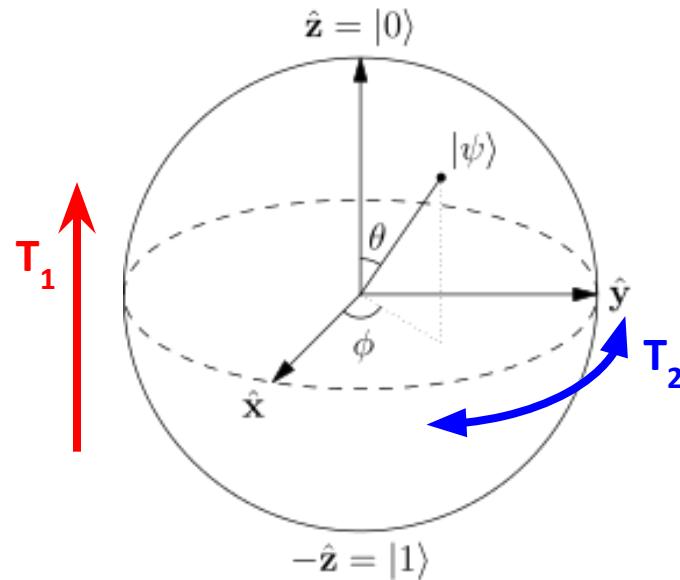


The Challenge: Qubit Errors

Physical qubits have T_1 , T_2 ,
Control Errors



Quantum gate errors: $\sim 10^{-2} - 10^{-4}$

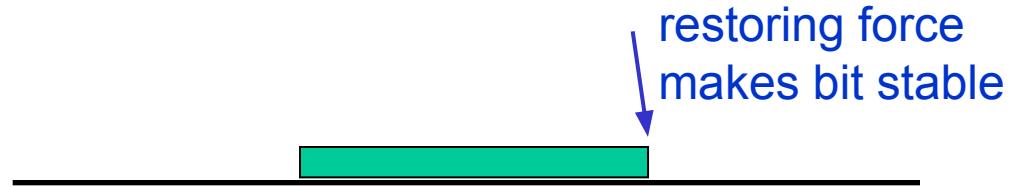
Error rate required: $\sim 10^{-10} - 10^{-20}$

Can make up 8+ orders of magnitude
with error correction: 100-1000 qubits

Digital Quantum Computing: Qubit Errors

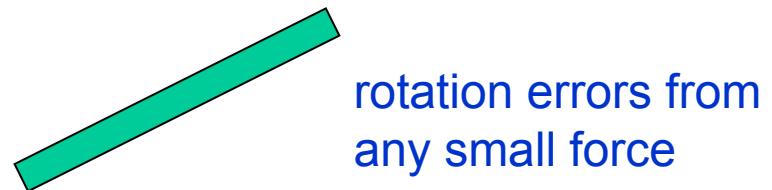
Classical bit:

Coin on table



Quantum bit (analogy):

Coin in space



Quantum bit (physics):

Qubit state $\psi = \cos\theta |0\rangle + \sin\theta e^{i\phi} |1\rangle$ described by **amplitude** and **phase**

1 qubit: measure **amplitude** randomizes **phase**

2 qubits: measure **amplitude** and **phase** **parity** at same time

Parity:
00,11 -> +1
01,10 -> -1

Error Digitization (bit-, phase-flip) with Measurement

Example: error from small X rotation:

$$|\Psi\rangle \rightarrow (\hat{I} + \varepsilon \hat{X}) |\Psi\rangle$$

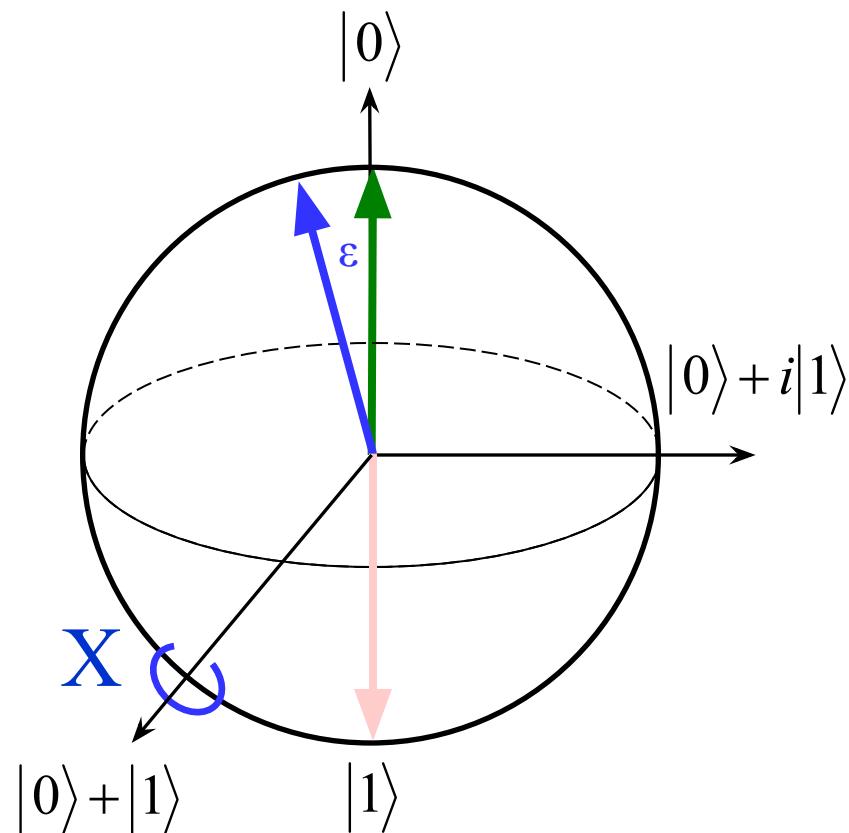
When measure Z, 2 outcomes:

1. Prob = $1 - \varepsilon^2$: erases error

$$(\hat{I} + \varepsilon \hat{X}) |\Psi\rangle \rightarrow |\Psi\rangle$$

2. Prob = ε^2 : X error

$$(\hat{I} + \varepsilon \hat{X}) |\Psi\rangle \rightarrow \hat{X} |\Psi\rangle$$



Error Detection Math

1) Heisenberg uncertainty depends on commutation $\hat{X}\hat{Z} - \hat{Z}\hat{X} \neq 0$

Bit flip $\hat{X} : |0\rangle \leftrightarrow |1\rangle$

Phase flip $\hat{Z} : |1\rangle \leftrightarrow -|1\rangle$

$$|0\rangle + |1\rangle \xrightarrow{\hat{X}} |1\rangle + |0\rangle \xrightarrow{\hat{Z}} -|1\rangle + |0\rangle$$

$$|0\rangle + |1\rangle \xrightarrow{\hat{Z}} |0\rangle - |1\rangle \xrightarrow{\hat{X}} +|1\rangle - |0\rangle$$

$$\hat{X}\hat{Z} + \hat{Z}\hat{X} = 0$$

2) Parities commute

simultaneous bit flip: $\hat{X}_{12} = \hat{X}_1 \hat{X}_2$

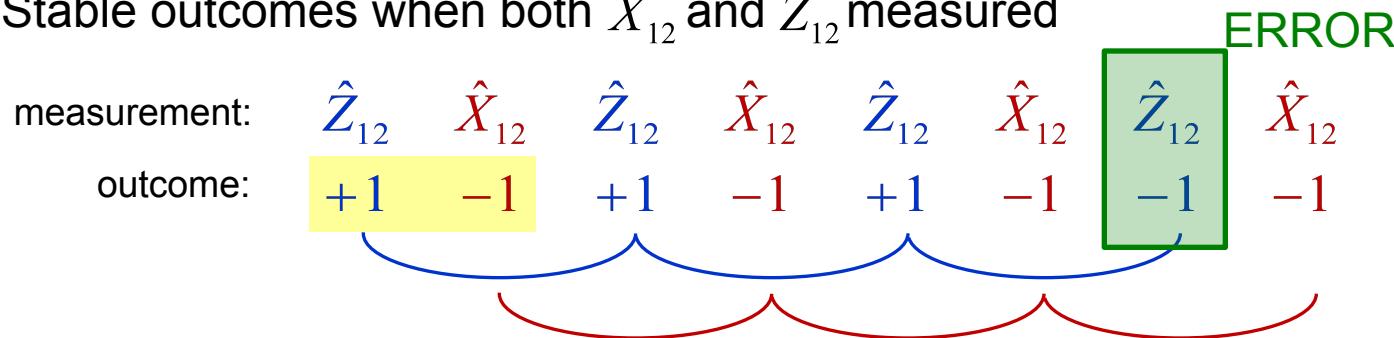
simultaneous phase flip: $\hat{Z}_{12} = \hat{Z}_1 \hat{Z}_2$

$$\begin{aligned} \text{new operators commute: } [\hat{X}_{12}, \hat{Z}_{12}] &= \hat{X}_1 \hat{X}_2 \hat{Z}_1 \hat{Z}_2 - \hat{Z}_1 \hat{Z}_2 \hat{X}_1 \hat{X}_2 \\ &= \hat{X}_1 \hat{Z}_1 \hat{X}_2 \hat{Z}_2 - (-\hat{X}_1 \hat{Z}_1)(-\hat{X}_2 \hat{Z}_2) \\ &= 0 \end{aligned}$$

Error Detection Basics

3) Parities behave classically (simultaneous eigenstates)

Stable outcomes when both \hat{X}_{12} and \hat{Z}_{12} measured



4) From detection to identification:

more qubits needed

① ② ③
 \hat{Z}_{12} \hat{Z}_{23}

Parities
12 23

Truth table:

0 0 0	0	0
1 0 0	1	0
0 1 0	1	1
0 0 1	0	1
0 1 1	1	0
1 0 1	1	1
1 1 0	0	1
1 1 1	0	0

} Unique decoding for single error

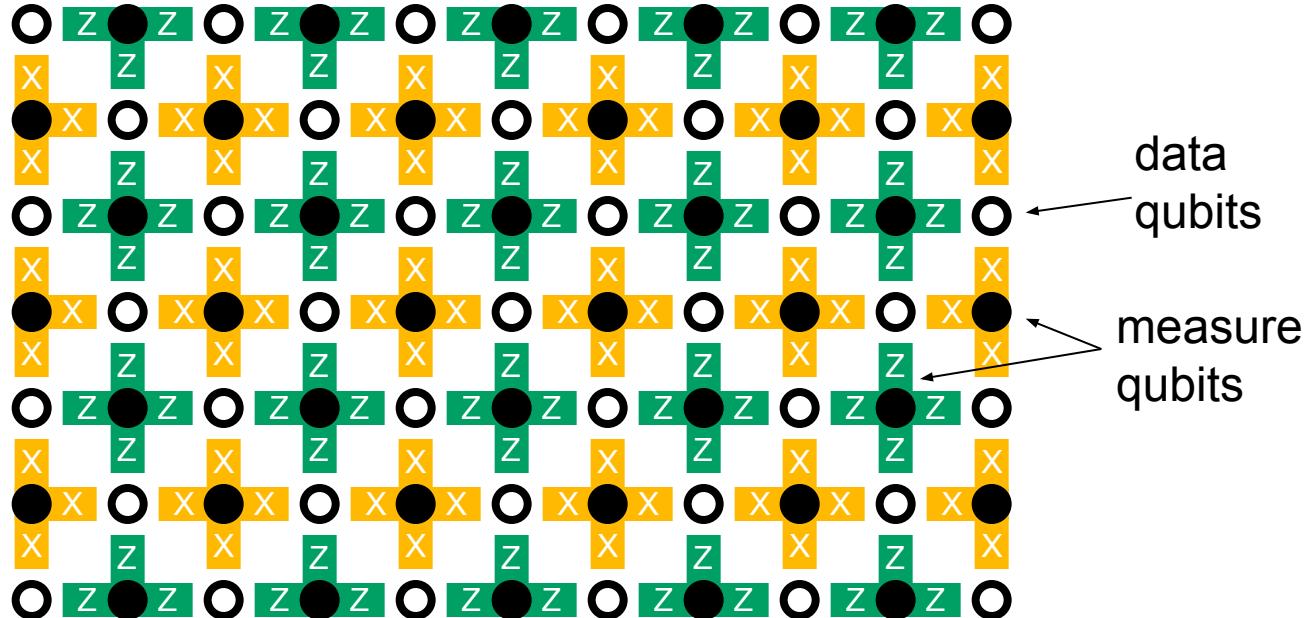
Surface Code Hardware

Toric code: Bravyi & Kitaev
arXiv:quant-ph/9811052 (1998)

Logical CNOT: Raussendorf et. al.,
PRL **98**, 190504 (2007)

Theory review: A. Fowler et. al.,
PRA **80**, 052312 (2009)

Surface code for mortals: Fowler,
Mariantoni, Martinis, Cleland
PRA **86**, 032324 (2012)

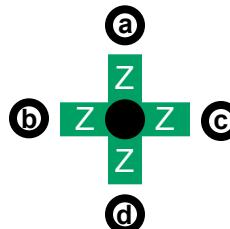


Measurement Symbol

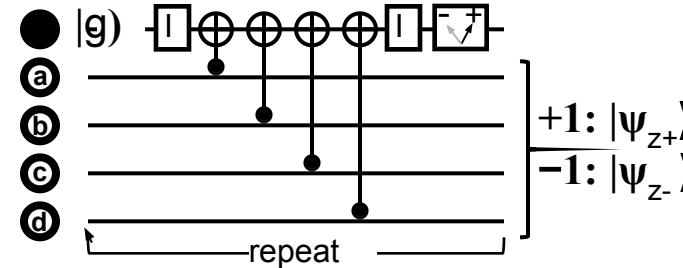
Physical Logic Sequence

4 bit parity

$$Z_{abcd} = Z_a Z_b Z_c Z_d$$

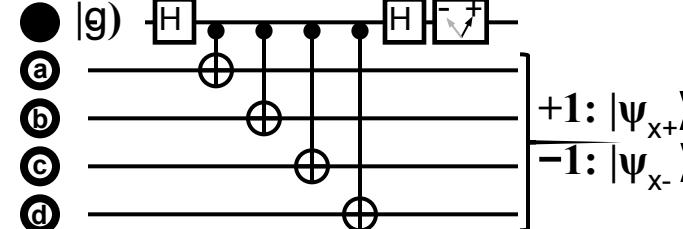
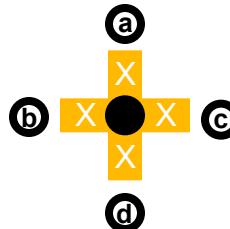


1 2 3 4 5 6 7 8



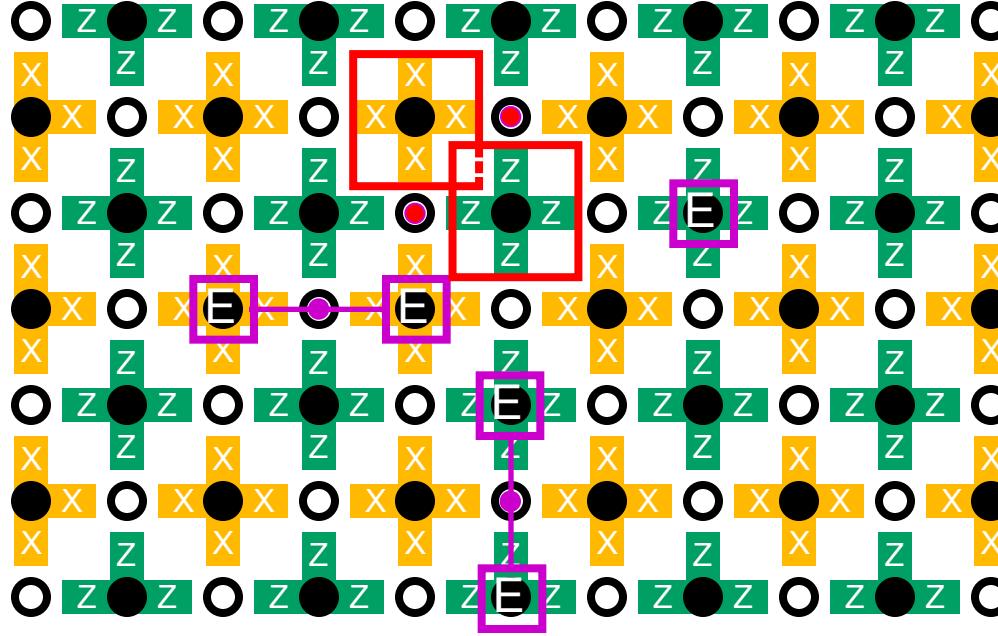
4 phase parity

$$X_{abcd} = X_a X_b X_c X_d$$



CNOT
= XOR

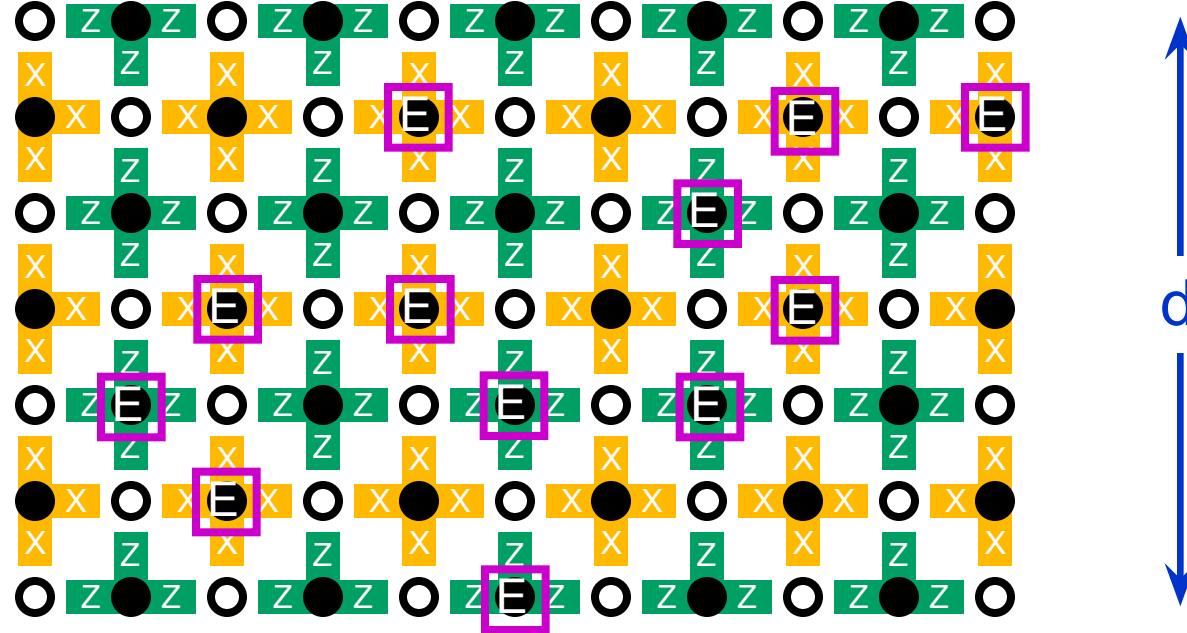
Stabilized State and Identifying Qubit Errors



All measurements **XXXX** and **ZZZZ** commute:
Measurement outcomes unchanging

When errors:
Data qubit errors – pairs in space
Measure errors – pairs in time

Stabilized State and Identifying Qubit Errors



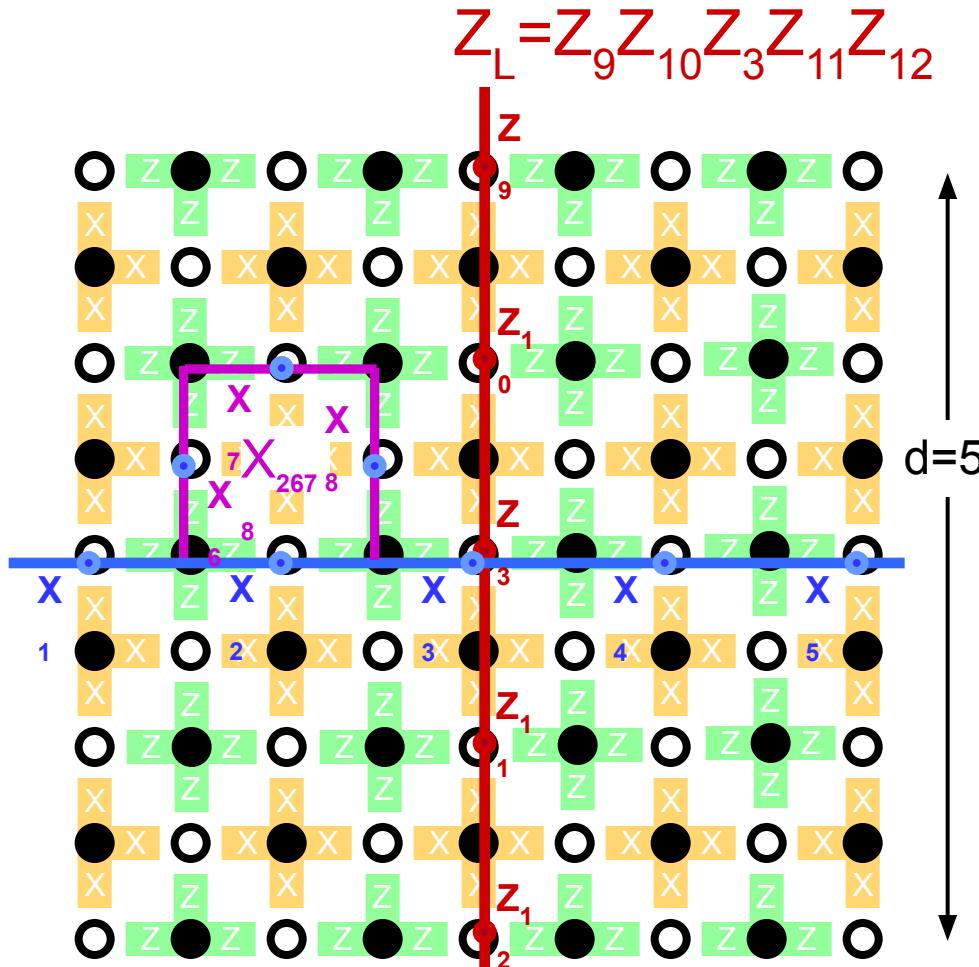
When large density -
Backing out errors may not be unique : logical error

Logical error drops exponentially with bigger arrays
when better than 99% fidelity threshold

$$\epsilon_{\text{logic}} \approx 0.01 \Lambda^{-(d+1)/2} \quad \Lambda \approx 0.01 / \epsilon_{\text{phy}} \quad (\text{error suppression factor})$$

Logical Qubit: 41 data qubits, 40 measure qubits

$$X_L = X_1 X_2 X_3 X_4 X_5$$



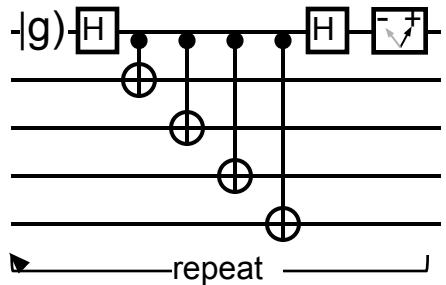
$$\cancel{X}_L \cancel{Z}_L = - \cancel{Z}_L \cancel{X}_L \quad (\text{so acts like qubit})$$

\cancel{X}_L and \cancel{Z}_L commute with measures (so stabilized)
 acts as extra degree of freedom

Logical Error Probability

Model: error p each step

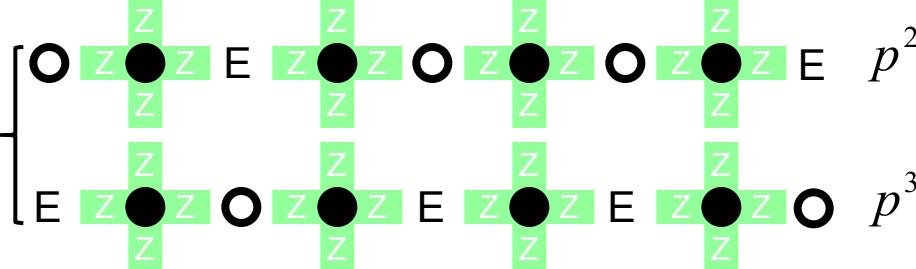
$p \ p \ p \ p \ p \ p \ p \ p$



measured error chain:



computed error of data qubits:



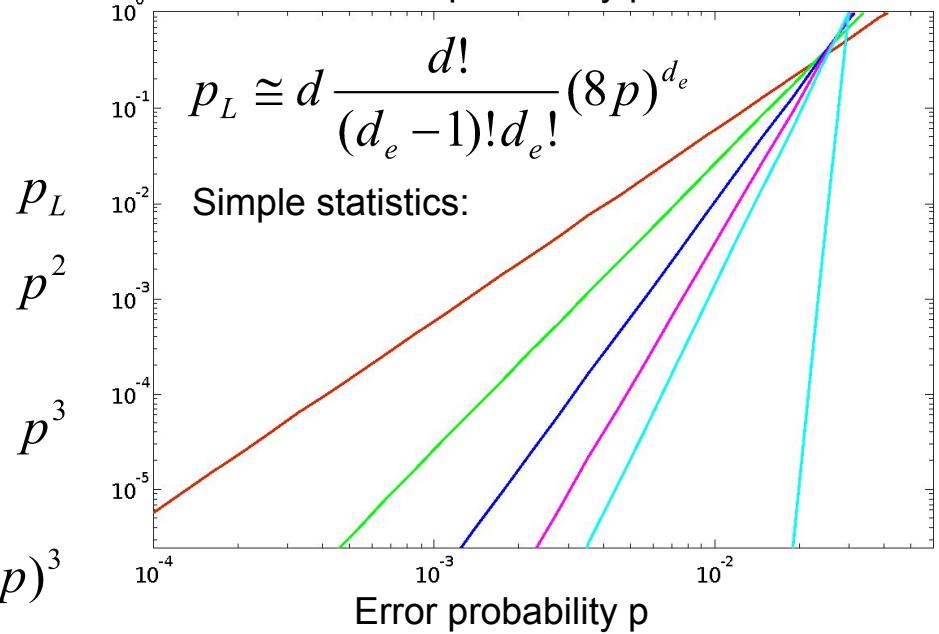
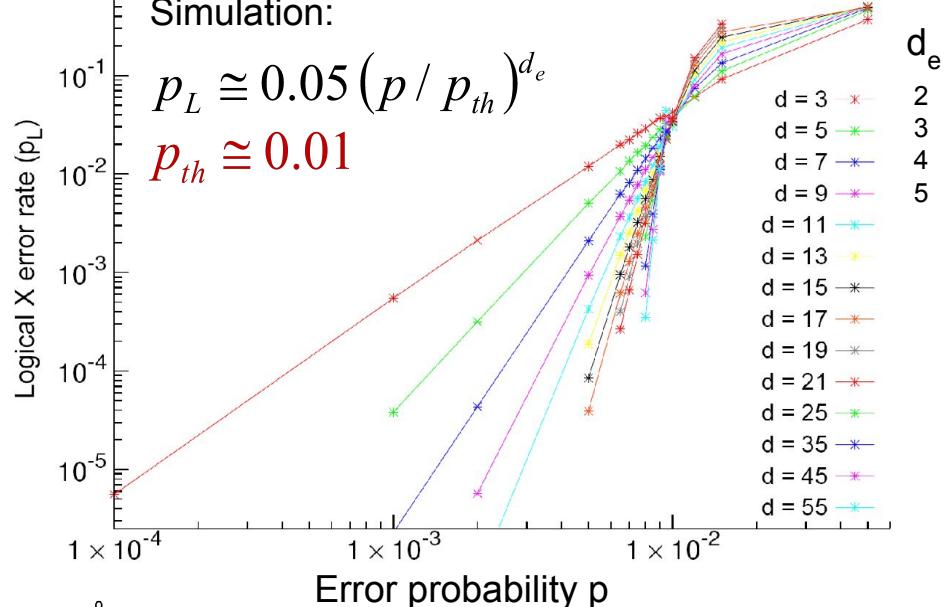
$$P_L \approx \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} (8p)^3$$

logical error improves with size

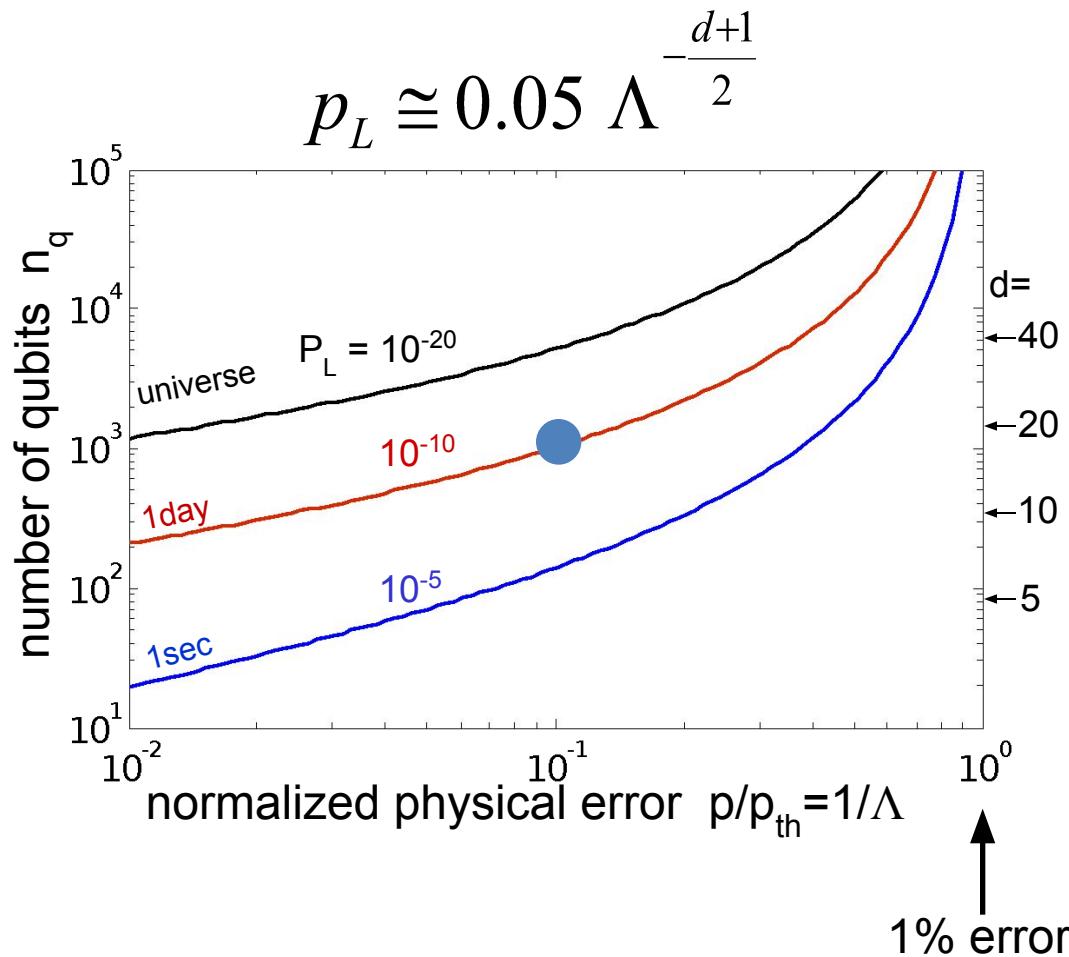
Simulation:

$$p_L \approx 0.05 (p / p_{th})^{d_e}$$

$p_{th} \approx 0.01$



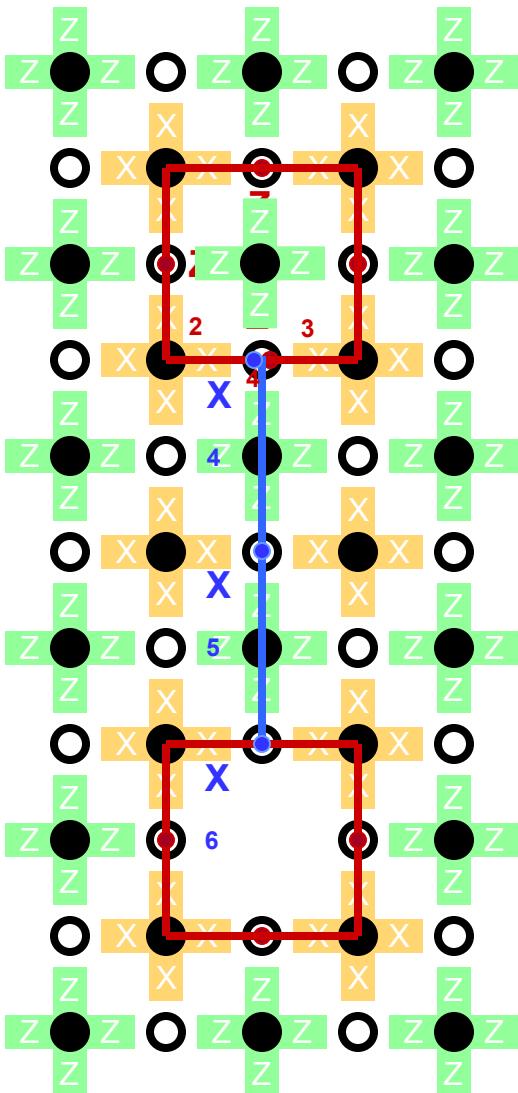
Size of Logical Qubits



Need ~ 1000 physical qubits per logical qubit

Logical Qubits and Gates

Interior surface qubit



$$Z_L = Z_1 Z_2 Z_3 Z$$

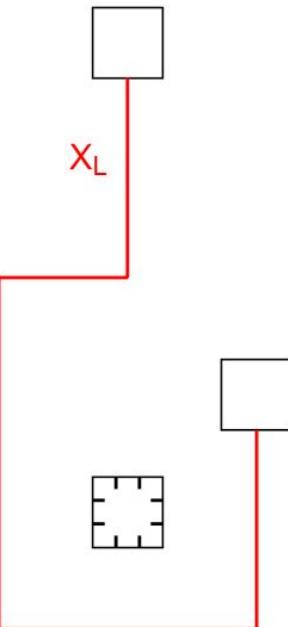
$$X_L = X_4 X_5 X$$

- Initialize Z_L with Z measurement
- Measure Z_L with Z measurement
- CNOT from braiding:

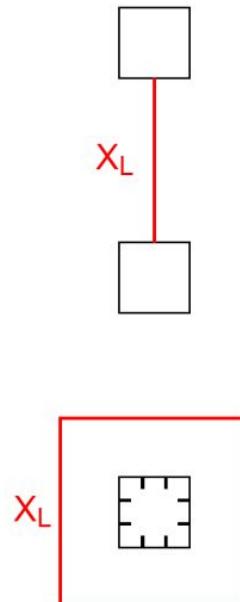
step 1



step 2



step 3



- Single gates: (X, Z) , H , $S = \sqrt{Z}$, $T = \sqrt{S}$

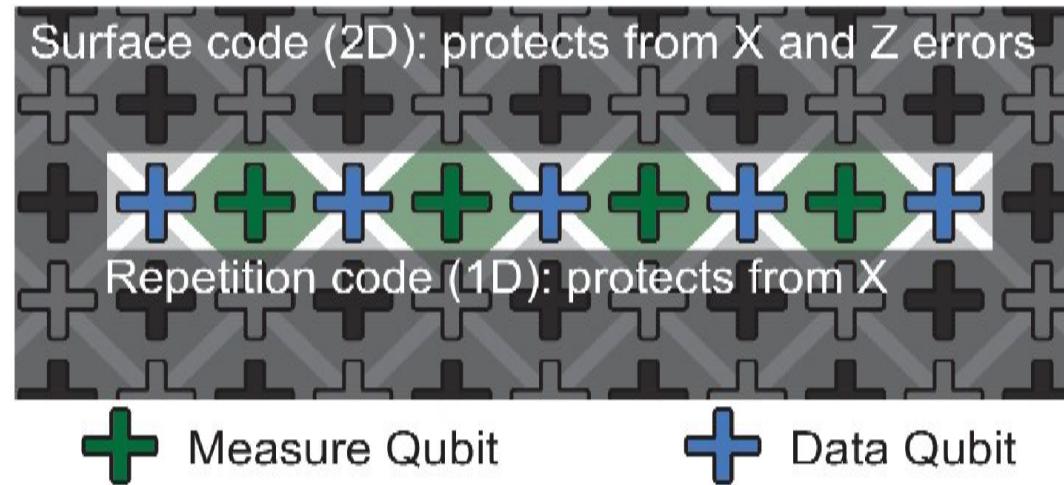
Surface Code Experiments: Top-Down Design

- 1) Memory with error 10^{-15} (correct order n=15, 1/10th of threshold)
Then show logical operations

- 2) Correct errors for n=3 to 5

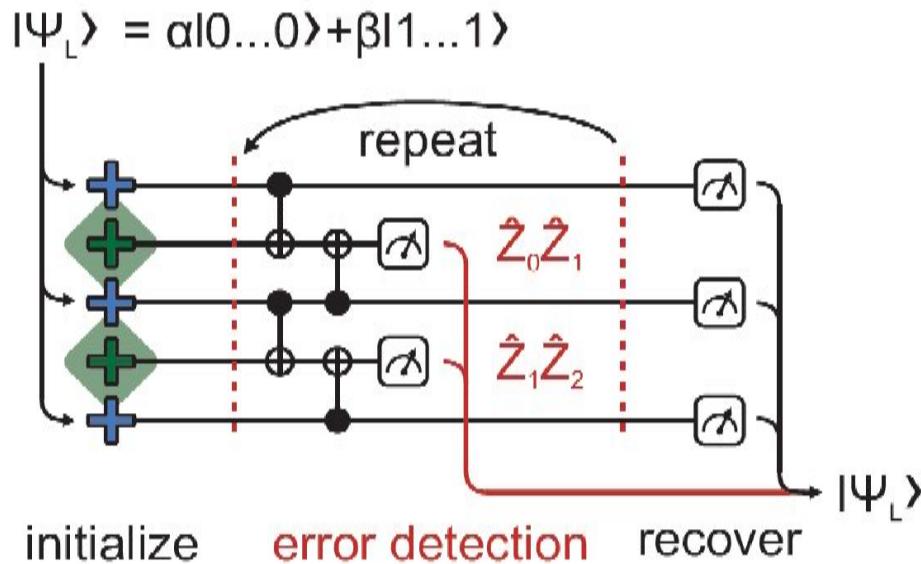
$$p_L = 0.01(p/p_{th})^n$$

Need **accurate** and **scalable**
Tests for correlated errors
Is theory correct (exp.)?

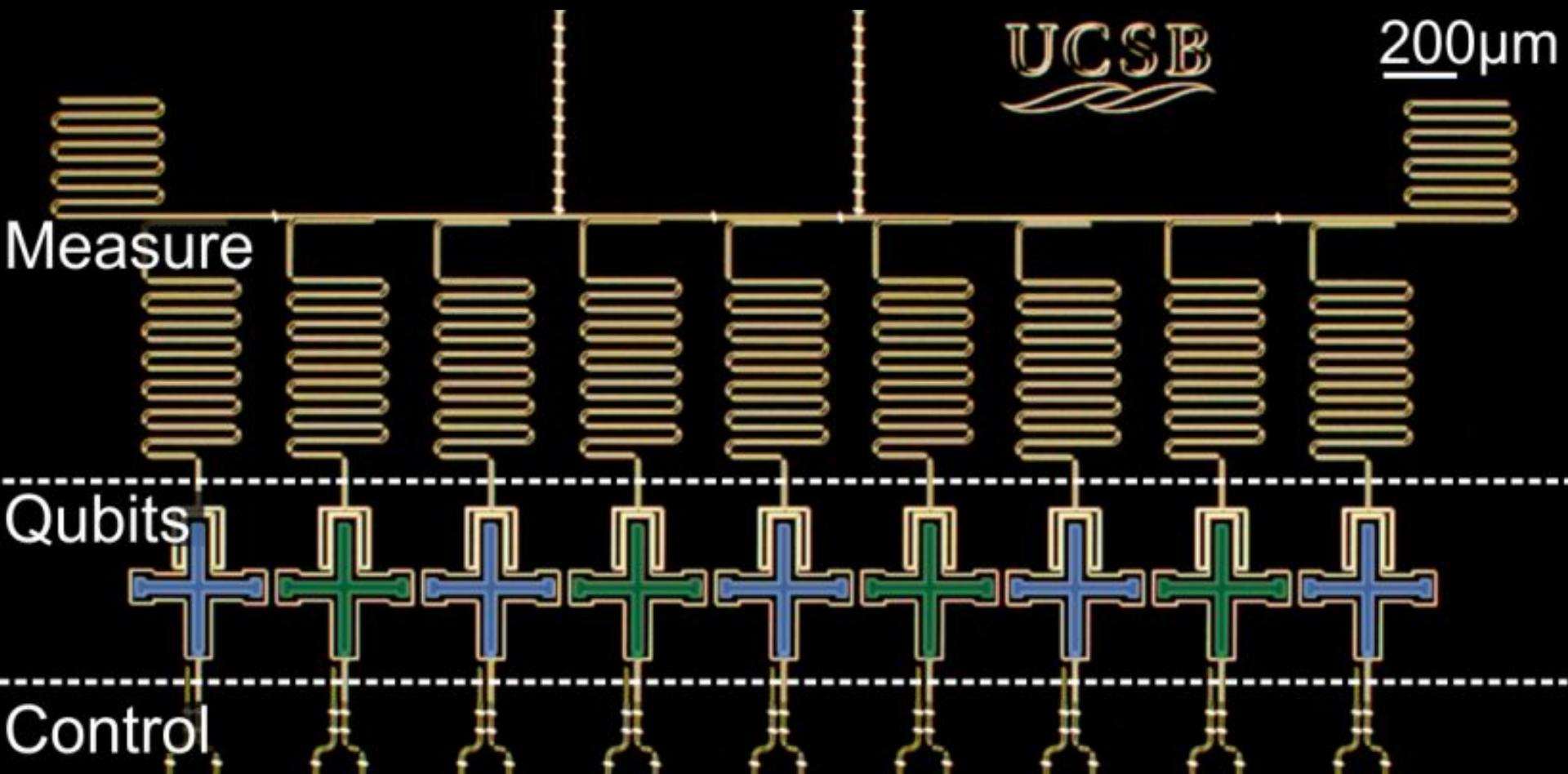


- 3) Here: bit-flip errors in linear chain

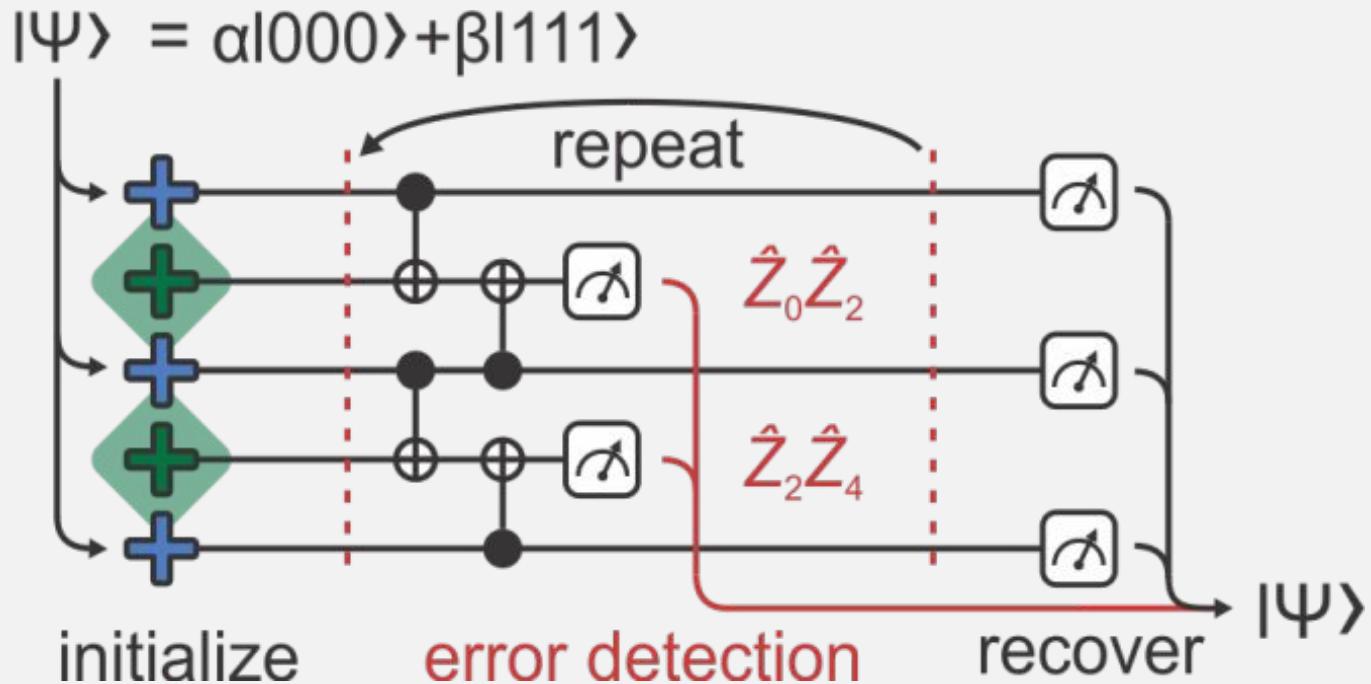
Quantum hardware that tests SC
Extend lifetime (**accurate**)
 $n = 1, 2$ errors (**scalable**)
Multiple measurement cycles!!
(usable error correction)
-This experiment can easily fail-



Repetition Code (bit-flip only): device



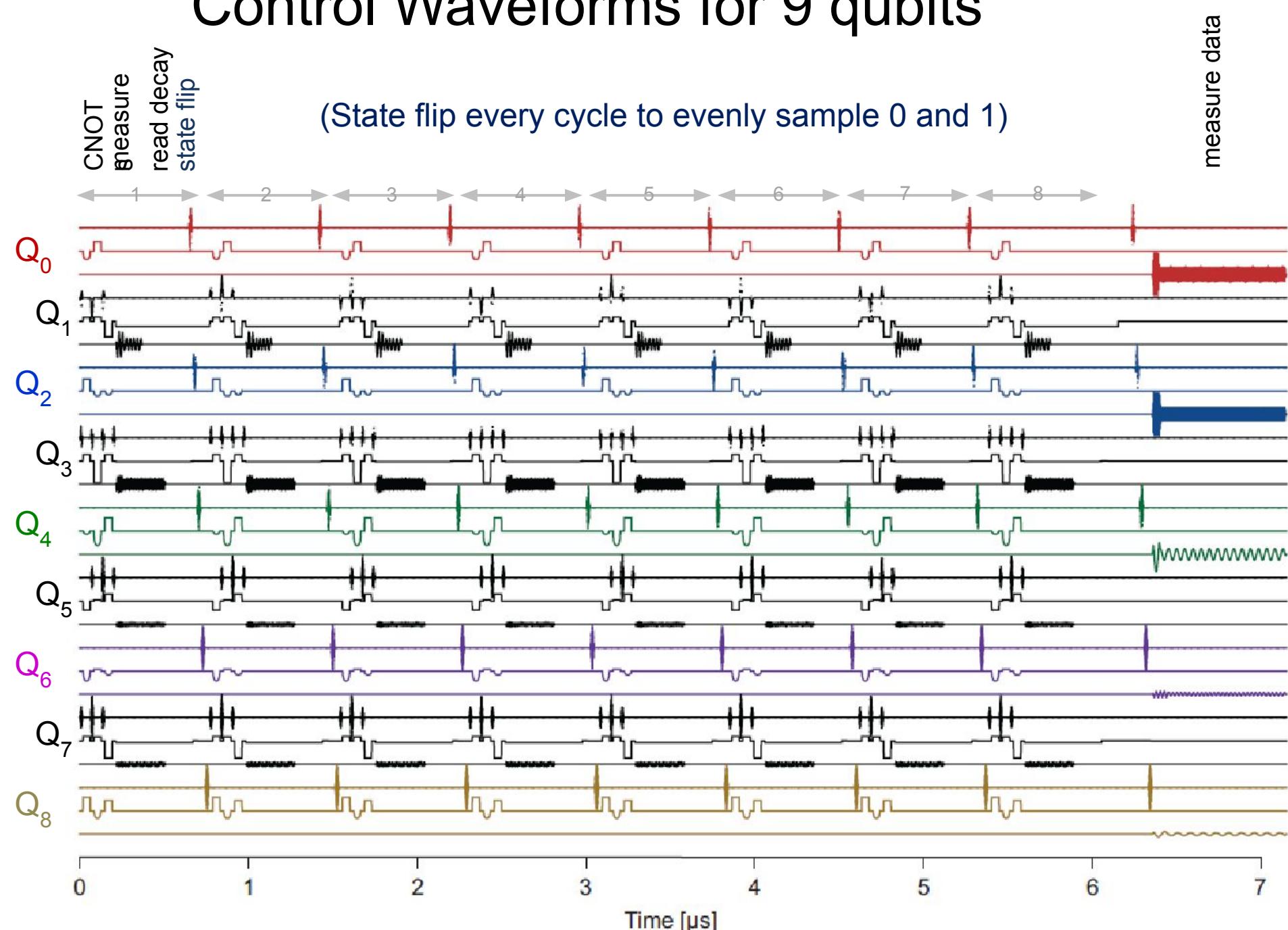
Tracking of Bit Flips: Experiment



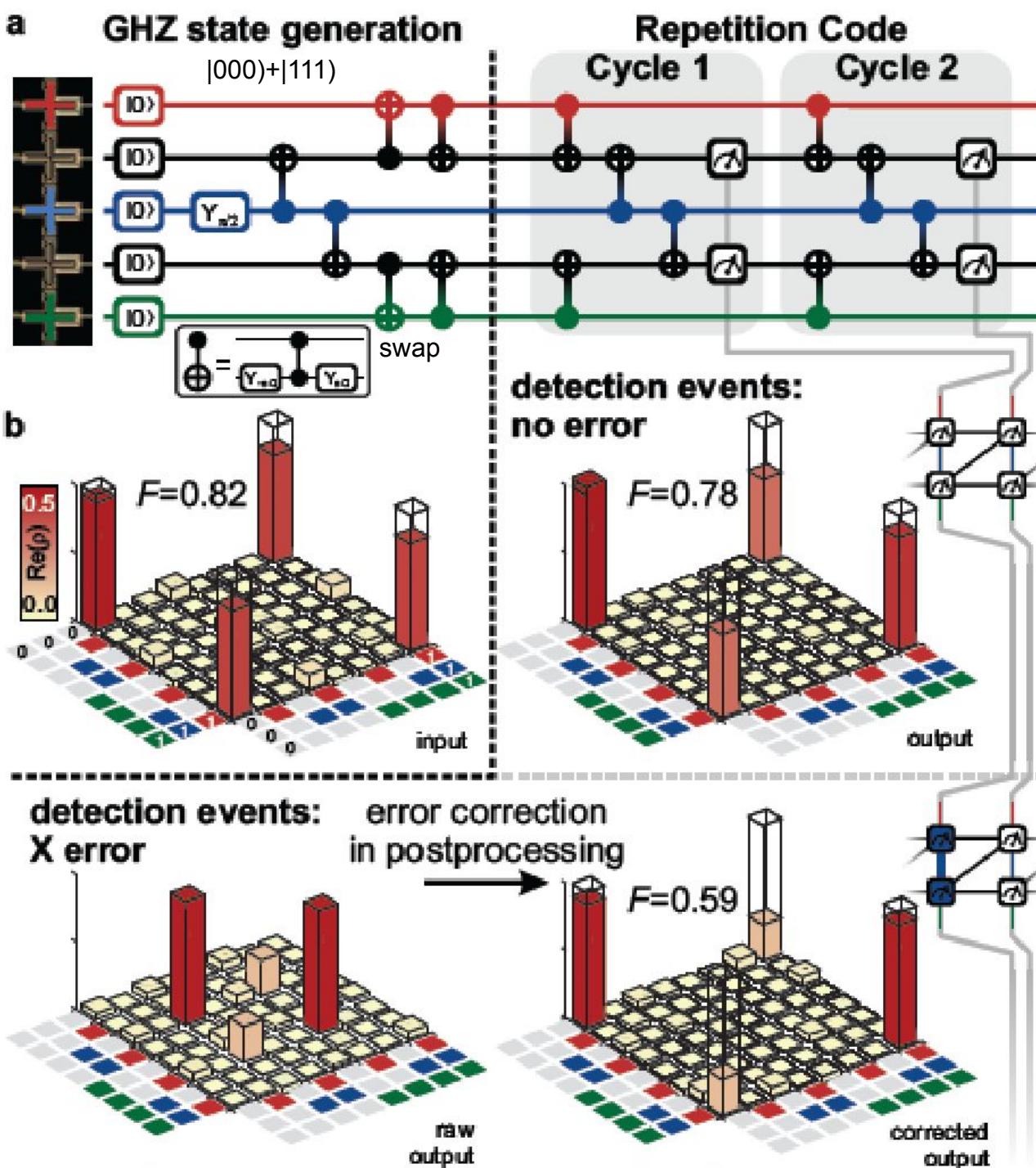
Use outcomes of Z_0Z_2 and Z_2Z_4 to protect from environmental bit-flip errors

Control Waveforms for 9 qubits

(State flip every cycle to evenly sample 0 and 1)



5 Qubit Data



2 examples for quantum state (GHZ)

no error – retains coherence

bit flip – corrects properly but lose coherence (as expected)

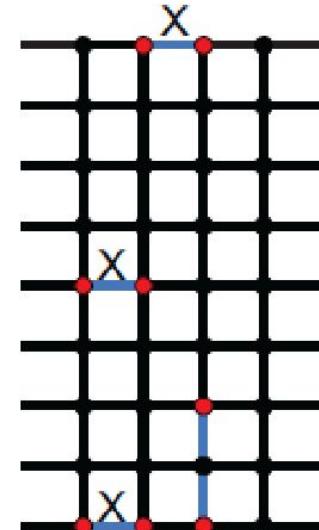
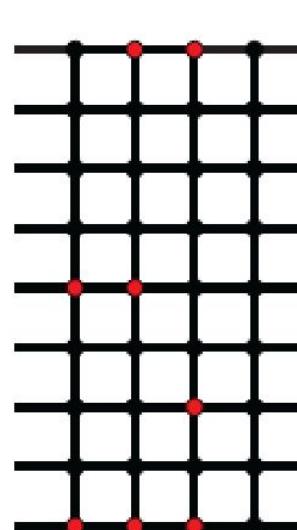
9 Qubit Experiment: Example data

Error detection and decoding:

in 000000000

		0 0 0 0	
		0 0 0 0	
1	0 1 1 0	0 1 1 0	. D D .
2	0 0 0 0	0 0 0 0
3	0 1 1 0	0 1 1 0
4	0 0 0 0	0 0 0 0
5	1 0 1 0	1 0 1 0	D D . .
6	0 0 0 0	0 0 0 0
7	1 0 0 0	1 0 0 0	. . D .
8	0 0 0 0	0 0 0 0
		0 1 1 0	D D D .

fin 0 0 1 0 0



raw data

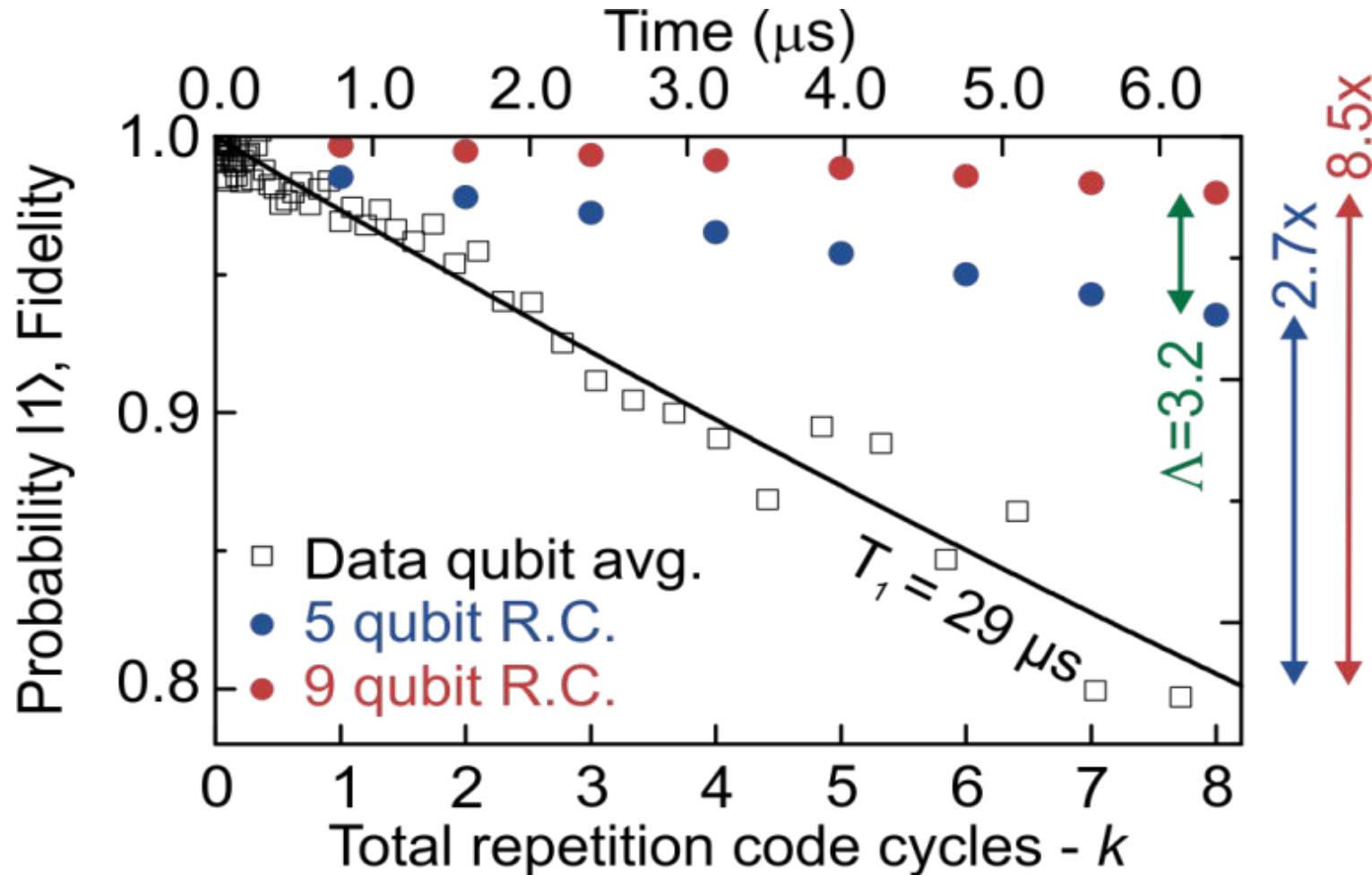
extended

detected
errors

graph

decoding

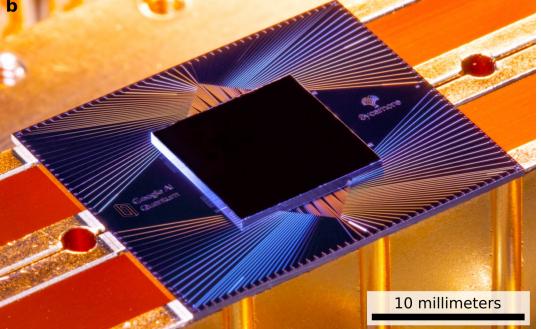
9 Qubit Data: Bit-Flip Error Correction Works!



Slower decay of logical
qubit state:

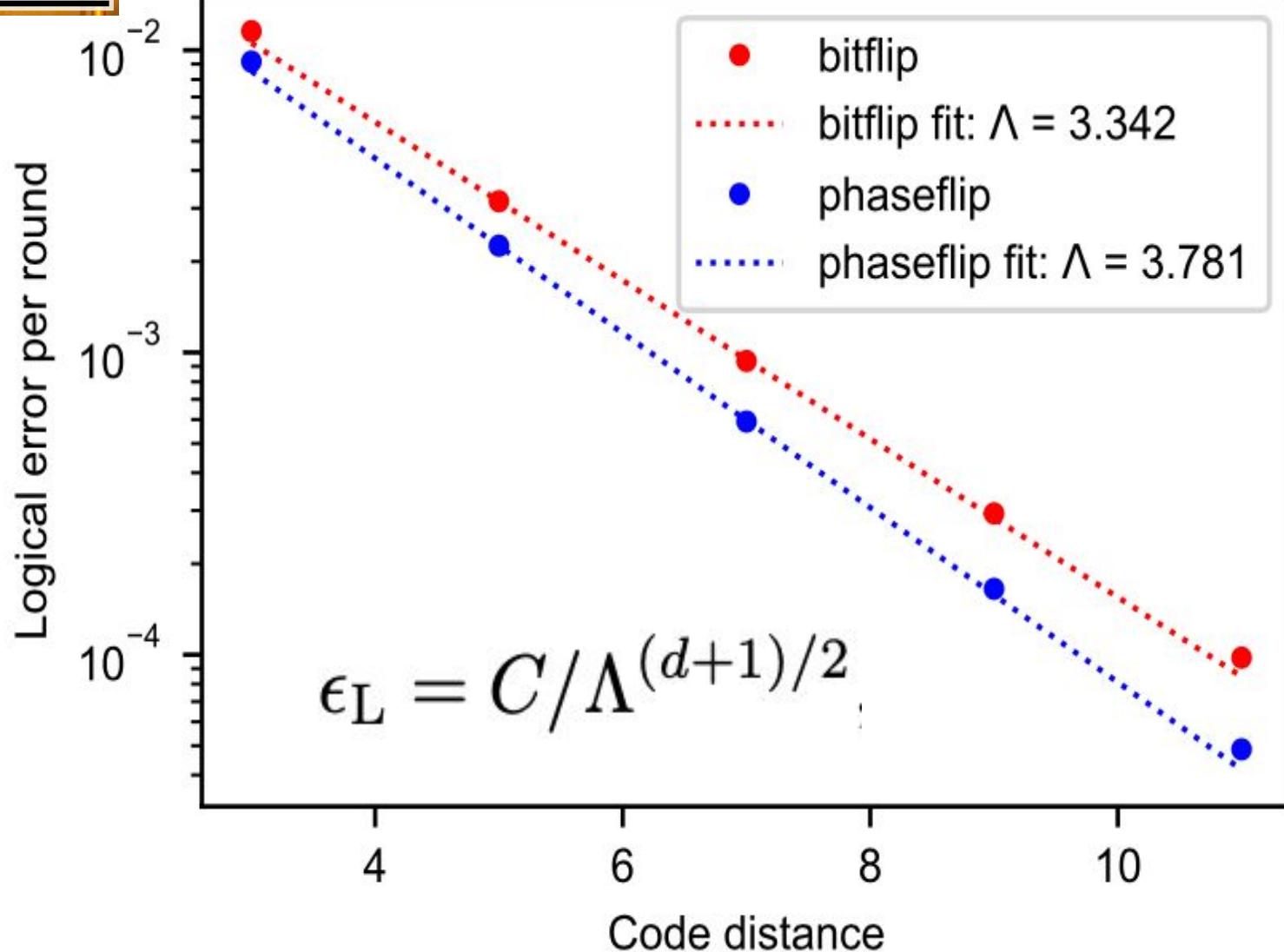
1st order: 2.7x
2nd order: 8.5x

$\Lambda = 3.2 > 1$, so better
memory for higher order
(fault tolerant behavior)



Logical Error per Round

Up to 21 qubits
Improvements



Summary

Understanding errors is important for quantum computing

Computation takes advantage of quantum amplitudes,
but errors understood as random bit or phase flips

Errors treated as classical bit- and phase-flip probabilities
(Assuming no error correlations)

Test theory/assumption with Quantum Supremacy experiment