22 Solutions to the Hierarchy Problem

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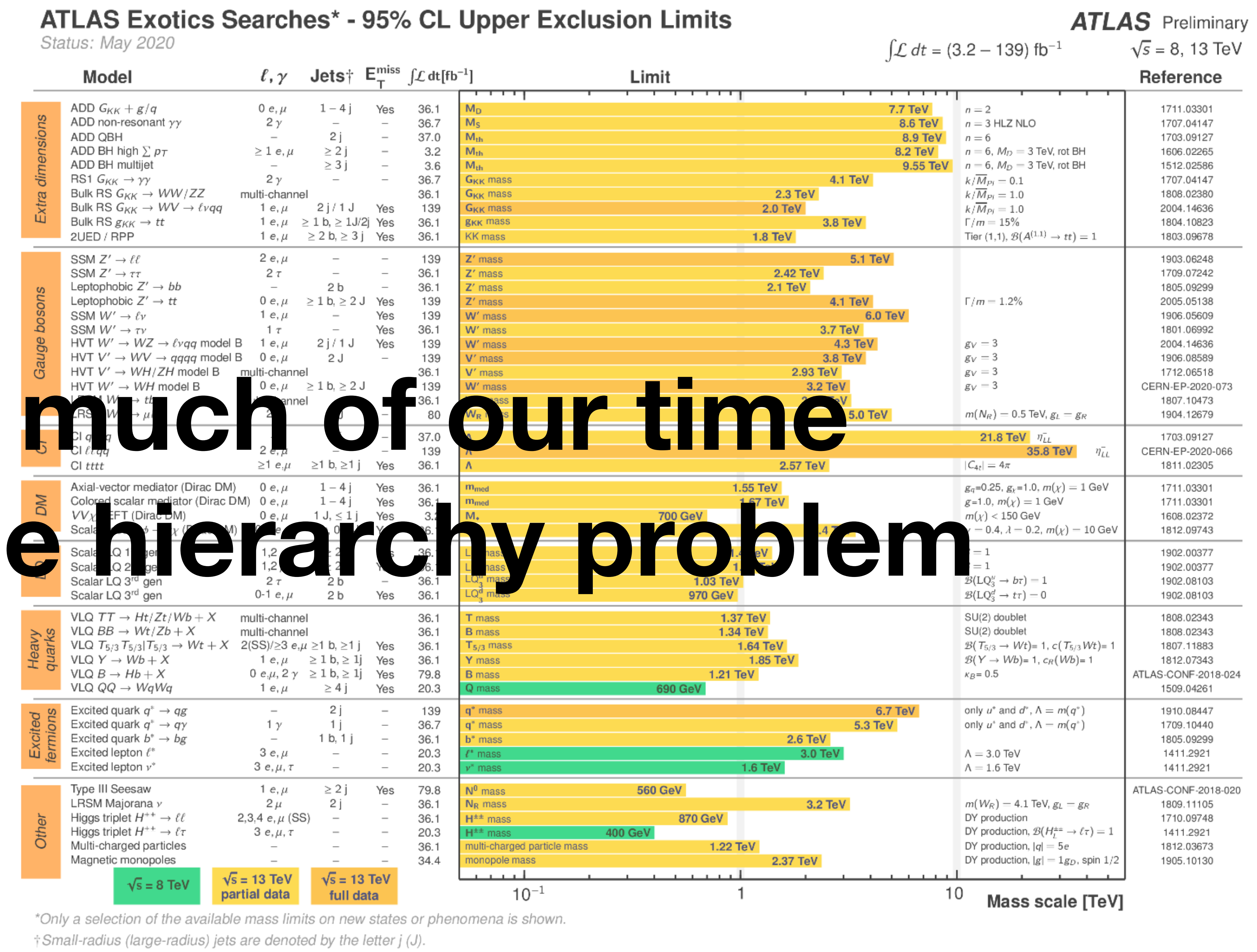
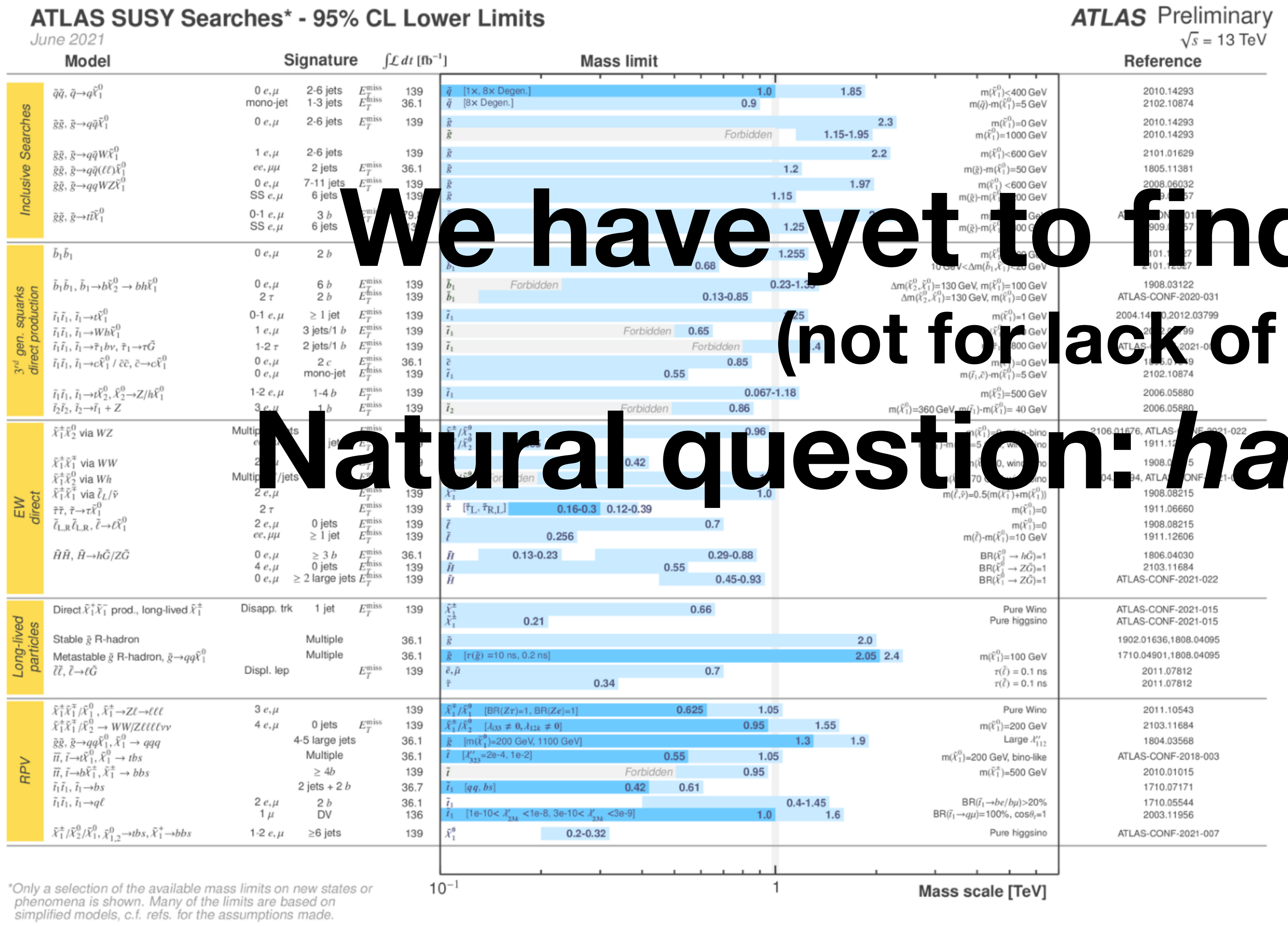
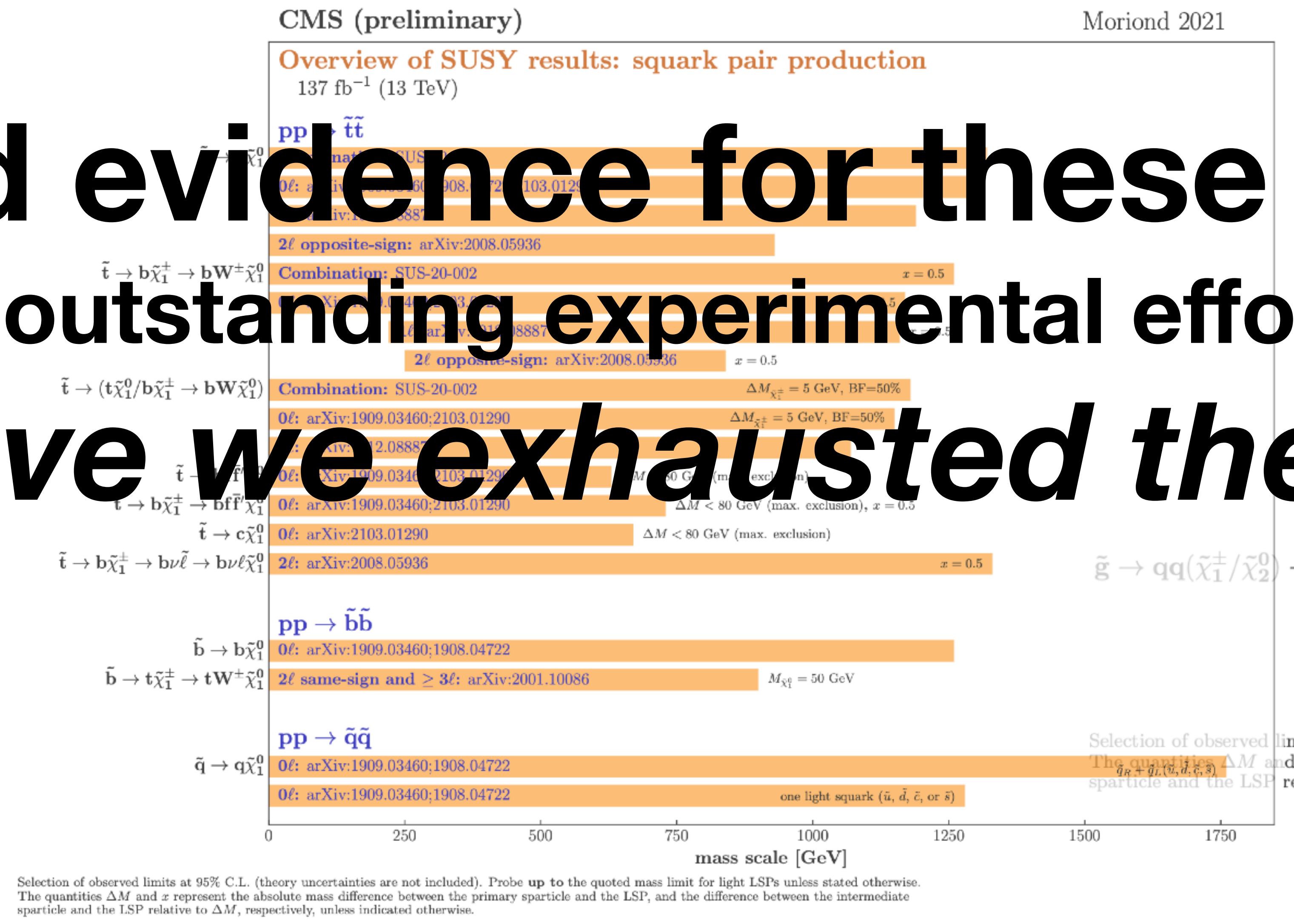
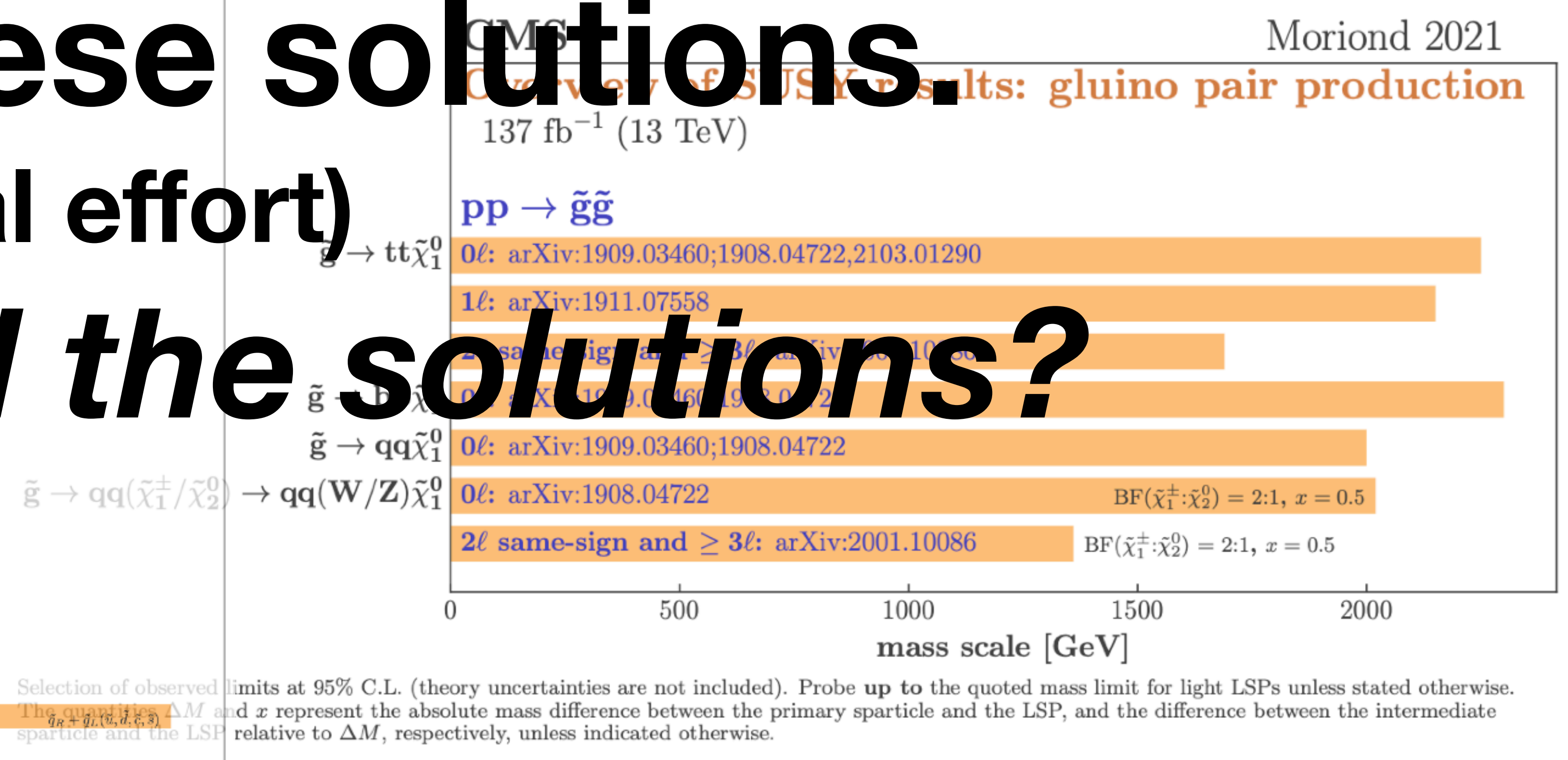


We (collectively) spend much of our time looking for solutions to the hierarchy problem





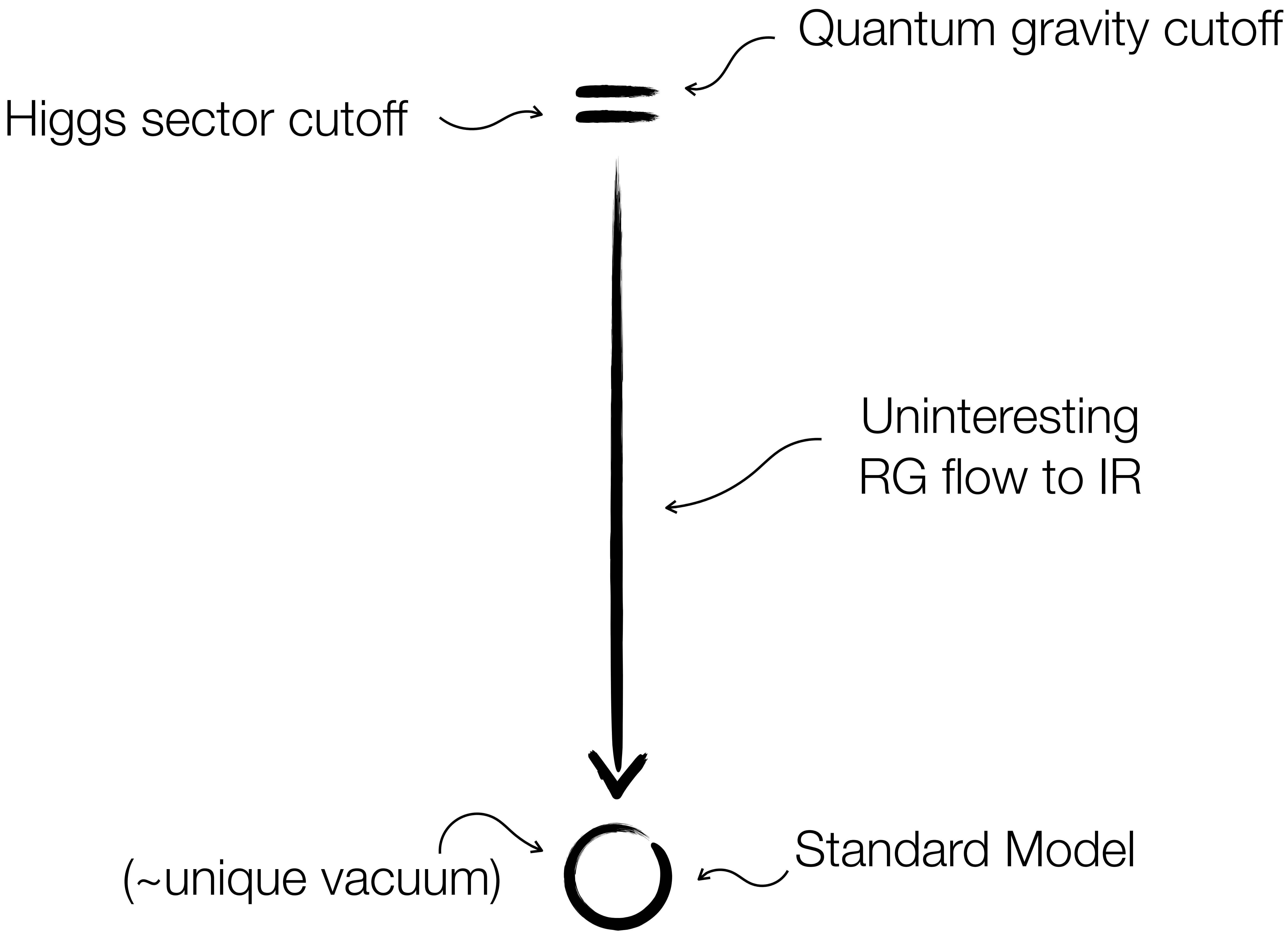


We have yet to find evidence for these solutions.

(not for lack of outstanding experimental effort)

Natural question: *have we exhausted the solutions?*

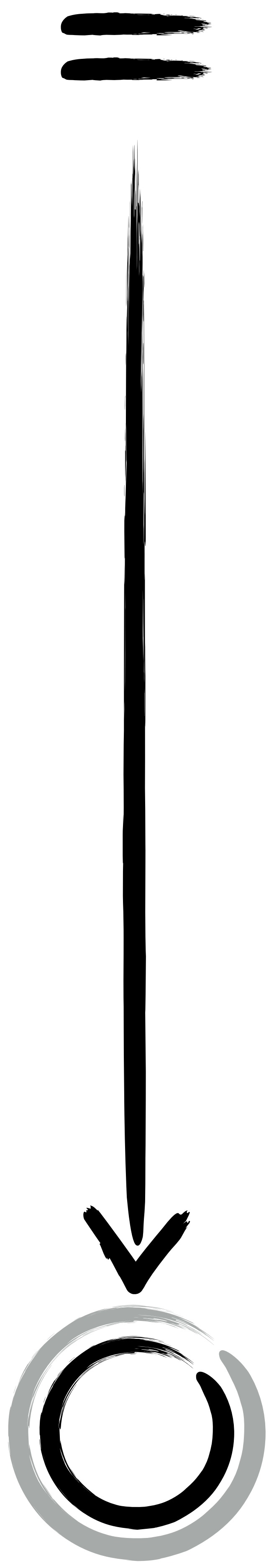
The Hierarchy Problem Cartoon



m_H is not technically natural ⇒ *hierarchy problem*

Adding a symmetry

…and (sometimes) breaking it softly

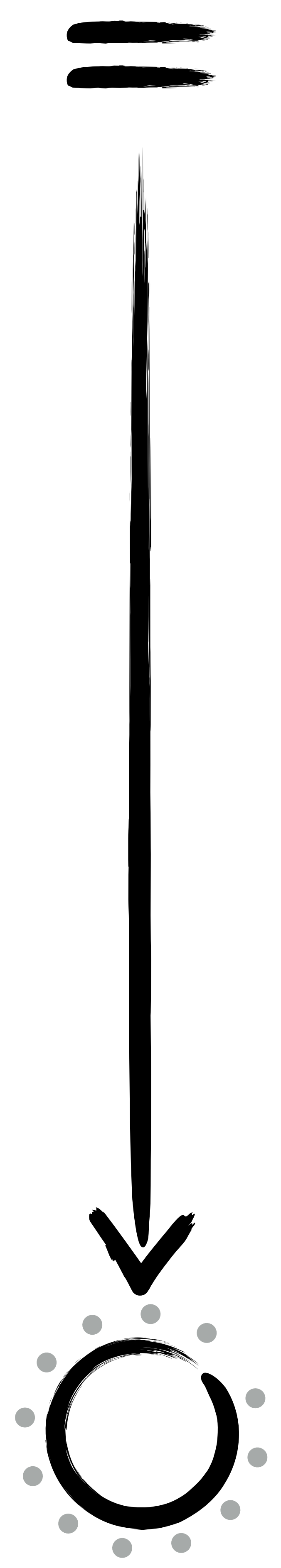


1. **Supersymmetry** (*a la* the electron)
2. **Global symmetry** (*a la* the pion)
3. **Discrete symmetry**
4. **Modular invariance**
   [Dienes et al. '94-'01, …]

Experimental signals: partner particles

- The familiar host of prompt signals (with or without missing energy)
- Rich variety of displaced decays (RPV, twin higgs, folded SUSY, …)

Discrete Symmetries



Consider a scalar *H* transforming as a fundamental under a global SU(4):

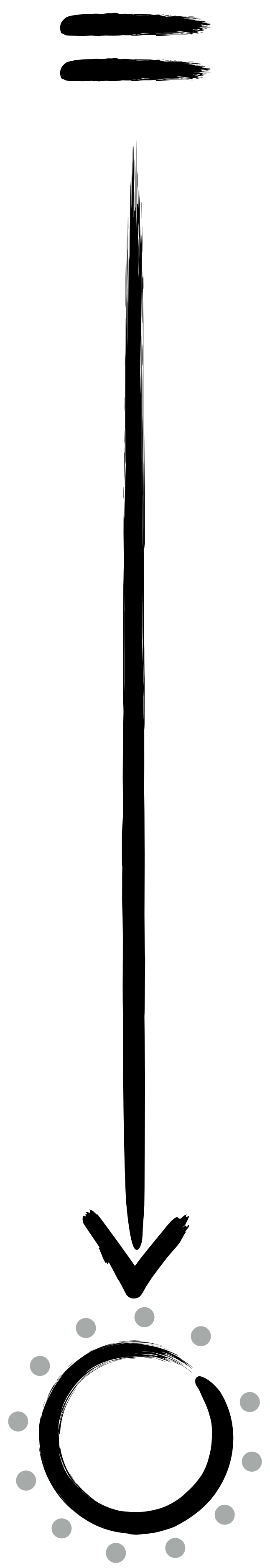
$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Potential leads to spontaneous symmetry breaking,

$$|\langle H\rangle|^2 = \frac{m^2}{2\lambda} \equiv f^2$$

$SU(4) \to SU(3)$ yields seven goldstone bosons.

Discrete Symmetries



Now gauge $SU(2)_A \times SU(2)_B \subset SU(4)$, w/ $$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$$

$\uparrow$ **Us** $\quad$ $\uparrow$ **Twins**

Then 6 goldstones are eaten, leaving one behind.

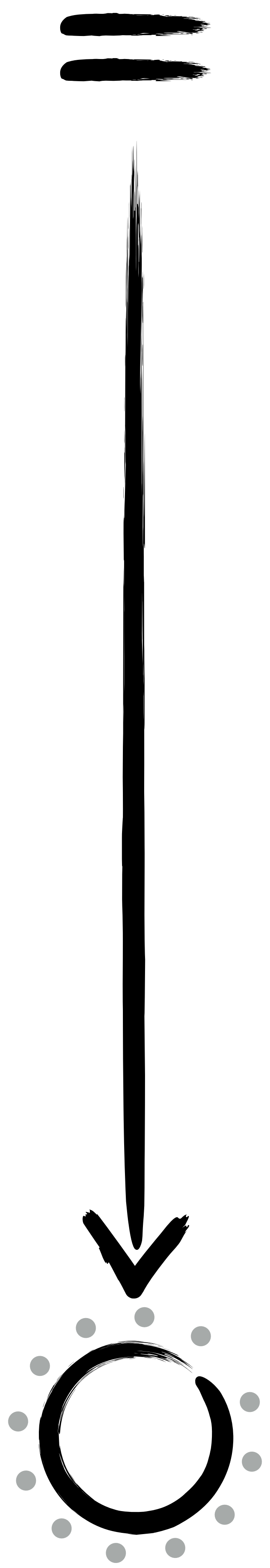
Explicitly breaks the SU(4); expect radiative corrections.

$$V(H) \supset \frac{9}{64\pi^2}\left(g_A^2\Lambda^2|H_A|^2 + g_B^2\Lambda^2|H_B|^2\right)$$

But these become SU(4) symmetric if $g_A = g_B$ from a Z_2

Quadratic potential has accidental SU(4) symmetry.

Discrete Symmetries



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Then 6 goldstones are eaten, leaving one behind.

Explicitly breaks the SU(4); expect radiative corrections.

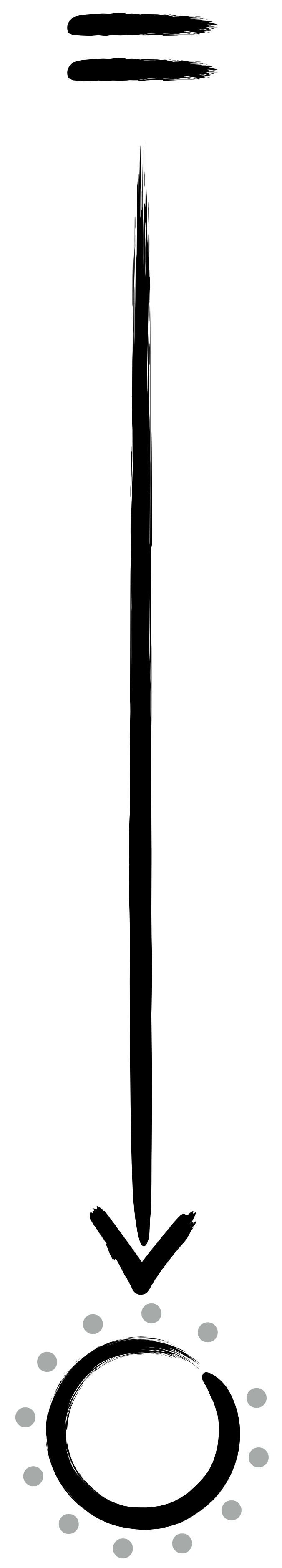
$$V(H) \supset \frac{9}{64\pi^2} g^2 \Lambda^2 \left(|H_A|^2 + |H_B|^2\right)$$

But these become SU(4) symmetric if $g_A = g_B$ from a Z_2

Quadratic potential has accidental SU(4) symmetry.

Twin Higgs

[Chacko, Goh, Harnik ‘05]



Z_2

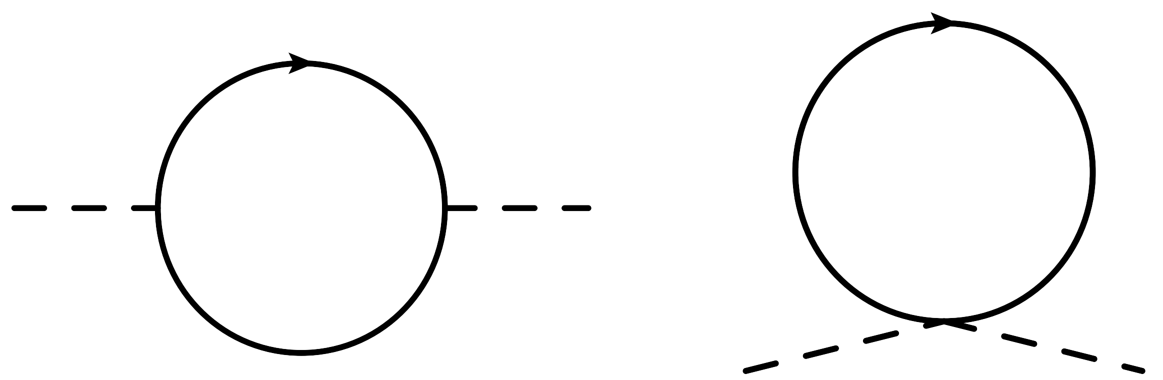
Standard Model ⟷ **Standard Model**

Radiative corrections to mass-squared are SU(4) symmetric thanks to Z_2:

$$V(H) \supset \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \ldots\right)\left(|H_A|^2 + |H_B|^2\right)$$

Higgs is a PNGB of ~SU(4), but partner states neutral under SM.

$$\mathcal{L} \supset -y_t \underbrace{H_A}_{h+\ldots} Q_3^A \bar{u}_3^A - y_t \underbrace{H_B}_{f - \frac{h^2}{2f} + \ldots} Q_3^B \bar{u}_3^B$$



[Curtin, Verhaaren '15]

		scalar	*fermion*
strong direct production	*QCD*	SUSY	Composite Higgs/ RS
DY direct production	*EW*	folded SUSY	Quirky Little Higgs
Higgs portal direct production	*singlet*	hyperbolic Higgs	Twin Higgs

Mirror Glueballs

Higgs portal observables

Higgs coupling shifts

$\sim$ tuning

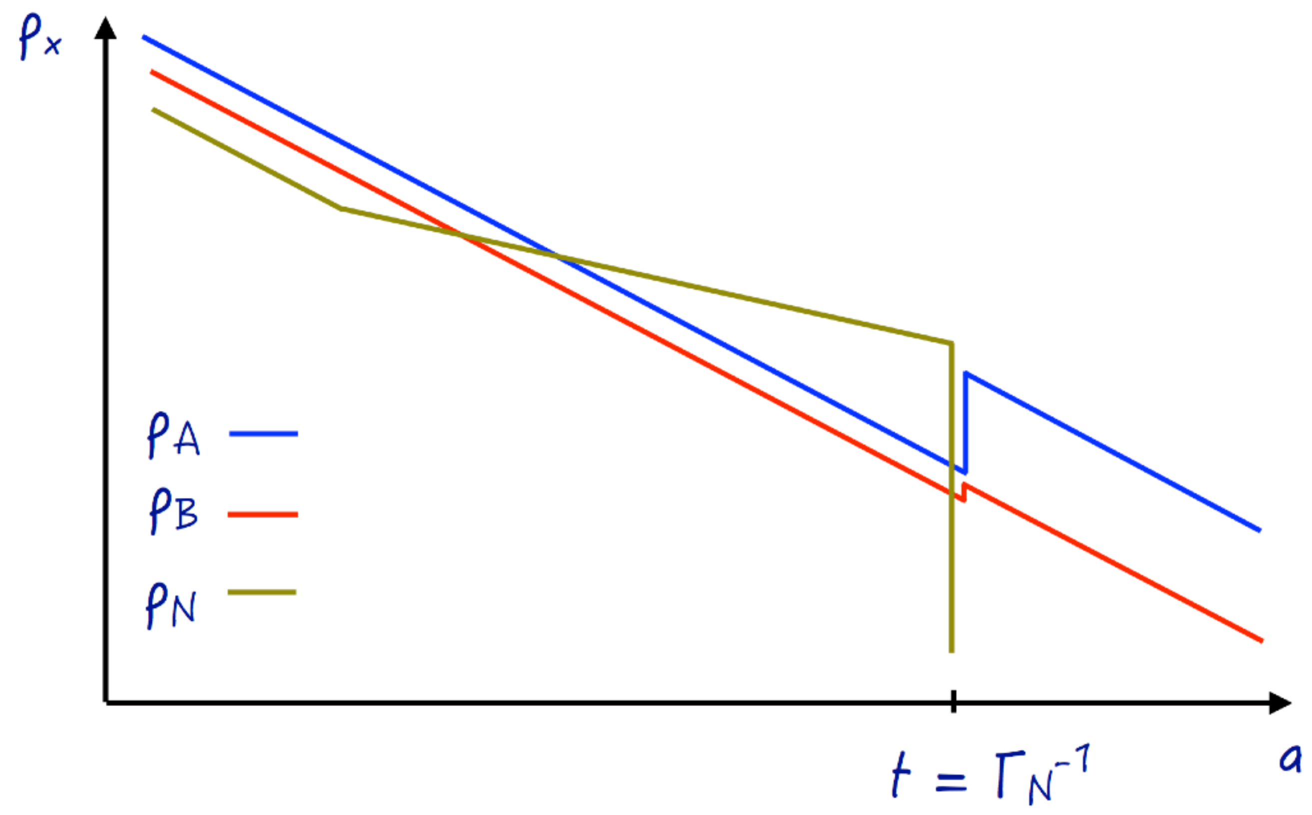
Cosmology is Key

The problem: thermal history of Z_2-symmetric theory has too much energy density in twin ν, γ

$$\Delta N_{\rm eff} \approx 7.4 \left.\frac{\rho_B}{\rho_A}\right|_{\rm BBN} \approx 5.6$$

Preserve symmetry & reconcile w/ current limits if energy density set by neutral particle *N* that

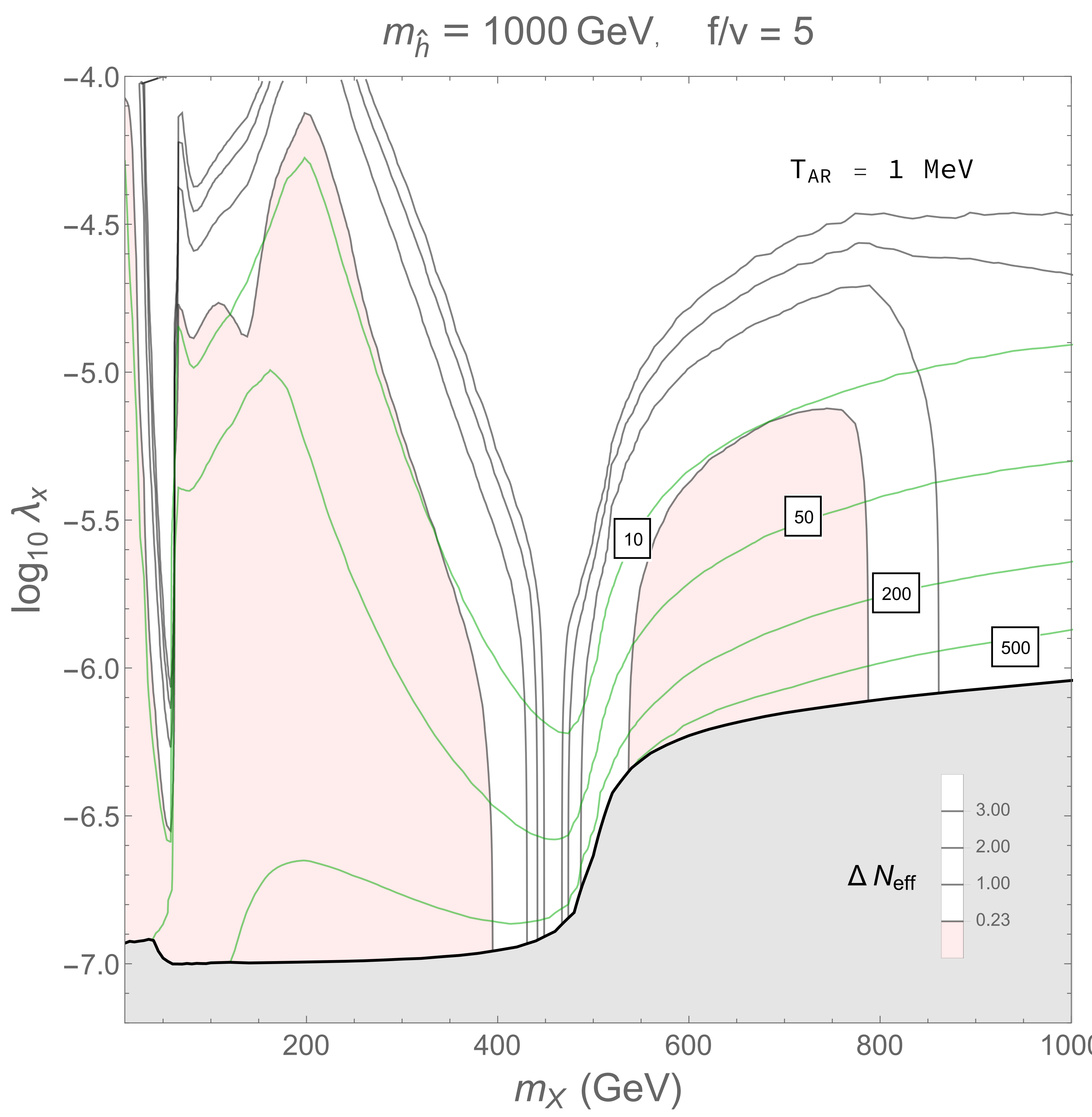
- decouples while relativistic
- decays some time thereafter
- decays primarily to A (SM)



Easy to do w/ symmetric coupling to H_A, H_B

[Chacko, NC, Fox, Harnik '16; NC, Koren, Trott '16]

Signals predominantly in CMB-S4



[Curtin, Gryba '21]

Modular Invariance

[Dienes et al. '94-'01, …]

At heart, solving hierarchy problem is about controlling $\delta m_H^2 \sim (\mathrm{Str}\mathcal{M}^0)\Lambda^2 + (\mathrm{Str}\mathcal{M}^2)\log\Lambda + ...$

For finite # of states $\mathrm{Str}\mathcal{M}^{2\beta} \equiv \sum_{\text{states } i} (-1)^F (M_i)^{2\beta}$ Cancellations require degenerate boson/fermion pairs

For infinite # of states, suitably regularize: $\mathrm{Str}\mathcal{M}^{2\beta} \equiv \lim_{y\to 0} \sum_{\text{states } i} (-1)^F (M_i)^{2\beta}\, e^{-yM_i^2}$

Then many *nondegenerate* spectra have vanishing supertraces

E.g. for masses $M_n = \sqrt{n}\mu$ and degeneracies g_n

$$g_n = \begin{cases} (-1)^n\, n^{2k} & \text{for any } k \geq 1,\ k \in \mathbb{Z} \\ (-1)^n\,(n^5 - n) \\ (-1)^n\,(n^5 + 2n^3) \end{cases}$$

n even: bosons
n odd: fermions

Generally satisfied when modular invariance controls the spectrum, degeneracies g_n given by the envelope of some suitable function $\Phi(n)$

Lowering the cutoff

…in diverse dimensions

5. **RS / Technicolor**
   [Randall, Sundrum '99; Weinberg '79; Susskind '79]
6. **LED / 10^{32} x SM**
   [Arkani-Hamed, Dimopoulos, Dvali '98; Antoniadis + ibid. '98; Dvali, Redi '09]
7. **LST / Clockwork**
   [Antoniadis, Dimopoulos, Giveon '01; Kaplan, Rattazzi '15; Giudice, McCullough '16]
8. **Classicalization**
   [Dvali, Giudice, Gomez, Kehagias '10]
9. **Disorder**
   [Rothstein '12]

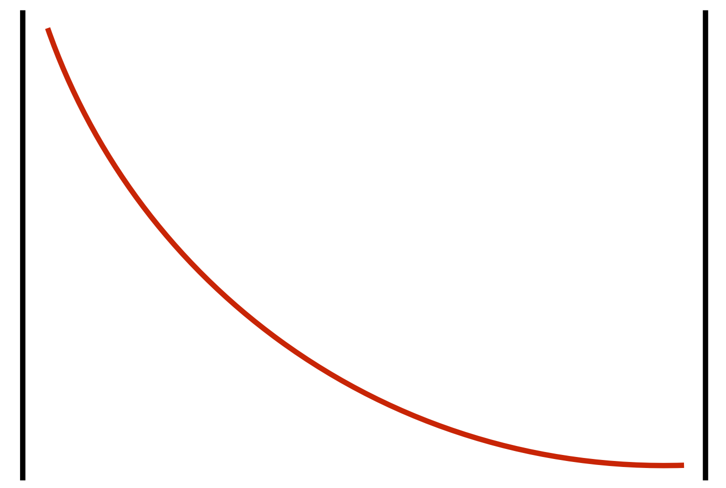
Experimental signals: resonances, …

- Primary distinctions are in spacing & coupling of resonances
- Potential goldmine of relatively unexplored signals for LST — e.g. perturbative string excitations

A Cutoff Solution?: Disorder

How does RS solve hierarchy problem?
Curvature localizes the graviton zero mode.

→ Fields localized at different points in 5th dimension see different fundamental scales



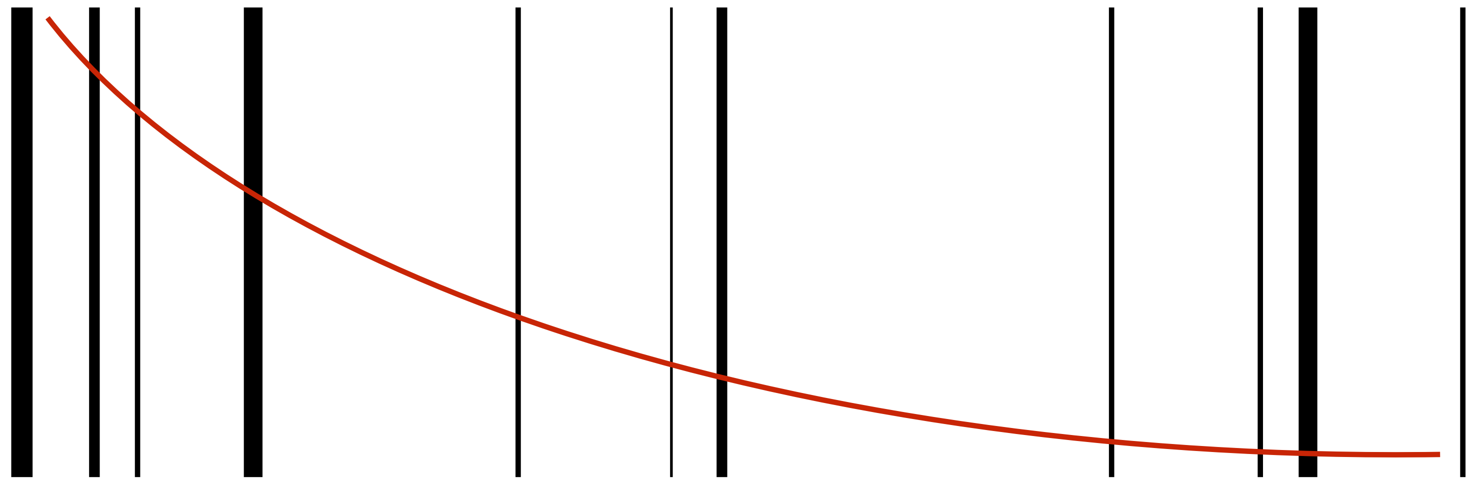
[Rothstein '12]: *Can achieve the same outcome in a flat fifth dimension by localizing graviton w/ disorder*

$$S = -\int d^5x \sqrt{G}(M_\star^3 \mathcal{R}) + \sum_{\langle ij \rangle} M_\star^4 V(|X_i - X_j|) - \sum_i \int d^4x \sqrt{g} f_i$$

In this case disorder = randomly spaced & tensioned branes

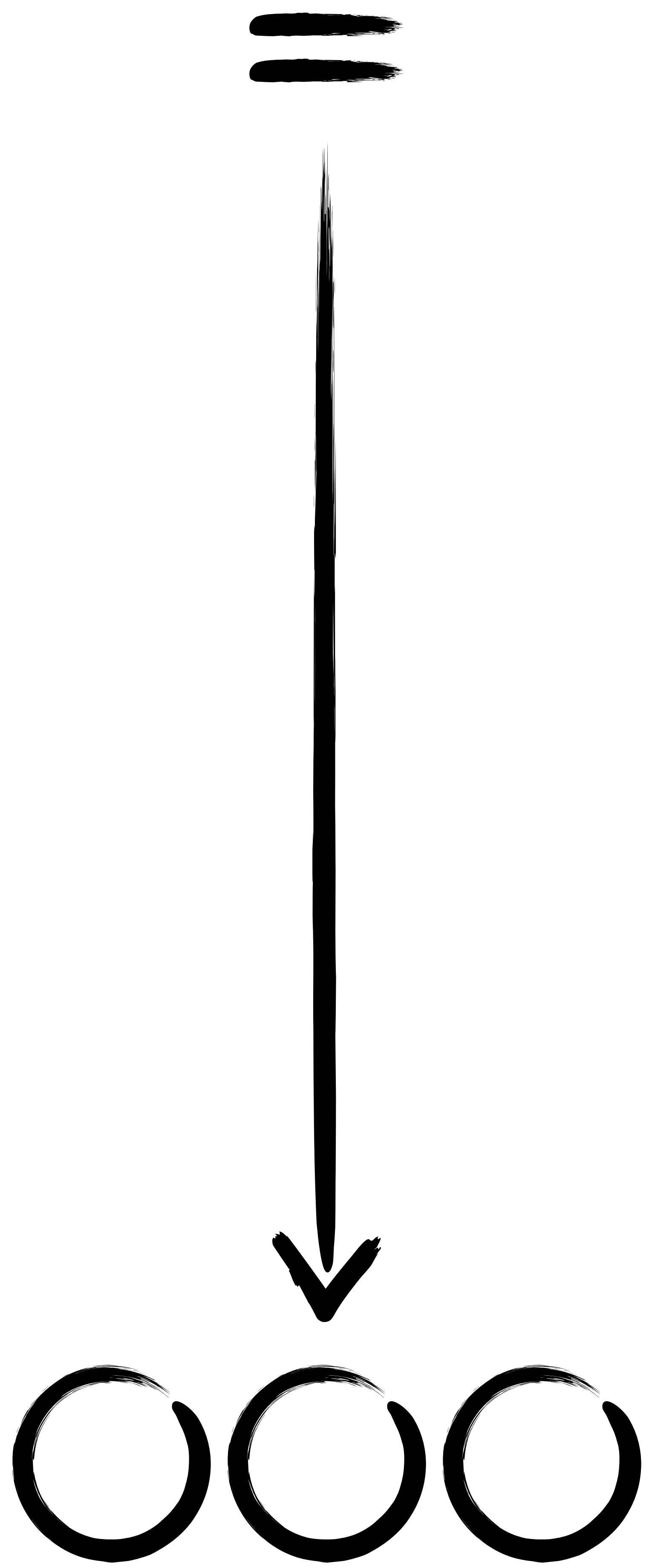
But: not obvious that it works in detail

An interesting source of exponential hierarchies for scalars [NC, Sutherland '17]



Selecting a vacuum

Vacuum is one of many; end up in observed vacuum through dynamical process or anthropic constraint.



10. Anthropics (pressure)

11. Relaxation (rolling) [Graham, Kaplan, Rajendran ‘15]

12. NNaturalness (reheating) [Arkani-Hamed et al ‘16]

13. Crunching away (collapse) [Csaki et al ’20, *see also* Geller, Hochberg, Kuflik ’18, Cheung & Saraswat ’18, …]

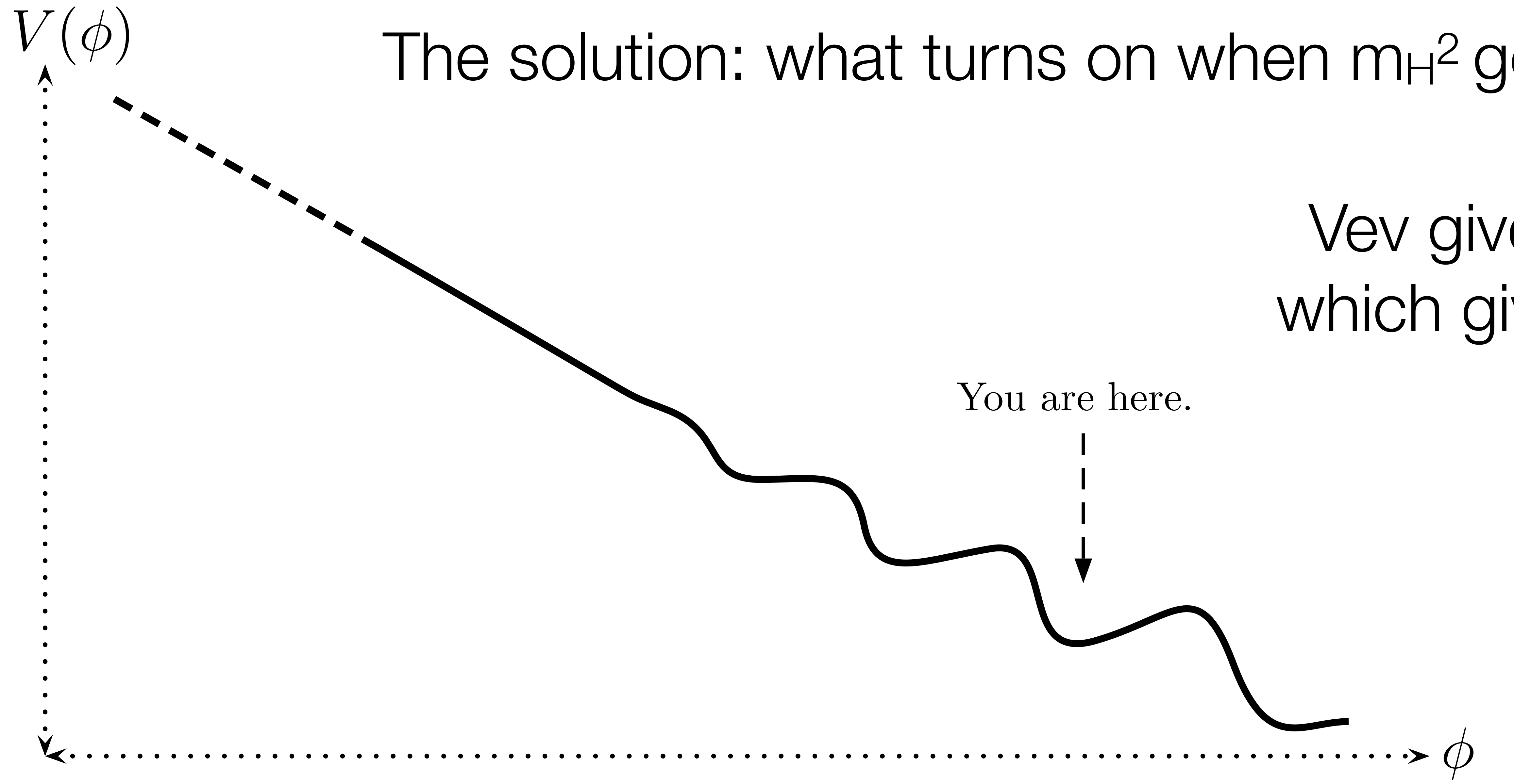
Experimental signals: Diverse, but typically

- Cosmology (Bubble collisions; axions; contributions to N_{eff} and $\sum m_\nu$)
- Exotic lab signals (displaced decays, hidden sector confinement, intensity frontier, …)

Relaxion

What if the weak scale is selected by dynamics, not symmetries?

The idea: couple Higgs to field whose minimum sets m_H=0
The problem: How to make m_H=0 a special point of potential?



The solution: what turns on when m_H^2 goes negative?

Vev gives quark masses which give axion potential.

"Relaxion"

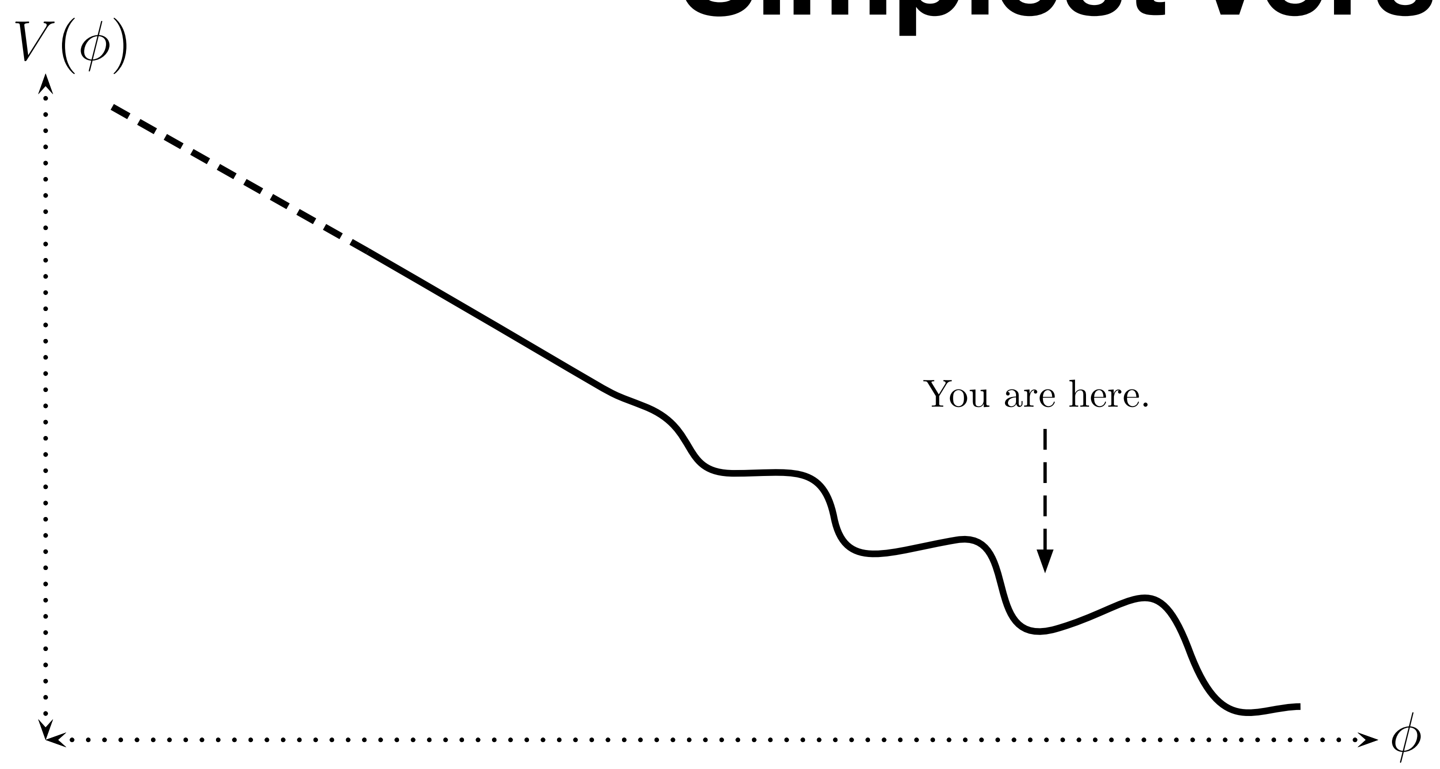
[Graham, Kaplan, Rajendran '15]

But: immense energy stored in evolving field, need dissipation.

[Graham, Kaplan, Rajendran ‘15]

Relaxion

Simplest version: an axion coupled to QCD during inflation.



$$\Lambda^4(H)\cos(\phi/f) + F(g\phi)$$
$$+\left(-M^2 + g\phi\right)|H|^2$$

Axion-like potential

Scanning term

Large bare Higgs mass

Viable for Higgs + non-compact axion + inflation w/

- Very low Hubble scale ($\ll \Lambda_{QCD}$)
- 10 Giga-years of inflation

Why not? Various other subtleties regarding technical naturalness, trans-Planckian field excursions, CC, fine-tuning to inflationary sector; need to solve strong CP problem. *New UV considerations.*

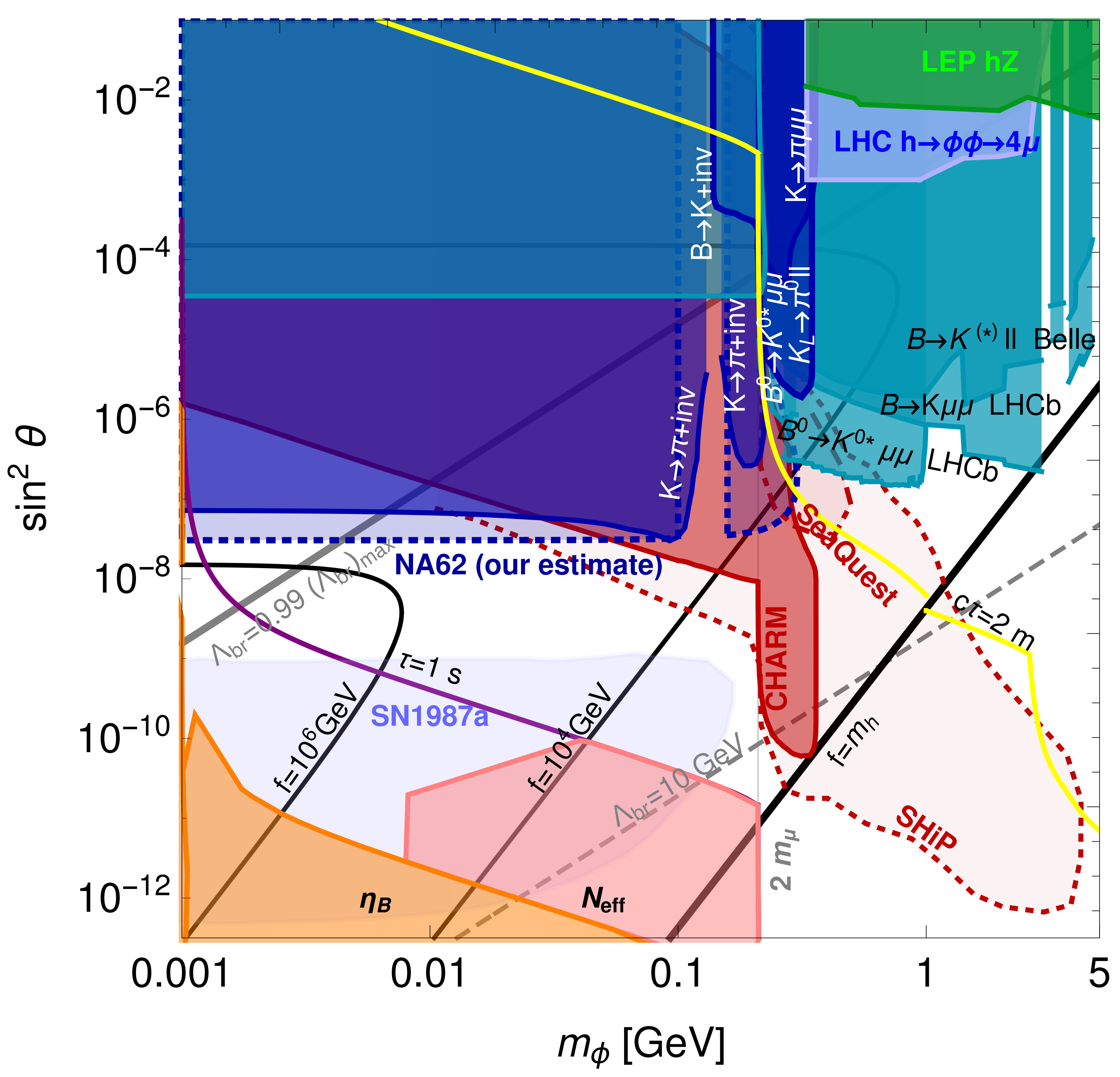
Extensive development, e.g. [Espinosa et al. ’15; Hardy ’15; Gupta et al ’15; Batell, Giudice, McCullough ’15; Choi, Im ’15; Kaplan, Rattazzi ’15; Di Chiara et al. ’15; Ibanez et al. ’15; Hook, Marques-Tavares ’16; Nelson, Prescod-Weinstein ’17; …]

New Signals

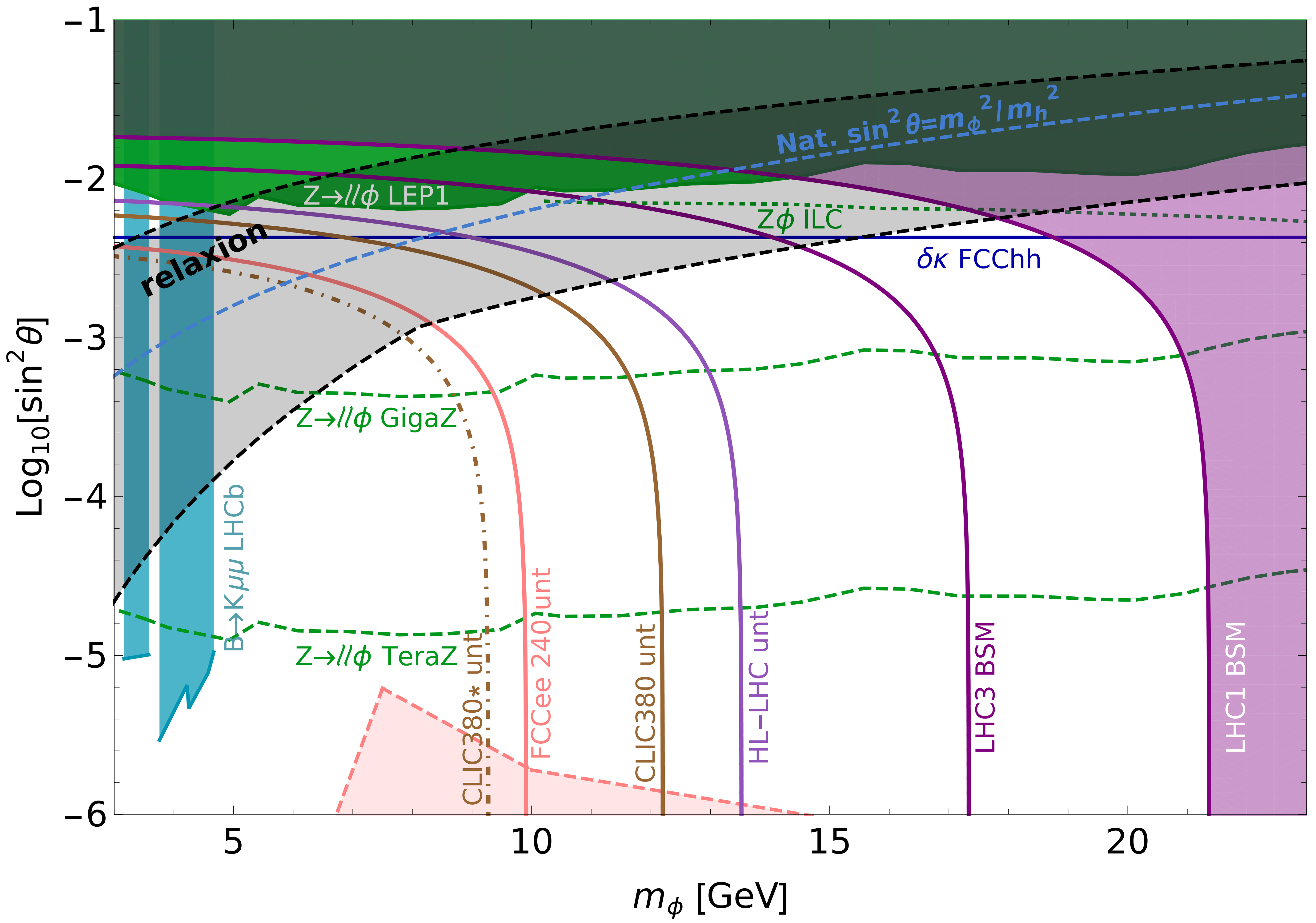
Higgs portals $g\phi|H|^2$ $\Lambda^4(H)\cos(\phi/f)$

$\Lambda^4(H)\cos(\phi/f)$ gives ϕ - H mixing* w/ $\sin\theta \approx \dfrac{\Lambda^4}{v f m_h^2}$

[Flacke, Friguele, Fuchs, Gupta, Perez ‘16]



[Fuchs, Matsedonskyi, Savoray, Schlaffer ’20]

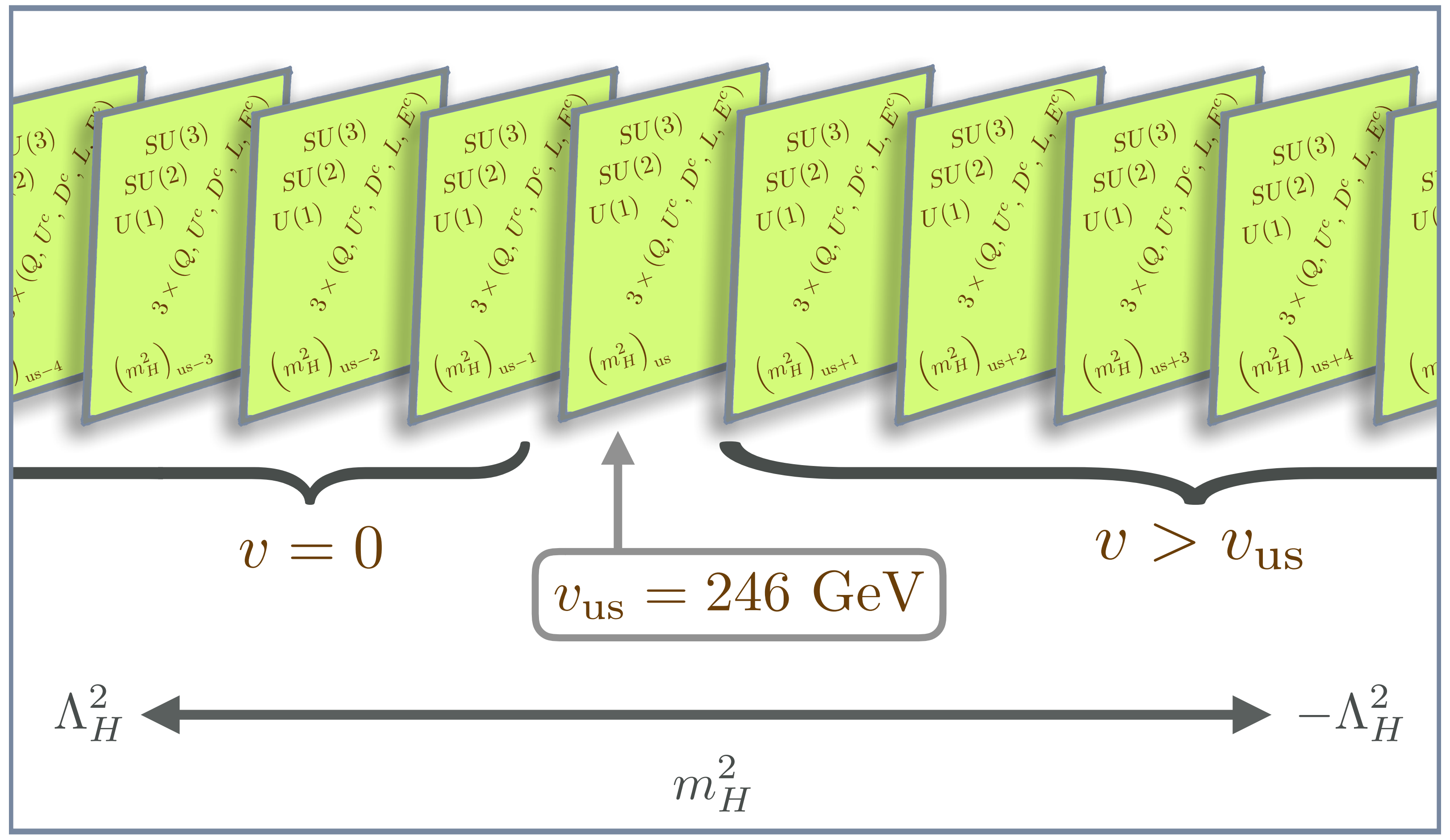


+5th force for $m_\phi <$ eV & cosmology for eV $< m_\phi <$ MeV

*assuming ⟨ϕ⟩ breaks CP

[Arkani-Hamed, Cohen, D'Agnolo, Hook, Kim, Pinner '16]

NNaturalness

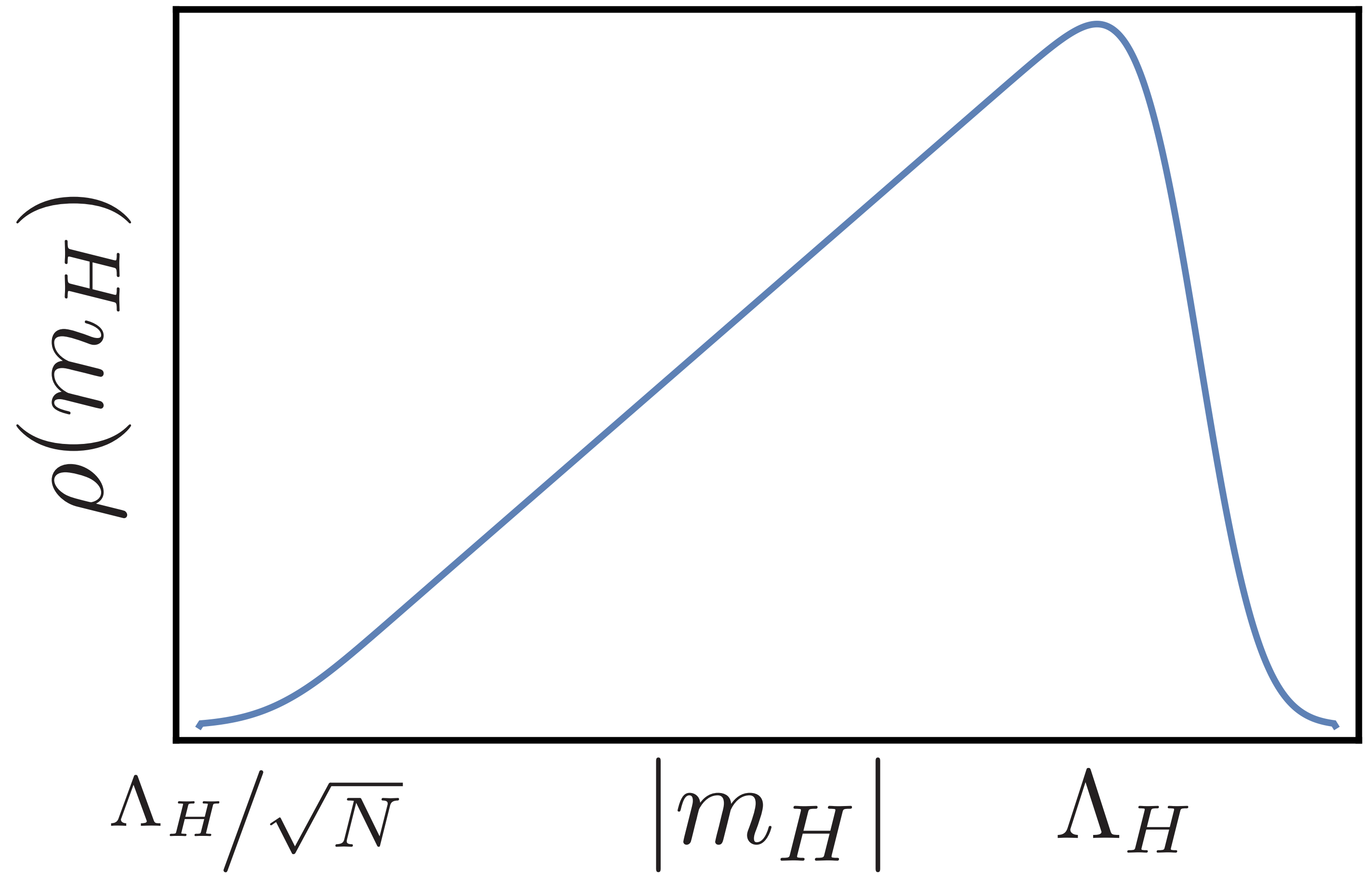


N copies of the SM

High Higgs cutoff Λ_H, high gravity cutoff Λ_G

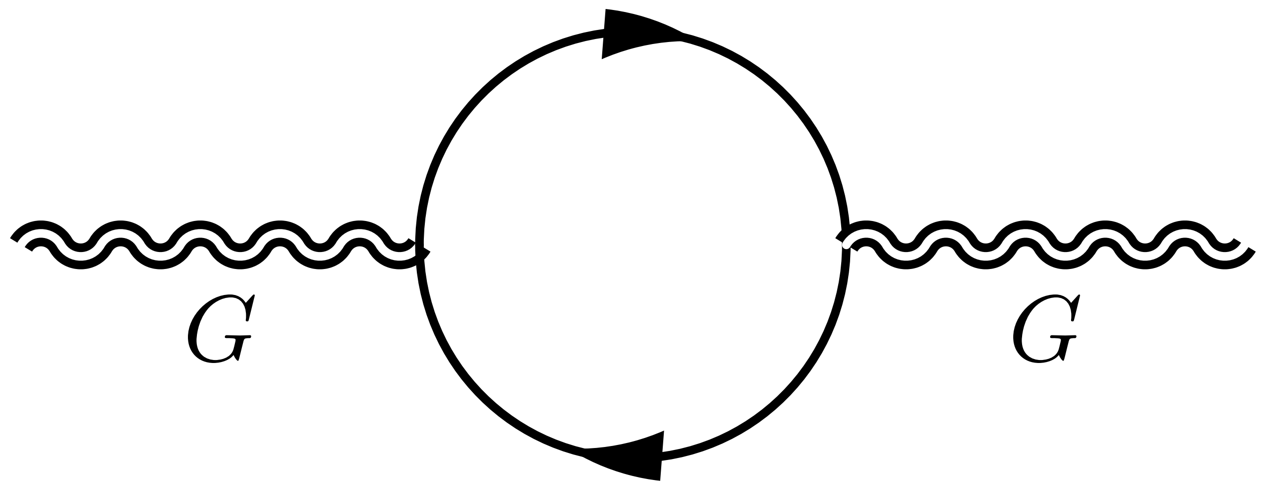
Two effects:

1. Random UV contributions → flat distribution of m_H^2 between $\pm\Lambda_H^2$



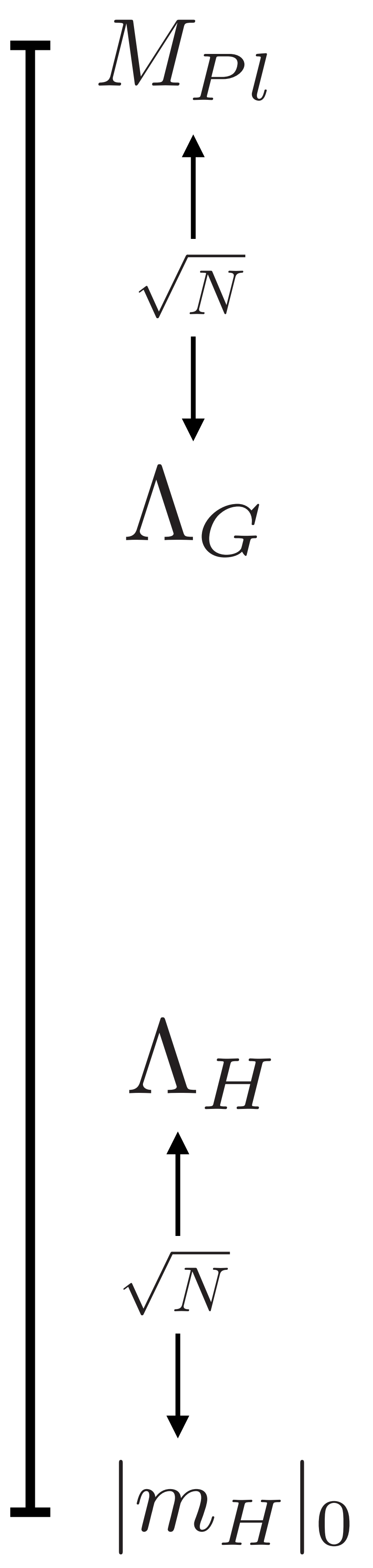
At least 1 copy w/ $|m_H| \sim \Lambda_H/\sqrt{N}$

2. Large number of species renormalizes Planck scale (e.g. graviton wavefunction renorm.)



Gravitational strong coupling scale Λ_G below M_{Pl} $\quad M_{Pl}^2 \sim N\Lambda_G^2$

NNaturalness



Scale separation

For example:
One copy w/ weak-scale Higgs for

N=10^{16}:
Λ_H=10^{10} GeV
Λ_G=10^{10} GeV
(That's it.)

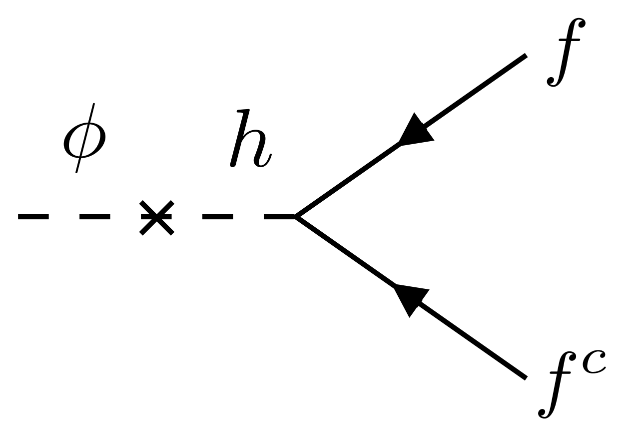
N=10^4:
Λ_H=10^4 GeV
Λ_G=10^{16} GeV
(SUSY/compositeness @ Λ_H)

Why does copy w/ smallest m_H dominate?
Cosmology.

Reheaton ϕ starts universe via ϕ $|H_i|^2$ couplings

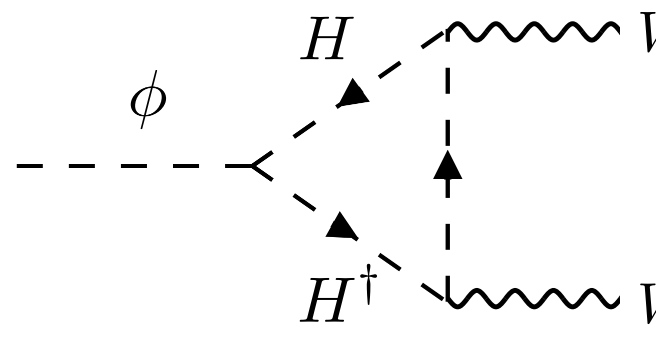
Decays (provided $m_\phi < |m_{H_i}|$)

$$m^2_{H,i} < 0$$



$$\Gamma \propto \frac{1}{m^2_{H,i}}$$

$$m^2_{H,i} \geq 0$$



$$\Gamma \propto \frac{1}{m^4_{H,i}}$$

Preferentially reheats copy w/ smallest $|m_H|$ & $m_H^2<0$

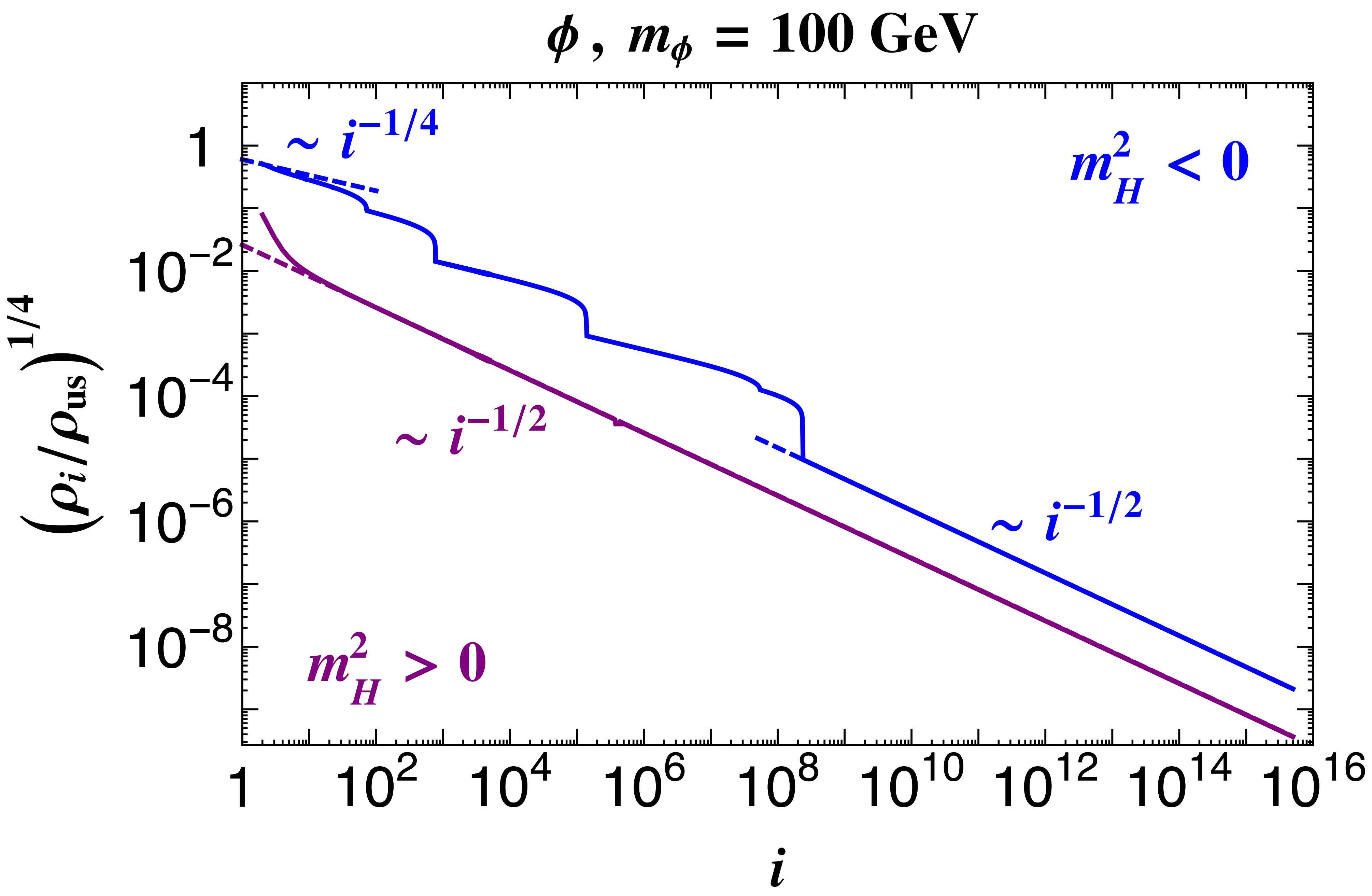
N Higgses…in the sky

All sectors reheated by some amount ⇒ dark radiation

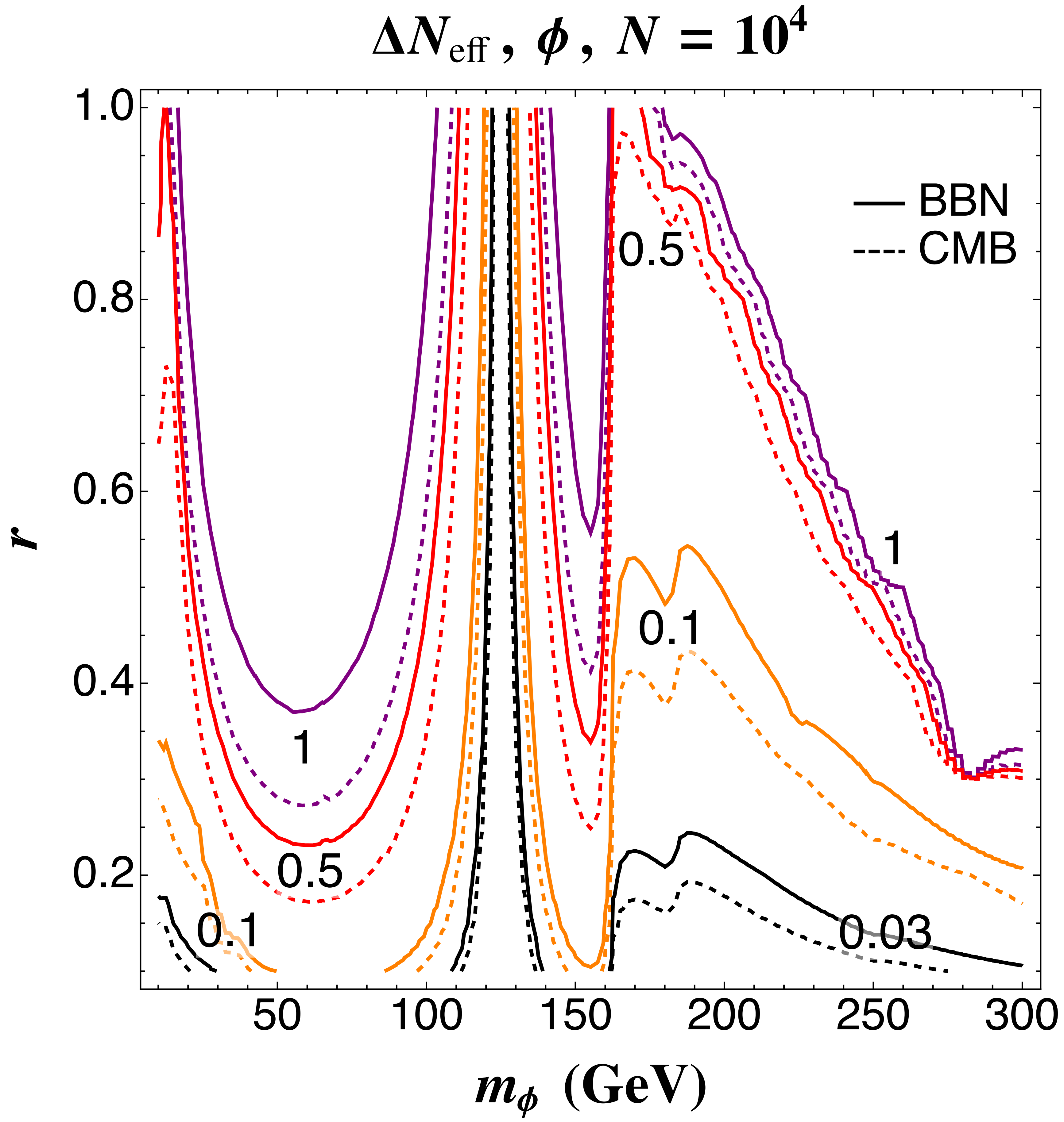
$$\frac{\rho_i}{\rho_{\rm us}} = \frac{\Gamma_i}{\Gamma_{\rm us}}$$

Dominated by sectors with similar scales

[Arkani-Hamed, Cohen, D'Agnolo, Hook, Kim, Pinner '16]



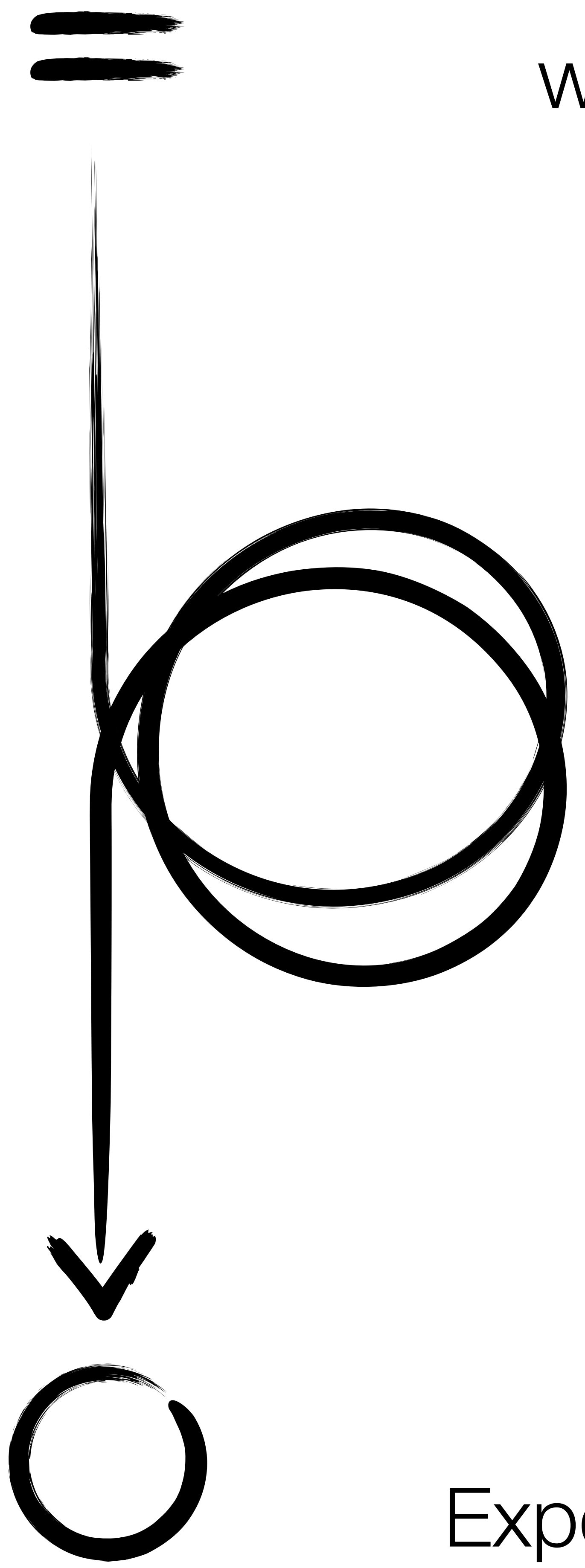
Primary signals in dark radiation, extensive coverage by CMB-S4



[Arkani-Hamed, Cohen, D'Agnolo, Hook, Kim, Pinner '16]

(r=1 ↔ flat m_H^2; r<1 ↔ larger splitting)

Complicating the flow



SM is reached from some intermediate fixed point
where, say, a generalized Veltman condition is satisfied

$$\delta m_H^2 = \sum_i c_i \frac{g_{i,\star}^2}{16\pi^2}\Lambda_i^2 = 0$$

This is a sense in which

14. Conformal symmetry

could address the hierarchy problem

Top-down: Embed SM in orbifold of N=4 SYM
[Frampton, Vafa '99; Csaki, Skiba, Terning '99]

Bottom-up: "Little conformal symmetry"
[Houtz, Colwell, Terning '16]

A challenge: how do fixed point couplings know about UV scale?

Experimental signals: Not fully explored, but expect new particles w/ SM quantum numbers around the TeV scale. Novelty is that statistics, irreps & couplings differ from more familiar solutions.

Exploding the cutoff

Gravity doesn't provide a UV scale & the SM takes care of itself

15. Asymptotic fragility
[Dubovsky, Gorbenko, Mirbabayi '13]

16. Agravity [Salvio, Strumia '14]

Scale M_{Pl} not associated with relevant operator becoming strong, not "felt" by non-grav physics.

In IR, looks like CFT perturbed by irrelevant operators; in UV, no UV fixed point; cannot define local observables.

Example in 2d, no proposal for 4d.

Gravity has no intrinsic length scale and is "renormalizable"

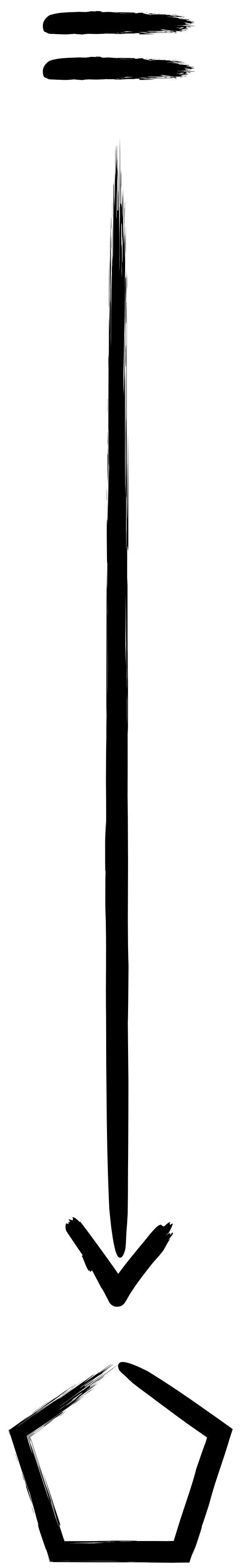
$$S \sim \int d^4x\sqrt{g}\left(\frac{R^2}{f_1^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \dots\right)$$

(E-H term via vev of some field)

Can be re-written in terms of 2-derivative fields w/ ghosts.

Experimental signals: Details of gravity sector might be irrelevant. Crucially, must render SM couplings asymptotically free. Not a property of the SM itself, so entails low-scale unification

Not actually the SM



17. Lee-Wick (higher derivative scalar)

[Grinstein, O'Connell, Wise '06]

Lee-Wick: higher-derivative theory

$$\sim \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2M^2}(\partial^2 \phi)^2 + \ldots$$

Write as normal field + new field w/ wrong-sign quadratic action

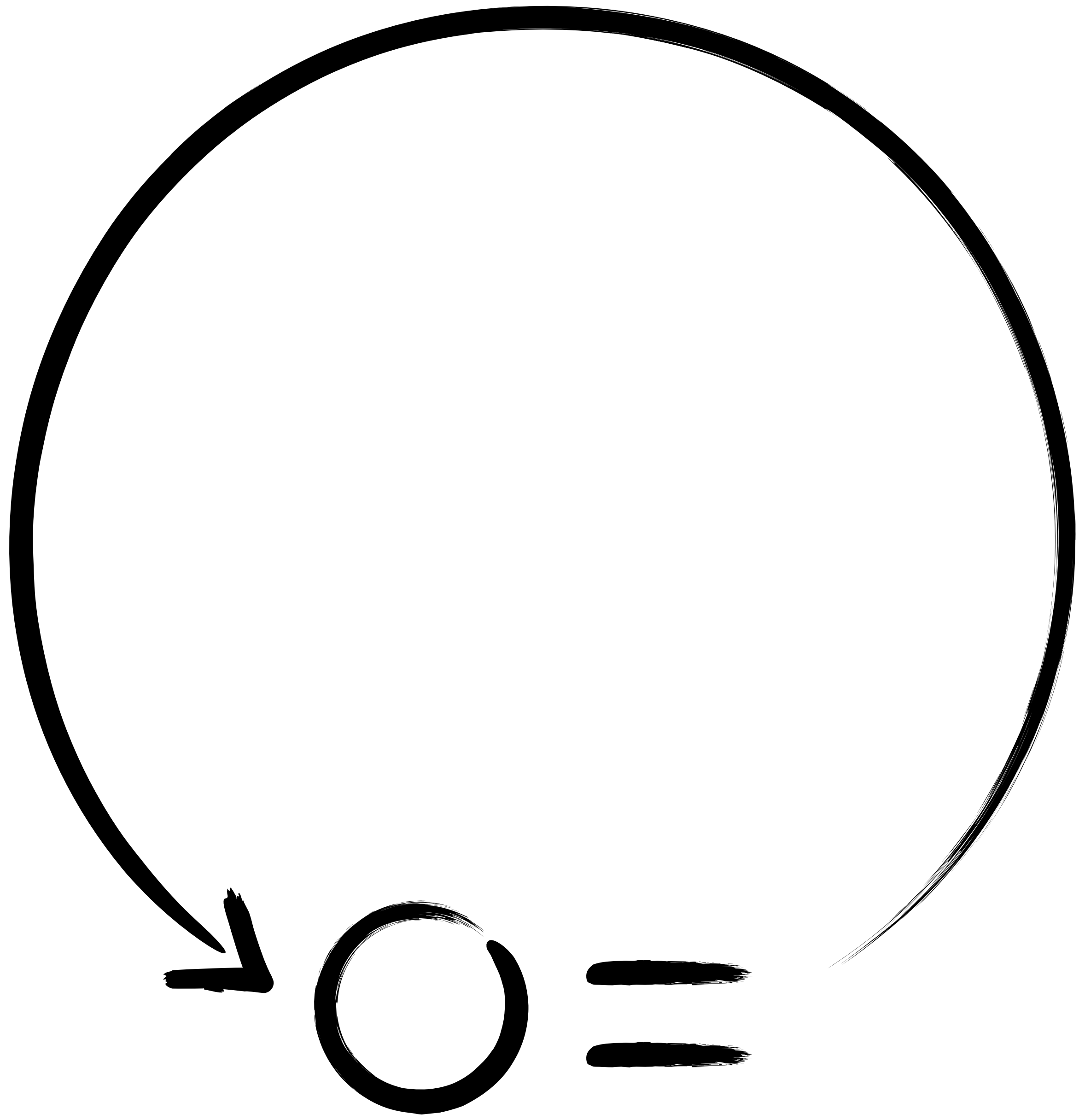
$$-\frac{1}{2}\partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \frac{1}{2}M^2 \tilde{\phi}^2 + \ldots$$

Improves UV convergence of diagrams, introduce for every SM field

$$\frac{1}{p^2 - m^2} - \frac{1}{p^2 - M^2} = \frac{m^2 - M^2}{(p^2 - m^2)(p^2 - M^2)}$$

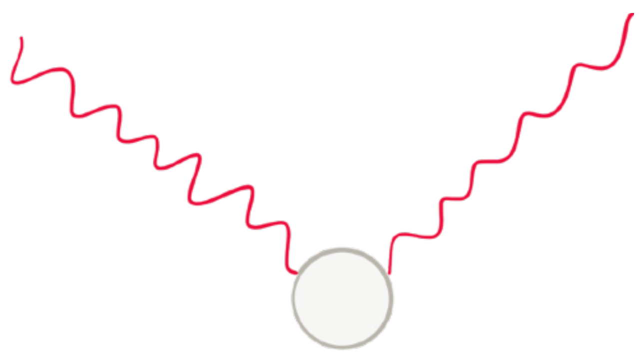
Can be defined in a unitary, Lorentz-invariant manner with only microscopic acausality. But who ordered that?

Connecting UV & IR

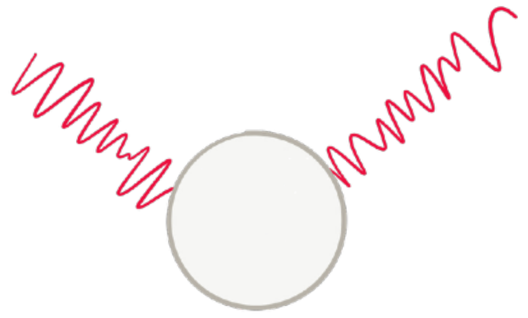


Essential feature of the hierarchy problem: the UV doesn't know about the IR…unless it does?

Two frameworks exhibiting UV/IR mixing: QG & NCQFT



QG (cartoon version): collide sufficiently energetic particles, make a black hole. More energetic particles → bigger black hole.



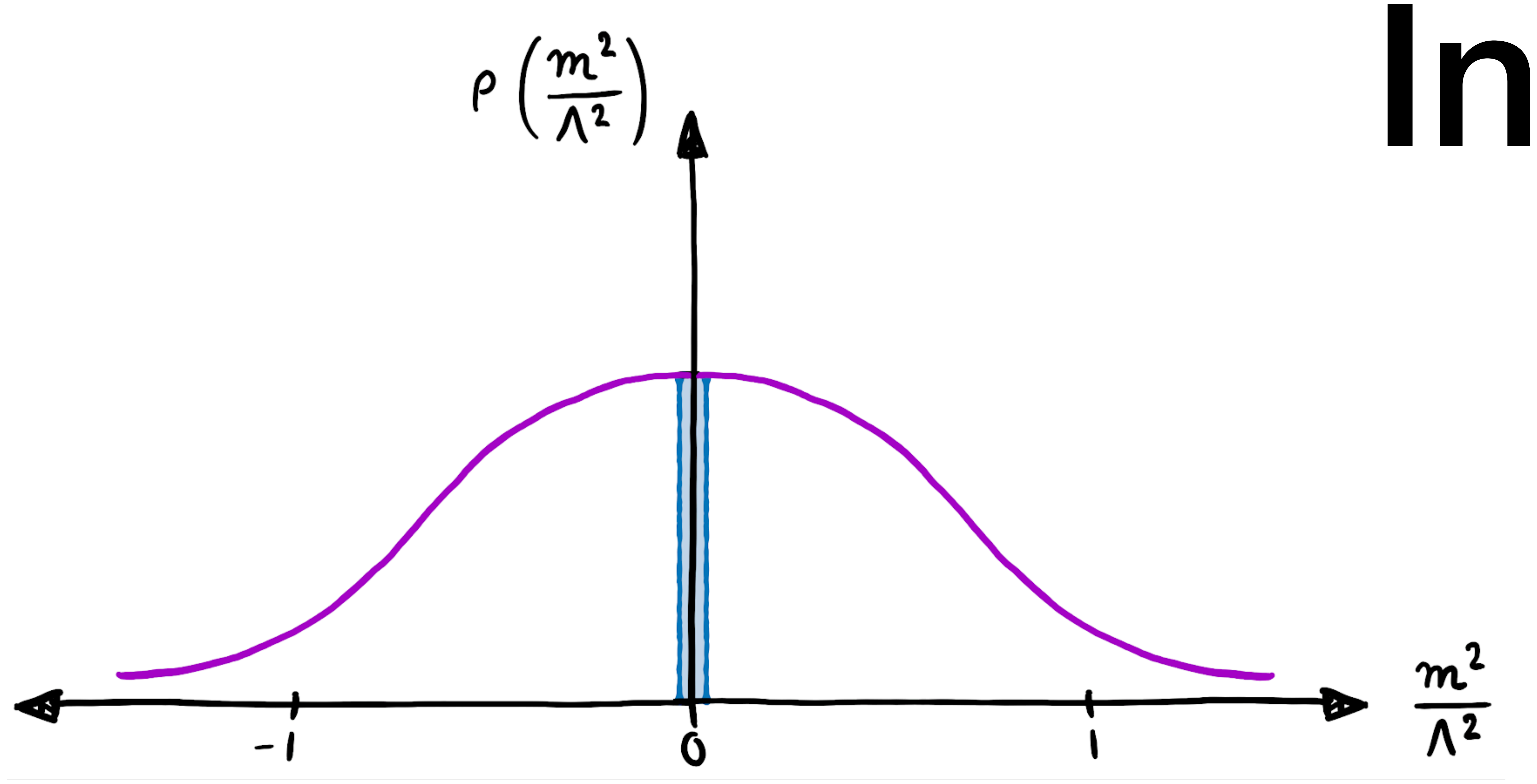
NCQFT (cartoon version): non-commutativity of the form $[x^\mu,x^\nu]=i\theta^{\mu\nu}$, qualitatively a position-position uncertainty principle $\Delta x^\mu \Delta x^\nu \geq \theta/2$ [Filk '96, Minwalla, Seiberg, Van Raamsdonk '99, NC, Koren '19]

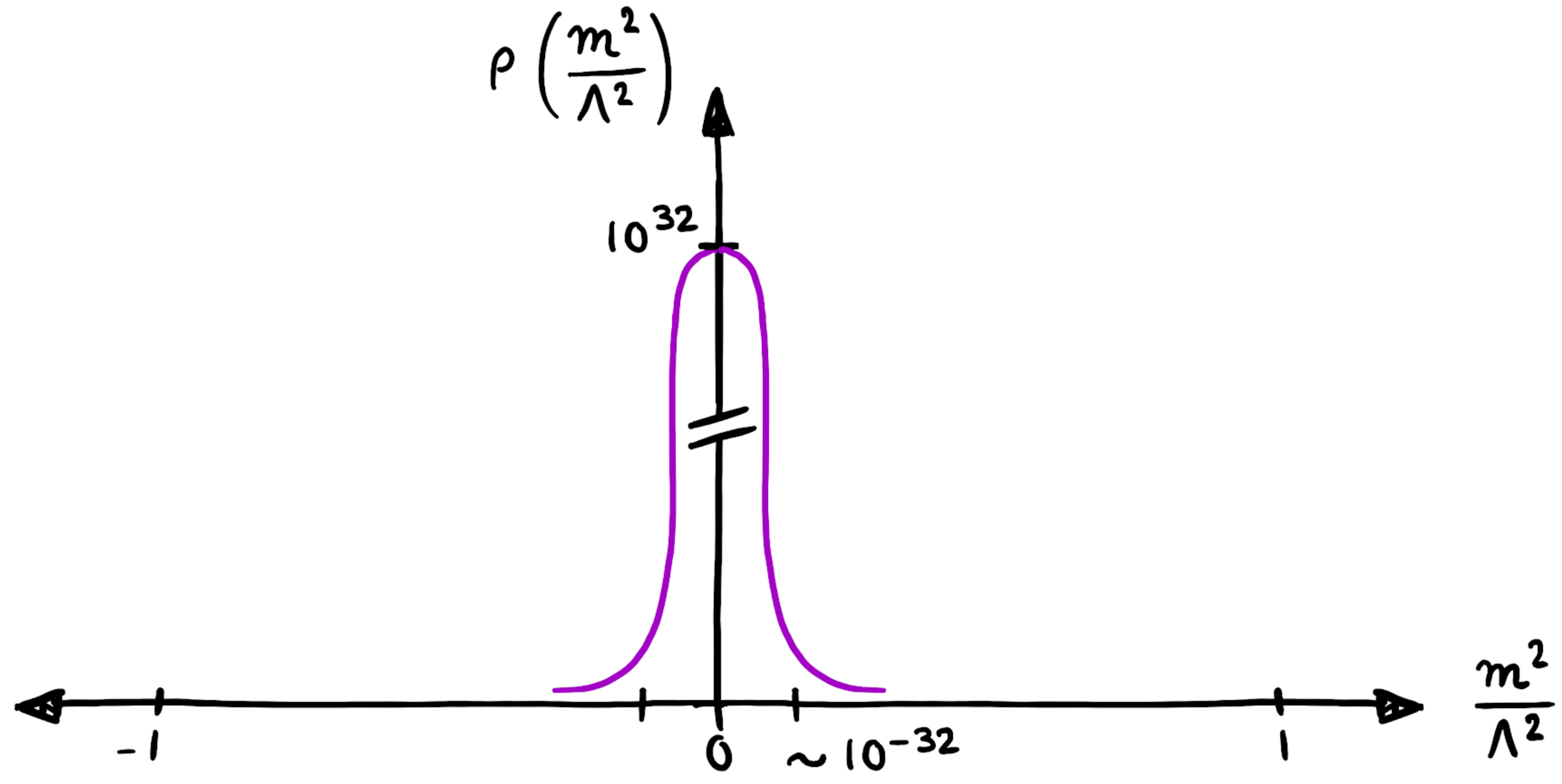
Two ways to put this to work for hierarchy problem:

18. Indirect UV/IR mixing

19. Direct UV/IR mixing

Indirect UV/IR





[Isabel Garcia Garcia, BSM Pandemic seminar 07/20]

Usual (EFT) logic of hierarchy problem: uncorrelated UV contributions give broad distribution of possible values of m_h up to cutoff; m_h well below cutoff "unlikely"

Usual (EFT) logic of hierarchy solution: lower the cutoff or eliminate sensitivity.

Alternately: consistency with gravity orchestrates correlations among UV parameters to satisfy bounds, changing the distribution.

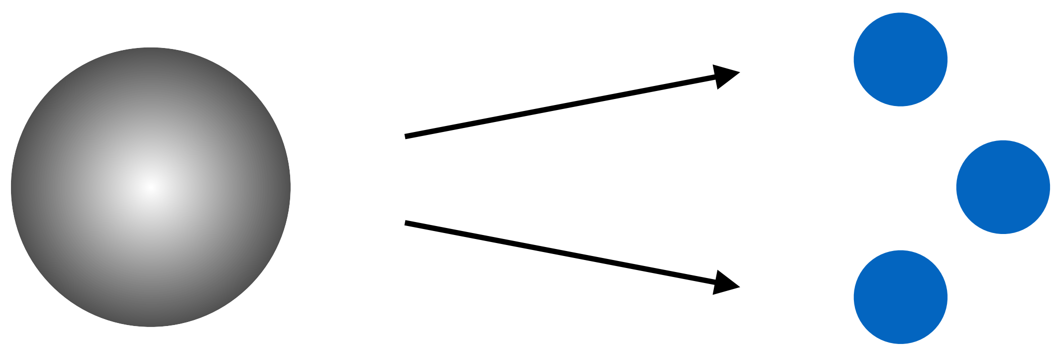
Indirect UV/IR: WGC

(Electric) weak gravity conjecture: an abelian gauge theory must contain a state of charge *q* and mass *m* satisfying

$$gq \geq \frac{m}{M_{\rm Pl}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '07]

"Justification": consider BH of charge *Q*, mass *M* decaying to this particle



\# particles produced = Q/q

Energy conservation: $mQ/q < M$

Then BH satisfies $\boldsymbol{Z = Q\, M_{Pl}/M < z = q\, M_{Pl}/m}$

Extremal BH (Z=1) stable unless there exists a state with z > 1

⇒ $\boldsymbol{q > m/M_{Pl}}$ to avoid stable black holes, remnants, in conflict w/ holography

A Family of Conjectures

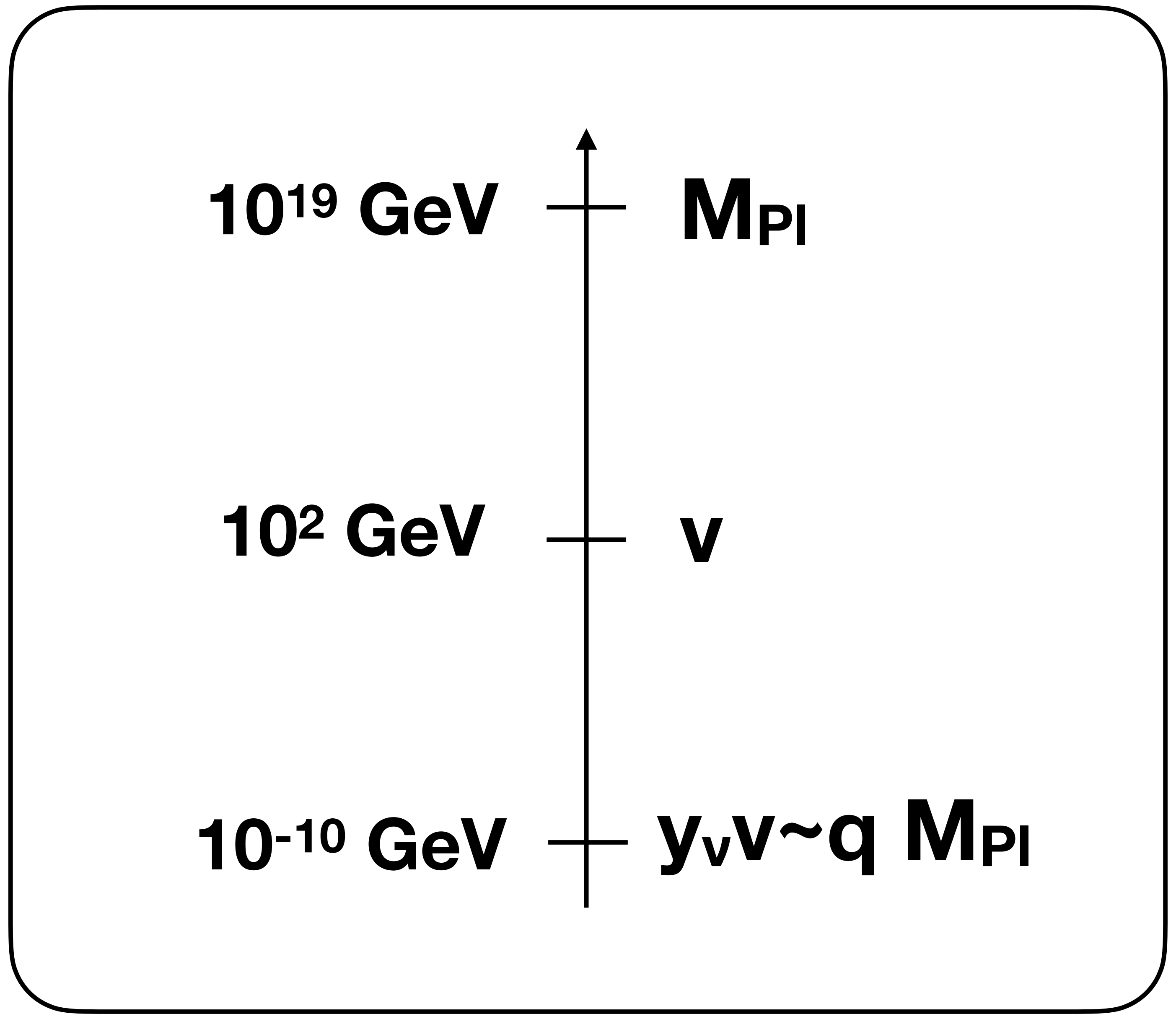
Electric WGC: [Arkani-Hamed, Motl, Nicolis, Vafa '07]	$m \leq (gq)M_{\rm Pl}$
Magnetic WGC: [Arkani-Hamed, Motl, Nicolis, Vafa '07]	$\Lambda \lesssim gM_{\rm Pl}$
+Scalar WGC: [Palti '17]	$m \leq \sqrt{g^2q^2 - \mu^2}M_{\rm Pl}$
dS WGC: [Montero, Van Riet, Venken '19]	$m^2 \gtrsim gqM_{\rm Pl}H$
Axion WGC: [Arkani-Hamed, Motl, Nicolis, Vafa '07]	$f \leq (1/S)M_{\rm Pl}$

New hierarchies from EFT + gravity.

Weak Gravity, Weak Scale?

[Cheung, Remmen '14]: If mass of WGC particle is UV sensitive, then for fixed UV-insensitive parameters, satisfying the WGC enforces fine-tuning. *(Or: would orchestrate correlations among UV contributions)*

Application to SM: charge SM fermions under weakly gauged (unbroken) $U(1)_{B-L}$ (bounds currently $q \lesssim 10^{-24}$). Cancel anomalies with RHN ν_R



Neutrino mass from EWSB

$$y_\nu H \bar{L} \nu_R \to m_\nu \sim y_\nu v$$

If lightest neutrino is WGC particle,

$m_\nu \sim 0.1$ eV, $q \gtrsim 10^{-29}$

For fixed y, q, satisfying WGC places an upper bound on v

See also: [Ibañez, Martin-Lozano, Valenzuela '17, …; March-Russell & Petrossian-Byrne '20, …]

Weak Gravity, Weak Scale?

Problem: Magnetic WGC implies Λ well below weak scale. Simple fix…

New U(1)$_X$ plus matter acquiring some mass from the Higgs. E.g…

[NC, Garcia Garcia, Koren '19]

	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
L	$\square$	$+1/2$	$+1$
L^c	$\square$	$-1/2$	-1
N	-	0	$+1$
N^c	-	0	-1

$$-\mathcal{L} \supset \left\{ m_L L L^c + m_N N N^c + y H^\dagger L N^x + y H L^c N \right\} + \text{h.c.}$$

Best option: $m_N < m_L$, lightest mass eigenstate χ_1 is WGC particle

Then for fixed (technically natural) g, m_L, m_N, y,

$$v^2 \lesssim \frac{2}{y^2}\left(m_{\chi_1}^2 + m_{\chi_1}(m_L - m_N) - m_L m_N\right)$$

Weak Gravity, Weak Scale?

Lightest particle charged under U(1)$_X$ is stable ⇒ **dark matter** candidate

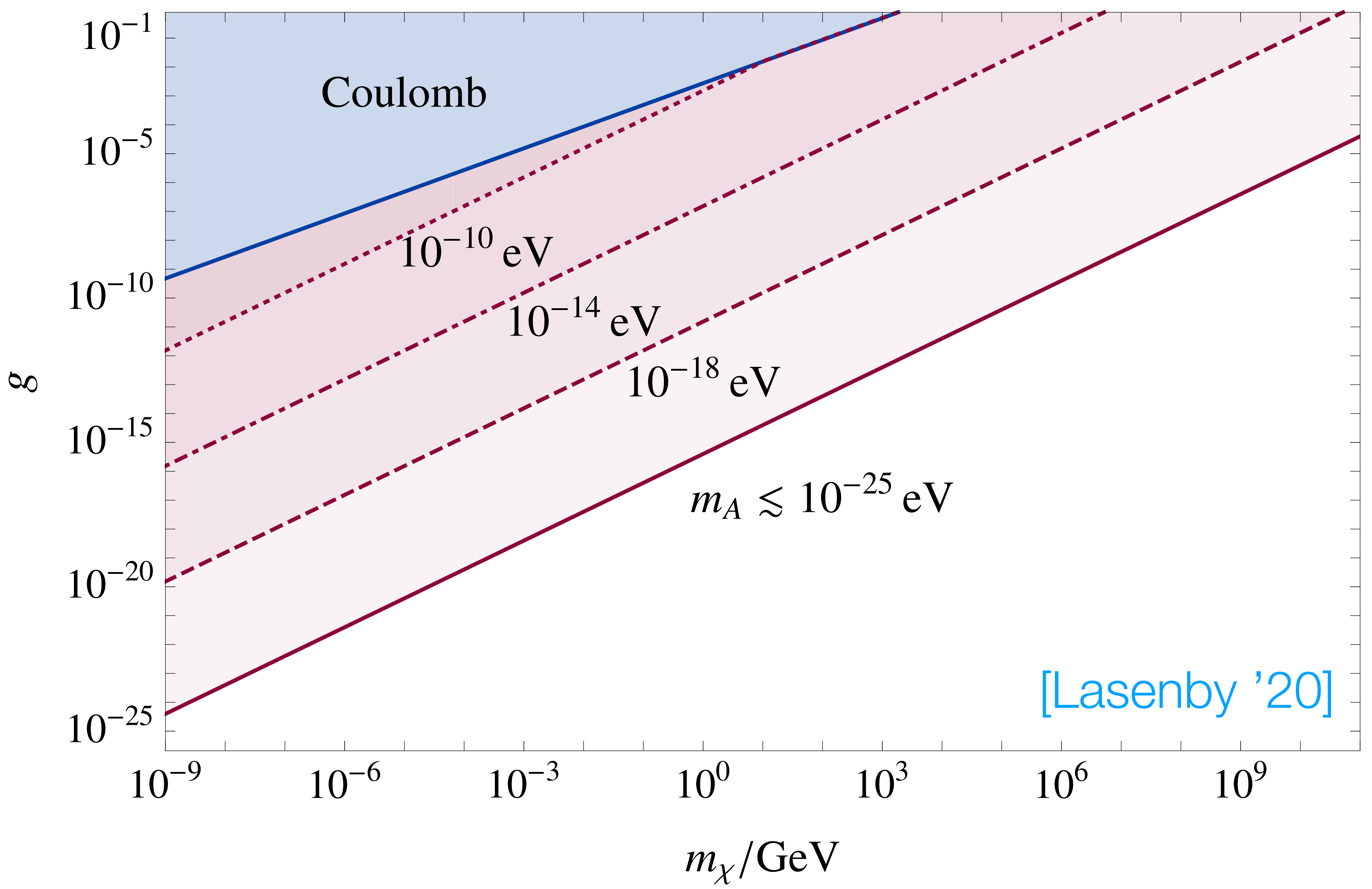
U(1)$_X$ gives a very weak, long-range force, too weak to influence individual collisions but relevant on scale of galaxy clusters

Galaxy cluster collisions can trigger plasma instabilities, making DM collisional on large scales [Ackerman, Buckley, Carroll, Kamionkowski '08; Heikinheimo, Raidal et al '15; Spethmann et al '16]

Timescale of plasma fluctuations set by plasma frequency,

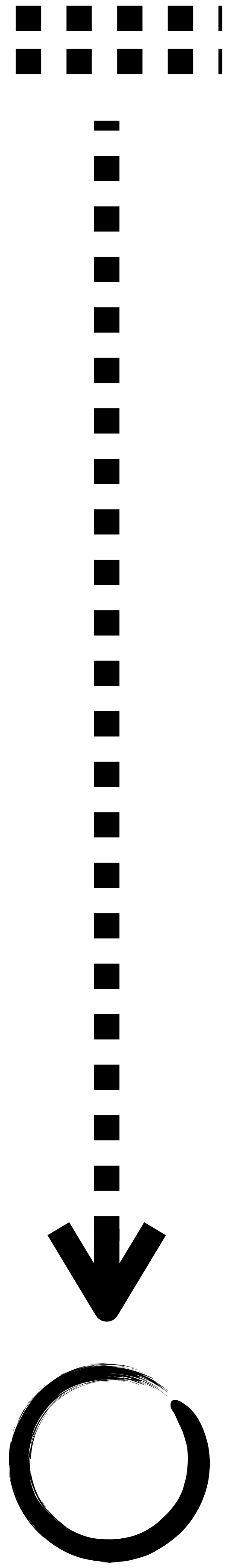
$$\omega_p = \sqrt{\frac{g^2 \rho}{m^2}} \geq \frac{\sqrt{\rho}}{M_{\rm Pl}} \qquad \omega_p^{-1} \lesssim 10^{15}\,\mathrm{s} \times \left(\frac{0.04\,\mathrm{GeV\,cm^{-3}}}{\rho}\right)^{1/2}$$

C.f. $\tau \sim 1\,\mathrm{Gyr} \sim 10^{16}\,\mathrm{s}$ for galaxy cluster collisions



“Other”

Things I can’t (yet) cleanly compartmentalize



20. Tune the CC to set the weak scale
[Arvanitaki, Dimopoulos, Gorbenko, Huang, Van Tilburg ‘16]

21. Massless moduli from explicitly broken SUSY
[Dong, Freedman, Zhao ’14, ’15]

Example: explicit marginal SUSY breaking involving $U(1)_R$ gauge fields on bdy of AdS_3

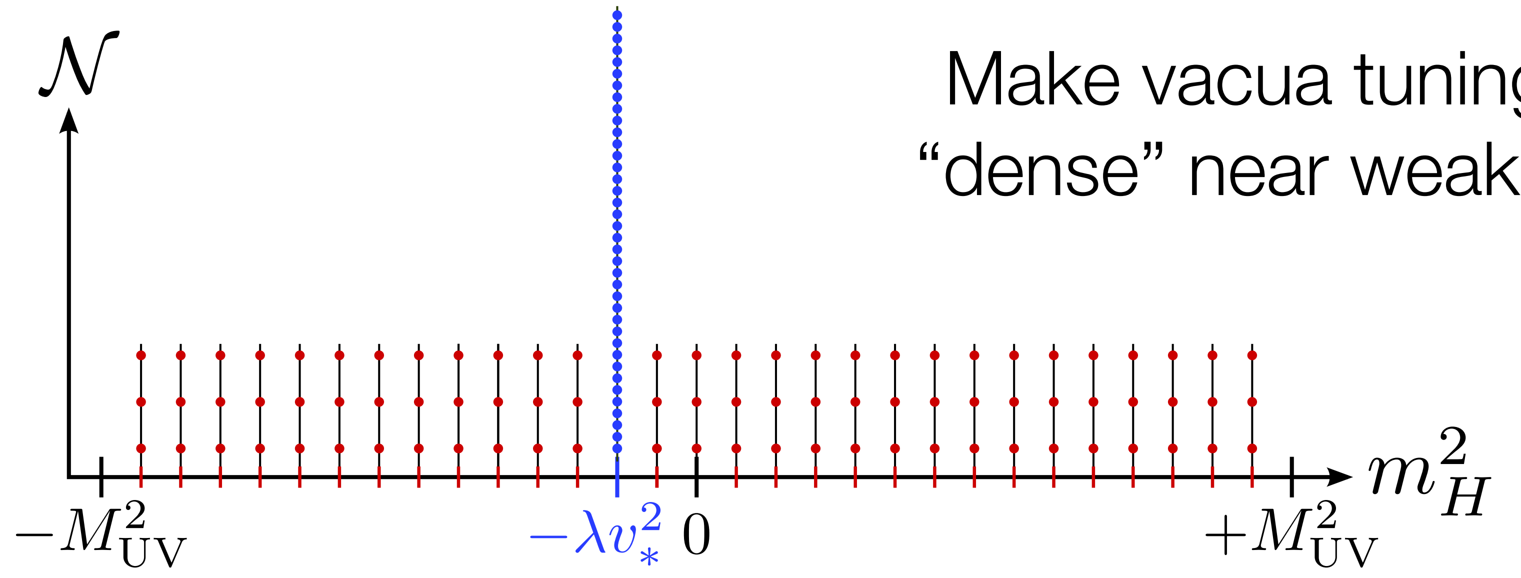
$$\delta S \sim \int_{bdy} A \wedge \tilde{A} \sim \int d^2 z J(z) \tilde{J}(\bar{z})$$

Induces splitting in R-charged multiplets. Feed to R-neutral multiplets w/ yukawa

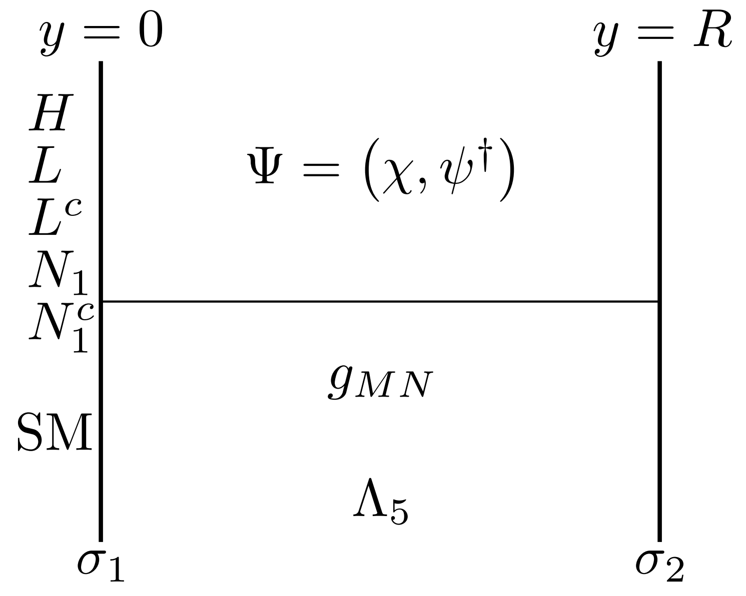
$$\lambda \phi_N \phi_R^\dagger \phi_R$$

R-neutral scalars massless to all orders

Analogous to $\quad y_t^2 m_t^2 - y_{\tilde{t}}^2 m_{\tilde{t}}^2 = 0$



Make vacua tuning CC “dense” near weak scale

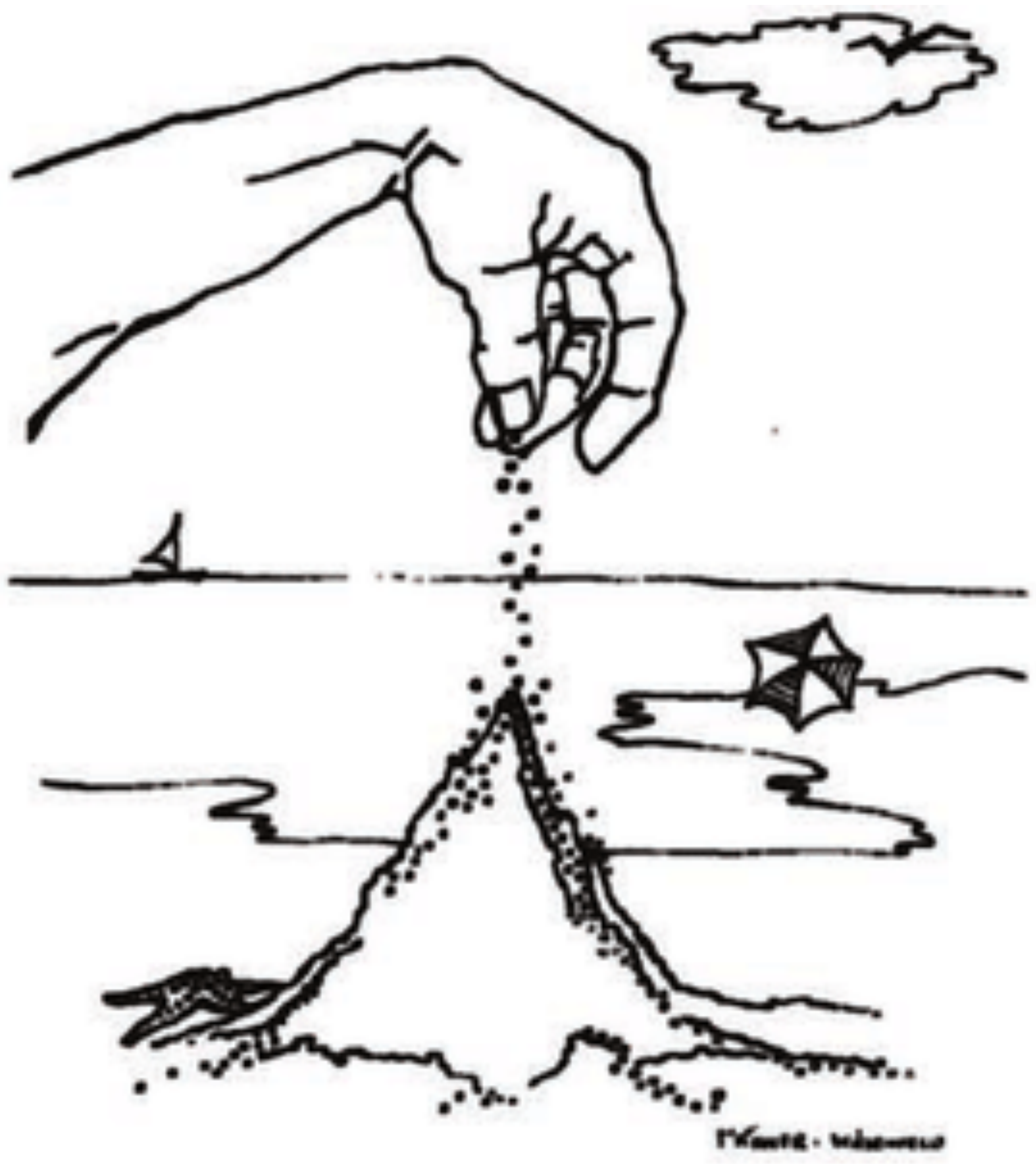


Signals

- Vector-like leptons (direct search, Higgs invisible width, precision electroweak)
- Super-light ($O(10^{-10}$ eV$)$) radion

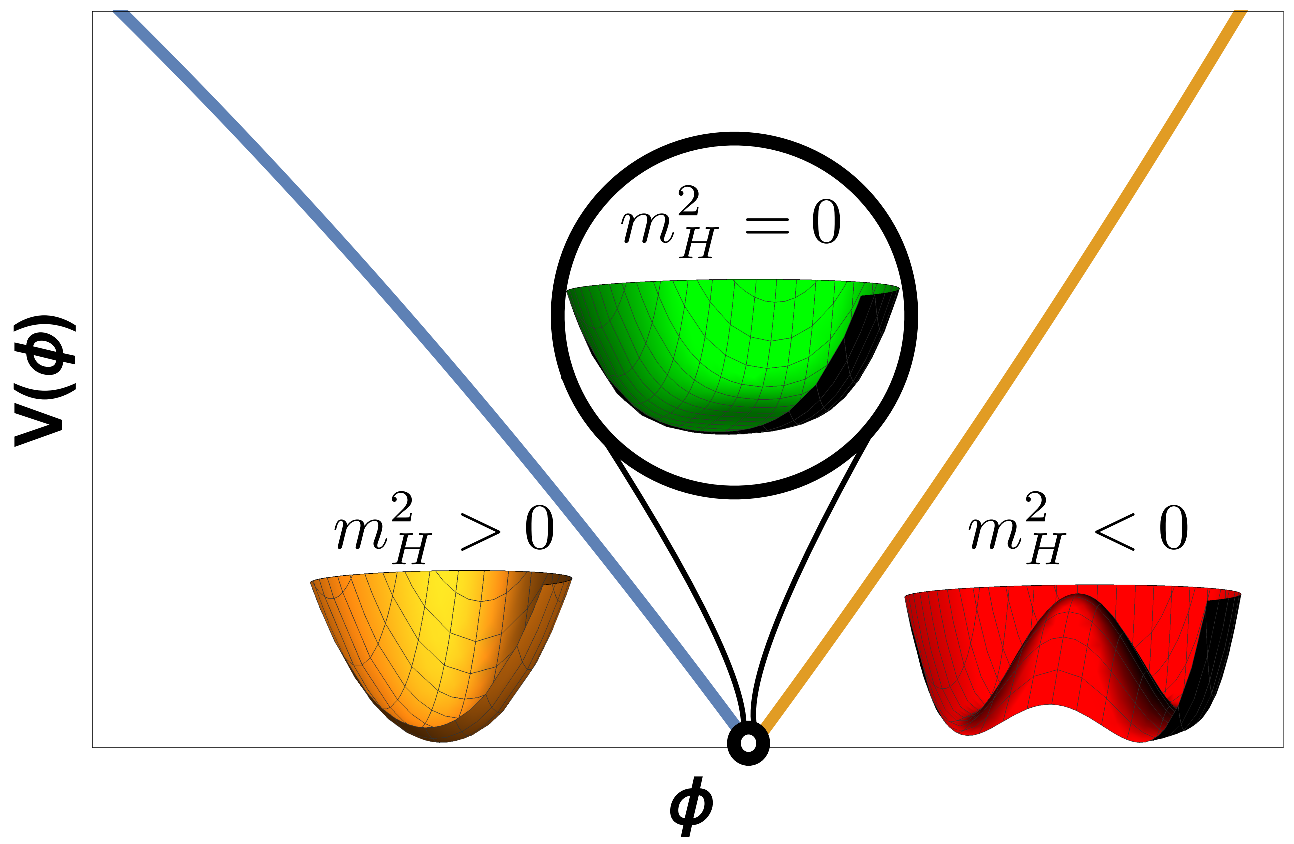
Self-Organized Criticality

Some systems evolve into critical states on their own (sandpiles, a la [Bak, Tang, Wiesenfeld '84]).
Wouldn't that be nice? [Giudice '08, Kaplan '97]

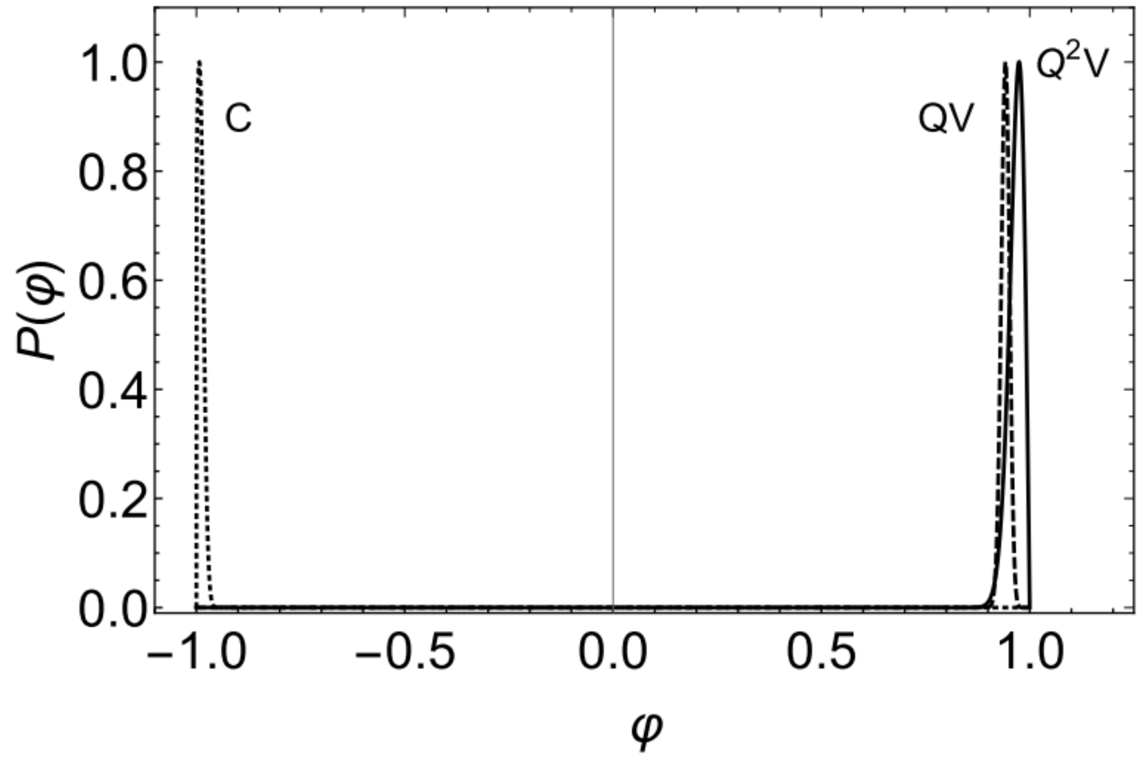


22. Self-organized Criticality

Vanishing Higgs mass coinciding with potential minimum for an extra-dimensional modulus field
[Eroencel, Hubisz, Rigo '18]



Localization of scalar fields exponentially close to critical points during eternal inflation
[Giudice, McCullough, You '21]
[Khoury et al. '19-'20]



1. Supersymmetry
2. Global symmetry
3. Discrete symmetry
4. Modular invariance
5. RS/Technicolor
6. LED/10^{32}xSM
7. LST/Clockwork
8. Classicalization
9. Disorder
10. Anthropics
11. Relaxation
12. NNaturalness
13. Crunching away
14. Conformal symmetry
15. Asymptotic fragility
16. Agravity
17. Lee-Wick Theory
18. Weak gravity conjecture
19. Non-commutative QFT
20. Weak scale from CC
21. AdS magic
22. Self-organized criticality
23. …

With apologies for the many omissions…

Conclusions

- Electroweak hierarchy problem remains one of the strongest motivations for BSM physics.
- Close to comprehensively understanding conventional solutions & searching accordingly. Should obviously keep searching for these as hard as possible, but…
- …at some point data tips the balance towards truly unconventional solutions. Many of these are a way of making sense of the apparent failure of Wilsonian EFT.
- Promising places to look: discrete symmetry; UV/IR mixing; self-organized criticality. But who am I to say? Lots to explore. Lively intersection of QFT, cosmology, quantum gravity.
- Experimental possibilities vast once we understand the full space of theories, cosmology playing an increasingly central role.
- Skepticism is justified, but imagine facing the ultraviolet catastrophe in the early 20th c…

Thank you!