Holographic Effective Actions and Bubble Nucleation

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Ares, Henriksson, Hindmarsh, C.H., Jokela; 2110.14442, 2109.13784, (2011.12878)

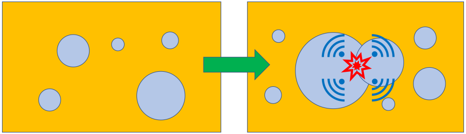
Motivation

- The Standard Model has no phase transitions at high temperatures
- A phase transition in the early Universe would be a signal of **new physics**

  Extensions of the EW theory or in a Dark Matter sector
- Generically, the transition will not happen at weak coupling

  Holography may be useful to describe the transition

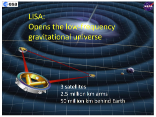
A phase transition in the early Universe may produce GWs through bubble collisions

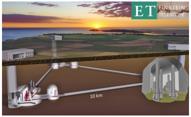


Alternative: "spinoidal evolution"

Bea, Casalderrey-Solana, Giannakopoulos, Jansen, Krippendorf 2112.15478

GW observatories sensitive to low frequencies might be able to observe them





GWs from neutron star mergers could also be observable in resonant cavities $f \sim 1\,\text{MHz}$

Casalderrey-Solana, Mateos, Sanchez-Garitaonandia 2210.03171

Thermal radiation seems to be relevant only at very high frequencies $f \sim 100\,\text{GHz}$

Castells-Tiestos, Casalderrey-Solana 2202.05241

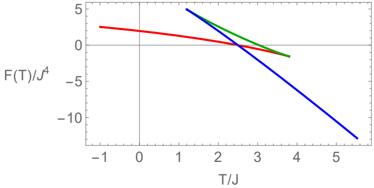
The GW power spectrum depends on the details of bubble nucleation

Nucleation rate per unit volume $= \frac{dN_b}{dtdV}$

- Transition starts at $T_0 < T_c$, when one bubble per Hubble volume per Hubble time nucleates
$$\left.\frac{dN_b}{dtdV}\right|_{T_0} = H^4$$
- Nucleation temperature T_n: fractional volume in metastable phase $= 1/e \approx 0.37$
$$\left.\frac{8\pi v_w^3}{\beta^4}\frac{dN_b}{dtdV}\right|_{T_n} \approx 1$$
- Transition rate:
$$\beta = \left.\frac{d}{dt}\log\frac{dN_b}{dtdV}\right|_{t=t_n} = -HT\left.\frac{d}{dT}\log\frac{dN_b}{dtdV}\right|_{T=T_n}$$
- Mean bubble separation:
$$R_*^3 \approx \frac{8\pi v_w^3}{\beta^3}$$

Quick review of bubble nucleation

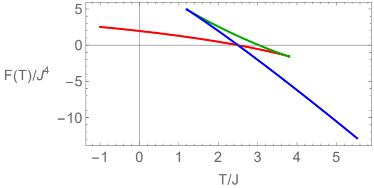
Nice review: Hindmarsh, Lüben, Lumma, Pauly 2008.09136



Stable phases:

- $T > T_c$: High-T phase
  $F_h = -V p_h$
- $T < T_c$: Low-T phase
  $F_l = -V p_l$

Classical Nucleation Theory



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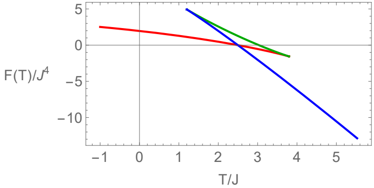
At $T < T_c$, $p_l > p_h$ and the high-T phase becomes metastable:

- Before the bubble forms: $F_{\text{hom}} = F_h = -Vp_h$
- After the bubble forms: $F_{\text{bub}} = -(V - V_b)p_h - V_b p_l + \sigma S_b$
  Spherical bubble volume: $V_b = \frac{4}{3}\pi r^3$, Bubble area: $S_b = 4\pi r^2$



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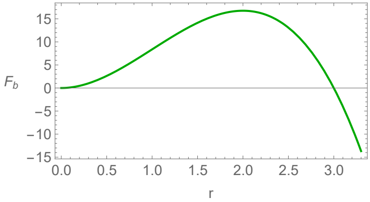
  Spherical bubble volume: $V_b = \frac{4}{3}\pi r^3$, Bubble area: $S_b = 4\pi r^2$
- Bubble free energy cost:

$$F_b = \Delta F = F_{\text{hom}} - F_{\text{bub}} = 4\pi r^2 \sigma - \frac{4}{3}\pi r^3 (p_l - p_h)$$

Classical Nucleation Theory

$$F_b(r) = 4\pi r^2\sigma - \frac{4}{3}\pi r^3(p_l - p_h)$$



$(\sigma = 1, p_l - p_h = 1)$

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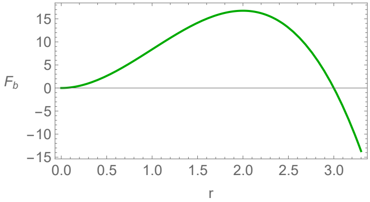

$(\sigma = 1, p_l - p_h = 1)$

Maximum of $F_b(r)$ at the critical radius:

$\left.\frac{\partial F_b}{\partial r}\right|_{r=r_c} = 0, \quad r_c = \frac{2\sigma}{p_l - p_h}$

For $r < r_c$: the bubble shrinks to $r = 0$

For $r > r_c$ the bubble expands

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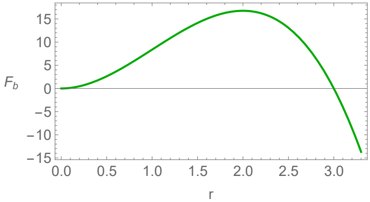

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- Phase transition realized through nucleation of critical bubbles
- The nucleation rate is given by

$$\boxed{\frac{dN_b}{dtdV} = \frac{n_S}{\tau} e^{-F_b(r_c)/T}}$$

n_S = density of nucleation sites, τ = mean free time

Thermal Field Theory

- Free energy $F = -T \log Z_T$:

  $Z_T = \mathrm{Tr}\left(e^{-H/T}\right) = \int \mathcal{D}\phi \exp\left(-\int_0^{1/T} d\tau \int d^3x \mathcal{L}_E[\phi]\right)$
- Euclidean action for a scalar

  $\mathcal{L}_E = \frac{1}{2}(\partial\phi)^2 - V_E(\phi), \quad V_E(\phi) = -V_T(\phi)$

  In principle thermal corrections may affect to kinetic terms as well

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- Saddle-point approximation:

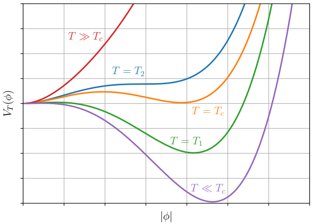
$$Z_T \simeq \sum_{\phi_{\rm cl}} \mathcal{Z}[\phi_{\rm cl}] = \sum_{\phi_{\rm cl}} \det\left(-\partial^2 + V_T''(\phi_{\rm cl})\right)^{-1/2} e^{-\int_0^{1/T} d\tau \int d^3x \mathcal{L}_E[\phi_{\rm cl}]}$$

- For a dominant homogeneous configuration $\phi_{\rm cl} = \phi_0$

$$F \simeq -T\log \mathcal{Z}[\phi_0] - T\sum_{\phi_{\rm cl}\neq\phi_0} \frac{\mathcal{Z}[\phi_{\rm cl}]}{\mathcal{Z}[\phi_0]} + \cdots$$

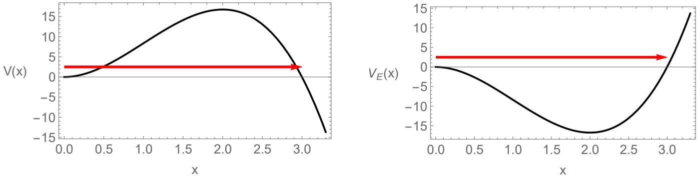
Effective theory with first order phase transitions

Thermal potential:



$T_1 < T < T_c$ transition between local and global minima

Particle in a potential



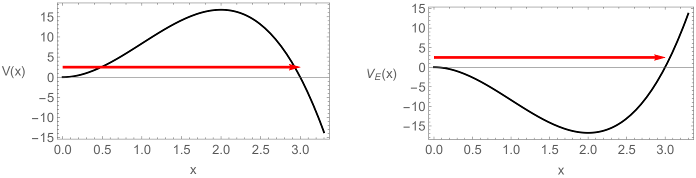
- Propagation amplitude

$$\langle x_f | \, e^{-H/T} \, | x_i \rangle = \int \mathcal{D}x \, e^{-S_E[x]} = \sum_n e^{-E_n/T} \langle x_f | E_n \rangle \langle E_n | x_i \rangle$$

- Saddle-point approximation to propagation amplitude:

$$\langle x = 0 | \, e^{-H/T} \, | x = 0 \rangle \simeq \sum_{x_{\rm cl}} \Delta(x_{\rm cl}) e^{-S_E[x_{\rm cl}]} \simeq e^{-E_0/T} | \langle 0 | E_0 \rangle |^2$$

$$\Delta(x_{\rm cl}) = \det(-\partial_\tau^2 + V''(x_{\rm cl}))^{-1/2}$$



- ‘Bounce’ solutions x_b give a contribution to the ground state energy

$$E_0 \simeq E(x=0) + T\Delta(x_{\rm cl})e^{-S_E[x_b]}$$

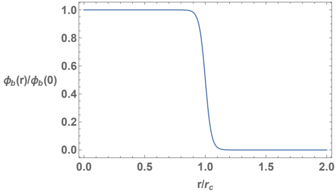
There is a negative eigenvalue of $-\partial^2 + V''(x_b) \;\Rightarrow\; \Delta = i|\Delta|$

- Decay probability per unit time Callan, Coleman '77

$$\boxed{\gamma = 2|\,\mathrm{Im}\, E_0|}$$

- Saddle-point approximation: critical bubble solution

$$\nabla^2\phi_b(r) - V_T'(\phi_b(r)) = 0$$
$$\lim_{r\to\infty}\phi_b(r) = \phi_0$$
$$\phi_b'(0) = 0$$



- Contribution to the free energy

$$\Delta F = -T\frac{\mathcal{Z}[\phi_b]}{\mathcal{Z}[\phi_0]} \simeq -T\frac{\Delta(\phi_0)}{\Delta(\phi_b)}e^{-S_c}$$

where

$$S_c = \frac{1}{T}\int d^3x\left[\frac{1}{2}(\nabla\phi_b)^2 + V_T(\phi_b) - V_T(\phi_0)\right]$$

- ϕ_b unstable: negative eigenvalue of $-\partial^2 + V_T''(\phi_b)$

$$\boxed{\frac{\gamma}{V} \sim \frac{1}{TV}|\,\mathrm{Im}\,F| \sim T^4 e^{-S_c(T)}}$$

Effective action

A reminder of the effective action in QFTs

- Partition function:

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, \exp\left[iS[\phi] + i \int d^4x J \Psi(\phi) \right]$$

- Generating functional:

$$\mathcal{W}[J] = -i \log \mathcal{Z}[J]$$

- "Classical field":

$$\frac{\delta \mathcal{W}[J]}{\delta J} = \langle \Psi \rangle_J \equiv \psi; \quad \psi_0 \equiv \langle \Psi \rangle_0 \, , \; \delta\psi \equiv \psi - \psi_0$$

- Effective action:

$$\Gamma[\langle \Psi \rangle_J] = \mathcal{W}[J] - \int d^4x \, \langle \Psi \rangle_J \, J$$

Euclidean Effective Action

- Thermal Partition Function = Euclidean Path Integral

$$Z_T[J] = e^{-W_E[J]} = \text{Tr}\left(e^{-H_J/T}\right) = \int \mathcal{D}\phi\, e^{-S_E[\phi]-\int_0^{1/T} d\tau \int d^3x \Psi(\phi) J}$$

- Euclidean Effective Action

$$\Gamma_E[\psi] = W_E[J] - \int_0^{1/T} d\tau \int d^3x \psi J$$

- The classical field satisfies the conditions

$$\psi(\tau = 0, \boldsymbol{x}) = \psi(\tau = 1/T, \boldsymbol{x}), \quad \lim_{|\boldsymbol{x}|\to\infty} \psi = \text{fixed}$$

- Free energy as function of the source and of the vev

$$F[J] = TW_E[J], \quad F[\psi] = T\Gamma_E[\psi], \quad F[J=0] = F[\psi_0]$$

- In general $\psi_0 = \psi_0(\tau, \boldsymbol{x})$ includes inhomogeneous configurations

$$\langle \Psi(\boldsymbol{x})\rangle_T = \psi_0(\tau = 0, \boldsymbol{x})$$

- Large-N saddle points of $W_E[J=0]$ coincide with extrema of Γ_E
  Note: $\mathrm{Re}\,\Gamma = O(N^2)$, $\mathrm{Im}\,\Gamma = O(1)$
- Homogeneous high- and low- T phases

$$F_{h,l} = T\Gamma_E[\psi_{h,l}] = V_3 V_T(\psi_{h,l}), \quad V_T'(\psi_{h,l}) = 0$$

- Bubble solutions $T < T_c$

$$\left.\frac{\delta\Gamma_E}{\delta\psi}\right|_{\psi_b} = 0, \quad \lim_{|\boldsymbol{x}|\to\infty} \psi_b = \psi_h, \quad F_{\text{with bubble}} = T\Gamma_E[\psi_b]$$

- Nucleation rate:

$$\frac{dN_b}{dtdV} \sim T^4 e^{-(\Gamma_E[\psi_b]-\Gamma_E[\psi_h])}$$

Expansion of the effective action

- No gapless degrees of freedom → local action

$$\Gamma[\psi] \simeq \int d^4x \left[-V_T(\psi) - \frac{1}{2} Z_T^{\mu\nu}(\psi)\partial_\mu\psi\partial_\nu\psi + \cdots \right]$$

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- Expanding around the extrema:

$$\Gamma[\psi] \simeq \int d^4x \left[-V_T(\psi_0) - \frac{1}{2} V_T''(\psi_0)(\delta\psi)^2 - \frac{1}{2} Z_T^{\mu\nu}(\psi_0)\partial_\mu\delta\psi\partial_\nu\delta\psi + \cdots \right]$$

Expansion of the effective action

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- Effective action $=$ generating functional of 1PI correlators $\Gamma^{(n)}$

$$\Gamma[\psi] \simeq \Gamma_0[\psi_0] + \frac{1}{2}\int d^4x_1 \int d^4x_2 \Gamma^{(2)}(x_1, x_2; \psi_0)\delta\psi(x_1)\delta\psi(x_2) + \cdots$$

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- The Fourier transform of the 1PI correlator will be

$$\widetilde{\Gamma}^{(2)}(k;\psi_0) \simeq -V_T''(\psi_0) - Z_T^{\mu\nu}(\psi_0)k_\mu k_\nu + \cdots$$

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$$F \to V_T \;\&\; \widetilde{\Gamma}^{(2)} \to Z_T^{\mu\nu}$$

1PI and connected correlators

- Generating functional:

$$\mathcal{W}[J] = \mathcal{W}_0 + \int d^4x \, \langle \Psi \rangle_0 \, J + \frac{1}{2} \int d^4x_1 \int d^4x_2 \, J(x_1) G_c(x_1, x_2) J(x_2) + \cdots$$

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- Classical field:

$$\langle\Psi\rangle_J (x) = \langle\Psi\rangle_0 + \int d^4x' G_c(x, x') J(x') + \cdots$$

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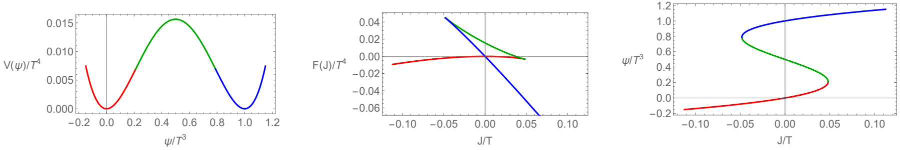
- 1PI correlator:

$$\boxed{\Gamma^{(2)}(x_1, x_2) = -G_c^{-1}(x_1, x_2)}$$

Strongly coupled QFT

Legendre transform at fixed temperature:



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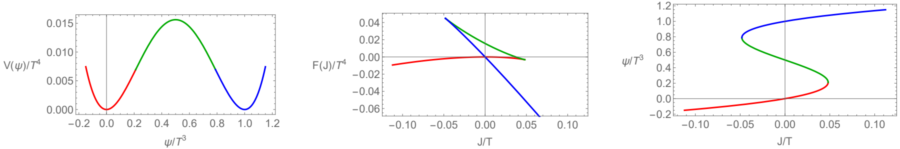
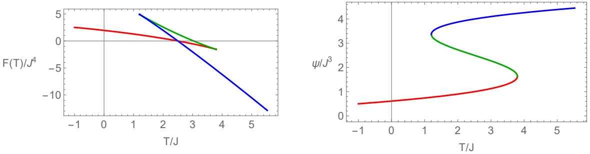


Strongly coupled QFT with phase transition:



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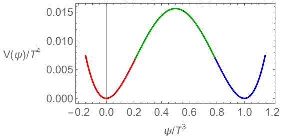

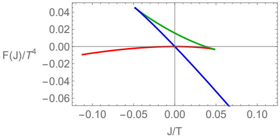

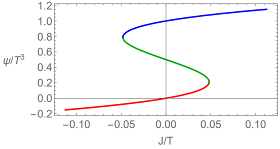

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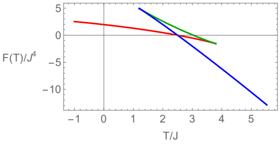
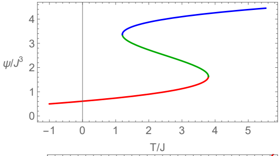
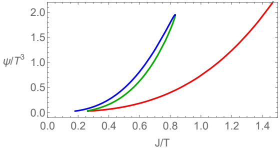




No obvious way to extract the effective potential



- The starting point is a strongly coupled large-N CFT (with a holographic dual)
- Ψ_Δ: relevant operator of conformal dimension $1 < \Delta < 4$

$$\text{e.g. } \Psi_2 = \frac{1}{N}\,\mathrm{tr}_{SU(N)}\left(\phi^2\right), \quad \Psi_3 = \frac{1}{N}\,\mathrm{tr}_{SU(N)}\left(\bar{\lambda}\lambda\right)$$

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- Introduce a source J

$$S[J] = S_{\text{CFT}} + \int d^4x J\Psi$$

We fix the temperature T and vary the source J

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We fix the temperature T and vary the source J

- Free energy density for a homogeneous J

$$f_{\mathrm{CFT}} = -w_{\mathrm{CFT}}(J)$$

- Classical field

$$\psi = w'_{\mathrm{CFT}}(J) \;\Rightarrow\; J(\psi)$$

- Effective potential

$$V_{\mathrm{CFT}} = -w_{\mathrm{CFT}}(J) + J\,\psi$$

The field theory

- We deform "by hand" the theory with multitraces: non-CFT

$$S_{QFT} = S_{CFT} - \int d^4x \Delta V(\Psi)$$

- In the large-N limit

$$\boxed{\langle \Delta V(\Psi) \rangle \simeq \Delta V(\langle \Psi \rangle)}$$

- Then, the effective potential in the deformed theory is

$$\boxed{V_T(\psi) = V_{\text{CFT}}(\psi) + \Delta V(\psi)}$$

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- Concerning the calculation using the holographic dual

  1) only V_{CFT} in the potential is computed

  2) $\tilde{\Gamma}^{(2)}$ is affected by the multitraces

Other approaches:

- Randall-Sundrum models

  Cline, Firouzjahi hep-ph/0005235; Creminelli, Nicolis, Ratazzi hep-th/0107141; Randall, Servant hep-ph/0607158; Konstandin, Nardini, Quiros 1007.1468; ...

- Confinement/Chiral symmetry breaking

  Bigazzi, Caddeo, Cotrone, Paredes 2008.02579; Janik, Jarvinen, Sonnenschein 2106.02642

- D-brane nucleation:

  Henriksson 2106.13254

A concrete example

Field theory:

- We take $\Delta = 4/3$ and a multitrace term

$$W(\Psi) \equiv \Delta V(\Psi) = \Lambda\Psi + \frac{f}{2}\Psi^2 + \frac{g}{3}\Psi^3$$

- Dimensionless quantities:

$$\widetilde{T} = \frac{T}{|\Lambda|^{3/8} + |f|^{3/4}}, \quad \Lambda_f = \frac{\Lambda}{f^2}, \quad g$$

Gravity dual:

- Holographic dual action to the CFT

$$S_{bulk} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[\mathcal{R} - \partial_\mu\phi\partial^\mu\phi - \mathcal{P}(\phi)\right], \quad N^2 = L^3/\kappa_5^2$$

$$\mathcal{P}(\phi) = -\frac{12}{L^2} + m^2\phi^2, \quad m^2L^2 = \Delta(\Delta - 4) = -32/9$$

- Multitraces are implemented through boundary conditions

Calculation of the effective action

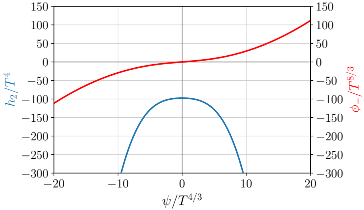
Metric:

$$ds^2 = -e^{-2\chi(r)}h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\vec{x}^2,\ \ h(r_H) = 0$$

Boundary expansions:

$$\phi = \frac{\phi_-}{r^{4/3}} + \frac{\phi_+}{r^{8/3}} + \dots$$

$$h = r^2 + \frac{4}{9}\frac{\phi_-^2}{r^{2/3}} + \frac{h_2}{r^2} + \dots$$



Numerical solutions

Holographic Dictionary:

Source: $J = \phi_+$

Classical field $\psi = -\frac{4}{3}\phi_-$

Temperature: $T = -\frac{e^{\chi(r_H)}\mathcal{P}(\phi_H)r_H}{12\pi}$

The gravitational action is

$$S = S_{bulk} + S_{GH} + S_{CT}$$

With

$$S_{GH} = \frac{1}{\kappa_5^2}\int d^4x\sqrt{-\gamma}K, \quad S_{CT} = \frac{1}{\kappa_5^2}\int d^4x\sqrt{-\gamma}\left(-3 + \frac{4}{3}\phi^2 + \phi\partial_r\phi\right)$$

The variation of the on-shell action is

$$\delta S = \frac{V_3}{T\kappa_5^2}\left(-\frac{4}{3}\phi_-\delta\phi_+\right) \equiv \frac{V_3}{T\kappa_5^2}\,\psi\delta J$$

The on-shell action is

$$S = \frac{V_3}{T\kappa_5^2}\left(-\frac{h_2}{2} - \frac{8}{27}\phi_+\phi_-\right) = \frac{V_3}{T\kappa_5^2}\left(-\frac{h_2(J)}{2} + \frac{2}{9}J\psi\right) \equiv \frac{V_3}{T\kappa_5^2}w(J)$$

Multitrace deformation

We add to the gravitational action a new term

$$S_{mult} = \frac{1}{\kappa_5^2} \int d^4x \left(\psi W'(\psi) - W(\psi)\right)$$

Variation of the action

$$\delta S_{tot} = \delta S + \delta S_{mult} = \frac{V_3}{T\kappa_5^2}\, \psi \left(\delta\phi_+ + W''(\psi)\delta\psi\right)$$

Source

$$J = \phi_+ + W'(\psi)$$

On-shell action

$$S_{tot} = S + S_{mult} = \frac{V_3}{T\kappa_5^2} \left(w(\phi_+) + \psi W'(\psi) - W(\psi)\right)$$

Momentum-dependent fluctuations

$$ds^2 = -e^{-2\chi(r)} h(r) \left(1 + e^{ikx} H_{tt}(r)\right) dt^2 + \frac{dr^2}{h(r)} + r^2 \left(1 + e^{ikx} H_{xx}(r)\right) dx^2$$
$$+ r^2 \left(1 + e^{ikx} H_{\perp}(r)\right) \left(dy^2 + dz^2\right)$$
$$\phi = \phi(r) + e^{ikx} \varphi(r) \ .$$

Gauge-invariant combinations

$$Z_\phi(r) = \varphi(r) - \frac{r}{4} \phi'(r) H_\perp(r)$$
$$Z_H(r) = -e^{-2\chi(r)} h(r) H_{tt}(r) - \frac{r}{4} e^{-2\chi(r)} \left[h'(r) - 2h(r)\chi'(r)\right] H_\perp(r)$$

Boundary expansions

$$Z_\phi(r) = \frac{Z_\phi^-}{r^{4/3}} + \frac{Z_\phi^+}{r^{8/3}} + \dots$$
$$Z_H(r) = Z_H^+ r^2 + \frac{Z_H^-}{r^2} + \dots$$

Two-point correlation functions

Low momentum expansion

$$Z_i(r) = Z_i^{(0)}(r) + k^2 Z_i^{(2)}(r) + \dots \qquad \text{with} \quad i \in \{\phi, H\}$$

Connected correlator

$$\widetilde{G}_c(k) = \frac{\delta\psi}{\delta J} = -\frac{4}{3}\frac{Z_\phi^{-(0)}}{Z_\phi^{+(0)} + \frac{4}{3}W''(\psi)Z_\phi^{-(0)}} - \frac{4}{3}\frac{Z_\phi^{-(2)}Z_\phi^{+(0)} - Z_\phi^{-(0)}Z_\phi^{+(2)}}{\left(Z_\phi^{+(0)} + \frac{4}{3}W''(\psi)Z_\phi^{-(0)}\right)^2}k^2 + \dots$$

1PI correlator

$$\widetilde{\Gamma}^{(2)}(k) = -\left(\frac{3}{4}\frac{Z_\phi^{+(0)}}{Z_\phi^{-(0)}} + W''(\psi)\right) - \frac{3}{4}\frac{Z_\phi^{-(2)}Z_\phi^{+(0)} - Z_\phi^{-(0)}Z_\phi^{+(2)}}{\left(Z_\phi^{-(0)}\right)^2}k^2 + \dots$$

Coefficient of the kinetic term

$$Z(\psi) = \frac{3}{4}\frac{Z_\phi^{-(2)}Z_\phi^{+(0)} - Z_\phi^{-(0)}Z_\phi^{+(2)}}{\left(Z_\phi^{-(0)}\right)^2}$$

CFT effective potential and kinetic term

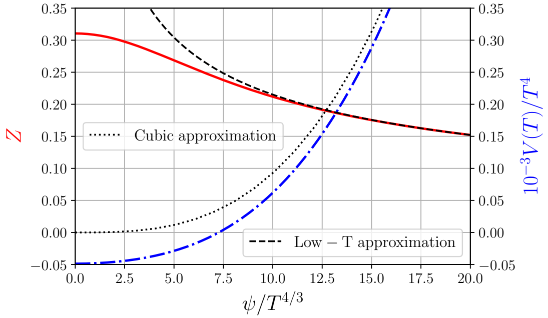
$$V_{CFT}(\psi) = -w(J) + J\psi = \frac{h_2(\psi)}{2} + \frac{7}{9}\phi_+(\psi)\psi$$

Small ψ: agrees with high-T CFT

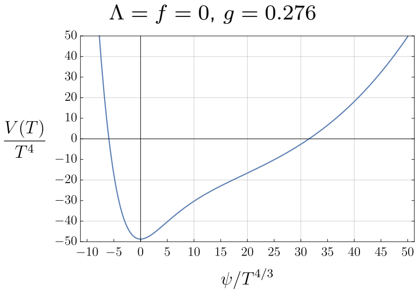
$$V_{CFT}(\psi) = T^4\left(V_0 + \frac{V_2}{2}\frac{\psi^2}{T^{8/3}} + \cdots\right), \quad V_0 = -\frac{\pi^4}{2}, \quad V_2 = \frac{9\pi^{17/6}}{\Gamma(1/6)^3}$$

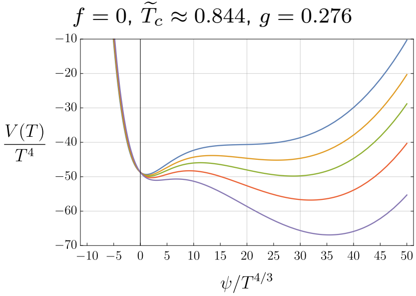
Large ψ: bound on triple-trace term: $g < \gamma_3$

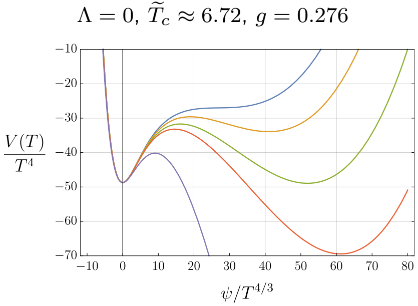
$$V_{CFT}(\psi) \sim \frac{\gamma_3}{3}|\psi|^3 \qquad \text{with} \qquad \gamma_3 \approx 0.278, \quad Z(\psi) \sim |\psi|^{-1/2}$$

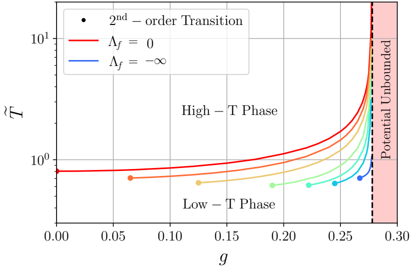


Effective potential









Warming exercise: domain wall solutions

- At $T = T_c$ the two minima of the effective potential have the same value $V(\psi_l) = V(\psi_h) = V_c$
- Mixed phases are possible
- Domain wall solution separating the two stable phases:

$$\lim_{x \to -\infty} \psi(x) = \psi_h, \quad \lim_{x \to \infty} \psi(x) = \psi_l$$

- Surface tension:

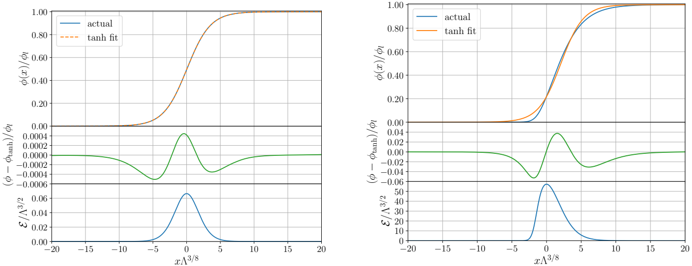
$$\sigma = \int_{-\infty}^{\infty} dx \left(\frac{1}{2} Z(\psi)(\partial_x \psi)^2 + V(\psi) - V_c \right)$$

Tanh approximation

$$\psi = \psi_h + \frac{\psi_l - \psi_h}{2}\left(1 + \tanh\left(\frac{x}{\ell} + \delta\right)\right)$$

Defining $\phi(x) = \psi(x) - \psi_h$, $\phi_l = \psi_l - \psi_h$

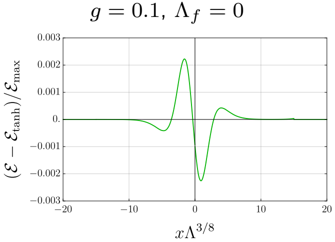
$g = 0.1,\ \Lambda_f = 0$ $\qquad$ $g = 0.2774,\ \Lambda_f = 0$



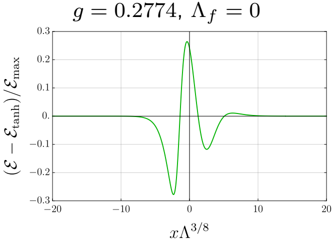
Warning: here and in the following the scale in the figures is $\Lambda^{3/8} \to |\Lambda|^{3/8} + |f|^{3/4}$

Domain wall profile

Energy density difference



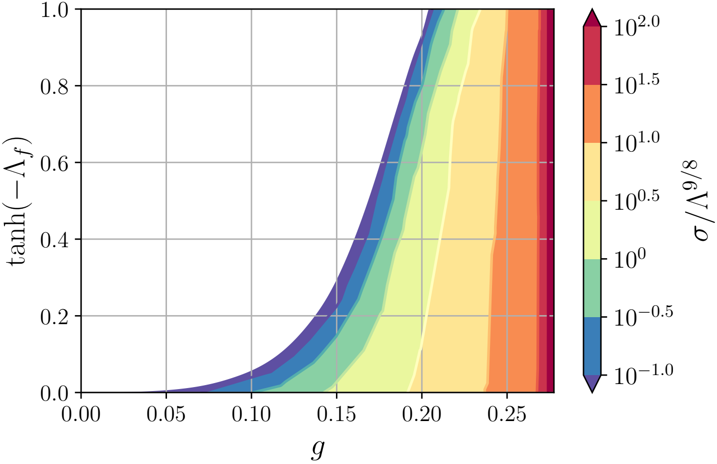
Minima of V close together



Minima of V far apart

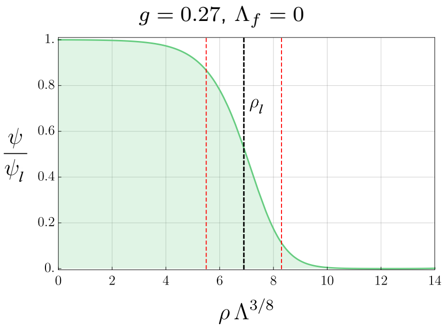
Deviations from quartic potential are reflected
in bad performance of tanh approximation

Scan of the domain wall tension



Bubble nucleation

Bubble solutions

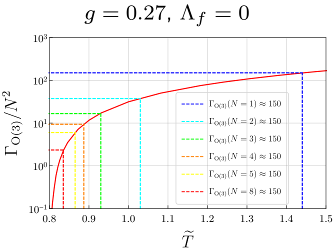


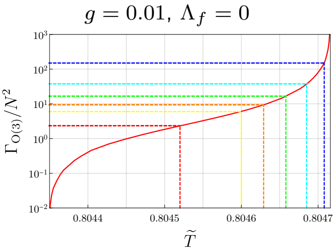
Euclidean effective action of a spherical bubble:

$$\Gamma_{O(3)} = \frac{4\pi N^2}{T}\int_0^\infty d\rho\, \rho^2 \left(\frac{1}{2}Z(\psi)\left(\frac{d\psi}{d\rho}\right)^2 + V(\psi)\right)$$

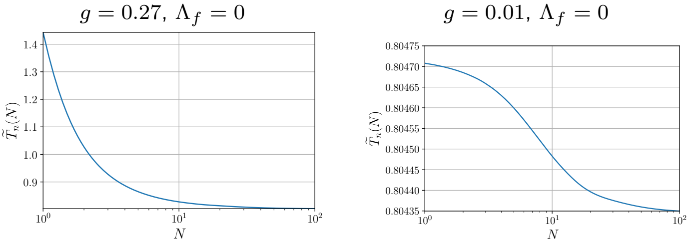
The N^2 dependence affects to the nucleation process

$$\Gamma_{O(3)}(T_n) \simeq 4\log(T_c/M_{\text{Pl}}) \underset{T_c\simeq 100\text{ GeV}}{\simeq} 150$$





N-dependence of nucleation temperature

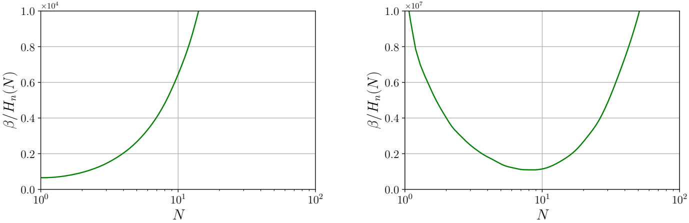


$$\Gamma \sim N^2 |T_n - T_*|^x \longrightarrow \frac{\beta}{H_n} \sim N^{2/x}$$

Thin wall: $T_n \simeq T_* = T_c$, $x = -2$, $\beta/H_n \sim N^{-1} \underset{N\to\infty}{\longrightarrow} 0$

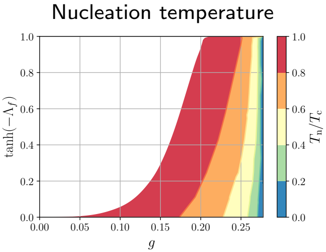
Thick wall: $T_n \simeq T_* = T_1$, $x > 0$, $\beta/H_n \underset{N\to\infty}{\longrightarrow} \infty$

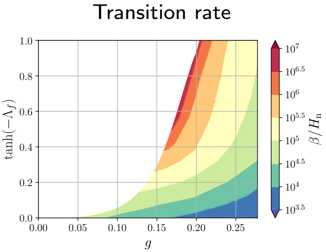
$g = 0.27$, $\Lambda_f = 0$ $\quad$ $g = 0.01$, $\Lambda_f = 0$



Application to GW production

Nucleation parameters for $N = 8$



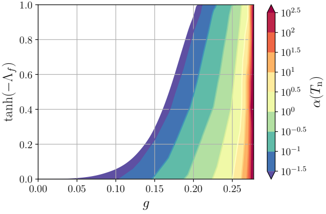


Opposite regimes:

- Larger g: stronger supercooling, lower transition rates
- Smaller g: $T_n \approx T_c$, higher transition rates

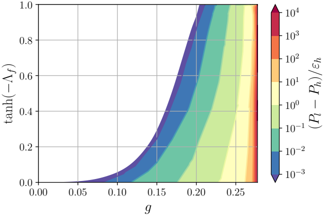
Nucleation parameters for $N = 8$

Transition strength



$$\alpha = \frac{1}{3}\frac{\varepsilon_h - 3p_h - (\varepsilon_l - 3p_l)}{\varepsilon_h + p_h}\Big|_{T_n}$$

Wall speed



$$u_w = \gamma_w v_w \simeq \frac{P_l - P_h}{\varepsilon_h}\Big|_{T_n}$$

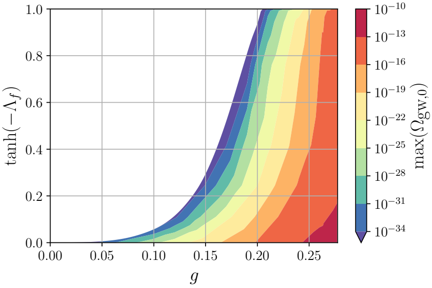
Formula for wall speed is an educated guess inspired by other holographic results

Bea, Casalderrey-Solana, Giannakopoulos, Mateos, Sanchez-Garitaonandia: 2104.05708; Bigazzi, Caddeo, Canneti, Cotrone 2104.12817; Janik, Jarvinen, Soltanpanahi, Sonnenschein 2205.06274

Enhanced production expected in the large supercooling region (larger g)

- Transition stronger
- Wall speeds larger

GW spectrum for $N = 8$



- Detectable signal in strong supercooling region $\Lambda_f \approx 0$, $g \approx 0.277$
- Peak frequency $f \sim 10^{-2} - 10^{-3}$ Hz when close to EW scale: 0.3 TeV $< T_c <$ 1.8 TeV
- Nearly-conformal dynamics Konstandin, Servant 1104.4791

Outlook

Possible extensions of the effective action approach:

- Coupling to hydrodynamics?
- Second order phase transitions? Higher order?
- Gravity dual solution: bubble provides initial condition for later dynamical evolution
- Phase transitions in realistic models: chiral symmetry breaking (easier), confinement (harder)
- Phase transitions at nonzero density, magnetic field, etc