Holographic Floquet on the brane

Alfonso V. Ramallo

Univ. Santiago

Holography for Astrophysics and Cosmology

Nordita, Stockholm, October 2022

Based on 2007.12115 and 2209.03884
with Martí Berenguer, Ana Garbayo and Javier Mas

Plan of the talk

- Introduction and motivation
- Simple examples
- Holographic brane model
- Results of the D3-D5 model
- Future directions

Floquet theorem in QM

Temporal analogue of Bloch theorem

For a hamiltonian periodic in time

$$H(t+T) = H(t)$$

The solutions of the Schrödinger equation are of the form:

$$|\psi(t)\rangle \;=\; e^{-i\epsilon\, t}|\Phi(t)\rangle$$

$$|\Phi(t+T)\rangle \;=\; |\Phi(t)\rangle \;\rightarrow\; \text{Floquet mode}$$

$$\epsilon \rightarrow \;\text{quasienergy}$$

It applies to periodically driven systems

There are specific techniques to study these systems:

Extended (Sambe) Hilbert space

High-frequency & Magnus expansions

...

Why is interesting?

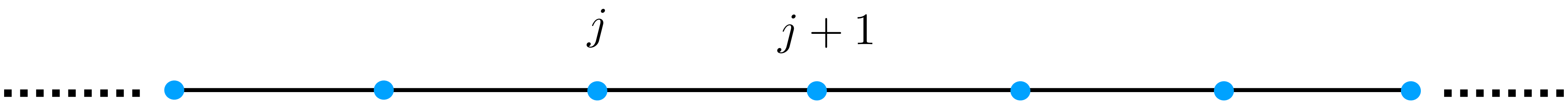
Floquet engineering → Control and design of quantum systems using periodic drivings

Examples

- New quantum (artificial) quantum materials induced by the driving (topological insulators, new superconductors, ...)
- Floquet time crystals
- Control of decoherence
- Artificial gauge fields
- New stable states out of equilibrium

We want to study these driven systems in holography

A simple example of a modulated system



Consider a 1d lattice with hamiltonian:

$$H_0 = -J \sum_j a^\dagger_{j+1} a_j + \text{h.c.}$$

$J \rightarrow$ hopping parameter $\rightarrow$ amplitude of the $j \rightarrow j+1$ transition

H_0 is the lattice version of a periodic one-particle hamiltonian in the tight-binding approximation

Add a periodic perturbation

$$H = H_0 + \omega\,\xi(t)\,V \qquad \xi(t) = \xi_0 \cos(\omega t)$$

$$V = \sum_j j\, a_j^\dagger\, a_j$$

V is the lattice version of the position operator x

The driving term can be absorbed by an unitary transformation

$$|\psi\rangle = R(t)\,\widetilde{|\psi\rangle} \qquad R(t) = e^{-i\xi_0\, \sin(\omega t)\, V}$$

New hamiltonian

$$\tilde{H}(t) = R^\dagger(t)\, H(t)\, R(t) - iR^\dagger(t)\partial_t\, R(t) = R^\dagger(t)\, H_0\, R(t)$$

$$\tilde{H}(t) = -J \sum_j \left(e^{i\alpha(t)}\, a^\dagger_{j+1}\, a_j + e^{-i\alpha(t)}\, a^\dagger_j\, a_{j+1} \right)$$

$$\alpha(t) = \xi_0 \sin \omega t$$

The driving has added a coupling to a gauge field

$$a^\dagger_{j+1}\, a_j \to a^\dagger_{j+1} \exp\left[-i \int_j^{j+1} A \right] a_j$$

Peierls substitution

Stroboscopic time evolution

$$U(T,0) = e^{-i\int_0^T \tilde{H}(t)dt} \qquad T = \frac{2\pi}{\omega}$$

Then

$$U(T,0) = e^{-iT\,\mathcal{H}_{eff}} \quad \longleftrightarrow \quad \mathcal{H}_{eff} = J_0(\xi_0)\,H$$

Equivalent to changing the hopping as:

$$J \rightarrow J_0(\xi_0)\,J \equiv J_{eff}(\xi_0)$$

Choosing ξ_0 such that $J_0(\xi_0) = 0$ we suppress the hopping

Dynamical localization

We want to study a (2+1)-dimensional gauge theory under the influence of a external electric field $\mathcal{E}$ rotating in the xy plane

$$\mathcal{E} = \mathcal{E}_x + i\,\mathcal{E}_y \;=\; E e^{i\Omega t}$$

Two physical regimes

- Weak field E $\rightarrow$ vacuum polarization in an insulating gapped phase

  A rotating polarization current $\vec{j}$ is induced with no dissipation ($\vec{j} \perp \vec{\mathcal{E}}$)

- For $E \geq E_c$ $\rightarrow$ vacuum is unstable by Schwinger pair production

  Dielectric breakdown, dissipation and Joule heating

The critical field depends of the frequency Ω $\rightarrow$ $E_c \;=\; E_c(\Omega)$

Remarkably, in our top-down holographic model

for some frequencies $\Omega = \Omega_c$

$$E_c(\Omega_c) = 0$$

Moreover, $E = 0$ and the current $j \neq 0$

in a finite range of frequencies $\Omega_c \leq \Omega \leq \Omega_m$

States with rotating current and zero external electric field

$$\Downarrow$$

Floquet condensate of vector mesons

Similar results in (3+1)-dim were previously obtained by

Hashimoto, Kinoshita, Murata and Oka [1611. 03702] [1712.06786]

Brane setup

	1	2	3	4	5	6	7	8	9
$D3:$	$\times$	$\times$	$\times$	_	_	_	_	_	_
$D5:$	$\times$	$\times$	_	$\times$	$\times$	$\times$	_	_	_

Gauge theory at the intersection

The D5-brane is treated as a probe in the $AdS_5 \times \mathbb{S}^5$ background

DBI action for the D5-brane

$$S = -N_f T_5 \int d^6\xi \sqrt{-\det\left(g_6 + 2\pi\alpha' \mathcal{F}\right)} \qquad \mathcal{F} = d\mathcal{A}$$

Bulk degrees of freedom

$\searrow$ Act as a bath for the flavor dof's

Background 10d metric (zero T)

$$ds^2_{10} \;=\; \frac{\rho^2+w_1^2+w_2^2+w_3^2}{R^2}\Big(-\,dt^2+dx^2+dy^2+dz^2\Big)+$$

$$+\,\frac{R^2}{\rho^2+w_1^2+w_2^2+w_3^2}\Big(d\rho^2+\rho^2\,d\Omega_2^2+dw_1^2+dw_2^2+dw_3^2\Big)$$

$$d\Omega_2^2 = d\theta^2+\sin^2\theta d\phi^2$$

We take $R=1$ from now on

D5-brane extended in $x, y, \rho, \theta, \phi \;\rightarrow\quad \xi^a \;=\; (t, x, y, \rho, \theta, \phi)$

Ansatz for the transverse scalars

$$z \;=\; \text{constant}\;, \qquad\qquad w_1 = w(t,\rho)\;, \qquad\qquad w_2 \;=\; w_3 \;=\; 0\;.$$

and the gauge field

$$2\pi\,\alpha'\,\mathcal{A} \;=\; a_x(t,\rho)\,dx \;+\; a_y(t,\rho)\,dy\;,$$

Complexify the gauge field in the rotating frame

$$a(t,\rho) \;=\; a_x + i a_y = b(t,\rho)\, e^{i(\Omega t+\chi(t,\rho))}$$

Important → The equations can be solved by

$$b \;=\; b(\rho) \qquad\qquad \chi \;=\; \chi(\rho) \qquad\qquad w \;=\; w(\rho)$$

No PDEs!

Lagrangian density

$$\mathcal{L} \;=\; \frac{\rho^2}{\rho^2+w^2}\sqrt{\Big((\rho^2+w^2)^2 \;-\; \Omega^2\, b^2\Big)\Big(1+b'^{\,2}+w'^{\,2}\Big) \;+\; (\rho^2+w^2)^2\, b^2\,\chi'^{\,2}}$$

UV boundary behavior

$$c(\rho) \;=\; b(\rho)\, e^{i\chi(\rho)} \;=\; \frac{i\,E}{\Omega} + \frac{j}{\rho} + \cdots ,$$

$$w(\rho) \sim m + \frac{\mathcal{C}}{\rho} + \cdots$$

j → Polarization current

$$j = \dot{\mathcal{P}}$$

$$\mathcal{P} \;=\; \langle \bar{\psi}(\gamma_x + i\gamma_y)\psi\rangle$$

χ is a cyclic variable

First integral $\rightarrow$ $$q = \Omega \frac{\partial \mathcal{L}}{\partial \chi'} = \Omega \frac{\rho^4}{\mathcal{L}} b^2 \chi'$$

Evaluating q at the boundary

$$q = = \mathrm{Re}(E\, j^*) = j_x E_x + j_y E_y$$ → Joule heating

Types of embeddings

Eoms are potentially singular when:

$$b_0 = \frac{w_0^2 + \rho_c^2}{\Omega} \qquad b_0 = b(\rho = \rho_c) \qquad w_0 = w(\rho = \rho_c)$$

What is the meaning of $\rho = \rho_c$?

$\rho = \rho_c \;\to$ pseudohorizon (of the open string metric)

Three possibilities

- Black hole embeddings

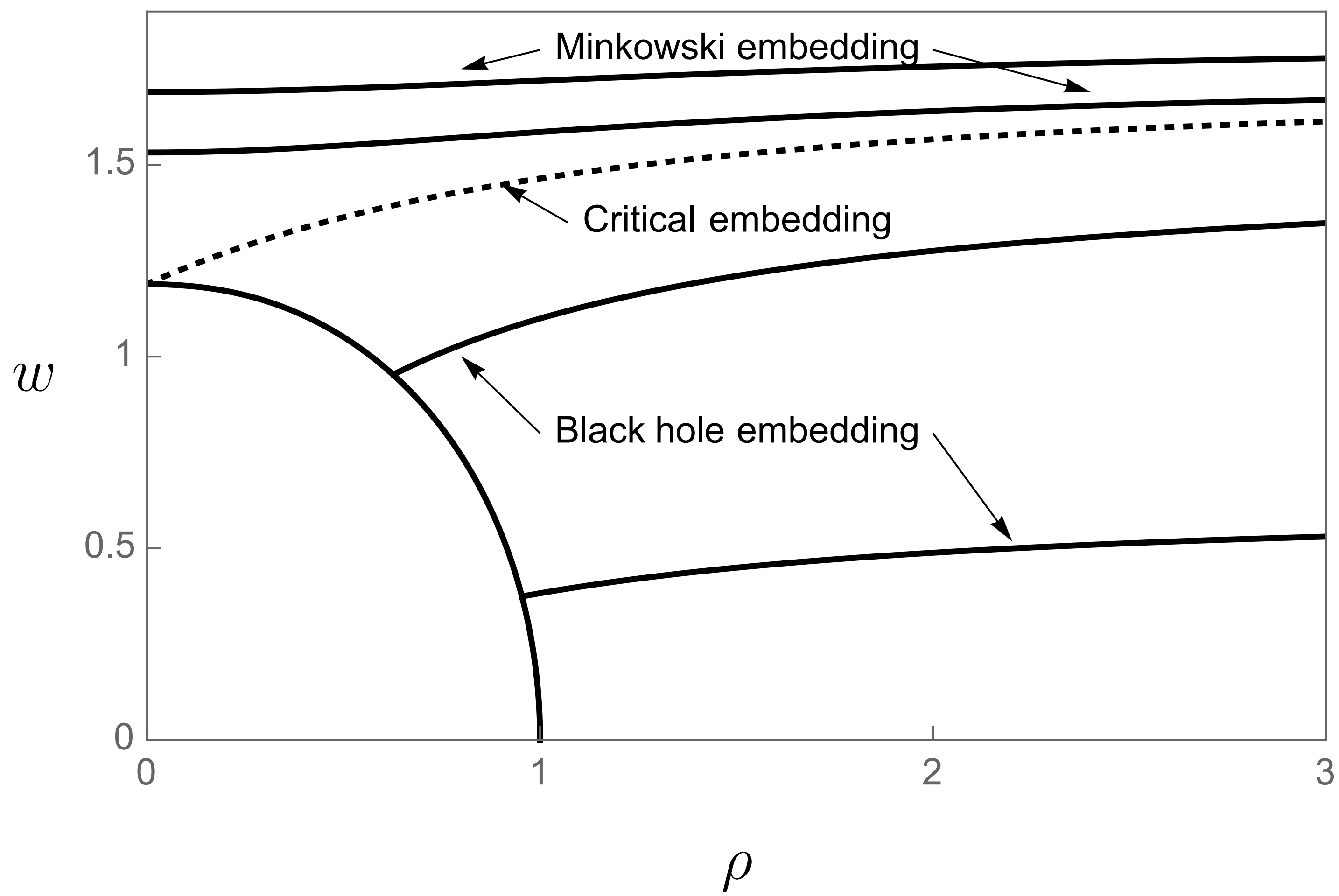
  Branes cross the pseudohorizon at $\rho_c \neq 0$

- Minkowski embeddings

  Branes do not cross the pseudohorizon and reach $\rho = 0$

- Critical embeddings

  Branes reach the pseudohorizon at $\rho_c = 0$



- Mink. embeddings $\rightarrow$ gapped insulating phase

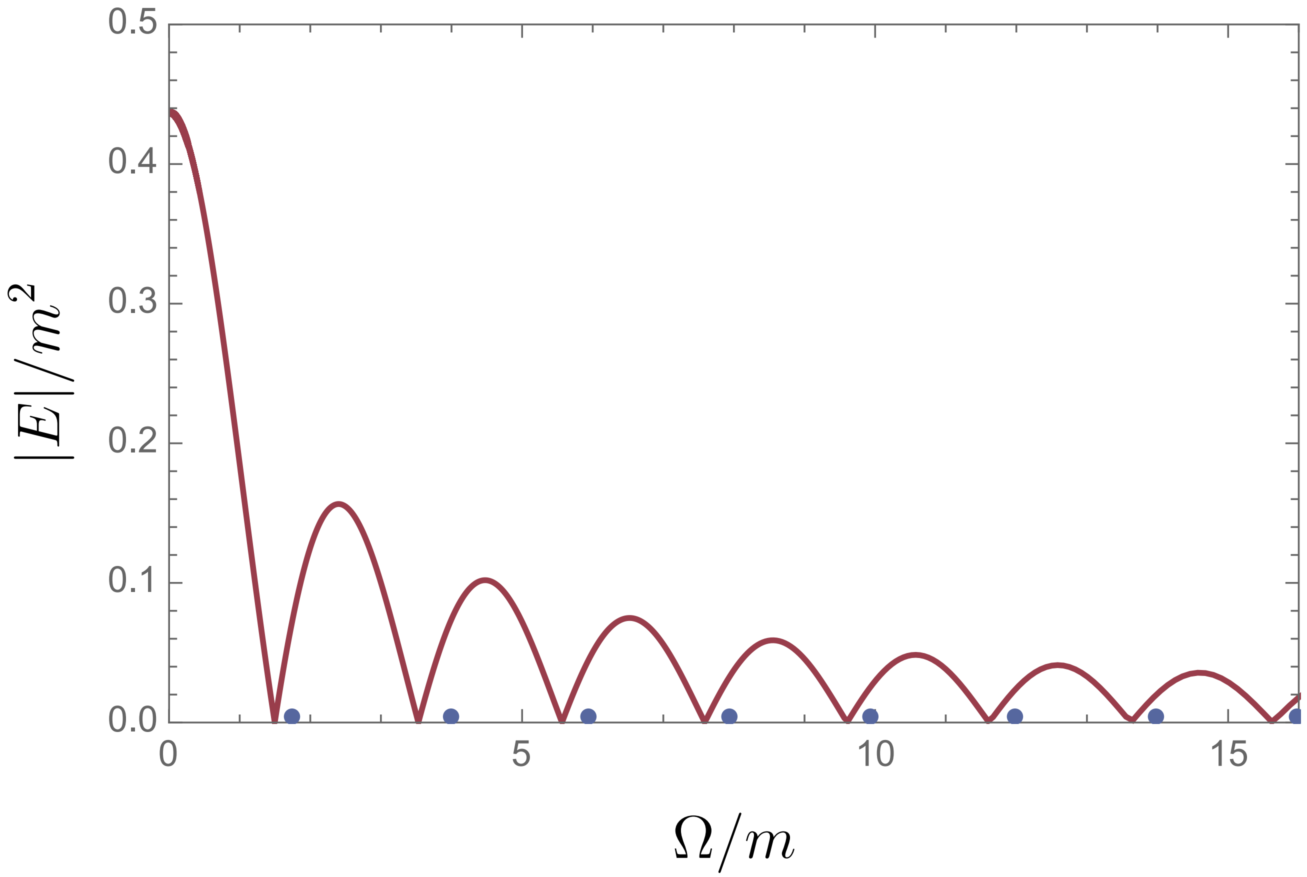
No dissipation $\rightarrow$ $q = 0$

- BH embeddings $\rightarrow$ metallic conductive phase

Dissipative $\rightarrow$ $q \neq 0$

$$q = \rho_c^2\,(w_0^2 + \rho_c^2)$$

Phase diagram (critical embeddings)

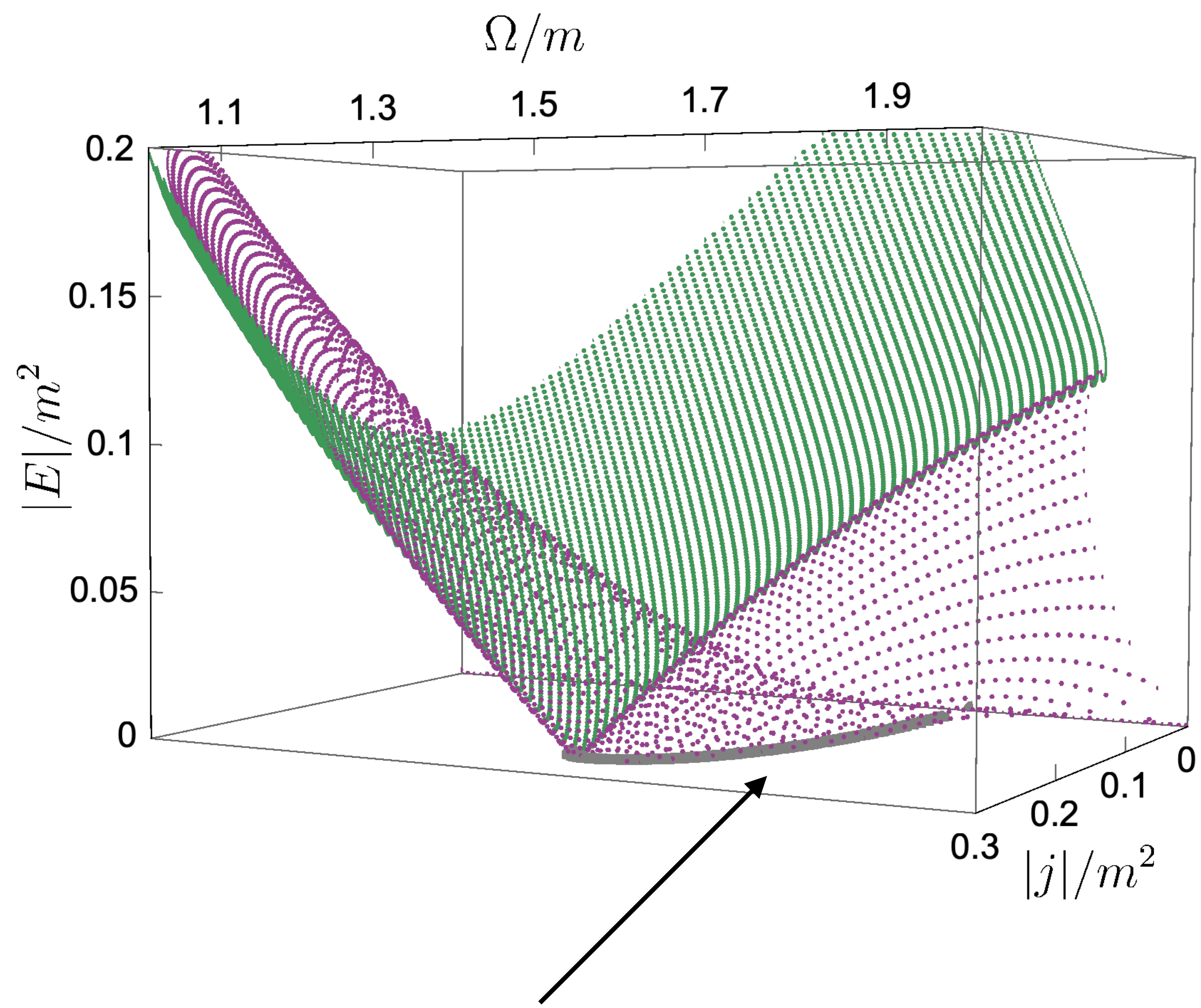


Critical frequencies $\longrightarrow$ $\Omega_c/m = 1.4965\,,\ 3.5308\,,\ 5.5676\,,\ 7.5851\ \cdots$

- $\longrightarrow\ \Omega_{meson}/m = 1.7320\,,\ 3.8730\,,\ 5.9161\,,\ 7.9372\,,\cdots\ .$

$$\Omega_{meson}/m = 2\sqrt{\Big(n+\frac{1}{2}\Big)\Big(n+\frac{3}{2}\Big)} \qquad n = 0, 1, 2, \cdots$$

3d phase diagram

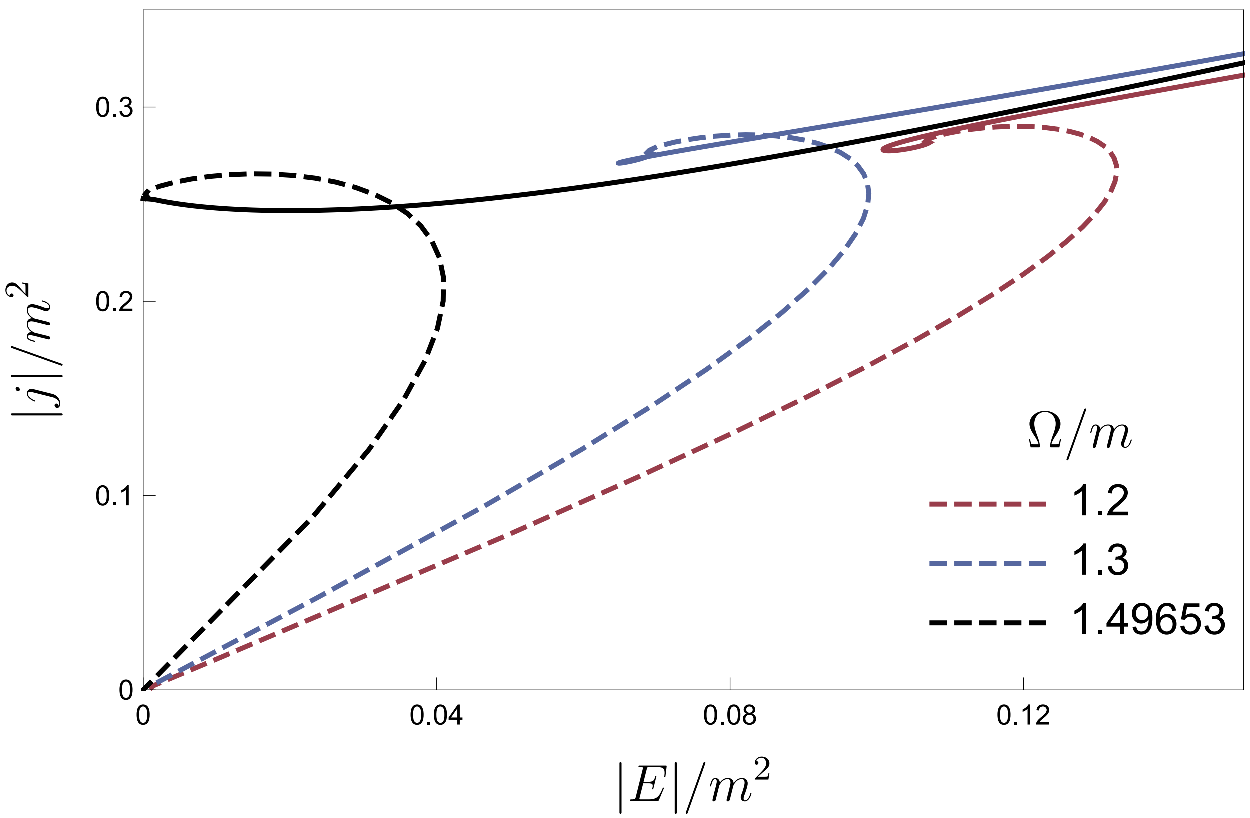


1st Floquet condensate $(\Omega \in [1.4965, 1.7320])$

Non-Equilibrium Steady State (NESS)

Energy injection=dissipation

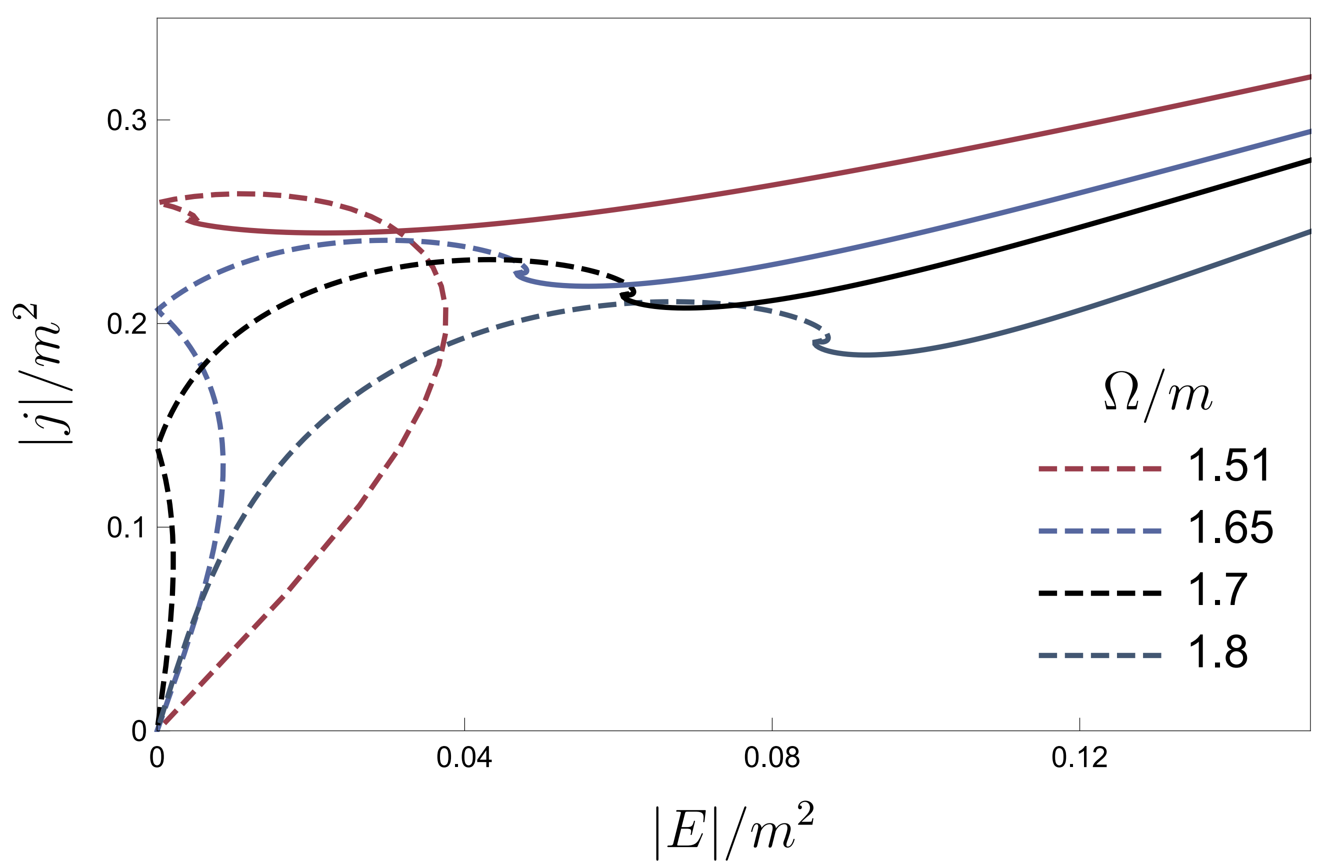
$\Omega \leq \Omega_c$



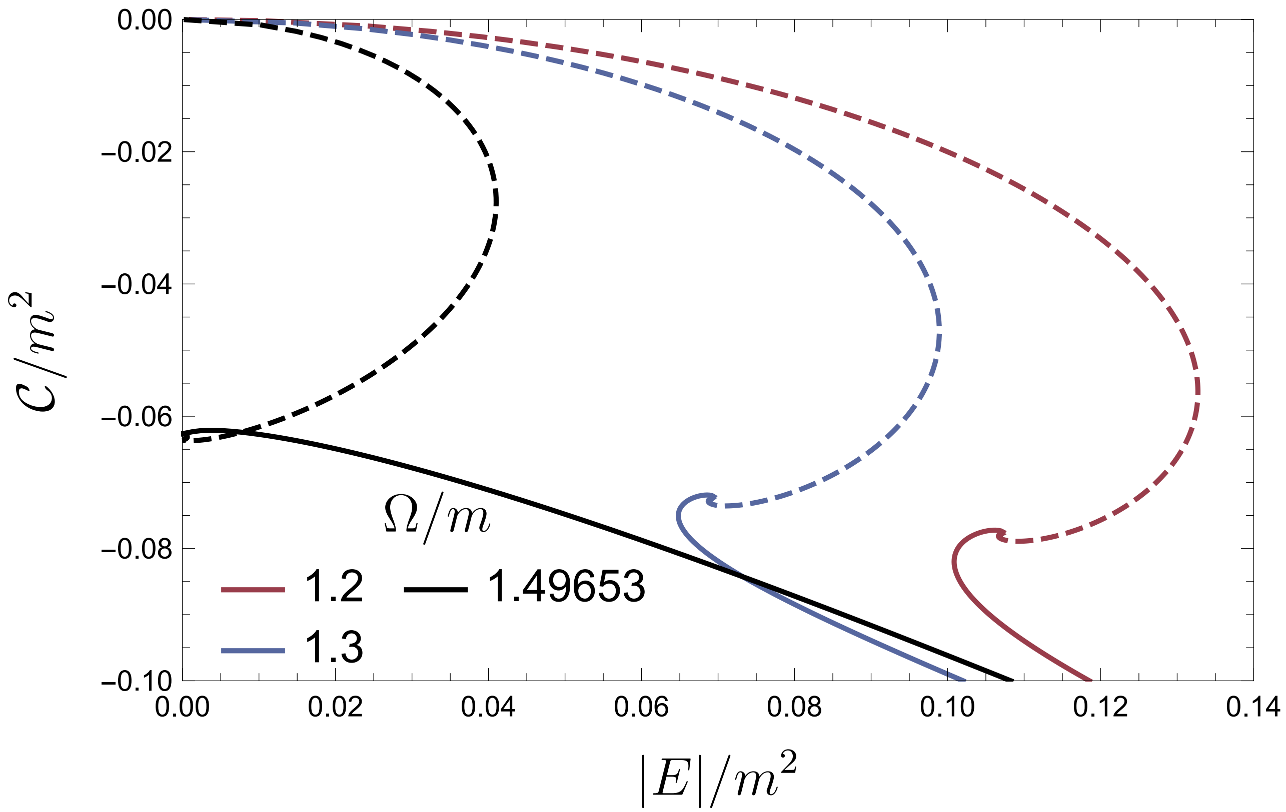
dashed $\rightarrow$ Mink.

$\Omega_c \leq \Omega \leq \Omega_m$

cont. $\rightarrow$ BH



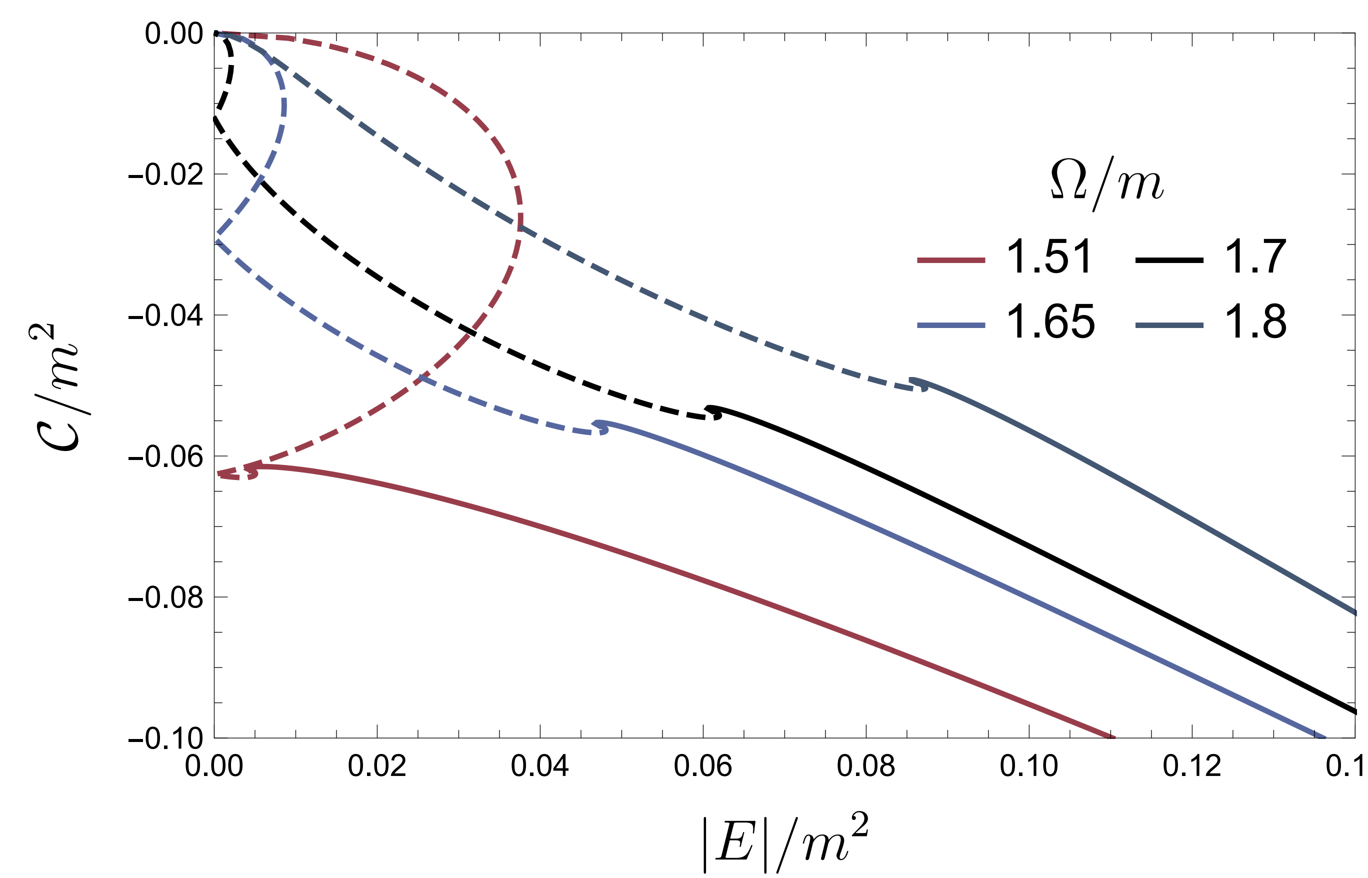
$\Omega \leq \Omega_c$



dashed $\rightarrow$ Mink.

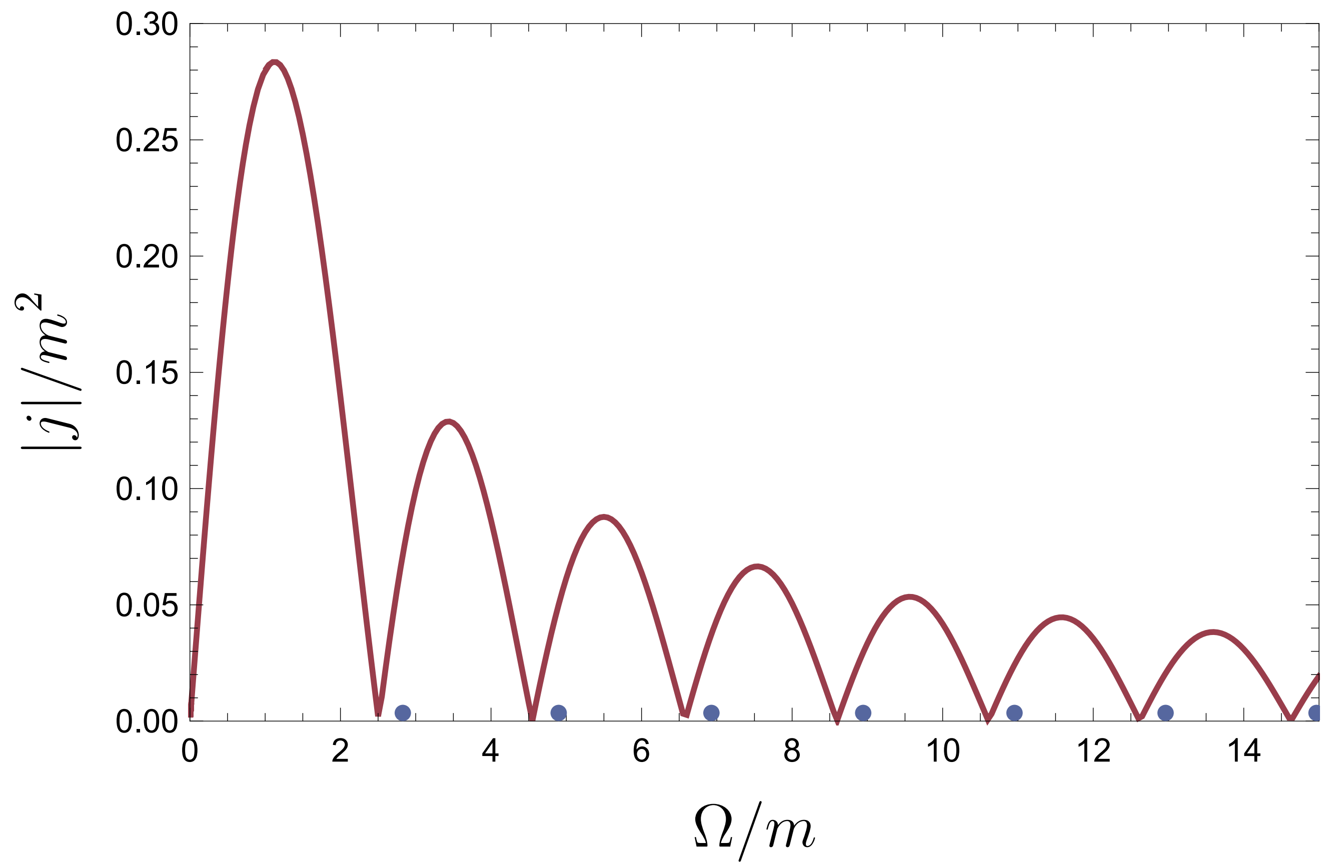
$\Omega_c \leq \Omega \leq \Omega_m$

cont. $\rightarrow$ BH



Floquet suppression points

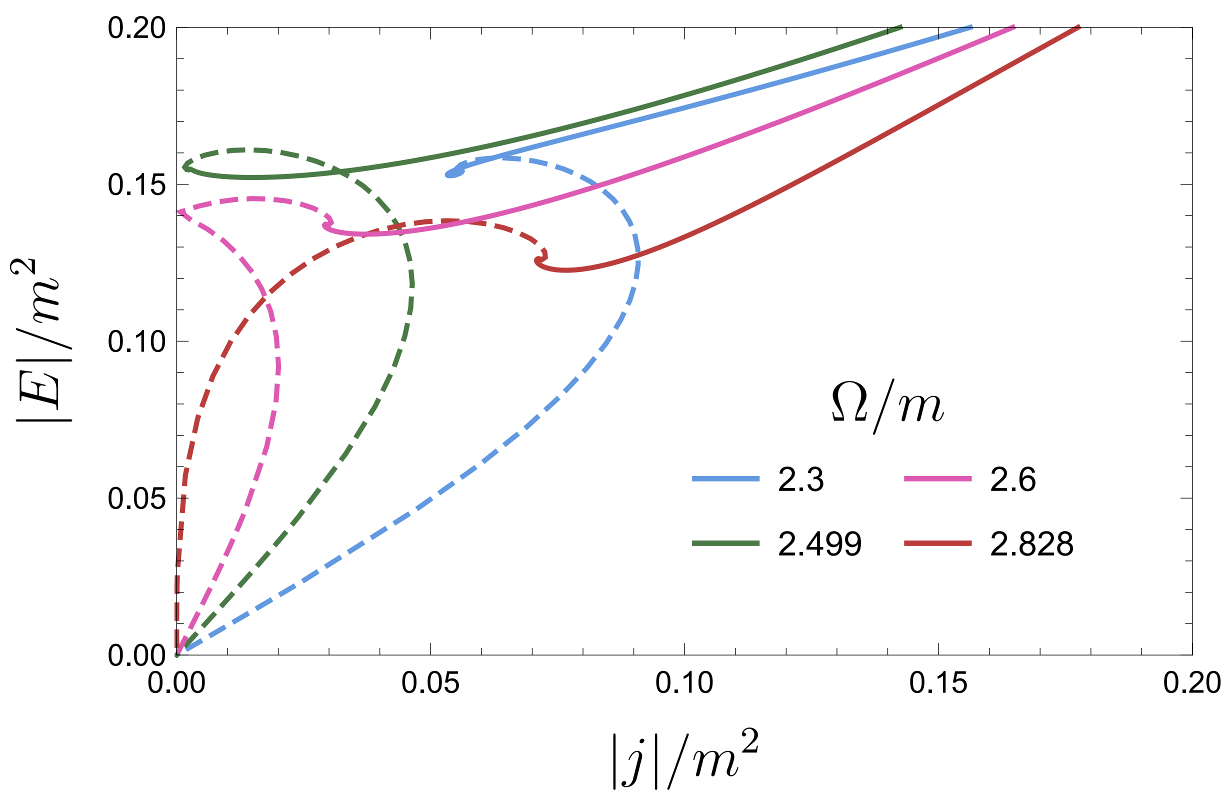
- j has also a lobe structure
- $j = 0$ close to the points where E is maximal



$\leftarrow$ Critical embeddings

Critical frequencies $\rightarrow \Omega_c/m = 2.499, 4.550, 6.580, \cdots$

- $\rightarrow \dfrac{\Omega_k}{m} = 2\sqrt{k(k+1)} = 2.828, 4.899, \cdots$



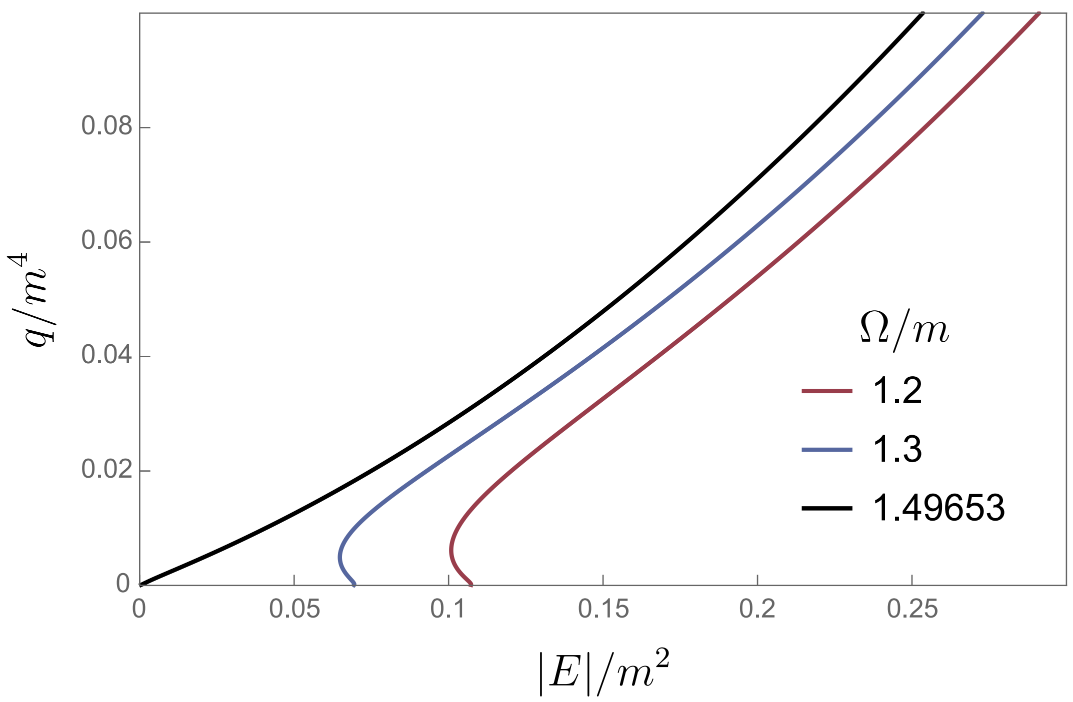
dashed $\rightarrow$ Mink.

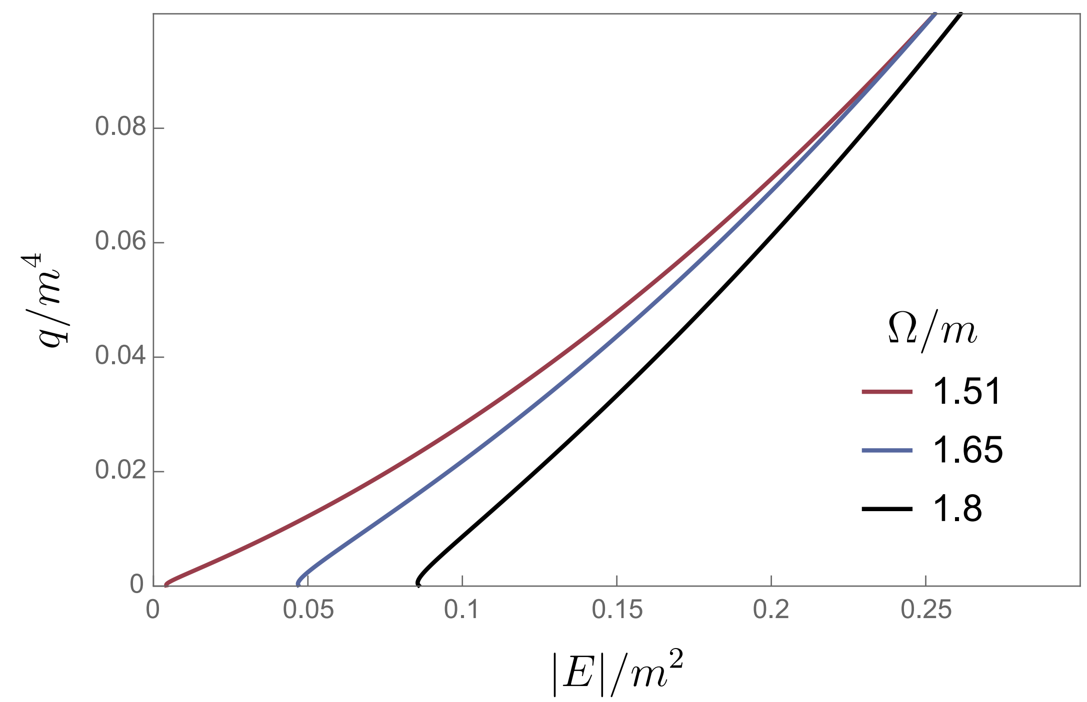
cont. $\rightarrow$ BH

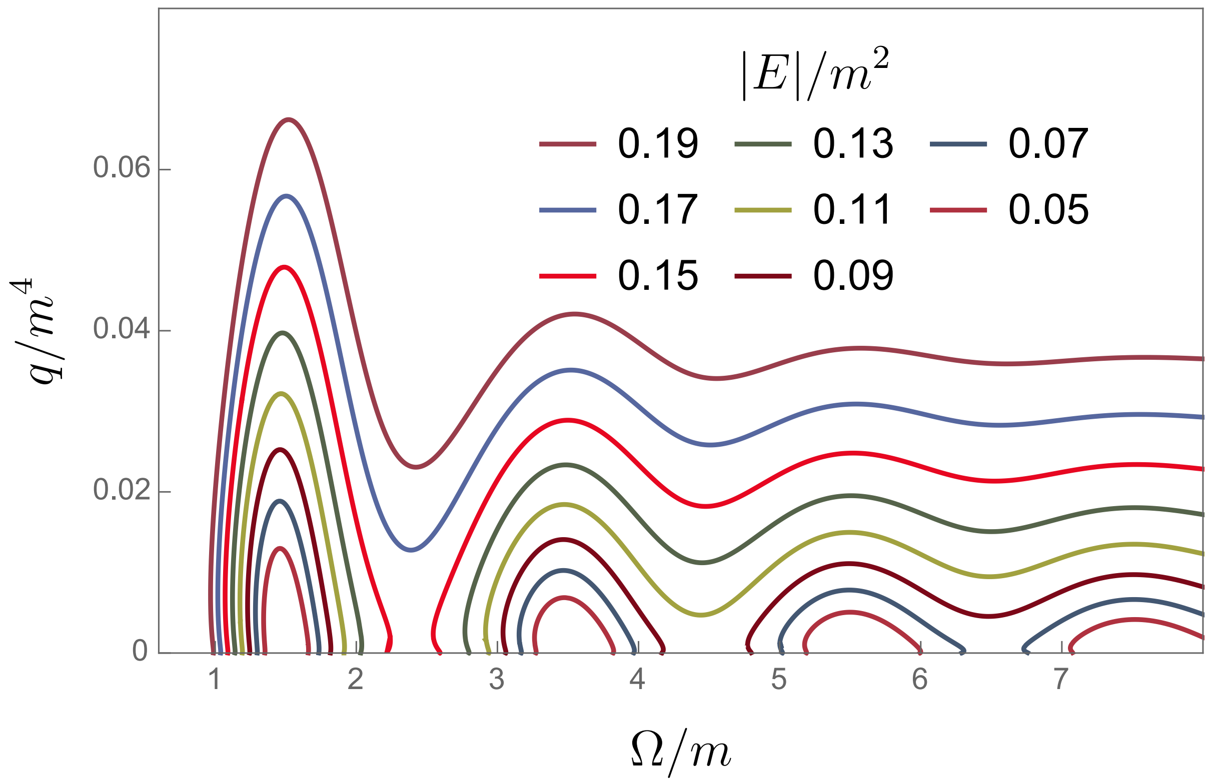
- $j = 0$ even with $E \neq 0$ (Floquet suppressed states)
- The polarization is dynamically suppressed for some frequencies

(Similar to tunneling suppression)

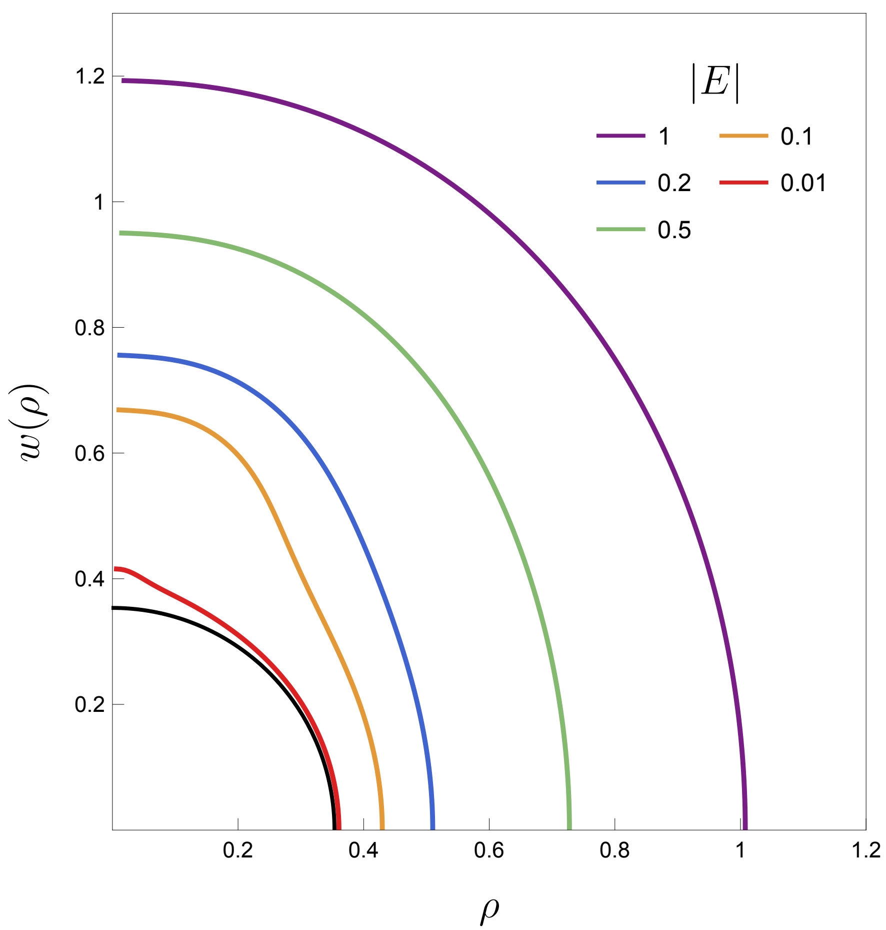
Joule heating for BH embeddings







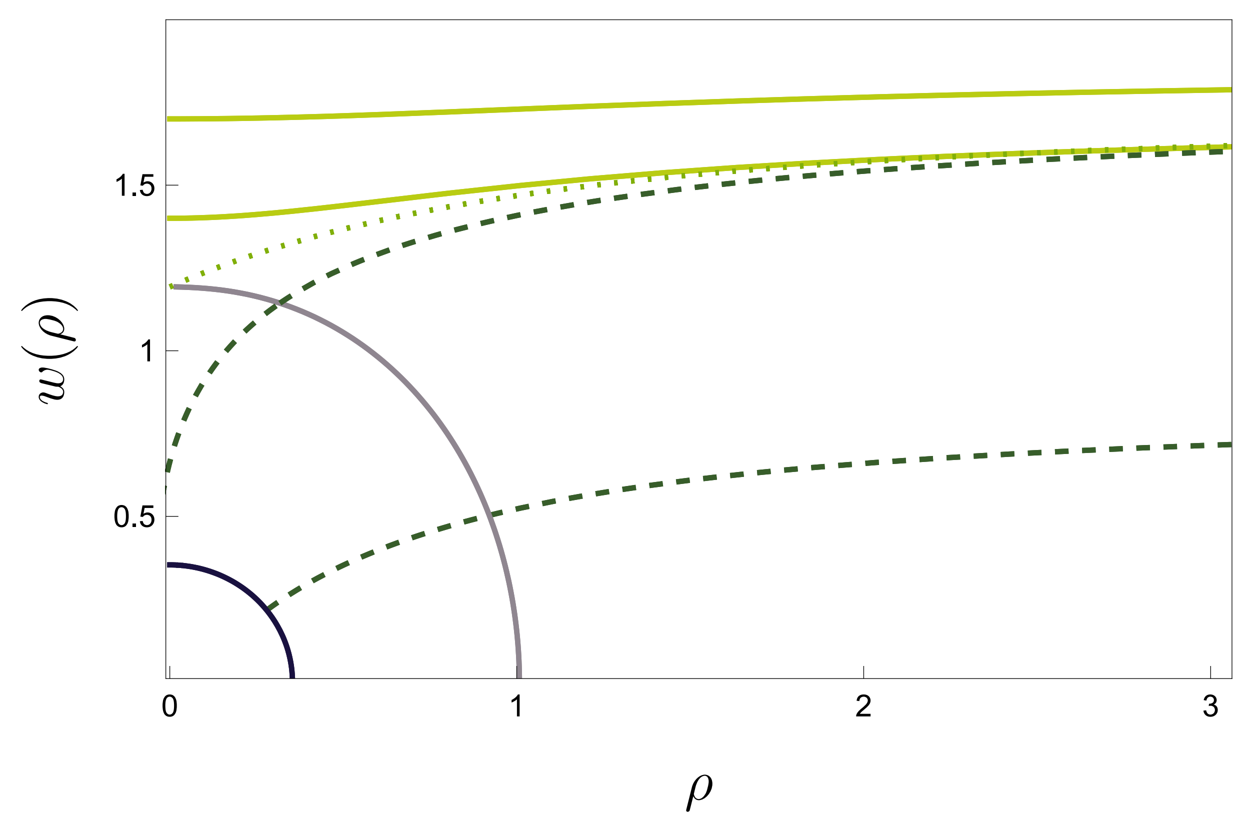
Non-zero temperature



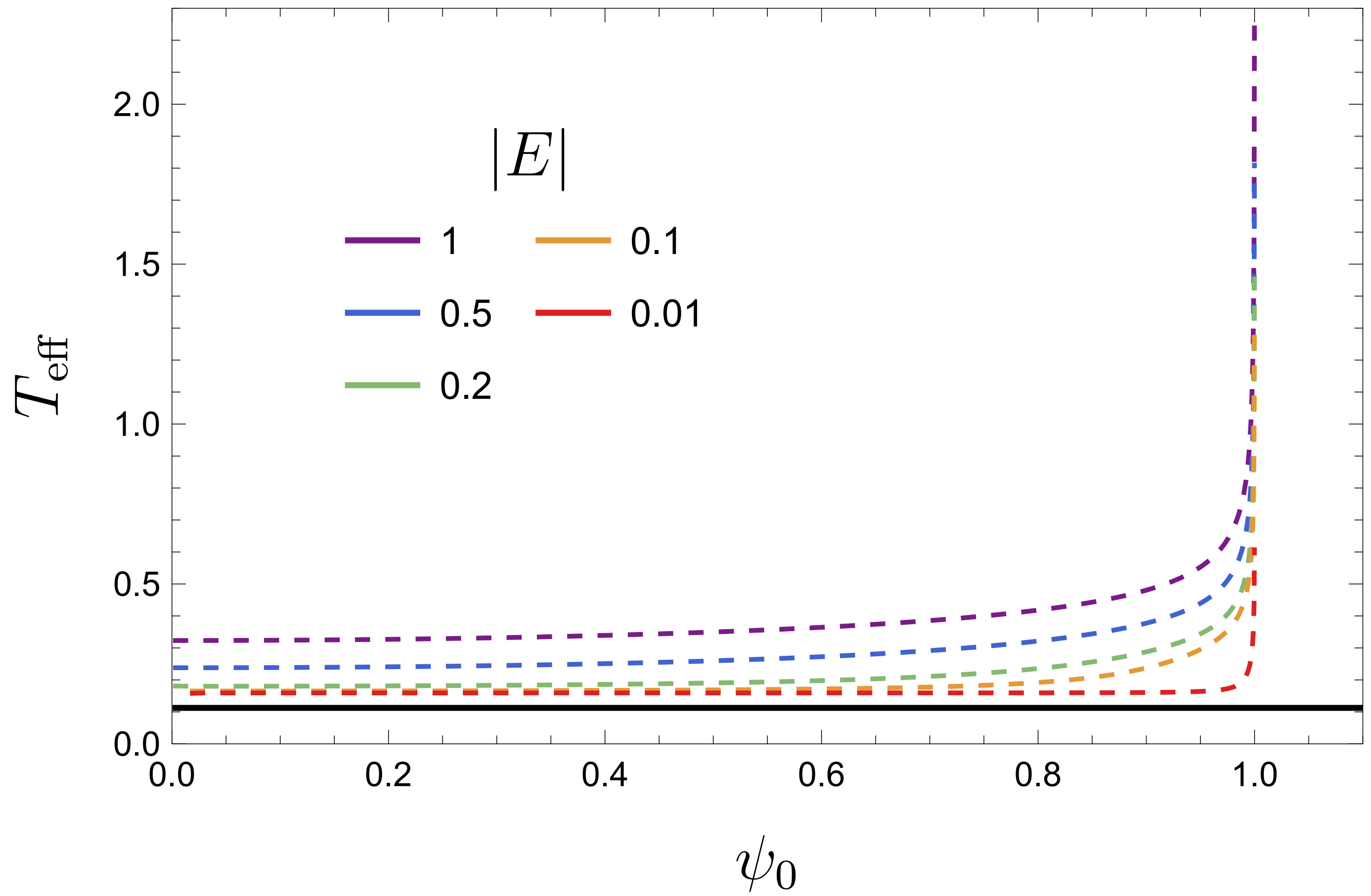
- Two horizons
- The BH horizon is always inside the singular shell

- Minkowski
- BH → { Thermal, Conical }
- Critical

Embeddings

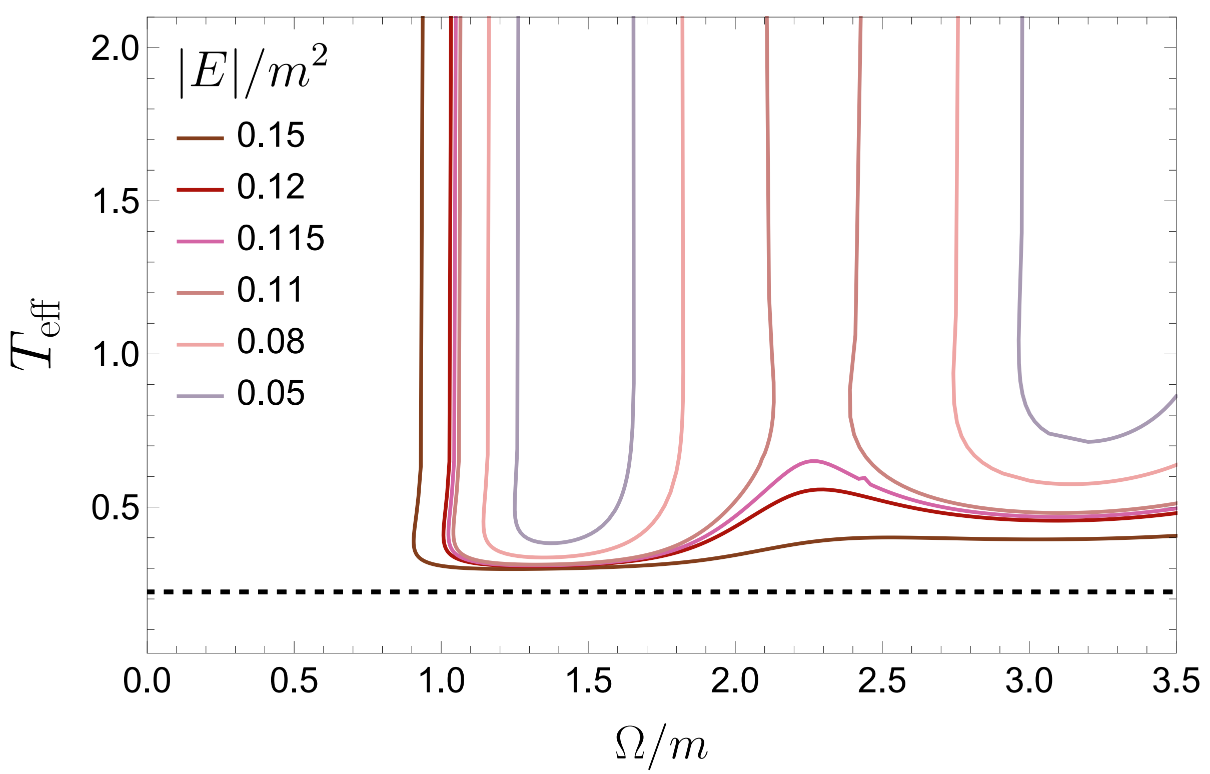


Effective temperature

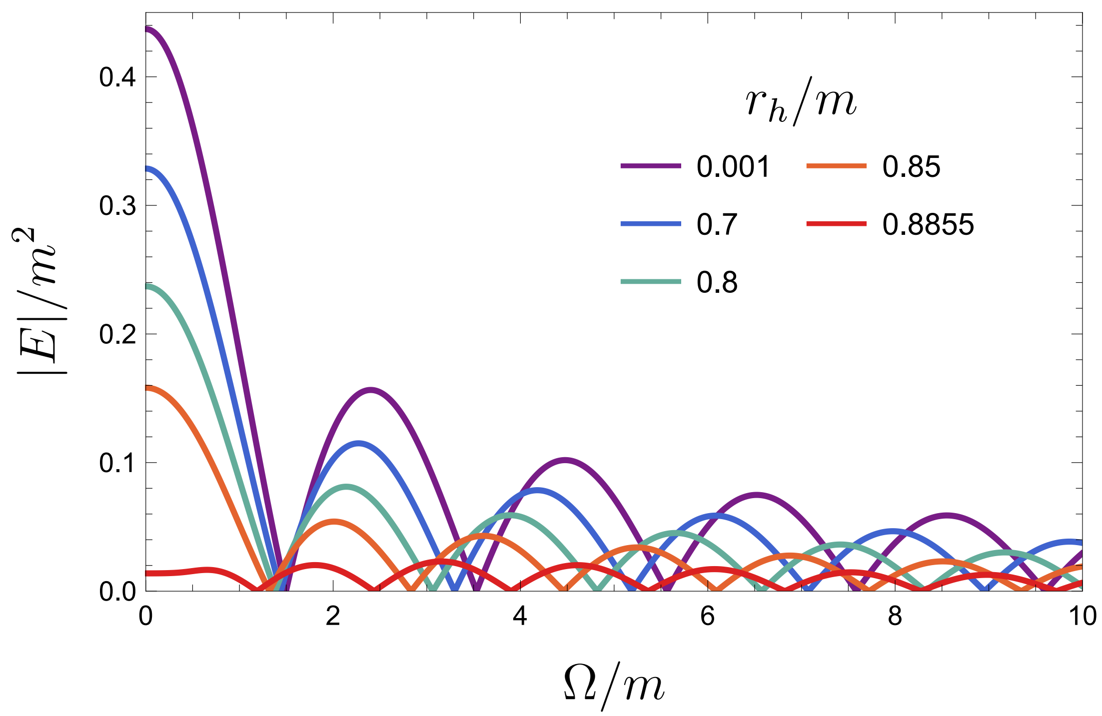


- Depends on the embedding (not just on E!)
- $T_{eff} > T$

$T_{eff} \to \infty$ at the critical embedding

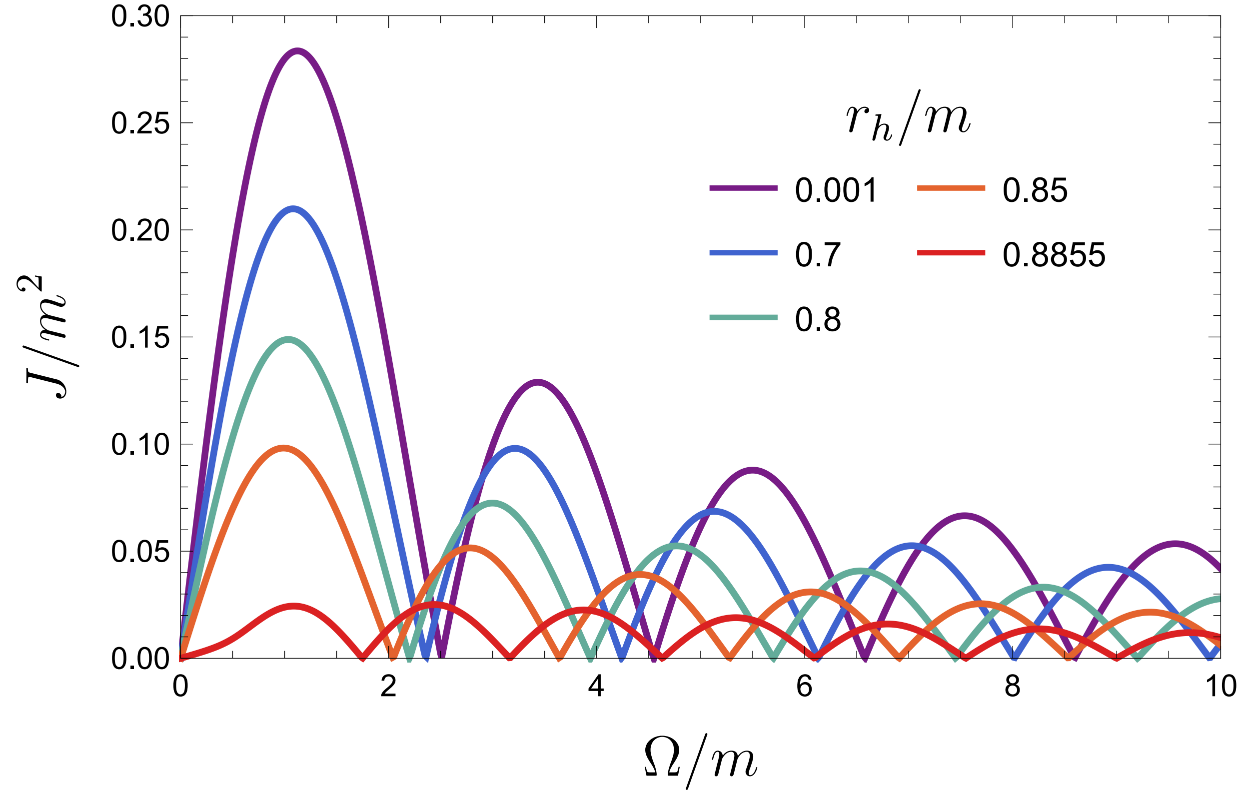


The lobes are deplected as T grows



Two bending effects (T and E)

$r_h \uparrow$ → Less E is needed



For $r_h/m > 0.8897$ all embeddings are BH

Future directions

- Add chemical potential & magnetic field
- Include backreaction
- Bottom-up models (see Ishii&Murata [1804. 06785]
- Clarify the role of topology
- Clarify the nature of the phase transition
  (Schwinger-Keldysh approach for open systems?)

That's it.

Thank *you for your attention!!*