} = (2e)\hat{n} = i\sqrt{\hbar\,\omega_0 C/2}\left[\hat{a}-\hat{a}^+\right]$$



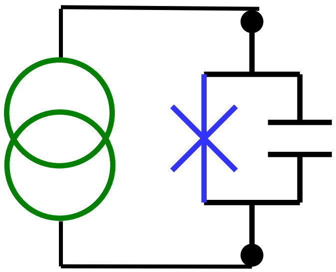
Non-linearity $\eta = E_c \sim 200$ MHz:

$$\Delta E_m = -E_J\langle m|\hat{\delta}^4|m\rangle/24$$

$$\Delta E_m - \Delta E_{m-1} = -mE_C$$

Zeroing Decoherence from Charge Noise

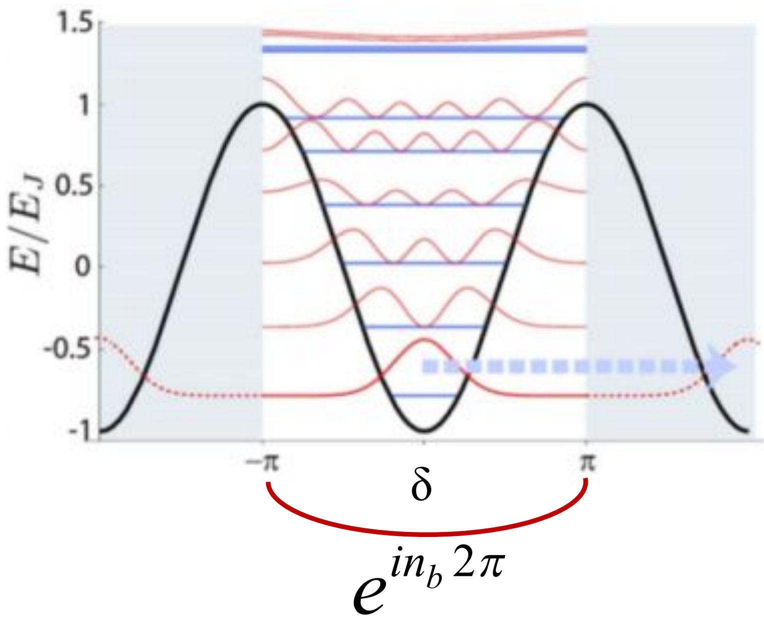
$Q_b = 2en_b$



$$\hat{H} = 4E_C(\hat{n} - n_b)^2 - E_J \cos\hat{\delta}$$

QP tunneling n_b={0,1/2}, defect motion in dielectrics produces charge noise.

In δ basis, described as displacement operator $e^{in_b\hat{\delta}}$



$$E(n_n) \propto \left|\Psi(\pi)\right|^2$$
$$\approx \exp\left[-(\omega C/\hbar)(\Phi_0/2)^2\right]$$
$$= \exp\left[-(\pi^2/8)\sqrt{8E_J/E_C}\right]$$
$$\approx \exp\left[-\sqrt{8E_J/E_C}\right]$$

H.O.

WKB

For exponential small dependence on Q_n, choose large C

For large energy non-linearity, small C

Good choice is E_C=200 MHz non-linearity E_J/E_C = 80: exp(-25)

$\omega_0/2\pi$ = 6 GHz
E_C= 200 MHz, C = 0.1 pF
E_J = 22.5 GHz, I_0 = 200 nA
$1/\omega_0 C$ = 250 Ω
Non-linearity $\eta/2\pi$ = 200 MHz

Numerical Solution via Matrix Eigenvalues

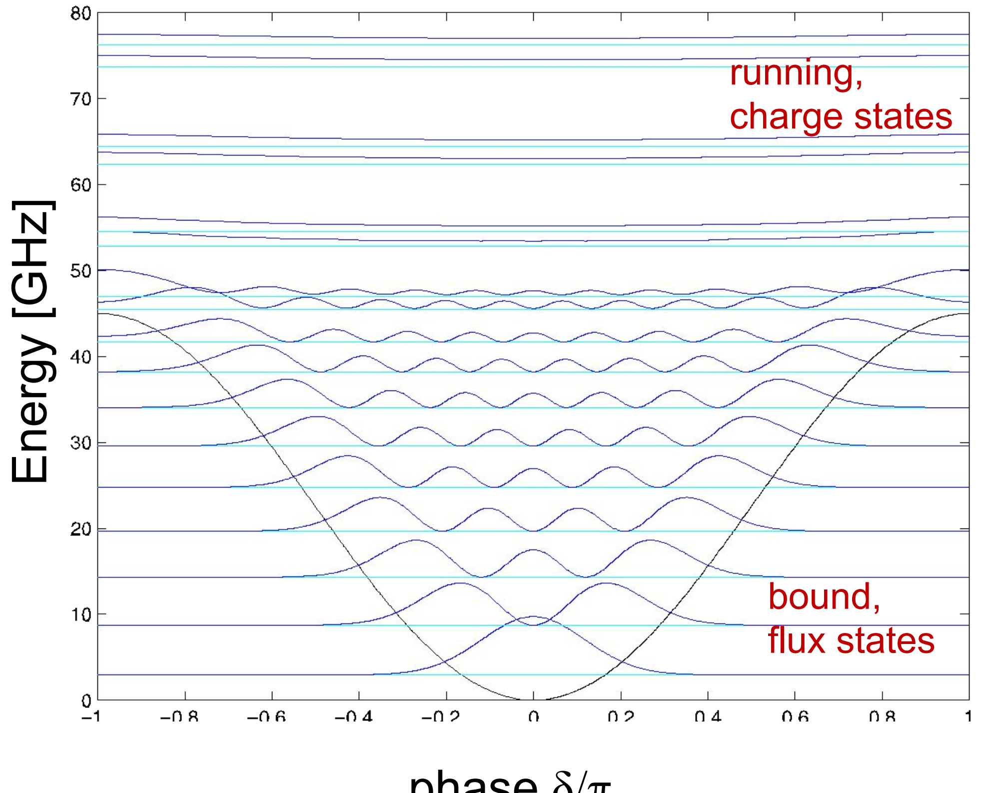
$$\hat{V} = \begin{pmatrix} V(\delta_1) & 0 & 0 & 0 \\ 0 & V(\delta_2) & 0 & 0 \\ 0 & 0 & V(\delta_3) & 0 \\ 0 & 0 & 0 & V(\delta_4) \end{pmatrix}$$

$$\hat{q} = i\frac{d}{d\delta} = \frac{i}{2\Delta\delta}\begin{pmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

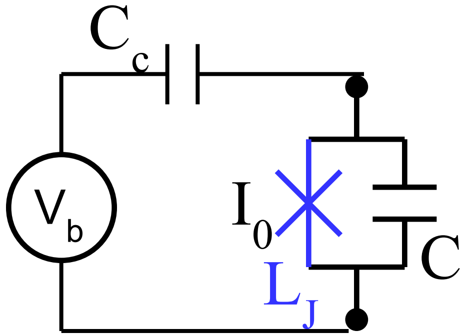
$$\hat{q}^2 = -\frac{d^2}{d\delta^2} = \frac{-1}{(\Delta\delta)^2}\begin{pmatrix} -2 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 1 & 0 & 1 & -2 \end{pmatrix}$$

Flux is natural basis set: $\Psi(\delta)$

Plot of $|\Psi(\delta)|^2$ at eigenenergies



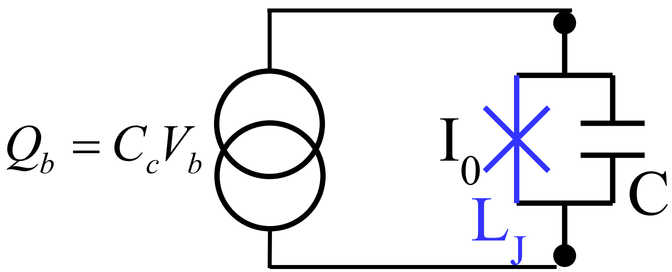
E_c=0.2 GHz
E_J=22.5 GHz
n_g=0.1

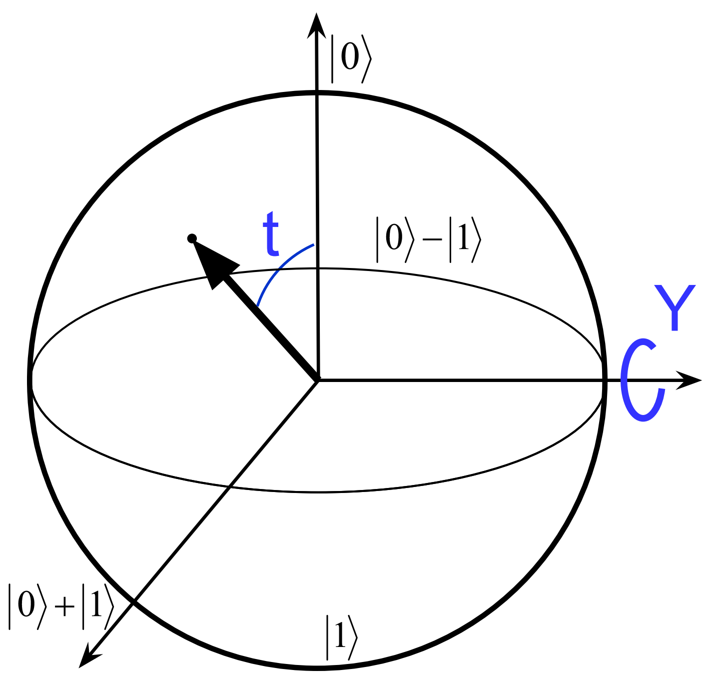


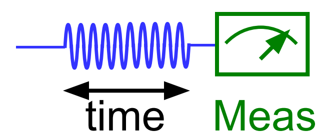
Qubit: Microwave Drive

Use weak coupling ($C_c << C$) but strong drive with large V_b

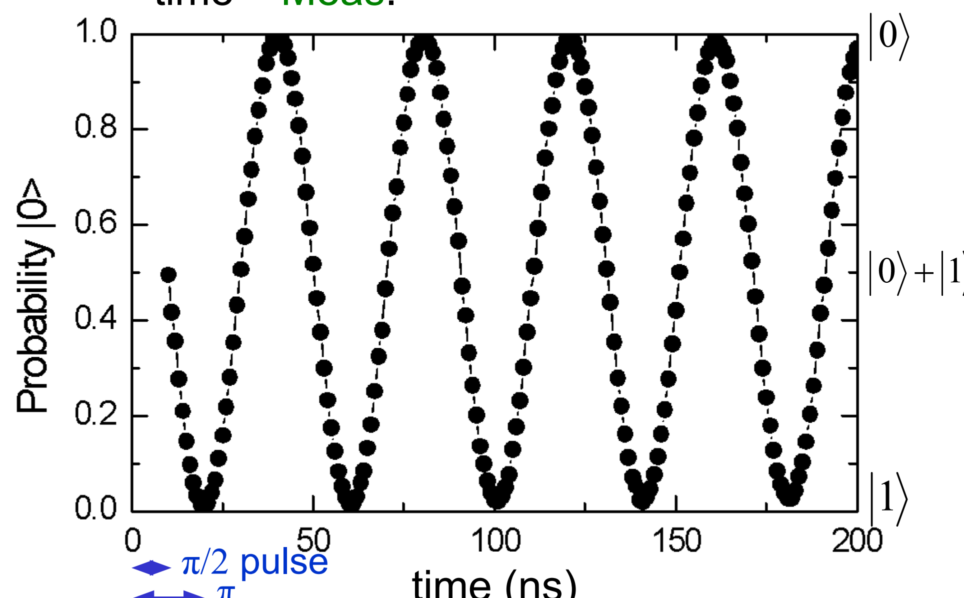
$$\hat{H}_b = Q_b\hat{V} = Q_b\hat{Q}/C$$
$$= iV_bC_c\sqrt{\hbar\omega_0/2C}\left[\hat{a}-\hat{a}^+\right]$$
$$\propto V_b\hat{\sigma}_y$$





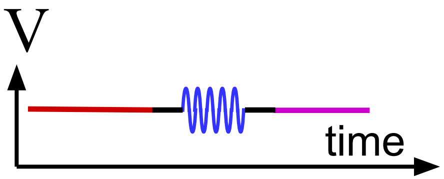


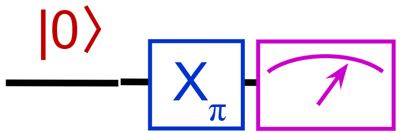
Microwave drive on resonance:

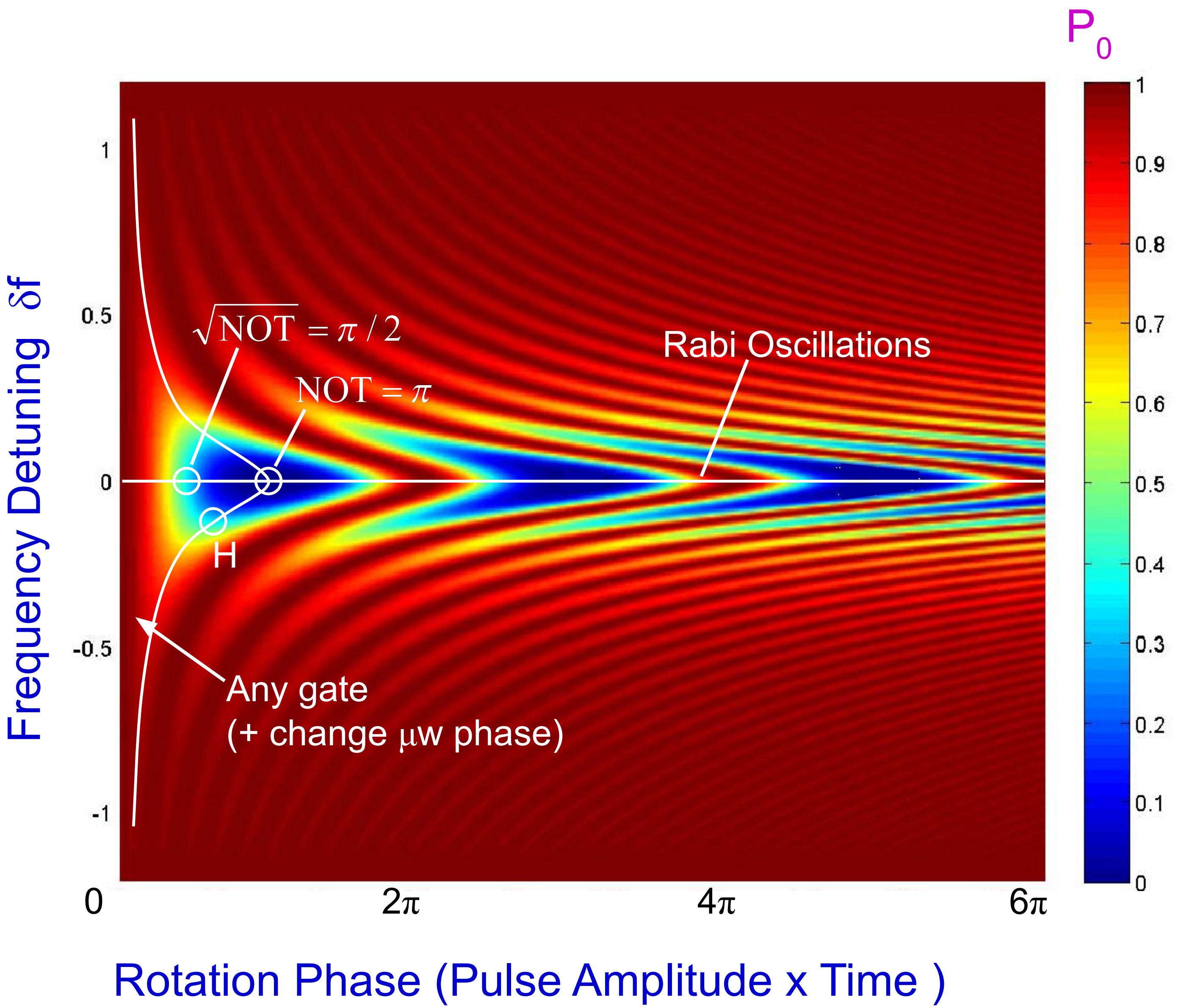


Probability Oscillations - Chevron Curve

Tune up sequence using measurement P_0



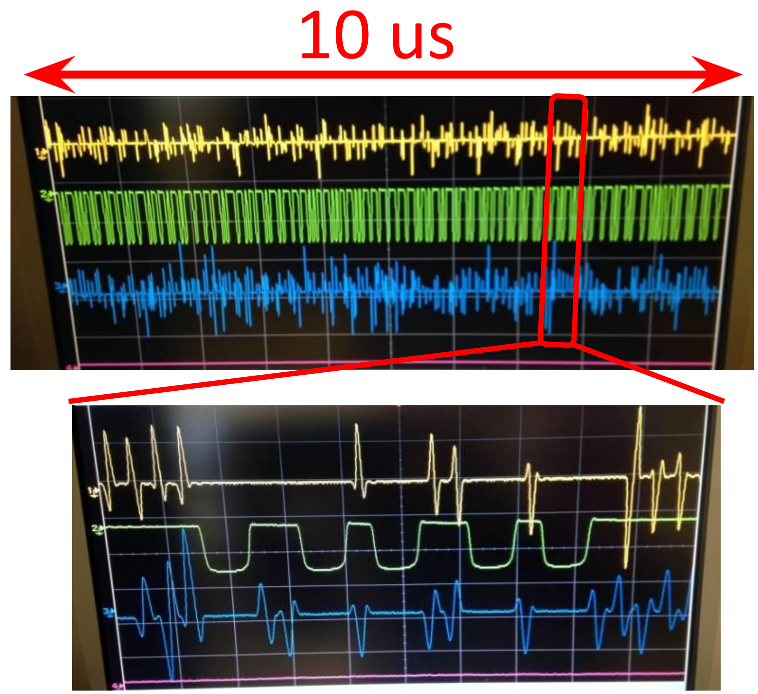


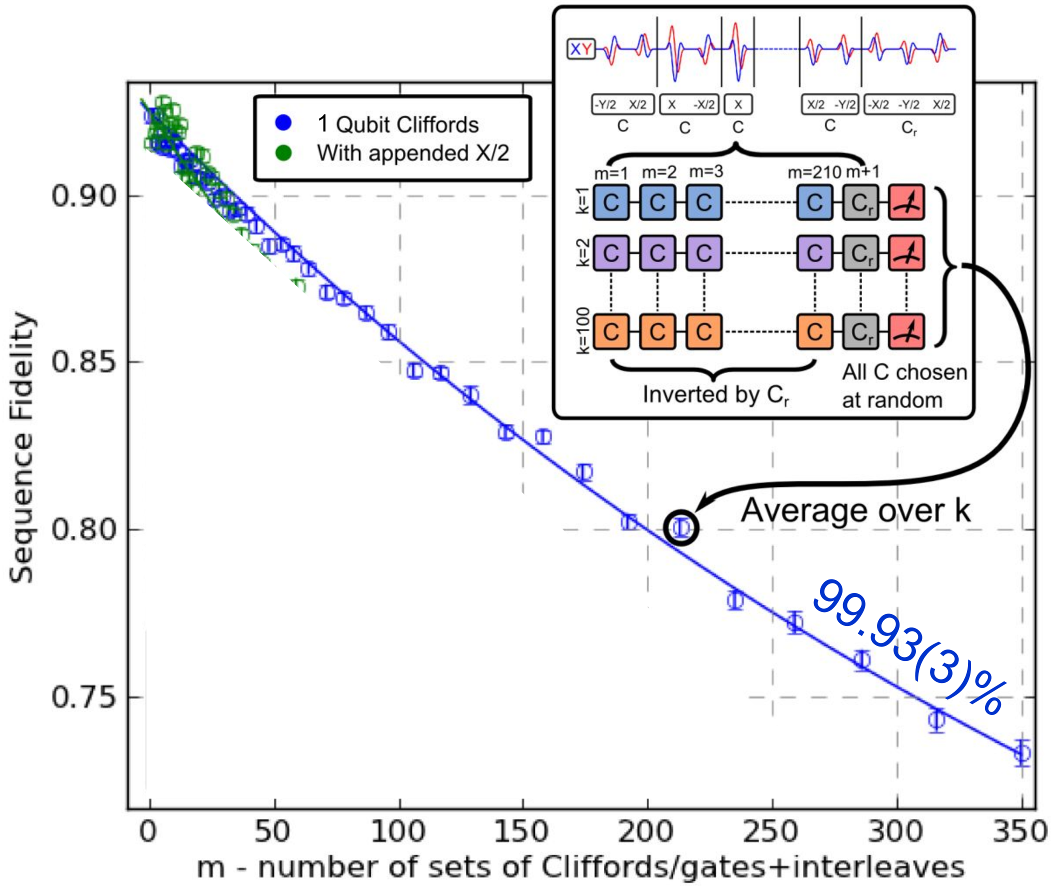


Basis of quantum logic

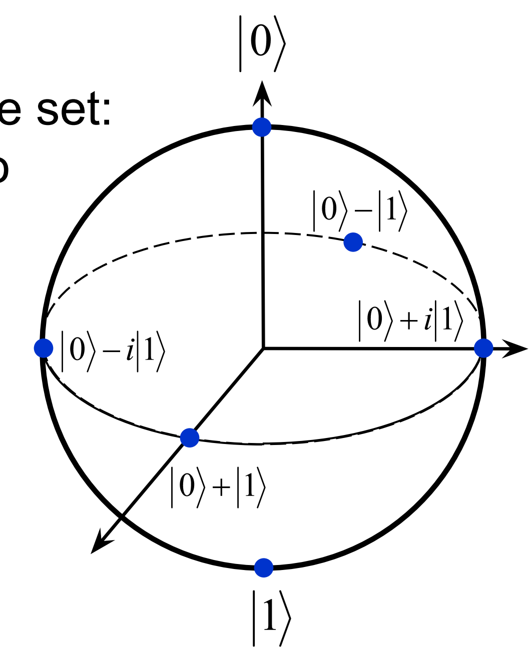
Randomized Benchmarking*

Realistic multi-qubit test of ***long*** algorithm
(1000+ gates)



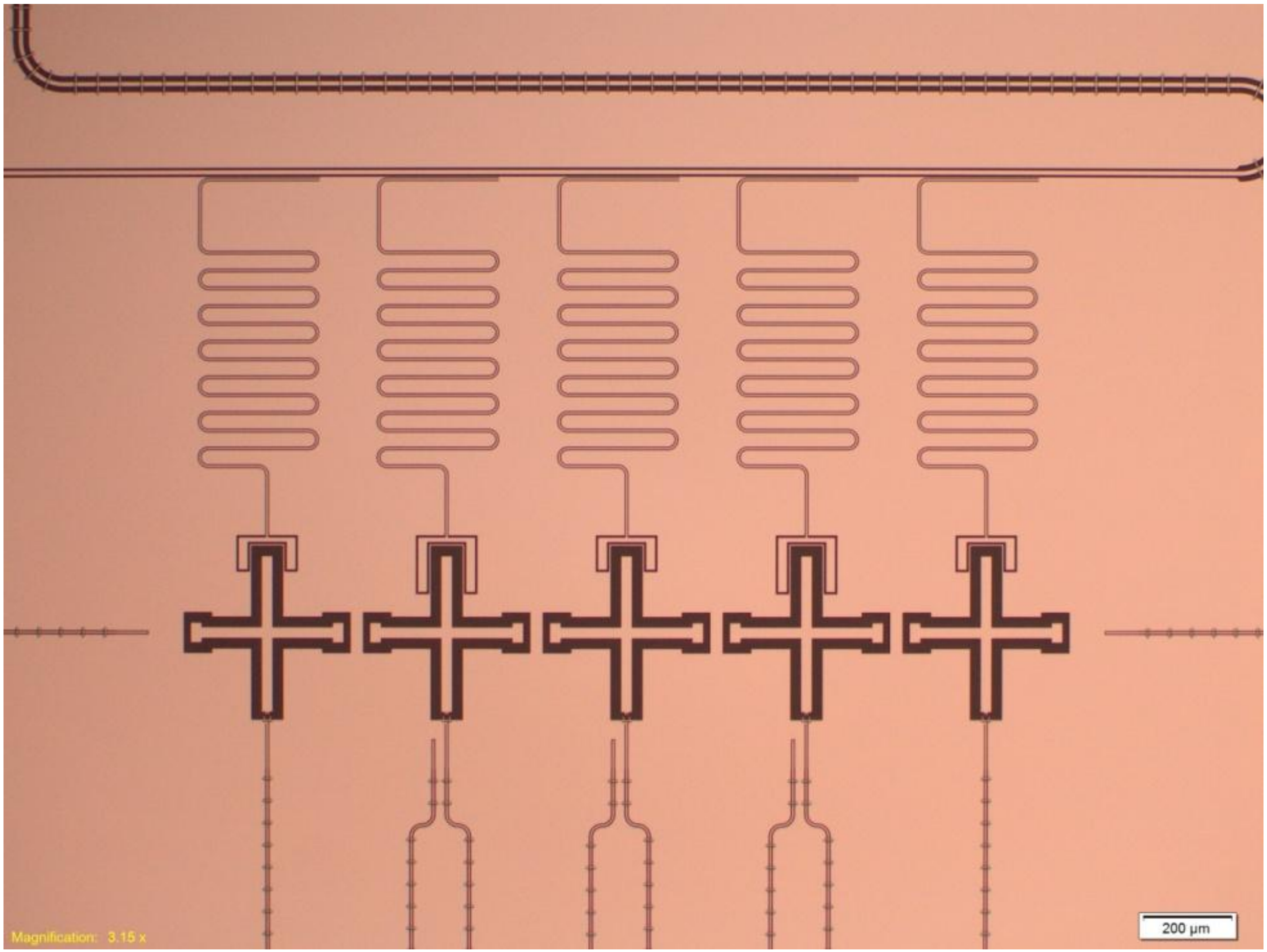


Clifford gate set:
“rotation” to
6 states



*Magesan et. al., *PRL* 106.18 (2011): 180504

Qubit Coupling with Nearest Neighbor capacitance (simplicity gives good performance)

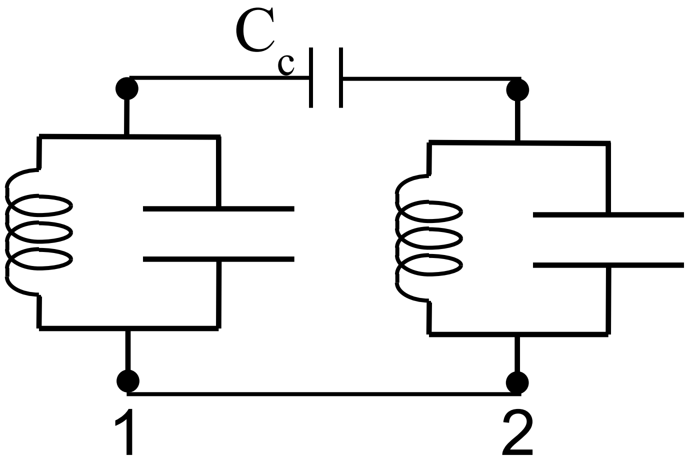


qubit readout (freq. mux'd)

5 Xmon qubits, C coupling

qubit control

Resonator – Resonator Coupling (Classical)



Equations of motion:

$$\int V_1 dt / L_1 + C_1 \dot{V}_1 + C_c(\dot{V}_1 - \dot{V}_2) = 0$$

$$\int V_2 dt / L_2 + C_2 \dot{V}_2 + C_c(\dot{V}_2 - \dot{V}_1) = 0$$

I sum at nodes

$$\begin{pmatrix} -1/\omega^2 L_1 + \overbrace{C_1 + C_c}^{\tilde{C}_1} & -C_c \\ -C_c & -1/\omega^2 L_2 + C_2 + C_c \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

Linearize ω

$$-\frac{1-\omega^2/\omega_1}{\omega^2 L_1} \cong -\frac{2(1-\omega/\omega_1)}{\omega_1^2 L_1} = -\frac{2\tilde{C}_1(\omega_1-\omega)}{\omega_1}$$

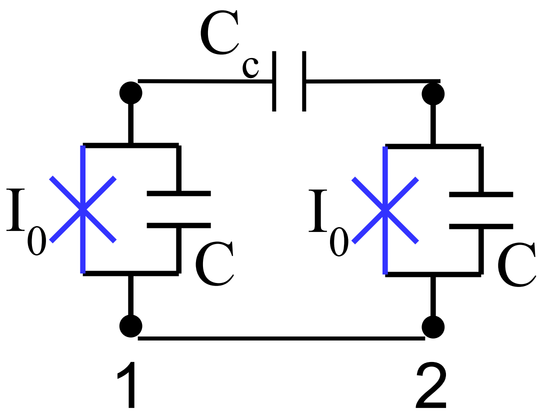
$$\omega_1 = 1/\sqrt{L_1 \tilde{C}_1}$$

$$\begin{pmatrix} -(2\tilde{C}_1/\omega_1)(\omega_1-\omega) & -C_c \\ -C_c & -(2\tilde{C}_2/\omega_2)(\omega_2-\omega) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

Normal modes (eigen-frequencies)

$$(\omega_1-\omega)(\omega_2-\omega) - \frac{\omega_1\omega_2}{4\tilde{C}_1\tilde{C}_2} C_c^2 = 0$$

Qubit – Qubit Coupling (Quantum)



$$\hat{H}_c = C_c \hat{V}_1 \hat{V}_2 = C_c (\hat{Q}_1 / C_1)(\hat{Q}_2 / C_2)$$

$$= -\hbar \frac{C_c}{2} \sqrt{\frac{\omega_1 \omega_2}{C_1 C_2}} \left(\hat{a}_1 - \hat{a}_1^+\right)\left(\hat{a}_2 - \hat{a}_2^+\right)$$

$$\cong -\hbar g \left(\hat{a}_1 \hat{a}_2^+ + \hat{a}_1^+ \hat{a}_2\right)$$

For weak coupling

$C_c << C$

coupling energy excitation swapping: |01) to |10)

Schrodinger equation

$$\begin{pmatrix} \hbar(\omega_1 - \omega) & -\hbar g \\ -\hbar g & \hbar(\omega_2 - \omega) \end{pmatrix} \begin{pmatrix} \Psi_{01} \\ \Psi_{10} \end{pmatrix} = 0$$

Eigen-frequencies

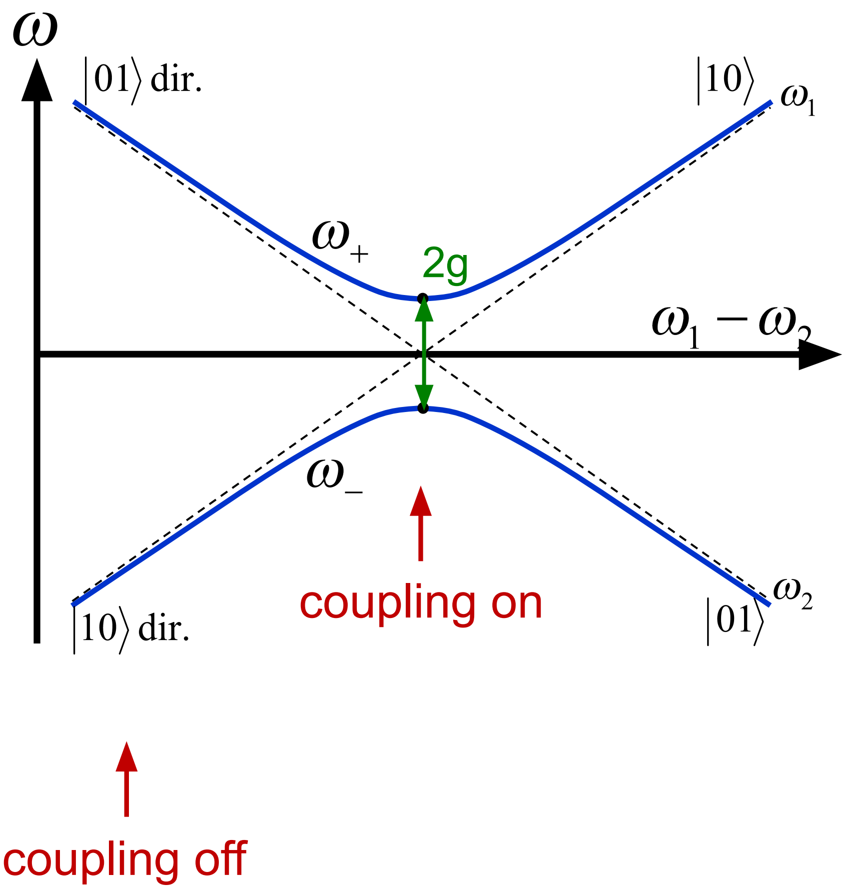
$$(\omega_1 - \omega)(\omega_2 - \omega) - g^2 = 0$$

Same formula as for classical normal modes!

Swapping Physics of States $|01\rangle, |10\rangle$

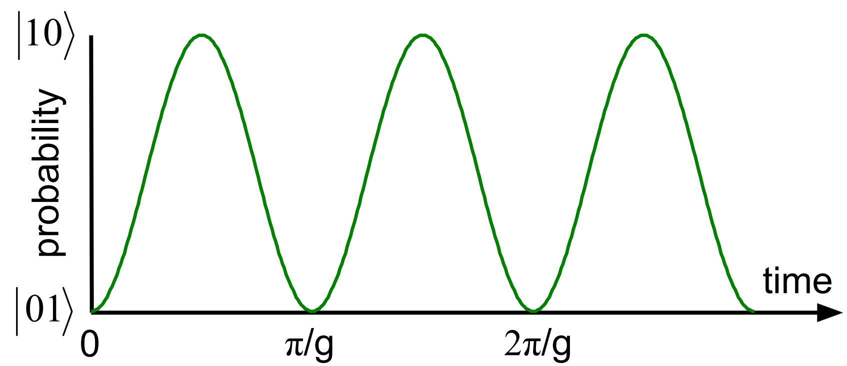
Avoided level crossing

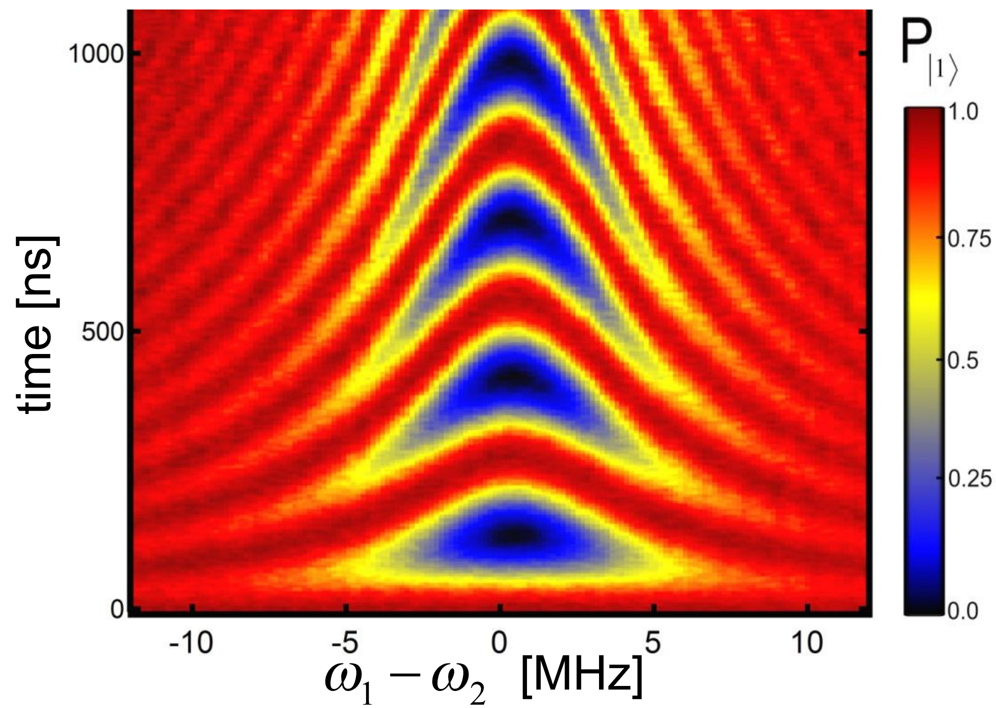
$$\omega = (\omega_1 + \omega_2)/2 \pm \sqrt{(\omega_1 - \omega_2)^2/4 + g^2}$$



Swapping: display of qubit-qubit coupling

$\omega_1 = \omega_2$





Coupling Physics of States $|01\rangle, |10\rangle$

Avoided level crossing

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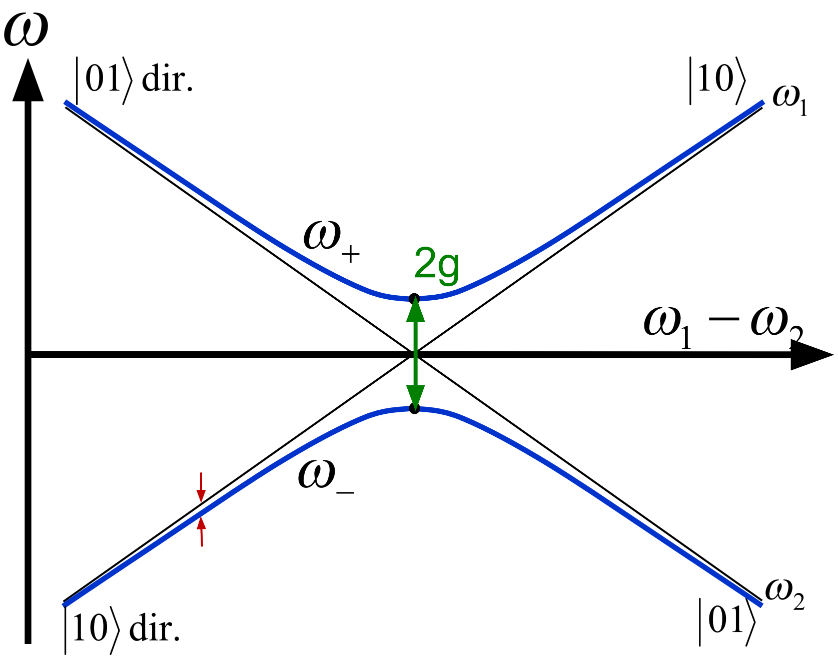
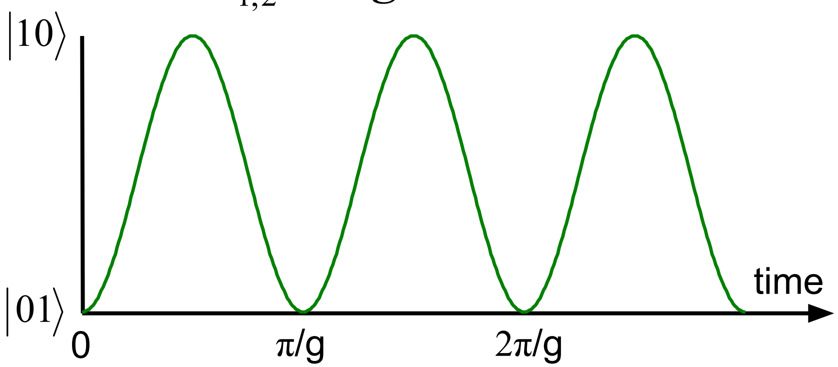


1.Swapping: basis of qubit-qubit gates

$$\omega_1 = \omega_2$$

$$\omega - \omega_{1,2} = \pm g$$

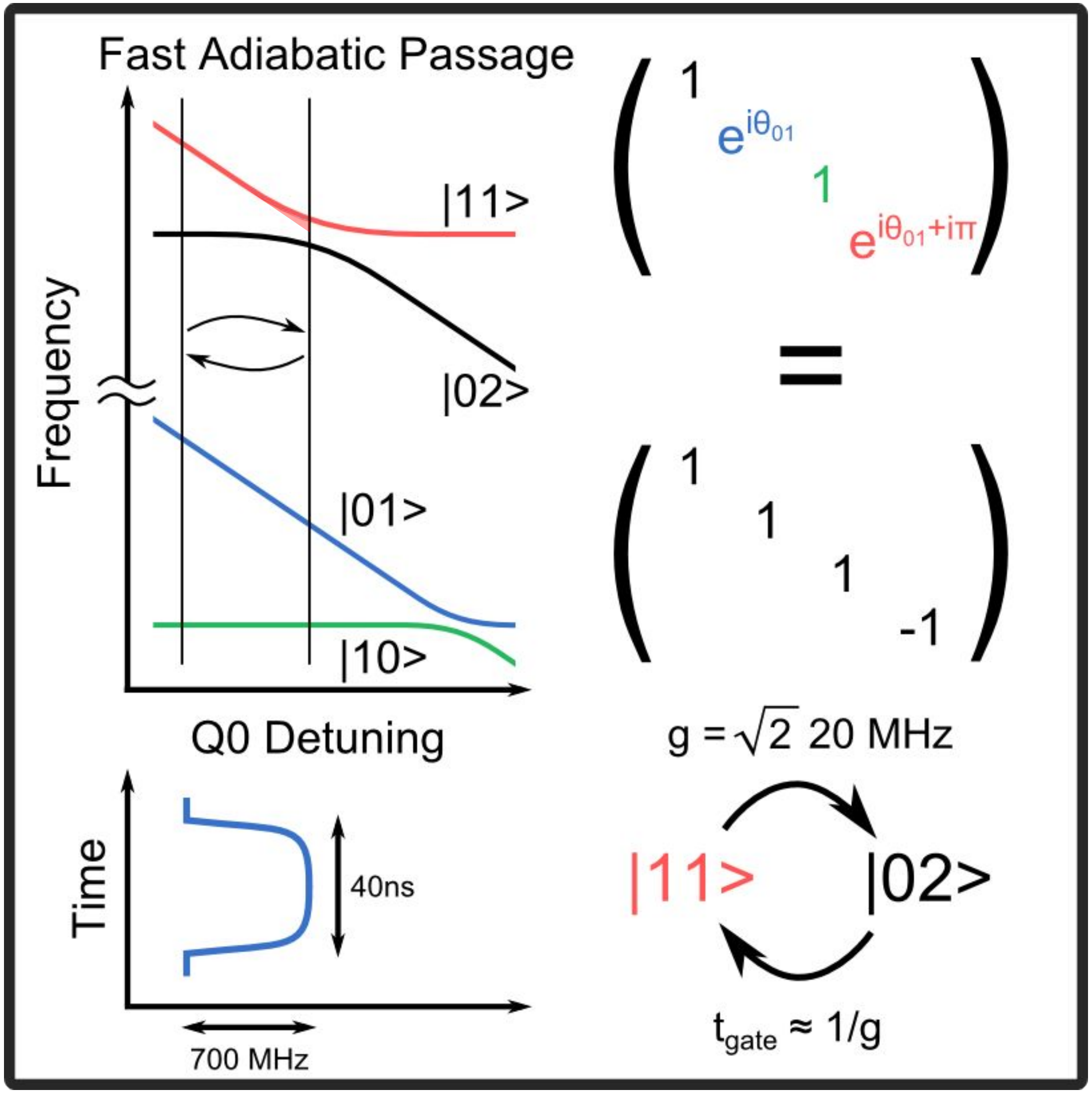


2.Dispersive frequency shift $|\omega_1 - \omega_2| >> g$

$$\omega_+ = \omega_1 + g^2/(\omega_1 - \omega_2)$$

$$\omega_- = \omega_2 - g^2/(\omega_1 - \omega_2)$$

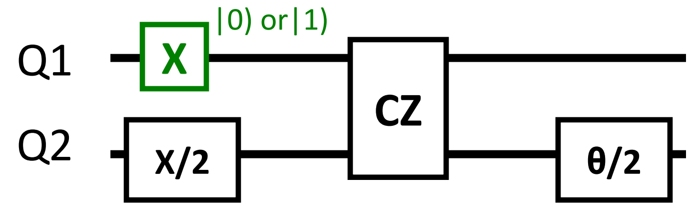
CZ entangling gate*

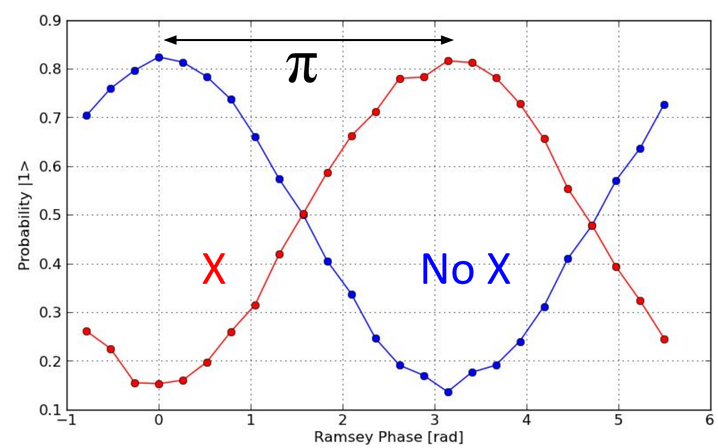


truth table

$$\begin{cases} |00\rangle \to |00\rangle \\ |01\rangle \to |01\rangle \\ |10\rangle \to |10\rangle \\ |11\rangle \to -|11\rangle \end{cases}$$

Functionality Check (truth table)





New theory for Fast Adiabatic Gate:

$t_{gate}\ 2g/2\pi = 1.1$ for 10^{-5} error

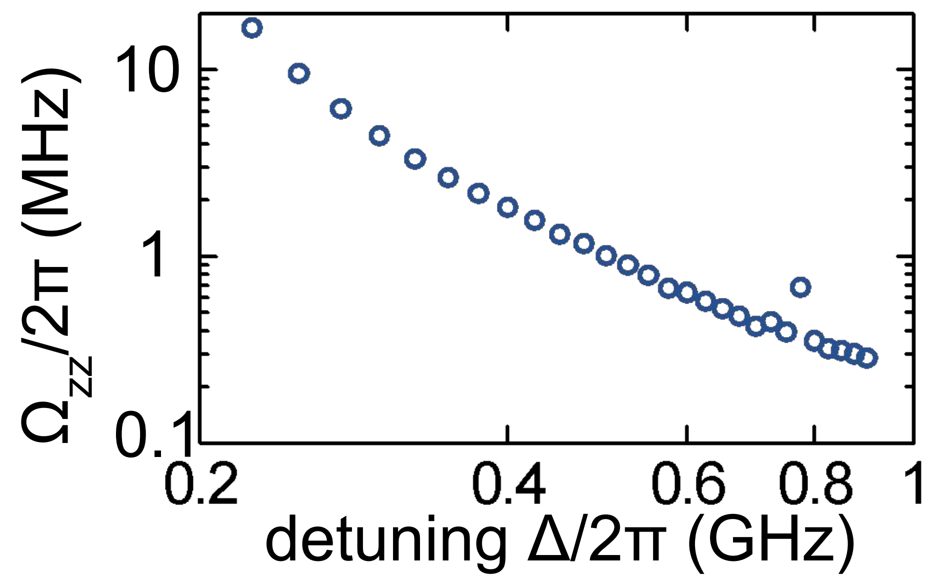
*Similar to Strauch; DiCarlo; Yamamoto

Detuning to Turn Off - How Much Off?

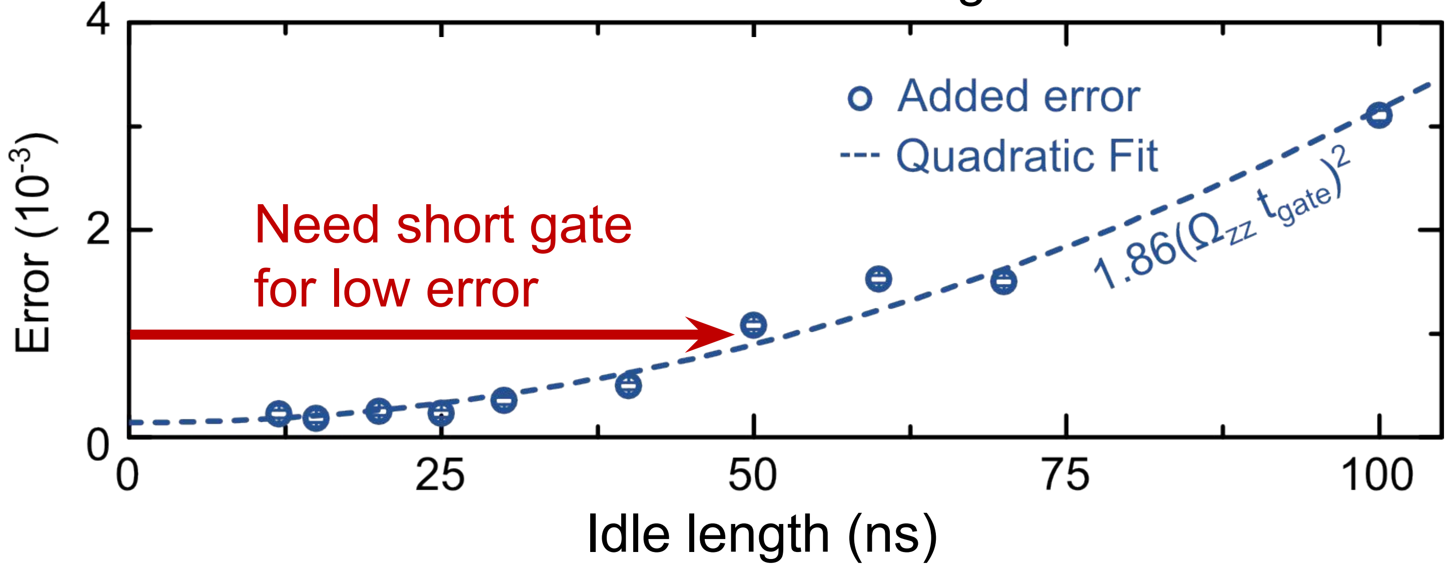
|11> has shifted frequency from coupling to |02>

Want small coupling so easier to turn off, $g/2\pi$ = 30 MHz

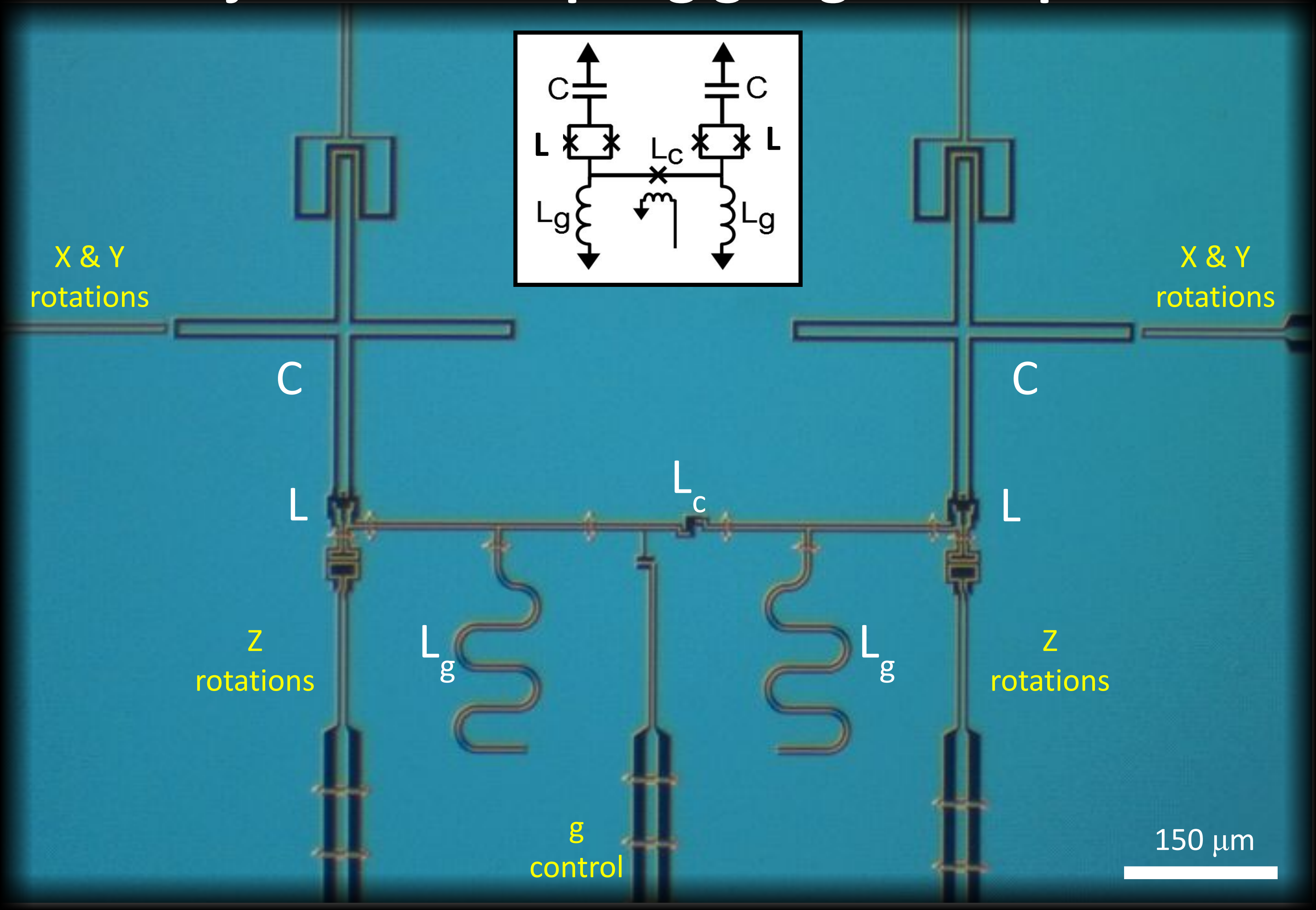
Use short gate times $P_{err} = (\varphi_{err})^2/4 \;\; \alpha \;\; (t_{gate})^2$



Small RB error at 800 MHz detuning



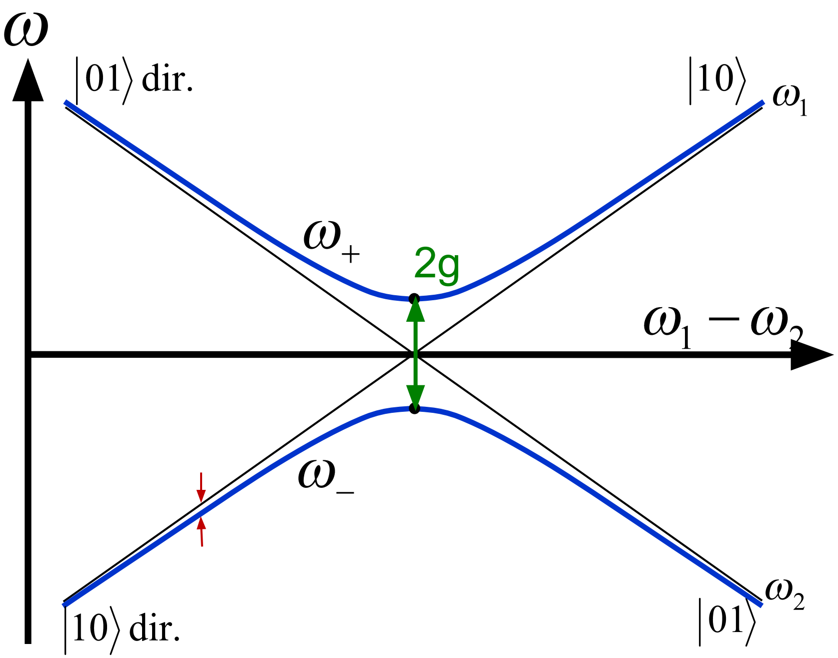
Adjustable Coupling g : “gmon” qubits



Coupling Physics of States $|01\rangle, |10\rangle$

Avoided level crossing

$$\omega = (\omega_1 + \omega_2)/2 \pm \sqrt{(\omega_1 - \omega_2)^2/4 + g^2}$$



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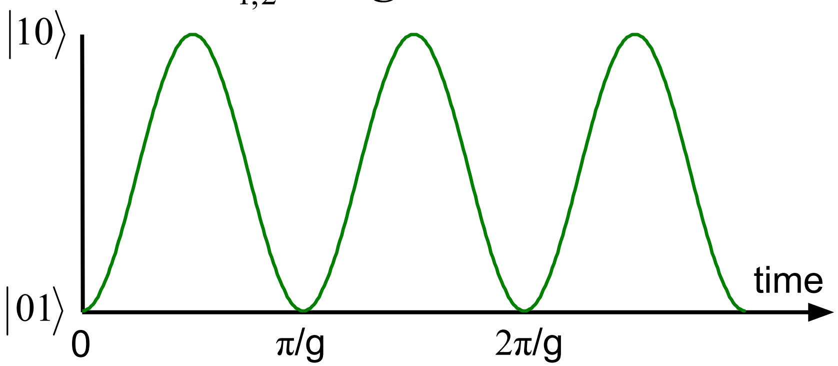

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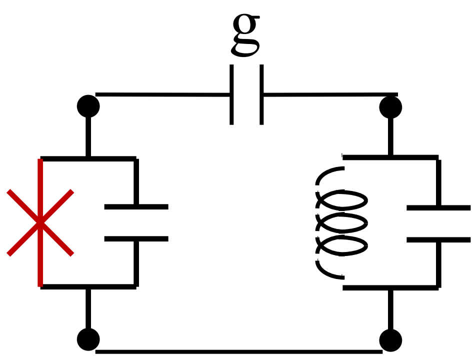
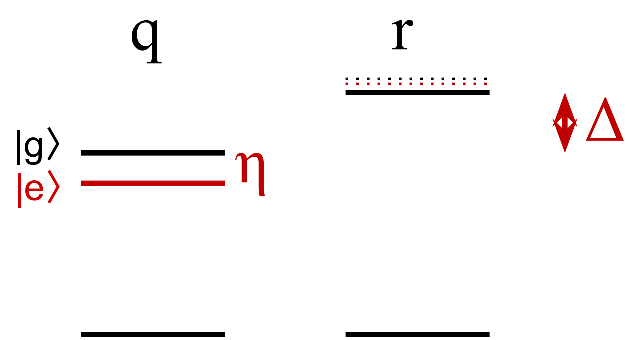
Dispersive Readout - Semiclassical





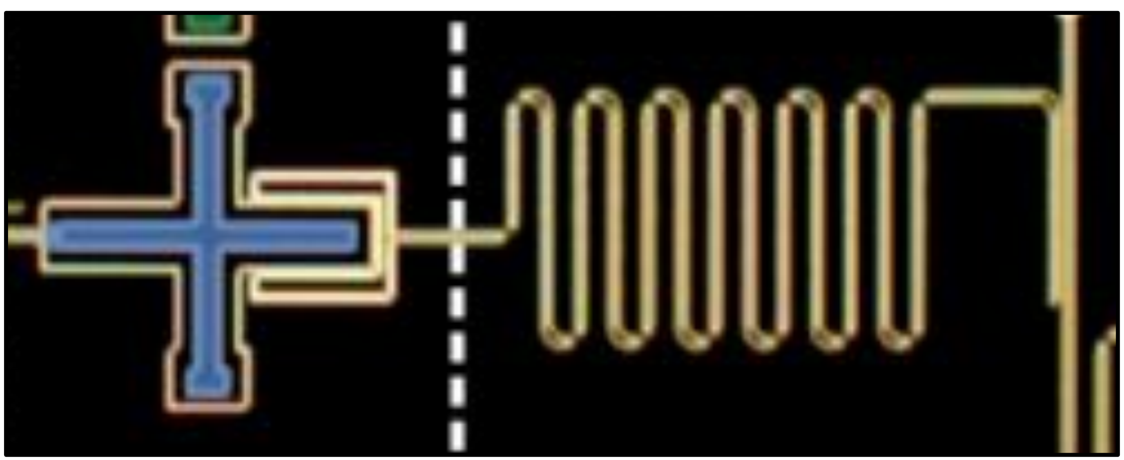
Resonator frequency changes for |g⟩ and |e⟩

|g⟩ |e⟩

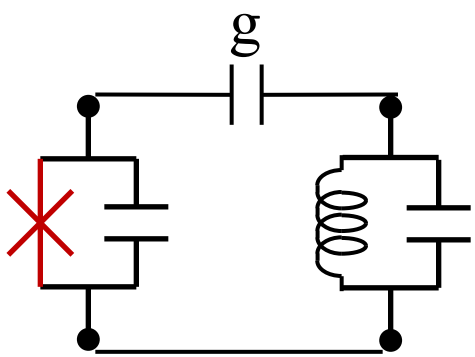
$$\frac{g^2}{\Delta} - \frac{g^2}{\Delta + \eta} \cong \eta \frac{g^2}{\Delta^2}$$

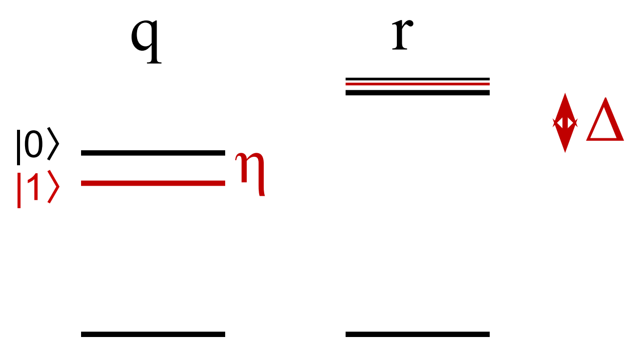
from avoided level crossing

xmon (6 / 5.8 GHz) λ/2 resonator (7 GHz) readout to amp



Dispersive Coupling (Qubit-Resonator) - Readout





Semiclassical (qualitative) understanding:

1. Frequency shift between |0⟩and |1⟩

$$\overset{|0\rangle}{\frac{g^2}{\Delta}} - \overset{|1\rangle}{\frac{g^2}{\Delta+\eta}} \simeq \eta \frac{g^2}{\Delta^2}$$

Avoided level crossing

2. Qubit energy from resonator energy $E_r = n\hbar\omega_r$

$$E_q = \frac{C_q}{C_r}\left|\frac{Z_r(\omega_r)}{Z_c(\omega_r)}\right|^2 n\hbar\omega_r$$

$$= \frac{g^2}{\Delta^2} n\hbar\omega_q$$

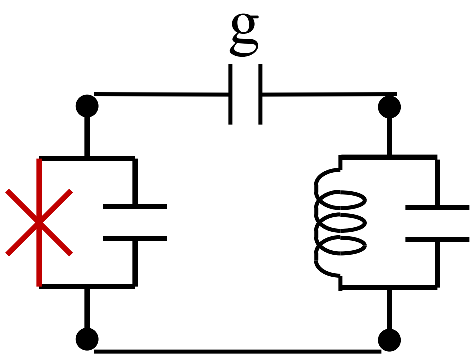
Voltage divider

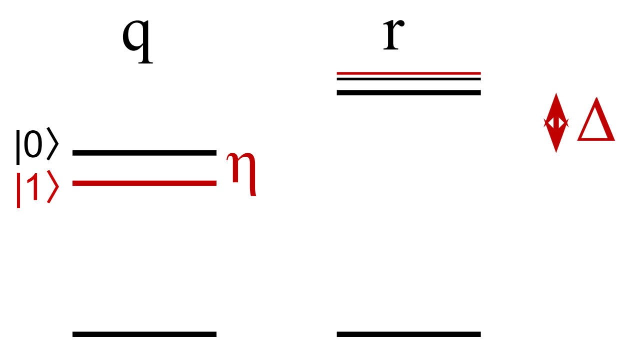
$$E_q/\hbar\omega_q \leq 1 \text{ to } 3$$

$$n_{\text{crit}} \leq (\Delta/g)^2 \times 1 \text{ to } 3$$

Limit on measure energy

Dispersive Coupling (Qubit-Resonator)- Readout





$$\hat{H}_c = g\left(\hat{a}_q \hat{a}_r^+ + \hat{a}_q^+ \hat{a}_r\right)$$

qubit-resonator coupling

$$\hat{H}_c^{(2)} = \sum_i \frac{g^2\left(\hat{a}_q \hat{a}_r^+ + \hat{a}_q^+ \hat{a}_r\right)\left(\hat{a}_q \hat{a}_r^+ + \hat{a}_q^+ \hat{a}_r\right)}{E_i}$$

$$= g^2\left(\frac{\hat{a}_q \hat{a}_q^+ \hat{a}_r^+ \hat{a}_r}{E_i} + \frac{\hat{a}_q^+ \hat{a}_q \hat{a}_r \hat{a}_r^+}{E_i'}\right)$$

2^{nd} order; dispersive (state conserving)

$$\langle 0n|\hat{H}_c^{(2)}|0n\rangle - \langle 1n|\hat{H}_c^{(2)}|1n\rangle = \overset{0\text{-}1\text{-}0}{g^2 n/\Delta} - \left[\overset{1\text{-}2\text{-}1}{2g^2 n/(\Delta+\eta)} + \overset{1\text{-}0\text{-}1}{g^2(n+1)/(-\Delta)}\right]$$

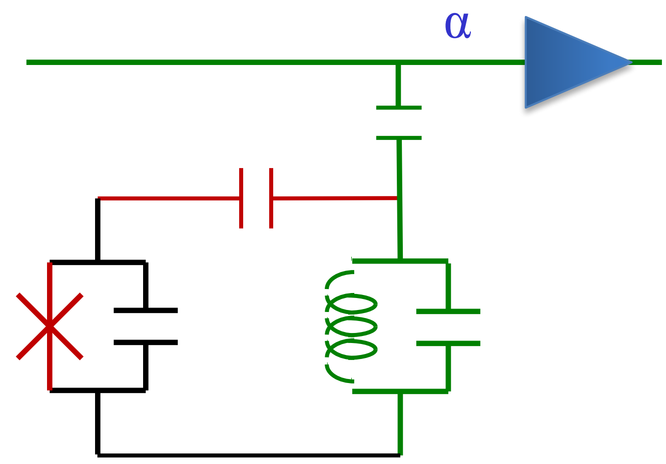
$$= 2n\eta g^2/\Delta^2 + \cancel{g^2/\Delta}$$

Energy shift proportional to n

$$\hat{H}_{\text{int}} = \chi \hat{\sigma}_z \hat{n} \ ; \quad \chi = \eta g^2/\Delta$$

Interaction Hamiltonian

Qubit Measurement



1. Measure with large signal / noise
   Resonator (large n) signal
   and quantum limited preamps

2. Shift of resonator frequency with qubit state
   Dispersive interaction

3. Phase shift of readout signal
   Good separation of states

4. Collapse of entanglement gives projection
   Coherent state is eigenstate of dissipation
   (a pointer state)

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

$$\left(|e\rangle + |g\rangle\right)|0\rangle \rightarrow |e\rangle|\alpha_e\rangle + |g\rangle|\alpha_g\rangle$$

$$\rightarrow \begin{cases} 50\%: & |e\rangle|\alpha_e\rangle \\ 50\%: & |g\rangle|\alpha_g\rangle \end{cases}$$