ITMO UNIVERSITY

Quantum Connections

faculty of PHYSICS

Photonic topological phases in superconducting circuits

Maxim Gorlach,

Assistant Professor,

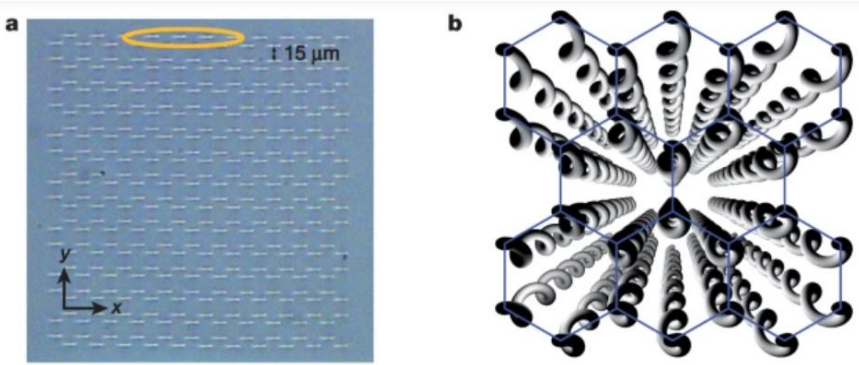
ITMO University

m.gorlach@metalab.ifmo.ru

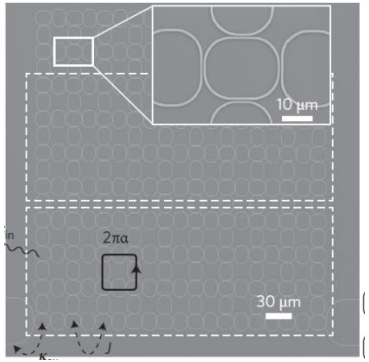
Stockholm, 2022

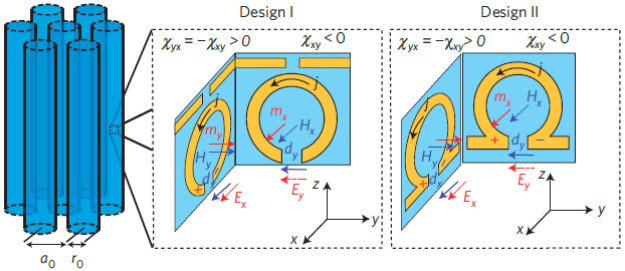
Topological photonics: from microwaves to the visible

Photonic (Floquet) topological insulators



M. Rechtsman *et al*, *Nature* **496,** 196–200 (2013)

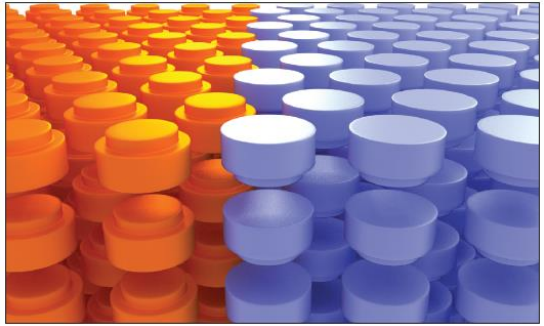


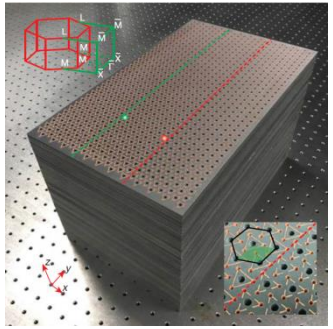


A.B. Khanikaev, *et al*. Nature Materials **12**, 233-239 (2013)

M. Hafezi, *et al*. Nature Physics **7**, 907-912 (2011).
M. Hafezi, *et al*. Nature Photonics **7**, 1001-1006 (2013)

disorder-robust routing and localization of light flows





Theory: A. Slobozhanyuk, *et al*. Nature Photonics **11**, 130-136 (2017)

Experiment: Y. Yang, *et al*. Nature **565**, 622-626 (2019)

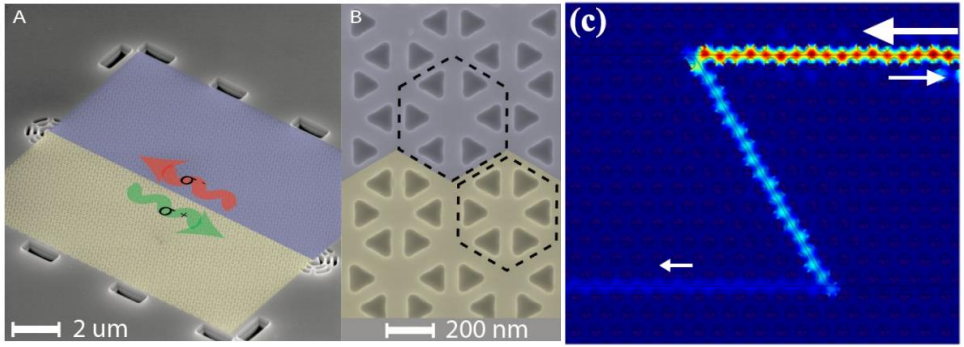
Bulk-boundary correspondence

Reviews on the topic:

T. Ozawa *et al*, Rev. Mod. Phys. **91**, 015006 (2019).
L. Lu, *et al*. Nature Photonics **8**, 821-829 (2014)
L. Lu, *et al*. Nature Physics **12**, 626-629 (2016)
A.B. Khanikaev, G. Shvets. Nature Photonics **11**, 763-773 (2017).
B. Xie, *et al*. Nature Reviews Physics **3**, 520-532 (2021).
and many other...

Topological photonics: enabling novel functionalities

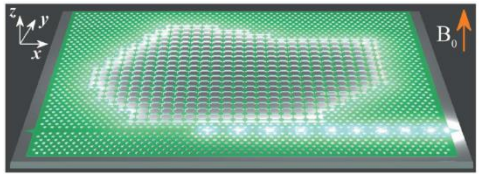
Topological waveguides

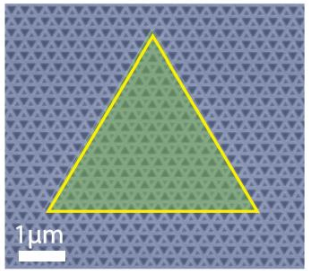


S. Barik *et al*, Science **359**, 666 (2018).
T. Ma, *et al*. Phys. Rev. Lett. **114**, 127401 (2015)

Disorder-robust routing

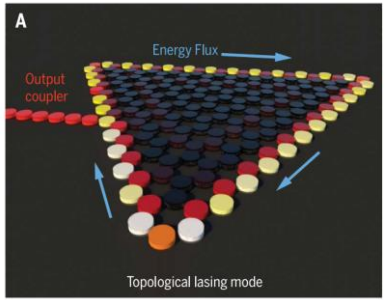
Topological resonators

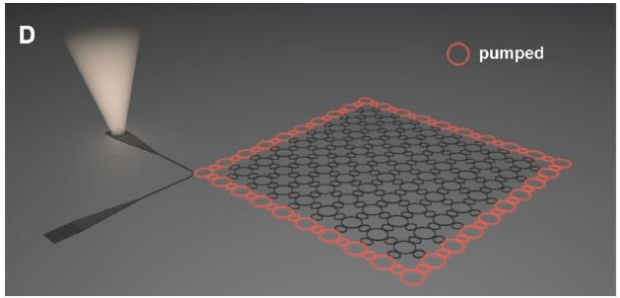




B. Bahari, *et al*. Science **358**, 636-640 (2017)
S. Barik *et al*, Phys. Rev. B **101**, 205303 (2020)

Topological lasers





Theory: G. Harari *et al*, Science **359**, eaar4003 (2018)

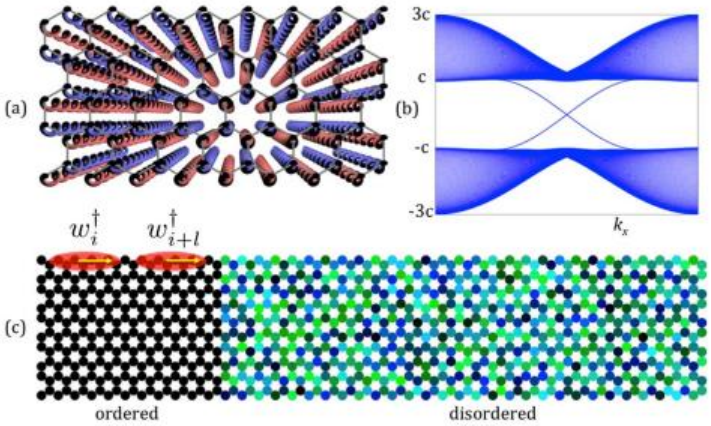
Experiment: M. Bandres, *et al*. Science **359**, eaar4005 (2018)

Further developments:
A. Dikopoltsev, *et al*. Science **373**, 1514 (2021)

Topological protection of quantum light: first theoretical proposals

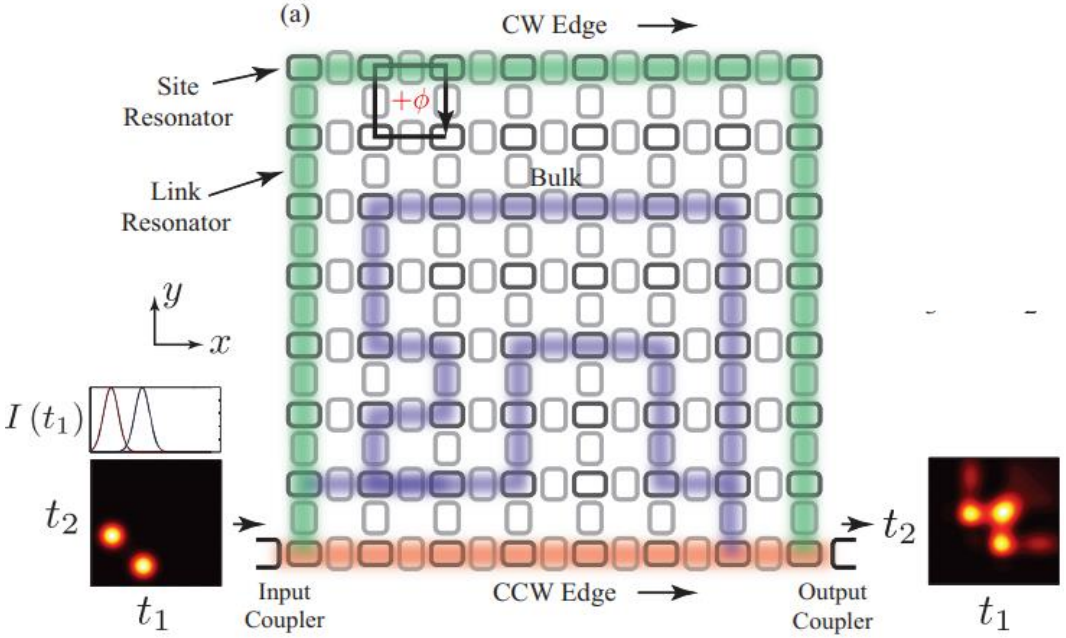
Can topology protect quantum states of light including nontrivial correlations?

Topological protection of photonic path entanglement (spatial correlations)



M. C. Rechtsman *et al*, Optica **3**, 925 (2016)

Transport through edge states preserves temporal correlations of entangled photons

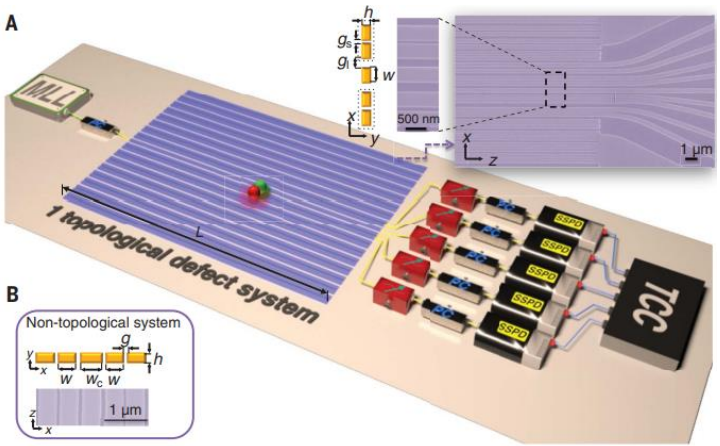


S. Mittal *et al*, Opt. Express **24**, 15631 (2016)

Topological protection of quantum light: experiments

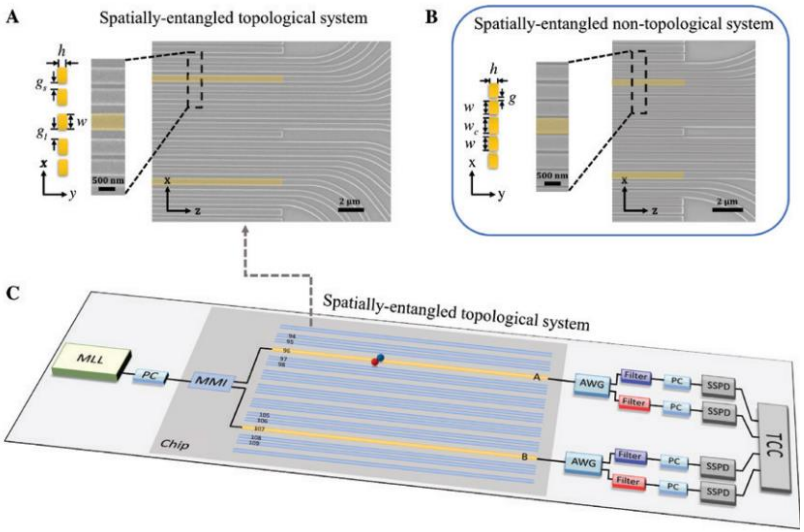
Robustness of spatial correlations, experiments

topological protection of the spatial correlations and the propagation constant of biphoton states



A. Blanco-Redondo *et al*, Science **362**, 568-571 (2018)

More on topological protection of spatial correlations

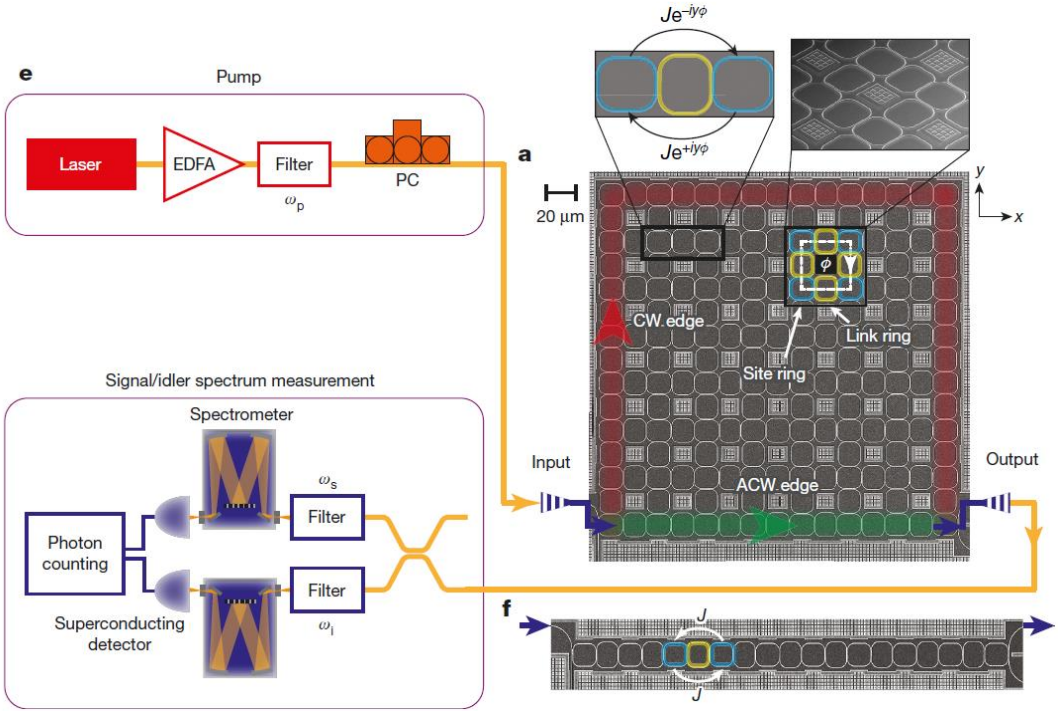


M. Wang *et al*, Nanophotonics **8**, 1327–1335 (2019)

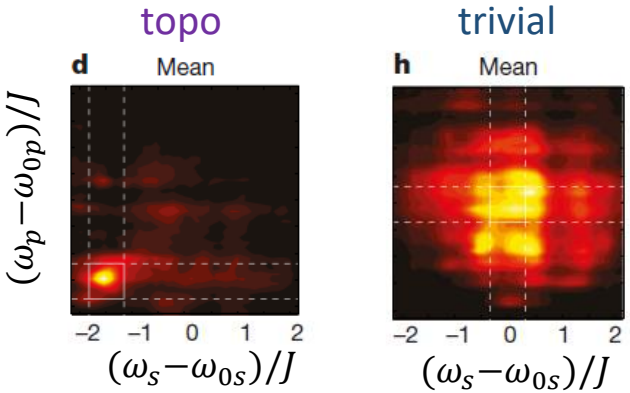
Review on quantum topological photonics

Q. Yan et al, Adv. Optical Mater. **9**, 2001739 (2021)

Topological protection of quantum light: spectral correlations



Spontaneous parametric four-wave mixing process.
The frequency of the generated signal is robust to disorder

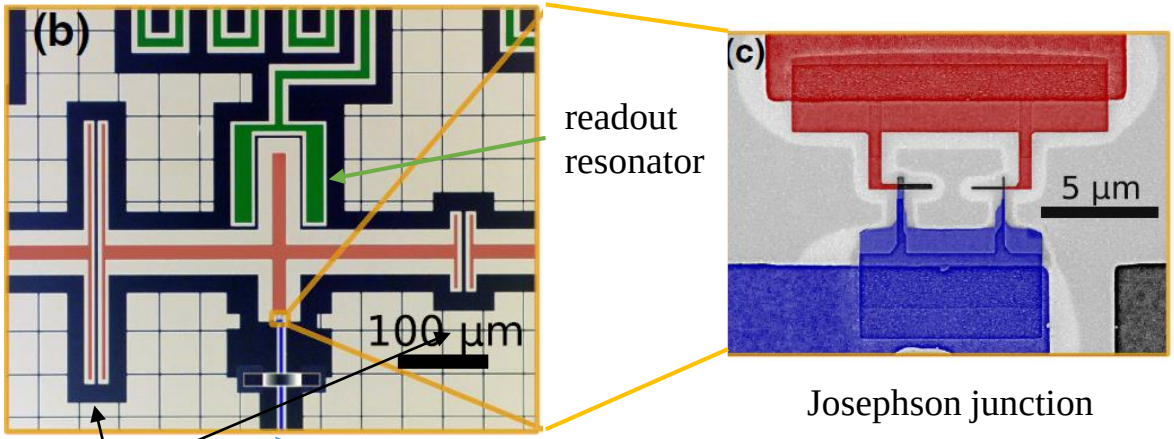


S. Mittal *et al*, Nature **561**, 502-506 (2018)

Could topology be useful for superconducting quantum processors?

Superconducting qubits and their arrays

Talk by Prof. John Martinis, Monday and Tuesday



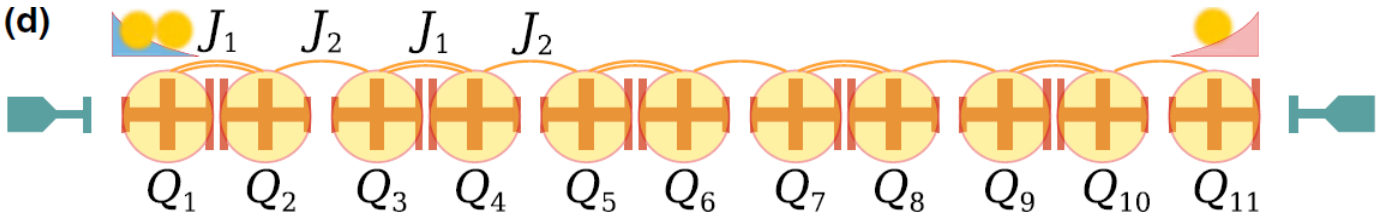
readout resonator

Josephson junction

capacitive couplings with other qubits

frequency control (DC)

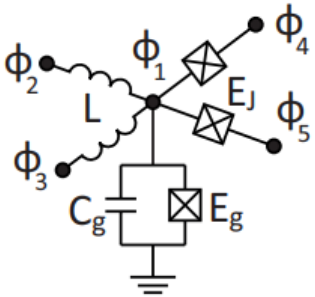
Such transmon qubits can be connected to operate together



Logical operations with qubits → Quantum computation algorithms

Theoretical description of the qubit array

See Monday lecture by Prof. John Martinis



1. Circuit Lagrangian via flux variables

$$\mathcal{L} = \underbrace{\frac{C_g}{2}\dot{\phi}_1^2}_{\text{capacitance}} + E_g \cos\phi_1 \underbrace{- \frac{(\phi_1-\phi_2)^2}{2L} - \frac{(\phi_1-\phi_3)^2}{2L}}_{\text{inductance}} \underbrace{+ E_J \cos(\phi_1-\phi_4) + E_J \cos(\phi_1-\phi_5)}_{\text{Josephson junction}}$$

node flux variable $\phi_i^\alpha \equiv \int^t V_i^\alpha(t')dt'$

2. Circuit Hamiltonian

conjugated variables (charges) $\pi_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}_i} = C_g \dot{\phi}_i$

$$\mathcal{H} = \frac{\pi_1^2}{2C_g} - E_g \cos\phi_1 + \frac{(\phi_1-\phi_2)^2}{2L} + \frac{(\phi_1-\phi_3)^2}{2L} - E_J \cos(\phi_1-\phi_4) - E_J \cos(\phi_1-\phi_5)$$

3. Quantize the circuit

elementary excitations are polaritons

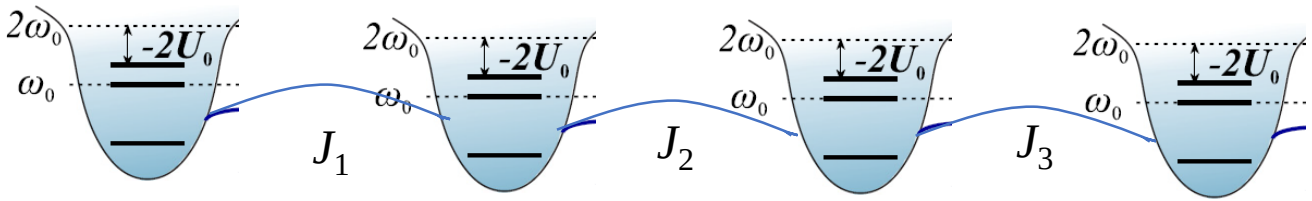
creation and annihilation operators

$$\phi_i = \sqrt{\frac{Z}{2}}(a_i^\dagger + a_i)\,, \quad \pi_i = i\sqrt{\frac{1}{2Z}}(a_i^\dagger - a_i)$$

Keep dominant nonlinearities via Taylor series expansion

$$\cos(\phi_i - \phi_j) \approx 1 - \frac{1}{2}(\phi_i-\phi_j)^2 + \frac{1}{24}(\phi_i-\phi_j)^4$$

Theoretical description of qubit array (continued)



$$\hat{H} = \sum_m \omega_m \hat{n}_m + \sum_m U_m \hat{n}_m(\hat{n}_m - 1) - \sum_m J_m \left(\hat{a}_m^\dagger \hat{a}_{m+1} + \hat{a}_{m+1}^\dagger \hat{a}_m\right)$$

qubit eigenfrequency — qubit anharmonicity — qubit anharmonicity

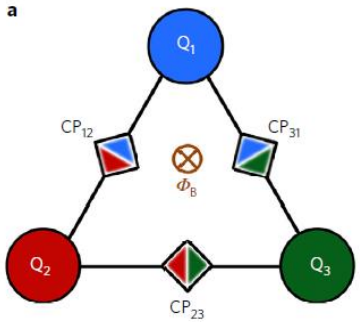
Bose-Hubbard model

Also appears in the context of cold atoms in optical lattice (see Monday talk by **Martin Zwierlein**)

But: qubits offer a greater flexibility in engineering on-site and coupling potentials. E.g. density-dependent coupling, cross-Kerr interaction, direct two-particle hopping, etc.

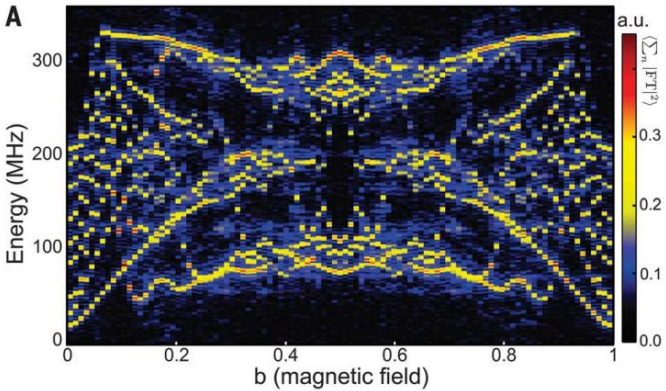
New physics in **extended** Bose-Hubbard models!

Current experiments with qubits



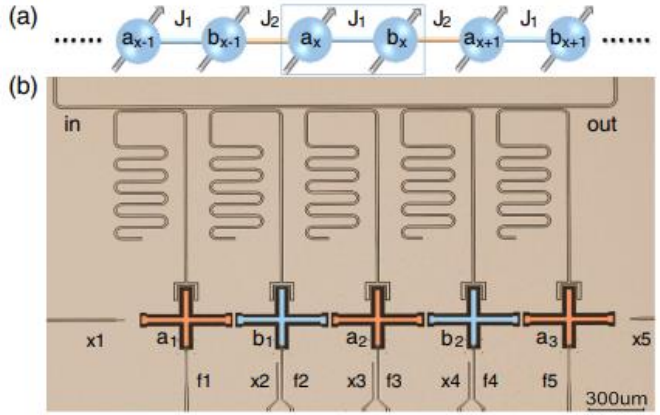
Temporal modulation of qubit couplings →complex effective couplings →artificial magnetic field → directional circulation of photons

P. Roushan *et al*, Nat. Phys. **13**, 146–151 (2017) (J. Martinis group)



P. Roushan *et al*, Science **358**, 1175–1179 (2017) (J. Martinis group)

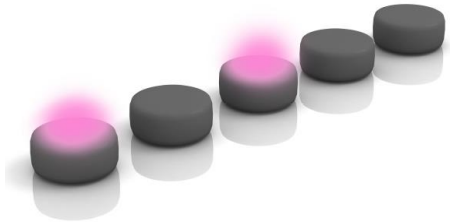
Resolving energy levels of interacting photons by tracing the temporal evolution



Topological single-photon excitations in 1D qubit array+measurement of topological winding numbers

W. Cai *et al*, Phys. Rev. Lett. **123**, 080501 (2019) (Tsinghua Univ., Beijing, L. Sun group)

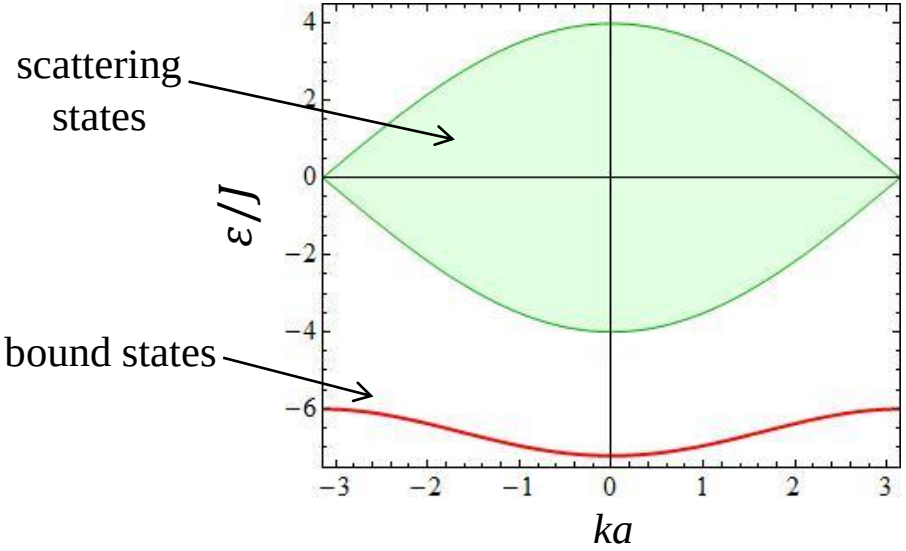
Two-photon Bose-Hubbard model in a simple 1D lattice



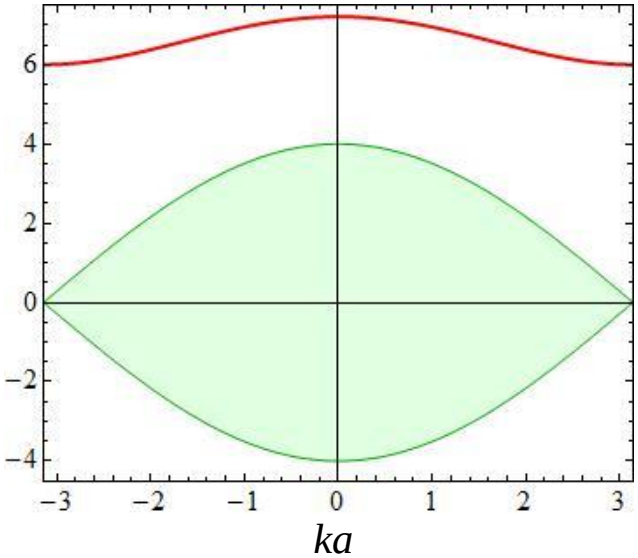
$$\hat{H} = \omega_0 \sum_j \hat{n}_j - J \sum_j \left(\hat{a}_j^\dagger \hat{a}_{j+1} + \hat{a}_{j+1}^\dagger \hat{a}_j\right) + U \sum_j \hat{n}_j \left(\hat{n}_j - 1\right)$$

$U < 0$ attractive interaction
$U > 0$ repulsive interaction

Attractive interactions, U/J=-3

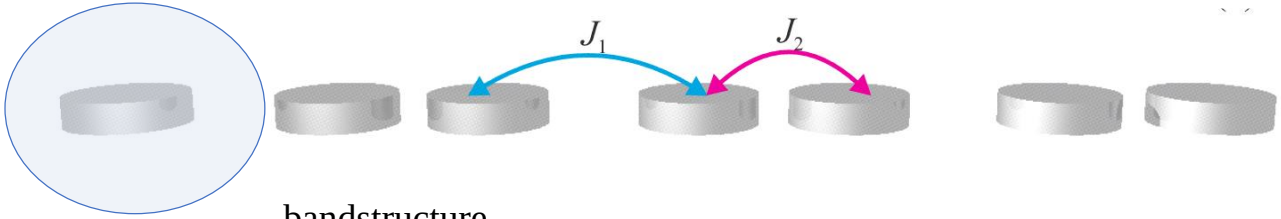


Repulsive interactions, U/J=3



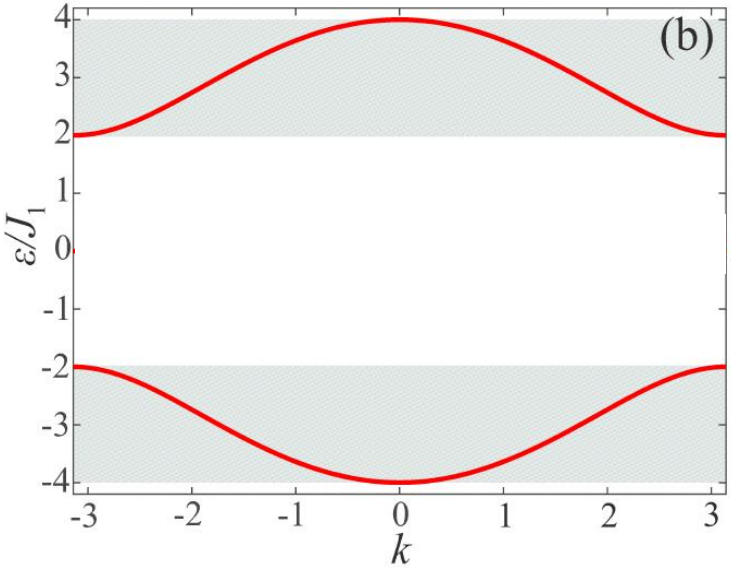
Winkler, *et al*. Nature **441**, pp. 853-856 (2006)

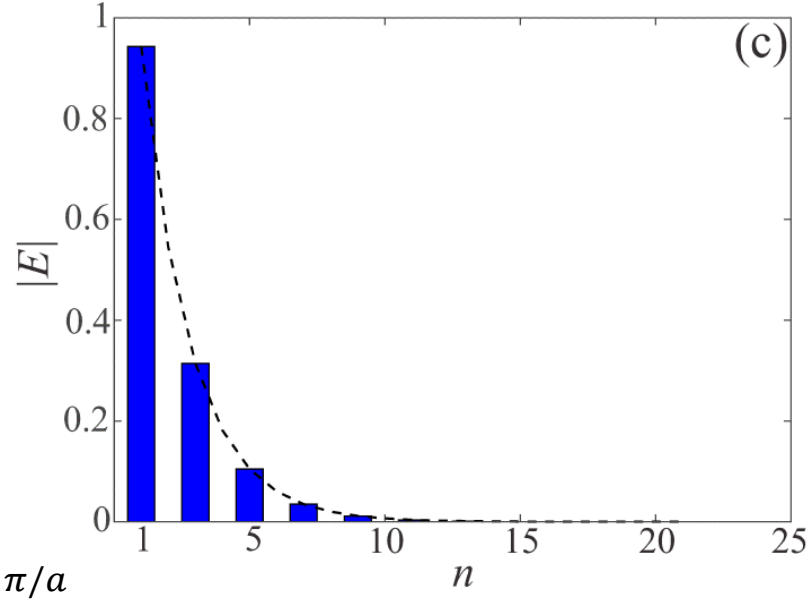
Su-Schrieffer-Heeger model: the simplest topological lattice



W. P. Su, J. R. Schrieffer, and A. J. Heeger. Phys. Rev. Lett. **42**, 1698 (1979).

bandstructure

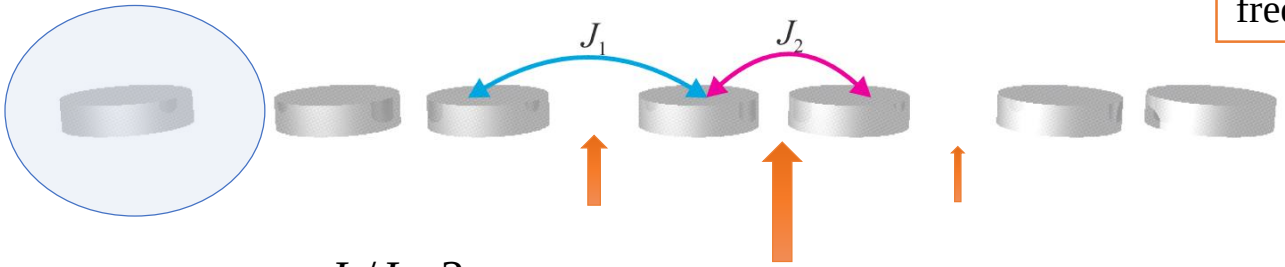




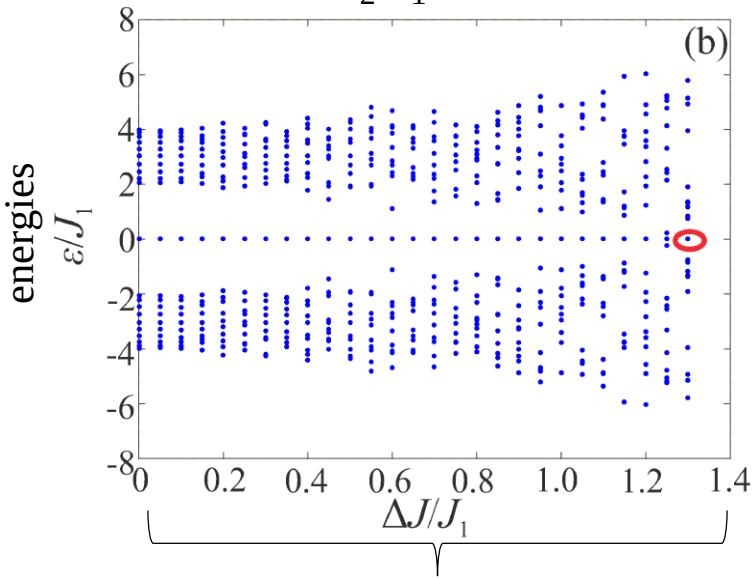
There is a nonzero invariant $$\gamma = i \int_{-\pi/a}^{\pi/a} \left\langle u_k \middle| \frac{\partial u_k}{\partial k} \right\rangle dk = \pi$$

What is special about Su-Schrieffer-Heeger model?

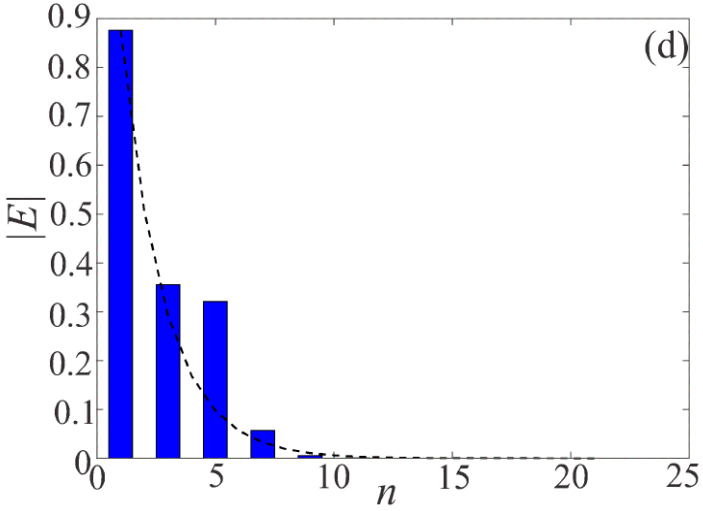
Disorder-robustness of the mode frequency and profile



$J_2/J_1=3$



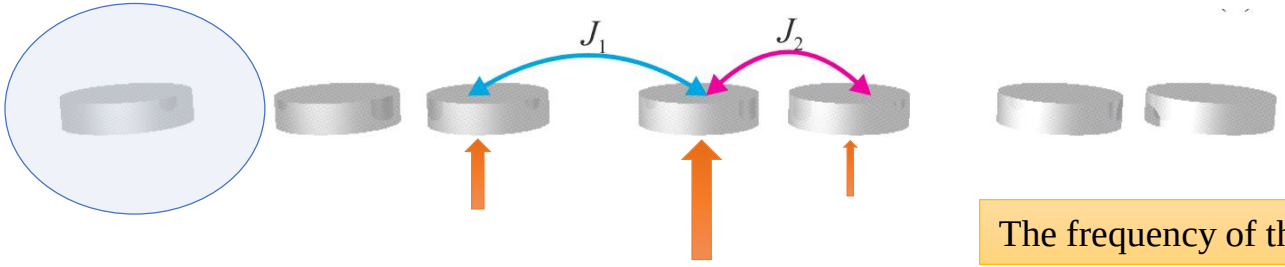
Strength of the coupling disorder



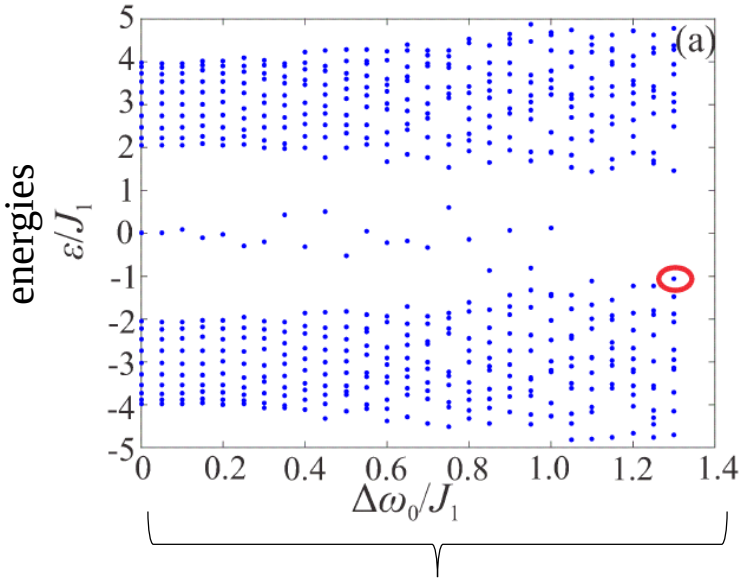
The edge mode persists!

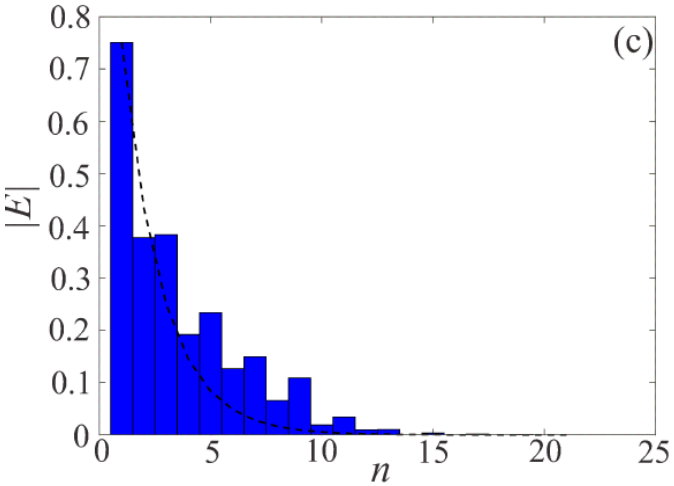
But there is no free lunch!

Random disorder in the eigenfrequencies



The frequency of the edge mode is no longer stable





Why? The underlying chiral symmetry is broken!

What happens if two photons travel in the Su-Schrieffer-Heeger array?

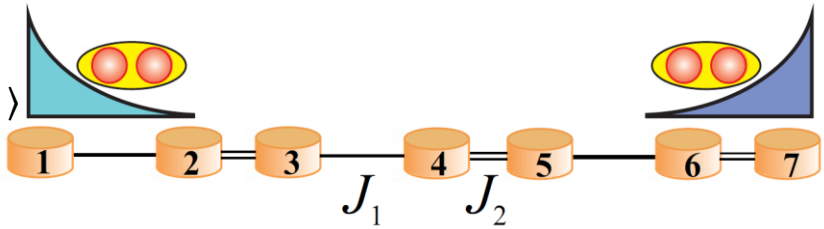
Bose–Hubbard Hamiltonian

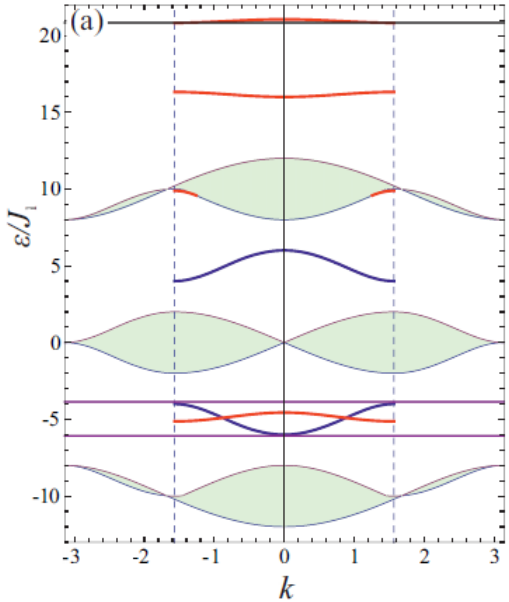
$$\hat{H} = \omega_0 \sum_m \hat{n}_m + U \sum_m \hat{n}_m(\hat{n}_m - 1) - \underbrace{\sum_m (J_1 \hat{a}^+_{2m-1}\hat{a}_{2m} + J_2 \hat{a}^+_{2m}\hat{a}_{2m+1})}_{\text{alternating coupling amplitudes, SSH}} - \text{H. c.}$$

search for the two-particle states in this model

photon number is conserved →ansatz for the wave function

$$|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{m,n} \beta_{mn}\, \hat{a}^\dagger_m \hat{a}^\dagger_n |0\rangle$$



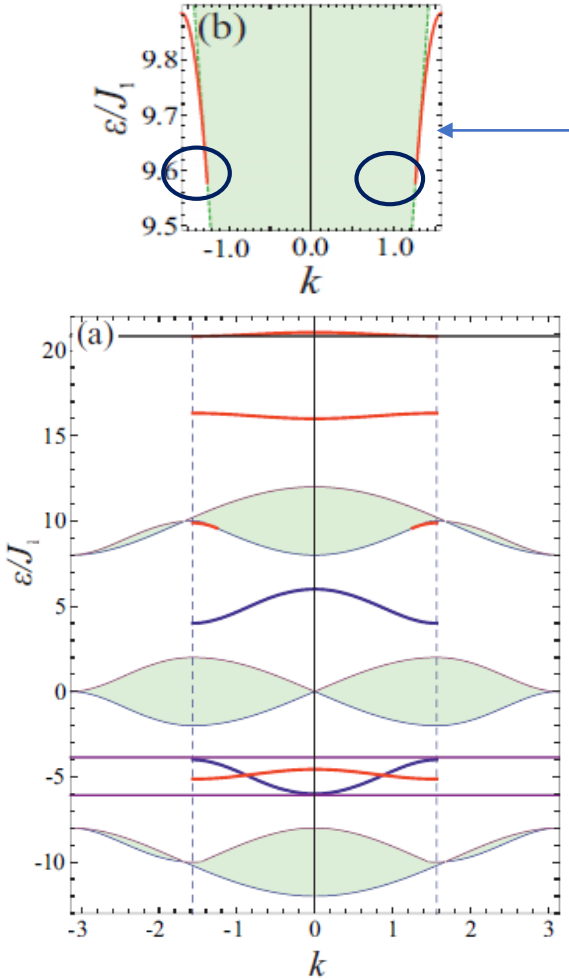


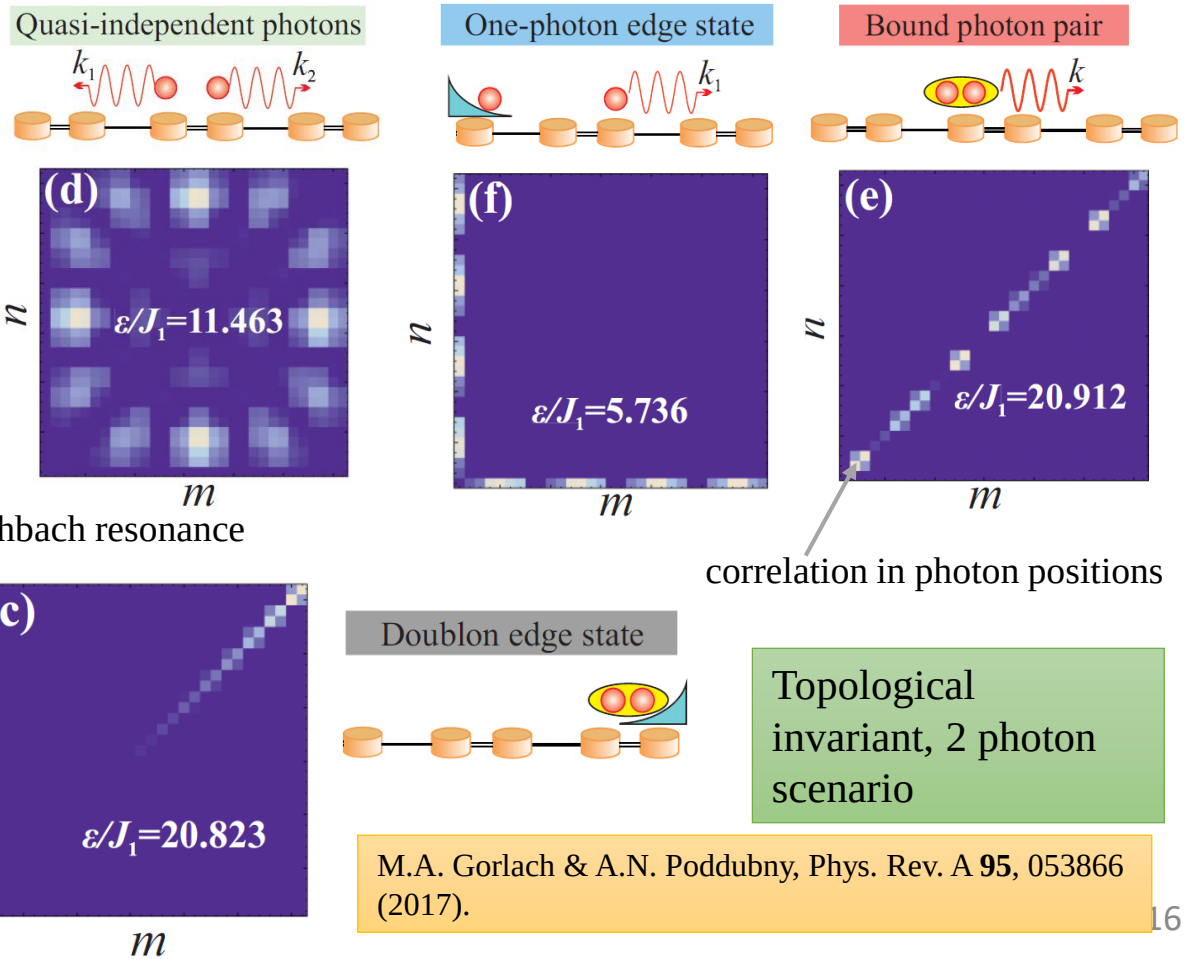
Bethe ansatz method

$$\beta_{mn} = e^{i\,k/2\,(m+n)} \underbrace{[C_{j(m,n)}\, e^{i\varkappa/2\,(n-m)} + \bar{C}_{j(m,n)}\, e^{i\bar{\varkappa}/2\,(n-m)}]}_{\text{relative motion of photons}}$$

M.A. Gorlach & A.N. Poddubny, Phys. Rev. A **95**, 053866 (2017)
M. Di Liberto, *et al*. Phys. Rev. A **94**, 062704 (2016)

What happens if two photons travel in the Su-Schrieffer-Heeger array?





Topological invariant, 2 photon scenario

M.A. Gorlach & A.N. Poddubny, Phys. Rev. A **95**, 053866 (2017).

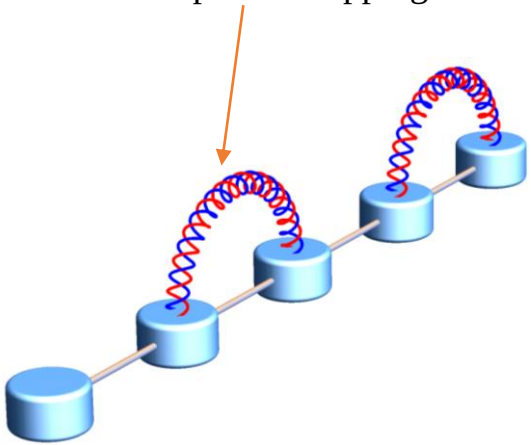
One step forward: interaction-induced topological states of photon pairs

Switch the topological properties due to interactions

Extended Bose–Hubbard model

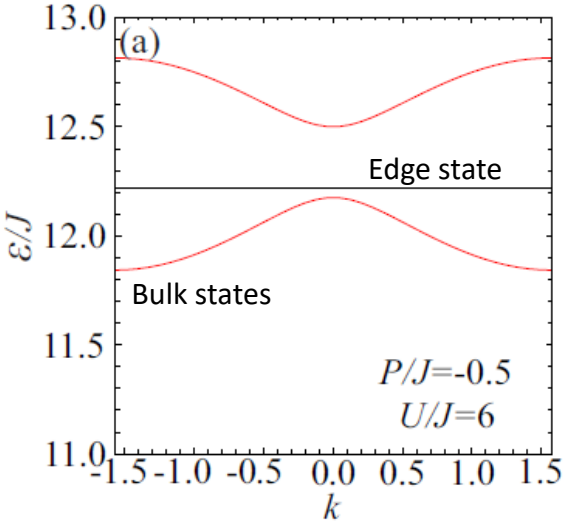
$$\hat{H} = \omega_0 \sum_m \hat{n}_m - J \sum_m (\hat{a}_m^+ \hat{a}_{m+1} + \hat{a}_{m+1}^+ \hat{a}_m) + U \sum_m \hat{n}_m (\hat{n}_m - 1)$$
$$+ \underbrace{\frac{P}{2} \sum_m (\hat{a}_{2m}^+ \hat{a}_{2m}^+ \hat{a}_{2m+1} \hat{a}_{2m+1} + \hat{a}_{2m+1}^+ \hat{a}_{2m+1}^+ \hat{a}_{2m} \hat{a}_{2m})}\,,$$

P – direct two-photon hopping



But: no single-photon edge states

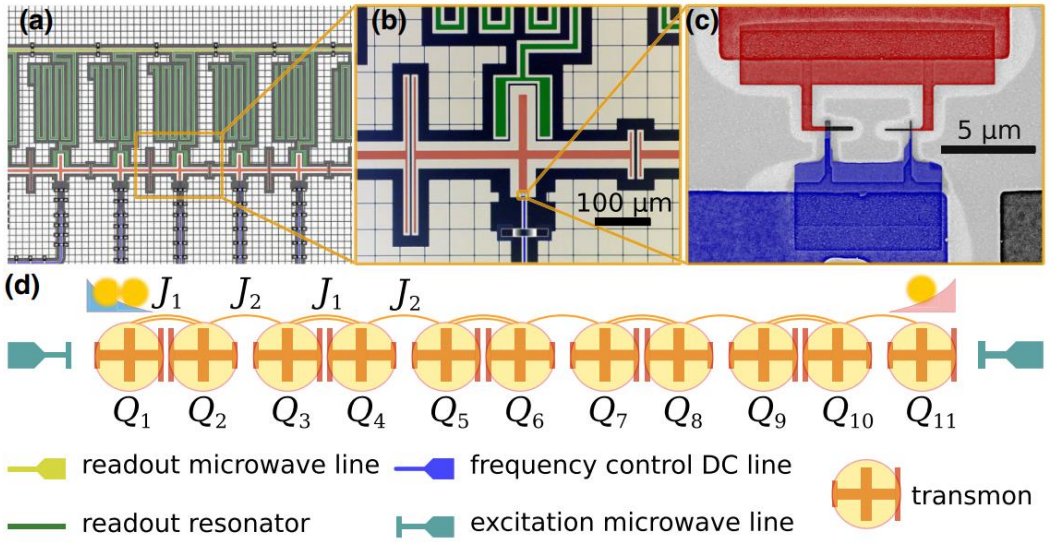
Dispersion:



Interaction-induced topological transition happens.
Need **extended** Bose-Hubbard model

A. Stepanenko & M. Gorlach, Phys. Rev. A **102**, 013510 (2020)

What about qubits?



Bose–Hubbard & SSH Hamiltonian

$$\hat{\mathcal{H}}_{\mathrm{BH}}/h = \sum_{q=1}^{N} \left[f_q\, \hat{n}_q + \frac{\delta_q}{2}\, \hat{n}_q\, (\hat{n}_q - 1) \right] + \sum_{q=1}^{(N-1)/2} \left[J_1\, \hat{a}_{2q}\, \hat{a}^{\dagger}_{2q-1} + J_2\, \hat{a}_{2q}\, \hat{a}^{\dagger}_{2q+1} \right] + \mathrm{H.c.},$$

$f_q = (3.73 \div 3.82)$ GHz

$\delta_q = -155$ MHz

$J_1 = 55.1$ MHz

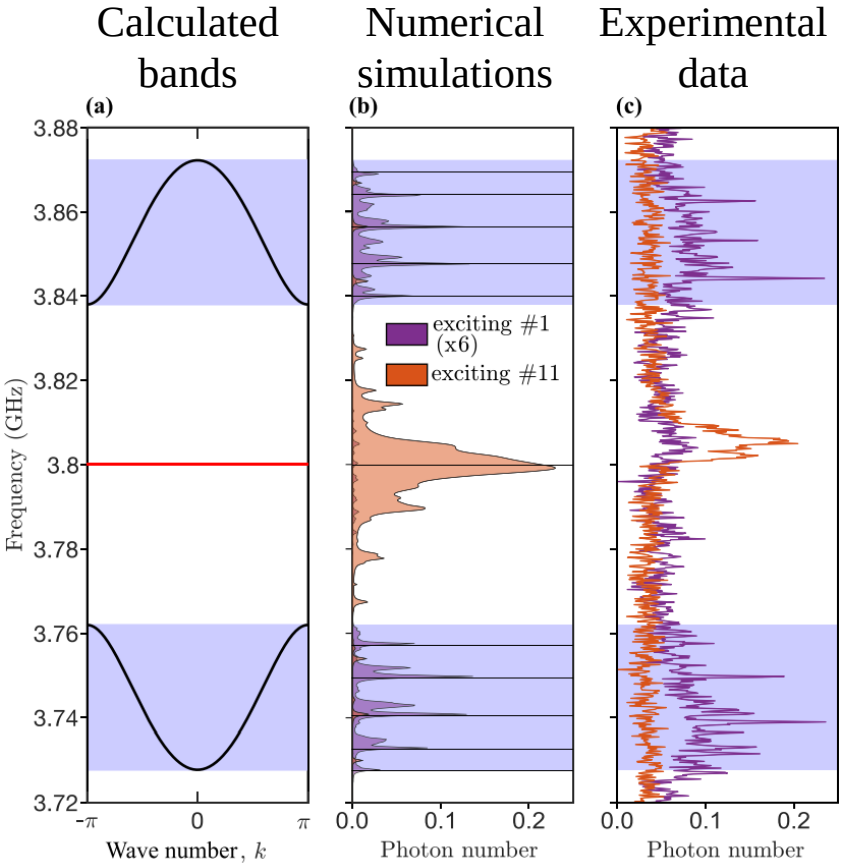
$J_2 = 17.1$ MHz

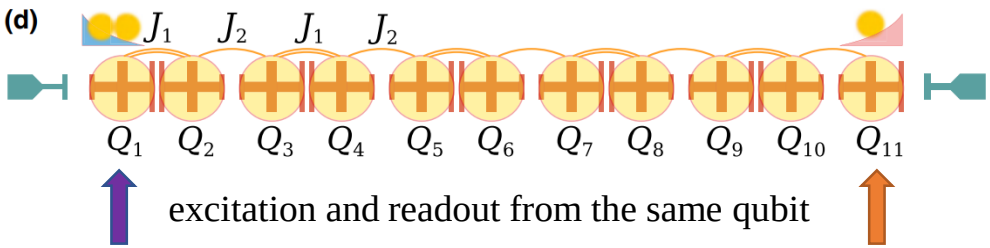
Can excite the qubits at the edge and measure $\langle n \rangle$ in all qubis

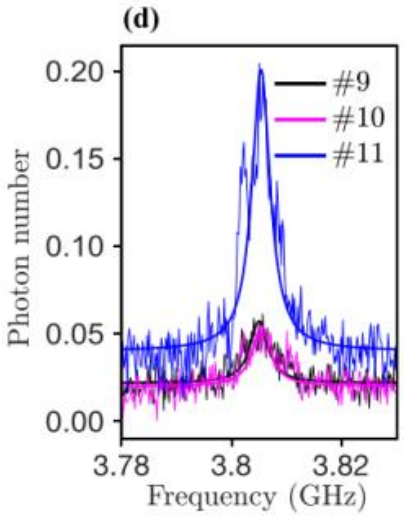
I.S. Besedin, M.A. Gorlach *et al*, Phys. Rev. B **103**, 224520 (2021)

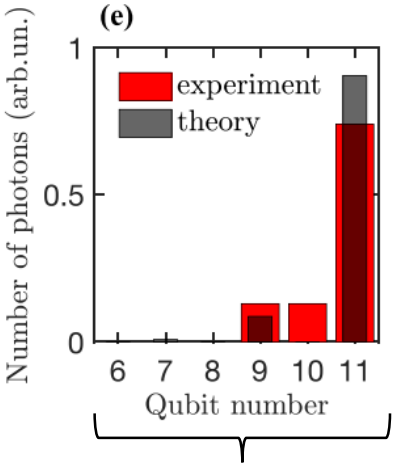
E. Kim, *et al*. Phys. Rev. X **11**, 011015 (2021) (O. Painter group)

Single-photon excitations: measurement results









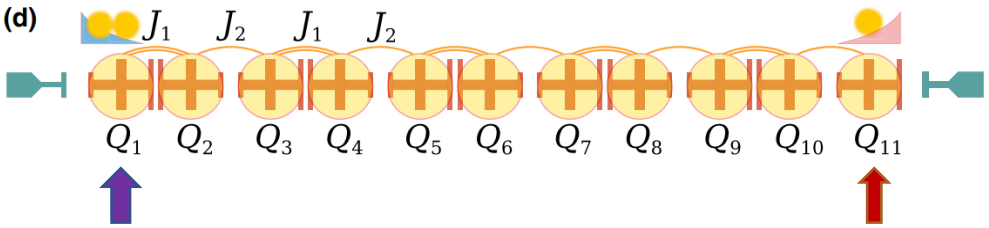
histogram for the edge mode

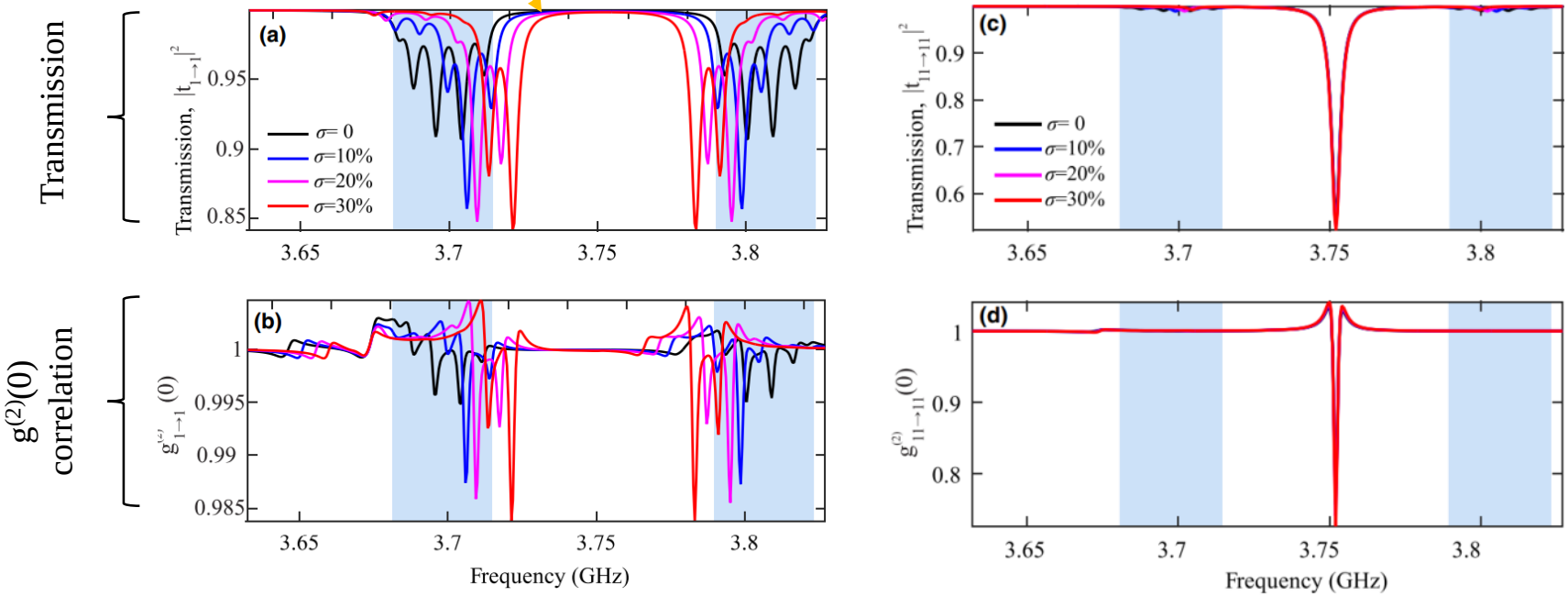
I.S. Besedin, M.A. Gorlach *et al*, Phys. Rev. B **103**, 224520 (2021)

How topological origin is manifested

excitation and readout from the same qubit

Different strengths of disorder in qubit couplings

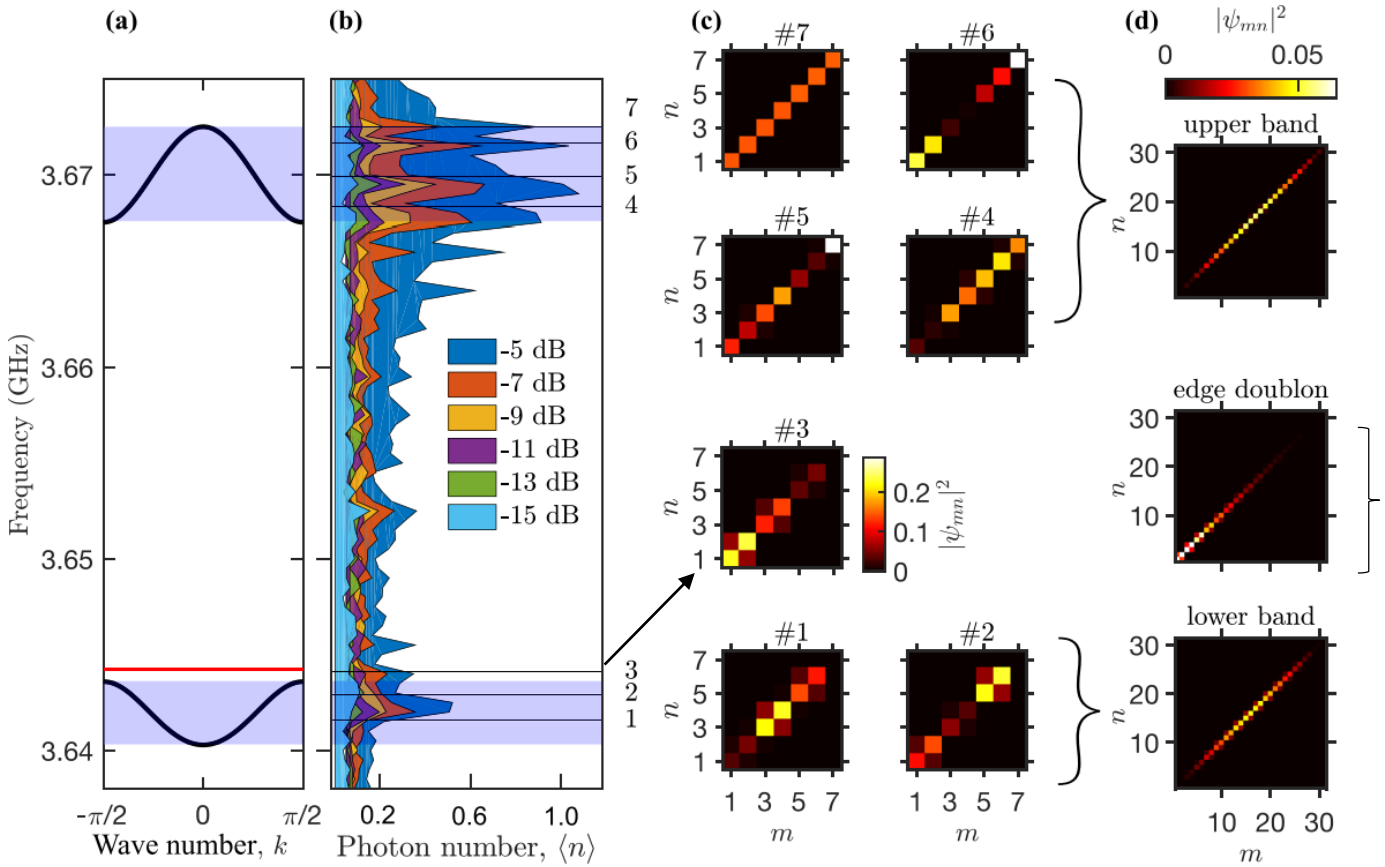




I.S. Besedin, M.A. Gorlach *et al*, Phys. Rev. B **103**, 224520 (2021)

Two-photon case – measurements

7 qubit array



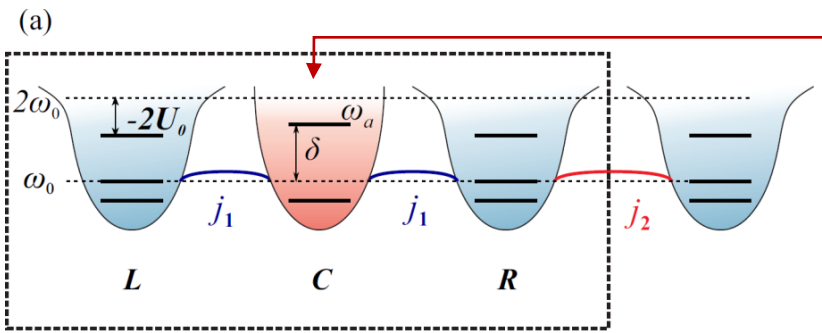
Two-photon bound states are shifted down to the scattering states

Characteristic peaks grow with the intensity of driving

Doublon edge mode is not manifested in short arrays

I.S. Besedin, M.A. Gorlach *et al*, Phys. Rev. B **103**, 224520 (2021)

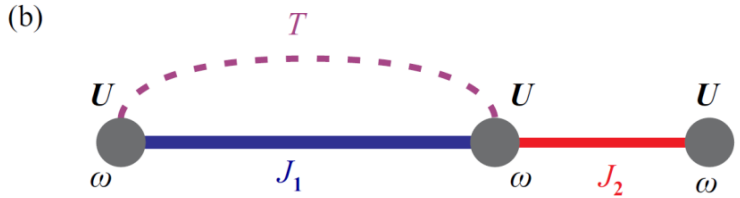
Extensions of Bose-Hubbard model in qubit arrays



auxiliary harmonic resonator detuned in frequency

$$|N_L+1,0,N_R\rangle \rightarrow |N_L,1,N_R\rangle \rightarrow |N_L,0,N_R+1\rangle$$

We do the perturbation theory excluding the redundant degree of freedom



Effective density-dependent coupling arises

$$J_{eff}(N_L,N_R) \simeq -\frac{j_1^2}{\delta} - \frac{j_1^2 U_0}{\delta^2}(N_L+N_R)$$

$$\hat{H}_T = \frac{T}{2}\left\{\hat{a}_1^\dagger(n_1+n_2)\hat{a}_2 + \hat{a}_2^\dagger(n_1+n_2)\hat{a}_1\right\}$$

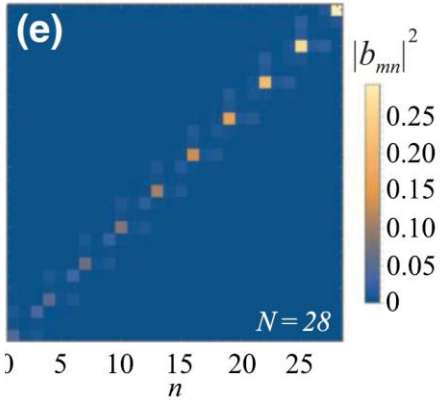
A.A. Stepanenko, M.D. Lyubarov, and M.A. Gorlach, Phys. Rev. Applied **14**, 064040 (2020)

Extensions of Bose-Hubbard model in qubit arrays: results

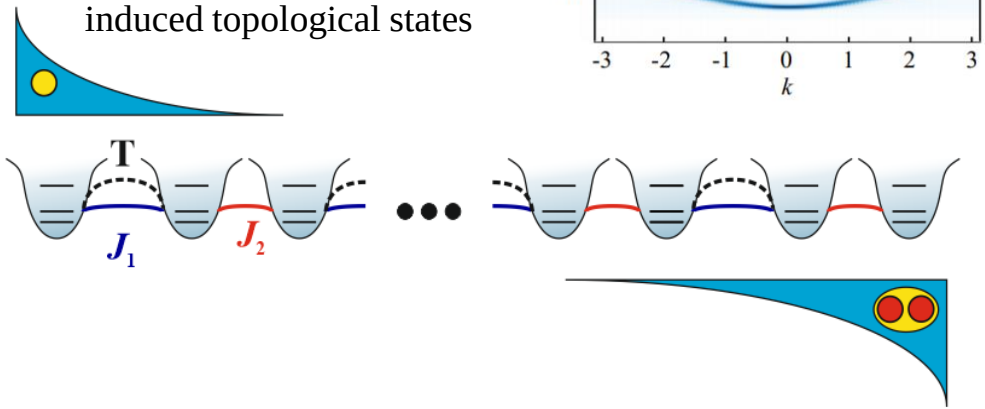
Extended Bose–Hubbard model

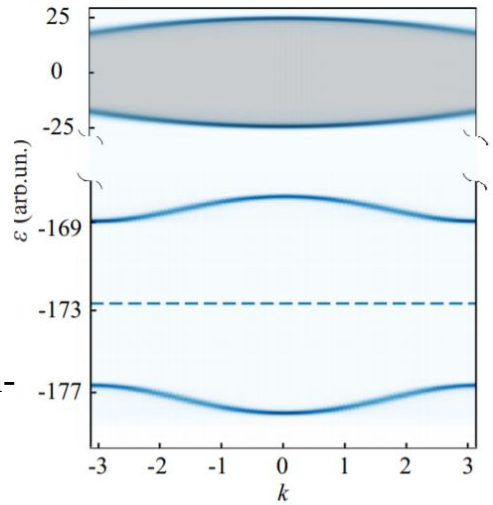
$$\hat{H} = \omega_0 \sum_m \hat{n}_m - J \sum_m (\hat{a}_m^+ \hat{a}_{m+1} + \hat{a}_{m+1}^+ \hat{a}_m) + U \sum_m \hat{n}_m (\hat{n}_m - 1)$$

$$+ \frac{T}{2} \sum_m (\hat{a}_{2m}^+ (\hat{n}_{2m} + \hat{n}_{2m+1}) \hat{a}_{2m+1} + \mathrm{H.c.}),$$



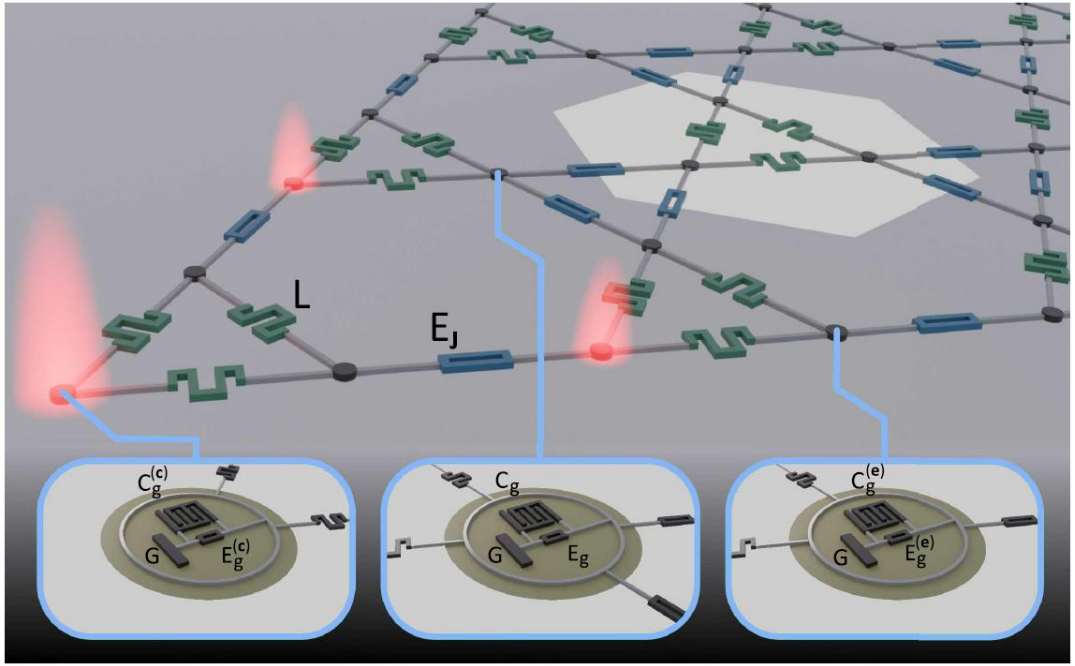
Result: density-dependent coupling enables interaction-induced topological states





A.A. Stepanenko, M.D. Lyubarov, and M.A. Gorlach, Phys. Rev. Applied **14**, 064040 (2020)

Inducing higher-order topology by interactions



Extended version of Bose-Hubbard model

$$\hat{H} = 2\pi f_0 \sum_{m,n,\alpha} \hat{n}^{\alpha}_{m,n} + \hat{V}_J + \hat{V}_I,$$

$$\hat{V}_J = \sum_{m,n,\alpha\neq\beta} \left[J_L (\hat{a}^{\alpha}_{m,n})^{\dagger} \hat{a}^{\beta}_{m,n} + J_J \sum_{m',n'} (\hat{a}^{\alpha}_{m,n})^{\dagger} \hat{a}^{\beta}_{m',n'} \right]$$

$$\hat{V}_I = \frac{E^k}{2} \sum_{m,n,\alpha} \hat{n}^{\alpha}_{m,n} (\hat{n}^{\alpha}_{m,n} - 1)$$

$$+ \frac{T}{2} \sum_{m,n,m',n',\alpha\neq\beta} (\hat{a}^{\alpha}_{m,n} \hat{a}^{\alpha}_{m,n})^{\dagger} \hat{a}^{\beta}_{m',n'} \hat{a}^{\beta}_{m',n'}$$

$$+ \frac{J^D}{\sqrt{2}} \sum_{m,n,m',n',\alpha\neq\beta} (\hat{a}^{\alpha}_{m,n})^{\dagger} (\hat{n}^{\alpha}_{m,n} + \hat{n}^{\beta}_{m',n'}) \hat{a}^{\beta}_{m',n'}$$

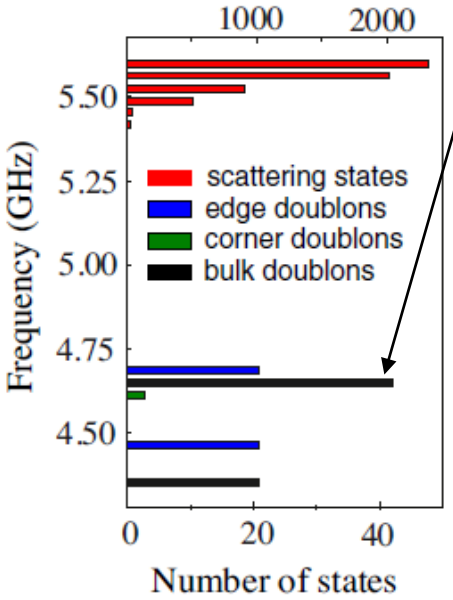
$$+ E^{ck} \sum_{m,n,m',n',\alpha\neq\beta} \hat{n}^{\alpha}_{m,n} \hat{n}^{\beta}_{m',n'},$$

Extra terms: **density-dependent coupling** J^D, **direct two-photon tunneling** T, **cross-Kerr interaction** E^{ck}

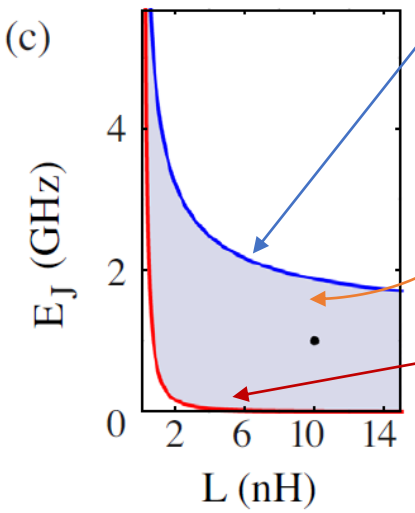
More flexibility compared to cold atoms: Dutta, et al. Rep. Prog. Phys. 78, 066001 (2015)

A.A. Stepanenko, M.D. Lyubarov, and M.A. Gorlach Phys. Rev. Lett. **128**, 213903 (2022)

Structure of the eigenstates



focus on bound photon pairs

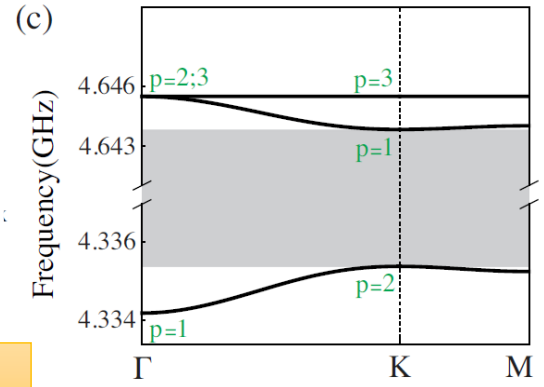


Topological transition, single-photon scenario (trivial below)

Topological transition, two-photon scenario (topo above)

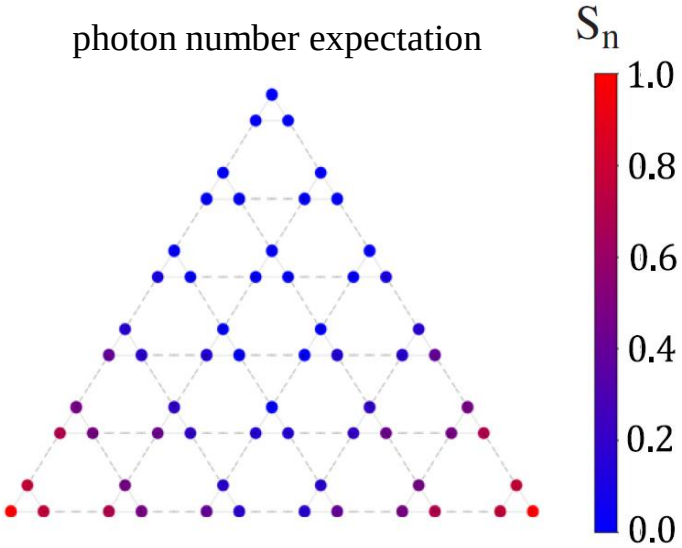
Single-photon states are trivial, two-photon states - topological

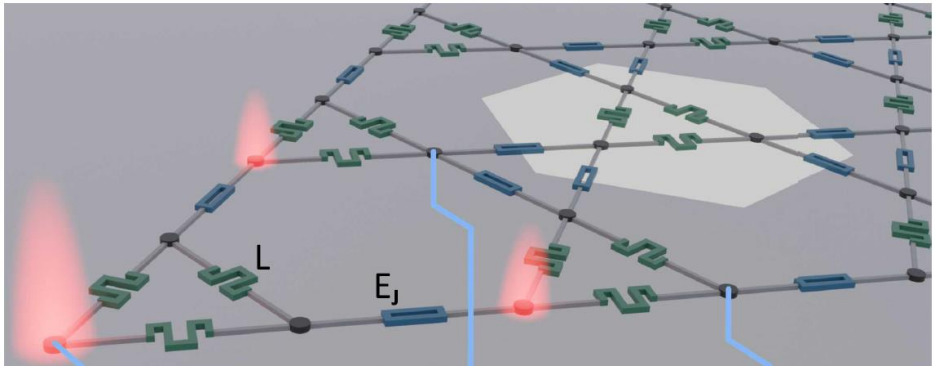
Topological nature of the phase→check of the symmetry indicators in high-symmetry points



A.A. Stepanenko, M.D. Lyubarov, and M.A. Gorlach Phys. Rev. Lett. **128**, 213903 (2022)

Interaction-induced corner state



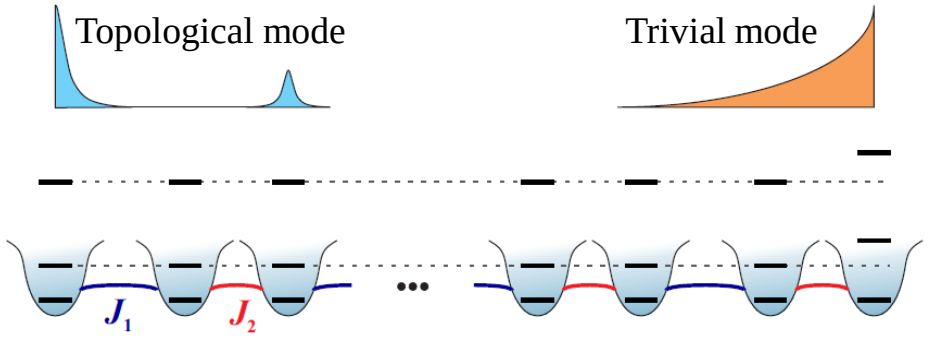


- Almost insensitive to disorder in linear couplings (L)
- Fully insensitive to disorder in Josephson energies (E_J)
- Feels disorder in grounding capacitances (C) and Josephson energies (E_g)

+zoo of other exotic modes: bulk-corner, bulk-edge, edge-corner

What these modes are good for?

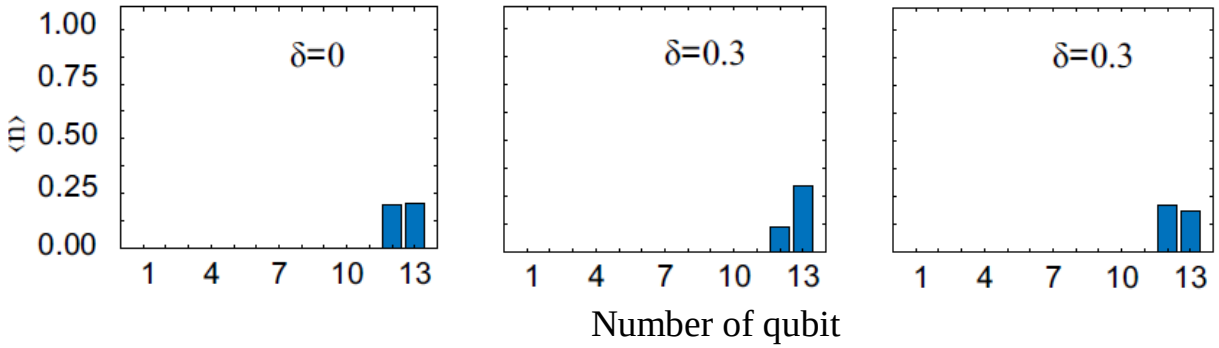
As an illustration, consider a **boson sampling** in a 1D topological array



1. Prepare initial state in a high-Q resonator.
2. Transfer the prepared state to the chosen qubit of the array.
3. Turn on the couplings between the qubits. Then turn off after some time Δt.
4. Read the populations of the qubits.

$$f_0 = 3.8 \text{ GHz}, E^k = -277.5 \text{ MHz}, J_1 = 10 \text{ MHz}, J_2 = 100 \text{ MHz}$$

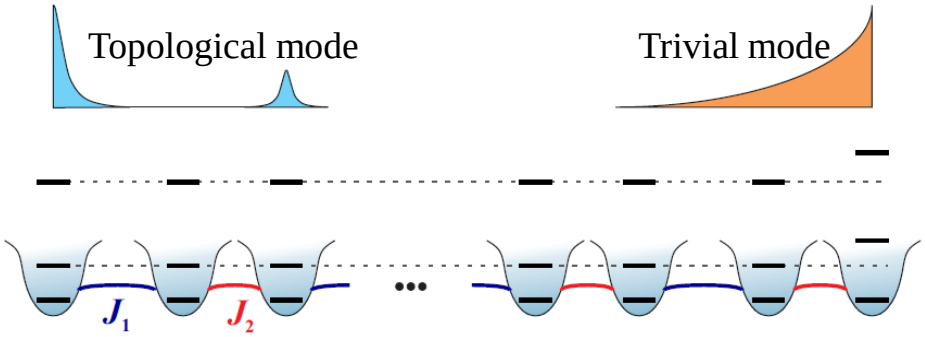
Launch an excitation in the trivial edge mode



δ quantifies the strength of the coupling disorder

What these modes are good for?

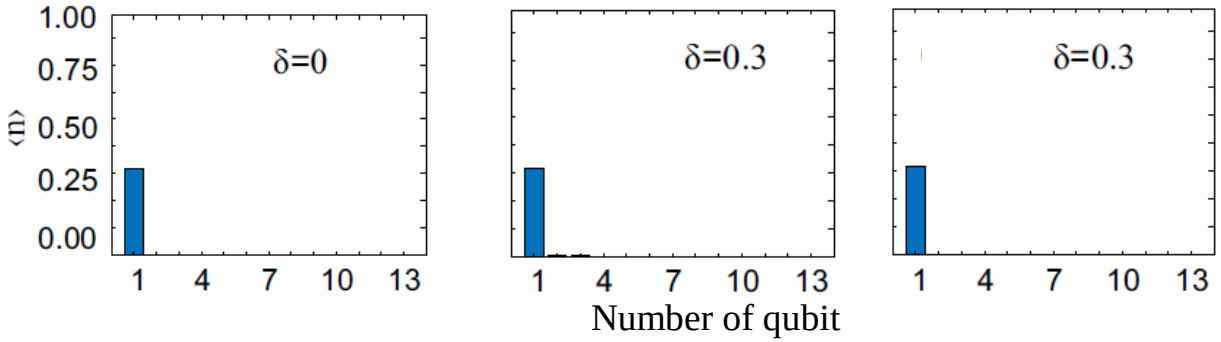
As an illustration, consider a **boson sampling** in a 1D topological array



$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}]$$
$$+\gamma_{\downarrow} \sum_m [\hat{a}_m \hat{\rho} \hat{a}_m^{+} - (\hat{a}_m^{+} \hat{a}_m \hat{\rho} + \hat{\rho} \hat{a}_m^{+} \hat{a}_m)/2]$$

$f_0 = 3.8$ GHz, $E^k = -277.5$ MHz, $J_1 = 10$ MHz, $J_2 = 100$ MHz, $\gamma_{\downarrow} = 1$ MHz

Launch an excitation in the topological edge mode



δ quantifies the strength of the coupling disorder

The results are stable with respect to the coupling disorder

Outlook

Rich physics: 1D & 2D qubit arrays, localized and propagating states

Enhanced disorder-robustness in boson sampling. Other quantum simulation protocols?

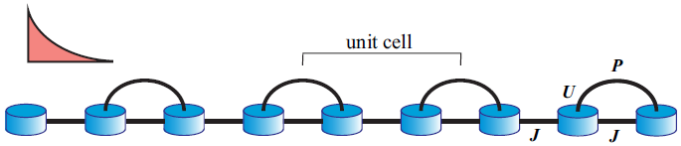
Trade-off: extra number of qubits vs additional robustness

m.gorlach@metalab.ifmo.ru

Thank you for attention

Supplementary slides

How to probe this physics?

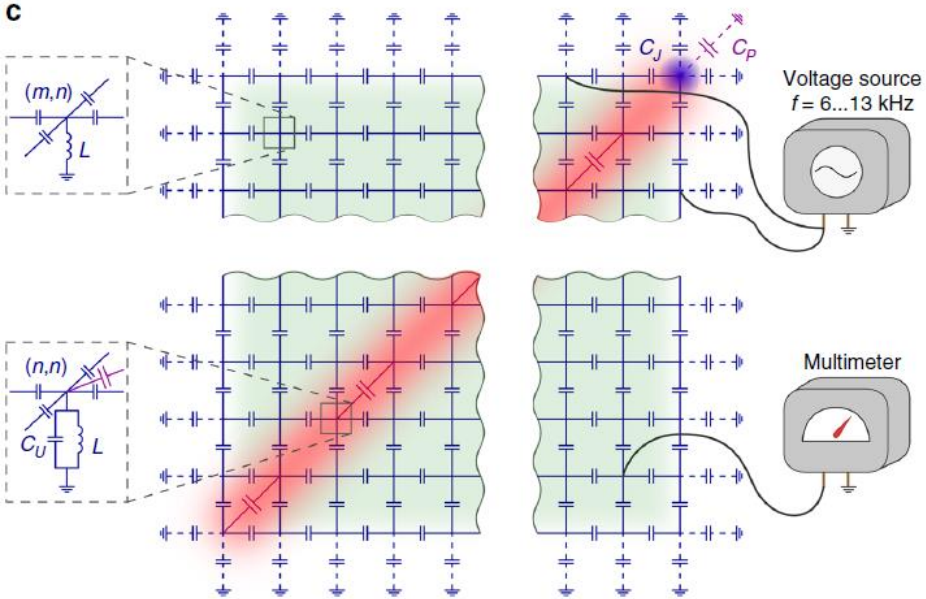


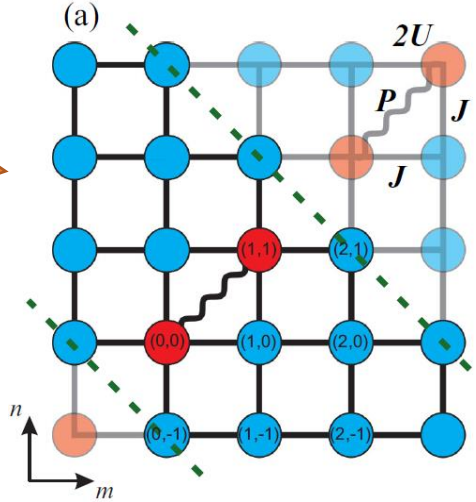
2D map of superposition coefficients

$$|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{m,n} \beta_{mn}\, \hat{a}_m^\dagger \hat{a}_n^\dagger |0\rangle$$

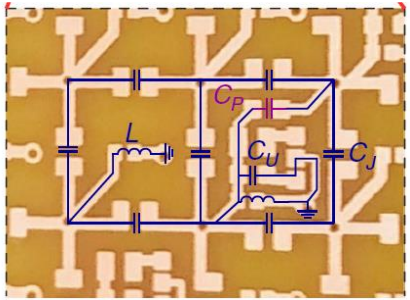
$$\hat{H}|\psi\rangle = \varepsilon|\psi\rangle$$

2D tight-binding equations for β_{mn} coefficients



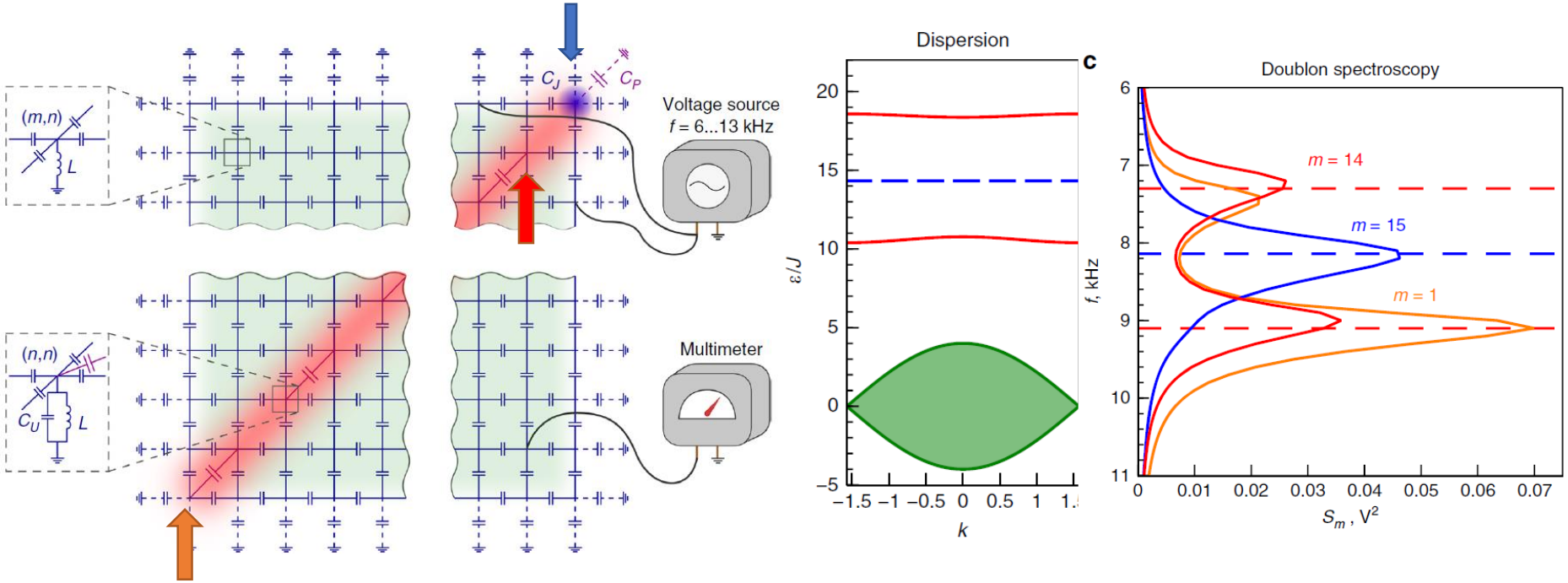


But this 2D system can be classical!



N.A. Olekhno, *et al*. Nature Communications **11**, 1436 (2020)

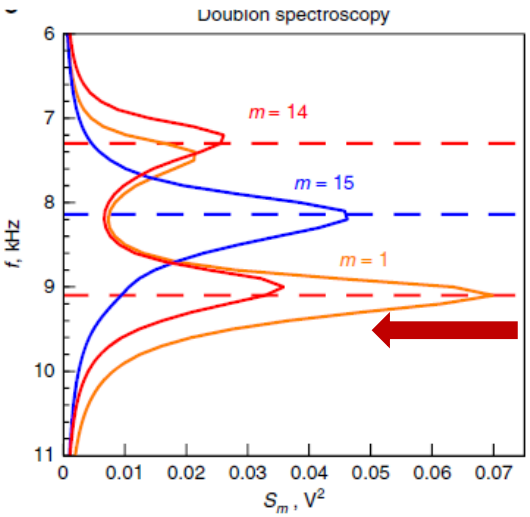
Features that correspond to the two-particle topological states

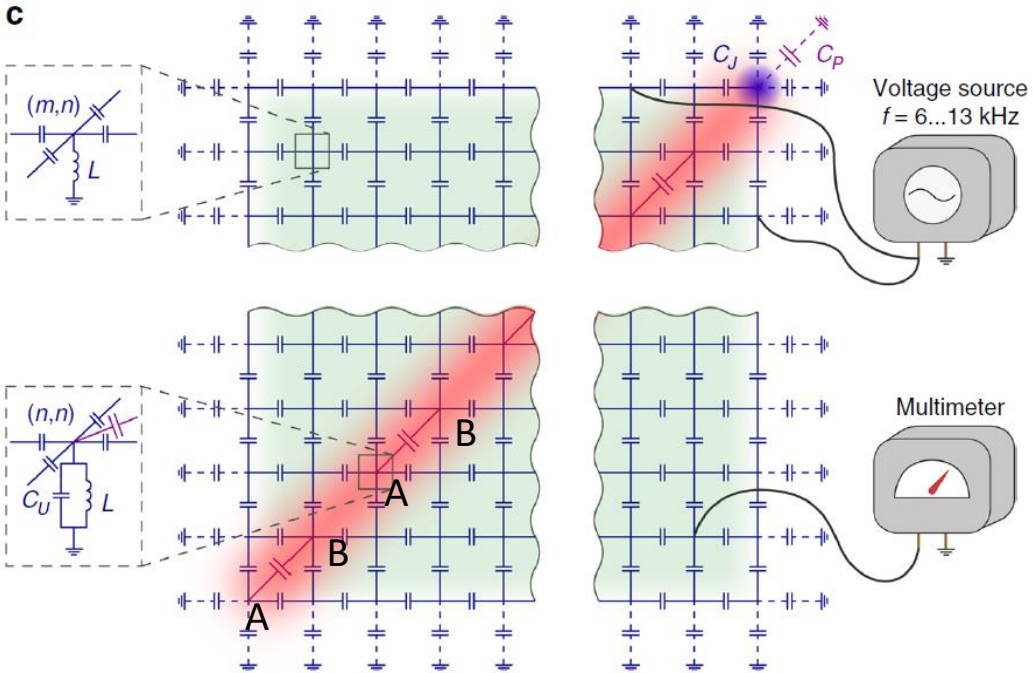


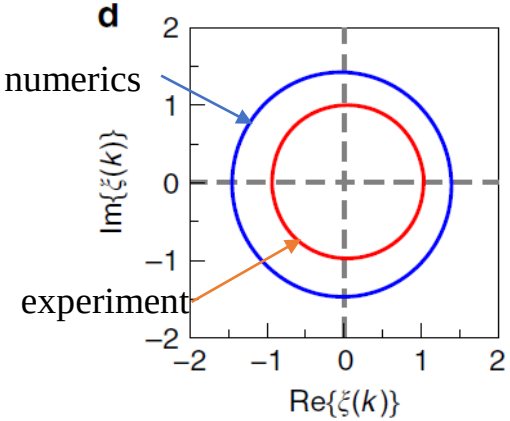
Characteristic peaks in $S_m(f)$ dependence→eigenmode frequencies

Field distribution at the chosen frequency→mode pattern

Retrieving the topological invariant







$$\xi(k) = U_A(k)/U_B(k)$$

Drive (7,7) node of the circuit

Winding number $w = 1$ → **topological origin** of the edge mode

N.A. Olekhno, *et al*. Nature Communications **11**, 1436 (2020)

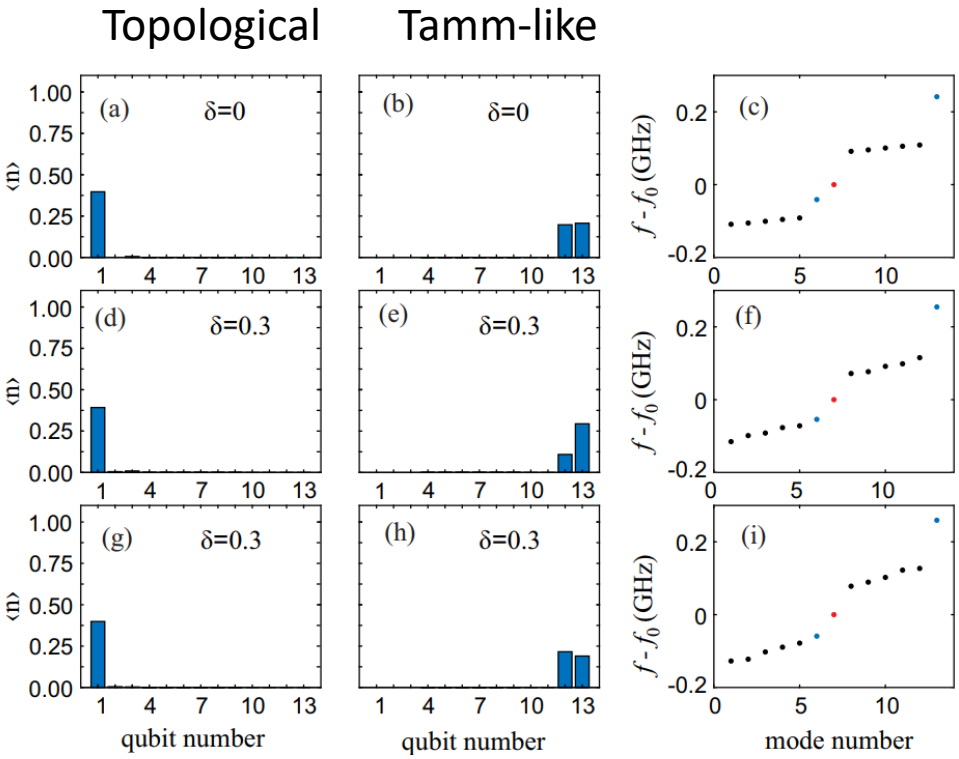
The impact of disorder on the boson sampling

Bose–Hubbard model with disorder

$$\hat{H} = f_0 \sum_m \hat{n}_m + E^k \sum_m \hat{n}_m(\hat{n}_m - 1)$$

$$- \sum_m (J_1^m \hat{a}_{2m-1}^+ \hat{a}_{2m}$$

$$J_{1,2}^m \in [J_{1,2}(1+\delta), J_{1,2}(1-\delta)]$$



Lindblad equation

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] + \gamma_\downarrow \sum_m [\hat{a}_m \hat{\rho} \hat{a}_m^+ - (\hat{a}_m^+ \hat{a}_m \hat{\rho} + \hat{\rho} \hat{a}_m^+ \hat{a}_m)/2]$$

$\gamma_\downarrow$ - dissipation

A.A. Stepanenko, M.D. Lyubarov, and M.A. Gorlach Phys. Rev. Lett. 128, 213903 (2022)

Topological transitions driven by quantum statistics

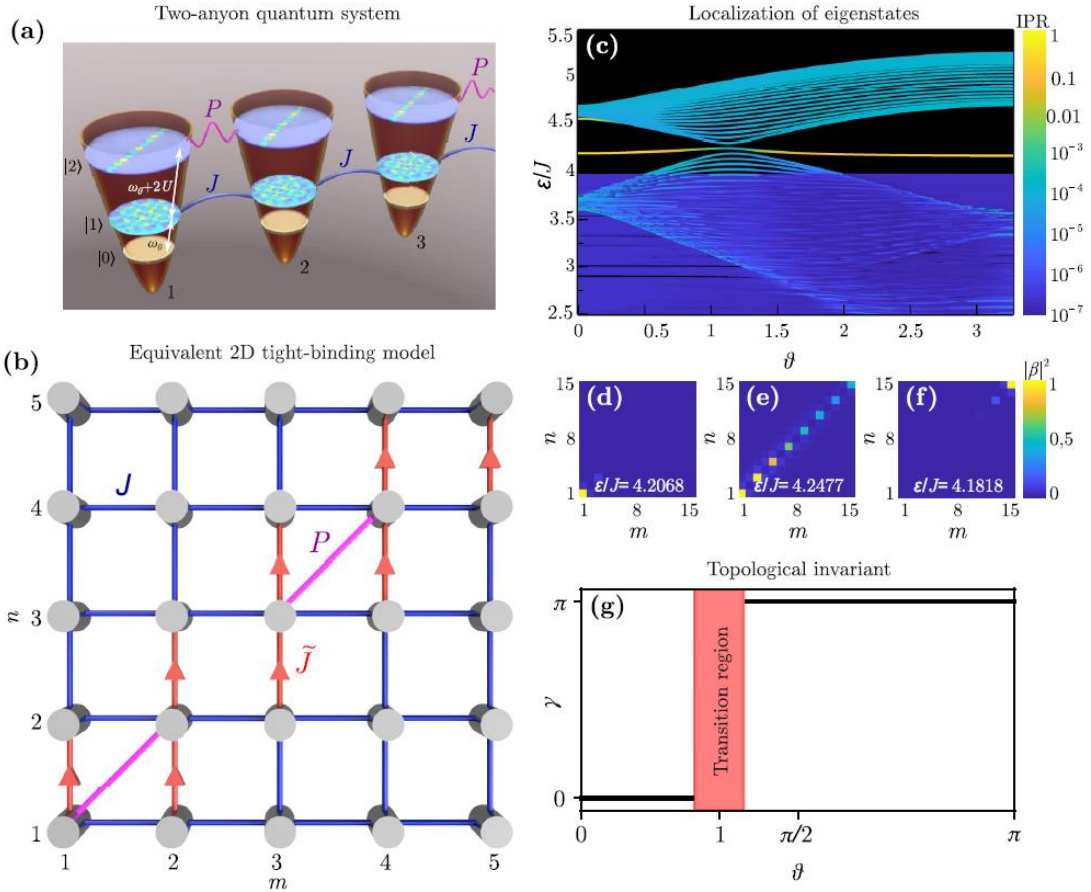
Hubbard model extended
by direct two-particle hopping

$$\hat{H} = \omega_0 \sum_{m=1}^{N} \hat{n}_m + U \sum_{m=1}^{N} \hat{n}_m(\hat{n}_m - 1) - J \sum_{m=1}^{N-1} (\hat{a}_m^\dagger \hat{a}_{m+1} + \text{H.c.}) + \frac{P}{2} \sum_{m=1}^{(N-1)/2} (\hat{a}_{2m-1}^\dagger \hat{a}_{2m-1}^\dagger \hat{a}_{2m} \hat{a}_{2m} + \text{H.c.}),$$

the commutation relations of anyon
creation and annihilation operators

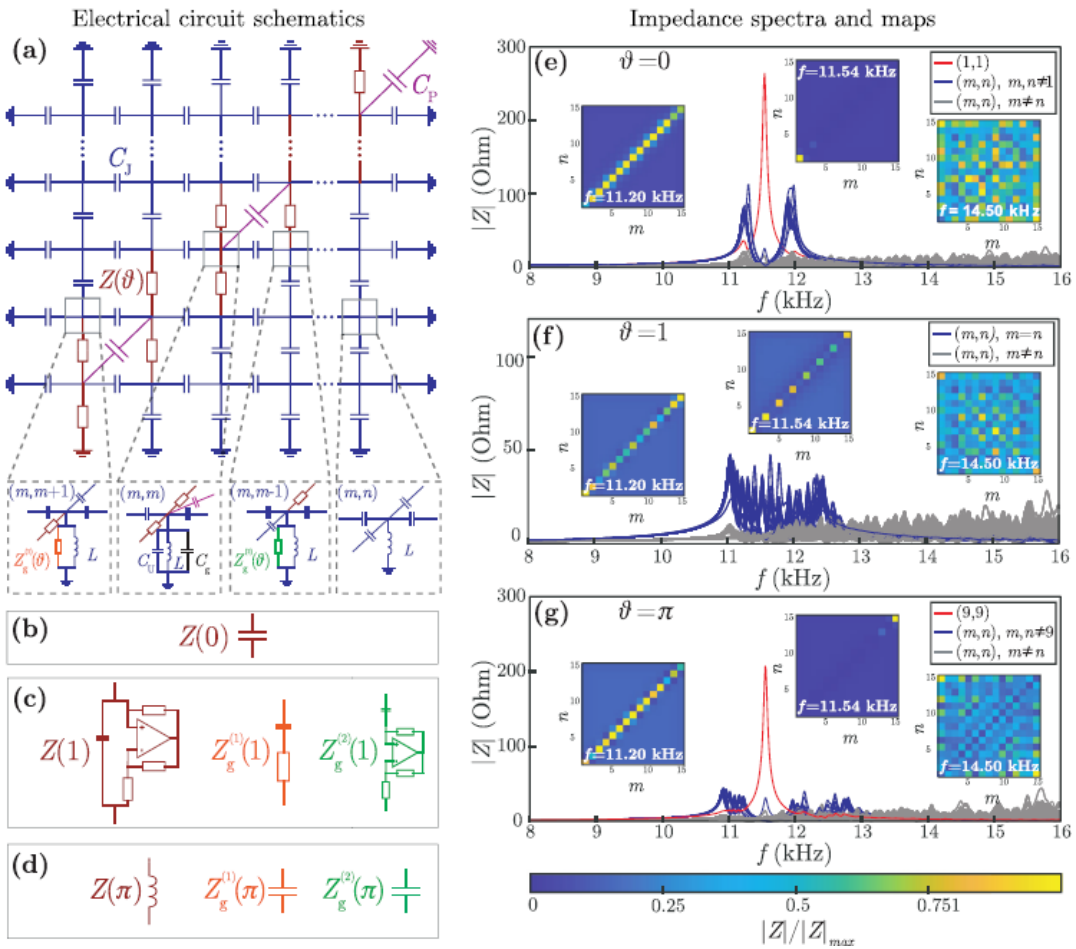
$$\hat{a}_l \hat{a}_k = \exp[i\theta\, \mathrm{sgn}(l-k)]\, \hat{a}_k \hat{a}_l,$$

$$\hat{a}_l \hat{a}_k^\dagger = \delta_{lk} + \exp[-i\theta\, \mathrm{sgn}(l-k)]\, \hat{a}_k^\dagger \hat{a}_l.$$



N.A. Olekhno et al, Phys. Rev. B **105**, 205113 (2022)

Emulating anyonic topological transitions



N.A. Olekhno *et al*, Phys. Rev. B **105**, 205113 (2022)