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Introduction to Quantum Error Correction

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Experiment
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GKP QEC: **+Alec Eickbusch**
+Vlad Sivak

Theory
SMG
Leonid Glazman
Shruti Puri

Liang Jiang
Mazyar Mirrahimi

+...

U.S. DEPARTMENT OF ENERGY
Office of Science
AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Take-home message:

Quantum error correction

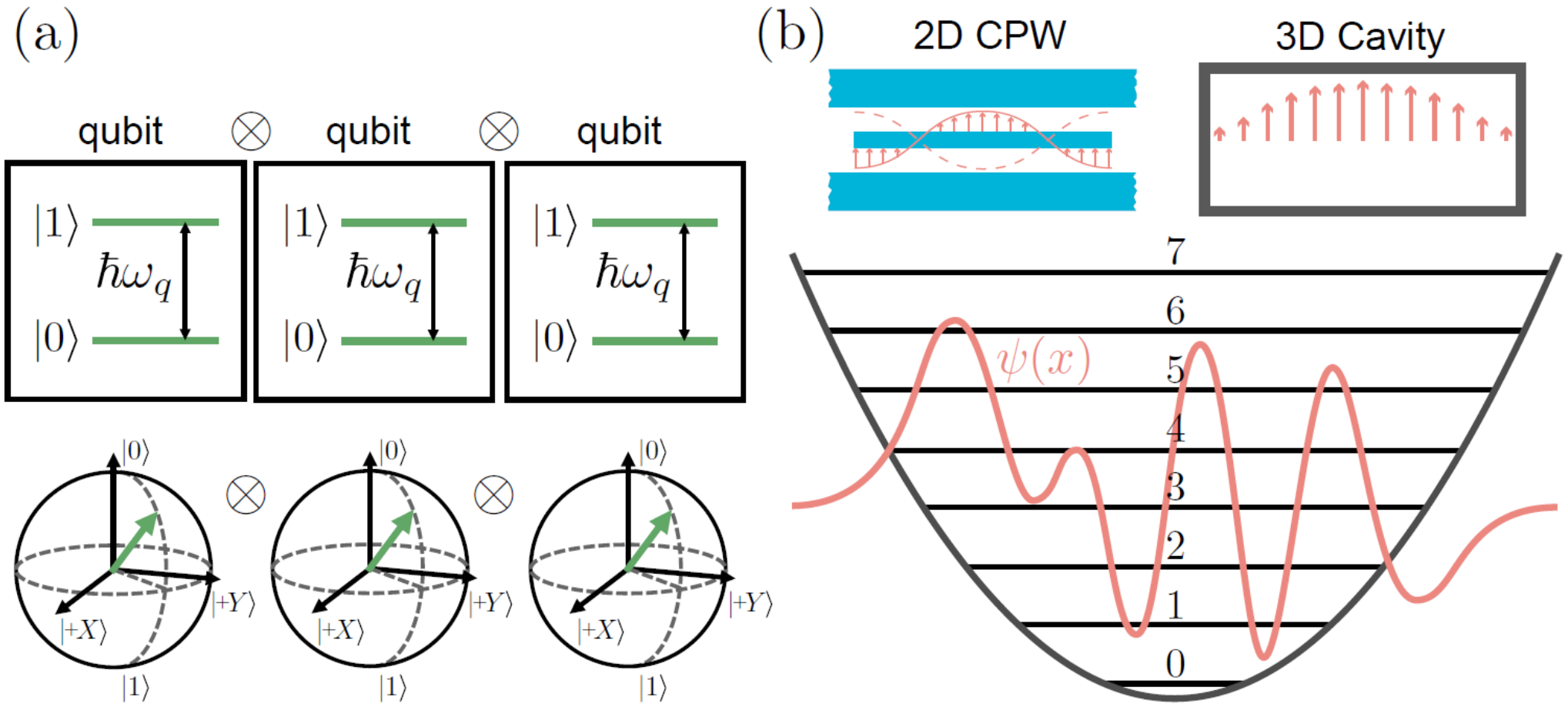
&

Quantum simulations of physical models containing bosons

are both vastly more efficient on hardware containing ‘native’ bosons

Discrete variable (transmon qubits)

Continuous variable (microwave or mechanical oscillators)



$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + a_3|011\rangle + a_4|100\rangle + a_5|101\rangle + a_6|110\rangle + a_7|111\rangle$$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle + a_5|5\rangle + a_6|6\rangle + a_7|7\rangle$$

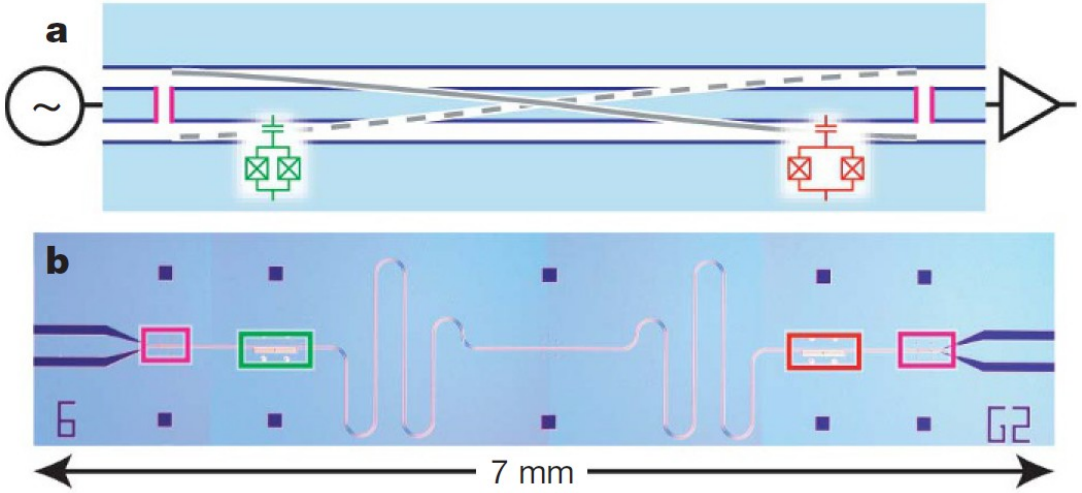
Boson Fock (photon number) states

Hybrid DV-CV hardware architectures

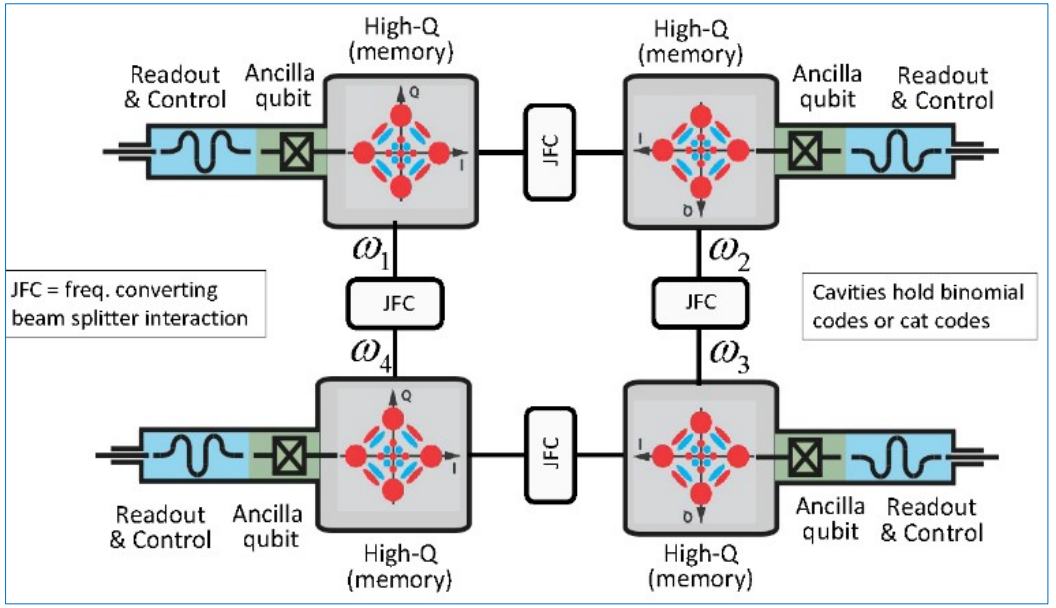
J. Majer et al., *Nature* **449**, 443 (2007)
L. DiCarlo et al., *Nature* **460**, 240 (2009)

DV = discrete variable = two-level qubit
CV = continuous variable = harmonic oscillator = boson

"DV-dominant:"
transmon qubits coupled through passive quantum busses (bosonic resonators)



"CV-dominant:"
bosonic encoded resonators controlled and coupled through ancilla transmons

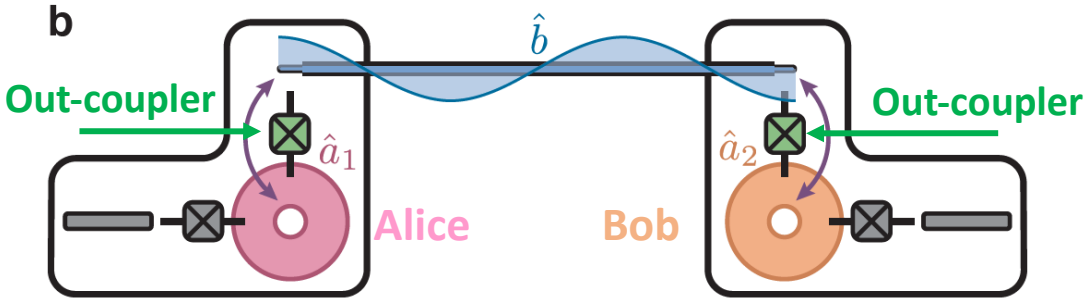


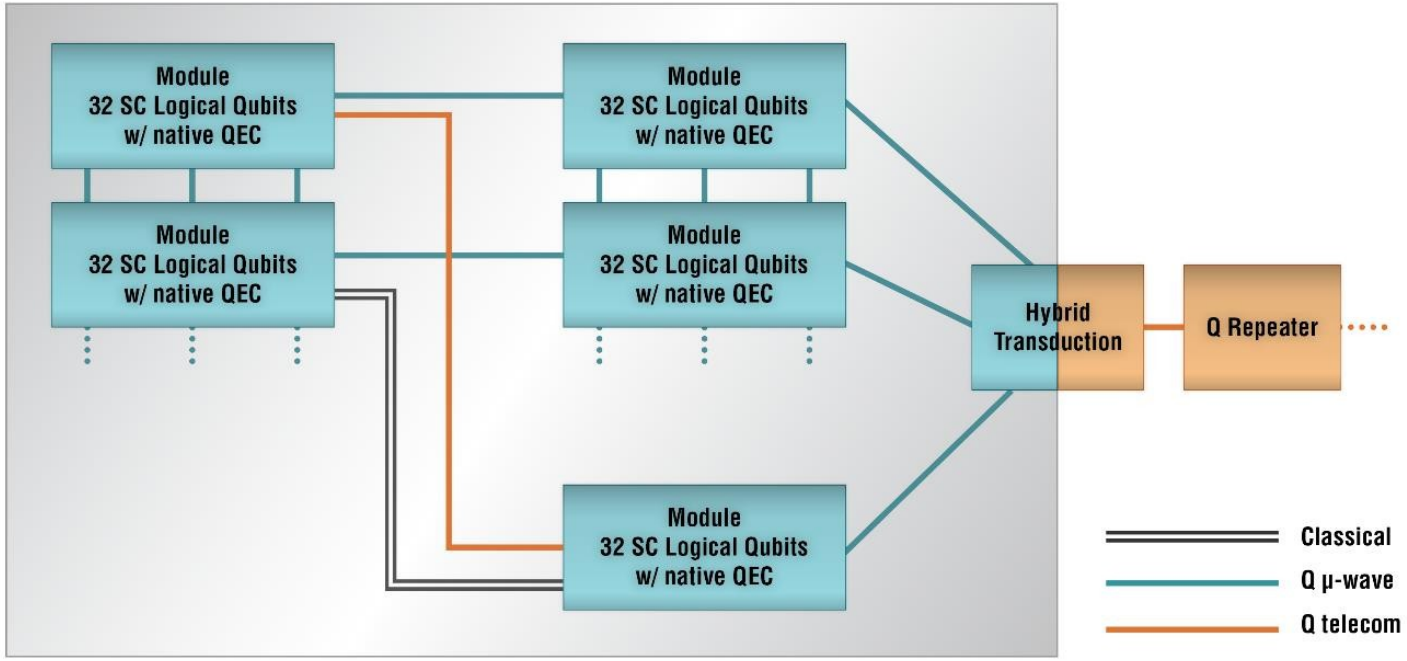
Benefits of bosonic encoding:

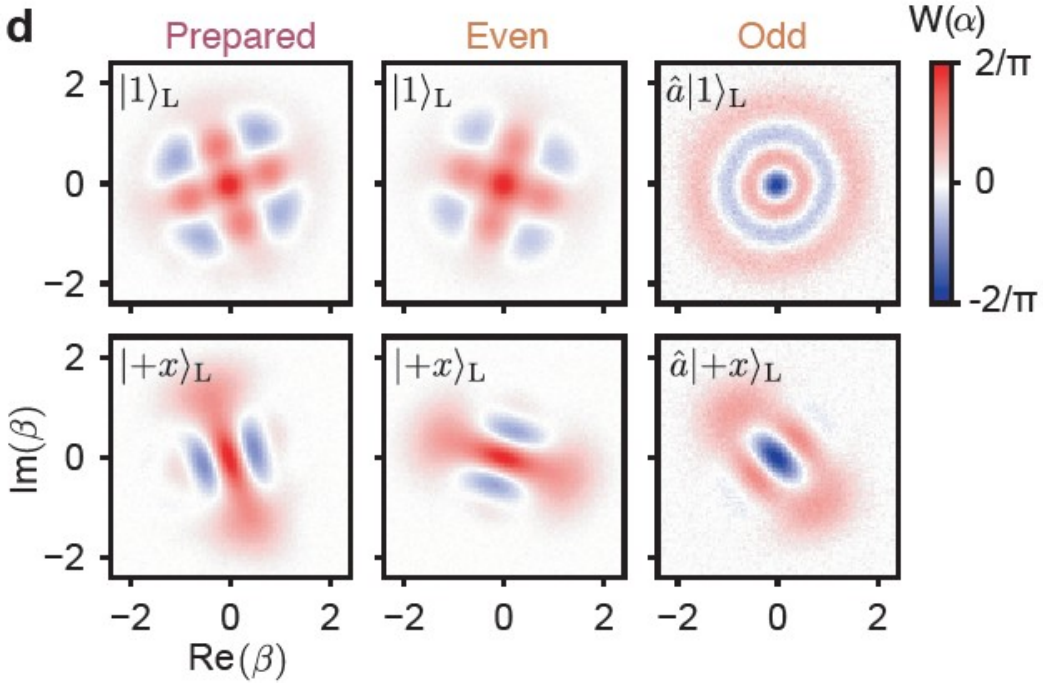
Bosonic QEC code words of 'standing' photons in resonators can be:

- Out-coupled as 'flying' photons for QEC local quantum communication
- Transduced to 'flying' photons for QEC at telecom wavelengths

'Error-detected state transfer and entanglement in a superconducting quantum network,'
L. Burkhardt et al., *PRX Quantum* **2**, 030321 (2021)







Quantum Control and Measurement of Hybrid DV-CV Systems

Recent theory papers:

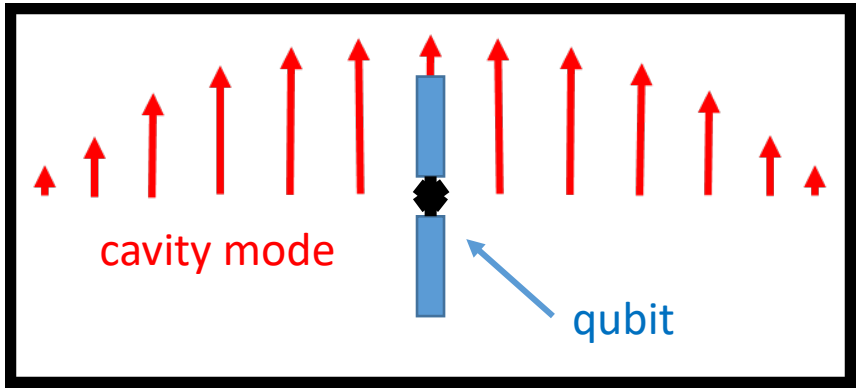
‘Quantum control of bosonic modes with superconducting circuits,’
Wen-Long Ma et al., arXiv:2102.09668

‘Photon-Number-Dependent Hamiltonian Engineering for Cavities,’
Chiao-Hsuan Wang et al. *Phys. Rev. Applied* **15**, 044026 (2021)

‘Constructing Qudits from Infinite Dimensional Oscillators by Coupling to Qubits,’
Yuan Liu et al., arXiv:2105.02896

Universal Control/Measurement required to
create and manipulate logical qubits. [Less power needed to correct errors.]

Universal quantum control of hybrid qubit-oscillator systems



Strong-dispersive coupling

$$H = \omega_c a^\dagger a + \frac{\omega_q}{2}\sigma^z + \chi\, \sigma^z\, a^\dagger a$$

$$H_{\text{control}} = \left[\alpha(t)a^\dagger + \alpha^*(t)a\right] + \left[\Omega(t)\sigma^+ + \Omega^*(t)\sigma_-\right]$$

cavity drive qubit drive

Strong-dispersive coupling permits:

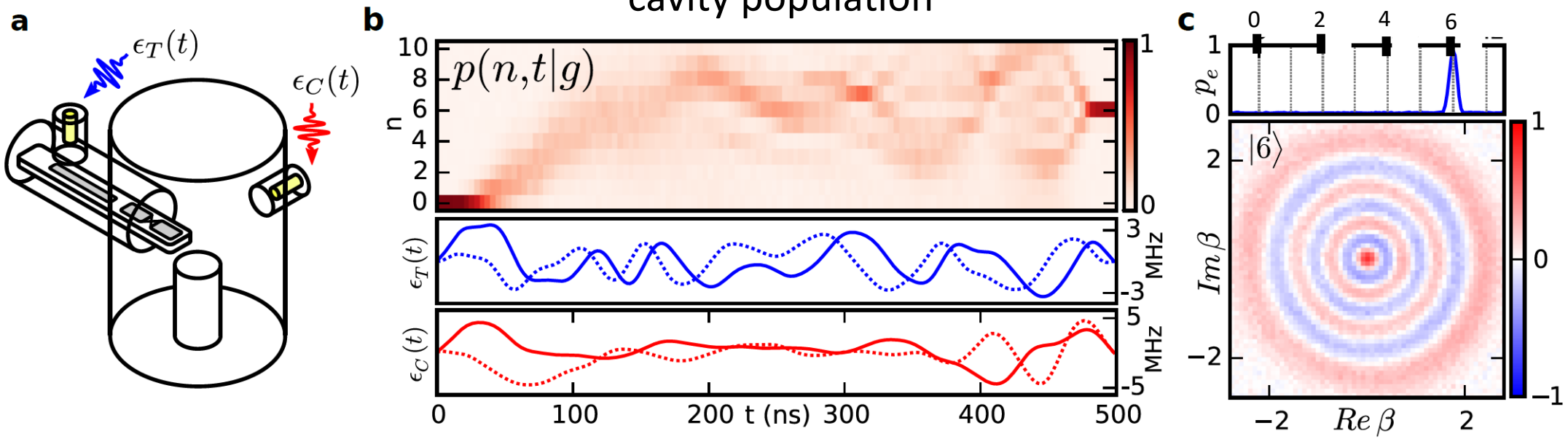
- Cavity displacement conditioned on qubit state
- Qubit rotation conditioned on cavity state

Provable universal control [Phys. Rev. A 92, 040303(R) (2015)]

Example of universal control: preparation of photon Fock state $|n=6\rangle$ using OCT GRAPE pulses

Definite number, completely indefinite phase

cavity population



ancilla and cavity quadrature drives

Measured Wigner function (quasi-probability distribution in phase space)

Heeres et al. (Schoelkopf lab)
Nature Communications 8, 94 (2017)

Key enabling resource for Wigner function: ability to measure photon number parity

Noise, Errors
and
Quantum Error Correction

The huge information content of quantum superpositions comes with a price:

$$|000\rangle + |001\rangle - |010\rangle - |011\rangle + |100\rangle + |101\rangle + |110\rangle - |111\rangle$$

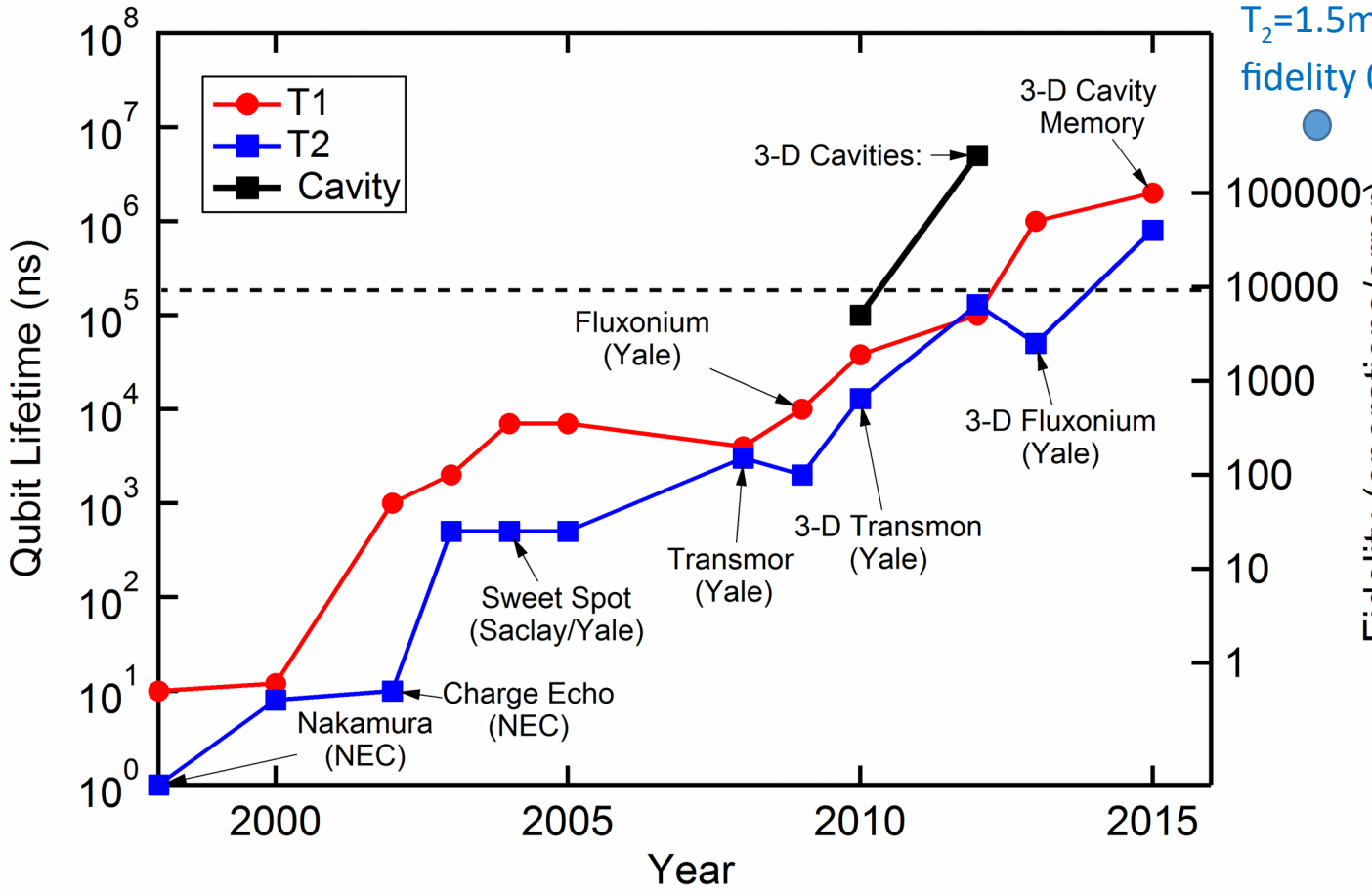
Great sensitivity to noise, perturbations and dissipation.

The quantum phase of superposition states is well-defined only for a finite 'coherence time' T_2

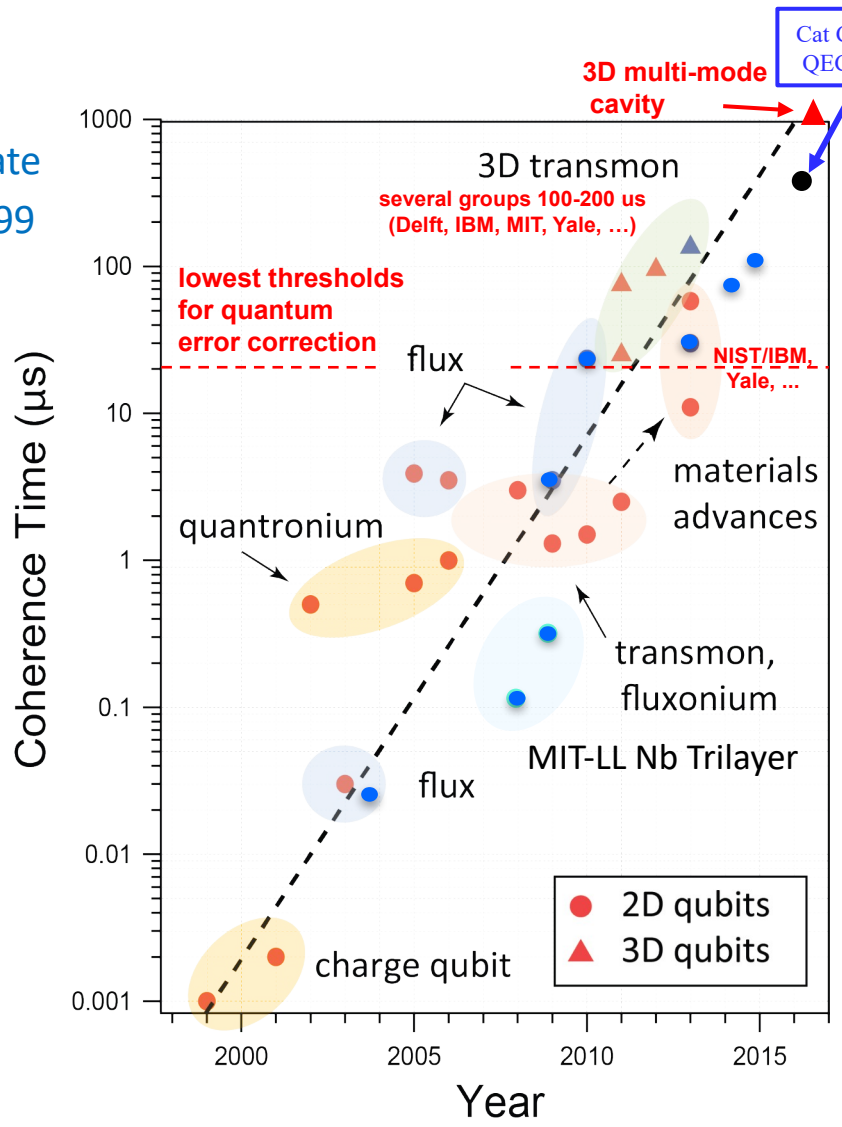
$$|0\rangle + |1\rangle \rightarrow |0\rangle - |1\rangle$$

Despite this sensitivity,
we have made exponential progress in qubit coherence times.

Exponential Growth in SC Qubit Coherence



Fluxonium
T_2=1.5ms, gate fidelity 0.9999



Oliver & Welander, MRS Bulletin (2013)

No matter how much progress there is in increasing coherence times, we still must contend with the fundamental law of quantum devices:

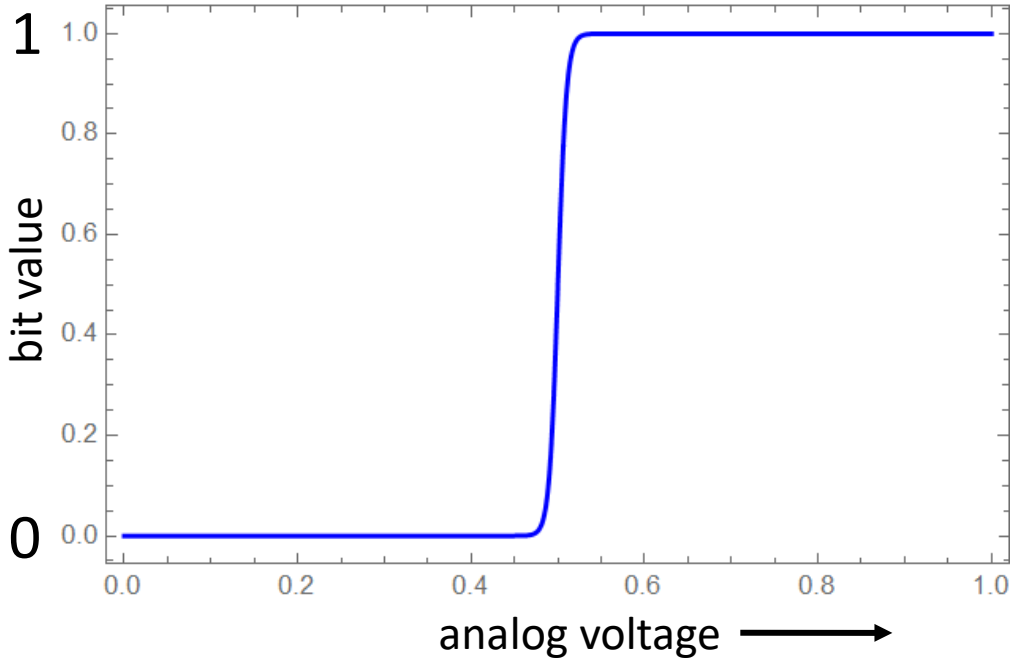
There is no such thing as too much coherence.

We need quantum error correction!

QUANTUM COMPUTERS ARE ANALOG MACHINES

Courtesy: Xavier Waintal

DIGITAL ≠ ANALOG



Engineer Karen Leadlay
General Dynamics, 1964

THE FUNDAMENTAL LAW OF ANALOG MACHINES

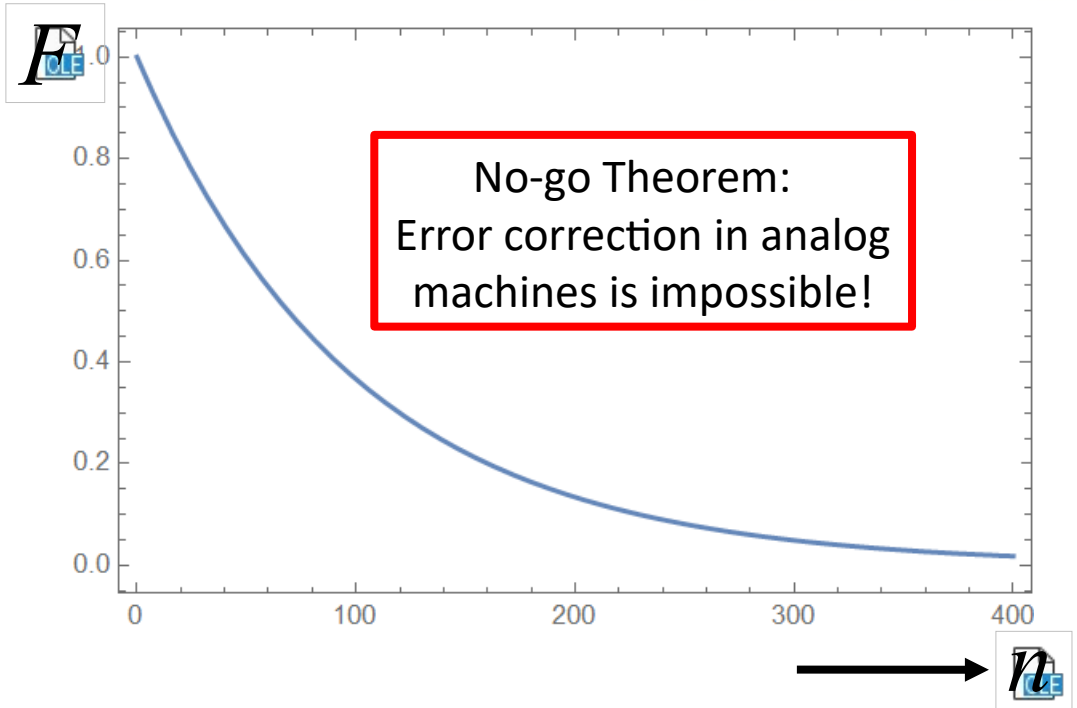
NUMBER OF GATES

$$F = f^n$$

FIDELITY

0 < FIDELITY PER GATE < 1

Courtesy: Xavier Waintal



No-go Theorem:
Error correction in analog machines is impossible!

The Quantum Error Correction Problem

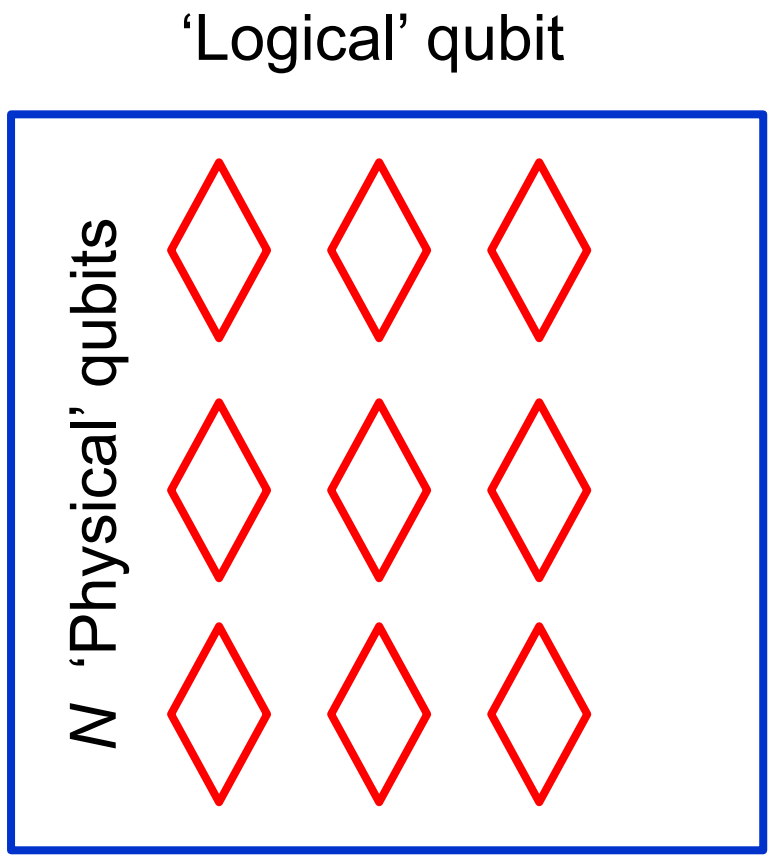
I am going to give you an unknown quantum state.

If you measure it, it will change randomly due to state collapse ('back action').

If it develops an error, please fix it.

***Mirable dictu*: It can be done!**

Quantum Error Correction for an unknown state requires storing the quantum information *non-locally* in (non-classical) *correlations* over multiple physical qubits.

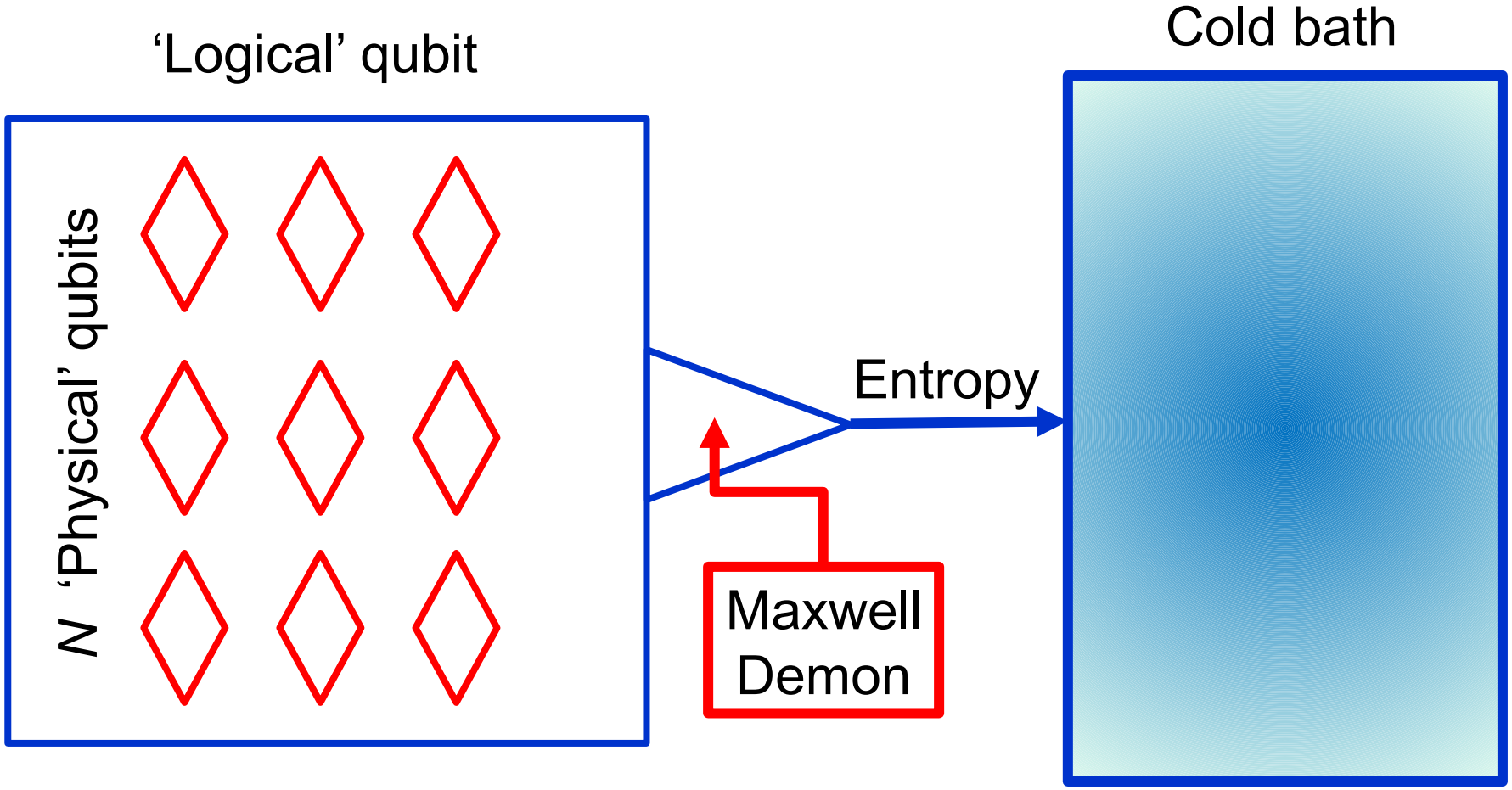


Non-locality: No single physical qubit can "know" the state of the logical qubit.

Special multi-qubit measurements can tell you about errors without telling you the logical state in which the error occurred!

<u>Miracle</u>: Quantum errors are analog (i.e. continuous). Measured errors are discrete (i.e. digital). State collapse is our friend!

Quantum Error Correction



N qubits have errors *N* times faster. Maxwell demon must overcome this factor of *N* – *and not introduce errors of its own! (or at least not uncorrectable errors)*

Stabilizer Codes

N qubits have 2^N states. Define a 2D logical code subspace: $\mathsf{C} = \text{span}\left\{|0_L\rangle, |1_L\rangle\right\}$ and logical operators

$$X_L = |0_L\rangle\langle 1_L| + |1_L\rangle\langle 0_L|, \quad Z_L = |0_L\rangle\langle 0_L| - |1_L\rangle\langle 1_L|, \quad Y_L = +iX_L Z_L$$

using $N-1$ stabilizers $\left\{S_j;\, j = 1, \ldots, N-1\right\}$ and imposing $N-1$ constraints

$$\mathrm{S}_j |\psi_{\text{code}}\rangle = (+1)|\psi_{\text{code}}\rangle, \forall j.$$

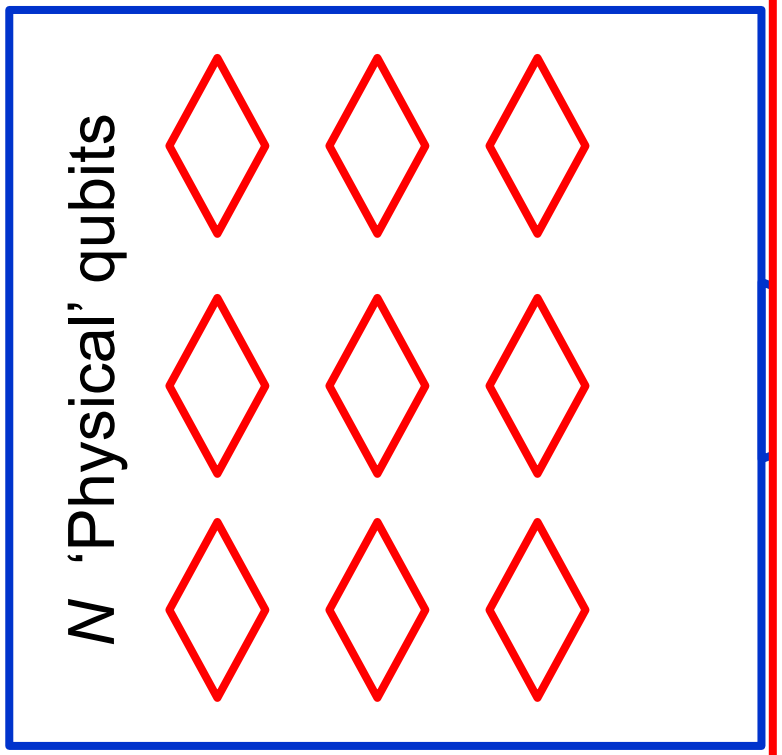
Stabilizers are mutually commuting and commute with logical operators.

[So can be measured simultaneously and without affecting logical state.]

Stabilizers anti-commute with physical errors so measurement of stabilizers give error syndromes that collapse the error state without collapsing the logical state.

Quantum Error Correction

'Logical' qubit



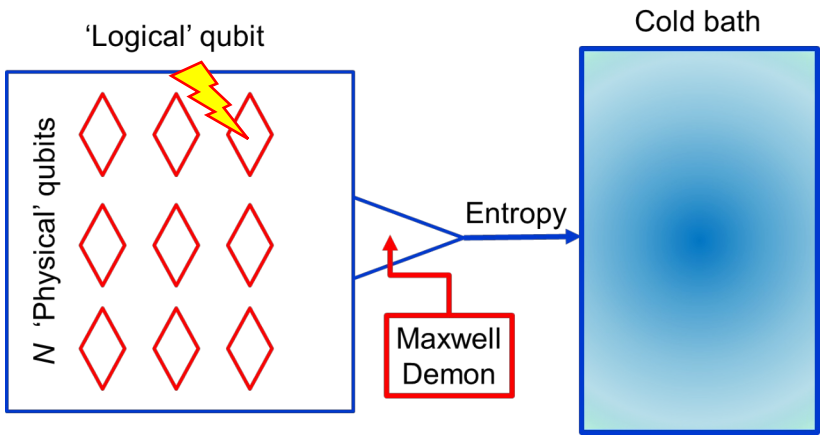
9 qubit Shor code can correct 1 error: X,Y, or Z

3 types of errors x 9 locations = 27 possible error states + (no-error state)

Code requires 8 stabilizer measurements

- Z_1Z_2, Z_2Z_3, Z_4Z_5, Z_5Z_6, Z_7Z_8, Z_8Z_9
  → Detect bit flip errors
- $X_1X_2X_3X_4X_5X_6$, $X_4X_5X_6X_7X_8X_9$
  → Detect phase flip errors

Very difficult multi-qubit measurements!
[N.B. cannot measure Z_1, Z_2 separately and multiply results! Need *joint* measurements.]



Idea:
Don't use material objects as qubits.

Use microwave photon states stored in high-*Q* superconducting resonators.

Cat code (first to exceed break-even):
Ofek, et al., *Nature* **536**, 441–445 (2016)

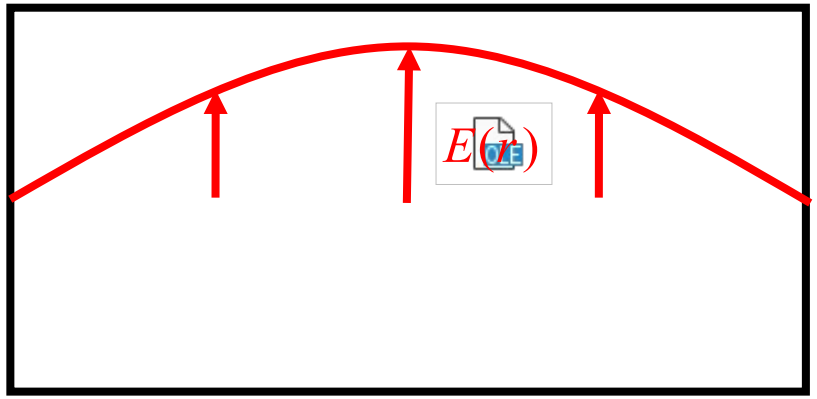
Binomial Code:
Michael et al., *Phys. Rev. X* **6**, 031006 (2016)
Hu et al., *Nature Physics* **15**, 503 (2019)

Autonomous Code (T4C truncated cat):
Gertler et al., *Nature* **590**, 243 (2021)

GKP Codes:
Campagne-Ibarcq et al. *Nature* **584**, 368 (2020)

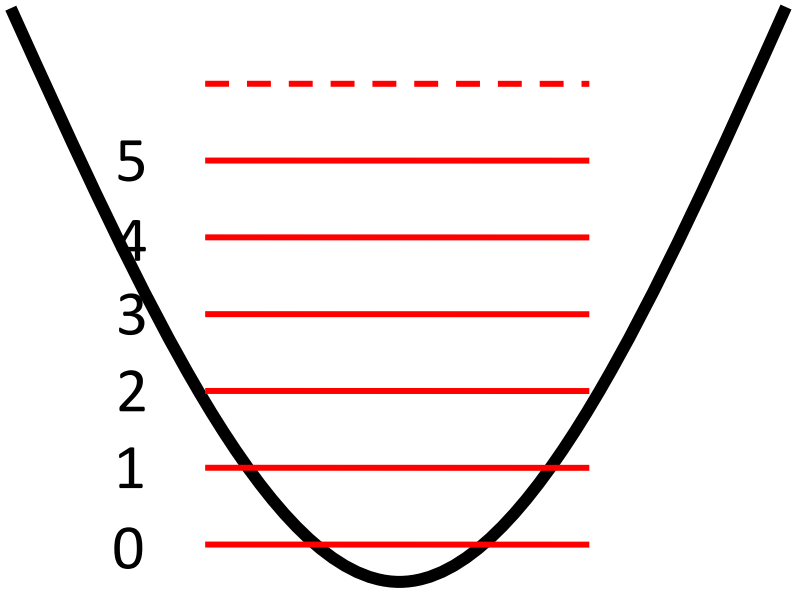
de Neeve et al., *Nature Physics* **18**, 296 (2022)
Royer et al., *Phys. Rev. Lett.* **125**, 260509 (2020)
PRX Quantum **3,** 010335 (2022)

Bosonic code reviews:
W. Cai et al., arXiv:2010.08699
A. Joshi et al., arXiv:2008.13471



Single-mode microwave resonators (harmonic oscillators) are empty boxes (vacuum surrounded by superconducting walls)

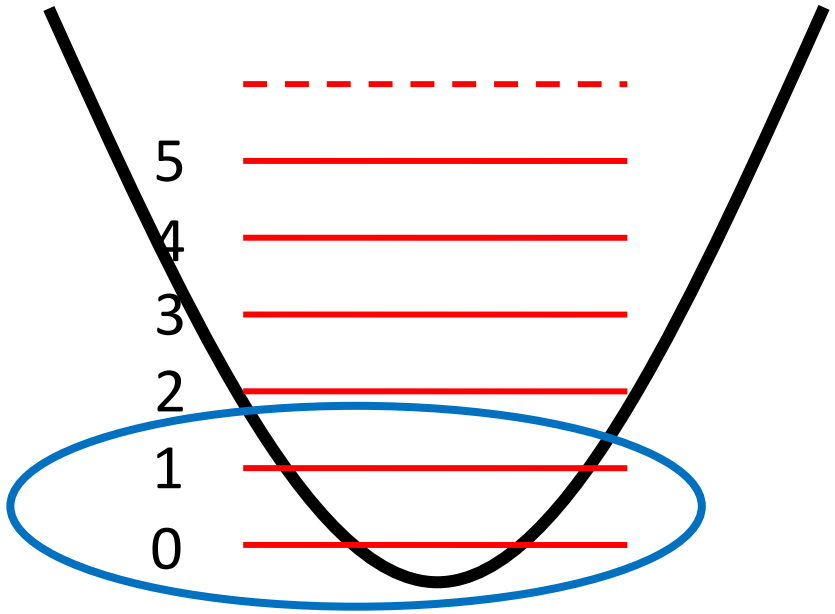
$$H = \hbar\omega\, a^{\dagger} a = \hbar\omega\, \hat{n}$$



"Hardware Efficiency"

Oscillators have many quantum levels so can replace multiple physical qubits without adding more 'moving parts.'

Bosonic Quantum Error Correction Codes



Harmonic oscillator has an infinite number of states. A qubit has only two states.

We need to pick out two orthogonal states to act as 'logical code words' to hold one qubit's worth of (protected) information.

Simplest code: $|0_L\rangle = |0\rangle$ $|1_L\rangle = |1\rangle$

$$\frac{dE}{dt} = -\kappa E \Rightarrow \frac{d\langle \hat{n} \rangle}{dt} = -\kappa \langle \hat{n} \rangle$$

Has smallest possible number of photons and therefore longest lifetime.

But <u>not</u> error correctable after photon loss: $\alpha|0\rangle + \beta|1\rangle \rightarrow |0\rangle$

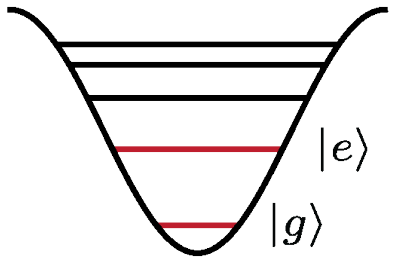
Experimental physics question

Can we leverage active quantum error correction to create

a "logical qubit" <u>better</u> than all constituent "physical qubits"?

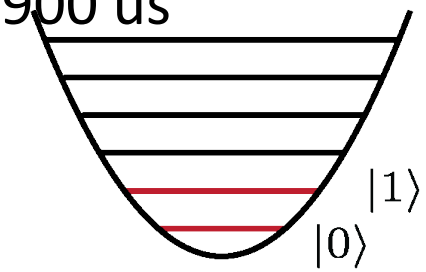
<u>Physical</u> Qubit #1: Transmon

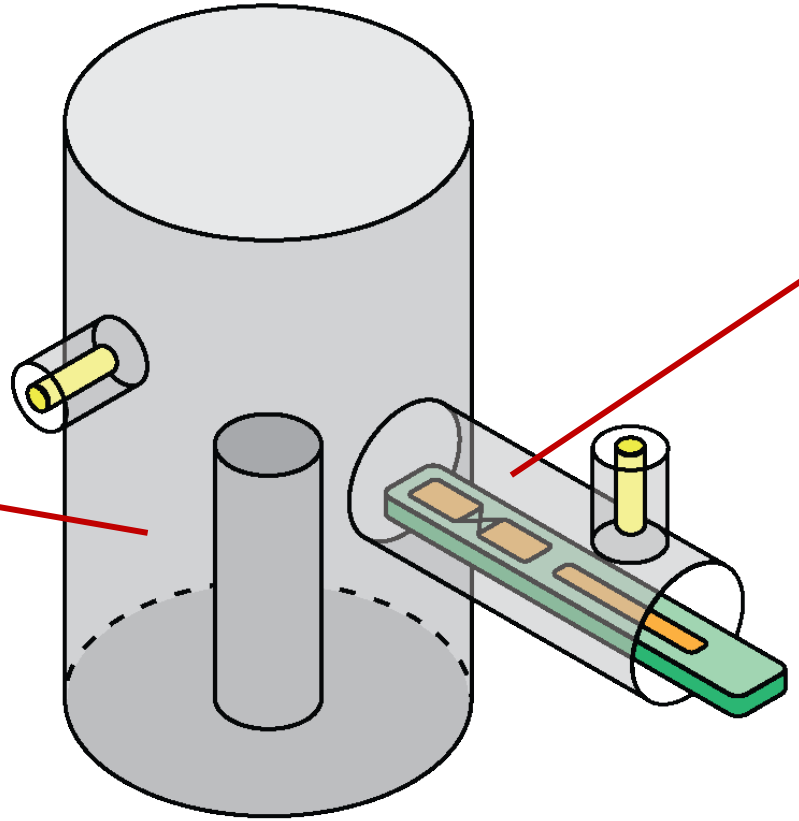
110 us
130 us



<u>Physical</u> Qubit #2: Fock |0>,|1>

550 us
900 us





Transferring QI from transmon to cavity strongly increases lifetime but does NOT constitute QEC "Gain." No QEC yet.

courtesy V. Sivak

Definition of "better"

Average channel fidelity

M. Nielsen, Phys. Lett. A (2002)

$$\overline{\mathcal{F}}[\mathcal{E}] = \int d\psi \langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle$$

Short time expansion

$$\overline{\mathcal{F}}(\delta t) = 1 - \frac{1}{2}\gamma_{\mathcal{E}}\,\delta t$$

Amplitude damping + dephasing

$$\gamma_{\mathcal{E}} = \frac{\gamma_1 + 2\gamma_2}{3}$$

Pauli channel

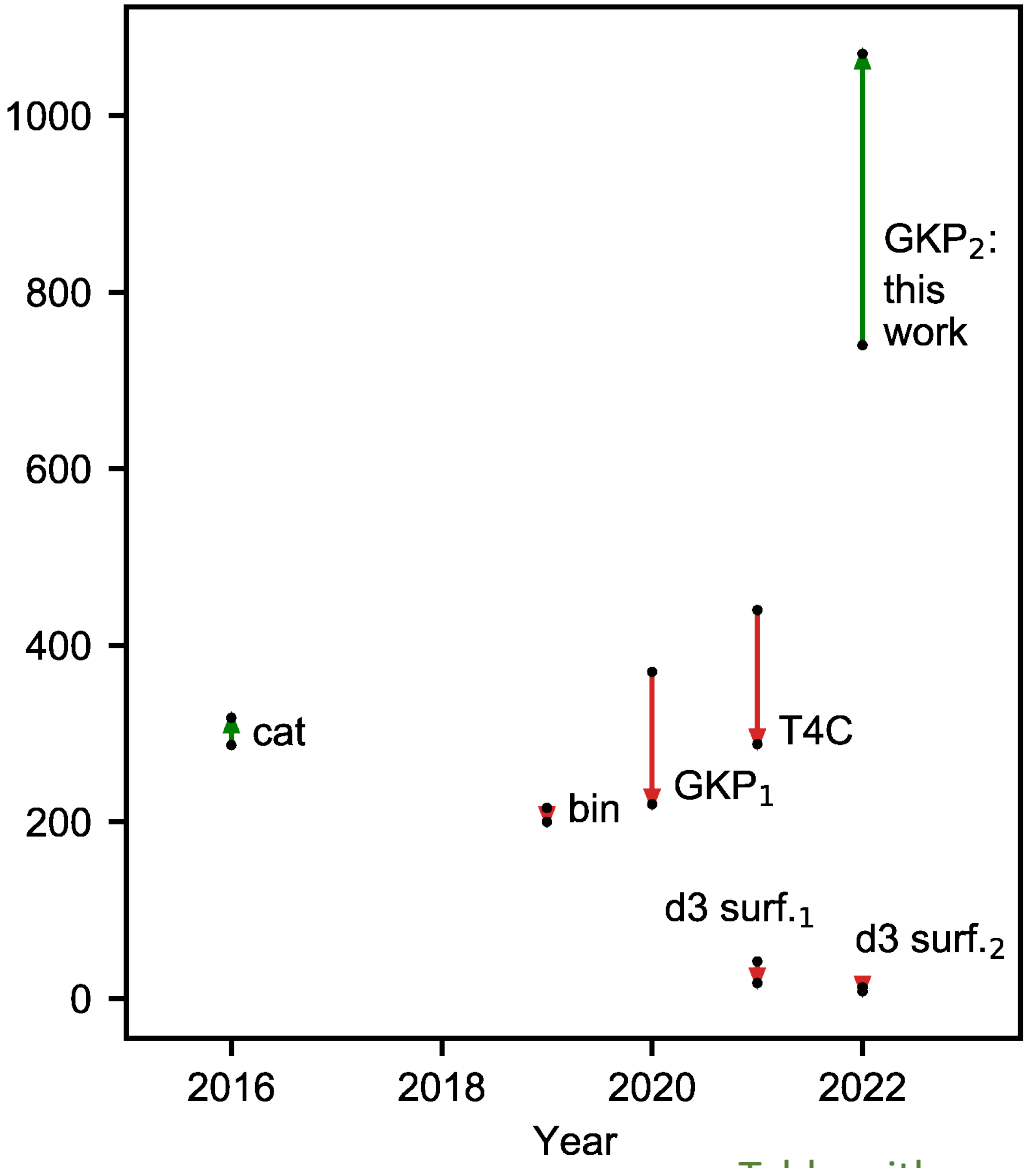
$$\gamma_{\mathcal{E}} = \frac{\gamma_X + \gamma_Y + \gamma_Z}{3}$$

<u>QEC gain</u> $G = \dfrac{\min_i[\gamma_{\mathcal{E}}^{(i)}]}{\gamma_L}$

"Break-even" $G = 1$

courtesy V. Sivak

Where do we stand? Qubit codes vs. Bosonic codes



	G	$1/\gamma_L$	$1/\min_i[\gamma_{\mathcal{E}}^{(i)}]$
4-legged cat code N. Ofek *et al.*, (Nature, 2016)	1.11	318 µs	287 µs
Binomial code L. Hu *et al.*, (Nature Physics, 2019)	0.93	200 µs	216 µs
GKP code (1) P. Campagne-Ibarcq *et al.*, (Nature, 2020)	0.59	220 µs	≥370 µs
T4C code J. Gertler *et al.*, (Nature, 2021)	0.65	288 µs	440 µs
D3 surface code (1) S. Krinner *et al*		17.5 µs	42.2 µs
D3 surface Y. Zhao *et al.*, (		8.1 µs	12.4 µs
GKP code (2) This work	1.45	1070 µs	740 µs

de Neeve et al.,
Nature Physics **18**, 296 (2022)
GKP Gain difficult to estimate

courtesy V. Sivak

Table with more QEC exps.: see SM of Google Quantum AI (Nature, 2021)

Where do we stand?

Process fidelity

M. Nielsen, Phys. Lett. A (2002)

$$\overline{\mathcal{F}}[\mathcal{E}] = \int d\psi \langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle$$

Short time expansion

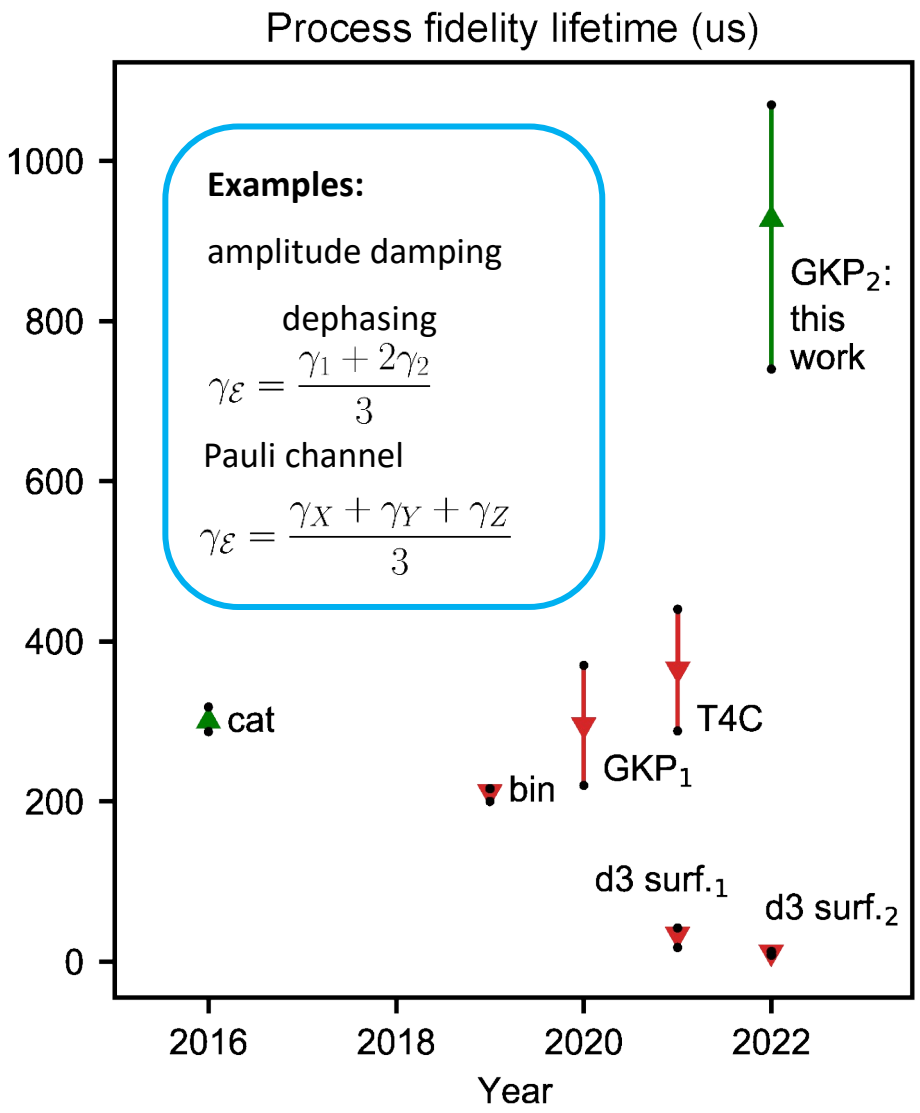
$$\overline{\mathcal{F}}(\delta t) = 1 - \frac{1}{2}\gamma_{\mathcal{E}}\,\delta t$$

Best physical qubit

QEC gain $G = \dfrac{\gamma_P}{\gamma_L}$

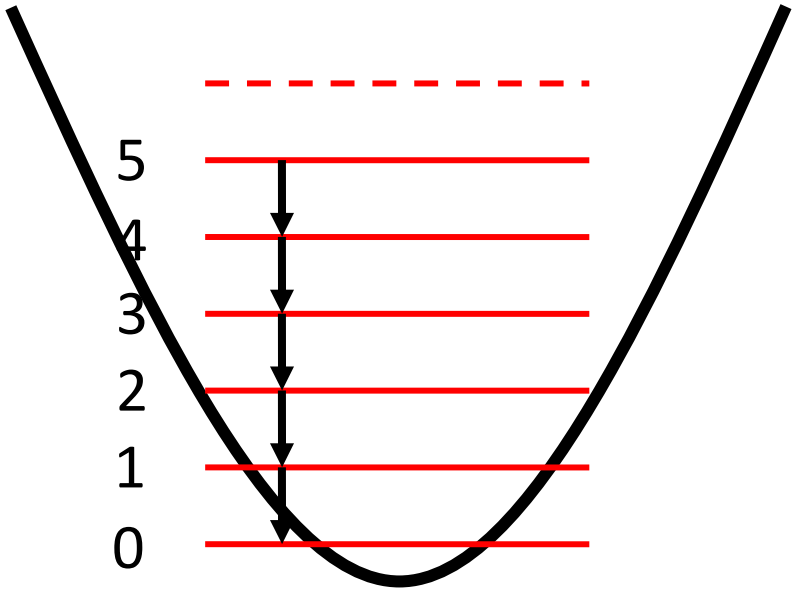
Logical qubit

"Break-even" $G = 1$



QEC experiment	G
4-legged cat code N. Ofek *et al.*, (Nature, 2016)	1.11
Binomial code L. Hu *et al.*, (Nature Physics, 2019)	0.93
GKP code (1) P. Campagne-Ibarcq *et al.*, (Nature, 2020)	0.59
T4C code J. Gertler *et al.*, (Nature, 2021)	0.65
D3 surface code (1) S. Krinner *et al.*, (arXiv:2112.03708, 2021)	0.41
D3 surface code (2) Y. Zhao *et al.*, (arXiv:2112.13505, 2022)	0.65
GKP code (2) This work	1.45

Table with more QEC experiments, see SM of Google Quantum AI (Nature, 2021)



Single-mode weakly damped oscillators have a very simple error model: photon loss

$$|\psi\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$$

Use 'code words' with definite photon number parity (e.g. even)

Only a single mode and only <u>one</u> kind of error—photon loss – NOT *3N* errors as for qubits.

Measurement of parity does not tell us the photon number so stabilizer commutes with logical operators.

Only need one simple code 'stabilizer,' Photon number parity:

$$\hat{P} = (-1)^{\hat{n}}$$

Example code:

$$|0_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$$

$$|1_L\rangle = |2\rangle$$

Photon loss error flips the parity:

$$\hat{P}a\hat{P} = -a$$

Easy to QND measure with high fidelity (unlike in ordinary quantum optics)

Parity stabilizer measurements 99.8% QND. L. Sun et al., *Nature* **511**, 444 (2014)

Simplest bosonic code example: 'binomial code' uses only 5 photon states 0-4 ($\ln_2$ 5 bits)
Phys. Rev. X 6, 031006 (2016) to correct errors to first order in $\delta = \kappa\,\delta t$.

Logical code words

(even parity)

$$|0_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$$

$$|1_L\rangle = |2\rangle$$

error words

(odd parity)

$$a|0_L\rangle = \sqrt{2}\,|3\rangle$$

$$a|1_L\rangle = \sqrt{2}\,|1\rangle$$

Error model Kraus ops.

$$\hat{K}_0 = \sqrt{\hat{I} - \kappa\,dt\,\hat{n}} \text{ where } \hat{n} = a^\dagger a$$ ← "no-jump"

$$\hat{K}_1 = \sqrt{\kappa\,dt}\;a$$ ← "jump"

number of photons

Universal control permits recovery after parity jump:

$$U|3\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |4\rangle\right]$$

$$U|1\rangle = |2\rangle$$

No jump evolution: $\theta = 2\delta = 2\kappa\,\delta t$

$$\frac{|0\rangle + |4\rangle}{\sqrt{2}} \to \cos\theta\,\frac{|0\rangle + |4\rangle}{\sqrt{2}} + \sin\theta\,\frac{|0\rangle - |4\rangle}{\sqrt{2}}; \quad |2\rangle \to |2\rangle$$

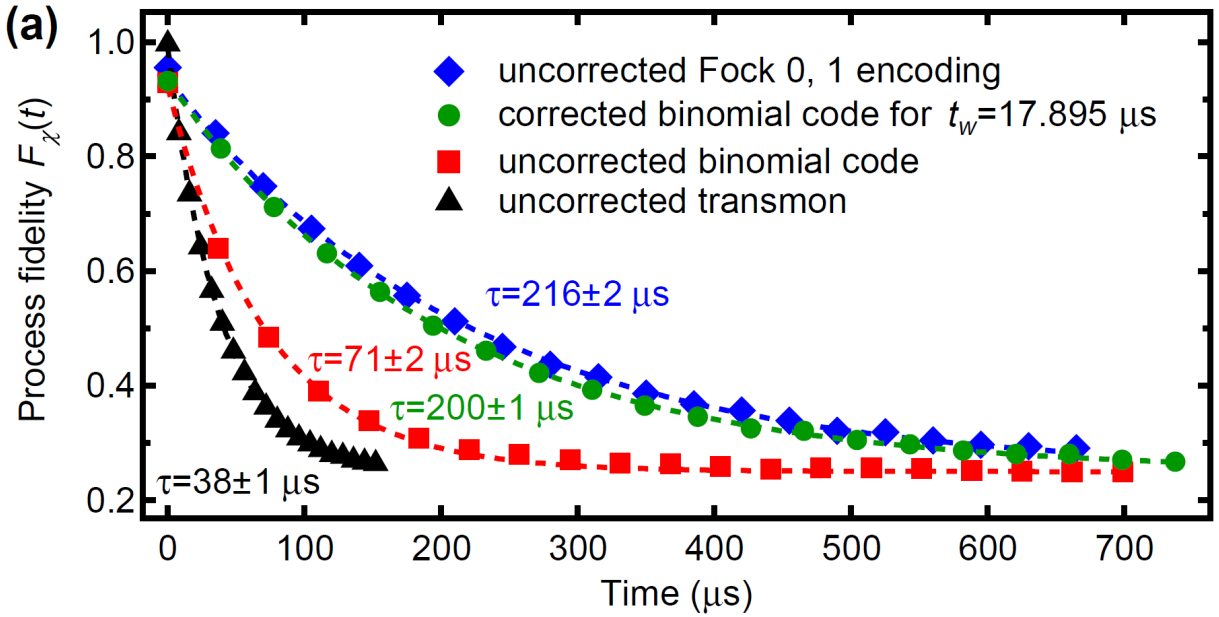
Simplest bosonic code example: 'binomial code' uses only 5 photon states 0-4 ($\ln_2 5$ bits)
Phys. Rev. X 6, 031006 (2016) to correct errors to first order in $\kappa\delta t$.

Logical code words	error words
(even parity)	(odd parity)
$\lvert 0_L\rangle = \frac{\lvert 0\rangle + \lvert 4\rangle}{\sqrt{2}}$	$a\lvert 0_L\rangle = \sqrt{2}\lvert 3\rangle$
$\lvert 1_L\rangle = \lvert 2\rangle$	$a\lvert 1_L\rangle = \sqrt{2}\lvert 1\rangle$

Break-even for bosonic codes is defined as beating the best uncorrectable bosonic code (0,1) photon Fock encoding:

$$\lvert\psi\rangle = \alpha\lvert 0\rangle + \beta\lvert 1\rangle$$

Reaches 92% of break even
Luyan Sun group (Tsinghua)
Nature Phys. **15**, 503 (2019)



Simplest bosonic code example: 'binomial code' uses only 5 photon states 0-4 ($\ln_2$ 5 bits)
Phys. Rev. X 6, 031006 (2016) to correct errors to first order in $\grave{O} = \kappa\,\delta t$.

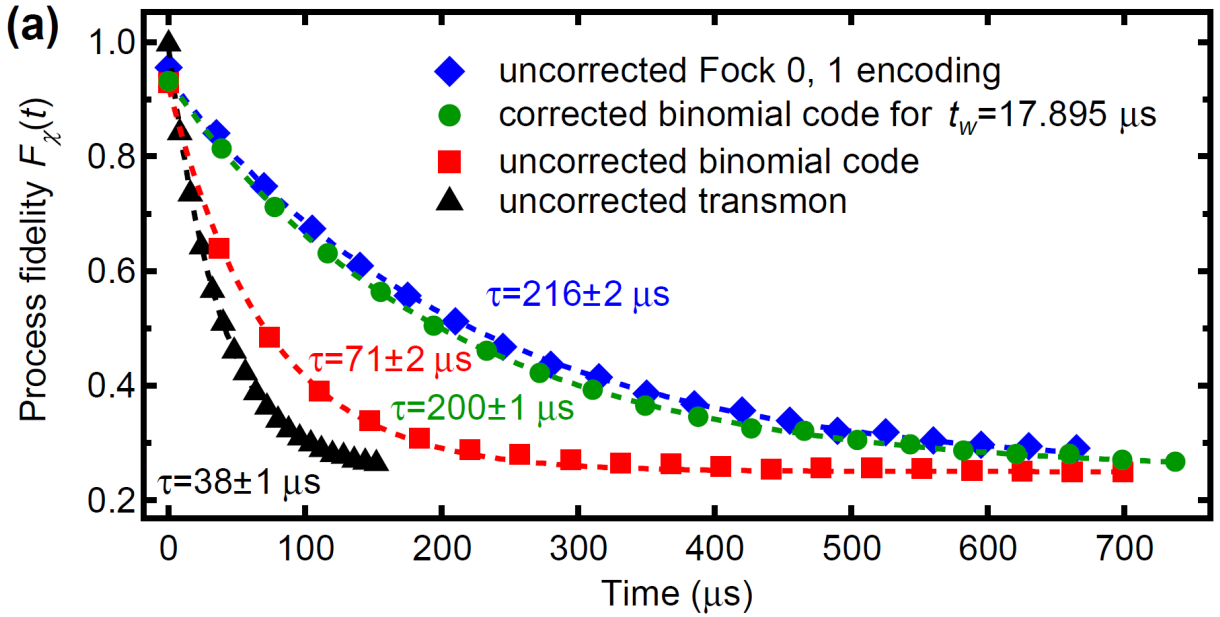
Logical code words	error words
(even parity)	(odd parity)
$\lvert 0_L \rangle = \frac{\lvert 0\rangle + \lvert 4\rangle}{\sqrt{2}}$	$a\lvert 0_L\rangle = \sqrt{2}\lvert 3\rangle$
$\lvert 1_L \rangle = \lvert 2\rangle$	$a\lvert 1_L\rangle = \sqrt{2}\lvert 1\rangle$

$\langle a^\dagger a\rangle = 2$ Loss is 4x larger!

Break-even for bosonic codes is defined as beating the best uncorrectable bosonic code (0,1) photon Fock encoding:

$\lvert\psi\rangle = \alpha\lvert 0\rangle + \beta\lvert 1\rangle$ $\langle a^\dagger a\rangle = 1/2$

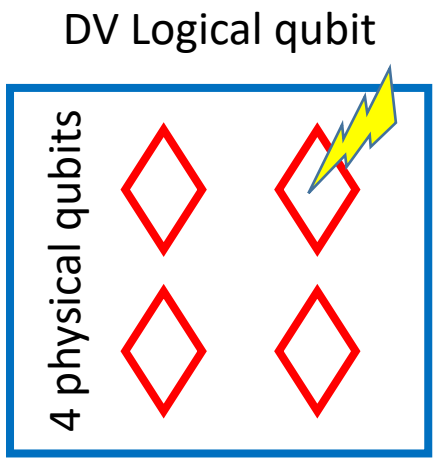
Reaches 92% of break even
Luyan Sun group (Tsinghua)
Nature Phys. **15**, 503 (2019)

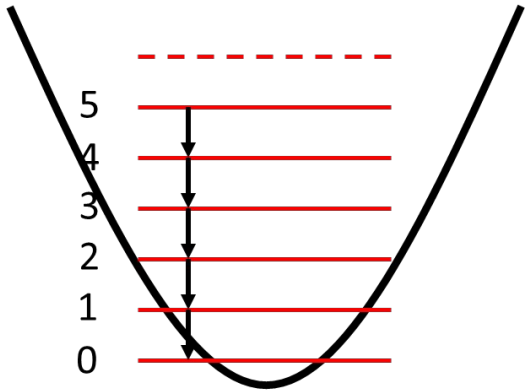


Head-to-head comparison of simplest codes that correct amplitude damping to first order:

DV: 4-qubit amplitude damping code:
(first to recognize approx. QEC: Knill-Laflamme does not have to be fulfilled exactly)
Debbie Leung et al., *Phys. Rev. A* **56**, 2567 (1997).

CV: Binomial bosonic code:
M. Michael et al., Phys. Rev. X **6**, 031006 (2016)





DV: 4-qubit amplitude damping code

QND stabilizer measurements are challenging

$$|0_L\rangle = \frac{1}{\sqrt{2}}\left[|1100\rangle + |0011\rangle\right]$$

$$|1_L\rangle = \frac{1}{\sqrt{2}}\left[|0000\rangle + |1111\rangle\right]$$

$$S_1 = Z_1 Z_2$$

$$S_2 = Z_3 Z_4$$

$$S_3 = X_1 X_2 X_3 X_4$$

Error model Kraus ops.

$$\hat{K}_0 = \sqrt{\hat{I} - \gamma\, dt\, \hat{n}} \quad \text{where } \hat{n} = \sum_{j=1}^{4} \sigma_j^+ \sigma_j^-$$

$$\hat{K}_1 = \sqrt{\gamma\, dt}\, \sigma_1^-$$

$$\hat{K}_2 = \sqrt{\gamma\, dt}\, \sigma_2^-$$

$$\hat{K}_3 = \sqrt{\gamma\, dt}\, \sigma_3^-$$

$$\hat{K}_4 = \sqrt{\gamma\, dt}\, \sigma_4^-$$

number of excited qubits

$$|0_L\rangle = |2\rangle$$

$$|1_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$$

$$S_1 = (-1)^{\hat{n}}$$

CV: binomial code

Parity stabilizer measurements 99.8% QND. L. Sun et al., *Nature* **511**, 444 (2014)

Error model Kraus ops.

$$\hat{K}_0 = \sqrt{\hat{I} - \kappa\, dt\, \hat{n}} \quad \text{where } \hat{n} = a^\dagger a$$

$$\hat{K}_1 = \sqrt{\kappa\, dt}\; a$$

number of photons

Comparison of amplitude damping codes: 4-qubit vs. bosonic (binomial)

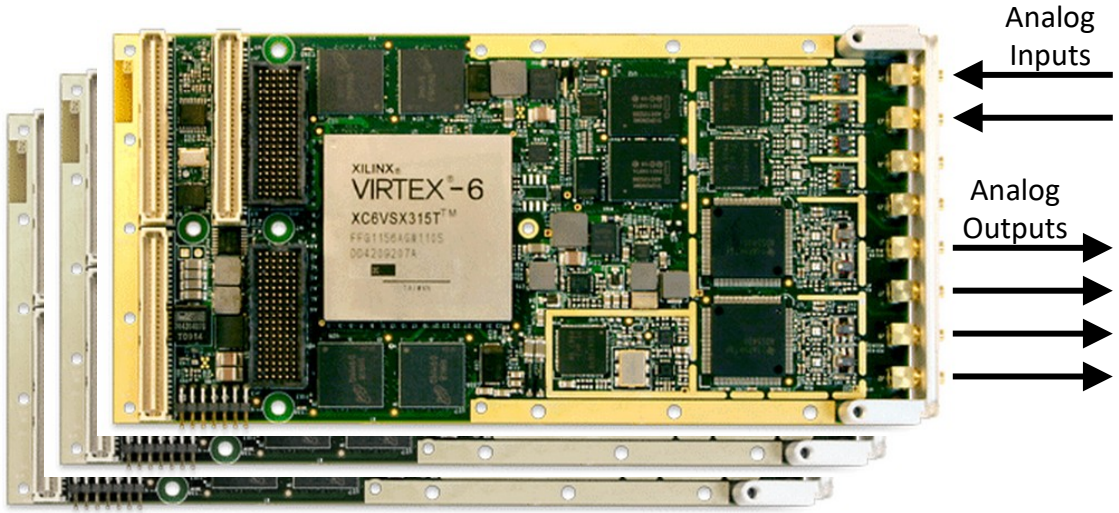
	4-qubit Code	Bosonic Kitten Code
Code word $\|1_L\rangle$	$\frac{1}{\sqrt{2}}(\|0000\rangle + \|1111\rangle)$	$\frac{1}{\sqrt{2}}(\|0\rangle + \|4\rangle)$
Code word $\|0_L\rangle$	$\frac{1}{\sqrt{2}}(\|1100\rangle + \|0011\rangle)$	$\|2\rangle$
Mean excitation number $\bar{n}$	2	2
Hilbert space dimension D	$2^4 = 16$	$\{0, 1, 2, 3, 4\} = 5$
N_{error}	$\{\hat{K}_0, \sigma_1^-, \sigma_2^-, \sigma_3^-, \sigma_4^-\} = 5$	$\{\hat{K}_0, a\} = 2$
Stabilizers	$S_1 = Z_1 Z_2,\ S_2 = Z_3 Z_4,\ S_3 = X_1 X_2 X_3 X_4$	$\hat{\Pi} = (-1)^{a^\dagger a}$
Number of Stabilizers N_{Stab}	3	1
Approximate QEC?	Yes, 1st order in γt	Yes, 1st order in κt

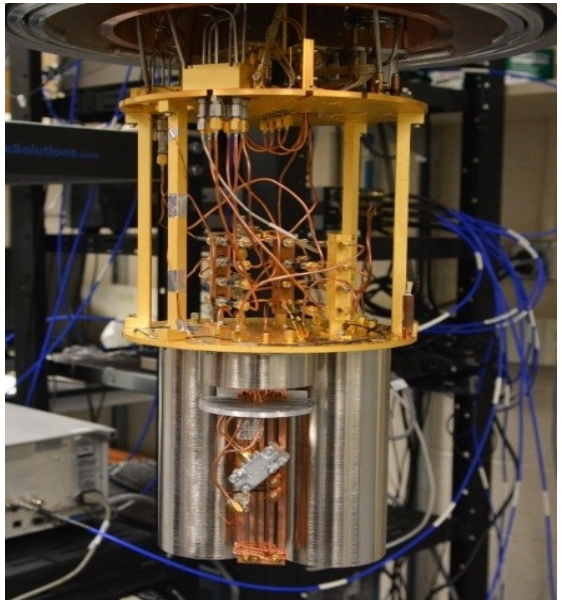
Qubit code has 4 distinct places errors can occur. Oscillator only 1.
Qubit code stabilizer measurements require multiple CNOT operations.
Boson parity is relatively easy and QND.

This is why, to date, only bosonic modes have reached break even!

‘cat code’ parity measurement and rapid feedback error correction engine

MAXWELL’S DEMON



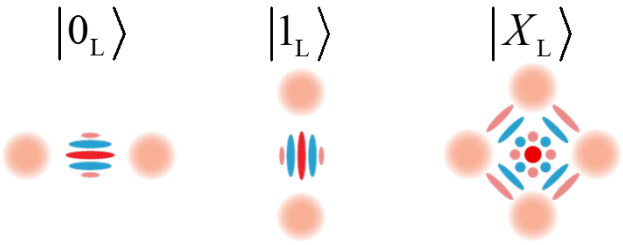


Ofek, et al., *Nature* **536**, 441–445 (2016)

‘Cat code’

$$|W_0\rangle = |0_L\rangle = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2}}$$

$$|W_1\rangle = |1_L\rangle = \frac{|i\alpha\rangle + |-i\alpha\rangle}{\sqrt{2}}$$



Store a ***qubit*** as a ***superposition*** of two cats of same ***parity***

Photon loss flips parity

Nice feature of this code: no-jump evolution need not be corrected on the fly.

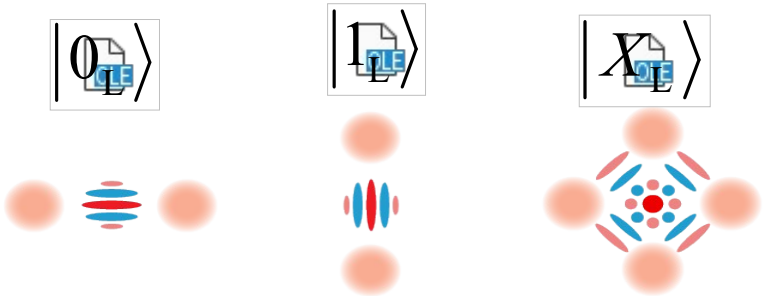
First code to (slightly) exceed break even: Schrödinger Cat Code

$$|\Psi\rangle = \psi_0 |0_L\rangle + \psi_1 |1_L\rangle$$

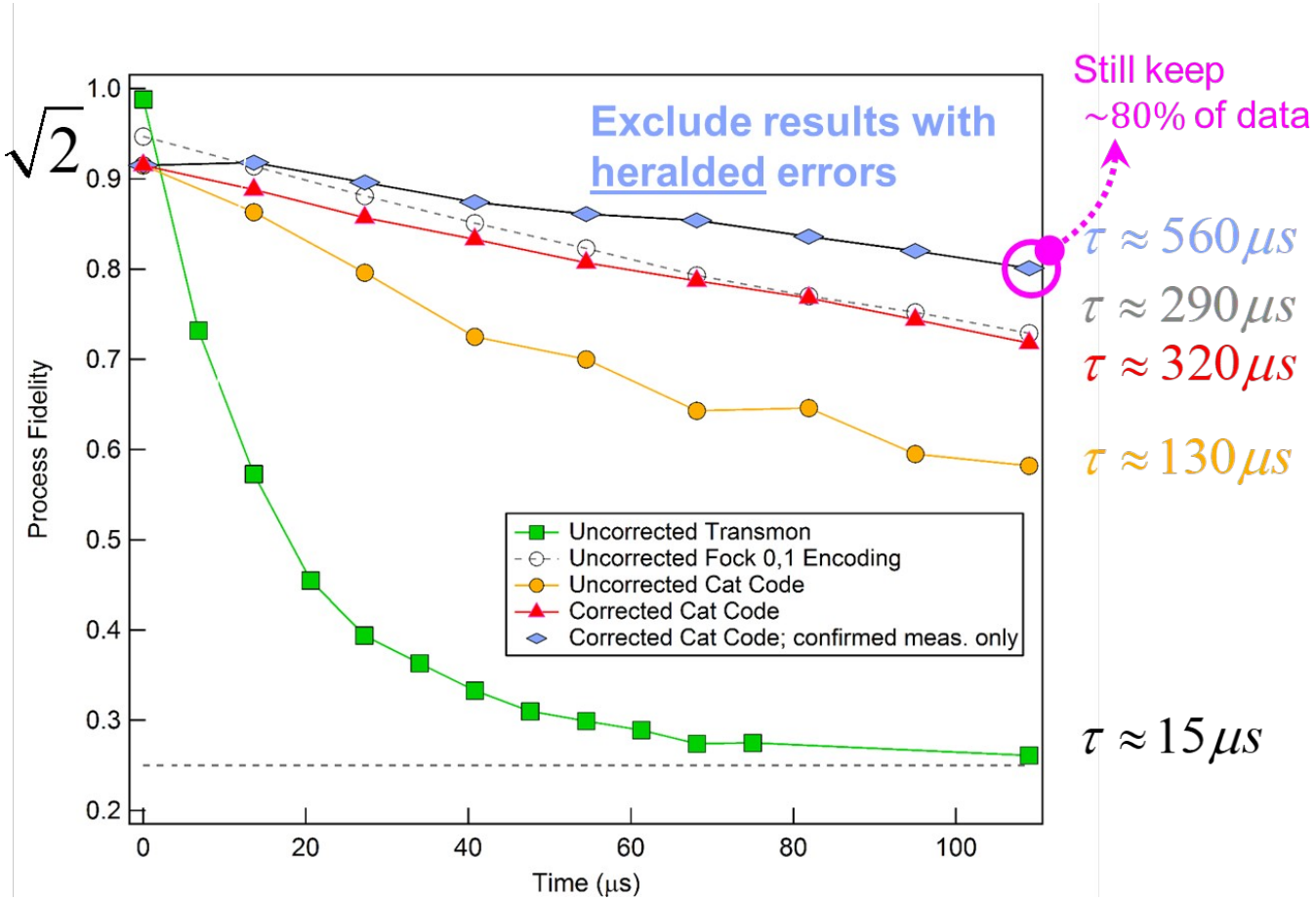
$$|0_L\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|1_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

$$\alpha = \sqrt{2}$$



Store a ***qubit*** as a ***superposition*** of two cats of same ***parity***



QEC Gain G:
1.1x break even (unheralded)
1.75x break even (heralded)

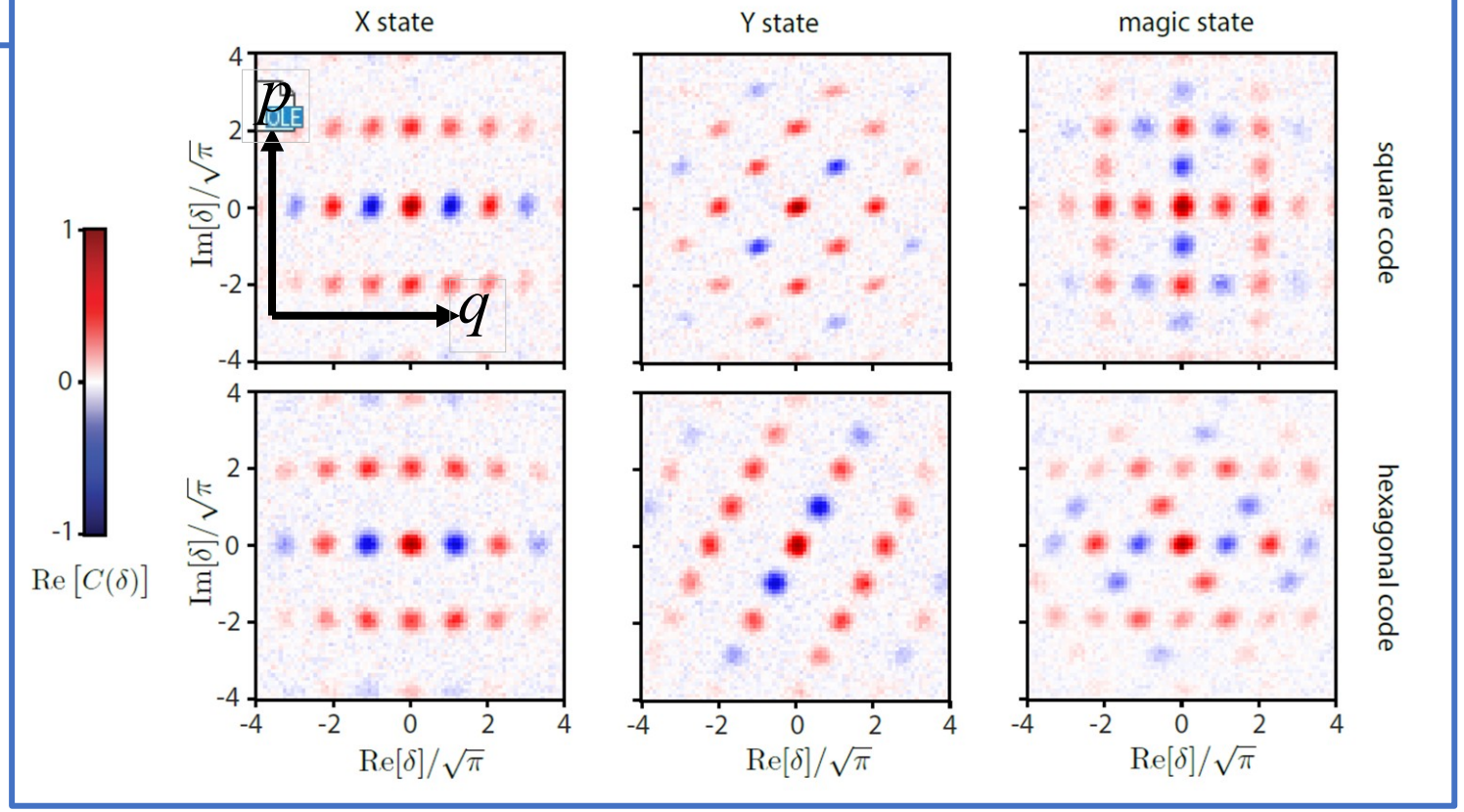
Theory: Leghtas, Mirrahimi, et al., *PRL* **111**, 120501 (2013)
Experiment: Ofek et al. *Nature* **536**, 441 (2016)

'GKP code': Coherent state lattice in phase space ("cat in 35 places at once")

C. Flühmann et al. (Home group) *Nature* **566**, 513 (2019) (state preparation)
P. Campagne-Ibarcq et al. (Devoret group) *Nature* **584**, 368 (2020) (QEC for X,Y,Z errors near break even)
de Neeve et al. (Home group) de Neeve et al. (J. Home group), *Nature Physics* **18**, 296 (2022)
Royer, Singh et al. (Girvin group) *Phys. Rev. Lett.* **125**, 260509 (2020)

Phase space map of oscillator states using Characteristic Function = FT of Wigner function

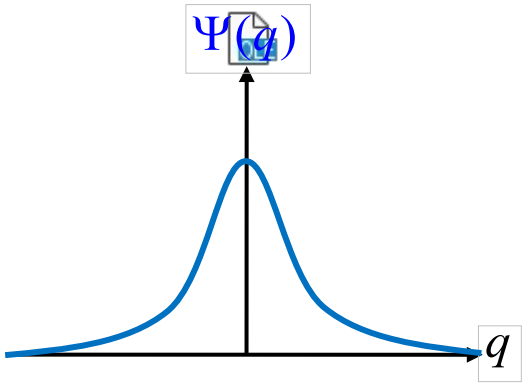
Stabilizers, errors, Clifford gates are all simple displacements!

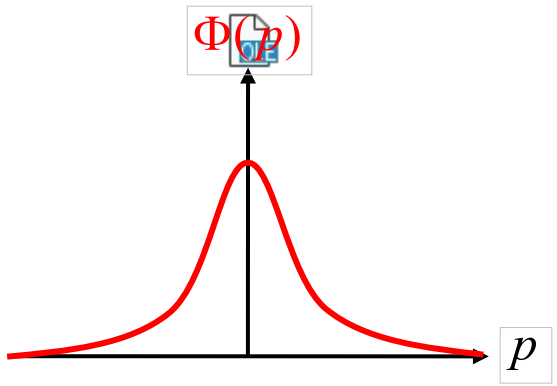


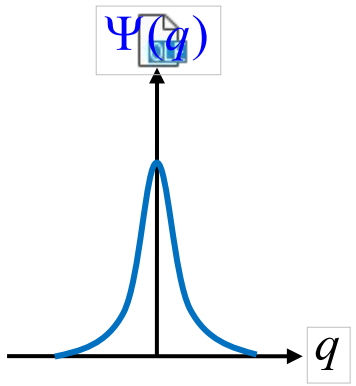
Coherent (vacuum) state of an oscillator

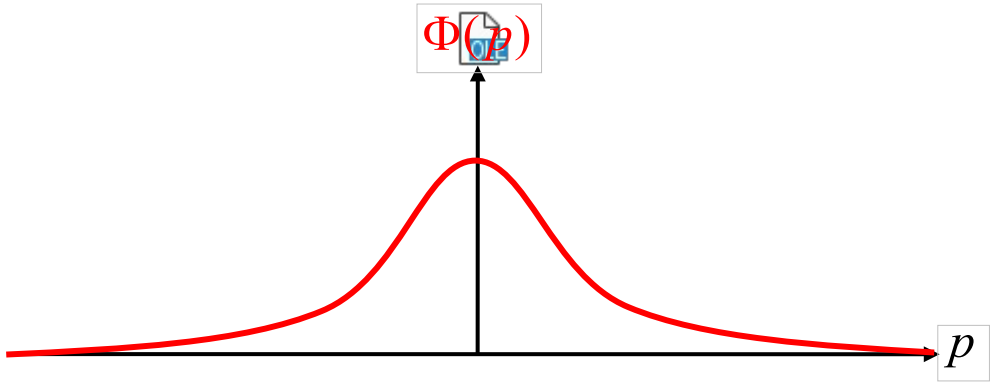
Squeezed state of an oscillator

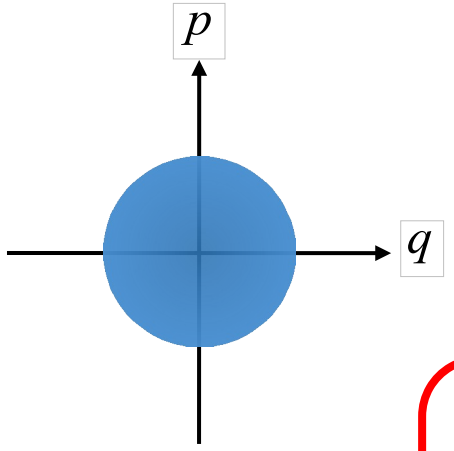
Understanding phase space….





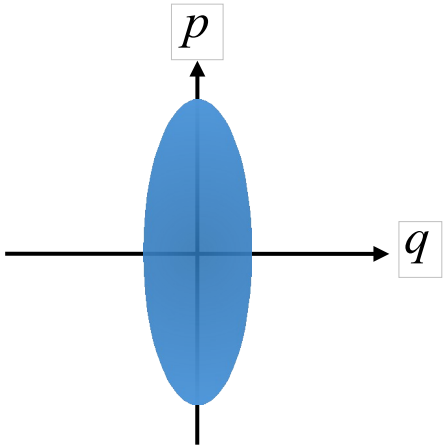






Heisenberg Uncertainty

$$[\hat{q},\hat{p}] = i \Rightarrow \Delta q \Delta p \geq \frac{1}{2}$$

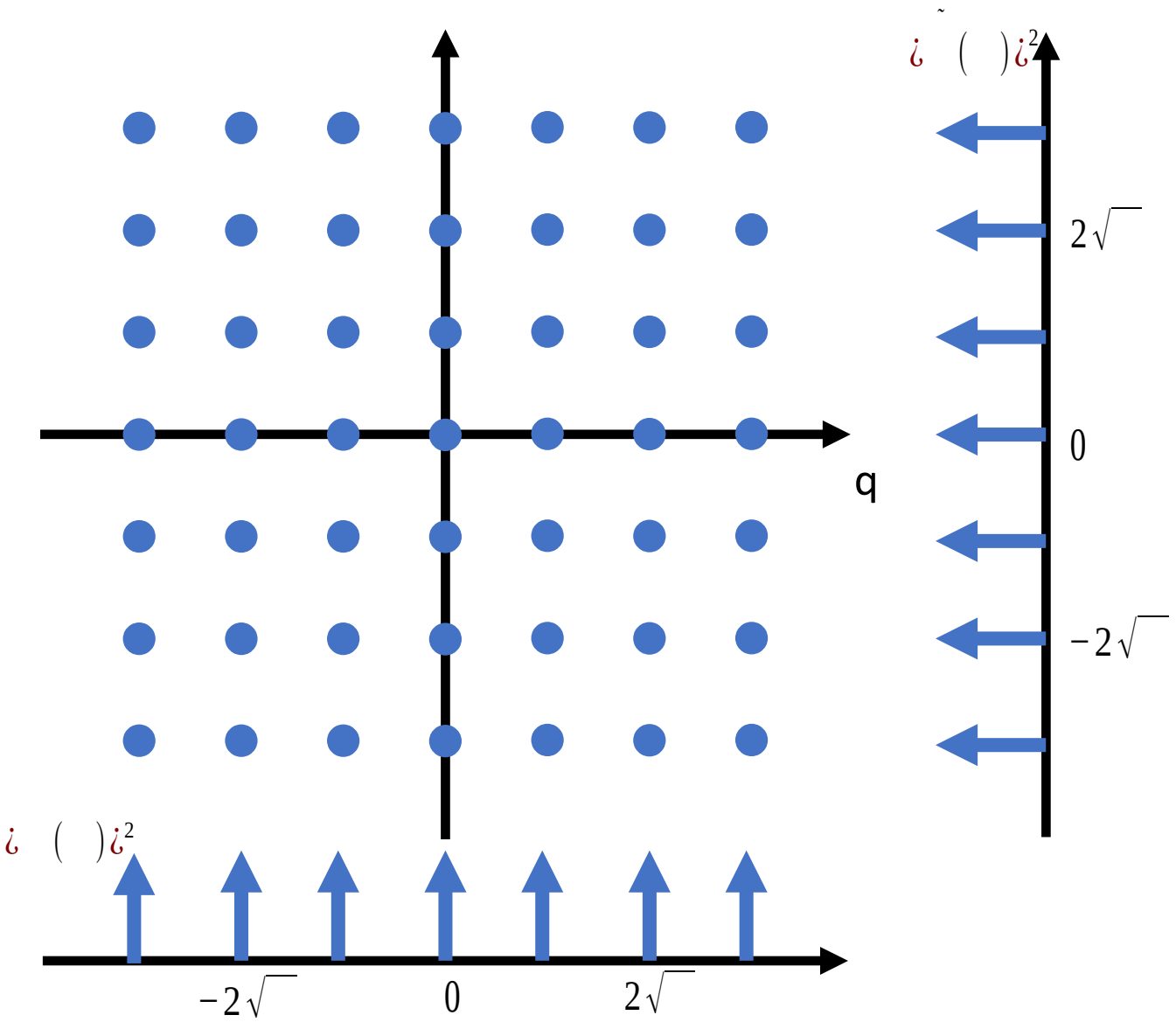


Phase space seems to be 'incompressible'

One state per area $h = 2\pi$

Note: squeezing can be achieved by simply measuring the position of the oscillator with uncertainty less than the zero-point motion.

But recall that a crystal lattice produces sharp Bragg peaks in x-ray diffraction.



Gottesman, Kitaev and Preskill, Phys. Rev. A 64, 012310 (2001)

Proposed encoding a logical qubit in oscillator ‘grid’ states.

How can the points in this phase space grid be smaller than the minimum uncertainty wave packet?

They seem to be squeezed in both position AND momentum!?

This is possible for special choices of lattice unit cell areas.

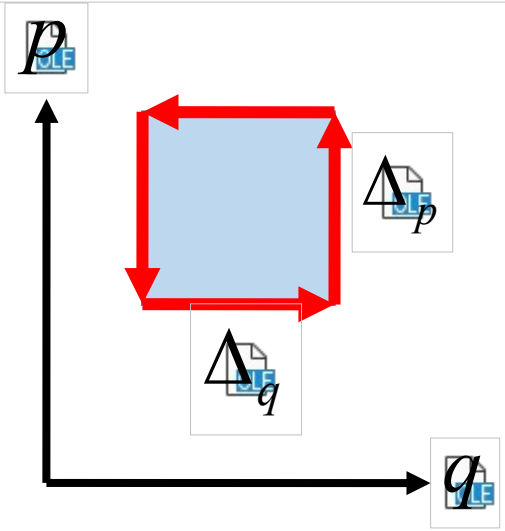
$[\hat{q},\hat{p}] = +i \quad \Rightarrow$ translations in phase space do not commute

$$\mathrm{D}(\Delta_q)\Psi(q) = e^{-i\Delta_q \hat{p}}\Psi(q) = \Psi(q-\Delta_q)$$

$$\mathrm{D}(i\Delta_p)\Psi(q) = e^{i\Delta_p q}\Psi(q)$$

$$\mathrm{D}(\Delta_q)\mathrm{D}(i\Delta_p) = e^{i\Delta_p\Delta_q}\mathrm{D}(i\Delta_p)\mathrm{D}(\Delta_q)$$

area



Harmonic Oscillator Phase Space

Inside the code space: *X,Y,Z* translations obey Pauli group

GKP code space is stabilized by special translations that do commute

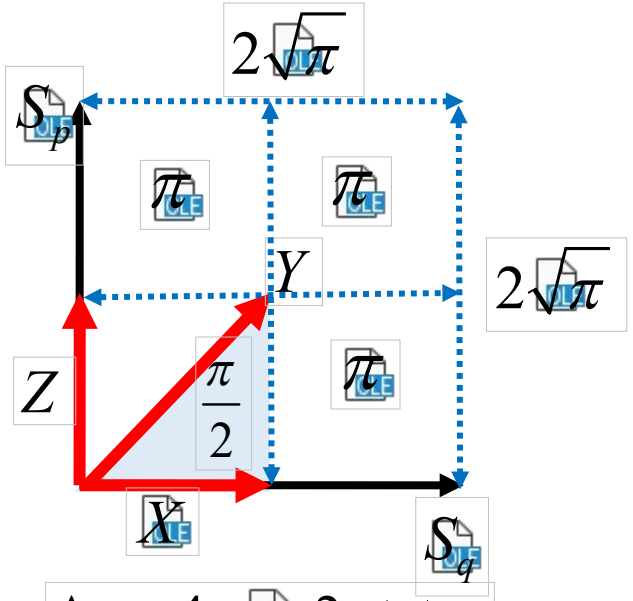
$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$

$$S_q = e^{i2\sqrt{\pi}\hat{p}}$$

$$S_q S_p = e^{i4\pi} S_p S_q$$

$$S|0_L\rangle = (+1)|0_L\rangle$$

$$S|1_L\rangle = (+1)|1_L\rangle$$



Area 4π = 2 states

$$S_p = S_q = 1$$

$$X^2 = S_q = 1$$

$$Z^2 = S_p = 1$$

$$ZX = e^{i\pi}XZ = -XZ$$

$$ZX = e^{i\pi/2}Y = iY$$

Exploring phase space with displacements of the oscillator controlled by the ancilla qubit

Qubit state

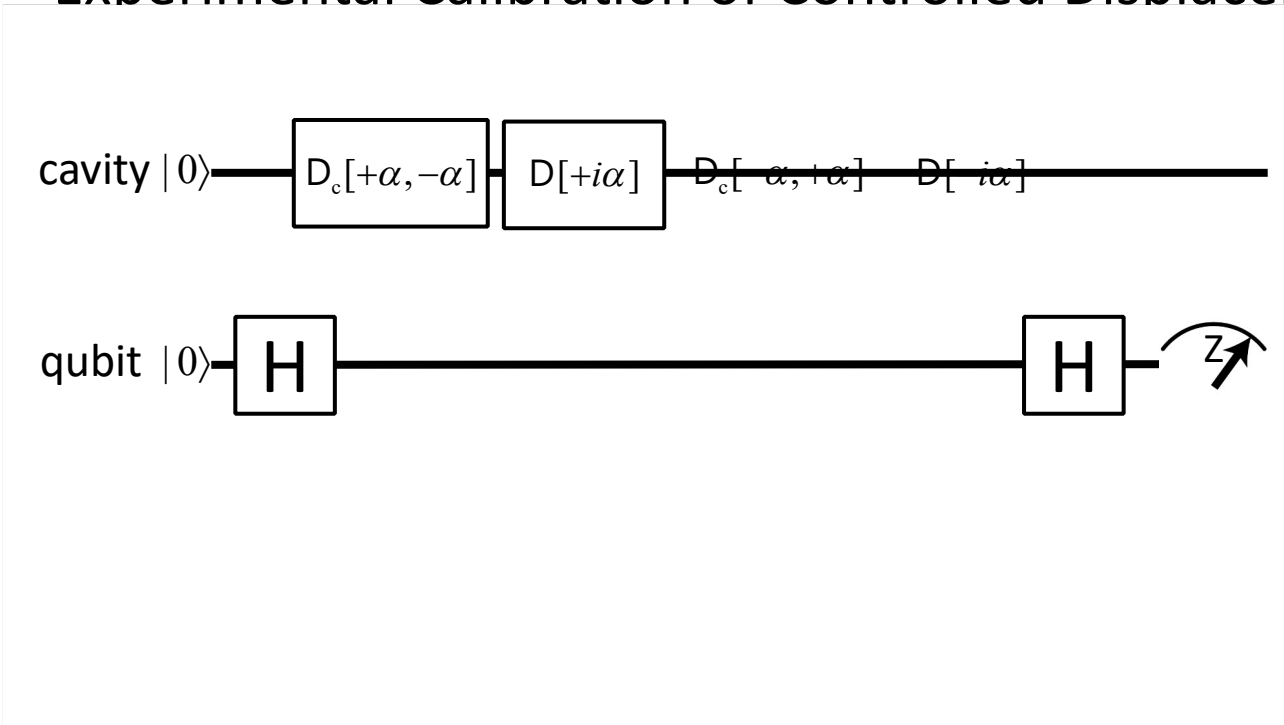
Cavity displacement

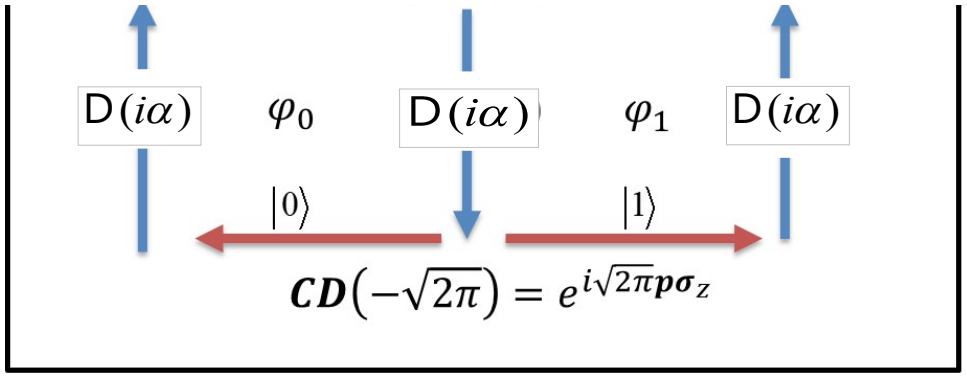
Key ancilla-controlled cavity operation:

Controlled Displacement gate: $\mathrm{D_c}[\alpha] = e^{Z[\alpha a^\dagger - \alpha^* a]} = |0\rangle\langle 0|\mathrm{D}[+\alpha] + |1\rangle\langle 1|\mathrm{D}[-\alpha]$

$Z = \sigma^z$ for ancilla qubit

Experimental Calibration of Controlled Displacements Non-Commutativity (Devoret Group)

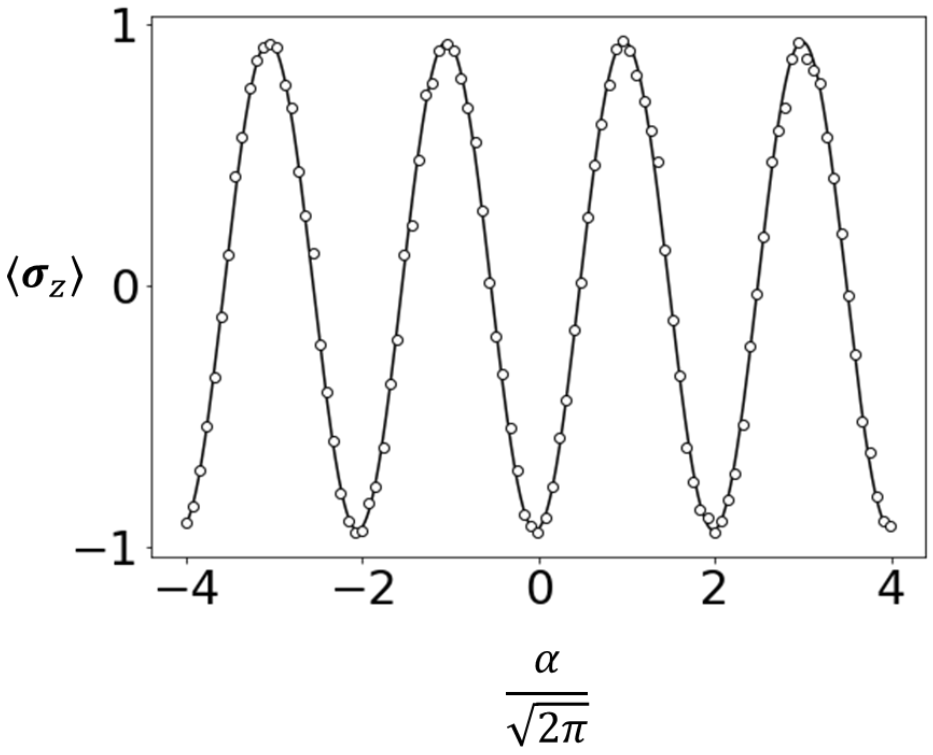




geometric phase acquired over 1 cycle:

$$|0\rangle + |1\rangle \rightarrow e^{-i\varphi_0}|0\rangle + e^{i\varphi_1}|1\rangle$$

EXP. DATA: BERRY PHASE OSC.



Campagne-Ibarcq, Eickbusch, Touzard, et al *Nature* **584**, 368 (2020)

Bosonic QEC with (idealized) GKP states of an oscillator

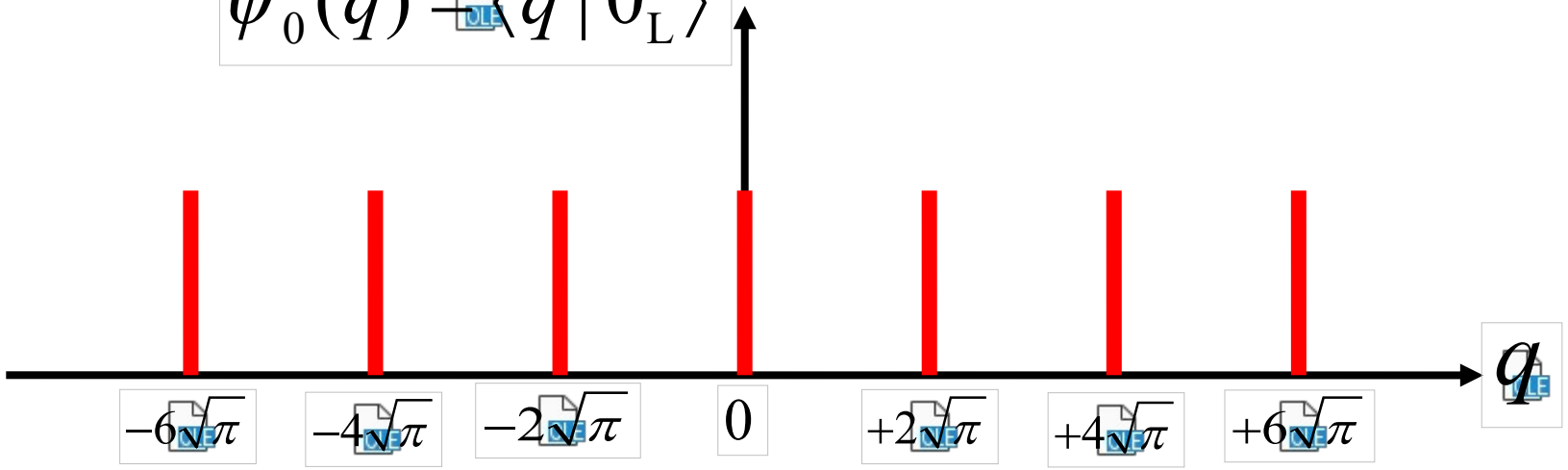
Stabilizers define code space:

$$S_p\,|0_L\rangle = S_q\,|0_L\rangle = (+1)|0_L\rangle$$
$$S_p\,|1_L\rangle = S_q\,|1_L\rangle = (+1)|1_L\rangle$$

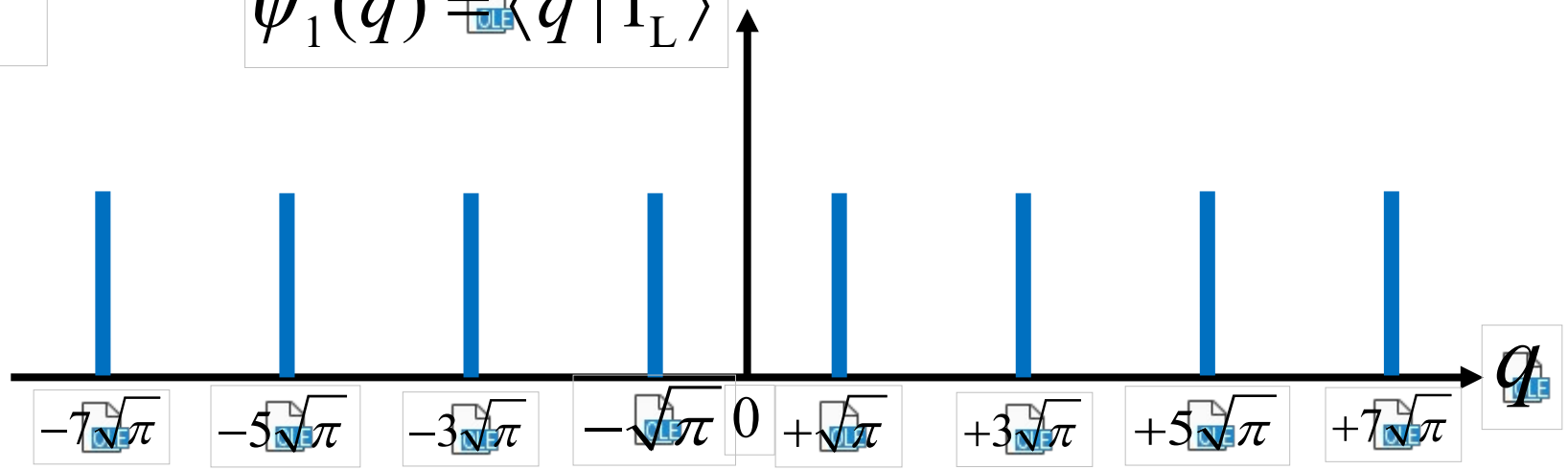
$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$
$$S_q = e^{i2\sqrt{\pi}\hat{p}}$$

Idealized GKP wave functions

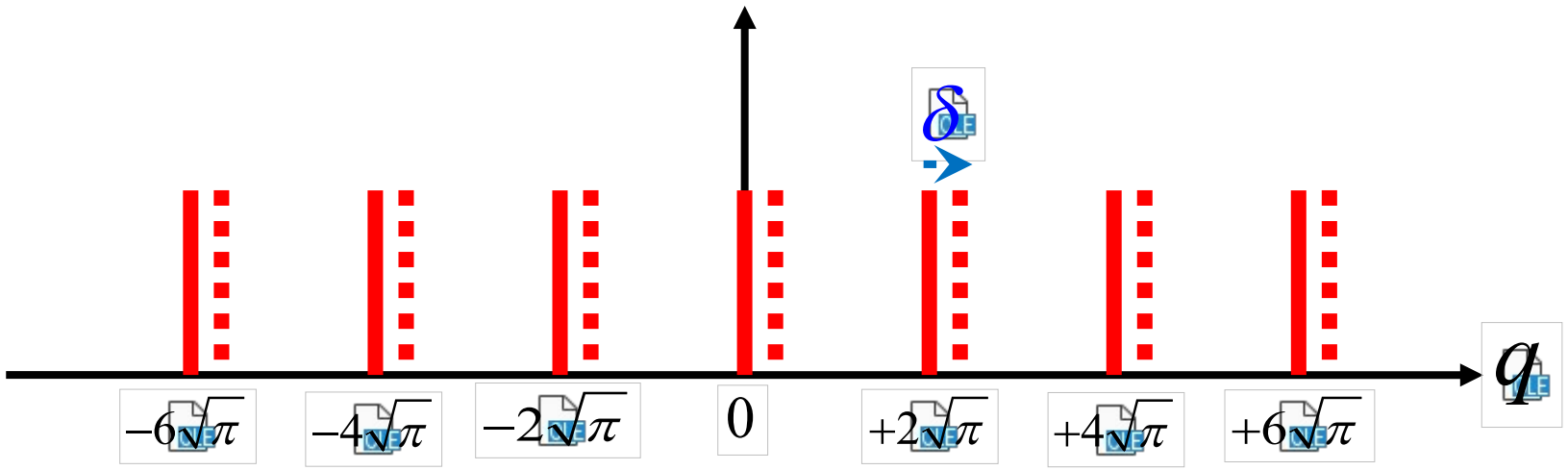
$$\psi_0(q) = \langle q\,|\,0_L\rangle$$



$$\psi_1(q) = \langle q\,|\,1_L\rangle$$



Bosonic QEC with GKP states of an oscillator



Code space is stabilized by:

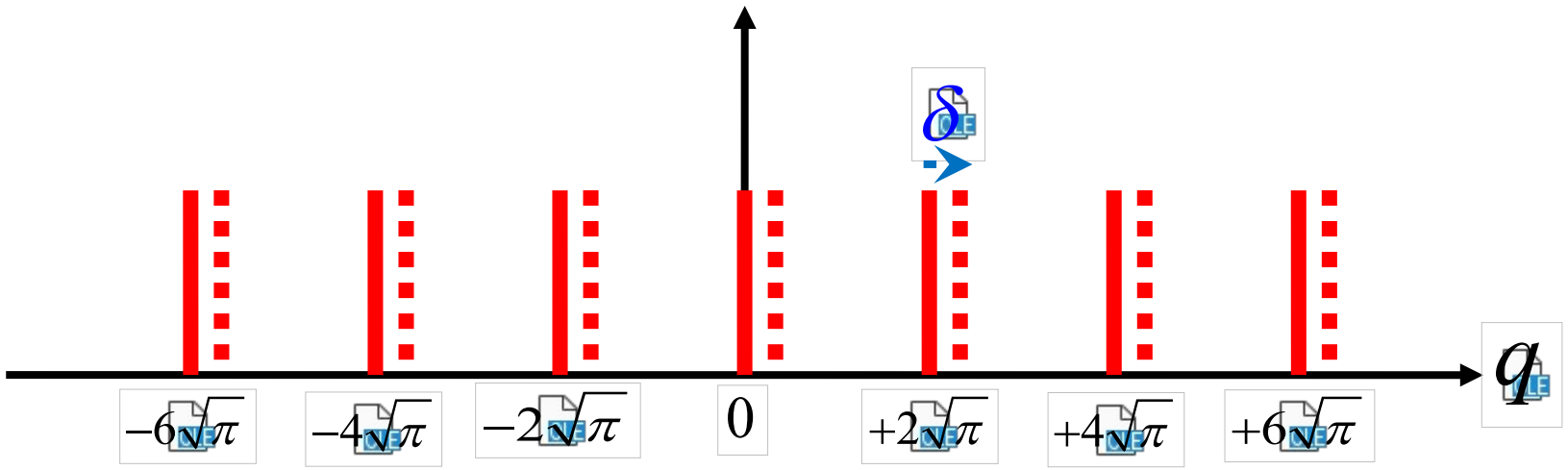
$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$

$$S_q = e^{i2\sqrt{\pi}\hat{p}}$$

$$S_p \left| \Psi_\delta \right\rangle = e^{i2\sqrt{\pi}\delta} \left| \Psi_\delta \right\rangle$$

Continuous stabilizer eigenvalue on the unit circle in the complex plane.

N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum.



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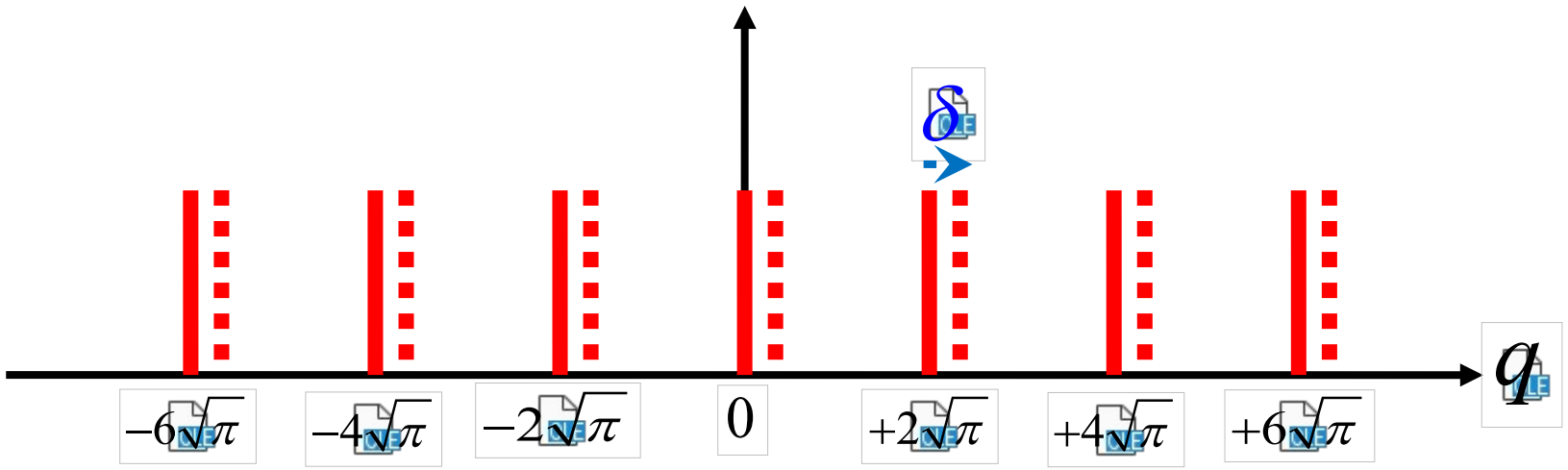
$$S_p \left| \Psi_\delta \right\rangle = e^{i2\sqrt{\pi}\delta} \left| \Psi_\delta \right\rangle$$

Continuous stabilizer eigenvalue on the unit circle in the complex plane.

ONLY 2 STABILIZERS NEEDED TO REDUCE INFINITE STATE SPACE DOWN TO 2 LOGICAL STATES!

Bosonic QEC with GKP states of an oscillator



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N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum.

Stabilization against drift errors in *position q*

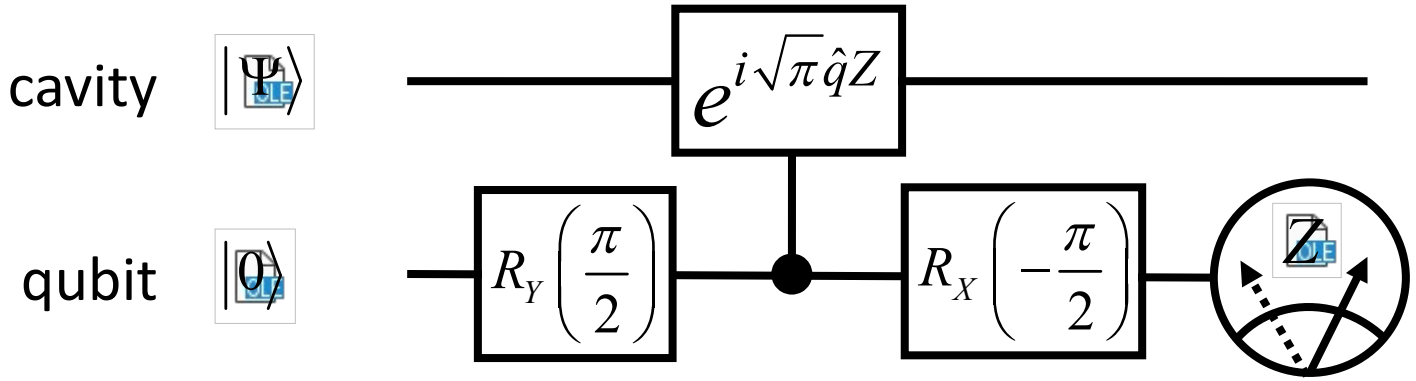
Measure stabilizer to detect error:

$$\mathrm{Im}\langle S_p \rangle = \langle \sin[2\sqrt{\pi}\hat{q}] \rangle$$

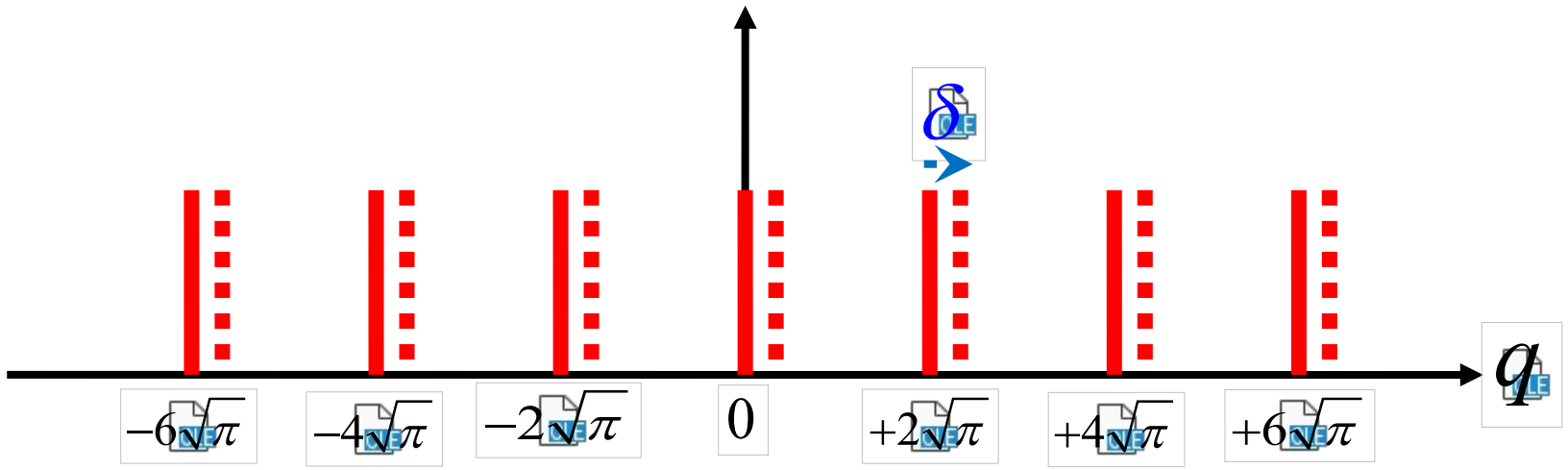
$$= \int dq \sin[2\sqrt{\pi q}]\,|\psi(q)|^2 = \sin[2\sqrt{\pi}\delta]$$

and feedback to correct.

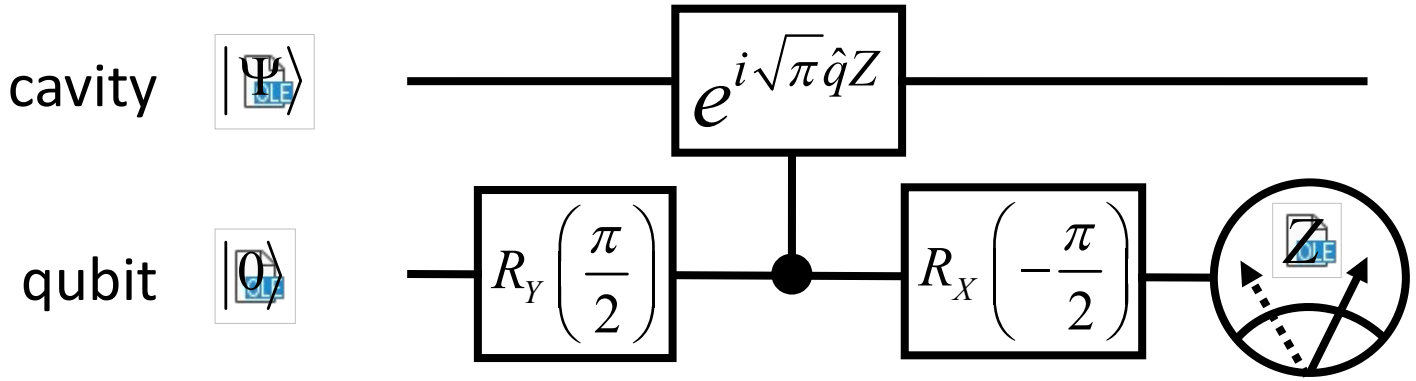
Measuring stabilizer using phase kickback from conditional displacement operation



$$\langle Z\rangle = \mathrm{Im}\langle S_p\rangle = \langle \sin 2\sqrt{\pi}\hat{q}\rangle = \langle \sin 2\sqrt{\pi}\delta\rangle$$



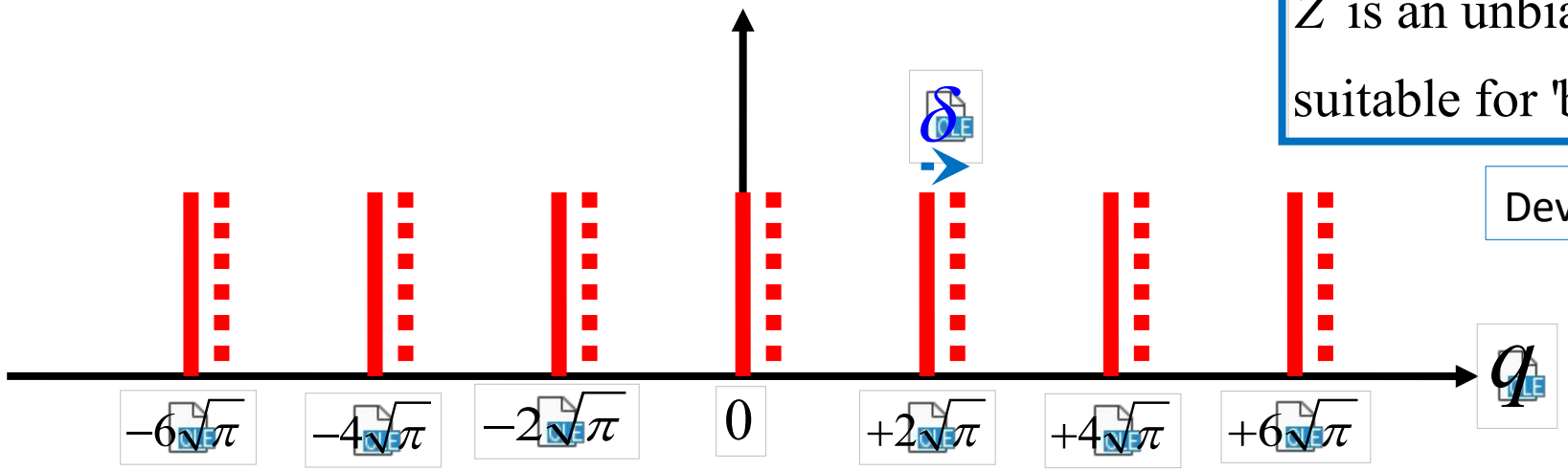
Measuring stabilizer using phase kickback from conditional displacement operation



$$\langle Z\rangle = \mathrm{Im}\langle S_p\rangle = \langle \sin 2\sqrt{\pi}\hat{q}\rangle = \langle \sin 2\sqrt{\pi}\delta\rangle$$

Z is an unbiased single-bit estimator of $\mathrm{Im}\, S_p$ suitable for 'bang-bang' feedback stabilization

Devoret group: *Nature* **584**, 368 (2020)



Quantum state tomography

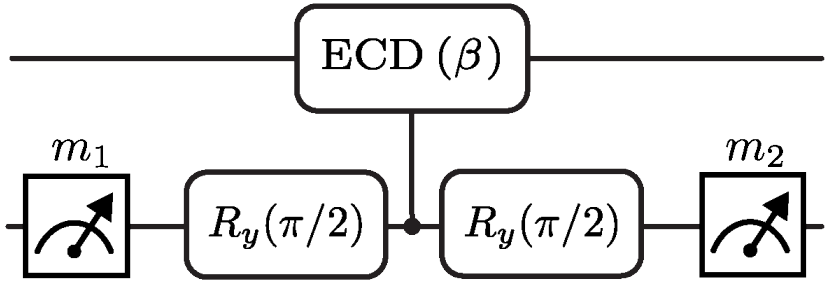
Characteristic function: $C(k) = \mathrm{Tr}[\rho\, W_k], \quad W_k \in \{I, X, Y, Z\}^{\otimes n}$

A. Eickbusch, **V. Sivak** *et al.,* arXiv:2111.06414

courtesy V. Sivak

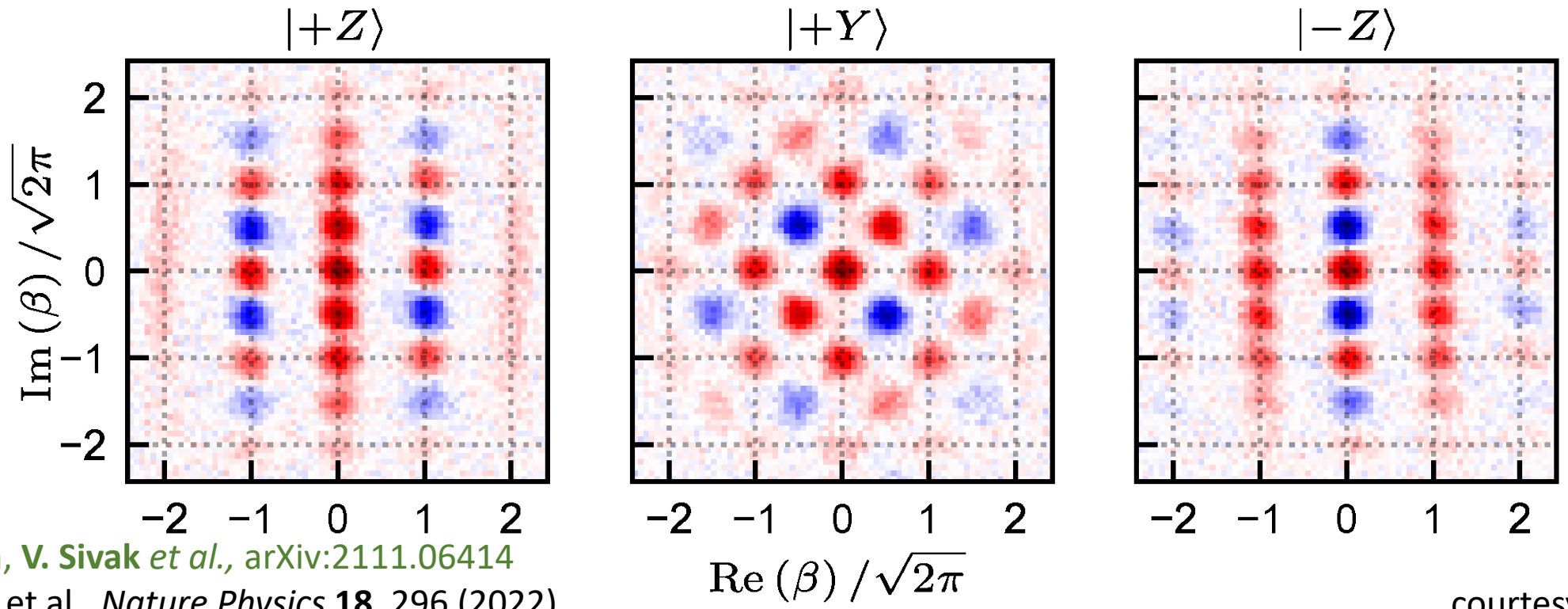
Quantum state tomography

Characteristic function: $C(\beta) = \mathrm{Tr}[\rho\, D(\beta)], \quad D(\beta) = \exp(\beta a^\dagger - \beta^* a)$



Fidelity

$\mathcal{F} = 0.8 - 0.85$



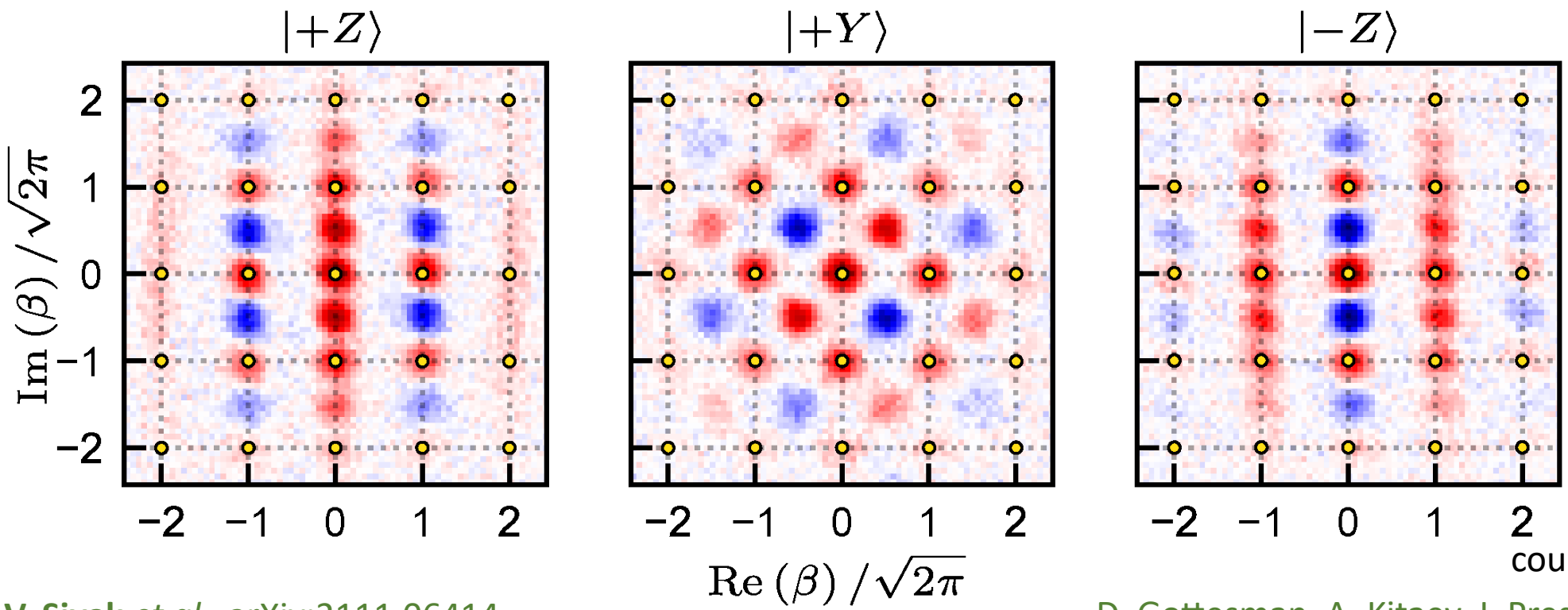
A. Eickbusch, **V. Sivak** *et al.*, arXiv:2111.06414
B. de Neeve et al., *Nature Physics* **18**, 296 (2022)

courtesy V. Sivak

Gottesman-Kitaev-Preskill code

Characteristic function: $C(\beta) = \mathrm{Tr}[\rho\, D(\beta)], \quad D(\beta) = \exp(\beta a^\dagger - \beta^* a)$

GKP code stabilizers: $S_X = D(\sqrt{2\pi}), \quad S_Z = D(\sqrt{2\pi} i)$



courtesy V. Sivak

A. Eickbusch, **V. Sivak** *et al.*, arXiv:2111.06414

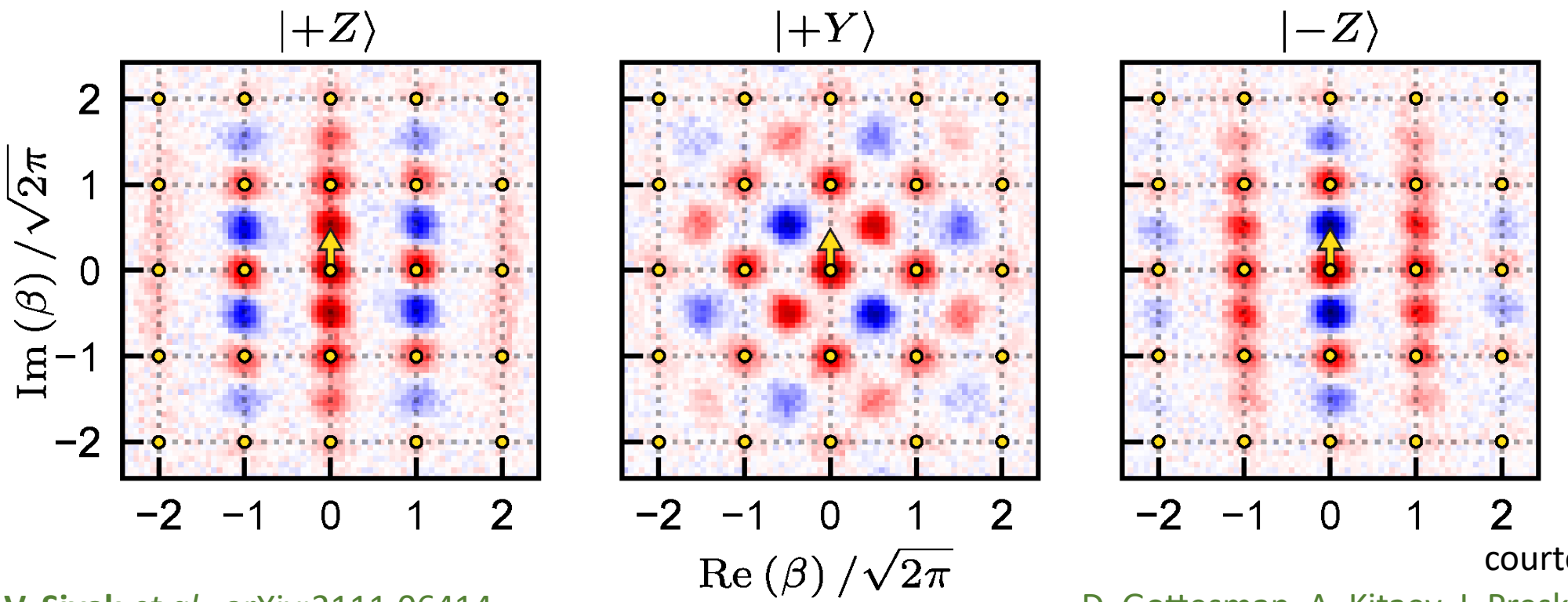
D. Gottesman, A. Kitaev, J. Preskill (PRA, 2001)

Gottesman-Kitaev-Preskill code

Characteristic function: $C(\beta) = \mathrm{Tr}[\rho\, D(\beta)], \quad D(\beta) = \exp(\beta a^\dagger - \beta^* a)$

GKP code stabilizers: $S_X = D(\sqrt{2\pi}), \quad S_Z = D(\sqrt{2\pi} i)$

GKP code Pauli operators: $X = D(\sqrt{\pi/2}), \quad Z = D(\sqrt{\pi/2} i)$



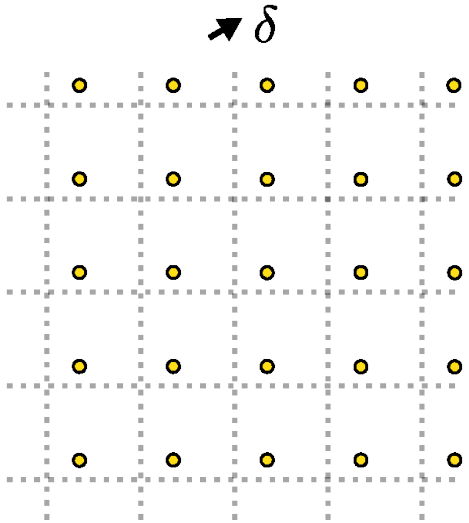
courtesy V. Sivak

A. Eickbusch, **V. Sivak** *et al.*, arXiv:2111.06414

D. Gottesman, A. Kitaev, J. Preskill (PRA, 2001)

Small-Big-Small (SBS) protocol (autonomous and tuned for finite-energy approximate GKP states)

Small displacement error: $|\psi_\delta\rangle = D(\delta)\,|\psi\rangle$ Still a grid state!

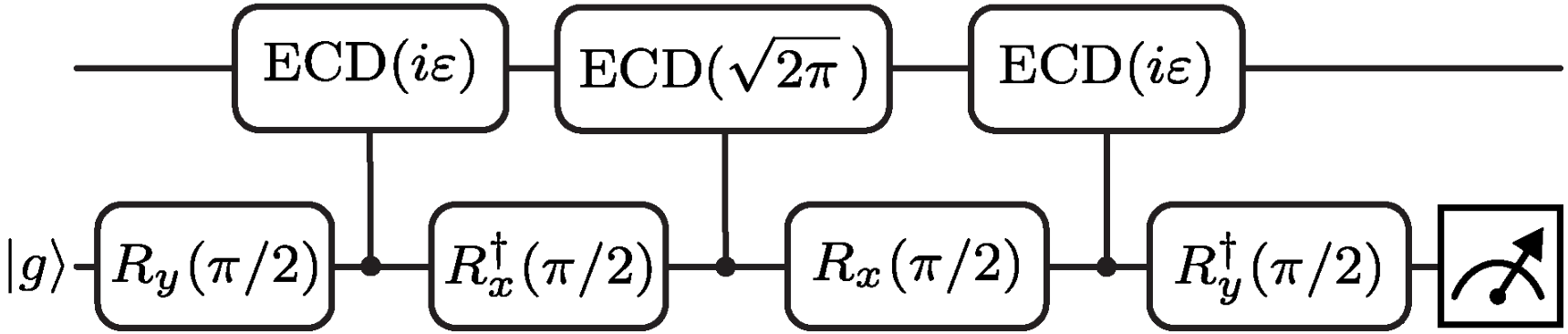


$$S_Z\,|\psi_\delta\rangle = e^{+2i\sqrt{2\pi}\mathrm{Re}[\delta]}|\psi_\delta\rangle$$
$$S_X\,|\psi_\delta\rangle = e^{-2i\sqrt{2\pi}\mathrm{Im}[\delta]}|\psi_\delta\rangle$$

Stabilizer phase estimation

Envelope pre-distortion

Displacement error correction

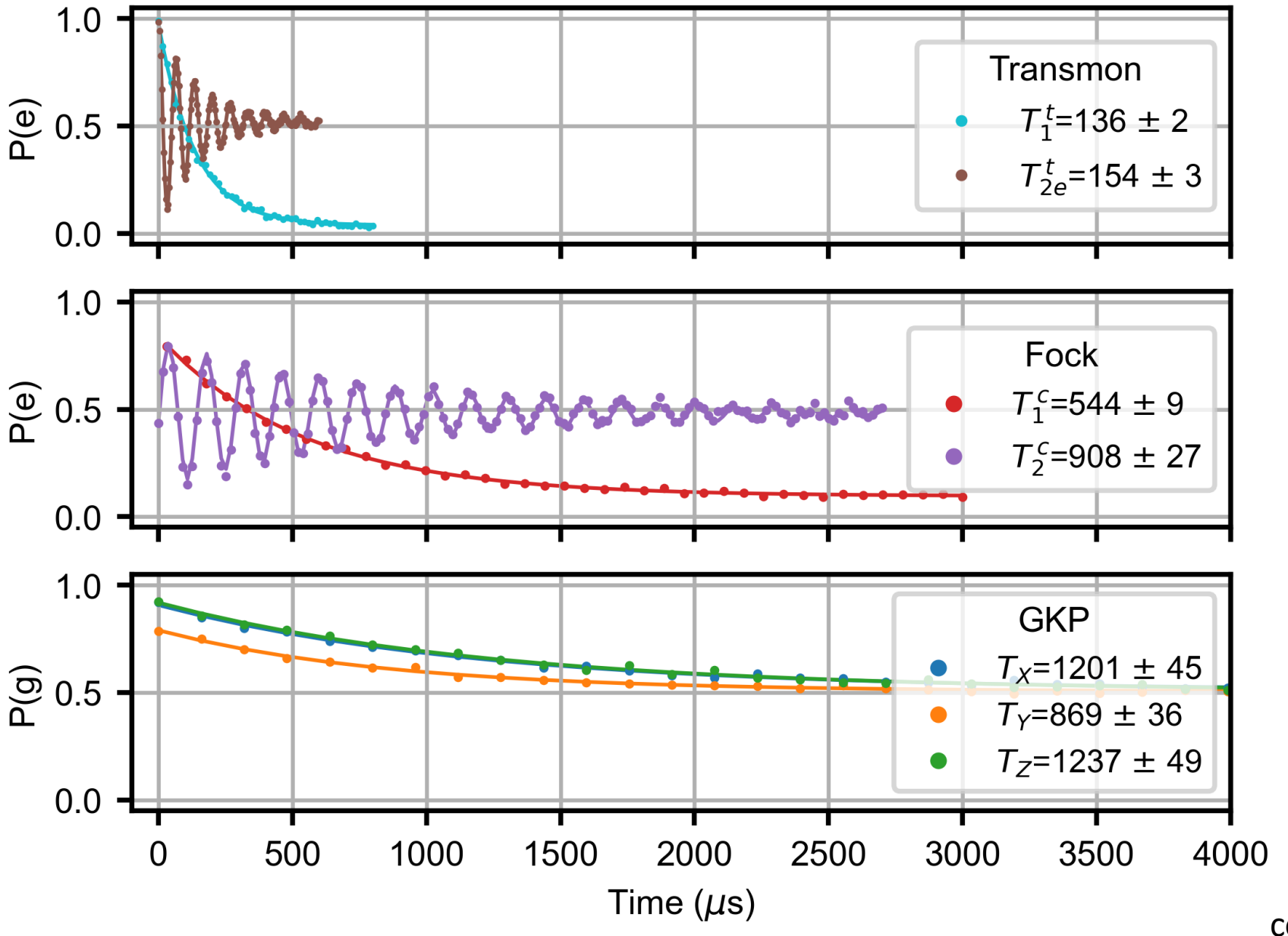


Ancilla reset

B. Royer *et al.*, (PRL, 2020); B. de Neeve *et al.*, (Nature Physics (2022); B. Terhal *et al.*, (PRA, 2016); P. Campagne-Ibarcq *et al.*, (Nature, 2020).

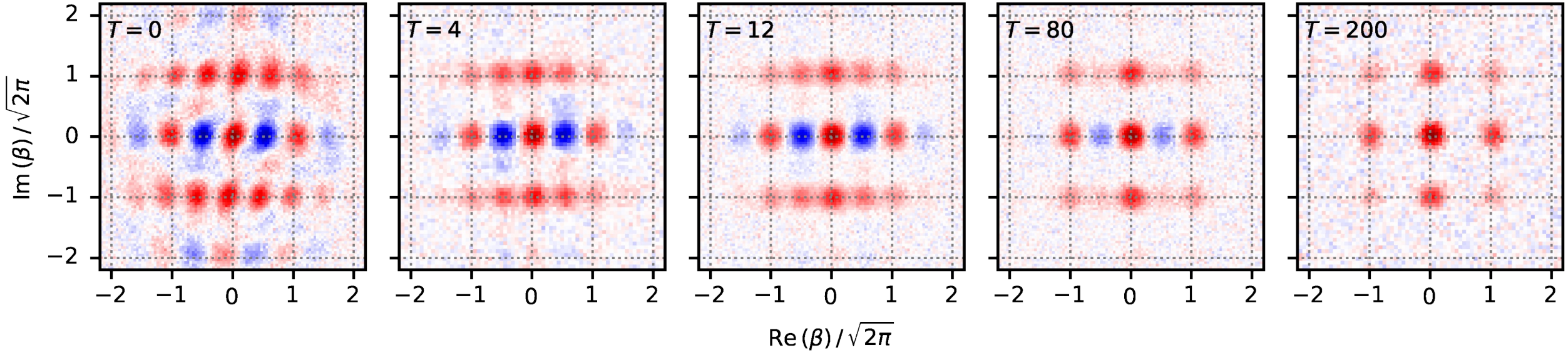
courtesy V. Sivak

QEC gain



courtesy V. Sivak

QEC in action



- Envelope too big
- Distorted

- Clip to the grid
- Resize envelope

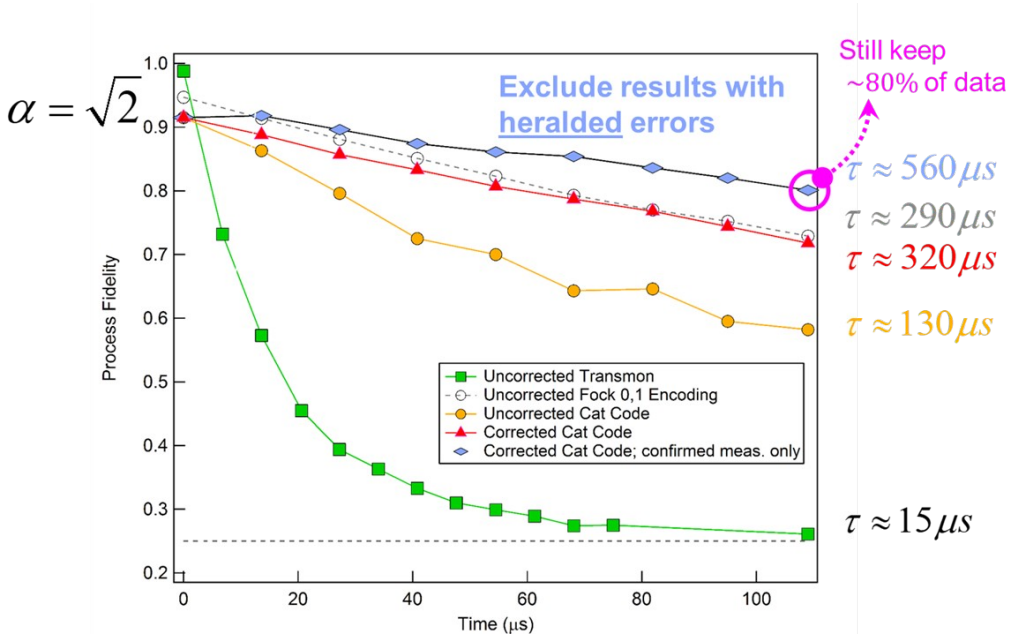
- Remove all deformations

- Depolarization in logical manifold

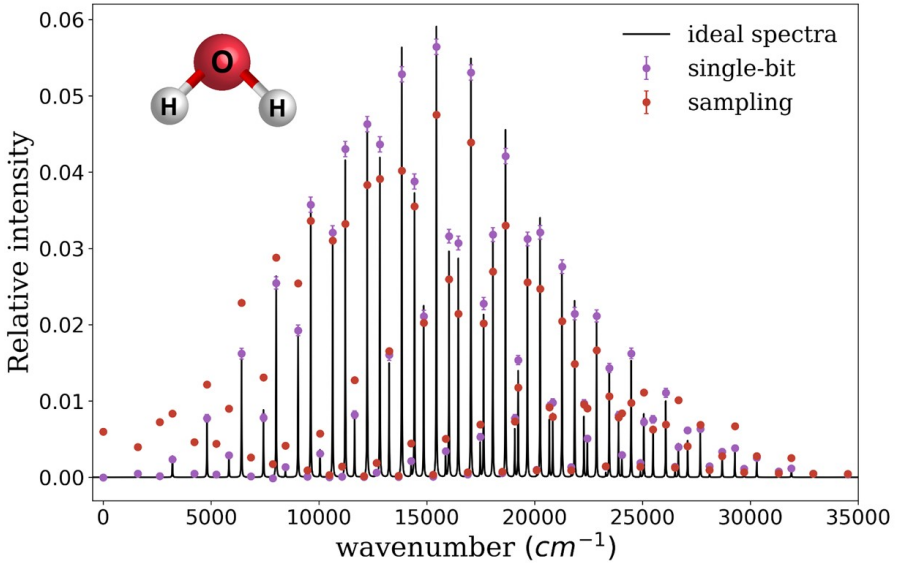
- Fully mixed logical state

courtesy V. Sivak

Take-home message:



Molecular Vibrational Spectra via Boson Sampling
Phys. Rev. X **10**, 021060 (2020)



Quantum error correction

&

Quantum simulations of physical models containing bosons

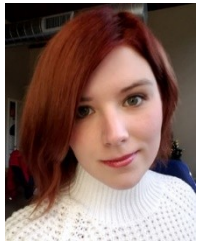
are both vastly more efficient on hardware containing 'native' bosons

Schoelkopf Lab


B. Lester

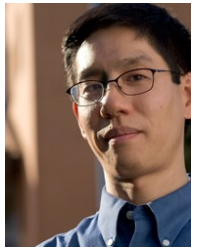

Y. Y. Gao


Y. Zhang


J. Freeze


V. Batista


P. Vaccaro


I. Chuang


L. Jiang


S. Girvin

U.S. DEPARTMENT OF ENERGY

QuantumInstitute.yale.edu

LUX ET VERITAS

& many others!

Devoret Lab