Neutron star interiors: model-independent lessons and unsolved problems

Aleksi Vuorinen

University of Helsinki & Helsinki Institute of Physics

Holography for Astrophysics and Cosmology Nordita, 21 October 2022







Perturbative QCD at high density:
1) Gorda, Kurkela, Paatelainen, Säppi, AV, PRL 127 (2021), 2103.05658
2) Gorda, Kurkela, Österman, Paatelainen, Säppi, Schicho, Seppänen,

AV, 2204.11893 Neutron-star applications:

3) Annala, Gorda, Kurkela, Nättilä, AV, Nature Phys. (2020), 1903.09121 4) Annala, Gorda, Katerini, Kurkela, Nättilä, Paschalidis, AV, PRX 12 (2022), 2105.05132

+ Ongoing work with AGKN & Hebeler, Hirvonen, Komoltsev, Schwenk



Dense QCD challenge: can we understand the composition and macroscopic properties of NSs using only first-principles field theory tools and robust observational data?

Link between micro and macro from GR and **Equation of State (EoS)**:

 $\frac{dM(r)}{dr}$

dp(r)



Dense QCD challenge: can we understand the composition and macroscopic properties of NSs using only first-principles field theory tools and robust observational data?



Clear need for systematic and model-independent approach to the microphysics of neutron stars, with EoS playing a special role

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r), \qquad [\text{Osel et al., Api B20 (2016)}]$$

$$\frac{dp(r)}{dr} = -\frac{G\varepsilon(r)M(r)}{r^2} \frac{(1+p(r)/\varepsilon(r))(1+4\pi r^3 p(r)/M(r))}{1-2GM(r)/r}$$

$$\varepsilon(r_r) \rightarrow M(R)$$

Rest of the talk:

- I. NS matter basics: what do we know about the composition of NSs from nuclear physics?
- II. Lessons from (ultra)high density: what can pQCD do for you?
- III. NS observations: from masses and radii to NS mergers
- IV. Putting it all together: how far can model-independent approaches take us right now?
- V. Future directions: what is to be expected in near future?

Rest of the talk:

- I. NS matter basics: what do we know about the composition of NSs from nuclear physics?
- II. Lessons from (ultra)high density: what can pQCD do for you?
- III. NS observations: from masses and radii to NS mergers
- IV. Putting it all together: how far can model-independent approaches take us right now?
- V. Future directions: what is to be expected in near future?

Recurring theme: where can holography make a difference? Please interrupt with questions and comments!

NS matter: from dilute crust to ultradense core

- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$



- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$

Beyond neutron drip point NN interactions important; then 3Ns, boost corrections, etc.



- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$

Beyond neutron drip point NN interactions important; then 3Ns, boost corrections, etc.

• Systematic effective theory framework: Chiral Effective Theory (CET)



	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Qº)	X H-I	—	—
NLO (Q²)	ХӨӨХД	—	—
N²LO (Q³)	$\mathbf{k} = \mathbf{k}$	HH HX X	—
N ³ LO (Q ⁴)	XMAX-	掛₩‡Х…	***
N⁴LO (Q⁵)	<! <!<!</td <td> 4 </td> <td> .≱+ +X/I••</td>	4 	.≱+ +X/I••

- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$

Beyond neutron drip point NN interactions important; then 3Ns, boost corrections, etc.

- Systematic effective theory framework: Chiral Effective Theory (CET)
- State-of-the-art CET EoSs NNNLO in χPT power counting but still long way from stellar centers [e.g. Tews et al., PRL 110 (2013)]



- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$
- At high density, asymptotic freedom \Rightarrow weakening coupling and deconfinement
- State-of-the-art pQCD EoS at partial NNNLO, with purely soft sector fully determined [Gorda et al., PRL 127 (2021)]
- Still remaining from full α³_s result: "purely hard" and "mixed" contributions



- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$
- ∴ Low- and high-density limits under control but extensive no-man's land at intermed.
 densities. Have to work with:
- 1) Astrophysical observations
- 2) Thermodynamic relations
- 3) Subluminality: $c_s \leq 1$



- μ_B increases gradually, starting from $\mu_{\rm Fe}$
- Baryon/mass density increase beyond saturation density $\approx 0.16/\text{fm}^3$
- Composition changes from ions to nuclei to neutron liquid and beyond
- Good approximations: $T \approx 0 \approx n_Q$
- ∴ Low- and high-density limits under control but extensive no-man's land at intermed.
 densities. Have to work with:
- 1) Astrophysical observations
- 2) Thermodynamic relations
- 3) Subluminality: $c_s \leq 1$

Barring of course a new ab-initio approach!



Possible way to proceed: build large ensembles of randomly generated interpolators with piecewise basis functions



Possible way to proceed: build large ensembles of randomly generated interpolators with piecewise basis functions

Require for all individual EoSs:

- 1) Smooth matching to nuclear and quark matter EoSs
- 2) Continuity of p and n_B with at most one exception (1st order transition)
- 3) Subluminality: $c_s < 1$
- 4) Stellar models constructed with interpolated EoSs agree with robust measurements of NS properties

[Kurkela et al., ApJ 789 (2014), Gorda et al., PRL 120 (2018); etc.]



Possible way to proceed: build large ensembles of randomly generated interpolators with piecewise basis functions

Require for all individual EoSs:

- 1) Smooth matching to nuclear and quark matter EoSs
- 2) Continuity of p and n_B with at most one exception (1st order transition)
- 3) Subluminality: $c_s < 1$
- Stellar models constructed with interpolated EoSs agree with robust measurements of NS properties

[Kurkela et al., ApJ 789 (2014), Gorda et al., PRL 120 (2018); etc.]



pQCD at high density and zero temperature: Hard (but not really) Thermal Loops

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_{\mu}e^{-\int d^3x\int_0^{1/T}d\tau\mathcal{L}_{\text{QCD}}},$$
$$\mathcal{L}_{\text{QCD}} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$



Andersen, Strickland, Su, JHEP 08 (2011) Ghiglieri, Kurkela, Strickland, AV, Phys. Rept. 880 (2020)

$$\Omega(T, \mu_u, \mu_d, \mu_s, m_s) = -T \log \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_{\mu} e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\text{QCD}}},$$
$$\mathcal{L}_{\text{QCD}} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_i (\gamma_\mu D_\mu + m_i - \mu_i \gamma_0) \psi_i$$

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_{\mu}e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\text{QCD}}},$$
$$\mathcal{L}_{\text{QCD}} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

- Sum-integrals get replaced by four-dimensional continuous integrals, with fermionic $p_0 \to p_0 i \mu$
 - Simplification from vanishing of diagrams with no fermion loops
 - $\circ~$ Technical challenge: how to deal with fermionic p_0 integrals in a systematic manner?

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_{\mu}e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\text{QCD}}},$$
$$\mathcal{L}_{\text{QCD}} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_{\mu}D_{\mu} + m_i - \mu_i\gamma_0)\psi_i$$

- Sum-integrals get replaced by four-dimensional continuous integrals, with fermionic $p_0 \to p_0 i \mu$
 - $\circ~$ Simplification from vanishing of diagrams with no fermion loops
 - $\circ~$ Technical challenge: how to deal with fermionic p_0 integrals in a systematic manner?
- IR sensitive modes no longer three-dimensional: all bosonic (Euclidean) four-momenta satisfying $|P| \leq m_E \sim g\mu_B$ need special treatment \circ Correct effective theory for IR modes: Hard Thermal Loops (HTL)

$$\mathcal{D}(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_{\mu}e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}_{\text{QCD}}},$$
$$\mathcal{L}_{\text{QCD}} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_{\mu}D_{\mu} + m_i - \mu_i\gamma_0)\psi_i$$

- Sum-integrals get replaced by four-dimensional continuous integrals, with fermionic $p_0 \to p_0 i \mu$
 - $\circ~$ Simplification from vanishing of diagrams with no fermion loops
 - $\circ~$ Technical challenge: how to deal with fermionic p_0 integrals in a systematic manner?
- IR sensitive modes no longer three-dimensional: all bosonic (Euclidean) four-momenta satisfying $|P| \leq m_E \sim g\mu_B$ need special treatment \circ Correct effective theory for IR modes: Hard Thermal Loops (HTL)
- No correct answer from lattice!

- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes



- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes



$$p = p_{\rm FD} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3$$
$$+ p_2^s \alpha_s^2 + p_3^s \alpha_s^3$$
$$+ p_3^m \alpha_s^3$$

- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes



$$p = p_{\rm FD} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3 + p_2^s \alpha_s^2 + p_3^s \alpha_s^3 + p_3^m \alpha_s^3 + p_3^m \alpha_s^3$$
Known since 1970's [Freedman, McLerran,

PRD 16 (1977)]

- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes





- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes



 $p = p_{\rm FD} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3 + p_2^s \alpha_s^2 + p_3^s \alpha_s^3$

Gorda, Kurkela, Paatelainen, Säppi, AV, PRL 127 (2021); Fernandez, Kneur, 2109.02410

- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes



 $p = p_{\rm FD} + p_1^h \alpha_s + p_2^h \alpha_s^2 + p_3^h \alpha_s^3$ $+ p_2^s \alpha_s^2 + p_3^s \alpha_s^3$ $+ p_3^m \alpha_s^3$ QED: Gorda, Kurkela, Österman, Paatelainen, Säppi, Seppänen, Schicho, AV, 2204.11893

QCD: Underway; results in 2022

- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes





- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes

Comparison of convergence at zero vs. high temperature:



- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes





- 1) Hard modes (scale μ_B) and their interactions: naïve loop expansion up to and including four loops
- 2) Soft modes (scale $m_E \sim g \mu_B$) and their interactions: one- and two-loop graphs in HTL effective theory
- 3) Mixing of soft and hard modes





What do we know from NS observations?



Radius (and combined *MR*) measurements more problematic, but recently important progress through X-ray observations:

- Cooling of thermonuclear X-ray bursts provide radii to ~ ± 400m [Nättilä et al., Astronomy & Astrophysics 608 (2017), ...]
- Pulse profiling (NICER) \Rightarrow nontrivial lower bounds for two stellar radii, including PSR J0740+6620 with $M \gtrsim 2M_{\odot}$ [Miller et al., Astrophysical Journal Letters 918 (2021),...]





Gravitational wave breakthrough: First observed binary NS merger GW170817 by LIGO & Virgo in 2017 (and many since then)

Three types of potential inputs:

- Tidal deformabilities of the NSs during inspiral – good measure of stellar compactness
- 2) Ringdown pattern sensitive to EoS (also at $T \neq 0$), but frequency too high for LIGO/Virgo
- 3) EM counterpart: indirect information on merger product

[LIGO and Virgo collaborations, PRL 119 (2017), PRL 121 (2018)]



Tidal deformability: How large of a quadrupolar moment a star's gravitational field develops due to an external quadrupolar field

$$Q_{ij} = -\Lambda \mathcal{E}_{ij}$$

Substantial effect on observed GW waveform during inspiral phase



[Read et al., PRD 88 (2013)]

Tidal deformability: How large of a quadrupolar moment a star's gravitational field develops due to an external quadrupolar field

$$Q_{ij} = -\Lambda \mathcal{E}_{ij}$$

LIGO & Virgo bound $70 < \Lambda(1.4M_{\odot}) < 580$ at 90% credence using low spin prior [LIGO and Virgo, PRL 121 (2018)]: useful test for EoSs



Gravitational wave breakthrough: First observed binary NS merger GW170817 by LIGO & Virgo in 2017 (and many since then)

Three types of potential inputs:

- Tidal deformabilities of the NSs during inspiral – good measure of stellar compactness
- 2) Ringdown pattern sensitive to EoS (also at $T \neq 0$), but frequency too high for LIGO/Virgo
- 3) EM counterpart: indirect information on merger product

[LIGO and Virgo collaborations, PRL 119 (2017), PRL 121 (2018)]



Ringdown pattern: Unlike in BH mergers, binary NS mergers expected to feature complex period of relaxation characterized by GW spectrum sensitive to both initial NS masses and the EoS



Post-merger dynamics can be studied with relativistic hydrodynamics simulations, showing marked sensitivity to first-order phase transitions, but frequency range (currently) too high for LIGO and Virgo



[Takami, Rezzolla, Baiotti, PRD 91 (2015)]

Gravitational wave breakthrough: First observed binary NS merger GW170817 by LIGO & Virgo in 2017 (and many since then)

Three types of potential inputs:

- Tidal deformabilities of the NSs during inspiral – good measure of stellar compactness
- 2) Ringdown pattern sensitive to EoS (also at $T \neq 0$), but frequency too high for LIGO/Virgo
- 3) EM counterpart: indirect information on merger product

[LIGO and Virgo collaborations, PRL 119 (2017), PRL 121 (2018)]



In GW170817, short gamma-ray burst 1.7s after GWs, followed by optical signal: Delayed collapse to a BH





In GW170817, short gamma-ray burst 1.7s after GWs, followed by optical signal: Delayed collapse to a BH

Constraints for maximal (TOV) mass of stable NSs from scenarios 2 and 3:

- 2) Differentially-rotating hypermassive NS: $M_{\text{remnant}} \ge M_{\text{crit}} = M_{\text{supra}}$ (HMNS-hyp below)
- 3) Uniformly-rotating supramassive NS: $M_{\text{remnant}} \ge M_{\text{crit}} = M_{\text{TOV}}$ (BH-hyp)





In GW170817, short gamma-ray burst 1.7s after GWs, followed by optical signal: Delayed collapse to a BH

Constraints for maximal (TOV) mass of stable NSs from scenarios 2 and 3:

- 2) Differentially-rotating hypermassive NS: $M_{\text{remnant}} \ge M_{\text{crit}} = M_{\text{supra}}$ (HMNS-hyp below)
- 3) Uniformly-rotating supramassive NS: $M_{\text{remnant}} \ge M_{\text{crit}} = M_{\text{TOV}}$ (BH-hyp)

HMNS-scenario more likely due to short delay between GW and EM signals; gives stronger constraints [Rezzolla et al, ApJ 852 (2018)]





Interpolation: combining all available information, what can we say about the EoS and the composition of massive NSs? Useful strategy: Implement interpolation starting from speed of sound and classify results in terms of maximal value c_s^2 reaches at any density [Annala et al., Nature Physics (2020) and PRX (2022)]



Useful strategy: Implement interpolation starting from speed of sound and classify results in terms of maximal value c_s^2 reaches at any density [Annala et al., Nature Physics (2020) and PRX (2022)]

Interesting because of tension between standard lore in nuclear physics and experience from other contexts



Useful strategy: Implement interpolation starting from speed of sound and classify results in terms of maximal value c_s^2 reaches at any density [Annala et al., Nature Physics (2020) and PRX (2022)]

Interesting because of tension between standard lore in nuclear physics and experience from other contexts

PHYSICAL REVIEW D 80, 066003 (2009)

Bound on the speed of sound from holography

Aleksey Cherman^{*} and Thomas D. Cohen[†] Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

Abhinav Nellore[‡]

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA (Received 12 May 2009; published 3 September 2009)

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single-scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.



- EoS must support $2M_{\odot}$ stars
- LIGO/Virgo 90% tidal deformability limit must be satisfied

[Annala et al., Nature Physics (2020)]



- EoS must support $2M_{\odot}$ stars
- LIGO/Virgo 90% tidal deformability limit must be satisfied
 [Annala et al., Nature Physics (2020)]

In recent work, also take into account:

- NICER data for PSR J0740+6620: $\circ R(2M_{\odot}) > 11.0$ km (95%) $\circ R(2M_{\odot}) > 12.2$ km (68%)
- BH formation in GW170817 via
 Supramassive or hypermassive NS



- EoS must support $2M_{\odot}$ stars
- LIGO/Virgo 90% tidal deformability limit must be satisfied
 [Annala et al., Nature Physics (2020)]

In recent work, also take into account:

- NICER data for PSR J0740+6620: $\circ R(2M_{\odot}) > 11.0 \text{km} (95\%)$ $\circ R(2M_{\odot}) > 12.2 \text{km} (68\%)$
- BH formation in GW170817 via
 Supramassive or hypermassive NS



- EoS must support $2M_{\odot}$ stars
- LIGO/Virgo 90% tidal deformability limit must be satisfied
 [Annala et al., Nature Physics (2020)]

In recent work, also take into account:

- NICER data for PSR J0740+6620: $\circ R(2M_{\odot}) > 11.0$ km (95%) $\circ R(2M_{\odot}) > 12.2$ km (68%)
- BH formation in GW170817 via
 Supramassive or hypermassive NS



- EoS must support $2M_{\odot}$ stars
- LIGO/Virgo 90% tidal deformability limit must be satisfied
 [Annala et al., Nature Physics (2020)]

In recent work, also take into account:

- NICER data for PSR J0740+6620: $\circ R(2M_{\odot}) > 11.0 \text{km} (95\%)$ $\circ R(2M_{\odot}) > 12.2 \text{km} (68\%)$
- BH formation in GW170817 via
 Supramassive or hypermassive NS



- EoS must support $2M_{\odot}$ stars
- LIGO/Virgo 90% tidal deformability limit must be satisfied
 [Annala et al., Nature Physics (2020)]

In recent work, also take into account:

- NICER data for PSR J0740+6620: $\circ R(2M_{\odot}) > 11.0$ km (95%) $\circ R(2M_{\odot}) > 12.2$ km (68%)
- BH formation in GW170817 via
 Supramassive or hypermassive NS





In particular the low- c_s EoSs suggest a two-phase structure

Distinguishing feature between phases: polytropic index (logarithm. slope) $\gamma \equiv \frac{d \ln p}{d \ln \epsilon} \approx 1$ in nearly conformal QM, ~2.5 in sub- n_s nuclear matter



Detailed comparison of interpolated EoSs with nuclear matter models and pQCD limit reveals M_{max} centres to reside closer to quark than nuclear-matter limit. Large QM-like cores for moderate latent heats and $\max(c_s^2)$.

This conclusion was significantly strengthened by new data in our 2022 PRX. [Annala et al., Nature Physics (2020); Annala et al., PRX 12 (2022)]



Detailed comparison of interpolated EoSs with nuclear matter models and pQCD limit reveals M_{max} centres to reside closer to quark than nuclear-matter limit. Large QM-like cores for moderate latent heats and $\max(c_s^2)$.

This conclusion was significantly strengthened by new data in our 2022 PRX.

[Annala et al., Nature Physics (2020); Annala et al., PRX 12 (2022)]



In addition to γ and c_s^2 , normalized trace anomaly $\Delta \equiv \frac{\epsilon - 3p}{3\epsilon} = \frac{1}{3} - \frac{p}{\epsilon}$ has been suggested as useful measure of conformality. [Fujimoto et al., 2207.06753]

To be conservative, demand that in QM $\gamma < 1.75$, $p/p_{\rm free} > 0.4 \& |\Delta| < 1/6$. Even then, likelihood of QM cores in $M_{\rm max}$ stars currently ~0.9!

[Annala, Gorda, Hirvonen, Komoltsev, Kurkela, Nättilä, AV, In preparation]

Future directions?

In near future, expect major advances from multiple fronts:

- Within CET, impressive efforts towards $2n_s$ limit
- In pQCD studies of cold QM, qualitative progress from inclusion of mixed contributions and resummations
- Astrophysical observations coming up:
 - GW observatory KAGRA started in 2020; Einstein Telescope in 2030s
 - On X-ray front NICER, to be complemented by eXTP around 2025
- Model-independent EoS studies:
 - Bayesian studies, enabling use of many more measurements
 - Incorporating explicit first-order transitions
- Yet, no ab-initio method with realistic chances between 2 and 20n_s
 Transport, out-of-equilibrium dynamics particular challenges