Dynamo effect in unstirred self-gravitating turbulence

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Magnetic field amplification during gravitational collapse – influence of turbulence, rotation and gravitational compression

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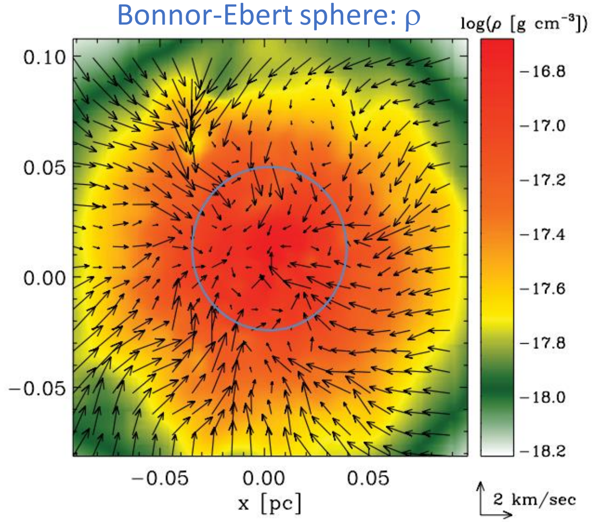

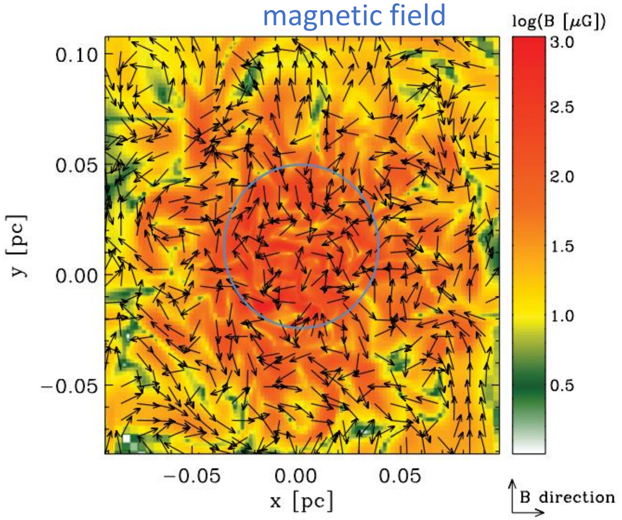

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Nonlinear Turbulent Dynamo during Gravitational Collapse

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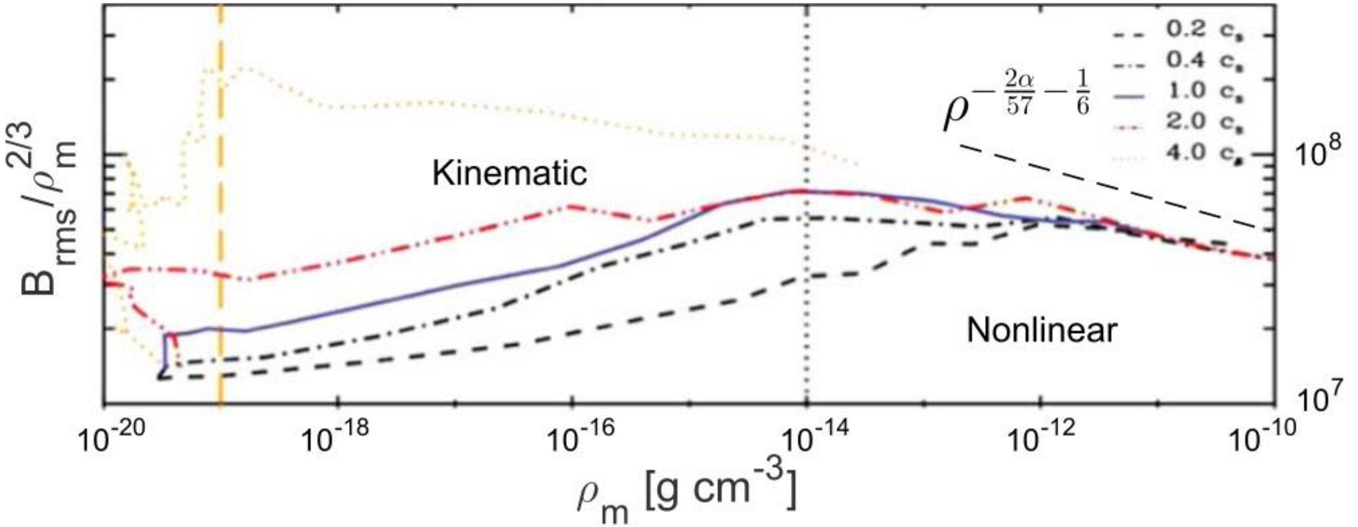
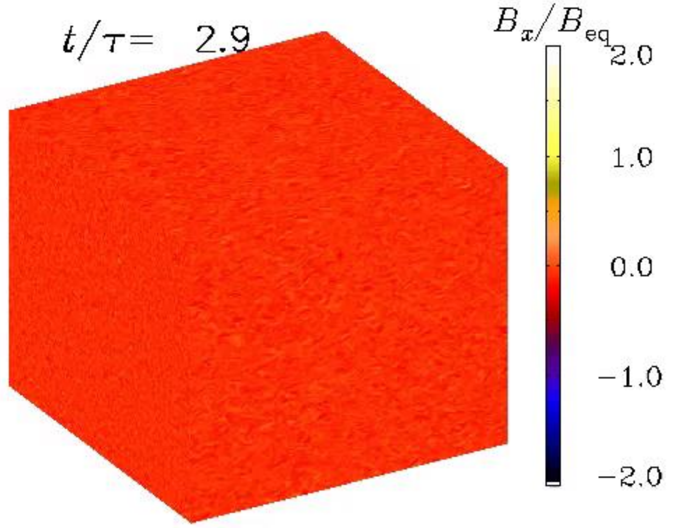
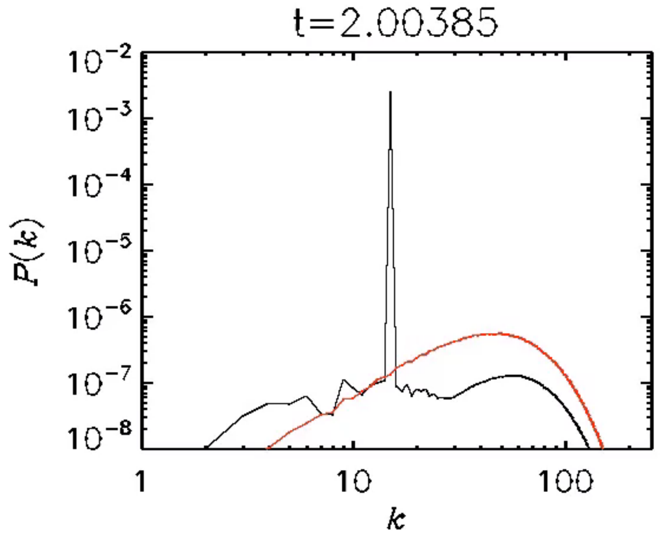
Abstract

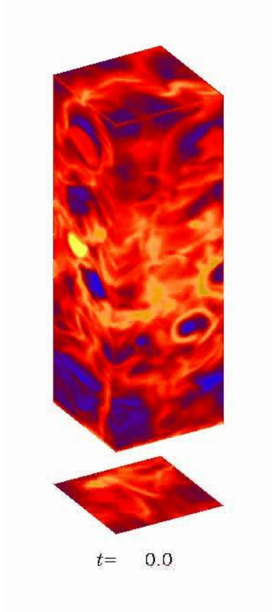
Via amplification by turbulent dynamo, magnetic fields can be potentially important for the formation of the first stars. To examine the dynamo behavior during the gravitational collapse of primordial gas, we extend the theory of the nonlinear turbulent dynamo to include the effect of gravitational compression. The relative importance between dynamo and compression varies during contraction, with the transition from dynamo- to compression-dominated amplification of magnetic fields with the increase of density. In the nonlinear stage of magnetic field amplification with the scale-by-scale energy equipartition between turbulence and magnetic fields, reconnection diffusion of magnetic fields in ideal magnetohydrodynamic turbulence becomes important. It causes the violation of the flux-freezing condition and accounts for (a) the small growth rate of the nonlinear dynamo, (b) the weak dependence of magnetic energy on density during contraction, (c) the saturated magnetic energy, and (d) the large correlation length of magnetic fields. The resulting magnetic field structure and the scaling of magnetic field strength with density are radically different from the expectations of flux freezing.

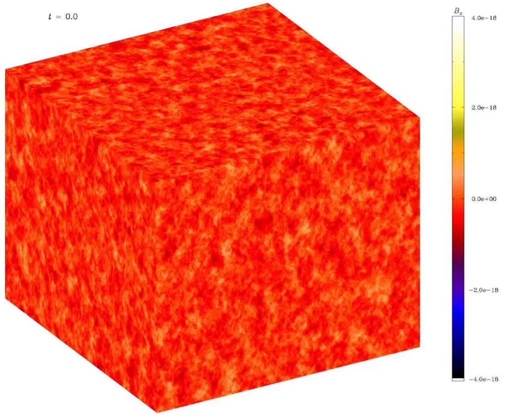


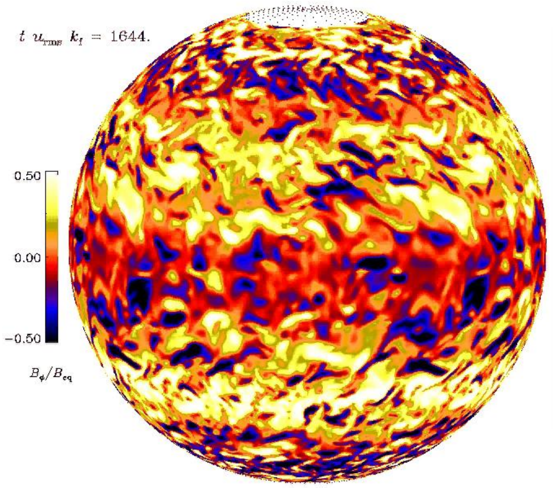
Dynamos in various other settings

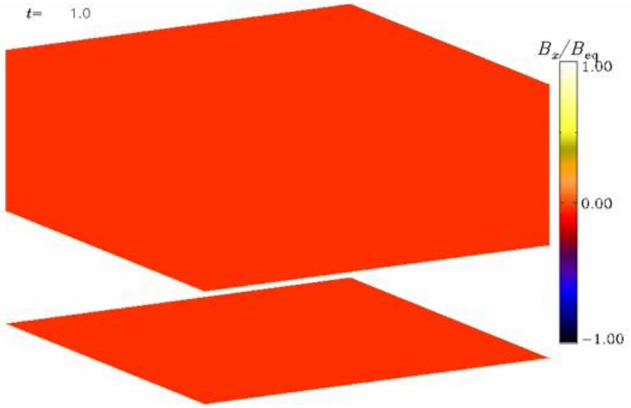


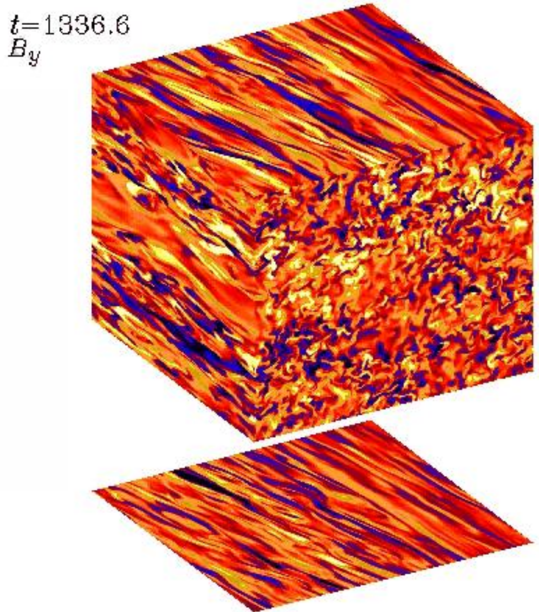


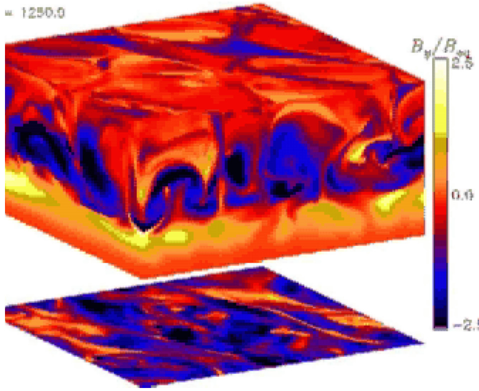












Self-gravitating MHD in unbounded space

$$\nabla^2 \Phi = 4\pi G \left(\rho - \rho_0\right),$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\boldsymbol{\nabla}\left(c_{\mathrm{s}}^2 \ln \rho + \Phi\right) + \frac{1}{\rho}\left(\boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{\nabla} \cdot 2\rho\nu\mathbf{S}\right),$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\boldsymbol{\nabla} \cdot \boldsymbol{u},$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B} - \eta\mu_0 \boldsymbol{J},$$

Jeans instability

$$\sigma^2 = \sigma_{\mathrm{J}}^2 - c_{\mathrm{s}}^2 k^2$$

Consider mostly σ_J=5
(but 2 in Run O2)

Earlier work

Jeans collapse in a turbulent medium

S. Bonazzola[1], E. Falgarone[2,3], J. Heyvaerts[1], M. Pérault[2,3], and J.L. Puget[2,3]

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$$\frac{\partial \Phi^i}{\partial t} = -\partial_m\left(\frac{\Phi^i \Phi^m}{\varrho_0}\right) - \frac{k_B T}{\mu}\,\partial_i \varrho + g^i \varrho + \partial_m\left[\nu(\varrho_0)\,\partial_m \frac{\Phi^i}{\varrho_0}\right] + F^i, \quad (20)$$

$$\frac{\partial \varrho}{\partial t} = -\partial_i \Phi^i, \quad (21)$$

$$\partial_i g^i = -4\pi G \varrho, \quad (22)$$

Homogeneous self-gravitating flows

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Received July 6, accepted October 1, 1987

$$a^2\partial_\tau \rho + \nabla\cdot(\rho \boldsymbol{u}) = 0$$
$$a^2\partial_\tau \boldsymbol{u} + \boldsymbol{u}\nabla\cdot\boldsymbol{u} = -M^{-2}a^{5-3\gamma}(\gamma\rho)^{-1}\nabla\rho^\gamma + (2/3)a\boldsymbol{g} + \mu a\nabla\cdot\boldsymbol{\Sigma}$$
$$\nabla\cdot\boldsymbol{g} = -(\rho - 1),$$

from an equation of state and a thermal equation. It is first found that the solution with uniform density, say *D*, is not stationary and corresponds to a uniformly expanding or contracting (homogeneous isotropic) flow. The equations of motion (4) and (1) are then written in the accelerated frame of the expansion and, with the help of some rescaling, evolution equations similar to (4)

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Why does the Jeans Swindle work?

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mass profile undefined; however, this trick has no formal justification. We show that when one includes the expansion of the Universe in the Jeans equation, a term appears which exactly cancels the divergent term from the background. We thereby establish a formal justification for using the Jeans Swindle.

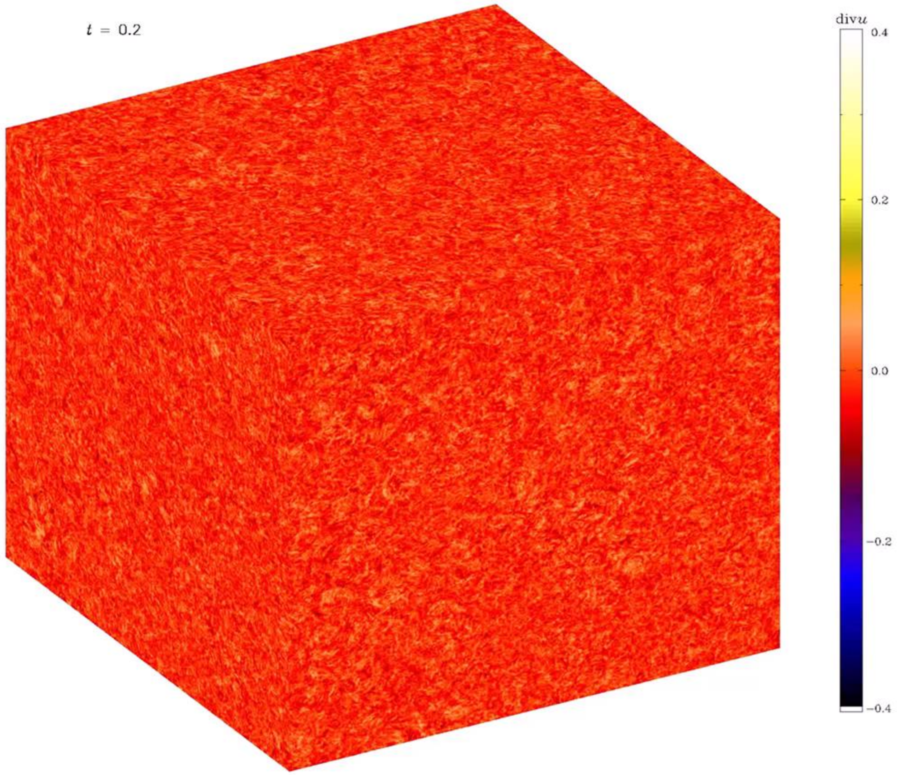
A **swindle** is a kind of fraud or confidence trick.

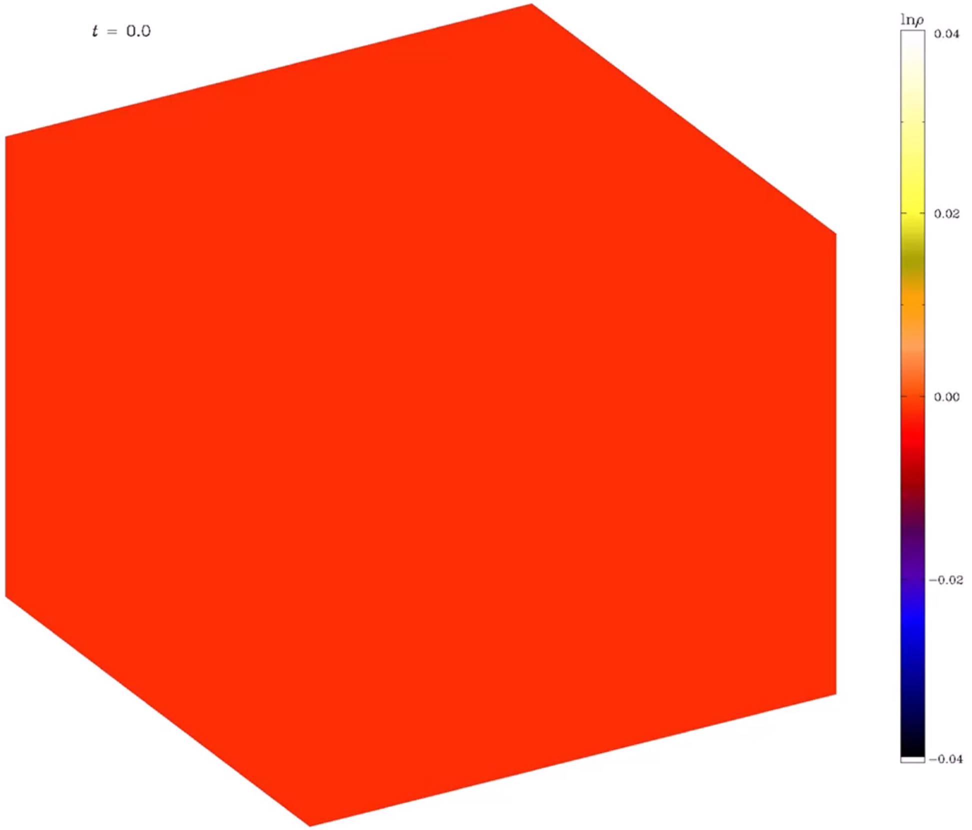
Confidence trick

Attempt to defraud a person or group after first gaining their confidence

Dynamo from selfgravitating turbulent collapse?

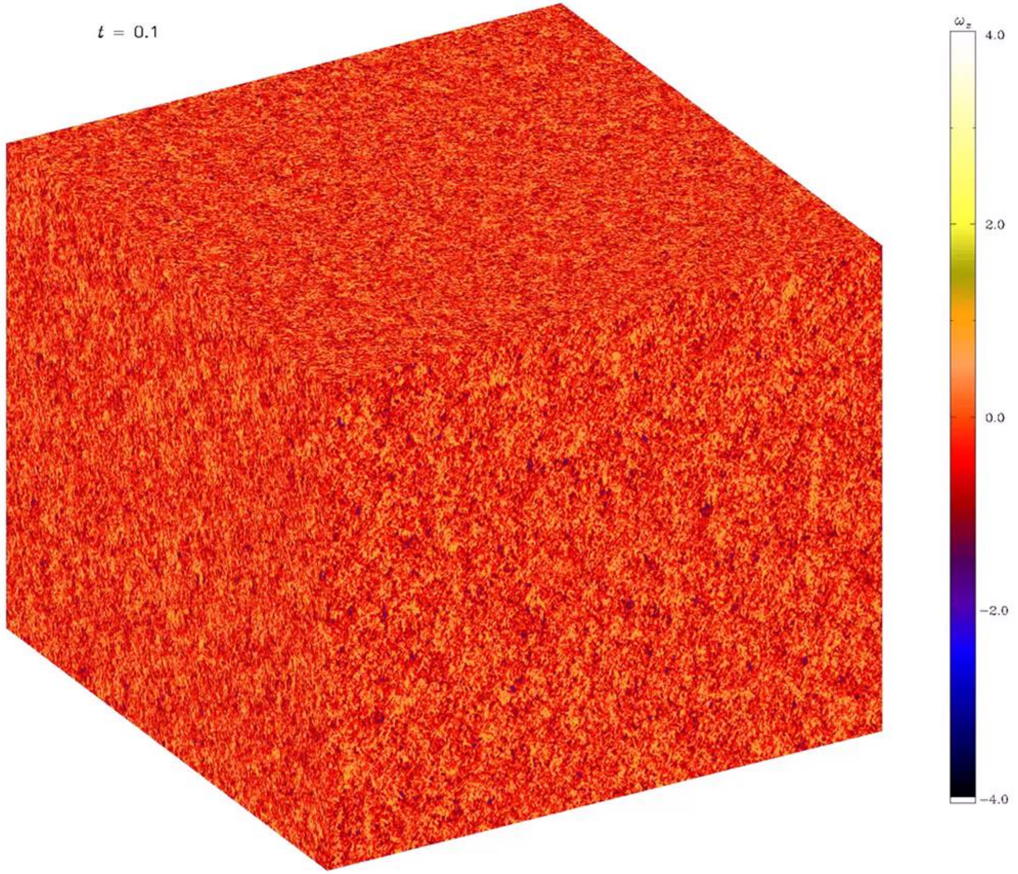
High density where large convergence

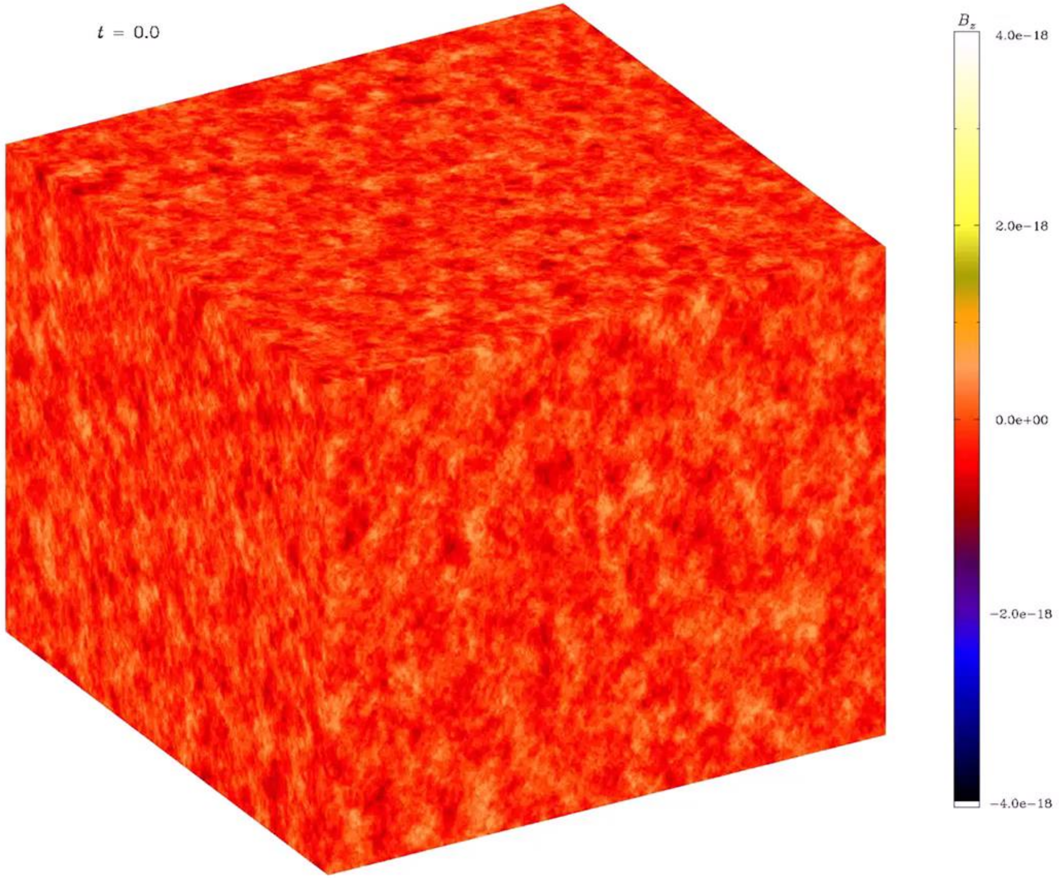




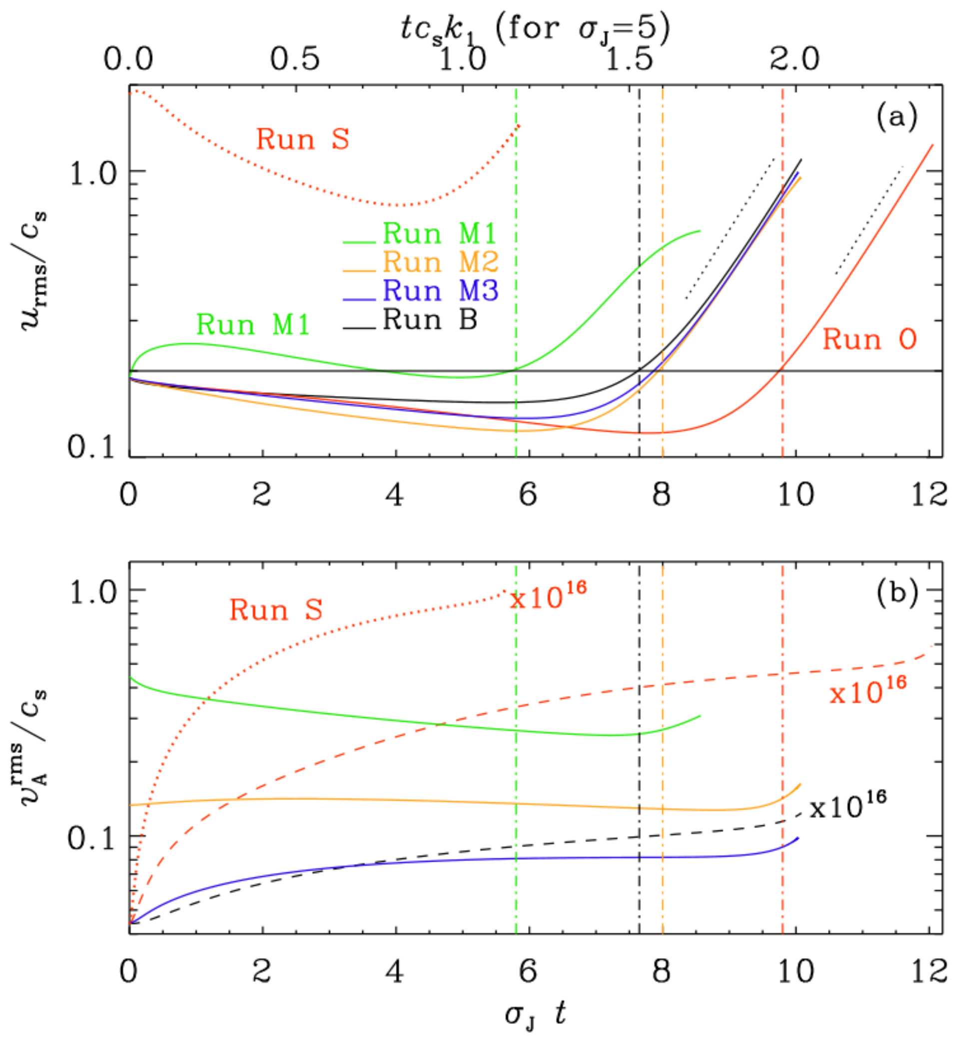
Vorticity & magnetic field

Collapse faster than turbulence





Summary of the runs



Run	k_J	k_f	Re_{k_f}	Re_{t_*}	Lu_{t_*}	ϵ_J^P	ϵ_J^{+K}	ϵ_J^{-K}	ϵ_J^L	ϵ_L^{+M}	ϵ_J^{-M}	ϵ_M^Δ	γ	N^3
O1	2	10	500	1000	2.3×10^{-13}	0.30	0.68	0.02	0.000	0.28	0.72	0.40	1.39	2048^3
O2	5	2	100	1000	3.9×10^{-14}	0.32	0.67	0.01	0.000	0.49	0.51	0.96	0.83	1024^3
A	5	10	500	1000	1.0×10^{-13}	0.33	0.64	0.03	0.000	0.35	0.65	0.70	3.0	2048^3
B	5	10	100	200	9.9×10^{-15}	0.34	0.63	0.03	0.000	0.21	0.79	0.26	0.44	2048^3
b	5	10	100	200	9.9×10^{-15}	0.31	0.66	0.03	0.000	0.20	0.76	0.32	0.54	1024^3
C	5	10	20	40	9.6×10^{-16}	0.34	0.63	0.03	0.000	0.06	0.94	0.06	0.08	2048^3
D	5	10	5	10	1.1×10^{-16}	0.31	0.66	0.03	0.000	−0.82	1.82	−0.48	−0.44	1024^3
E	5	10	1	2	5.6×10^{-18}	0.25	0.73	0.02	0.000	−11.5	12.5	−0.96	−1.13	1024^3
S	5	10	500	1300	8.7×10^{-14}	0.31	0.60	0.09	0.000	0.34	0.66	0.79	2.47	1024^3
M1	5	10	500	1000	1.3×10^{3}	0.36	0.46	0.13	0.05	−17.6	18.6	−0.94	−0.38	2048^3
M2	5	10	500	1000	6.4×10^{2}	0.31	0.67	0.01	0.01	−0.97	1.97	−0.45	−0.16	2048^3
M3	5	10	500	1000	4.1×10^{2}	0.32	0.65	0.01	0.02	0.03	0.97	0.17	0.11	2048^3
M4	5	10	100	200	9.8×10^{0}	0.33	0.65	0.02	0.00	0.20	0.80	0.25	0.42	2048^3
I1	5	10	500	1000	1.6×10^{3}	0.29	0.68	0.01	0.02	0.67	0.33	2.63	0.05	2048^3
I2	5	10	500	1000	7.5×10^{2}	0.31	0.67	0.01	0.01	0.20	0.80	0.26	0.03	2048^3
I3	5	10	500	1000	4.3×10^{2}	0.32	0.65	0.01	0.02	0.32	0.68	0.60	0.33	2048^3

To characterize the flow of energy, it is convenient to define the fractions $\epsilon_J^P \equiv -W_P/W_J$, $\epsilon_J^L \equiv -W_L/W_J$, $\epsilon_J^{+K} \equiv \dot{\mathcal{E}}_K/W_J$, and $\epsilon_J^{-K} = Q_K/W_J$. Likewise, we define the fractions $\epsilon_L^{+M} \equiv \dot{\mathcal{E}}_M/(-W_L)$, and $\epsilon_J^{-M} = Q_M/(-W_L)$. To characterize the growth or decay of the magnetic field, we define the nondimensional ratio $\epsilon_M^\Delta = (-W_L - Q_M)/Q_M$. A related

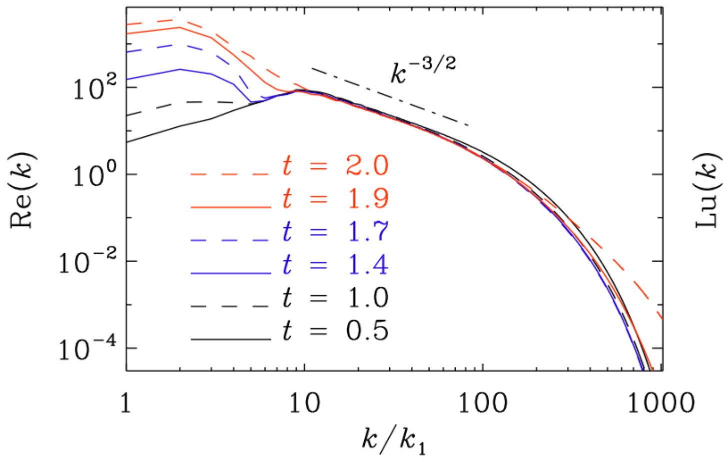
Reynolds number dependence

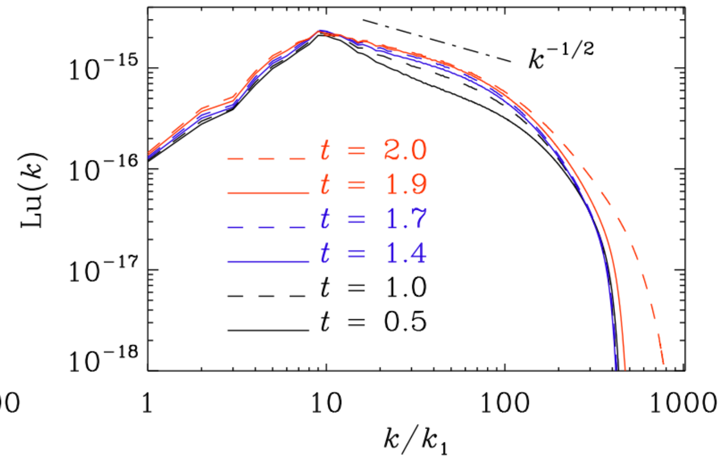
$$u_k(t) = \sqrt{2kE_{\rm K}(k,t)/\rho_0}, \quad B_k(t) = \sqrt{2\mu_0 k E_{\rm M}(k,t)}, \quad (13)$$

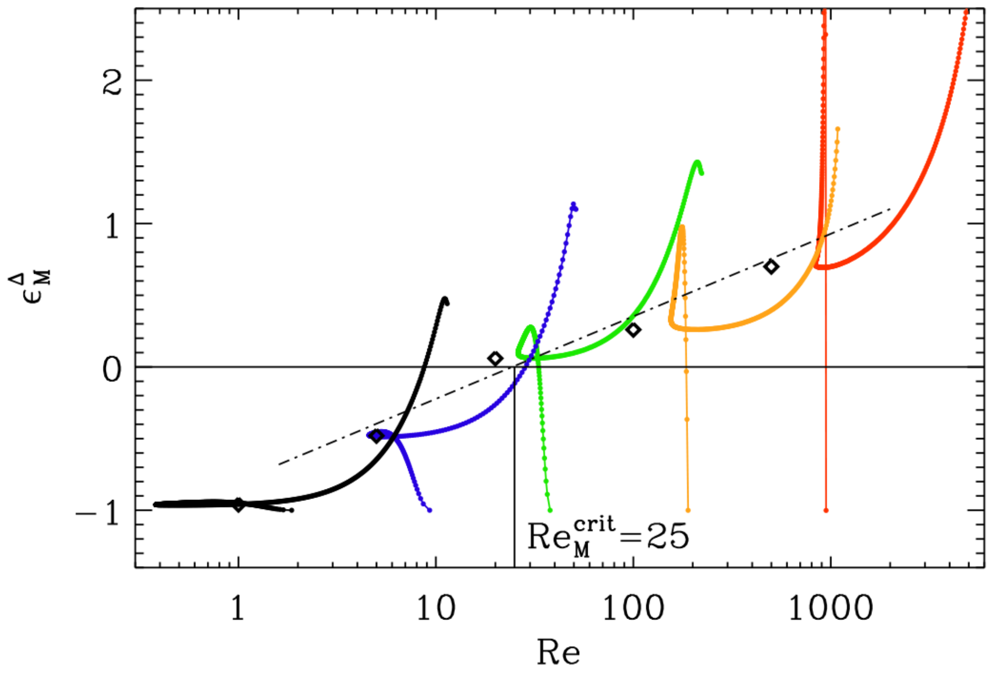
respectively. We then define

$$\mathrm{Re}_k(t) = u_k(t)/\nu k \quad \text{and} \quad \mathrm{Lu}_k(t) = B_k(t)/(\sqrt{\mu_0\rho_0}\,\eta k). \quad (14)$$

A Kolmogorov-type spectrum with $E_{\rm K}(k) \propto k^{-5/3}$ corresponds then to $u_k \propto k^{-1/3}$ and $\mathrm{Re}_k \propto k^{-4/3}$. In the following,







of $\mathrm{Re}_{k_{\rm f}}$ and Re_t are close to the Taylor microscale Reynolds number (Tennekes & Lumley 1972), which is universally defined as $\mathrm{Re}_\lambda = v'\lambda_{\rm Tay}/\nu$. Here, $v' = u_{\rm rms}/\sqrt{3}$ is the one-dimensional rms velocity and $\lambda_{\rm Tay} = \sqrt{15\nu\rho_0/Q_{\rm K}}\,v'$ is the Taylor microscale.

enhanced dissipation very late

Reynolds number definition

> consistent definition of the Reynolds number. The correct definition of the
> Reynolds number is not the one used here in Eq. (14), or in the many papers
> led and co-authored by the lead author. I understand that this dates back
> to many works, but the Reynolds number is defined in any major textbook as
> Re (or Rm) = L * v(L) / viscosity, where the viscosity is kinetic or
> magnetic, for Re and Rm, respectively, with L being the length scale on
> which the Re (or Rm) is defined or measured and v(L) being the velocity
> (dispersion) on that length scale (see many textbooks or for quick
> reference https://en.wikipedia.org/wiki/Reynolds_number). The definition
> used here in Eq. (14) is inconsistent with that general definition of
> Reynolds number, because it misses a factor 2pi. In other words, this
> incorrect definition of Reynolds number evaluates L and v inconsistently,
> i.e., the velocity (here denoted as u_k in Eq. 14) is not the correct
> velocity on the respective length scale (here 1/k). An Re has to be defined
> with length and velocity being consistent, such that it is the velocity on
> that very length scale used to define Re. This is not the case here in Eq.
> 14, where v and L are on different scales. This means that the definition
> of Re in Eq. 14 is incorrect, i.e., it cannot be called a 'Reynolds
> number', because it misses a factor 2pi, and therefore, it cannot be

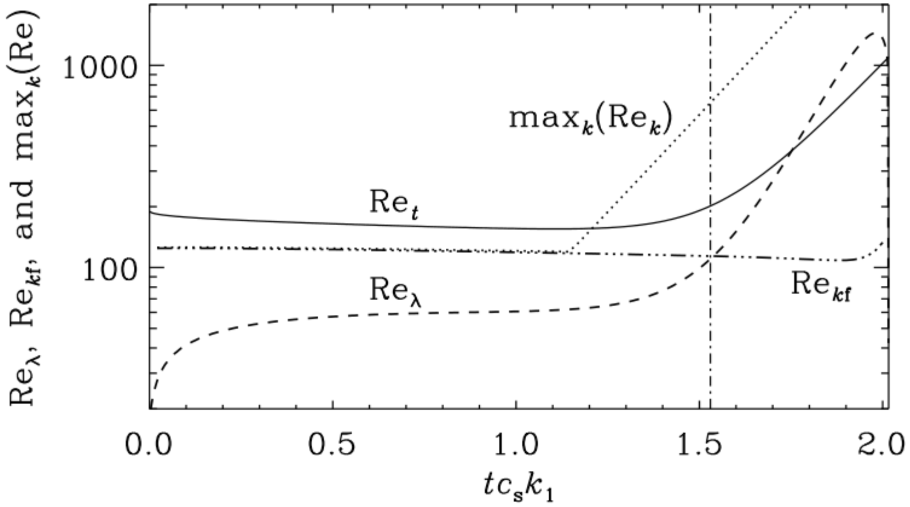
As we explained above, the only universally defined Reynolds number is Re_lambda, the Taylor microscale Reynolds number. We compare it now with ours in Appendix C. The values by our definition are close to it. We hope that this addresses this point. They are also consistent in the sense mentioned by the referee, because both urms and kf are integral scale quantities.

$$u_k(t) = \sqrt{2kE_\mathrm{K}(k,t)/\rho_0}, \quad B_k(t) = \sqrt{2\mu_0 k E_\mathrm{M}(k,t)}, \quad (13)$$

respectively. We then define

$$\mathrm{Re}_k(t) = u_k(t)/\nu k \quad \text{and} \quad \mathrm{Lu}_k(t) = B_k(t)/(\sqrt{\mu_0\rho_0}\,\eta k). \quad (14)$$

A Kolmogorov-type spectrum with $E_\mathrm{K}(k) \propto k^{-5/3}$ corresponds then to $u_k \propto k^{-1/3}$ and $\mathrm{Re}_k \propto k^{-4/3}$. In the following,



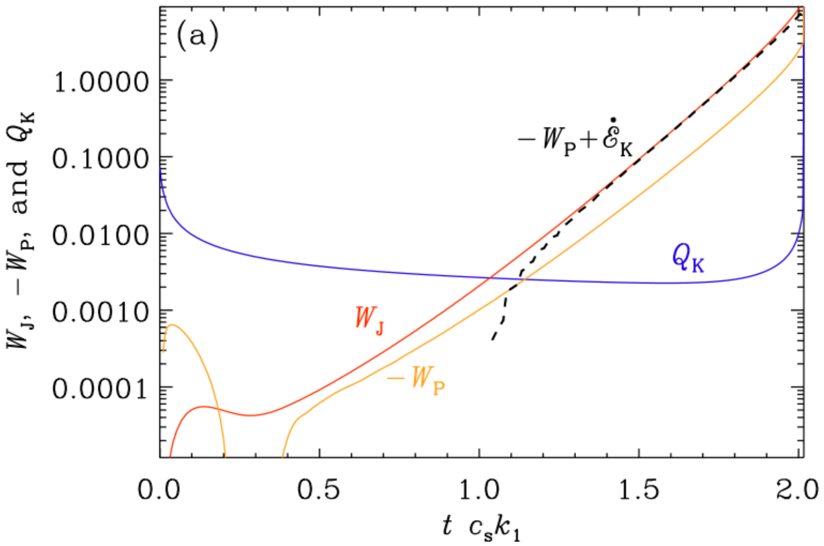
of $\mathrm{Re}_{k_\mathrm{f}}$ and Re_t are close to the Taylor microscale Reynolds number (Tennekes & Lumley 1972), which is universally defined as $\mathrm{Re}_\lambda = v'\lambda_\mathrm{Tay}/\nu$. Here, $v' = u_\mathrm{rms}/\sqrt{3}$ is the one-dimensional rms velocity and $\lambda_\mathrm{Tay} = \sqrt{15\nu\rho_0/Q_\mathrm{K}}\,v'$ is the Taylor microscale.

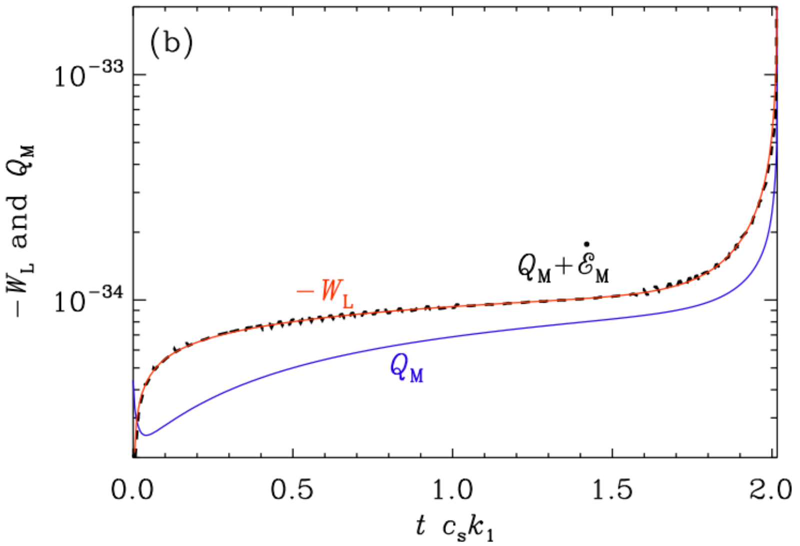
Flow of energy

$$\frac{\mathrm{d}\mathcal{E}_{\mathrm{P}}}{\mathrm{d}t} = -W_{\mathrm{J}},$$

$$\frac{\mathrm{d}\mathcal{E}_{\mathrm{K}}}{\mathrm{d}t} = W_{\mathrm{P}} + W_{\mathrm{J}} + W_{\mathrm{L}} - Q_{\mathrm{K}},$$

$$\frac{\mathrm{d}\mathcal{E}_{\mathrm{M}}}{\mathrm{d}t} = -W_{\mathrm{L}} - Q_{\mathrm{M}},$$





$$-\langle\rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi\rangle = -\langle p\boldsymbol{\nabla}\cdot\boldsymbol{u}\rangle + \frac{\mathrm{d}\mathcal{E}_{\mathrm{K}}}{\mathrm{d}t}\ldots,$$

$$-\langle\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle = \langle\mu_0\eta\boldsymbol{J}^2\rangle + \frac{\mathrm{d}\mathcal{E}_{\mathrm{M}}}{\mathrm{d}t}.$$

$$\mathcal{E}_{\mathrm{P}} = -\langle(\boldsymbol{\nabla}\Phi)^2\rangle/8\pi G$$

$$\mathcal{E}_{\mathrm{K}} = \langle\rho\boldsymbol{u}^2\rangle/2$$

$$\mathcal{E}_{\mathrm{M}} = \langle\boldsymbol{B}^2\rangle/2\mu_0$$

where $W_{\mathrm{P}} = -\langle\boldsymbol{u}\cdot\boldsymbol{\nabla}p\rangle = \langle p\boldsymbol{\nabla}\cdot\boldsymbol{u}\rangle$ is the work done by the pressure force, $W_{\mathrm{J}} = -\langle\rho\boldsymbol{u}\cdot\boldsymbol{\nabla}\Phi\rangle$ is the work done by the gravity term, $W_{\mathrm{L}} = \langle\boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B})\rangle$ is the work done by the Lorentz force, and $Q_{\mathrm{K}} = \langle 2\rho\nu\mathbf{S}^2\rangle$ and $Q_{\mathrm{M}} = \langle\mu_0\eta\boldsymbol{J}^2\rangle$ are the viscous and Joule dissipation terms. The thermal energy density is sourced by the terms $-W_{\mathrm{P}}+Q_{\mathrm{K}}+Q_{\mathrm{M}}$, but with the

Virial parameter

$$\alpha_{\rm vir} = 2\mathcal{E}_{\rm K}/|\mathcal{E}_{\rm P}|$$

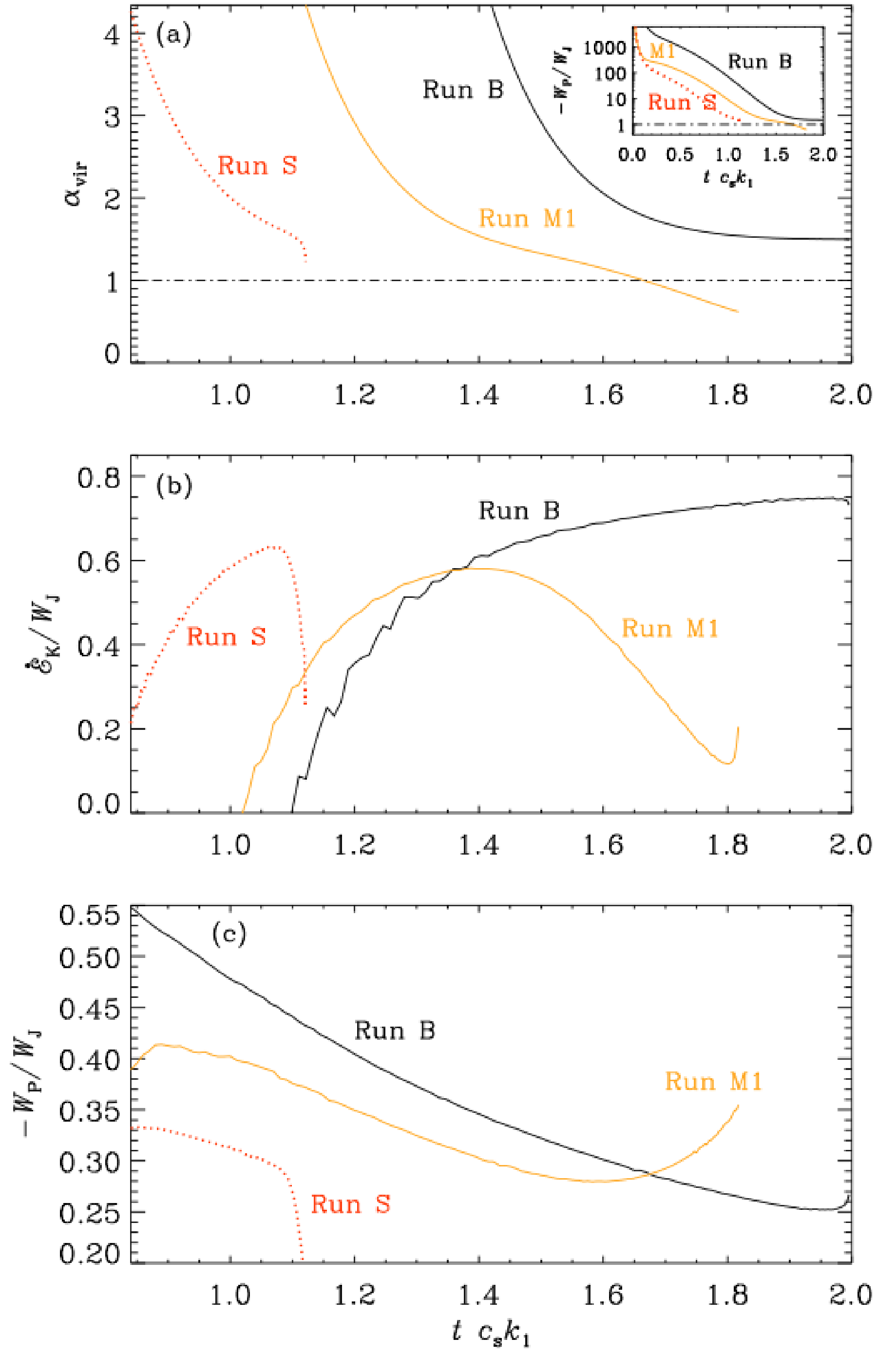
$$\mathcal{E}_{\rm K} = \langle \rho \boldsymbol{u}^2 \rangle/2$$

$$\mathcal{E}_{\rm P} = -\langle (\boldsymbol{\nabla}\Phi)^2 \rangle/8\pi G$$

$$-\langle \rho \boldsymbol{u} \cdot \boldsymbol{\nabla}\Phi \rangle = -\langle p \boldsymbol{\nabla} \cdot \boldsymbol{u} \rangle + \frac{{\rm d}\mathcal{E}_{\rm K}}{{\rm d}t} \ldots,$$

$$-W_{\rm P} \approx \frac{1}{3} W_{\rm J}, \quad \dot{\mathcal{E}}_{\rm K} \approx \frac{2}{3} W_{\rm J}. \qquad (16)$$

The latter can be integrated to give $\mathcal{E}_{\rm K} \approx (2/3)\int W_{\rm J}\,{\rm d}t$. Likewise, integrating Eq. (5) gives $-\mathcal{E}_{\rm P} \approx \int W_{\rm J}\,{\rm d}t$, which implies $\alpha_{\rm vir} = 2\mathcal{E}_{\rm K}/|\mathcal{E}_{\rm P}| \approx 4/3$. Its value would be unity, if only half



Vortical & irrotational parts

To characterize the compressive and solenoidal flow components, it is convenient to compute the rms velocity divergence, $(\boldsymbol{\nabla}\cdot\boldsymbol{u})_{\rm rms} = \left\langle(\boldsymbol{\nabla}\cdot\boldsymbol{u})^2\right\rangle^{1/2}$, and the rms vorticity, $\omega_{\rm rms} = \langle\boldsymbol{\omega}^2\rangle^{1/2}$, where $\boldsymbol{\omega} = \boldsymbol{\nabla}\times\boldsymbol{u}$, and to define

$$k_{\boldsymbol{\nabla}\cdot\boldsymbol{u}} = (\boldsymbol{\nabla}\cdot\boldsymbol{u})_{\rm rms}/u_{\rm rms}, \tag{9}$$

$$k_{\boldsymbol{\omega}} = \omega_{\rm rms}/u_{\rm rms}, \tag{10}$$

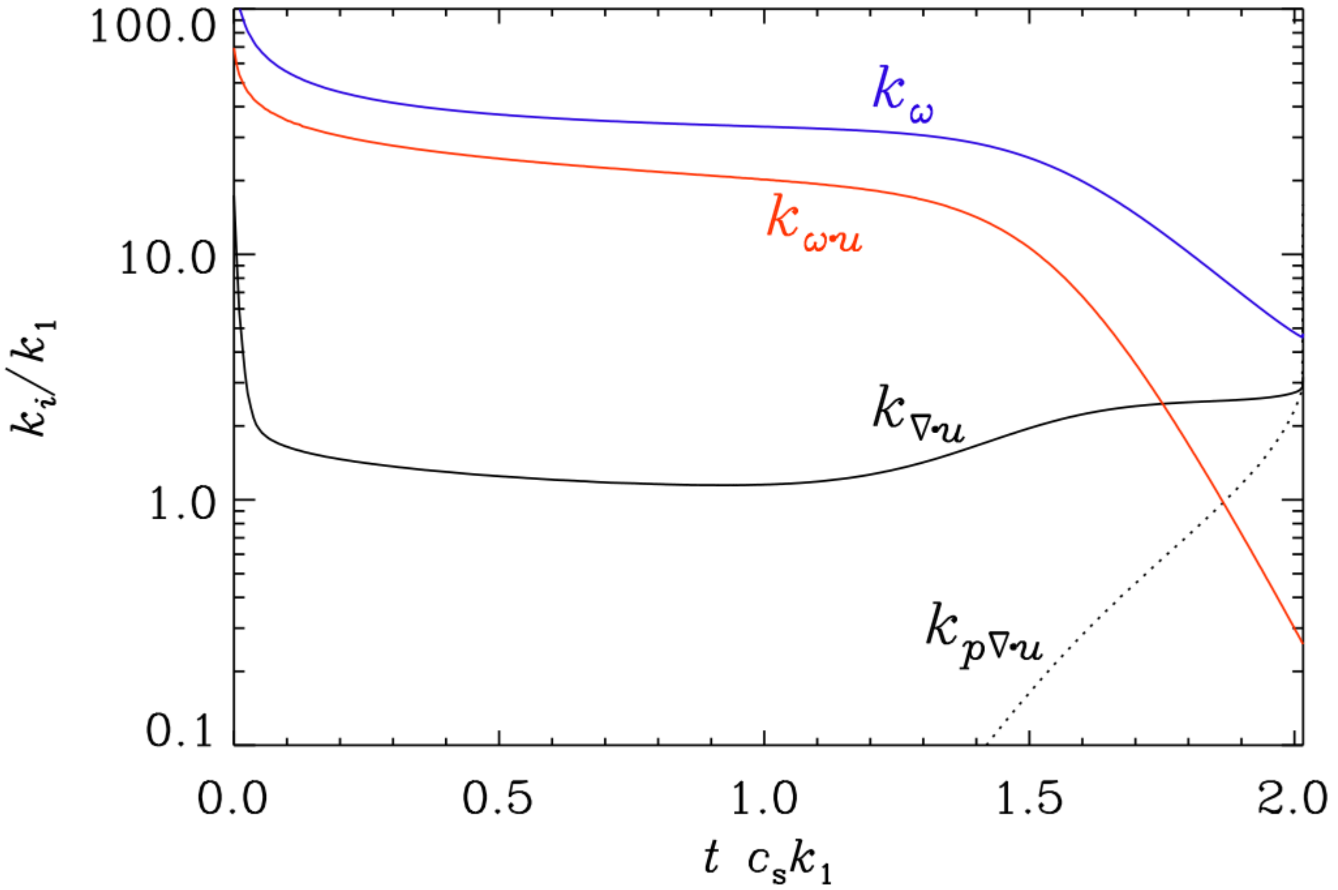
which have the dimension of a wavenumber. Since the flow is helical, we can also define the wavenumber

$$k_{\boldsymbol{\omega}\cdot\boldsymbol{u}} = |\langle\boldsymbol{\omega}\cdot\boldsymbol{u}\rangle|/u_{\rm rms}^2, \tag{11}$$

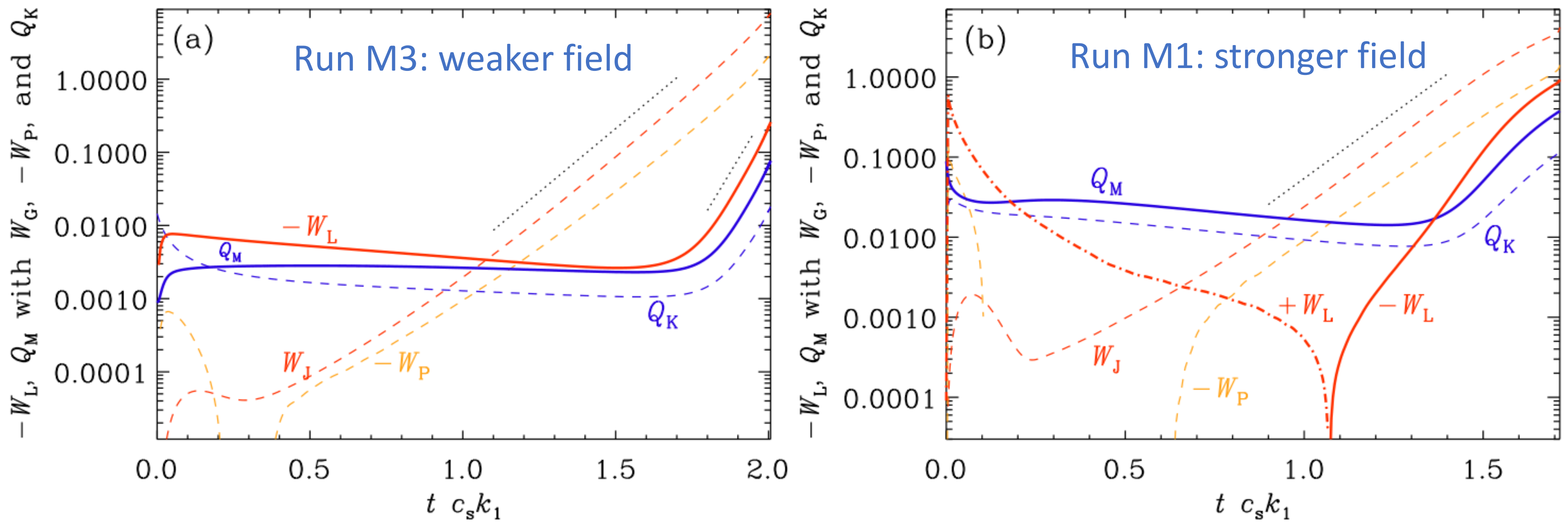
which characterizes the typical wavenumber where helicity plays a role. Large values of $k_{\boldsymbol{\nabla}\cdot\boldsymbol{u}}$, $k_{\boldsymbol{\omega}}$, and $k_{\boldsymbol{\omega}\cdot\boldsymbol{u}}$ imply strong flow divergences or compressions, strong vortices, and strong swirls, respectively. To characterize the flow compression from the gravitational collapse, we also define

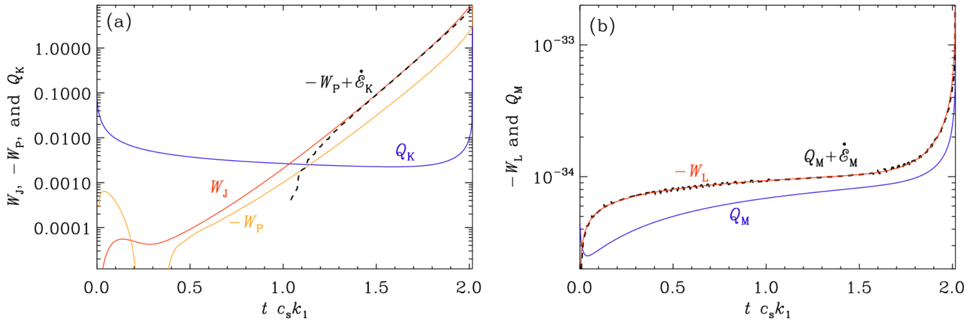
$$k_{p\boldsymbol{\nabla}\cdot\boldsymbol{u}} = -\langle p\boldsymbol{\nabla}\cdot\boldsymbol{u}\rangle/p_0 u_{\rm rms} \quad (\text{when } k_{p\boldsymbol{\nabla}\cdot\boldsymbol{u}} > 0), \tag{12}$$

Vorticity becomes relatively weaker



Cases with stronger magnetic field

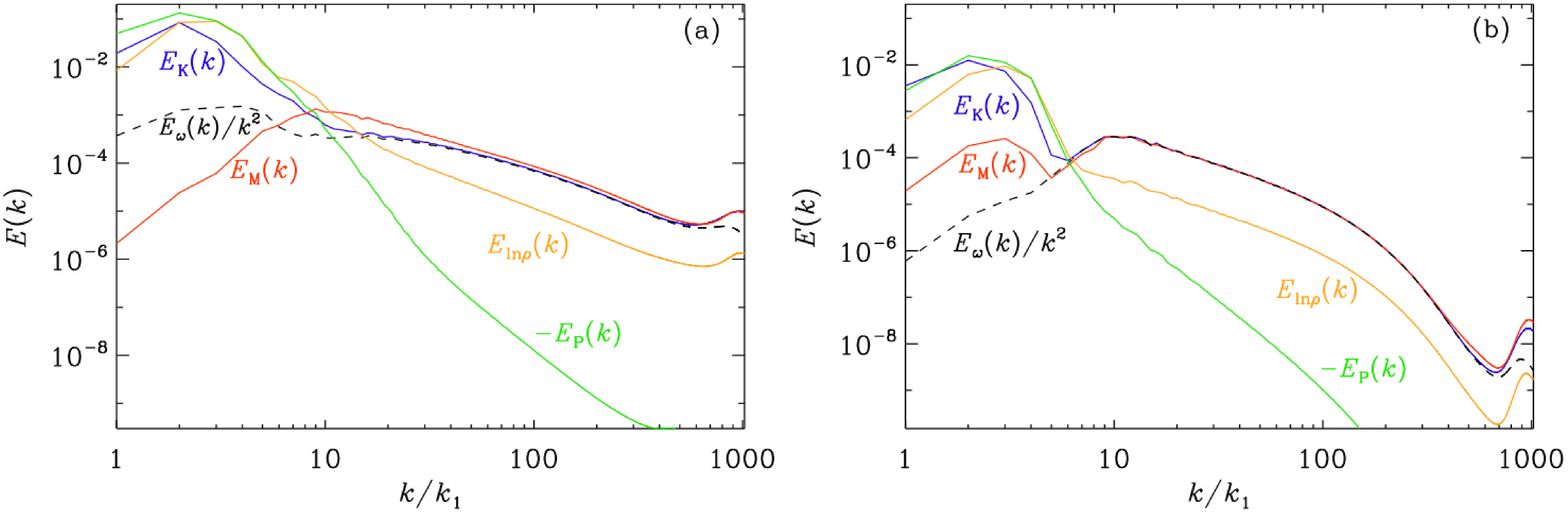




Lorentz work initially reversed:
magnetic field drives flow

Comparison with
very weak field

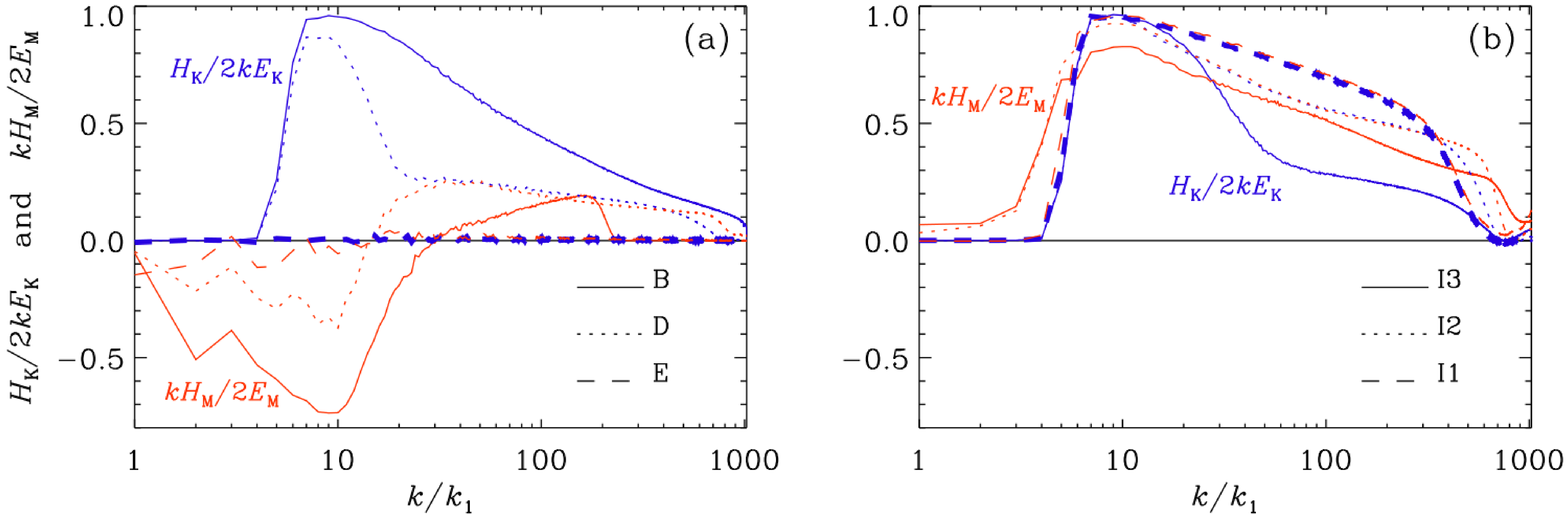
Spectra: comparison w/ imposed field



Turbulent magnetic field:
large-scale vorticity production

Imposed field:
no vorticity production

Helicity spectra: comparison w/ imposed field



Both signs of magnetic helicity

Only one sign of magnetic helicity

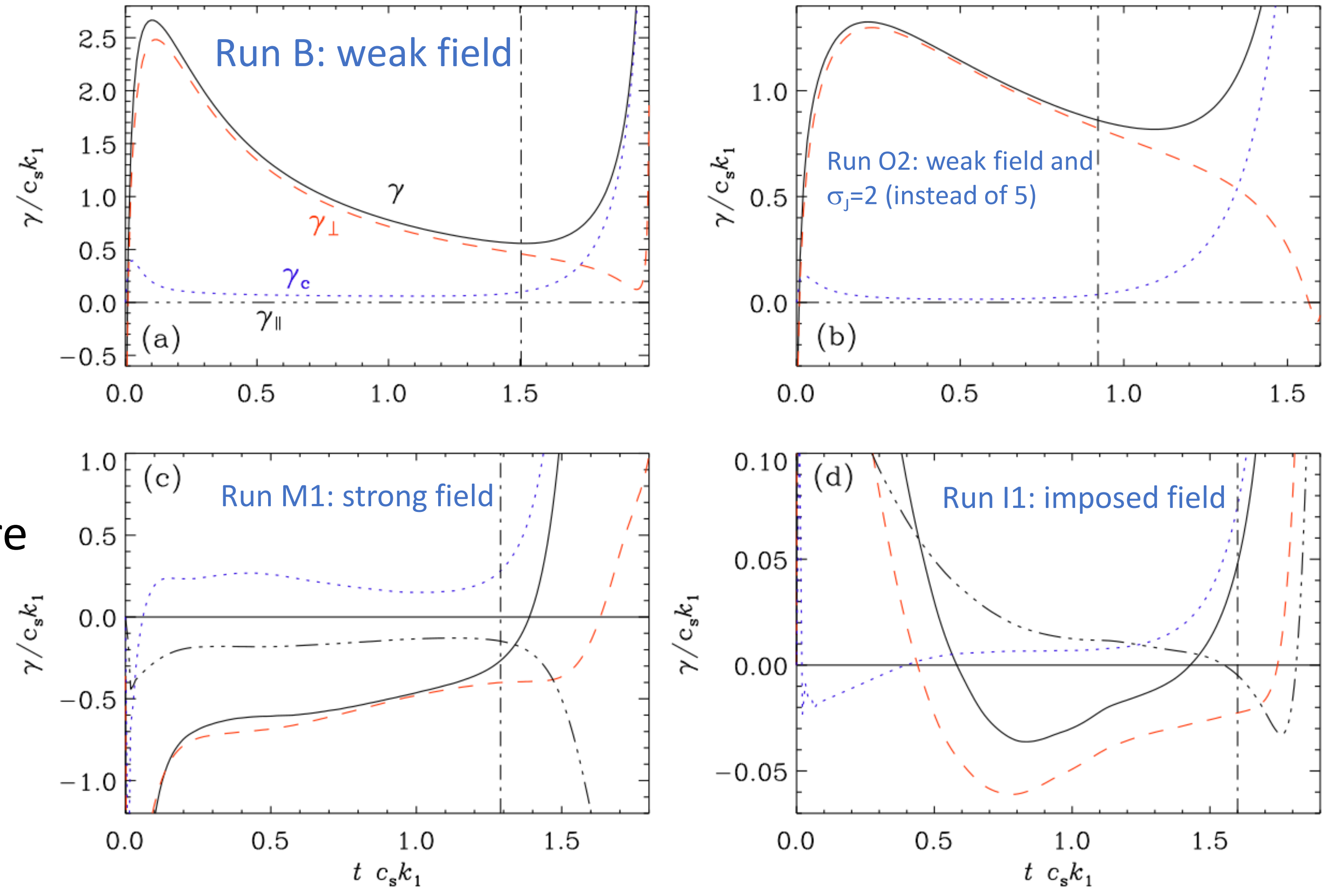
Work against compression, stretching, & curvature force

$$-\langle \boldsymbol{u} \cdot (\boldsymbol{J} \times \boldsymbol{B}) \rangle = \langle \mu_0 \eta \boldsymbol{J}^2 \rangle + \frac{\mathrm{d}\mathcal{E}_\mathrm{M}}{\mathrm{d}t}.$$

The W_L term can be split into three constituents: $W_\mathrm{L}^\mathrm{c} = -\langle \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{B}^2/2\mu_0 \rangle$, $W_\mathrm{L}^\parallel = \langle \boldsymbol{u} \cdot (\boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B}/\mu_0)_\parallel \rangle$, and $W_\mathrm{L}^\perp = \langle \boldsymbol{u} \cdot (\boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B}/\mu_0)_\perp \rangle$. Here, $-\boldsymbol{\nabla} \boldsymbol{B}^2/2\mu_0$ is the magnetic pressure contribution of $\boldsymbol{J} \times \boldsymbol{B}$, and $(\boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B}/\mu_0)_\parallel$ and $(\boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B}/\mu_0)_\perp$ are the stretching terms along and perpendicular to the mag-

$$\gamma = (-W_\mathrm{L} - Q_\mathrm{M})/\mathcal{E}_\mathrm{M}$$

$$\gamma_\perp = (-W_\mathrm{L}^\perp - Q_\mathrm{M})/E_\mathrm{M}$$



$$\gamma_\mathrm{c} + \gamma_\parallel + \gamma_\perp = \gamma$$

- Early growth dominated by curvature
  - But declines with time
  - Compression just at late times
  - Possibly not by dynamo action
- Strong initial magnetic field
  - Compression term negative
  - But strong compression at late times

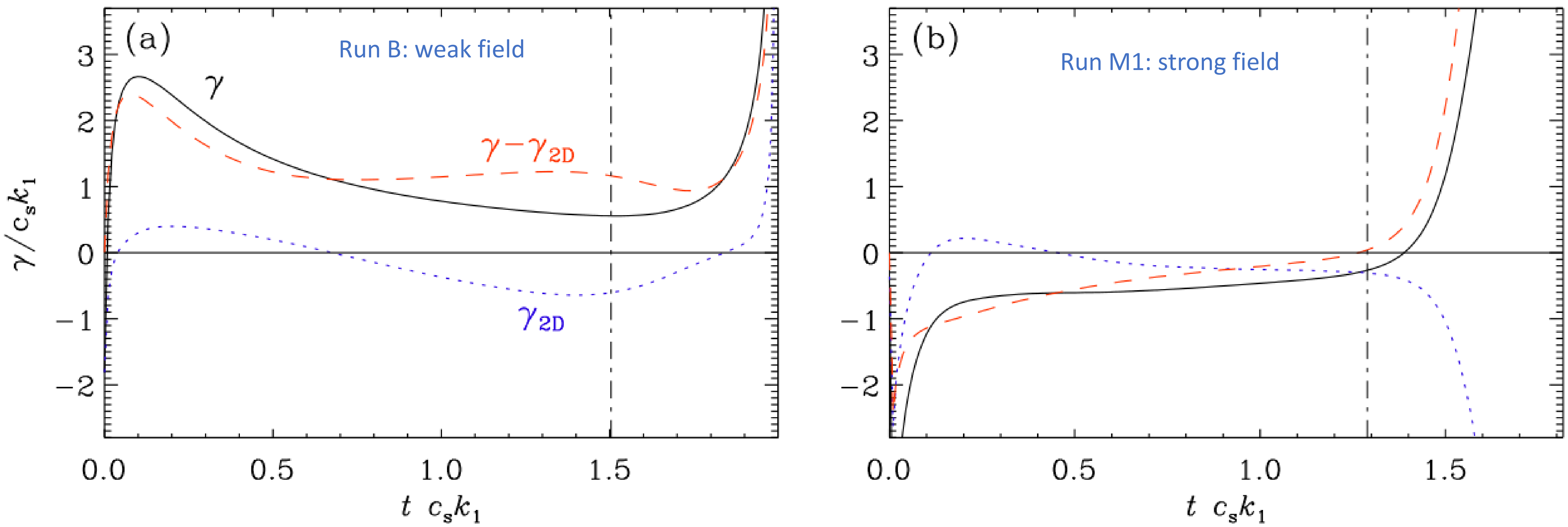
Work terms for 2-D and 3-D fields

In the following, we also decompose $W_{\rm L}$ by writing it as $W_{\rm L} = -\langle \boldsymbol{J} \cdot (\boldsymbol{u} \times \boldsymbol{B})\rangle$ and expanding the curl to get

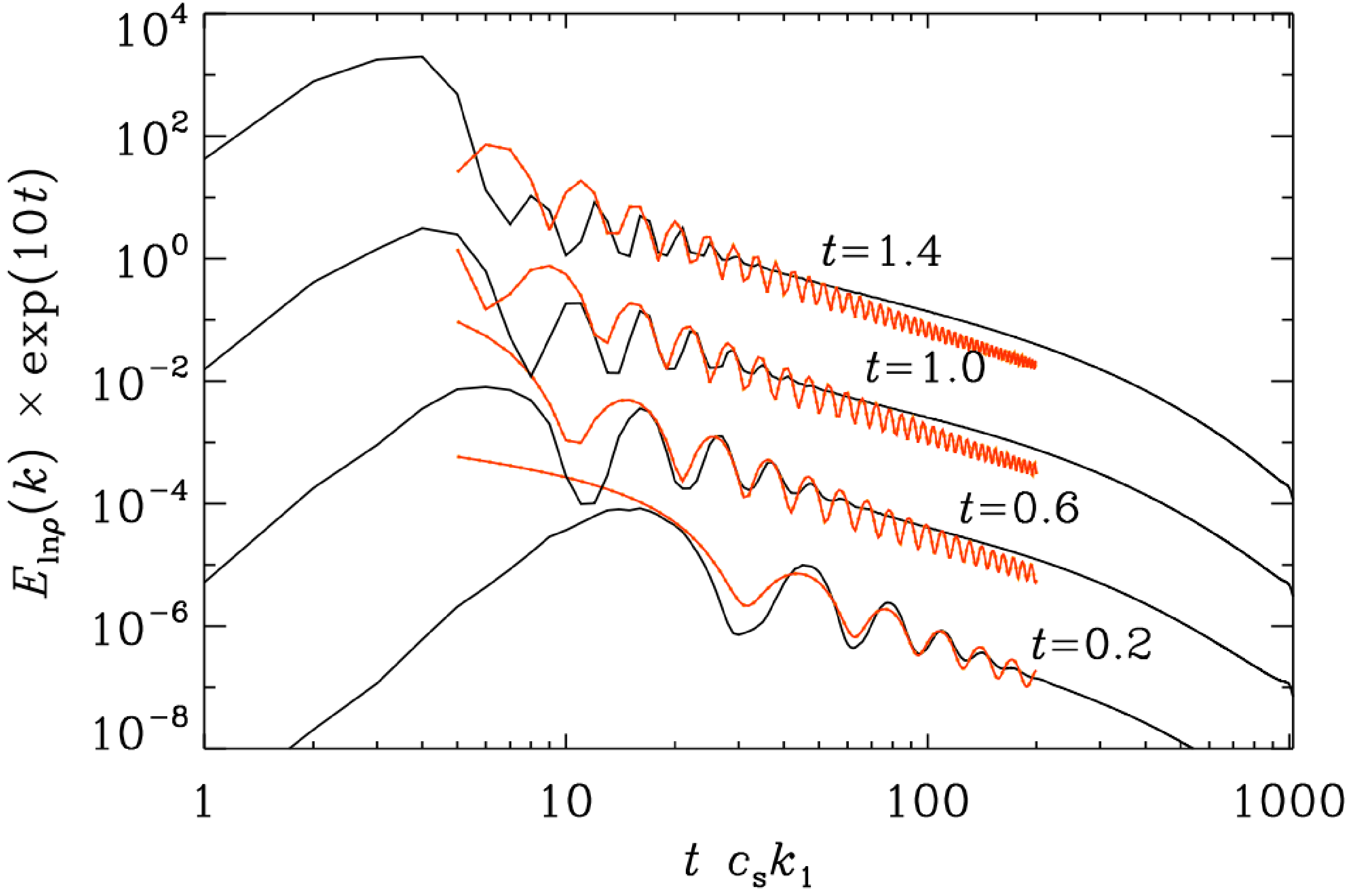
$$-\langle \boldsymbol{J} \cdot (\boldsymbol{u} \times \boldsymbol{B})\rangle = \langle J_i u_j (A_{i,j} - A_{j,i}\rangle \equiv W_{\rm L}^{\rm 2D} + W_{\rm L}^{\rm 3D}. \quad (8)$$

netic fields. Likewise, we define $\gamma_{\rm 2D} = -(W_{\rm L}^{\rm 2D} + Q_{\rm M})/E_{\rm M}$ and $\gamma_{\rm 3D} = -W_{\rm L}^{\rm 3D}/E_{\rm M}$, so that $\gamma_{\rm 2D} + \gamma_{\rm 3D} = \gamma$.

Here, we make use of the fact that the Weyl gauge has been used in Eq. (4). In two dimensions, the magnetic field can be represented as $\boldsymbol{B} = \boldsymbol{\nabla} \times A_z \hat{\boldsymbol{z}}$, with its x and y components lying in the xy plane. Then the term $W_{\rm L}^{\rm 3D} = -\langle J_i u_j A_{j,i}\rangle$ vanishes in 2-D. Thus, we can identify $W_{\rm L}^{\rm 3D}$ with a contribution that characterizes the 3-D nature of the system and can therefore be a proxy for dynamo action, provided $W_{\rm L}^{\rm 3D}$ is large enough.



Oscillations in the spectra!?



Waves in k-space:

$$\cos k\xi(t)$$

Radius of expansion waves launched at t=0

$$\xi(t) = c_s t$$

Red line: a simple fit that captures the change of phase at early times

$$E_{\ln\rho}(k,t) = E_\rho^{(0)}(k)\ [1 + g(k,t)\,(1 - \cos k c_s t)]$$

Conclusions

- Dynamo question not obvious
  - Dynamos in collapsing flows previously taken for granted
  - Now: collapse responsible for driving irrotational flows
  - Such flows never produced dynamos (so far!)
- Could be different for Bonnor-Ebert spheres
  - Used in Sur+10,12; Federrath+11
  - Collapse might be sufficiently slow to allow dynamo to establish
  - Might also produce more vorticity (at least via rotation and B-fields)
- Work term analysis
  - 1/3 into heating, 2/3 into kinetic energy
  - At later times more like 1/4 and 3/4 for heating and kinetic energy
  - Implies virial parameters of 4/3 and 3/2, respectively
- Next?
  - Do 1-D Ebert sheet (solutions by Spitzer-42 and Ledoux-51)