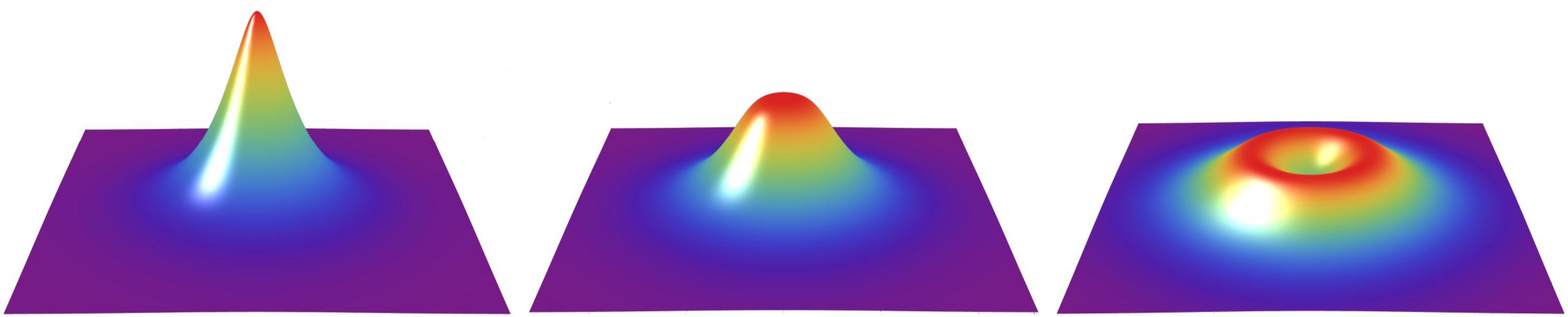
Novel aspects in relativistic hydrodynamics

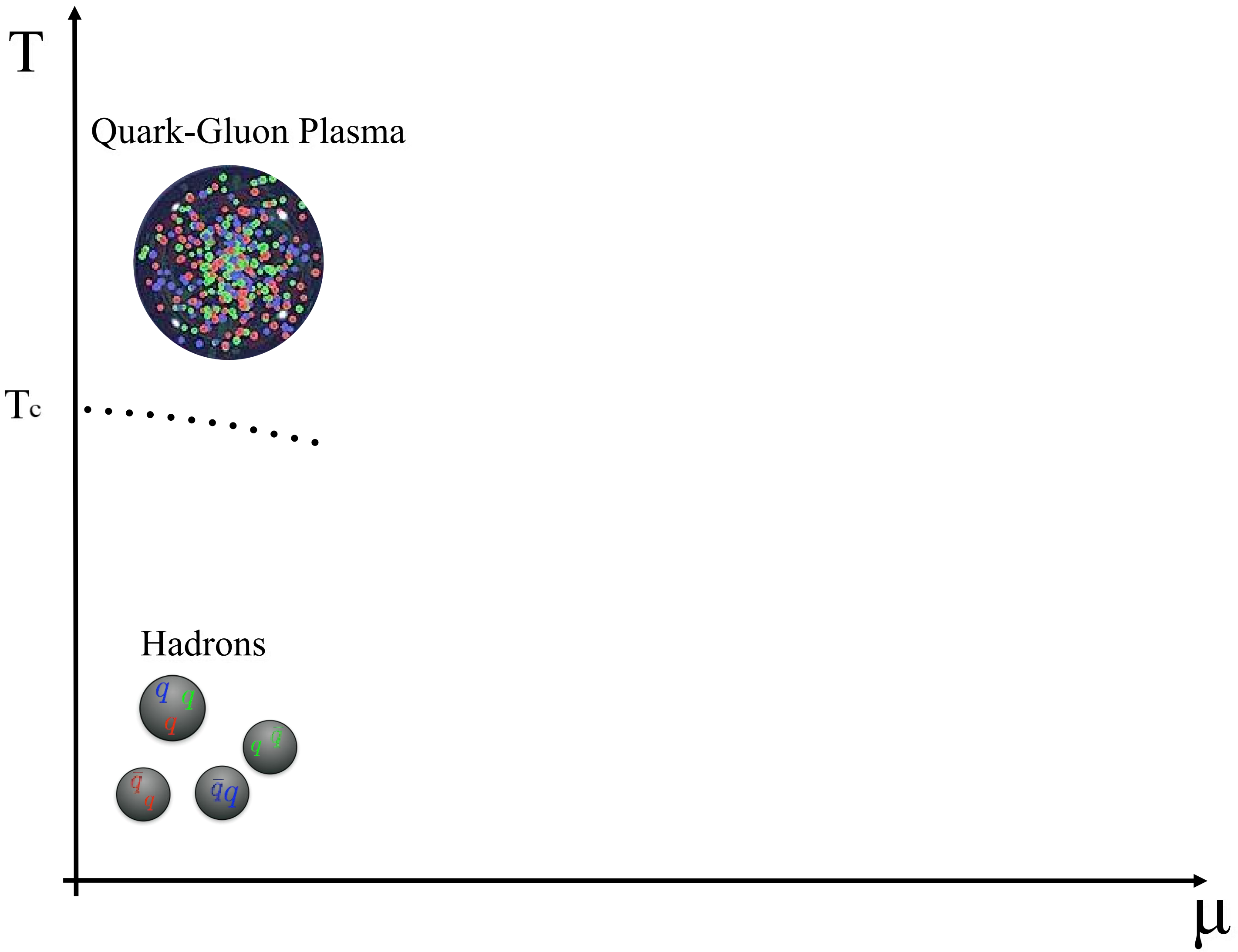
Yago Bea

University of Barcelona



With Hans Bantilan and Pau Figueras

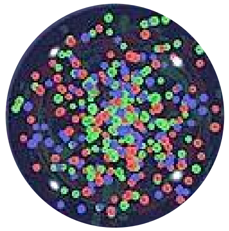
QCD phase diagram



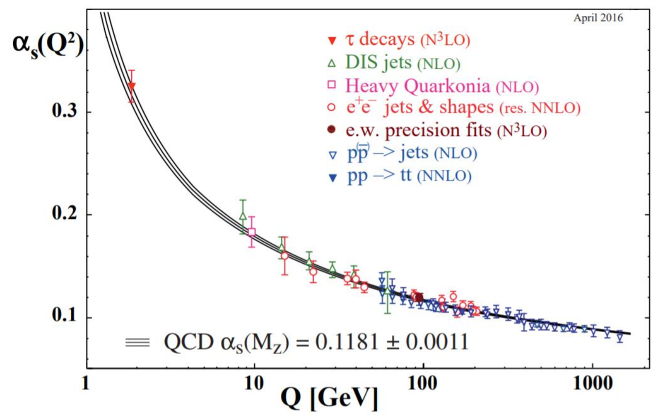
QCD phase diagram

T

Quark-Gluon Plasma



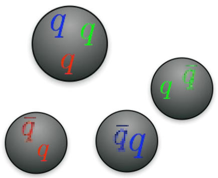
Tc



- Perturbative QCD

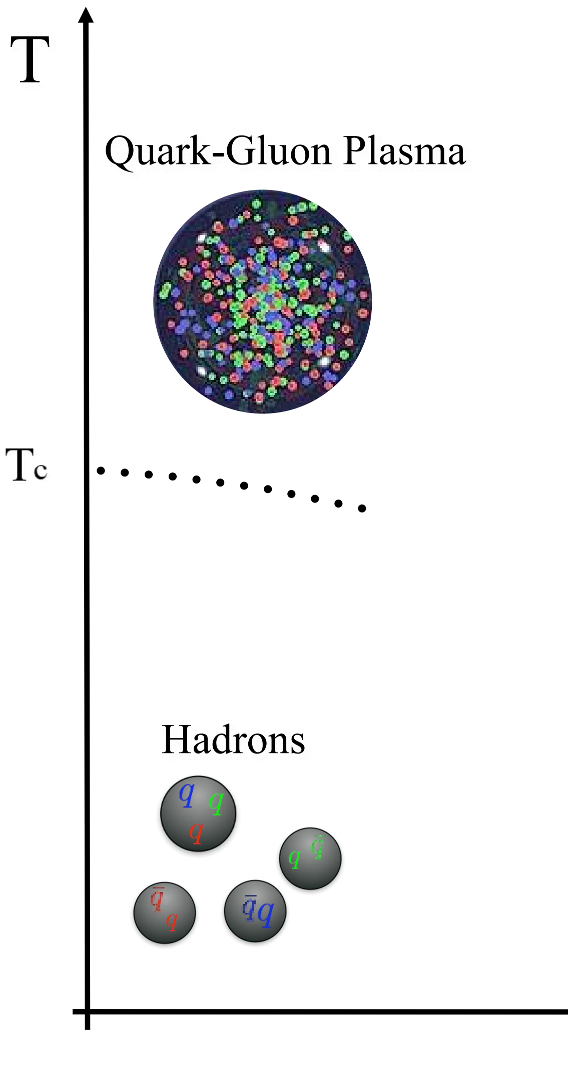
→ Limited applicability

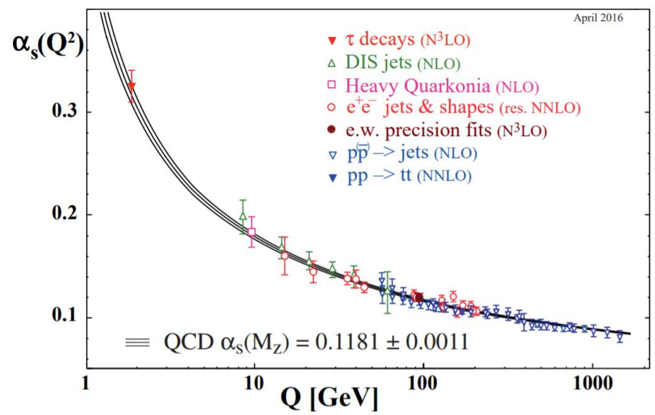
Hadrons



μ

QCD phase diagram

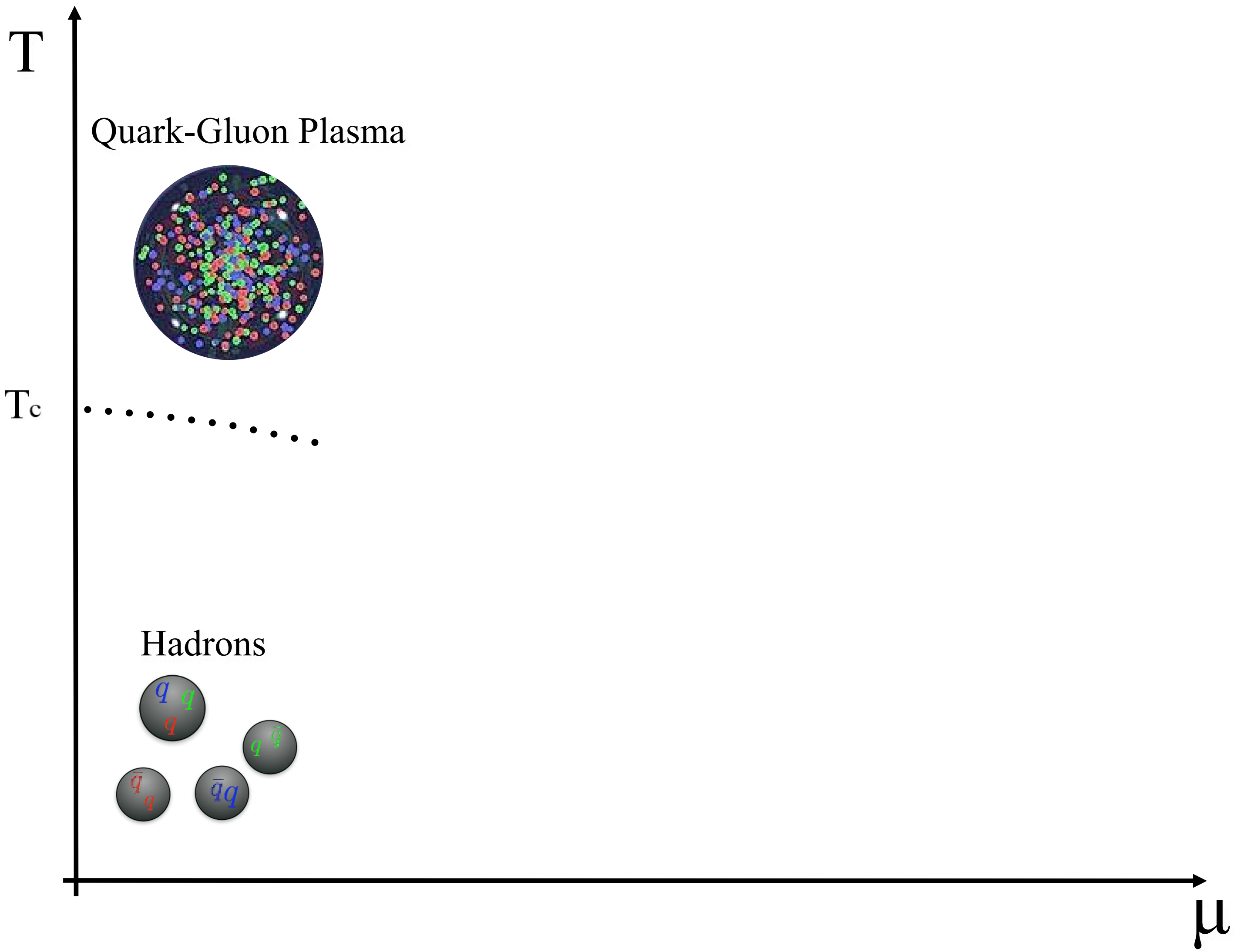




- Perturbative QCD
- Lattice QCD

→ Limited applicability

QCD phase diagram



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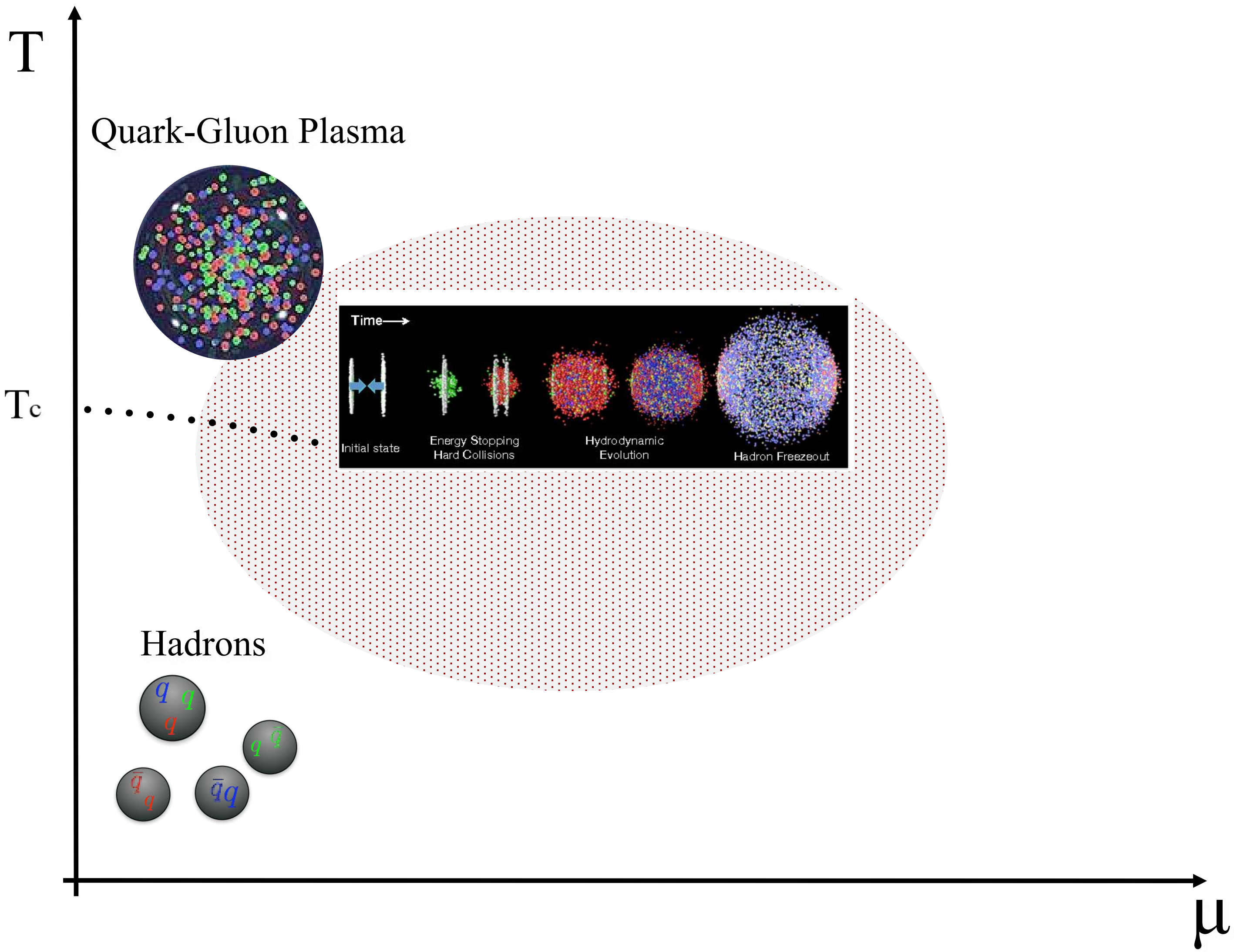

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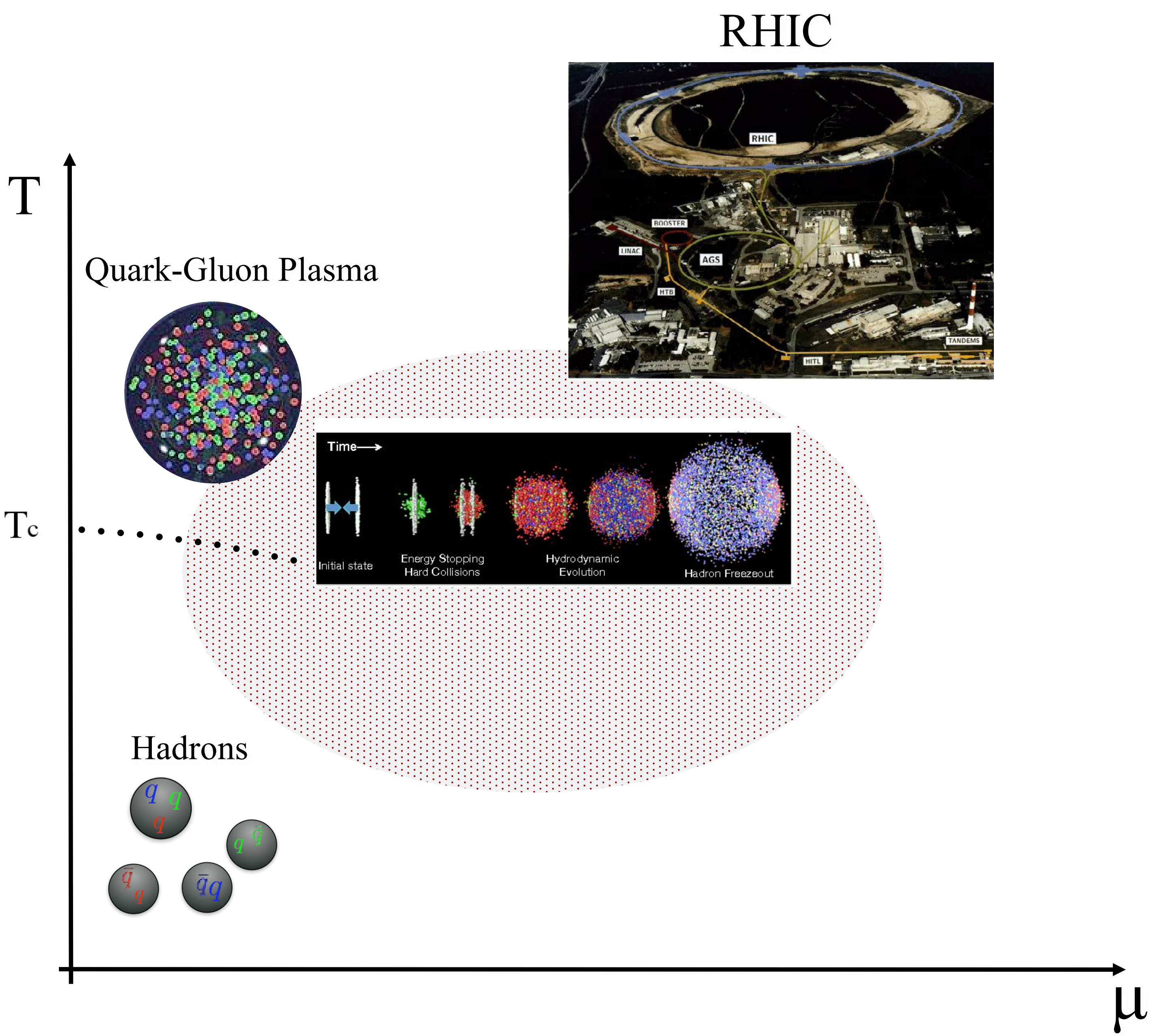

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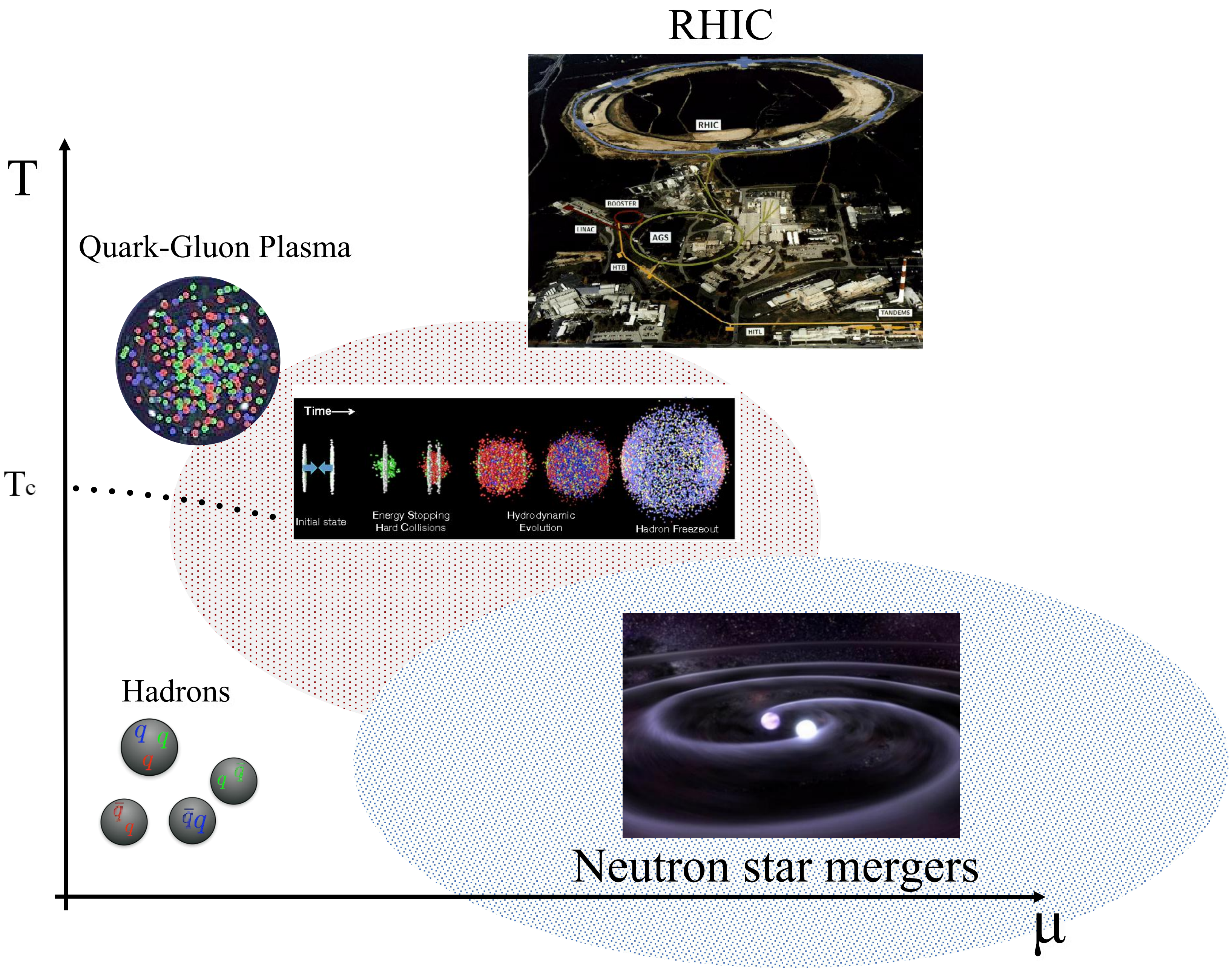

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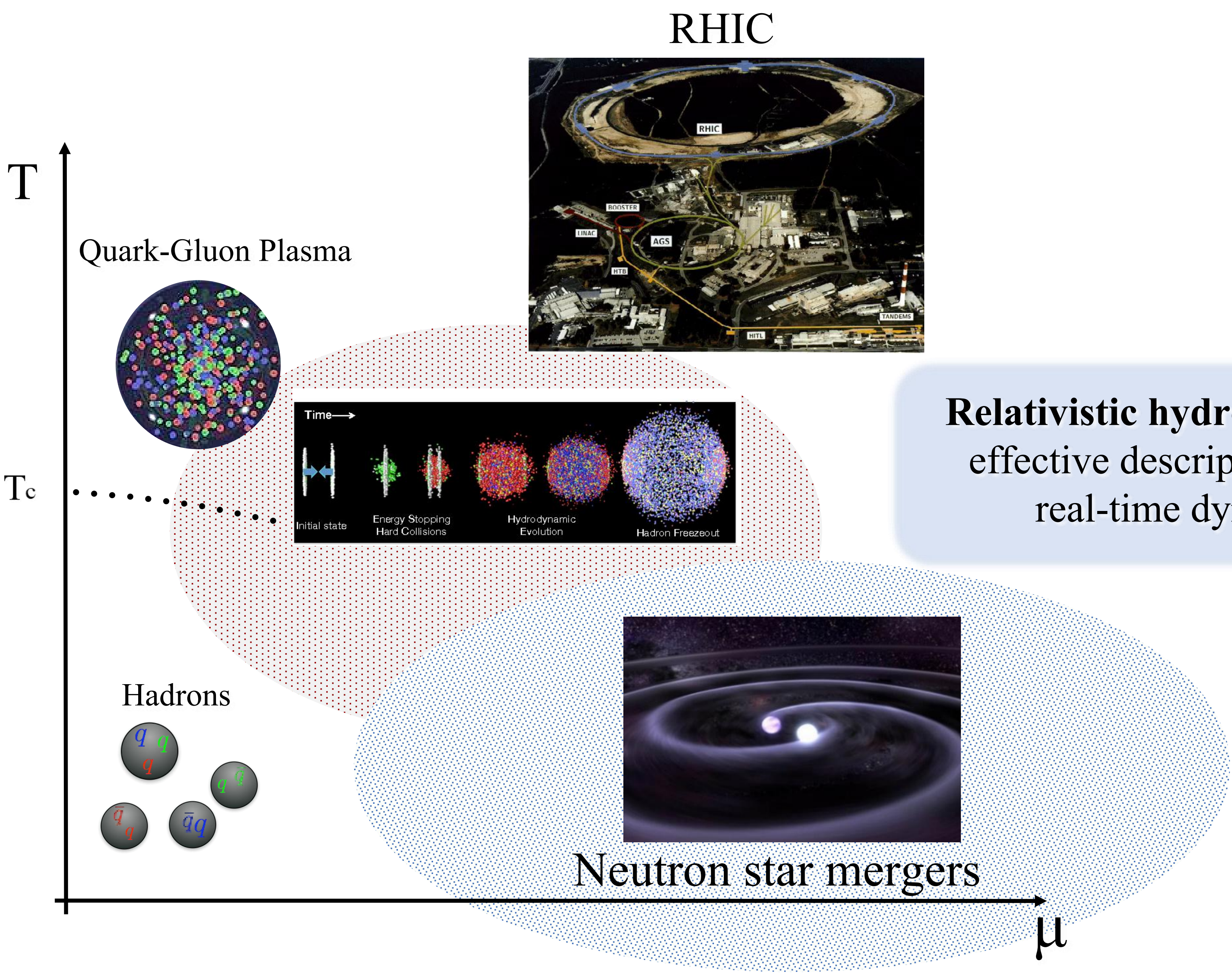
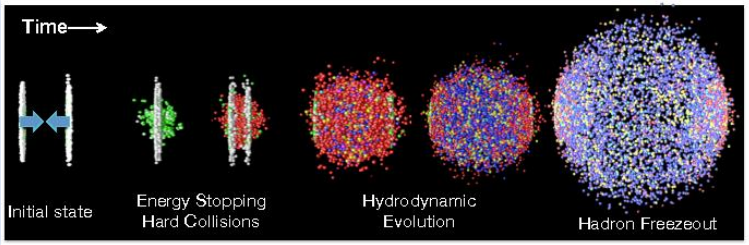


Relativistic hydrodynamics: effective description of the real-time dynamics

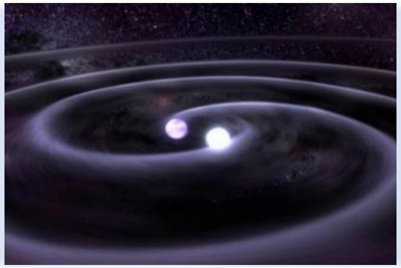
Relativistic Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

Quark-Gluon Plasma



Neutron star mergers



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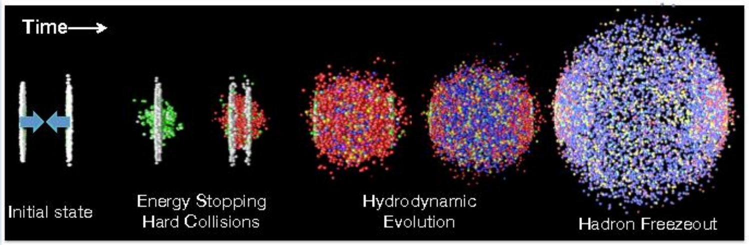

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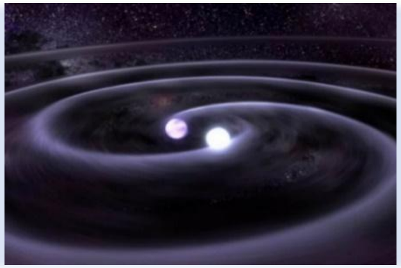
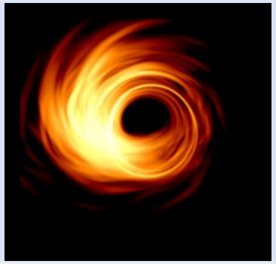
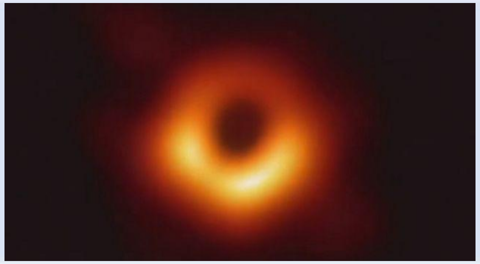


Black hole accretion disk





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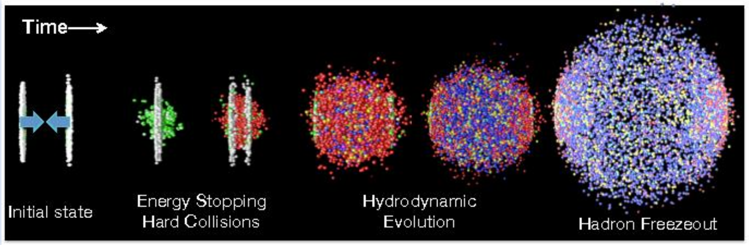

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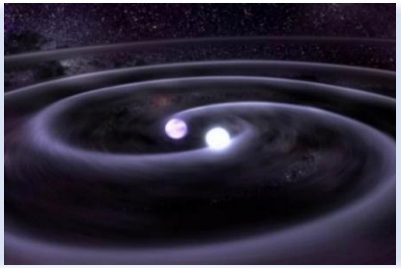

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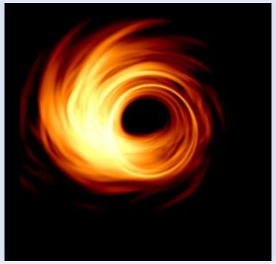
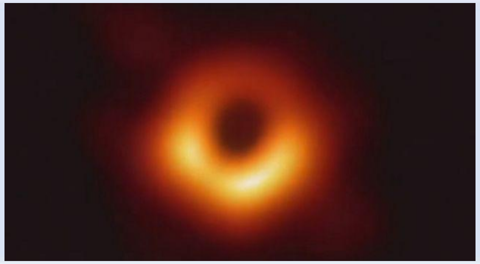
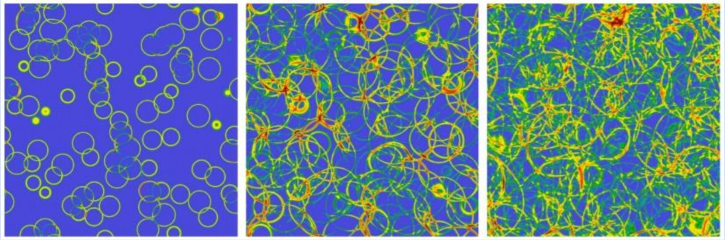




Early universe



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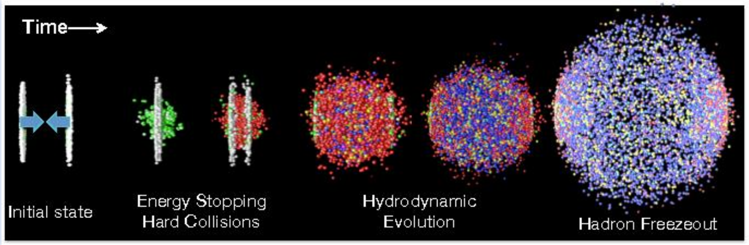

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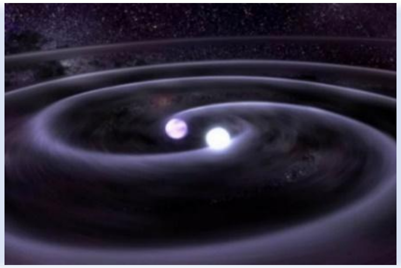

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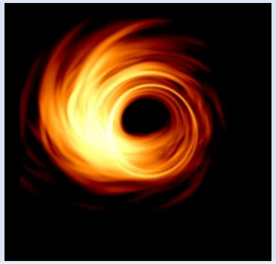

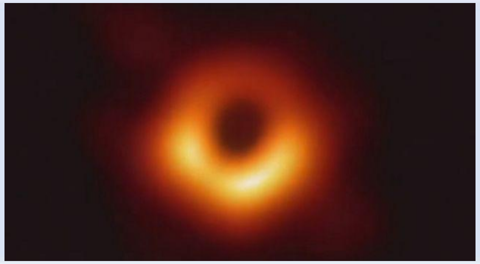

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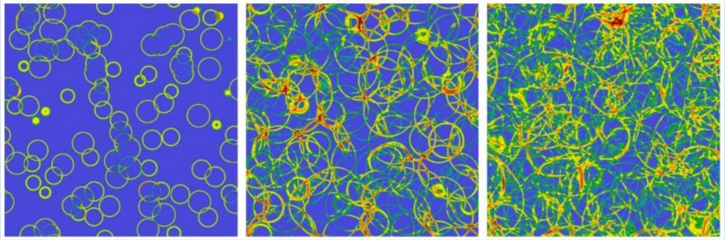
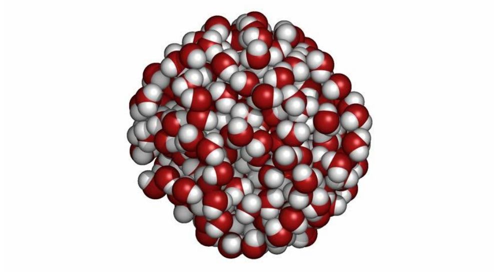


⟶ Relevant for groundbreaking research!

Hydrodynamics

What is hydrodynamics? ⟶ **Effective theory**

Water



Complicated molecular dynamics

Collective description: <u>hydrodynamics</u>

Hydrodynamics

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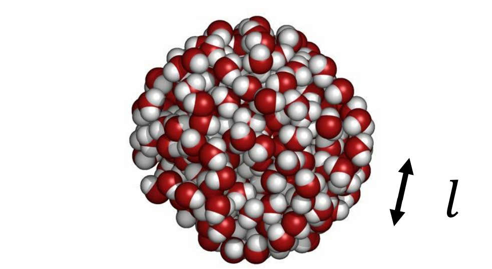


l

Complicated molecular dynamics

L

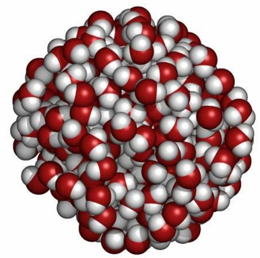
Collective description: hydrodynamics

- Two scales well separated: $l \ll L$

Hydrodynamics

What is hydrodynamics? $\longrightarrow$ **Effective theory**

Water



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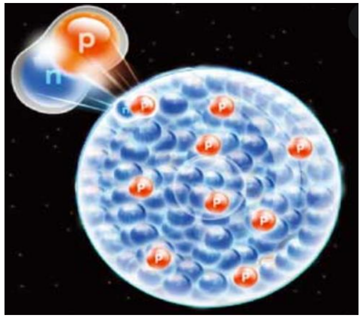
L

Collective description: <u>hydrodynamics</u>

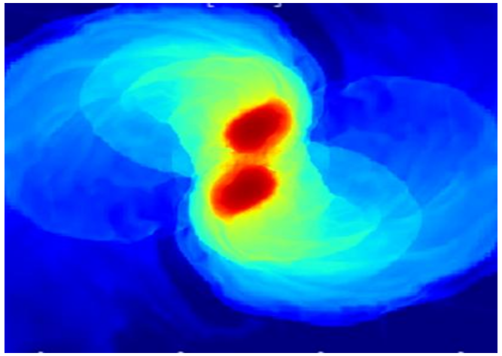
- Two scales well separated: $l \ll L$

Neutron star merger

Complicated QCD dynamics



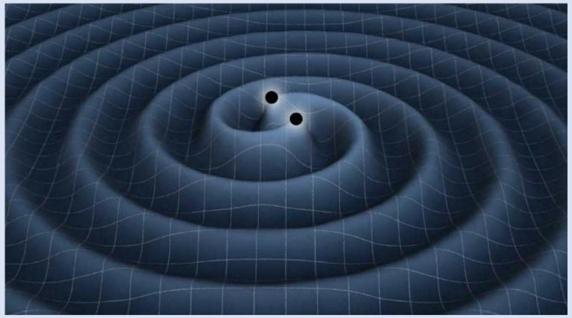
<u>Hydrodynamics</u>



Neutron star mergers

Black hole mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \Rightarrow R_{\mu\nu} = 0$$



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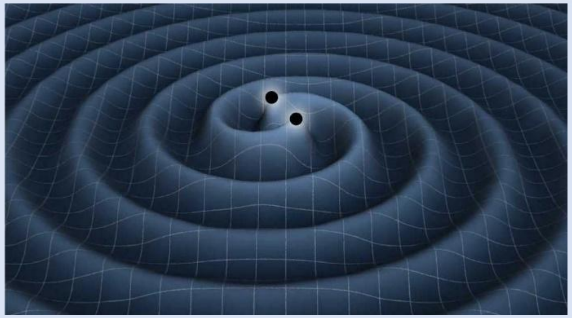

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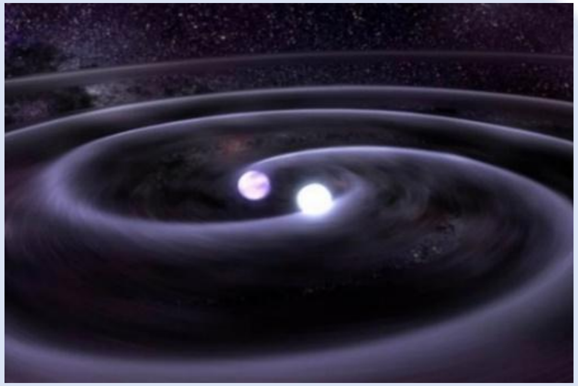

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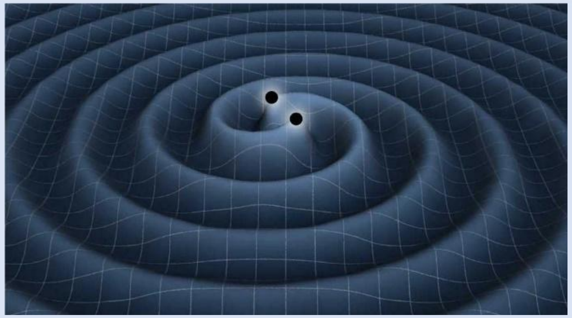
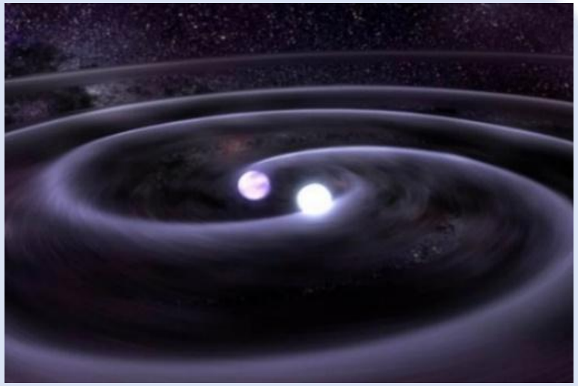


Neutron star mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

Matter must be specified



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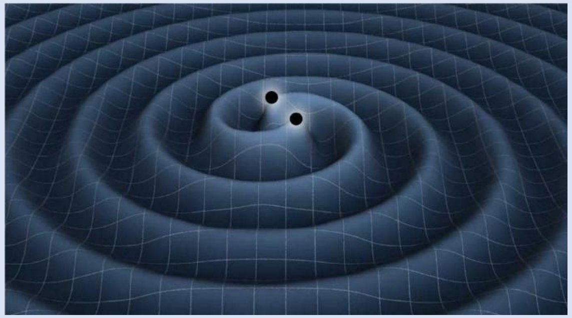

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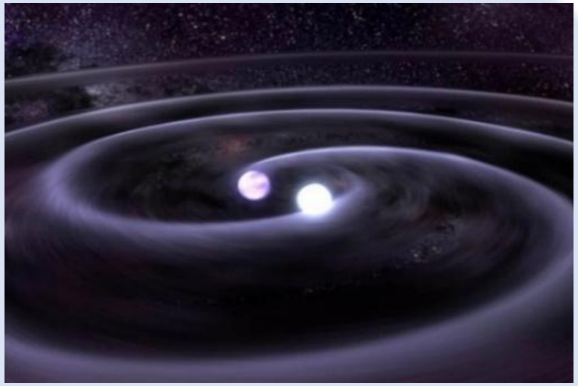

$\longrightarrow$ Gravity coupled to QCD

Neutron star mergers

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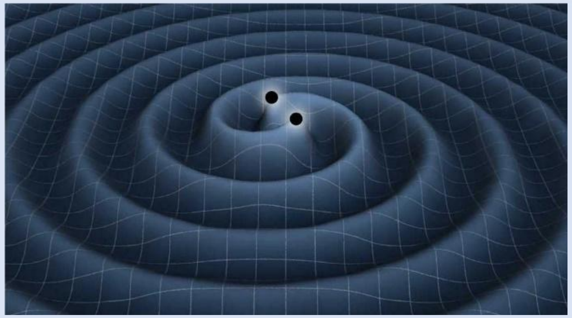

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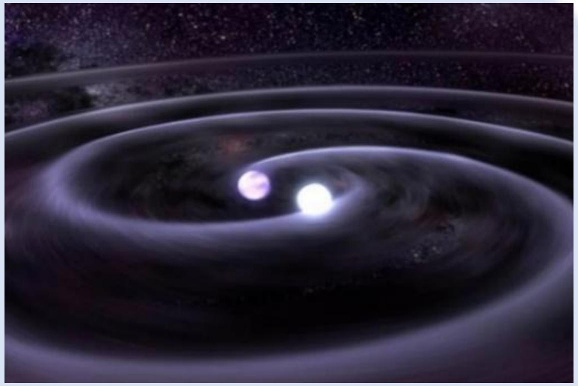

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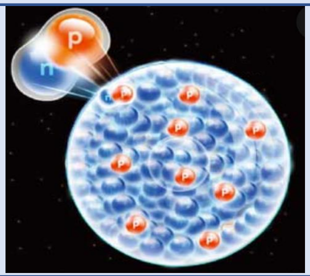
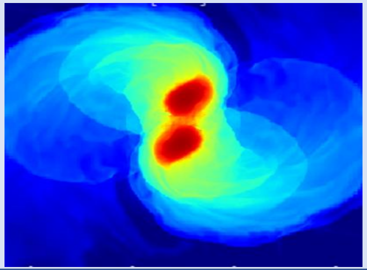


$l \sim$ fm $\ll$ L $\sim$ km

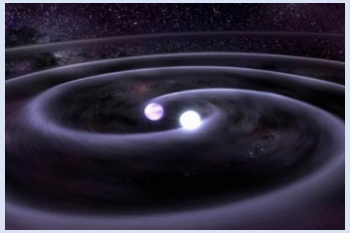


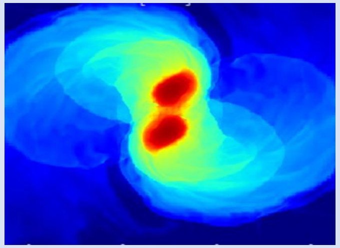
$\Rightarrow$ Hydrodynamics provides a good description

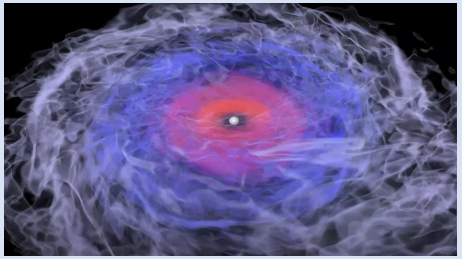
Neutron star mergers

Picture from simulations:

NS + NS → **HMNS** → **BH+torus**







Highly dynamical post merger region

Neutron star mergers

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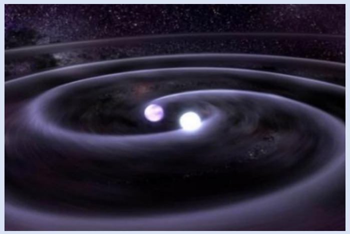

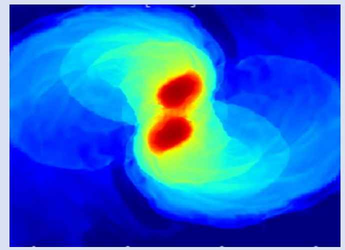

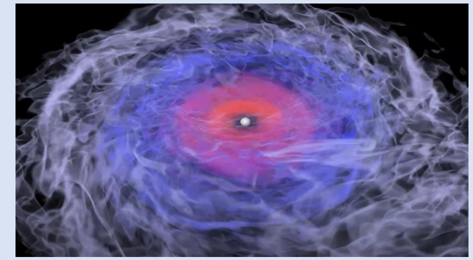

Highly dynamical post merger region

→ If we aim to precision physics, we must include all the **relevant physics**

Neutron star mergers

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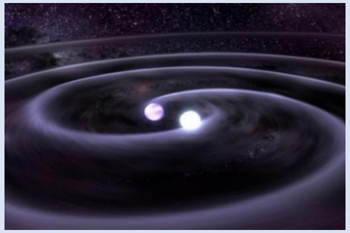
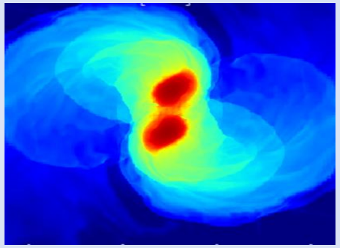
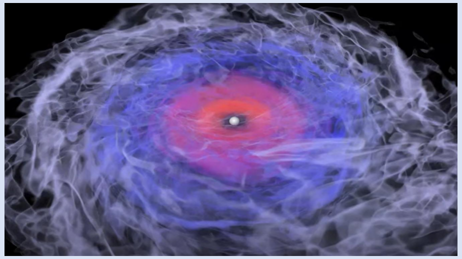


HMNS



BH+torus



Highly dynamical post merger region

→ If we aim to precision physics, we must include all the **relevant physics**

Weak processes are relevant!

$$n \to p + e^- + \bar{\nu}_e$$

$$p + e^- \to n + \nu_e$$

Neutron star mergers

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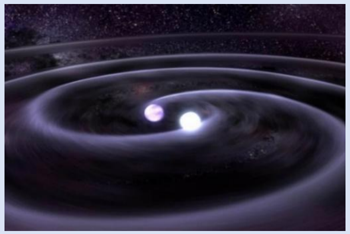

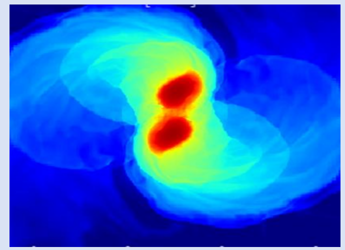

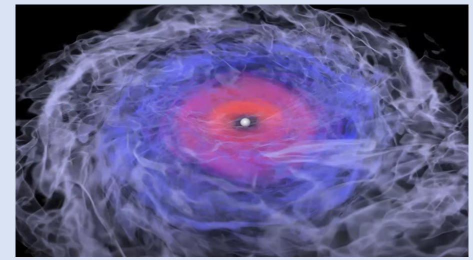

Highly dynamical post merger region

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Weak processes are relevant! → **Effective bulk viscosity**!

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M. Alford, A. Harutyunyan, A. Sedrakian ‘22
E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford, J. Noronha‘22
Alford, Haber, Harris, Zhang‘21
Alford, Harutyunyan, Sedrakian ‘21
Most, Harris, Plumberg, Alford, Noronha, Noronha-Hostler, Pretorius, Witek, Yunes‘21
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Neutron star mergers

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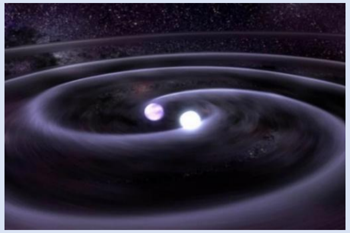

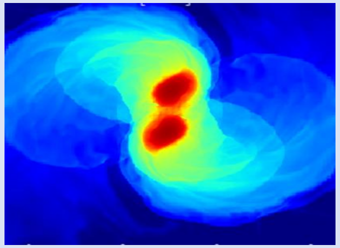

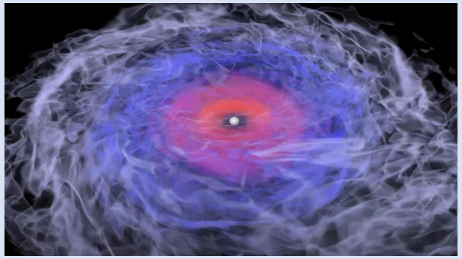

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Beyond state of the art: Introduce viscosity in the hydrodynamic equations

Neutron star mergers

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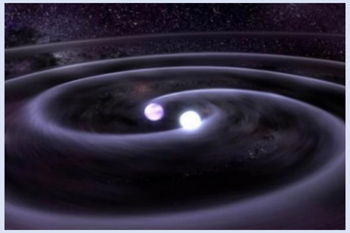

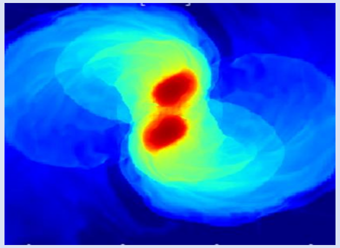

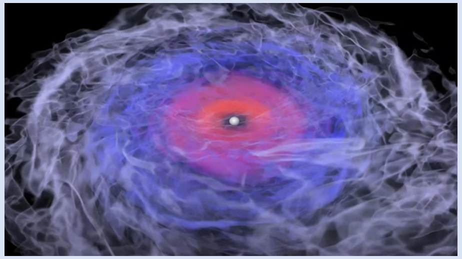

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→ If we aim to precision physics, we must include all the **relevant physics**

Weak processes are relevant! → **Effective bulk viscosity**!

$$n \to p + e^- + \bar{\nu}_e$$

$$p + e^- \to n + \nu_e$$

M. Alford, A. Harutyunyan, A. Sedrakian '22
E. R. Most, A. Haber, S. P. Harris, Z. Zhang, M. G. Alford, J. Noronha'22
Alford, Haber, Harris, Zhang'21
Alford, Harutyunyan, Sedrakian '21
Most, Harris, Plumberg, Alford, Noronha, Noronha-Hostler, Pretorius, Witek, Yunes'21
....

Beyond state of the art: Introduce viscosity in the hydrodynamic equations

Including viscosity in relativistic hydrodynamics involves difficulties...

Hydrodynamics: equations and causality

Constitutive relations

$$T^{\mu\upsilon} = T^{\mu\upsilon}_{ideal} + \partial + \partial^2 + \dots$$

0th order 1st 2nd

Gradient expansion

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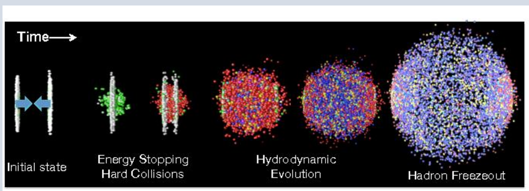
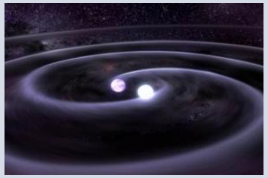
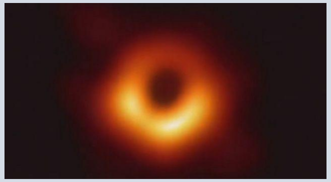
Real-time evolutions are required!!







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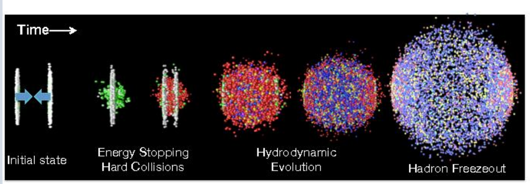

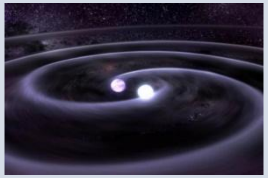

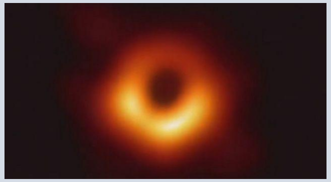

Ideal hydro $\longrightarrow$ Well posed

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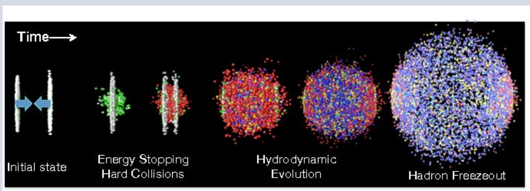

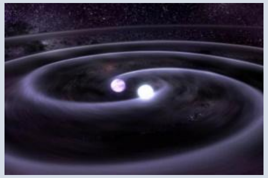

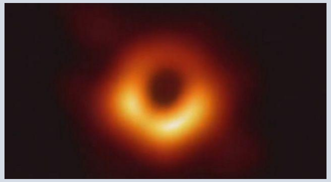

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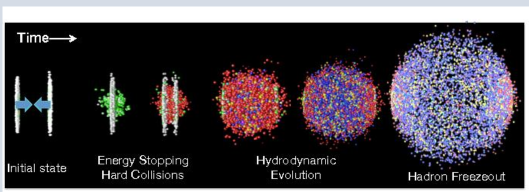

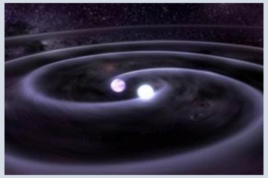

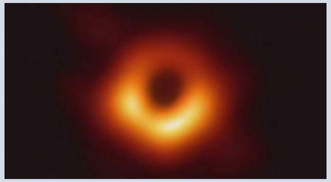

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Müller-Israel-Stewart (MIS) ⟶ Well posed

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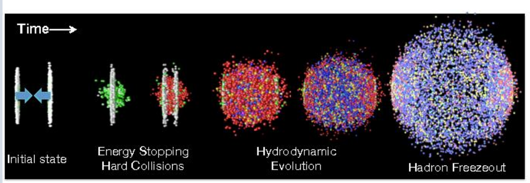

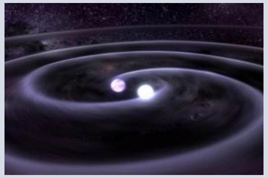

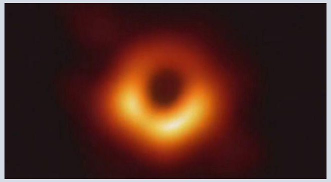

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Usual fix

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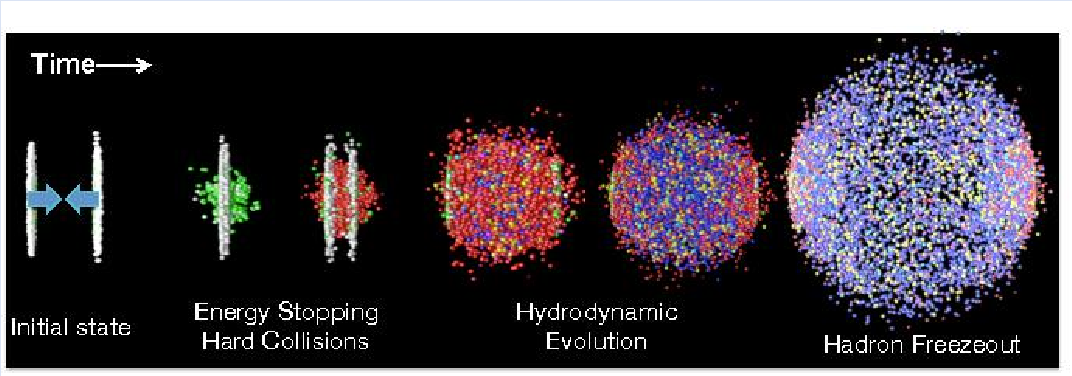
MIS is not unique:

- MIS
- BRSSS
- DNMR
- Divergence type
- …

Viscosity in the quark-gluon plasma

⟶ In the quark gluon plasma **viscosity is relevant**



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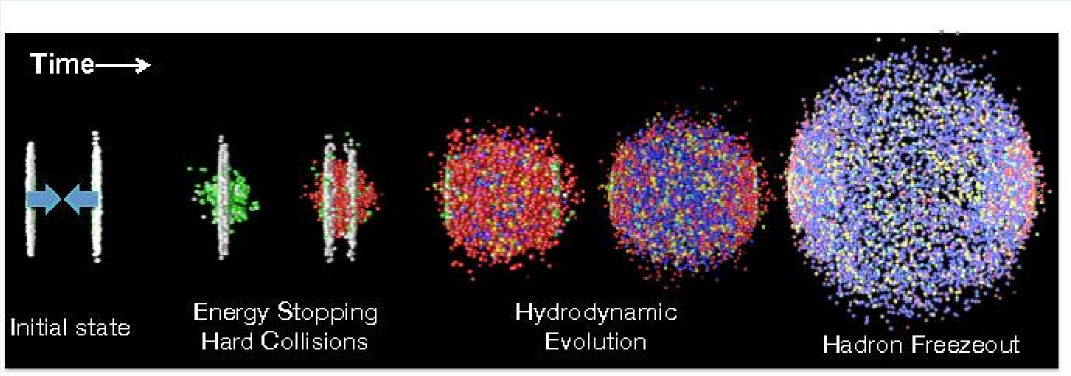

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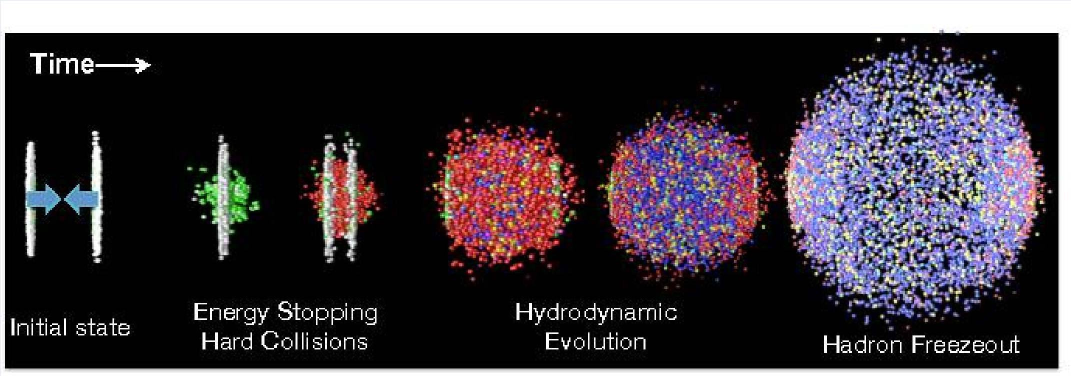

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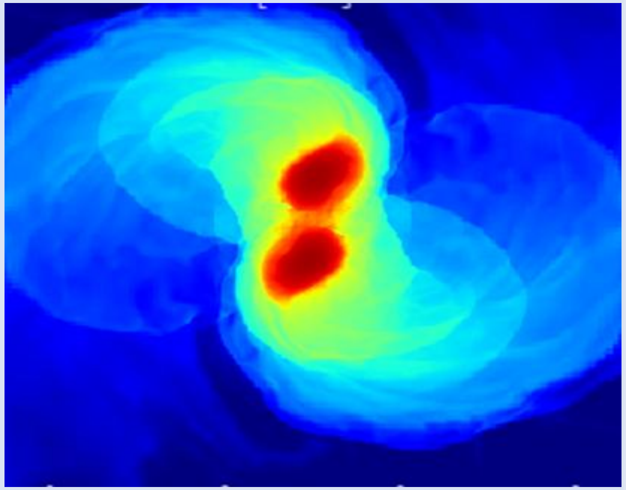
⟶ But there are issues....

Issues with shocks in MIS

→ We have seen that **viscosity is relevant in neutron star mergers.** This would require to use MIS-type theories.



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Olson, Hiscock ’90
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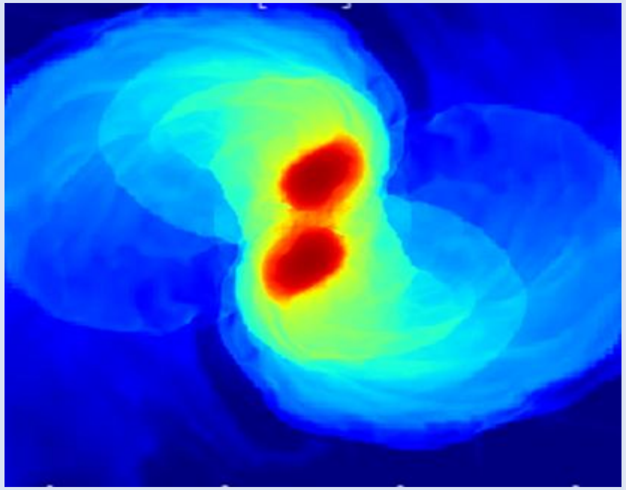

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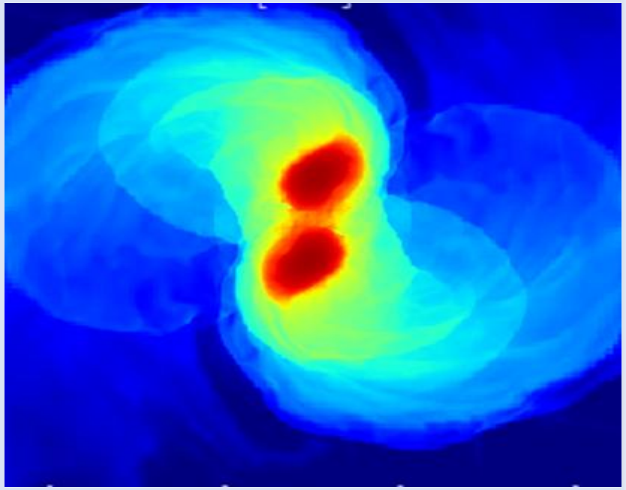

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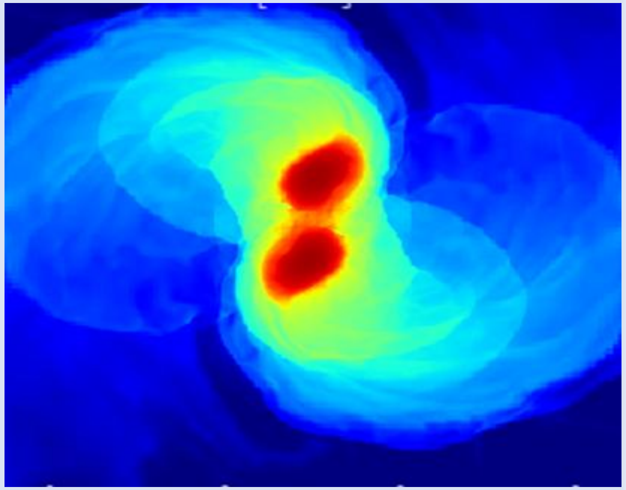

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It could happen that **viscous neutron star mergers are not viable by using MIS**....

Causality issues in MIS

→ Recent studies point in the direction that MIS present causality issues

Causality issues in MIS

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- Non linear causality conditions for MIS unknown until 2020.

Nonlinear Constraints on Relativistic Fluids Far From Equilibrium

Fábio S. Bemfica,[1] Marcelo M. Disconzi,[2] Vu Hoang,[3] Jorge Noronha,[4] and Maria Radosz[3]

[1] *Escola de Ciências e Tecnologia, Universidade Federal do Rio Grande do Norte, 59072-970, Natal, RN, Brazil**
[2] *Department of Mathematics, Vanderbilt University, Nashville, TN, USA*†
[3] *Department of Mathematics, The University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249, USA*‡
[4] *Department of Physics, University of Illinois, 1110 W. Green St., Urbana IL 61801-3080, USA*§

(Dated: May 26, 2020)

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0$$

$$\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,$$

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$$\varepsilon + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0, \quad a \neq d$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d + \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} + (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$

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- Constraints quite restrictive ⟶ **Relevant for heavy ions? YES**

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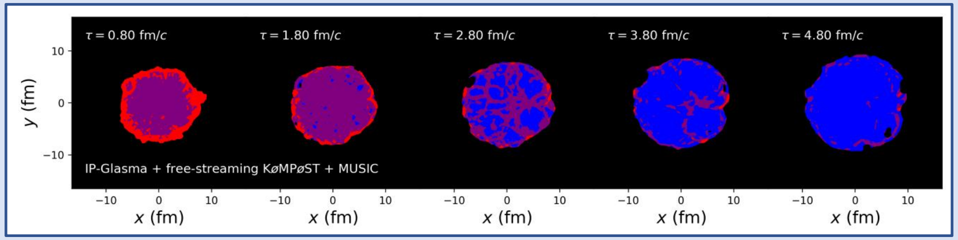
Causality violations in realistic simulations of heavy-ion collisions

Christopher Plumberg,[1] Dekrayat Almaalol,[2] Travis Dore,[1] Jorge Noronha,[1] and Jacquelyn Noronha-Hostler[1]

[1]*Illinois Center for Advanced Studies of the Universe, Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA*
[2]*Department of Physics, Kent State University, Kent, OH 44242, USA*

(Dated: March 31, 2021)

→ **Significant causality violations!**



BDNK

A novel formulation of viscous hydrodynamics

BDNK vs MIS

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Bemfica, Disconzi, Noronha ’17 ’19

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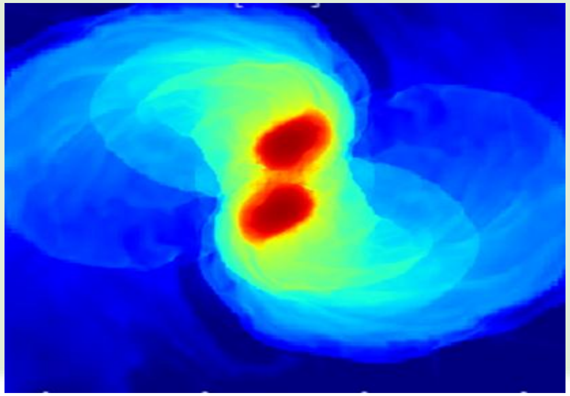

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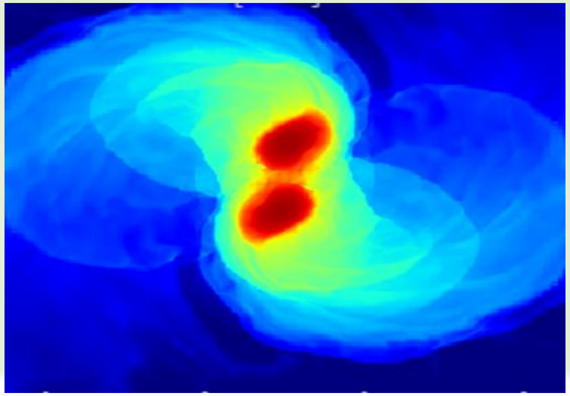

Real-time evolutions using this formulation

Bantilan, Bea, Figueras ’22

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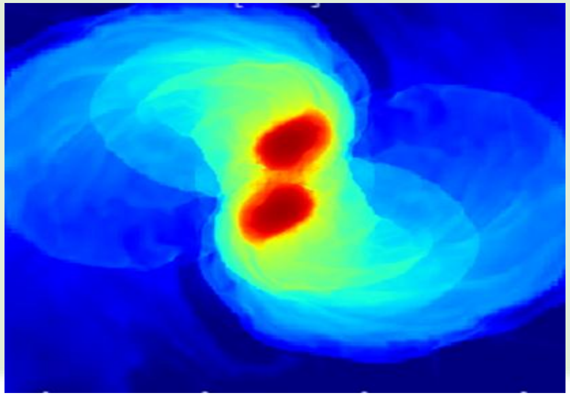

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The BDNK equations

Hydro equations

- **Conformal theory**

Hydro equations

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- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

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$$\Downarrow$$

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$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \text{2nd order}$$

New equation

$\Rightarrow$

Hyperbolic!!

$$\nabla_\mu T^{\mu\nu} = 0$$

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Hydro equations

- **Conformal theory**

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

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- First order hydro: **Landau frame**

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$$T^{\mu\nu} = (\epsilon \qquad)\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \qquad - \eta\, \sigma^{\mu\nu}$$

→ Include all 1st order terms compatible with Poincare symmetry

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→ Freedom choosing the out of equilibrium variables.

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Constants specifying the frame

a1=a2=0 → Landau frame

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a1=a2=0 ⟶ Landau frame

$$a_2 > 1\,, \qquad a_1 > \frac{4\, a_2}{a_2 - 1}\,.$$

$$\Rightarrow \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

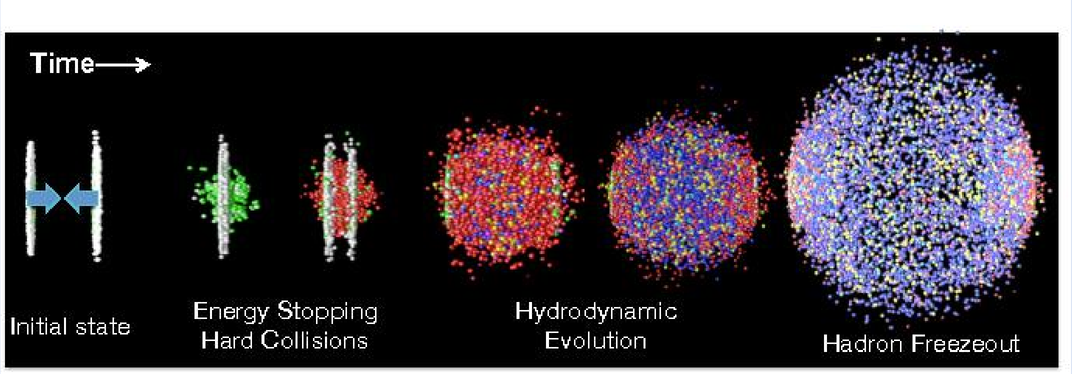
BDNK equations

Bemfica, Disconzi, Noronha ’17’19
Kovtun ’19

BDNK: Dynamical evolutions

Motivation

⟶ Motivation: In heavy-ion collisions dissipative terms comparable to ideal terms



⟶ Starts exploring the UV of the theory…

Motivation

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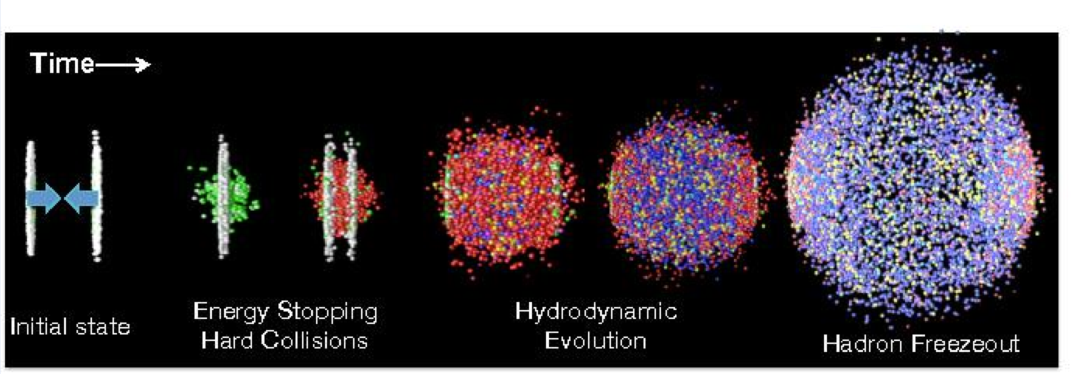

→ Starts exploring the UV of the theory…

→ We would like to explore the non-linear, and far from equilibrium regimes of the BDNK equations

Holography

Holography

→ Excellent framework to study the applicability of hydrodynamics.

→ Far from equilibrium strongly coupled field theories from first principles.

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→ Far from equilibrium strongly coupled field theories from first principles.

- Strongly coupled QFT
- Out of equilibrium physics

- Dual of QCD not known…
- Not precision holography

  → Qualitative aspects

Holography: Our model

- Gravity with Λ in 3+1 dim :

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

Holography: Our model

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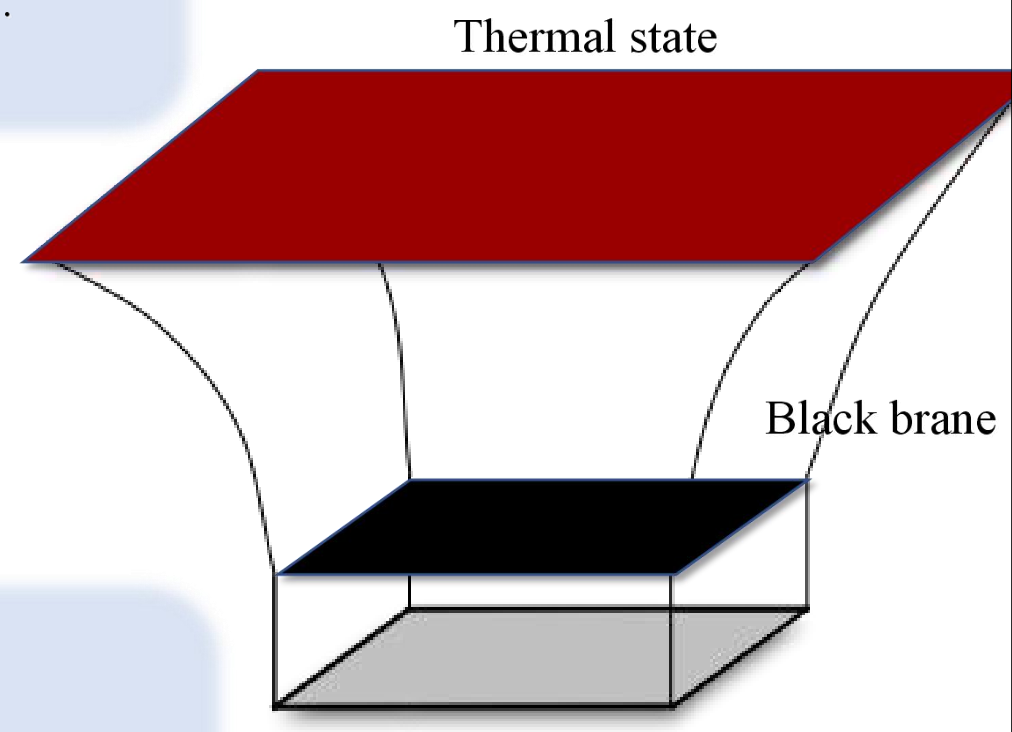

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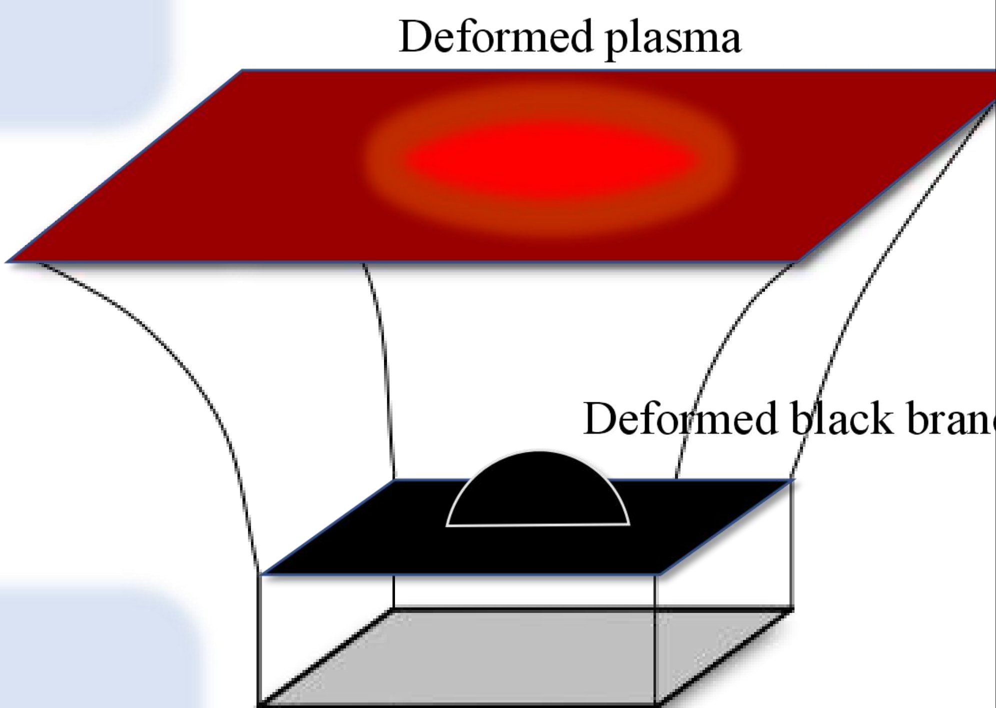

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- CFT on Minkowski in 2+1 dim
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Real-time quantum dynamics

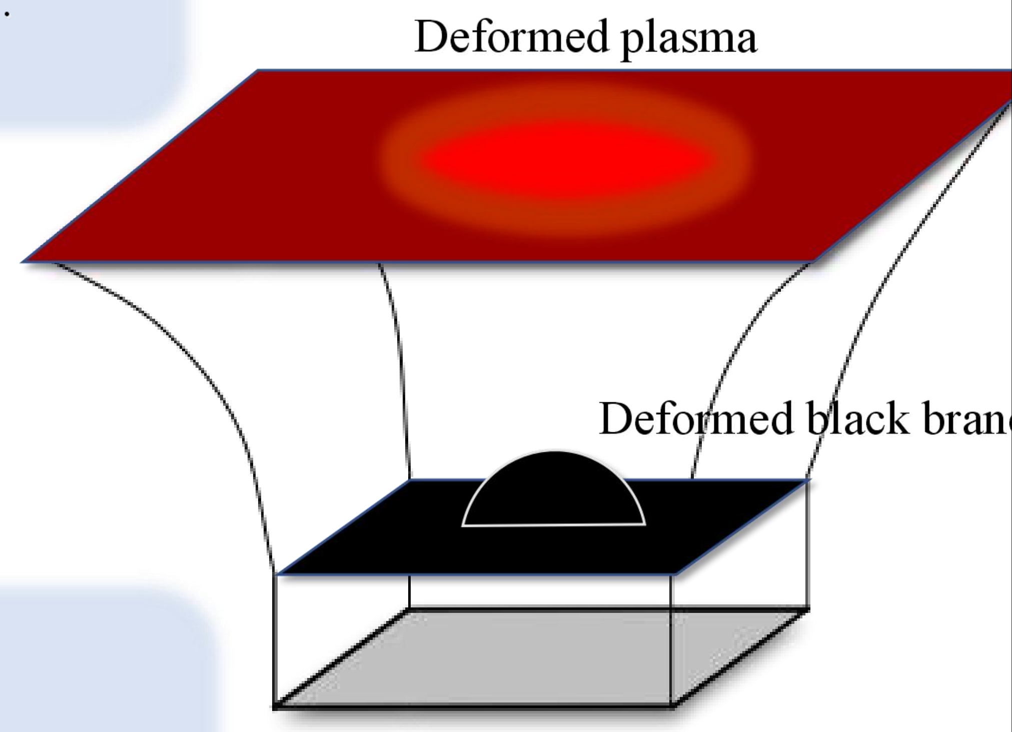
$\Updownarrow$ Holography

Numerical Relativity

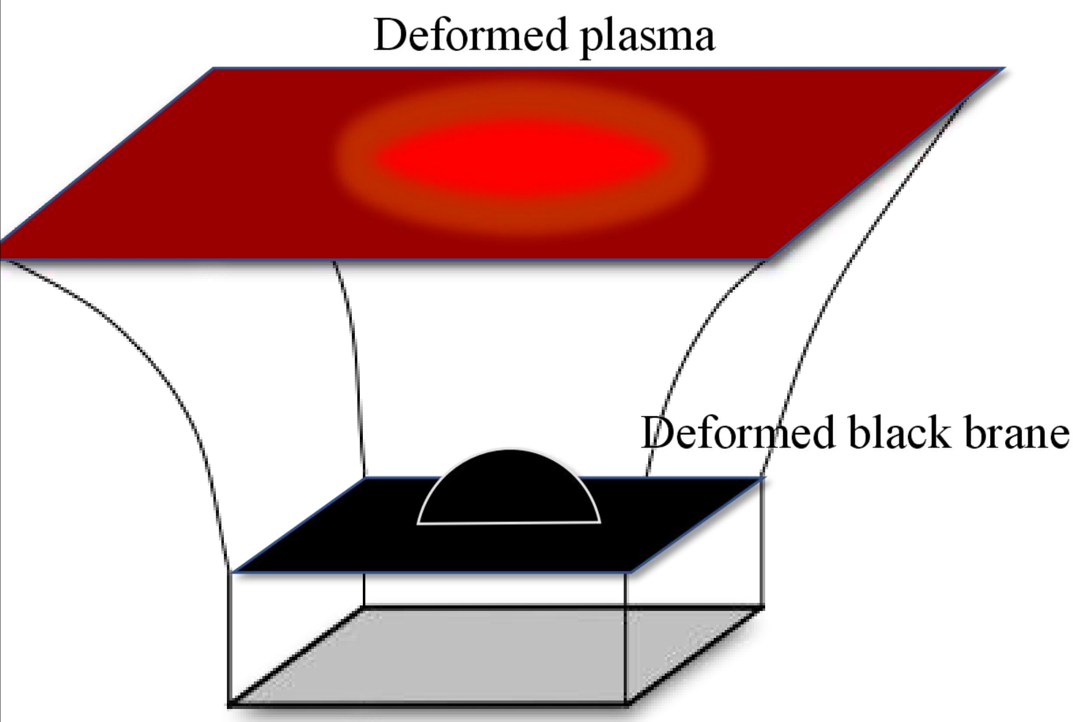
Dynamical classical gravity

- Gravity with Λ in 3+1 dim :

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$



Holographic solution



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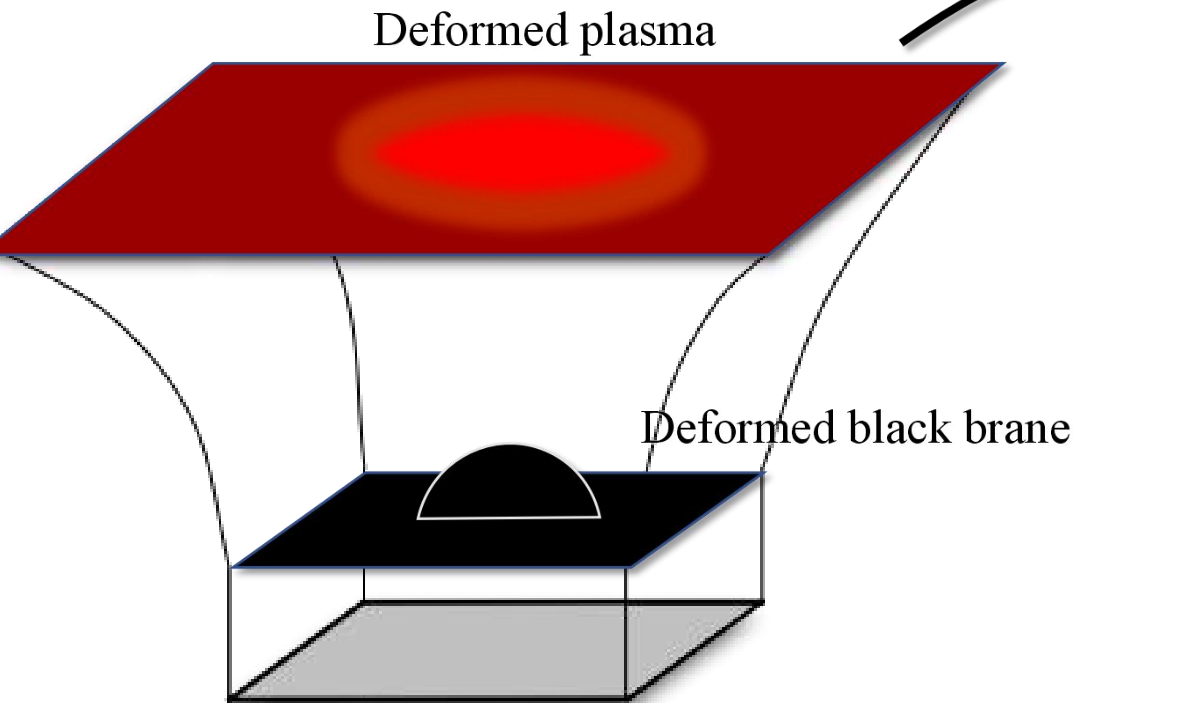

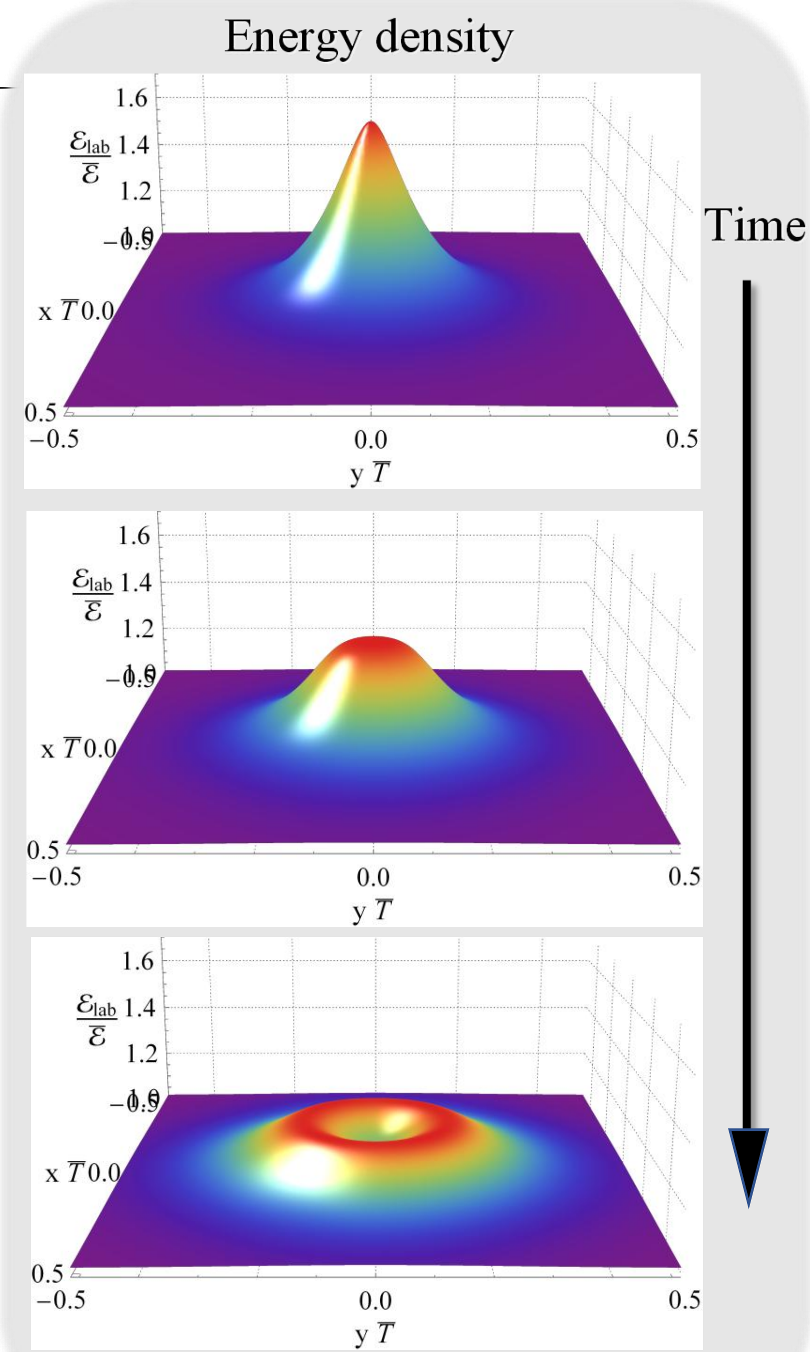

Bantilan, Bea, Figueras ‘22

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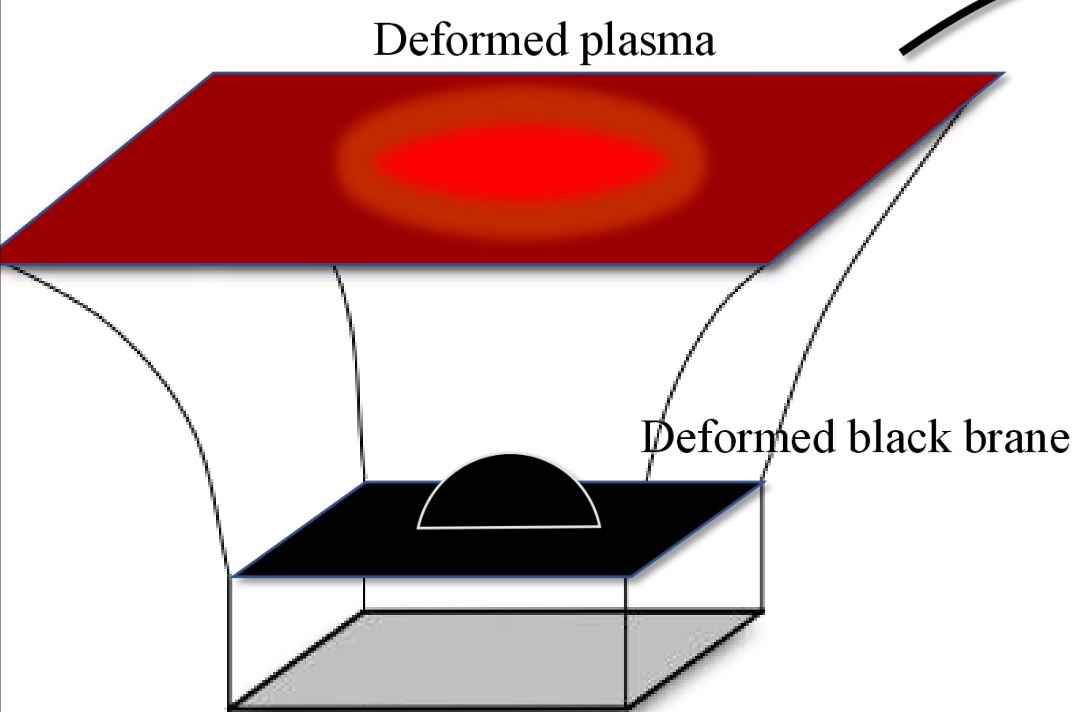

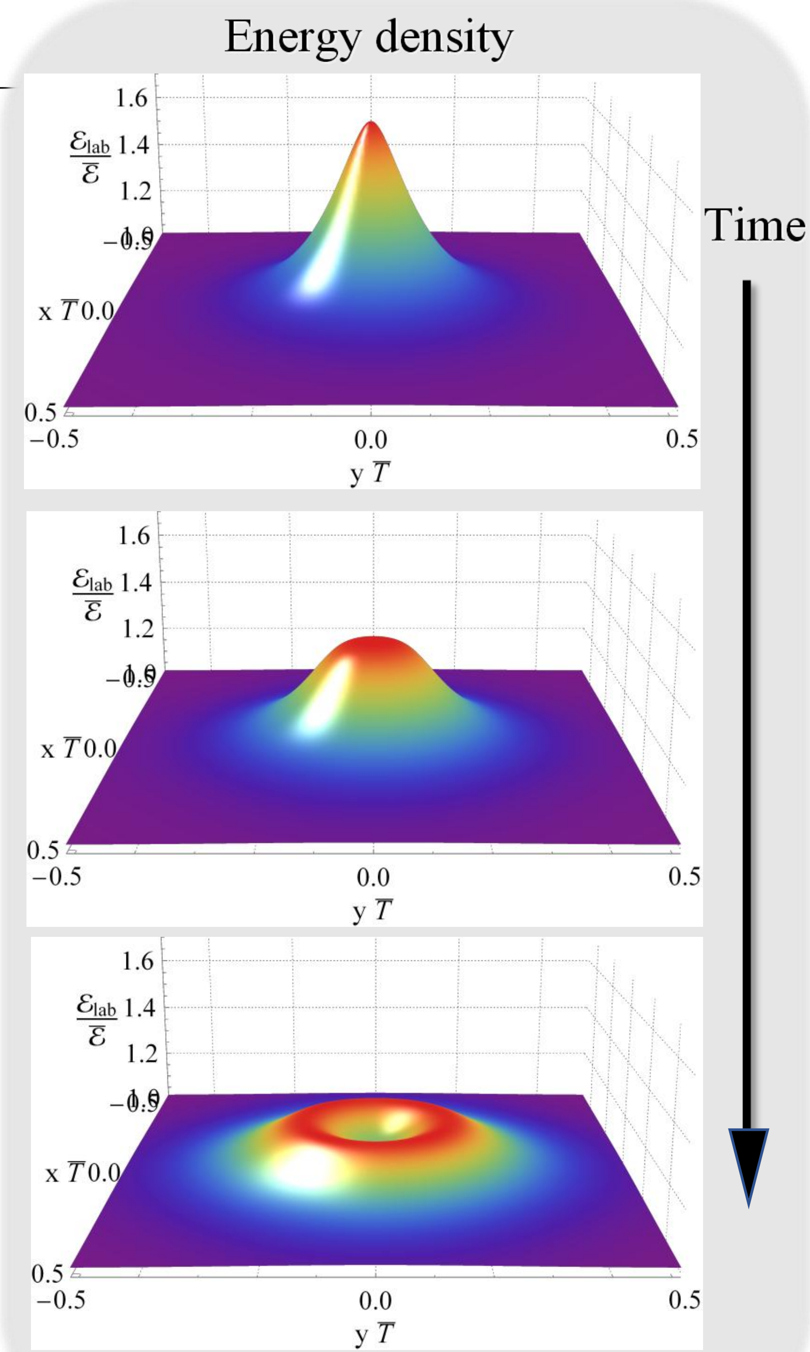

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→ We have a microscopic solution

→ We want to test the aplicability **hydrodynamics**:

- Check constitutive relations
- Time evolutions in hydro

Hydro: Constitutive relations

$$\boldsymbol{T_{\mu\nu}} = \underbrace{\boldsymbol{T^{ideal}_{\mu\nu}}}_{\text{0th}} + \underbrace{\boldsymbol{\partial}}_{\text{1st}} + \underbrace{\boldsymbol{\partial^2}}_{\text{2nd}} + \ldots$$

Gradient expansion

Hydro: Constitutive relations

$$\boldsymbol{T_{\mu\nu} = T^{ideal}_{\mu\nu} + \partial + \partial^2 + \ldots}$$

Gradient expansion

0th 1st 2nd

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\,\Delta^{\mu\nu} \quad -\eta\sigma^{\mu\nu} \quad -\eta\tau_\pi\left(\dot{\sigma}^{<\mu\nu>} + \frac{3}{2}\sigma^{\mu\nu}\nabla\cdot u\right) + \ldots$$

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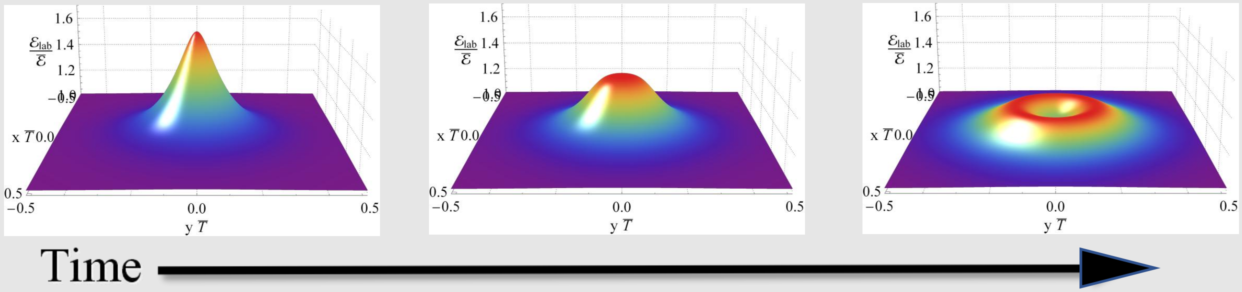

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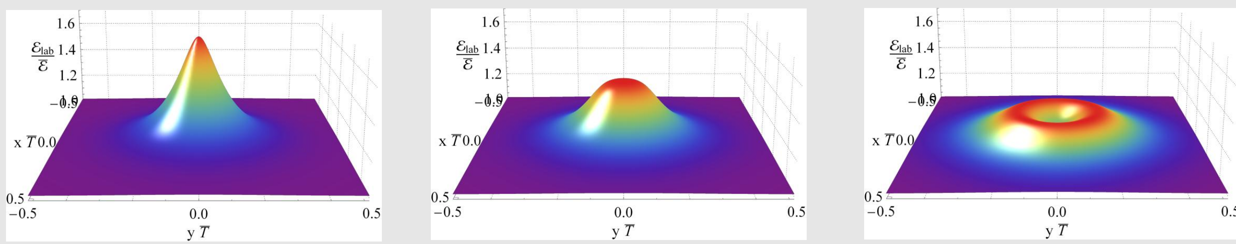
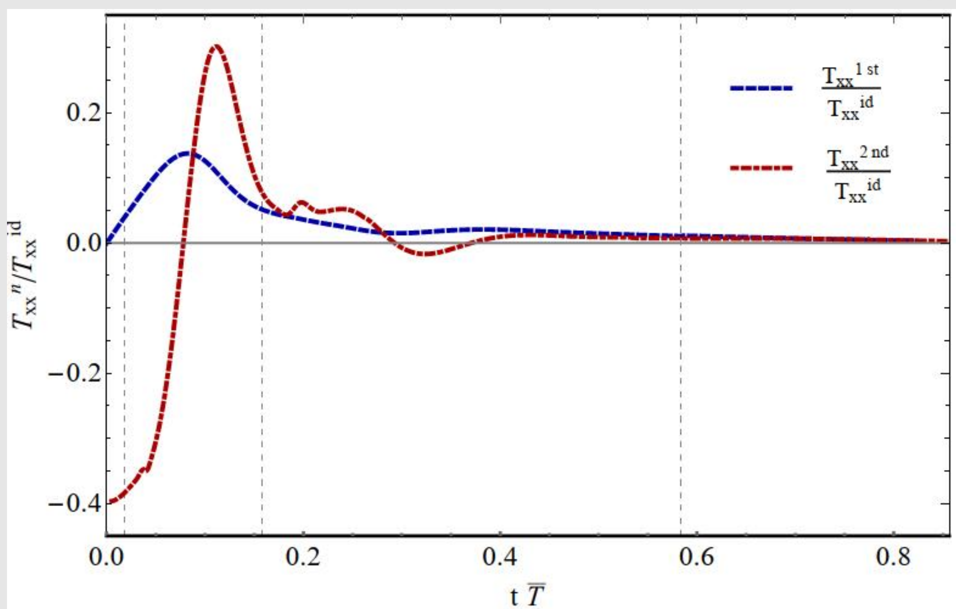


Time



- Off center point
- T_{xx} component

Hydro: Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$

Gradient expansion

0th 1st 2nd

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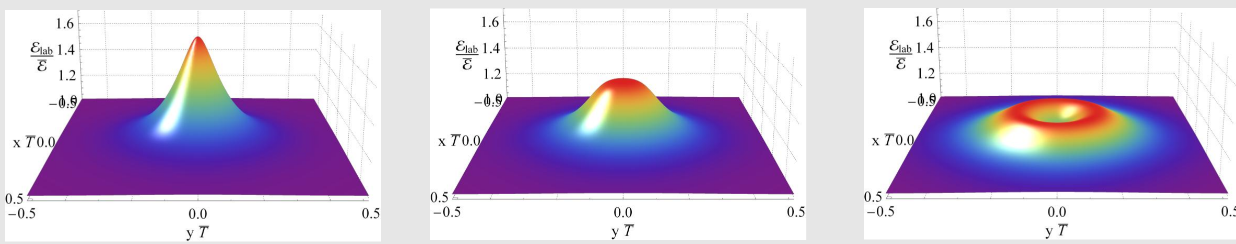
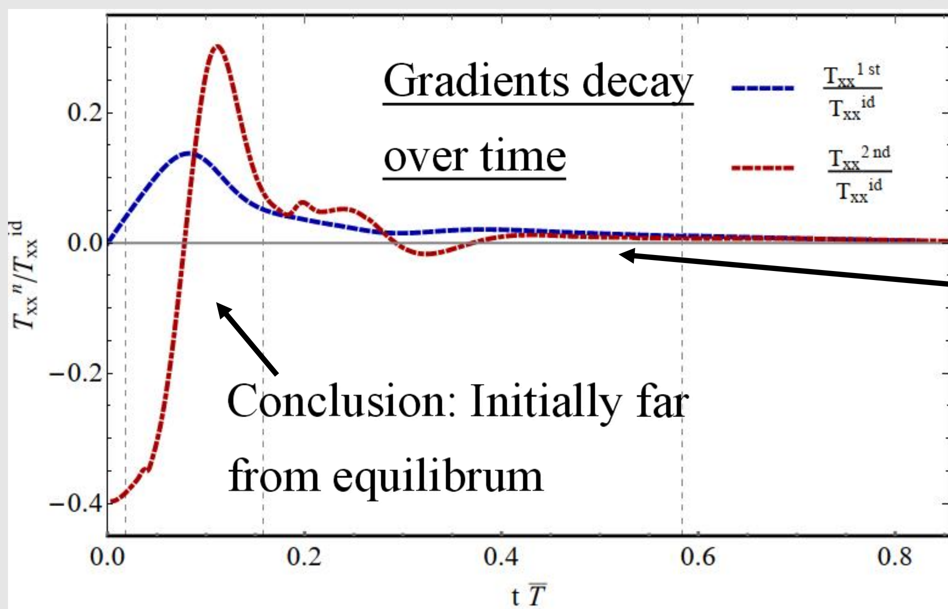


Time

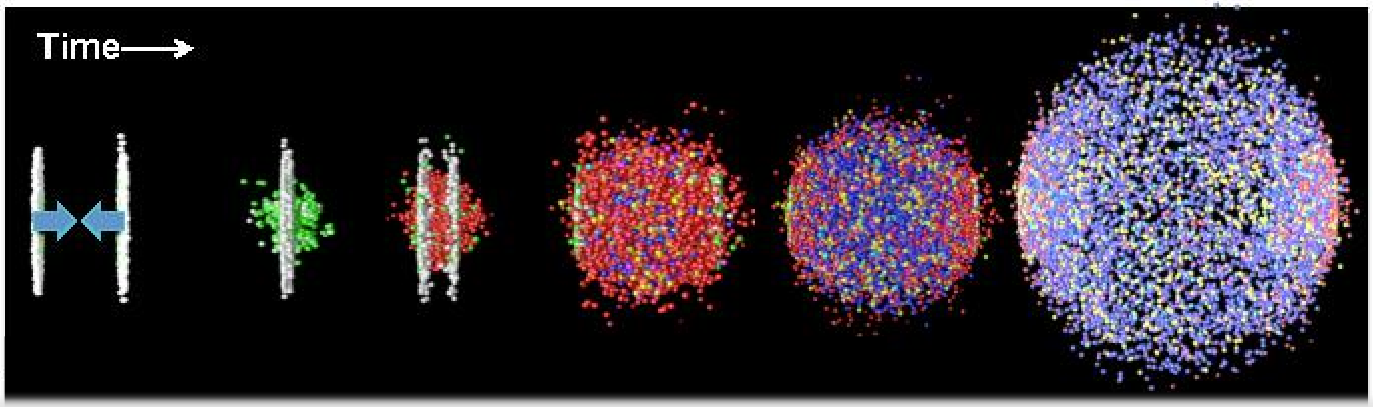
- Off center point
- $\boldsymbol{T_{xx}}$ component



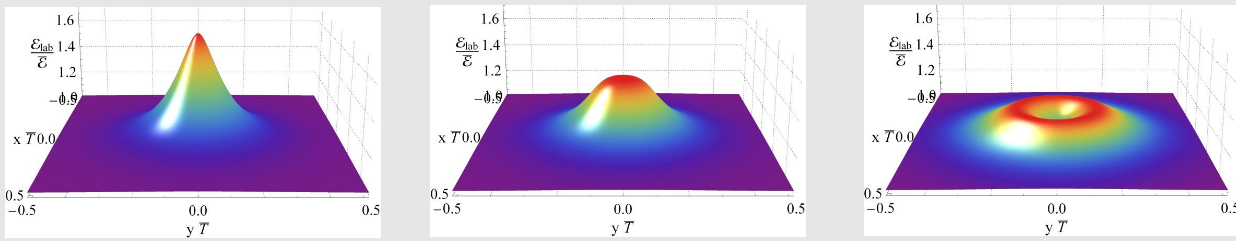
Hydrodynamizes around tT=0.5

Hydro: Constitutive relations

Heavy-ion collision in QCD



Localized perturbation in our holographic theory



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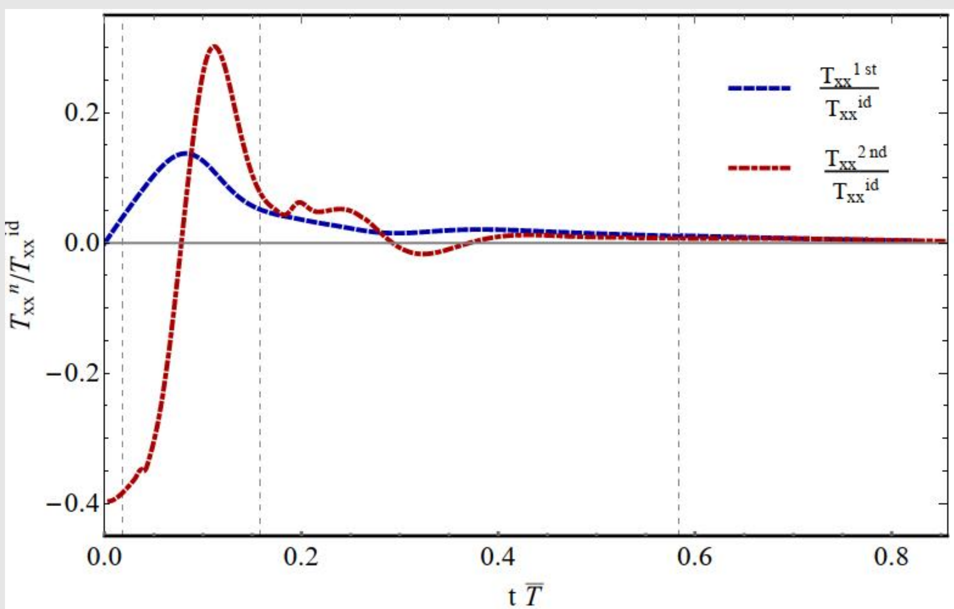

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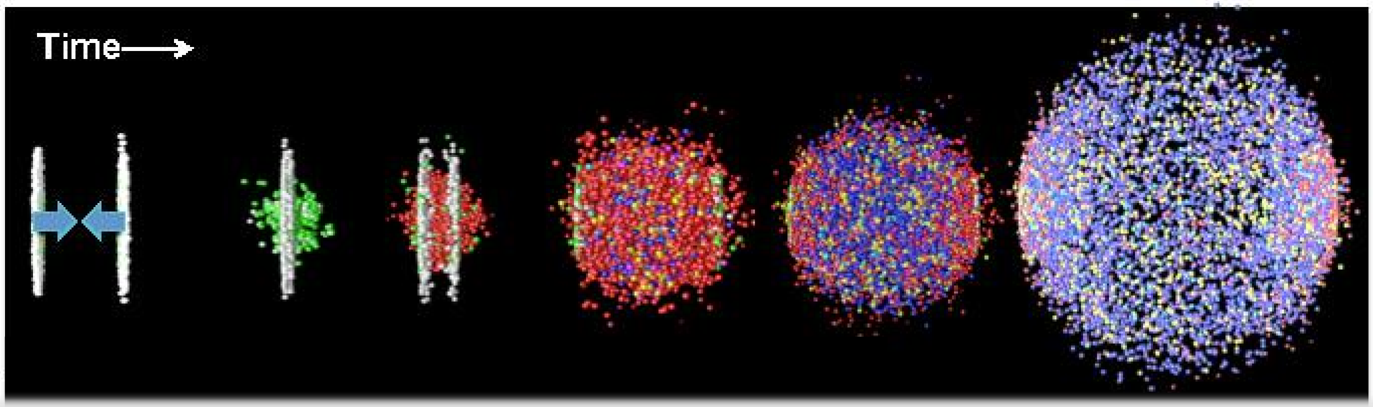

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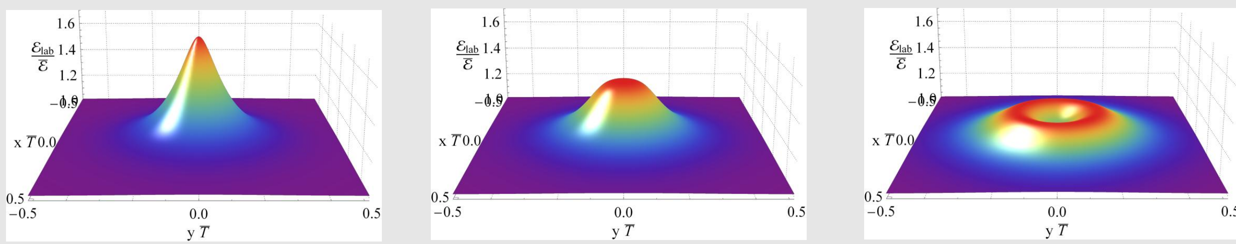

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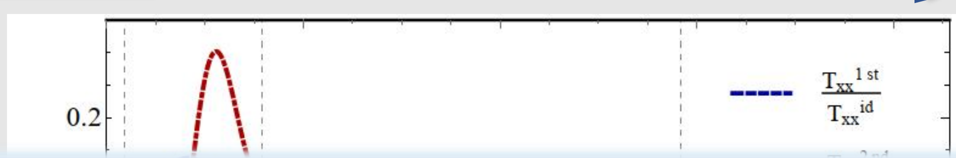

Theories and states are VERY DIFFERENT, but share one aspect:

→ Far from equilibrium and relaxes to a hydro regime

Hydro: Constitutive relations

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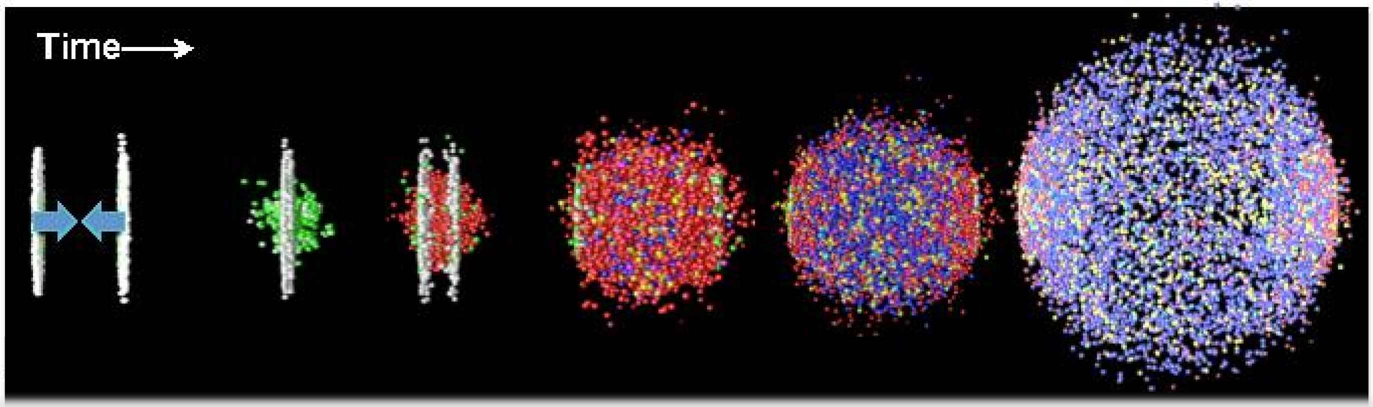

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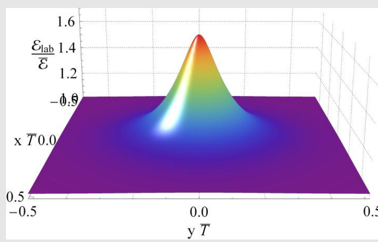

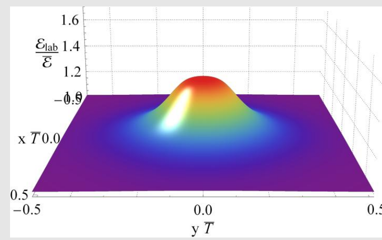

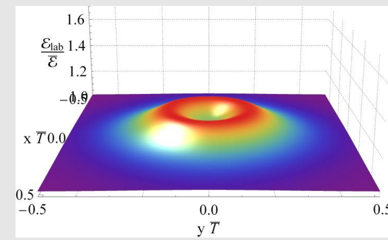

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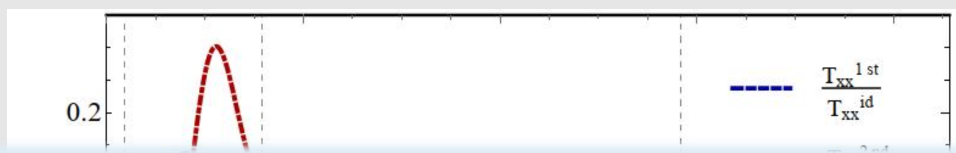

Theories and states are VERY DIFFERENT, but share one aspect:

→ Far from equilibrium and relaxes to a hydro regime

→ We initialize the hydro codes at different times, in analogy to the QGP

Evolutions: holography vs hydrodynamics

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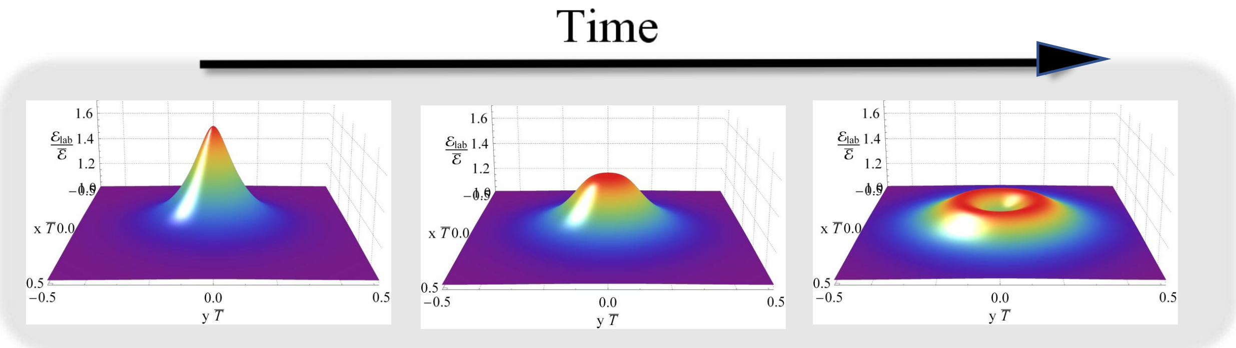
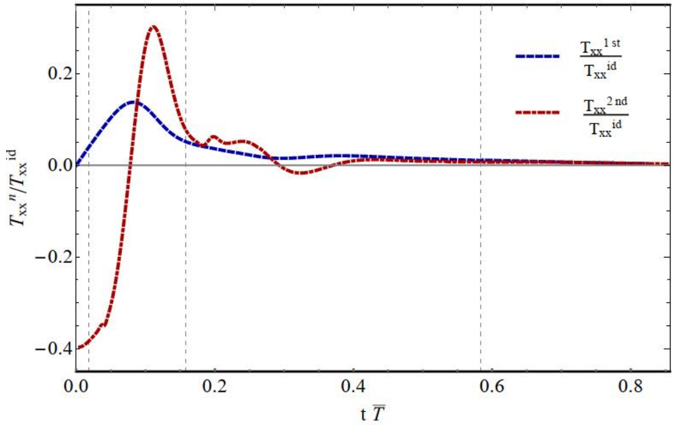
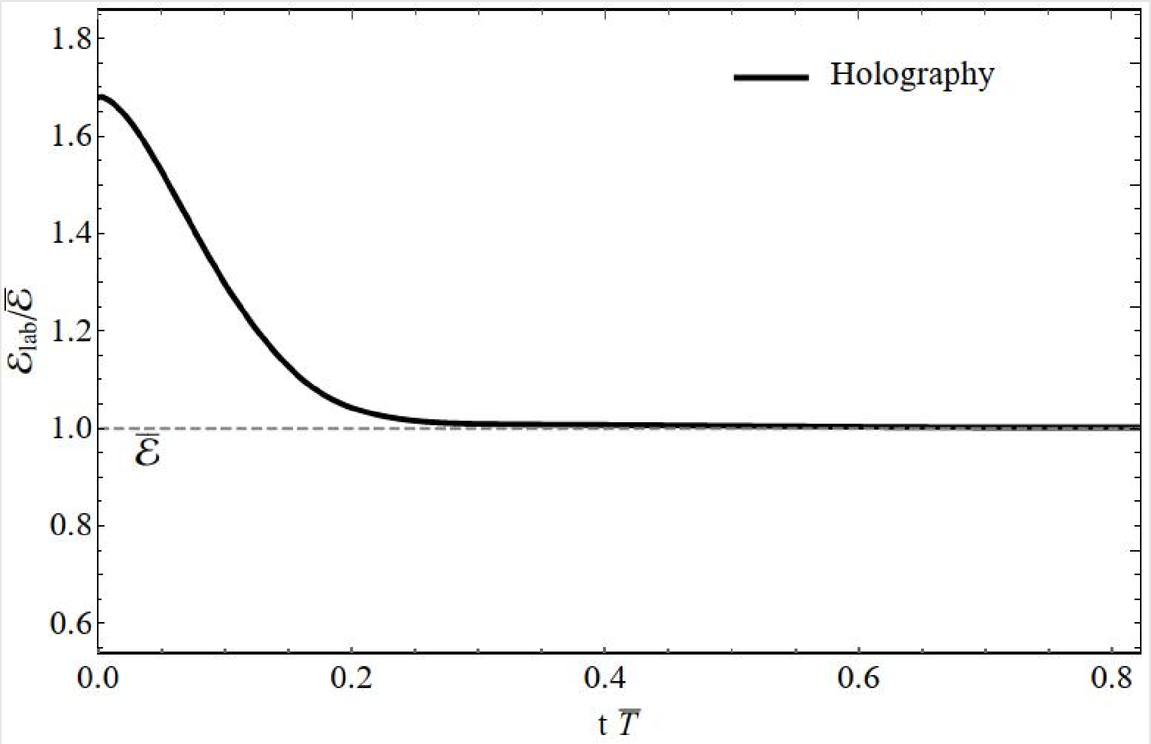




Energy at the center of the domain:



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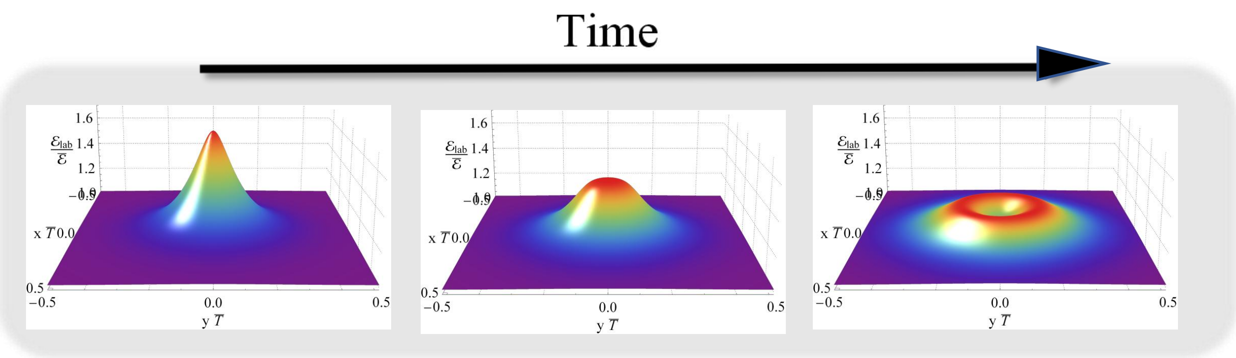
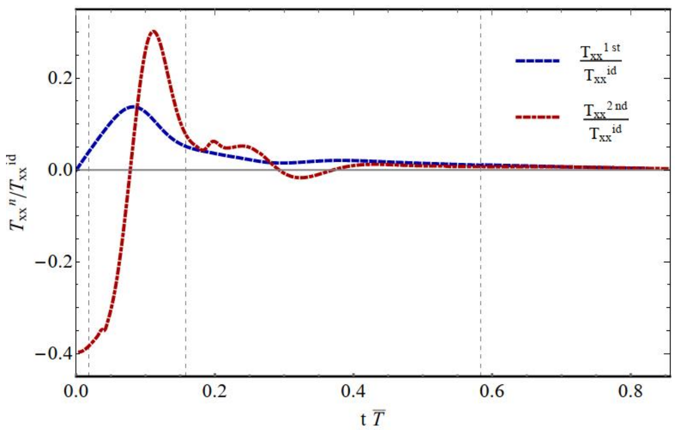
Evolutions: holography vs hydrodynamics





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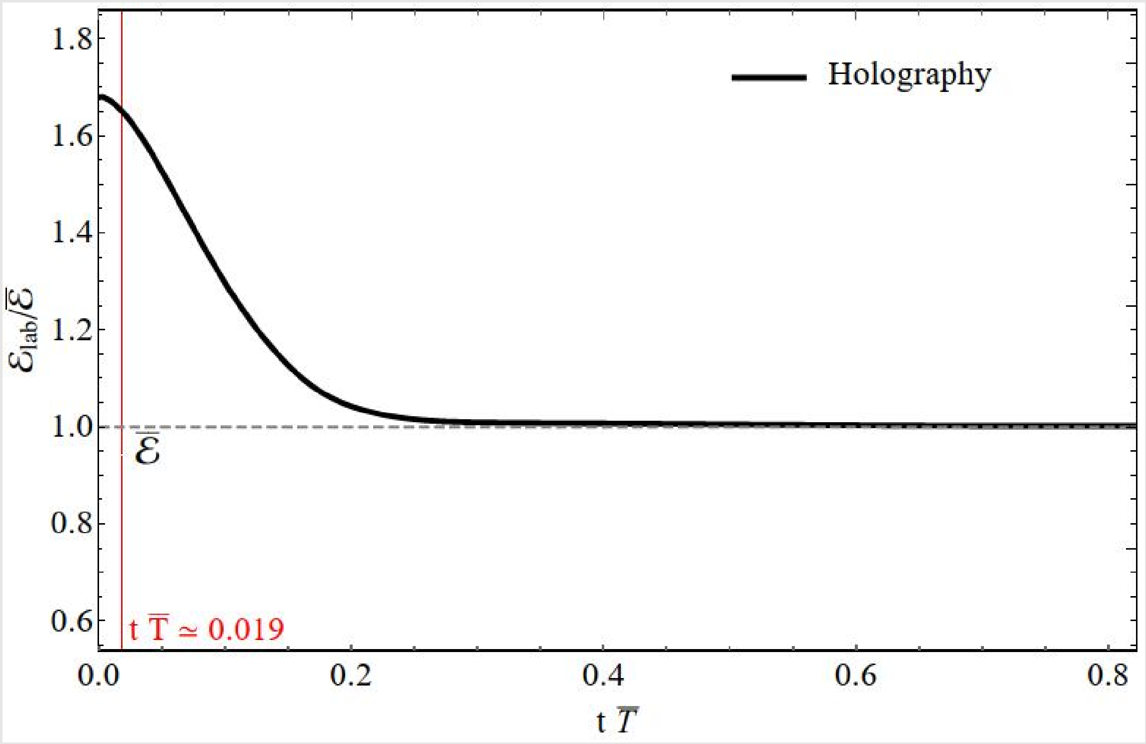

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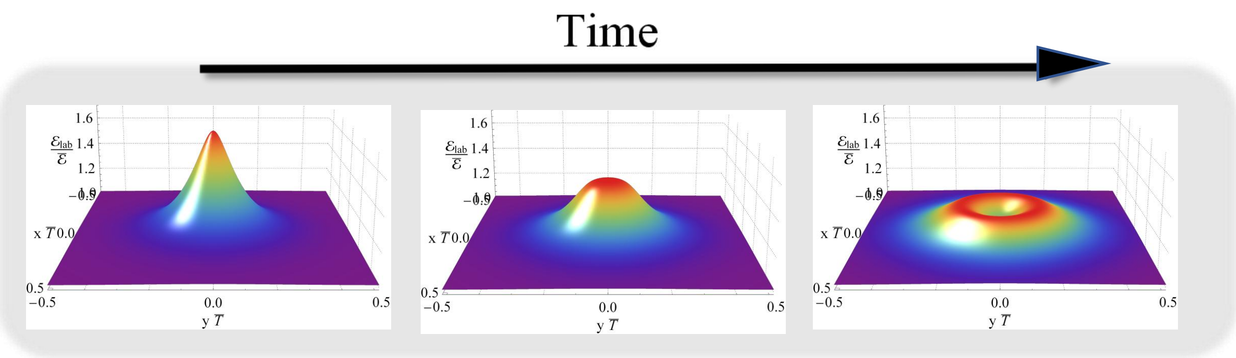

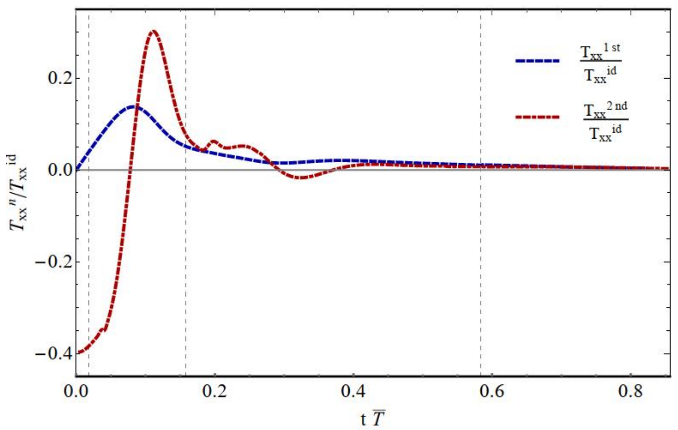

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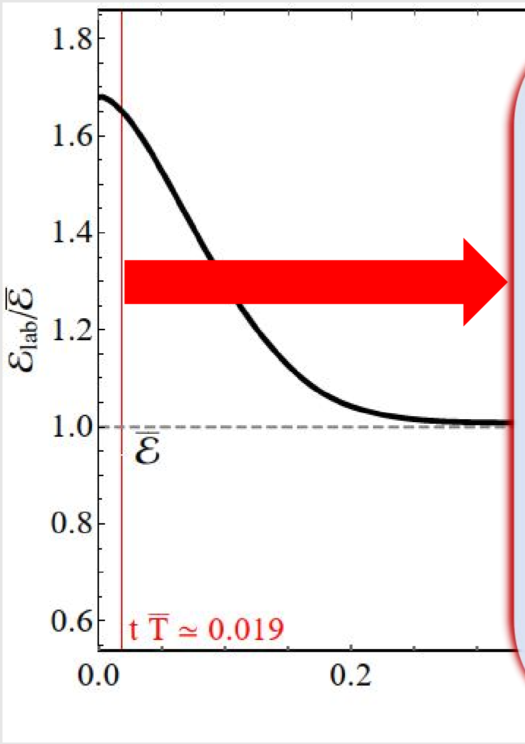

Initial data for hydro codes

MIS-type ε_{loc} u_x u_y Π_{xx} Π_{xy}

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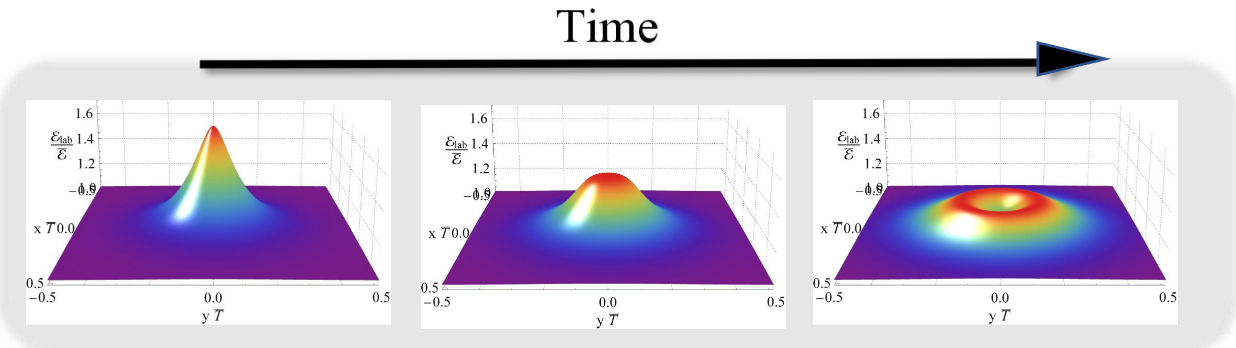
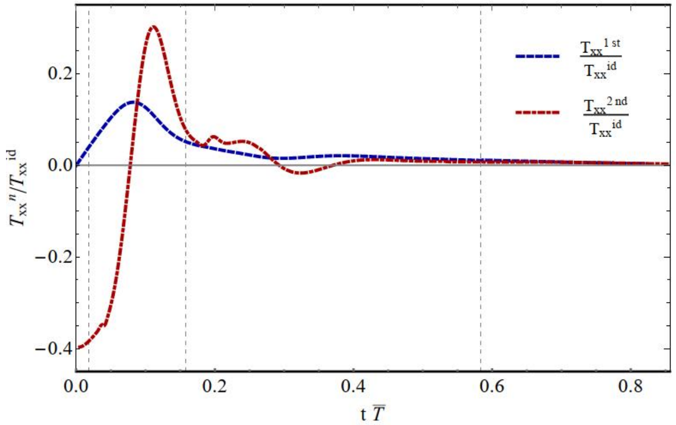
Evolutions: holography vs hydrodynamics





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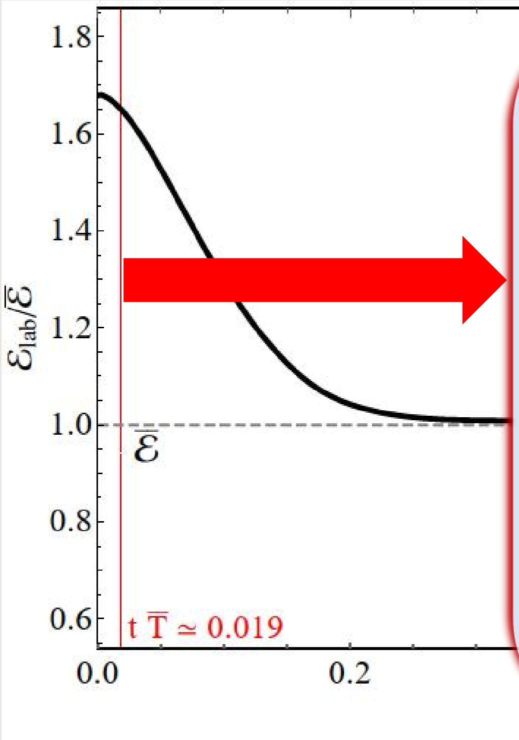

Initial data for hydro codes

MIS-type ε_{loc} u_x u_y Π_{xx} Π_{xy}

BDNK Change to causal frame $a_1 = 6, a_2 = 4$

ε_{loc} u_x u_y

$\partial_t \varepsilon_{loc}$ $\partial_t u_x$ $\partial_t u_y$

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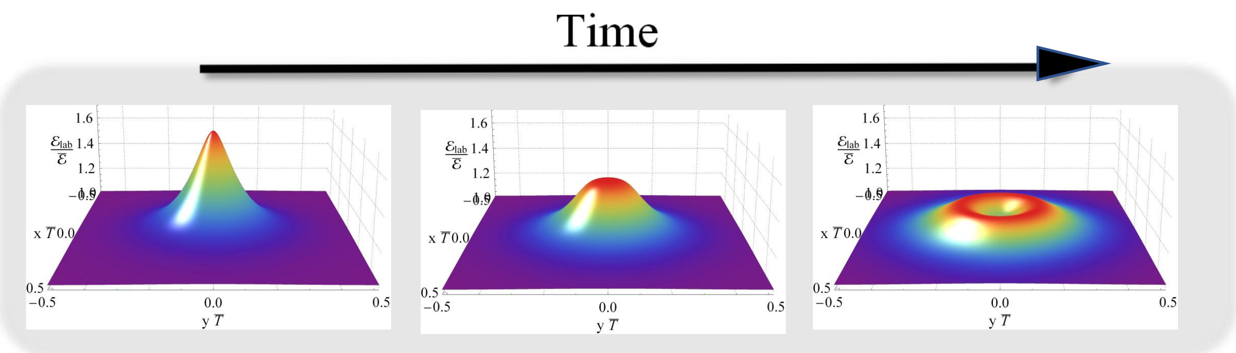
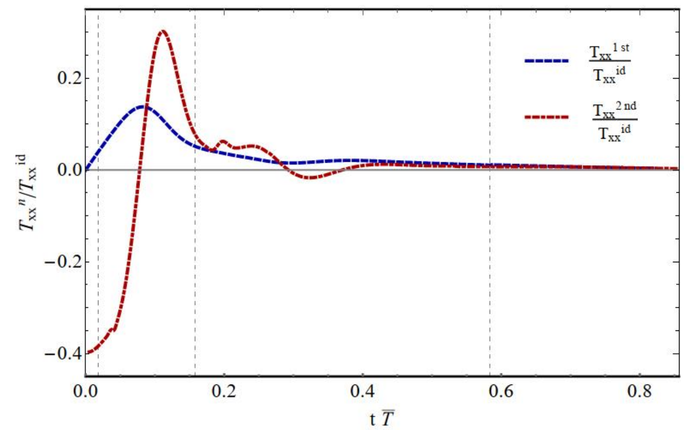
Evolutions: holography vs hydrodynamics





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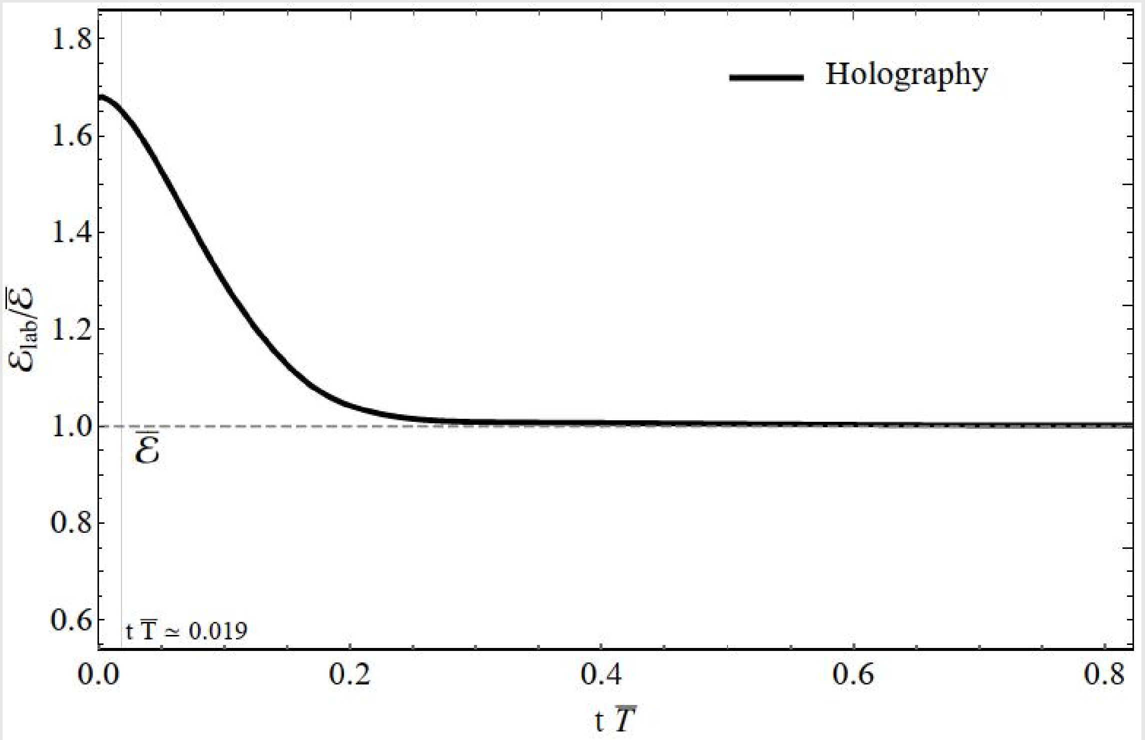

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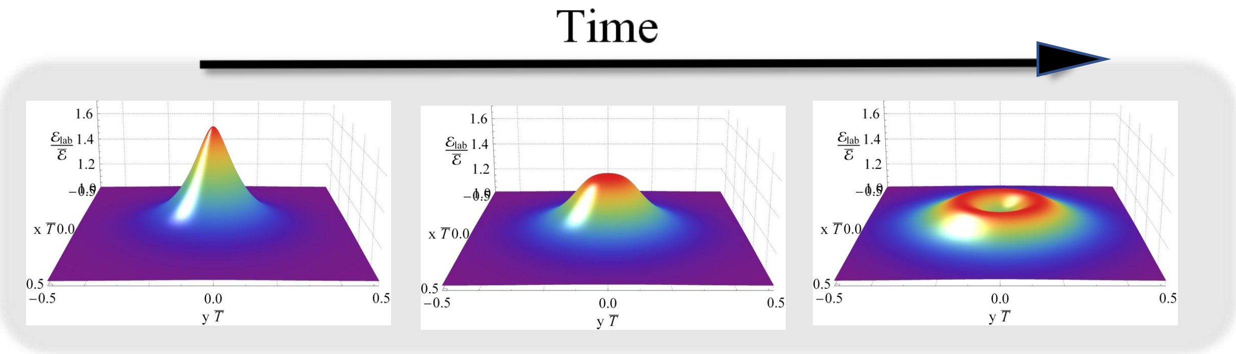
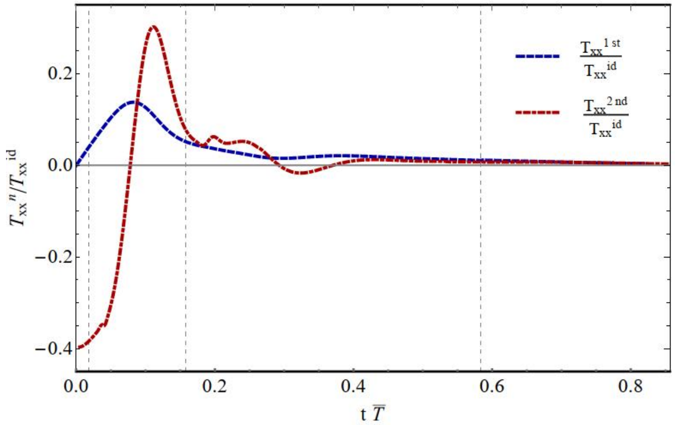
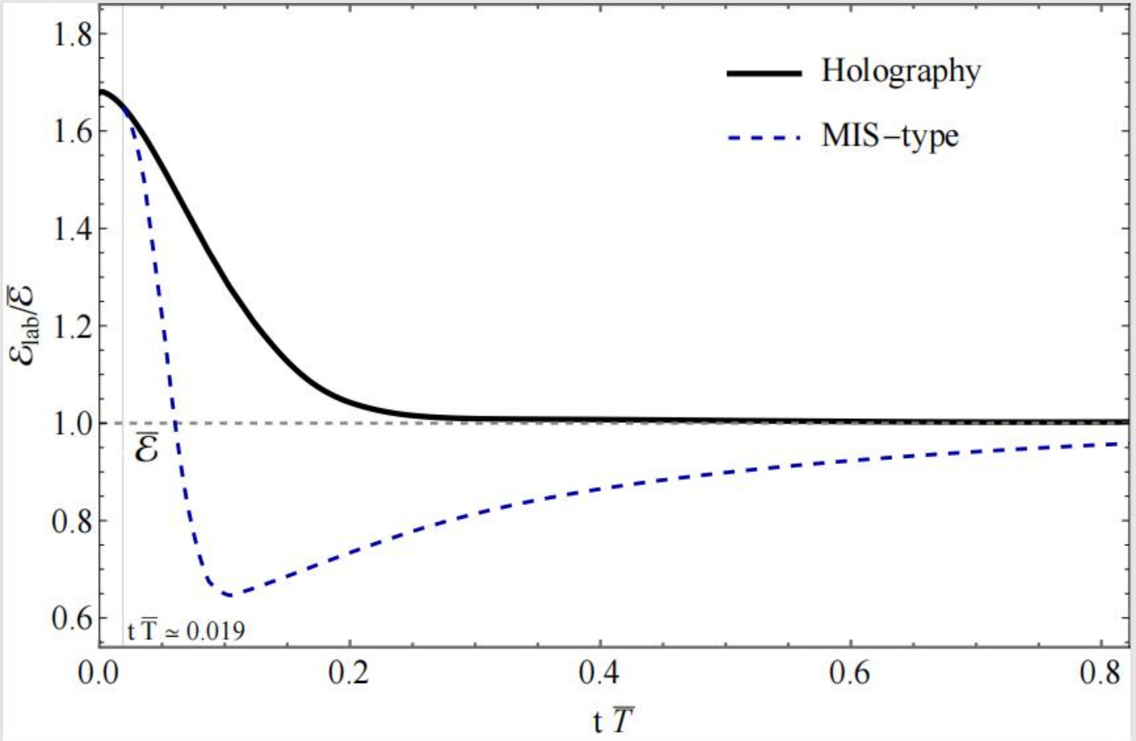
Evolutions: holography vs hydrodynamics



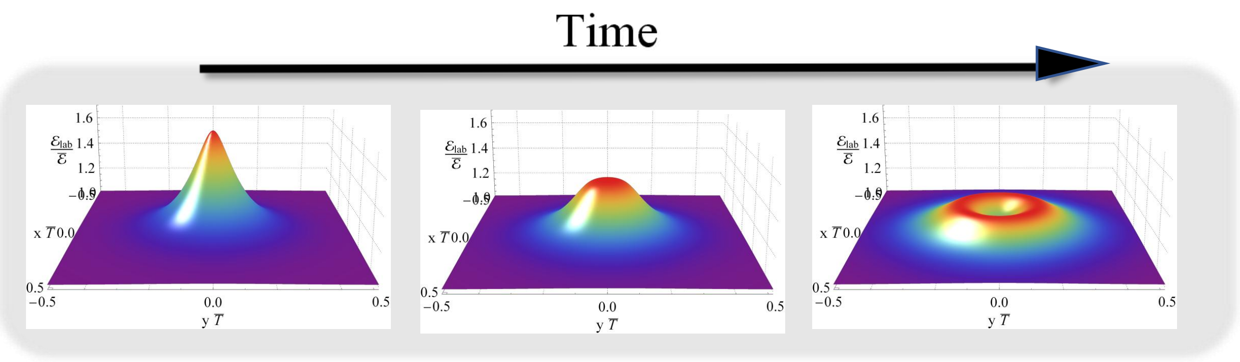


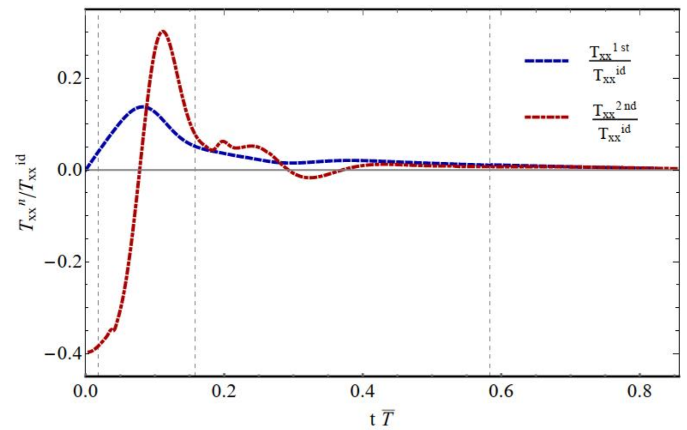
Energy at the center of the domain:



Bantilan, Bea, Figueras ‘22

Evolutions: holography vs hydrodynamics





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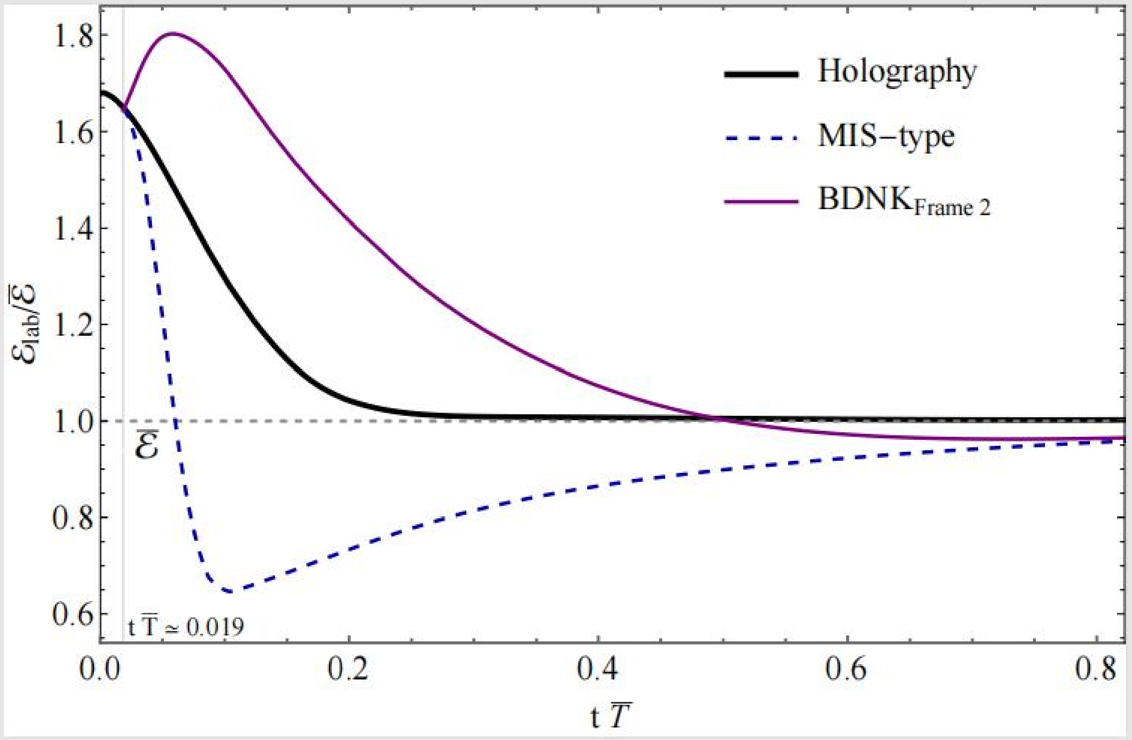
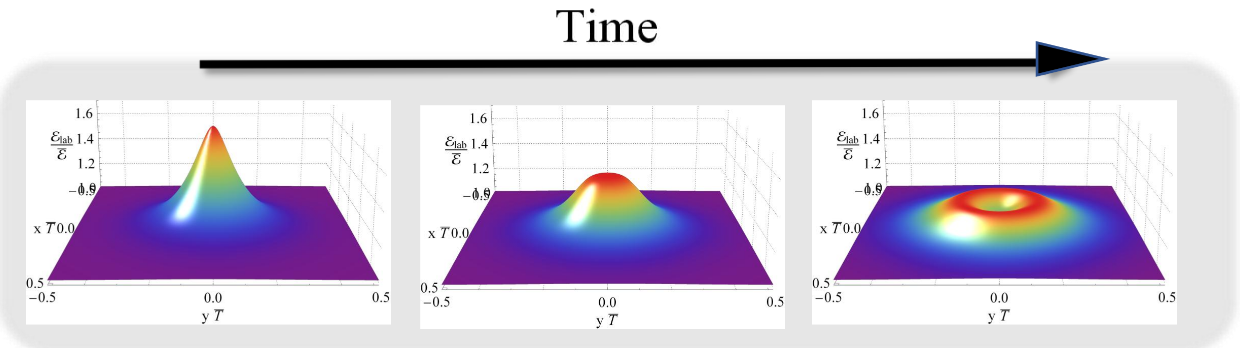
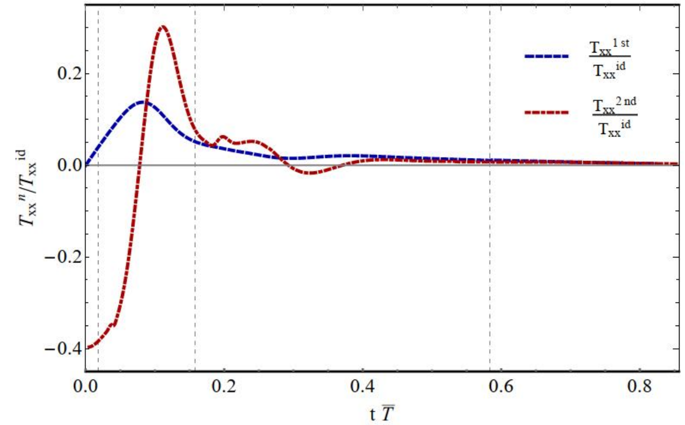


Bantilan, Bea, Figueras ‘22

Evolutions: holography vs hydrodynamics





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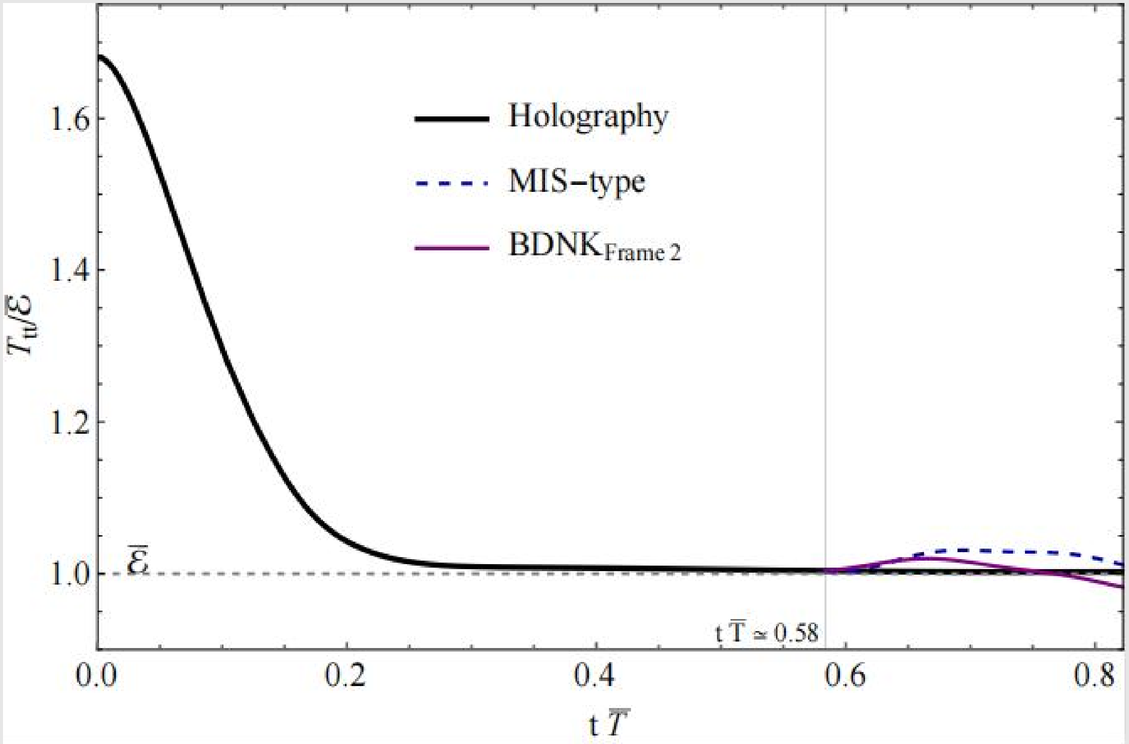

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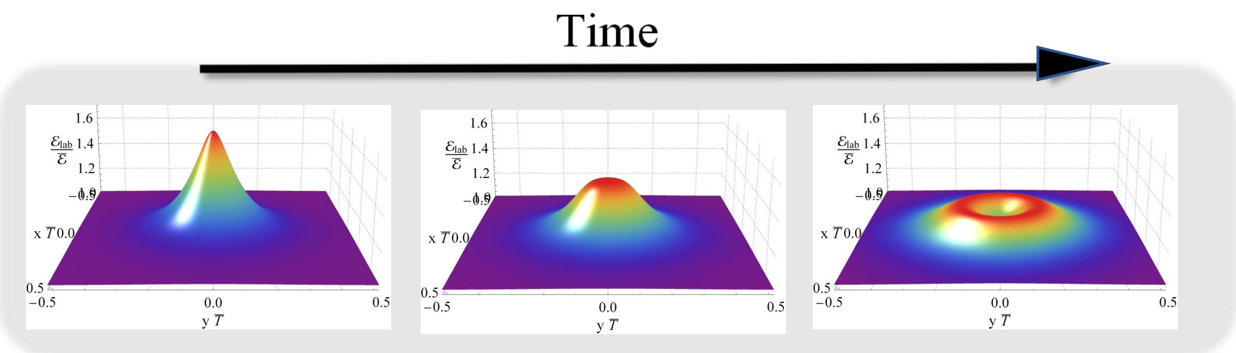
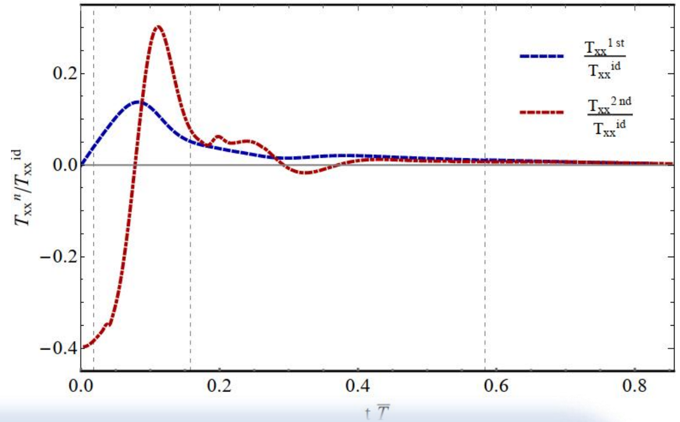
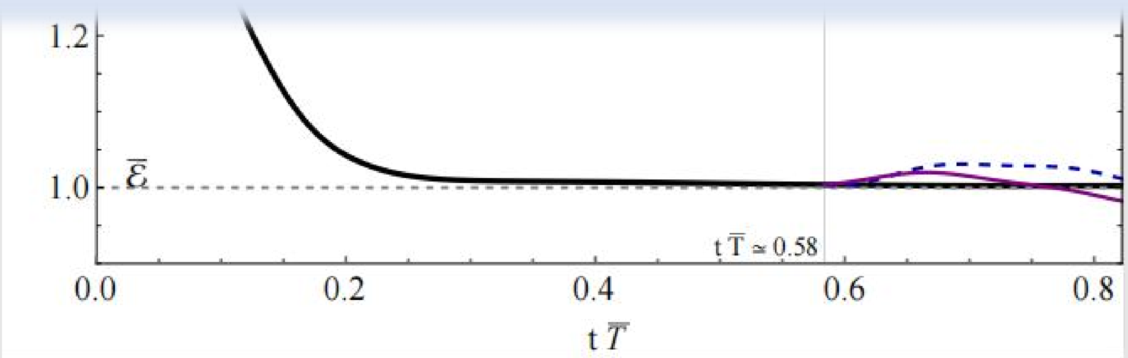
Evolutions: holography vs hydrodynamics





Main conclusions:

- Gradients dilute with time ⟶ hydro evolutions provide a better description at late times.
- Evolutions in BDNK: provides a physically sensible description of the system in the hydro regime, and compatible with MIS.



Bantilan, Bea, Figueras '22

Future directions

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→ BDNK might be the only way to include viscosity in neutron star mergers.

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→ My research program aims to develop the numerical infrastructure required to implement BDNK into neutron star merger simulations.

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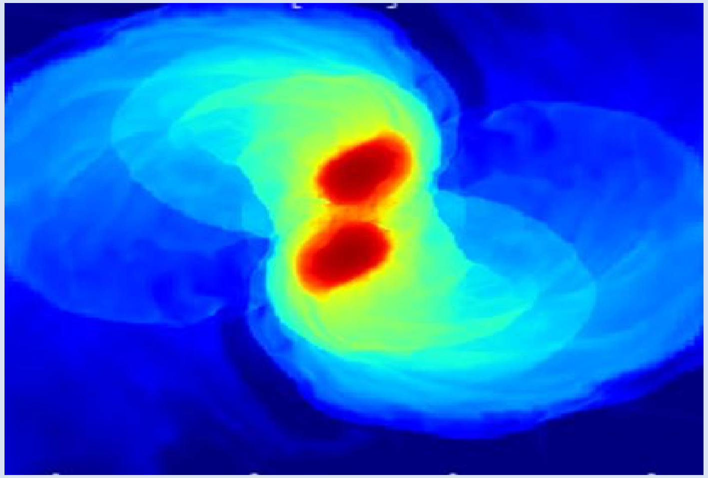
Goal:
First viscous neutron star merger simulation

Important step towards precision physics



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→ BDNK might be the only way to include viscosity in neutron star mergers.

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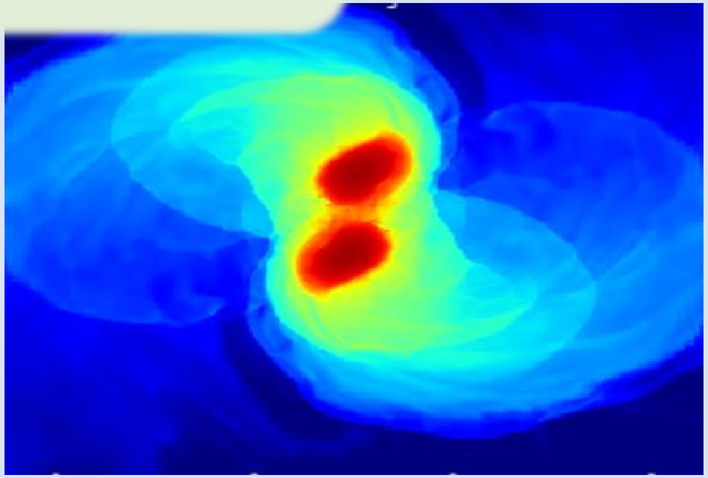
Thank you!

Goal:
First viscous neutron star merger simulation

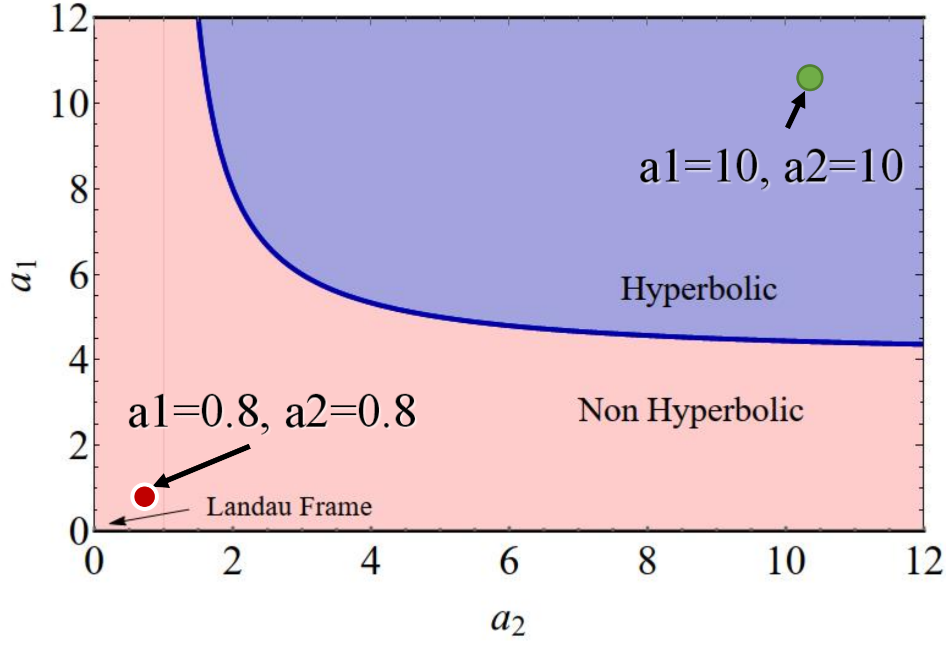
Important step towards precision physics

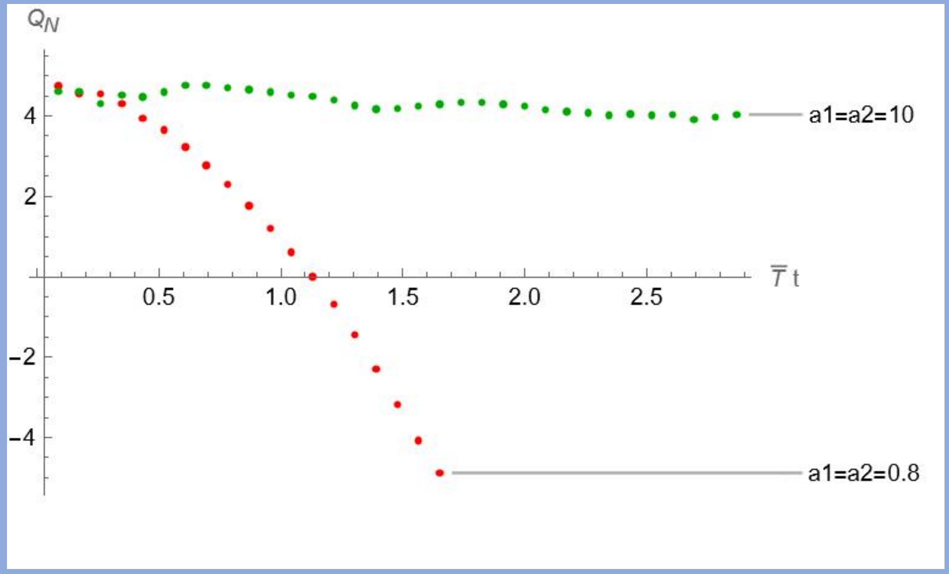


BDNK: Acausal region

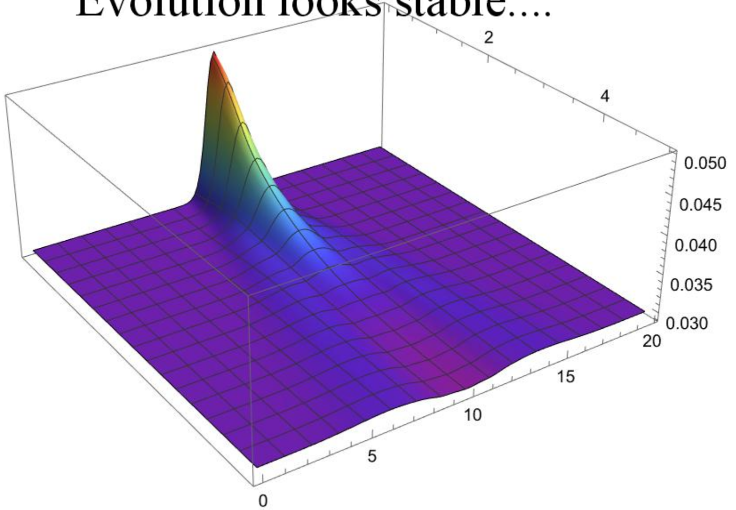
- BDNK eqs. are hyperbolic if:

$$a_2 > 1\,, \qquad a_1 > \frac{4\,a_2}{a_2 - 1}\,.$$





Evolution looks stable....

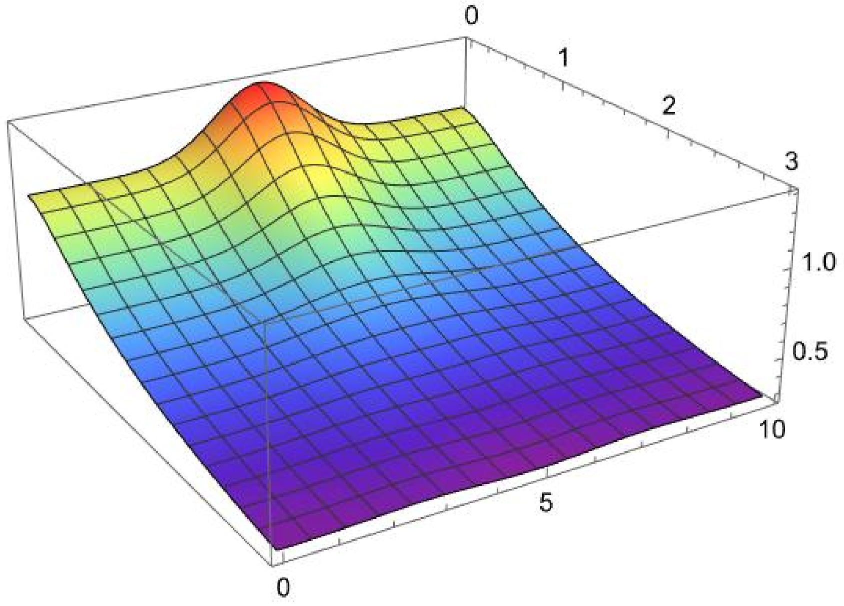


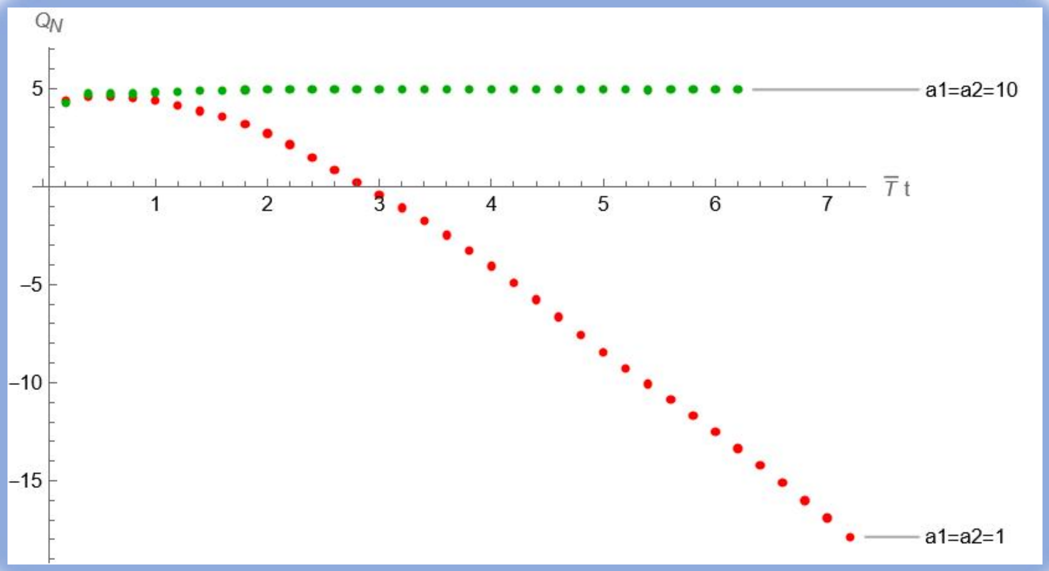
Backup slides: BDNK: 3+1, boost invariant

- 3+1 theory
- Boost invariant, 2+1 dynamics

⟶ Similar to hydro codes used to describe the QGP

Evolution looks stable....





⟶ Similar conclusions!

Hydro equations

- Conformal theory in 2+1 dimensions

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \;\Rightarrow\; \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

- First order hydro: **Landau frame**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} - \eta\sigma^{\mu\nu} \;\Rightarrow\; \nabla_\mu T^{\mu\nu} = 0$$

Not hyperbolic…

- Usual fix: **MIS-type**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \eta\tau_\pi \left(\dot\sigma^{<\mu\nu>} + \frac{3}{2}\sigma^{\mu\nu}\nabla\cdot u \right)$$

$\Downarrow$

New variable

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left(\dot\Pi^{<\mu\nu>} + \frac{3}{2}\Pi^{\mu\nu}\nabla u \right)$$

New equation

$\Rightarrow$

Hyperbolic!!

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left(\dot\Pi^{<\mu\nu>} + \frac{3}{2}\Pi^{\mu\nu}\nabla u \right)$$

Backup slides: Hydro equations

- **Conformal theory**

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\,\Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

- First order hydro: **Landau frame**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\,\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

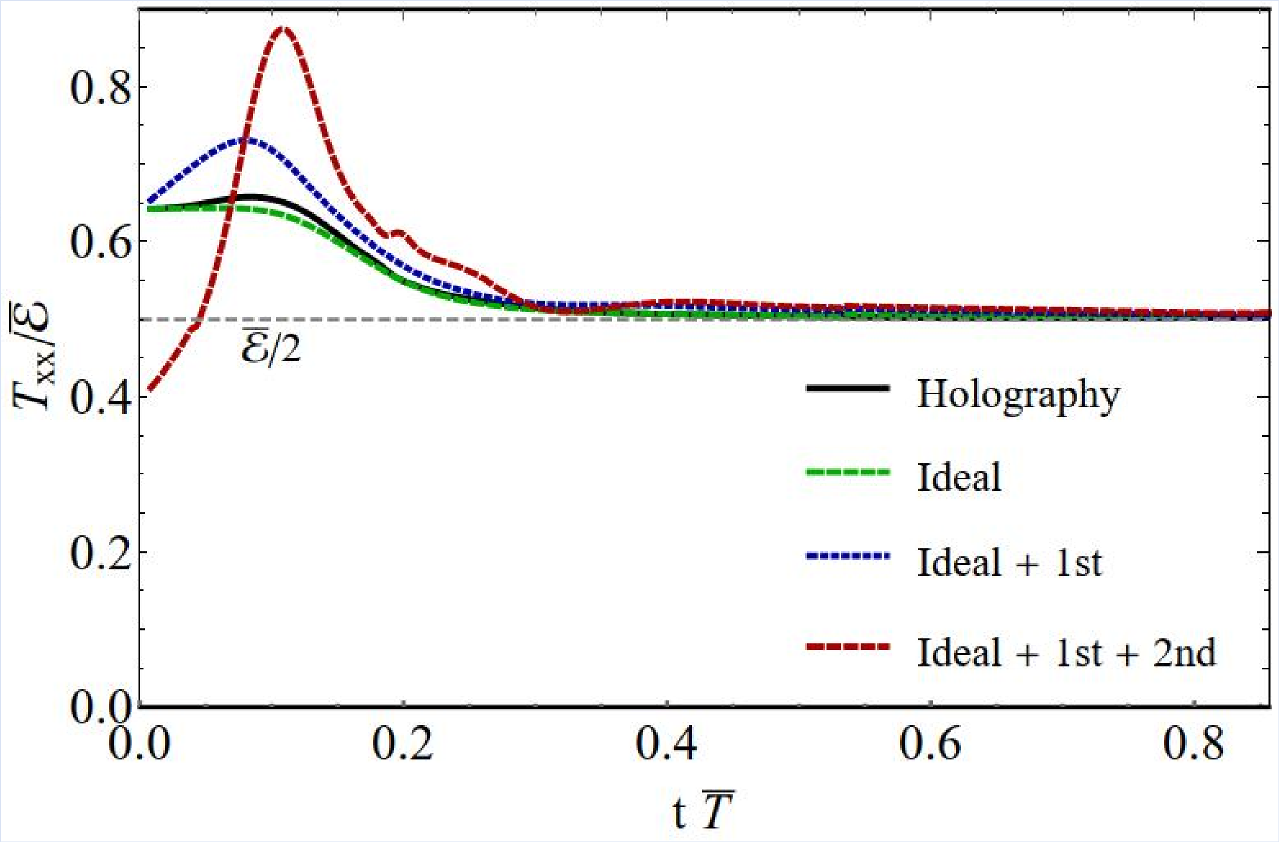
Not hyperbolic…

- First order hydro: **general frame**

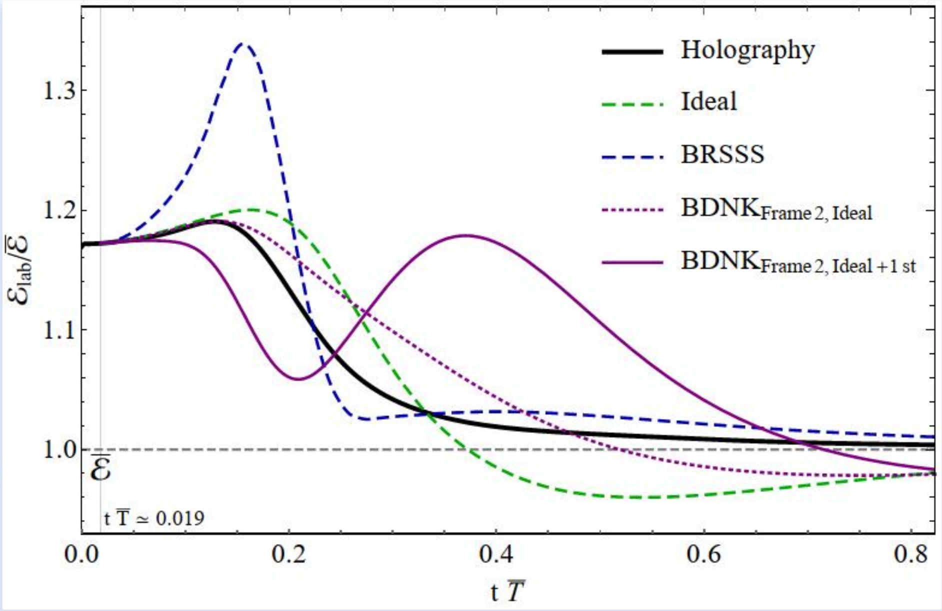
$$T^{\mu\nu} = \left[\epsilon + \boxed{2\,a_2\eta\left(\frac{2}{3}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u\right)}\right]\left(u^\mu\, u^\nu + \frac{\Delta^{\mu\nu}}{2}\right)$$
$$\boxed{+\, a_1\eta\left[\left(\dot u^\mu + \frac{1}{3}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right)u^\nu + (\mu \leftrightarrow \nu)\right]}$$
$$\boxed{-\,\eta\,\sigma^{\mu\nu}}$$

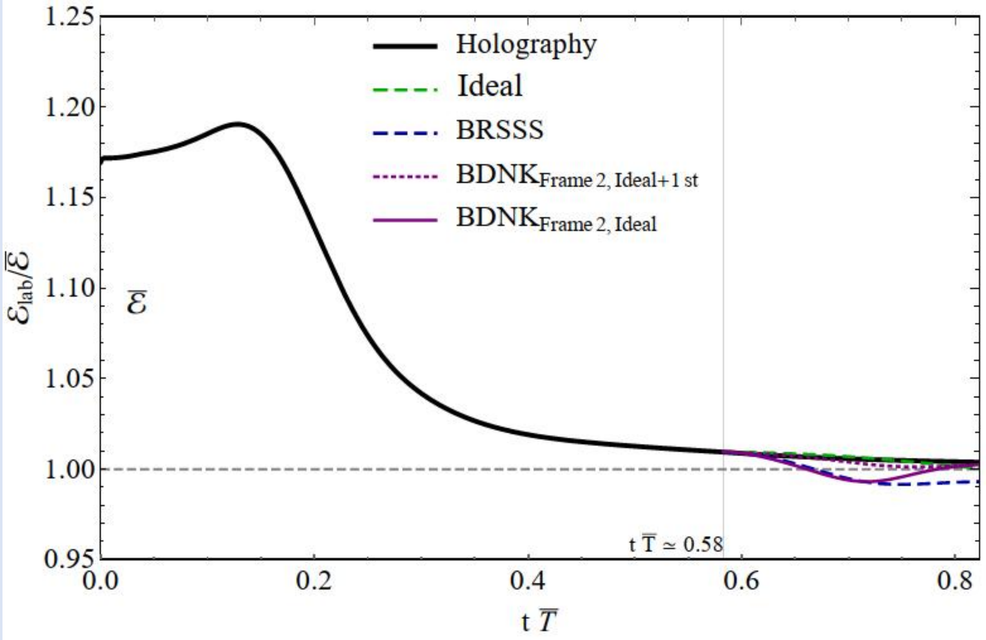
→ Include all 1st order terms compatible with Poincare symmetry

Backup slides: Constitutive relations

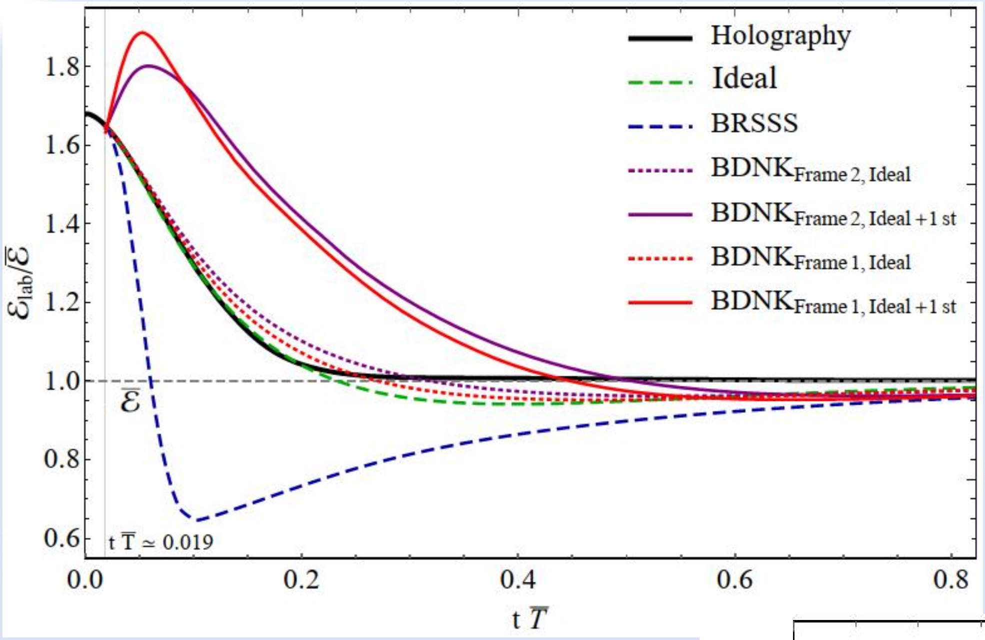


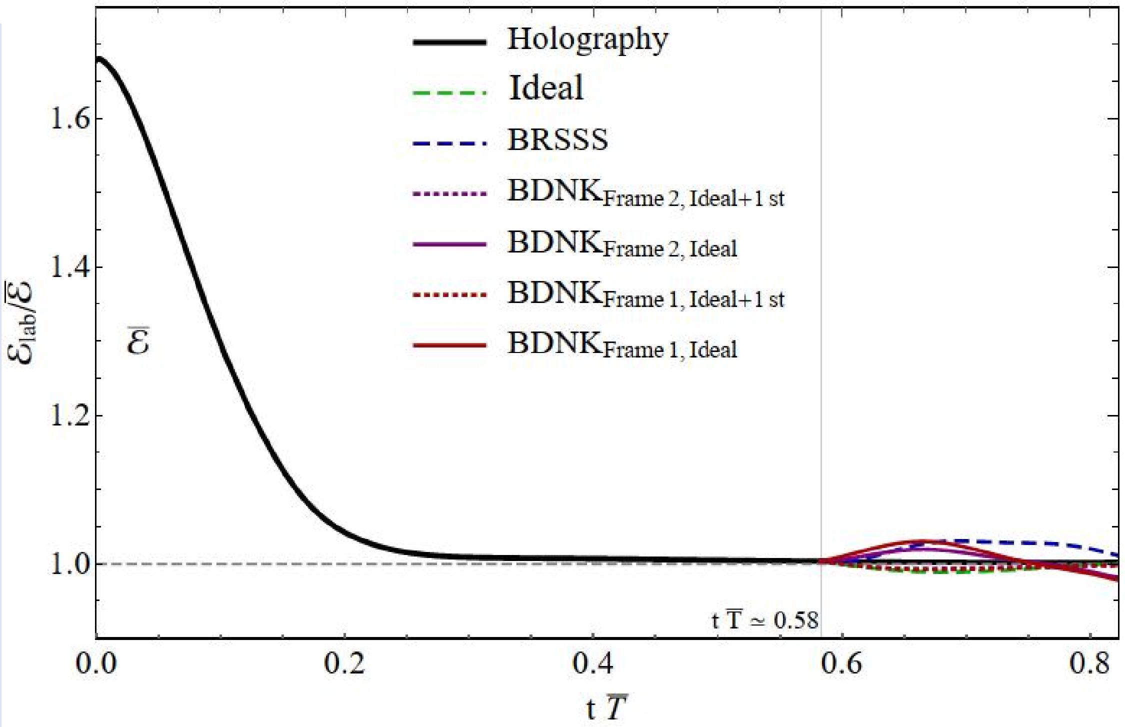
Backup slides: Off center





Backup slides: Frames 1 and 2





Hydro equations

- Conformal theory in 2+1 dimensions

$$\frac{2}{3}\frac{\dot\epsilon}{\epsilon} + \nabla_\lambda u^\lambda = 0\,, \qquad \dot u^\mu + \frac{1}{3}\frac{\nabla_\perp^\mu \epsilon}{\epsilon} = 0\,.$$

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\,\Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

- First order hydro: **Landau frame**

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Not hyperbolic…

- First order hydro: **general frame**

$$T^{\mu\nu} = \left[\epsilon + 2\,a_2\eta\left(\frac{2}{3}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u\right)\right]\left(u^\mu\, u^\nu + \frac{\Delta^{\mu\nu}}{2}\right) + a_1\eta\left[\left(\dot u^\mu + \frac{1}{3}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right)u^\nu + (\mu\leftrightarrow\nu)\right] - \eta\,\sigma^{\mu\nu}$$

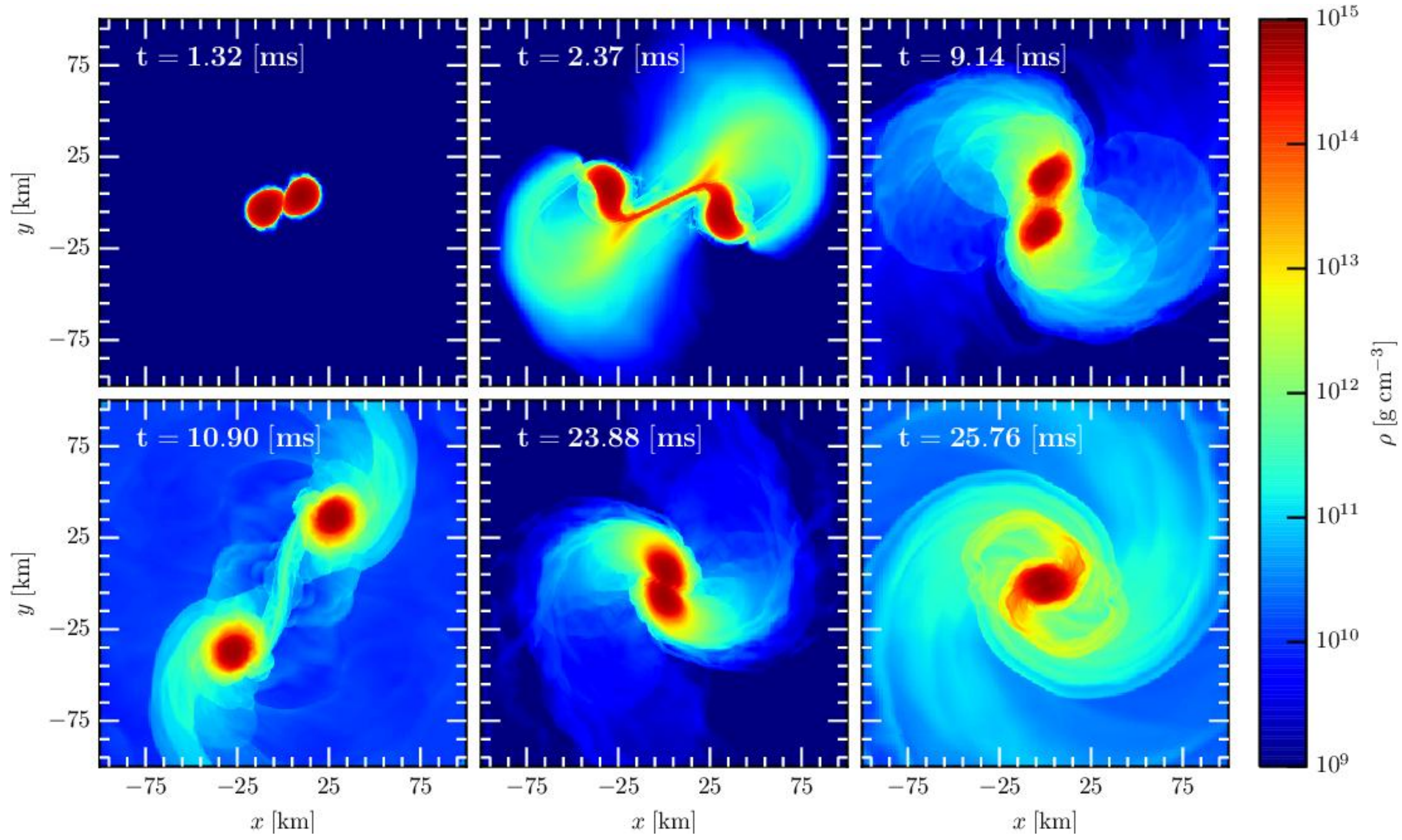
$$a_2 > 1\,, \qquad a_1 > \frac{4\,a_2}{a_2 - 1}\,.$$

$$\nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

BDNK equations

Viscosity can be relevant



BDNK: Integration method

Recall: BDNK eqs. are 2nd order in time

- Reduce to 1st order. RK4 unstable → Unstable…

- Implicit integration method → STABLE!!

- Explicit integration method RKNG34 → STABLE and FASTER!!

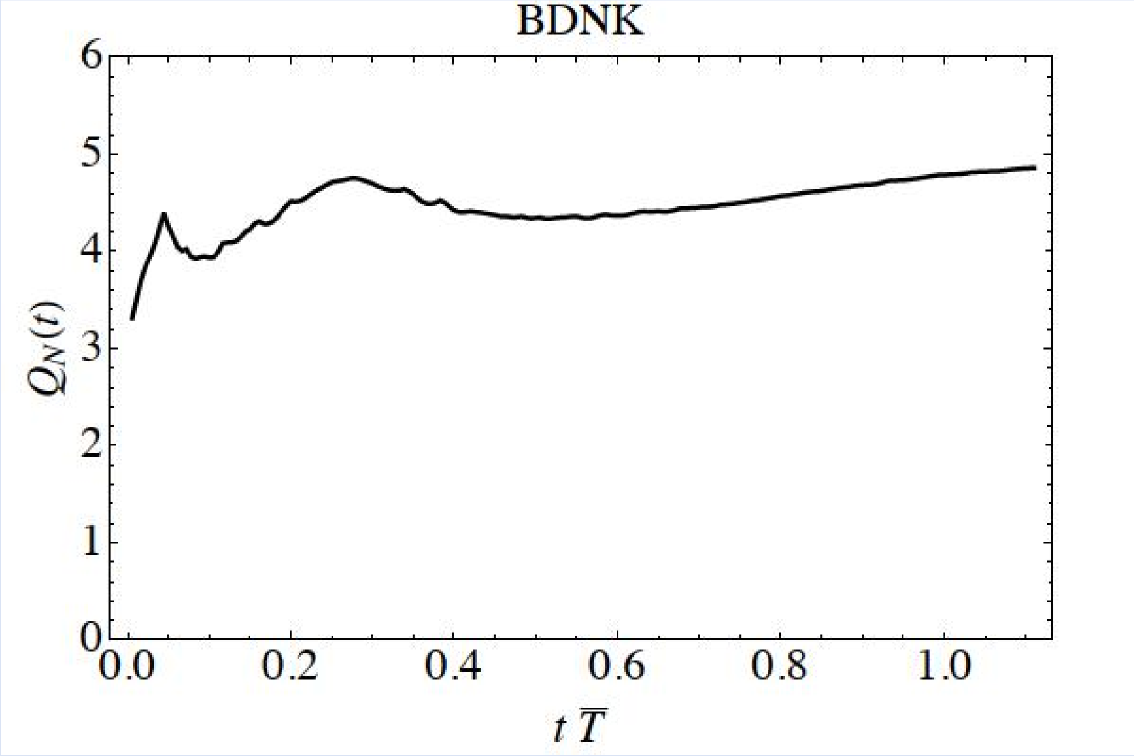
→ We use RKNG34 in our simulations

BDNK: Convergence tests

→ Convergence test for BDNK

→ Performed for evolutions of Gaussian profiles

Quantity capturing the convergence order



BDNK: We are not the first ones

- Main differences:

A numerical exploration of first-order relativistic hydrodynamics

Alex Pandya* and Frans Pretorius†
Department of Physics, Princeton University, Princeton, New Jersey 08544, USA.
(Dated: April 5, 2021)

We present the first numerical solutions of the causal, stable relativistic Navier-Stokes equations as formulated by Bemfica, Disconzi, Noronha, and Kovtun (BDNK). For this initial investigation we restrict to plane-symmetric configurations of a conformal fluid in Minkowski spacetime. We consider evolution of three classes of initial data: a smooth (initially) stationary concentration of energy, a standard shock tube setup, and a smooth shockwave setup. We compare these solutions to those obtained with a code based on the Müller-Israel-Stewart (MIS) formalism, variants of which are the common tools used today to model relativistic, viscous fluids. We find that for the two smooth initial data cases, simple finite difference methods are adequate to obtain stable, convergent solutions to the BDNK equations. For low viscosity, the MIS and BDNK evolutions show good agreement. At high viscosity the solutions begin to differ in regions with large gradients, and

Evolutions in first-order viscous hydrodynamics

Hans Bantilan, Yago Bea, and Pau Figueras
School of Mathematical Sciences, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom

We perform real-time evolutions using the first-order viscous relativistic hydrodynamic equations formulated by Bemfica, Disconzi, Noronha and Kovtun (BDNK) in three-dimensional conformal theories. For comparison, we also perform evolutions using the ideal and viscous BRSSS equations of hydrodynamics. Moreover, motivated by the physics of the quark-gluon plasma, we use holography to obtain the microscopic dynamical evolution of a system relaxing to equilibrium in a strongly-coupled field theory that we use to study the applicability of hydrodynamics.

Introduction. Dynamical evolutions of the rel-
in noticing that if we change from these frames

- 1+1 dynamics
- 3+1 theory
- Motivated by neutron star mergers
- Integration method: conservative methods (HRSC)

- 2+1 dynamics
- 2+1 theory
- Motivated by heavy-ion collisions
- Integration method: Explicit RKNG34
- Microscopic solution

⟶ Our works only partially overlap, and they are complementary

⟶ With our work and Pretorius paper, we are paving the way to the implementation in relevant physical systems.

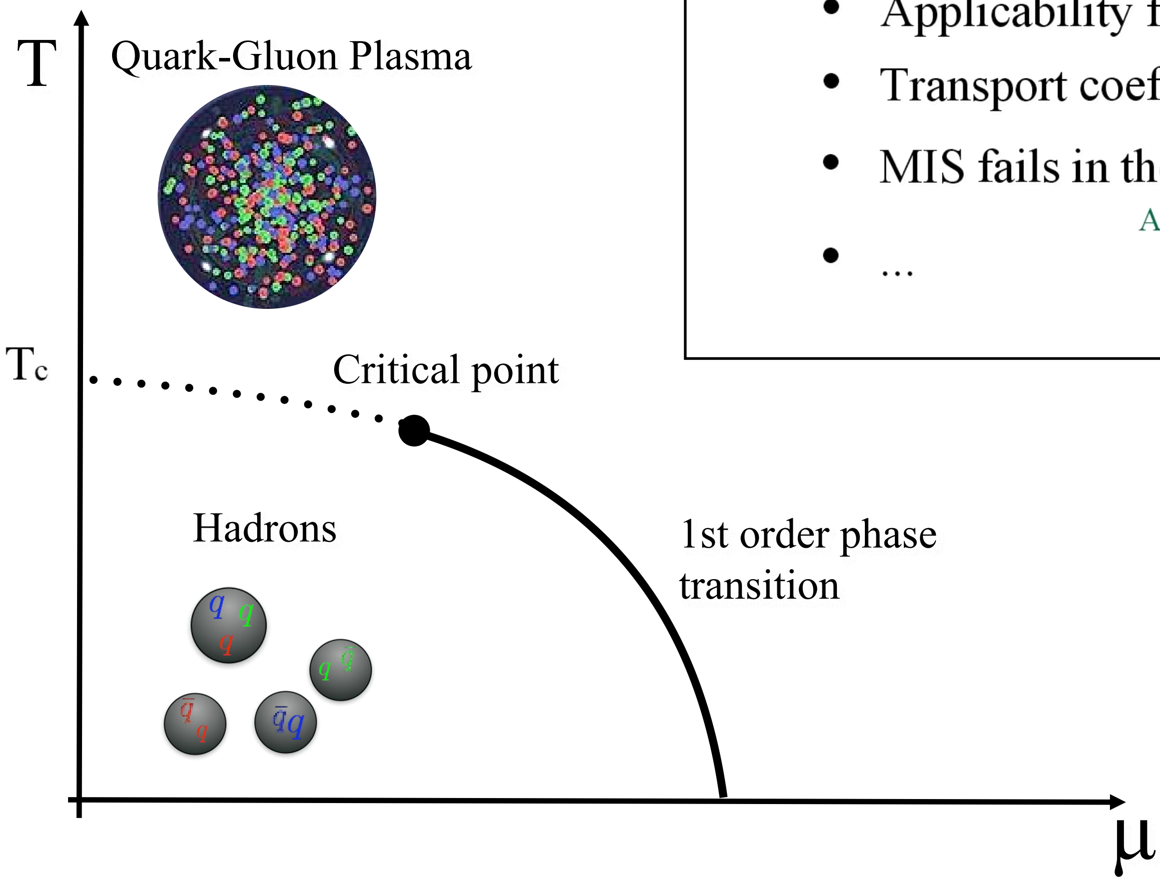
QCD & Holography

<u>What have we learned from holography so far?</u>

Chesler, Yaffe, Casalderrey, Mateos, Heller, van der Schee, …

- Early hydrodynamization times
- Applicability with large gradients
- Applicability for small systems
- Transport coefficients
- MIS fails in the presence of a phase transition

  Attems, Bea, Mateos, Casalderrey, Triana, Zilhao ’19, ‘20
- …



Holography: Our model

- CFT on Minkowski in 2+1 dim
- Decoupled sector of the stress tensor $T^{\mu\nu}$.

Real-time quantum dynamics

$\Updownarrow$ Holography

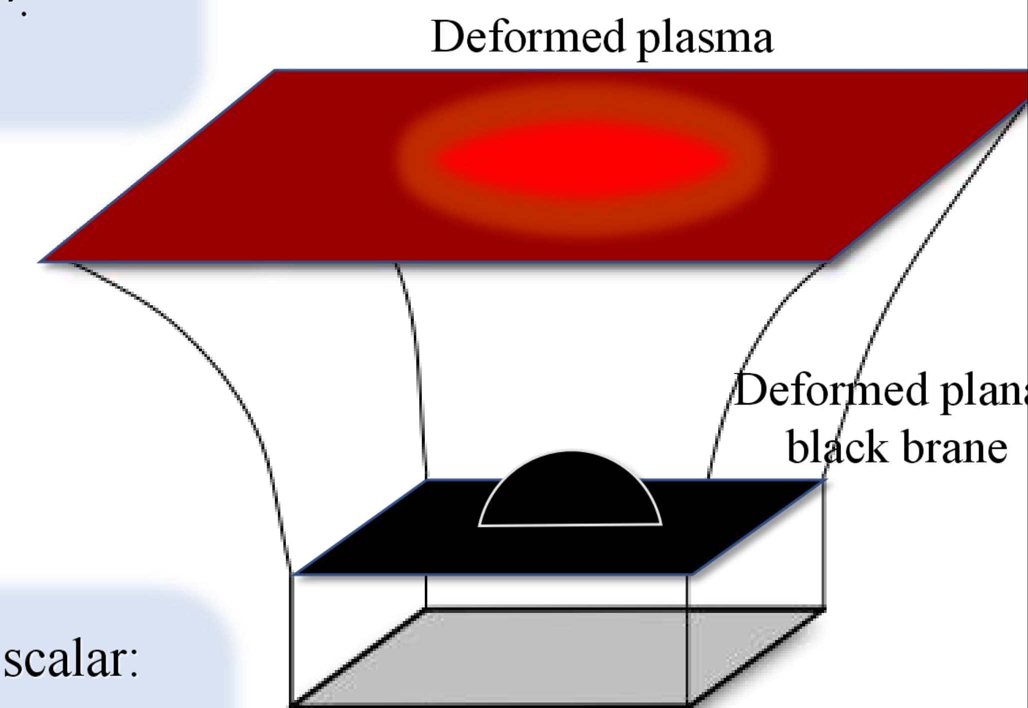
Dynamical classical gravity

Numerical Relativity

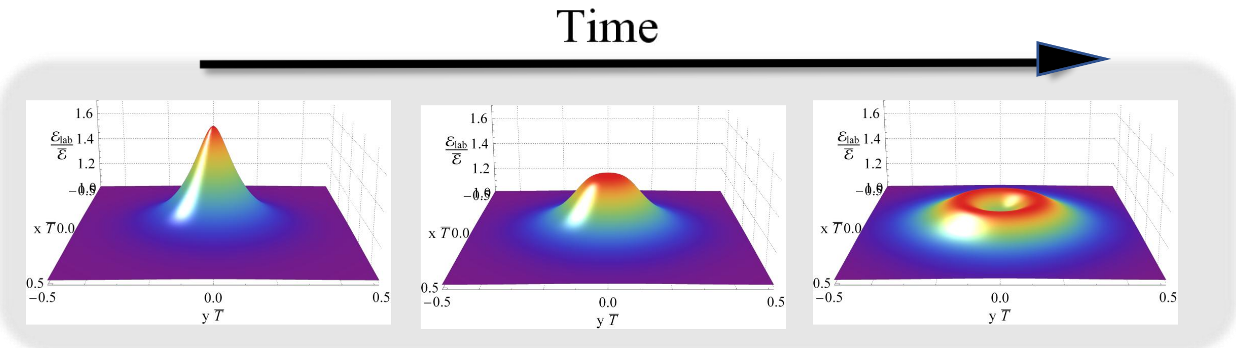
- Gravity with Λ in 3+1 dim plus massless scalar:

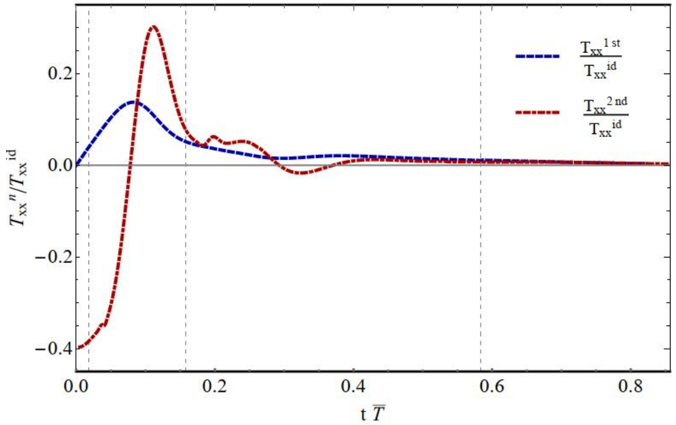
$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda + (\partial\phi)^2)$$

- We focus on the Poincare patch of AdS.



Evolutions: holography vs hydrodynamics





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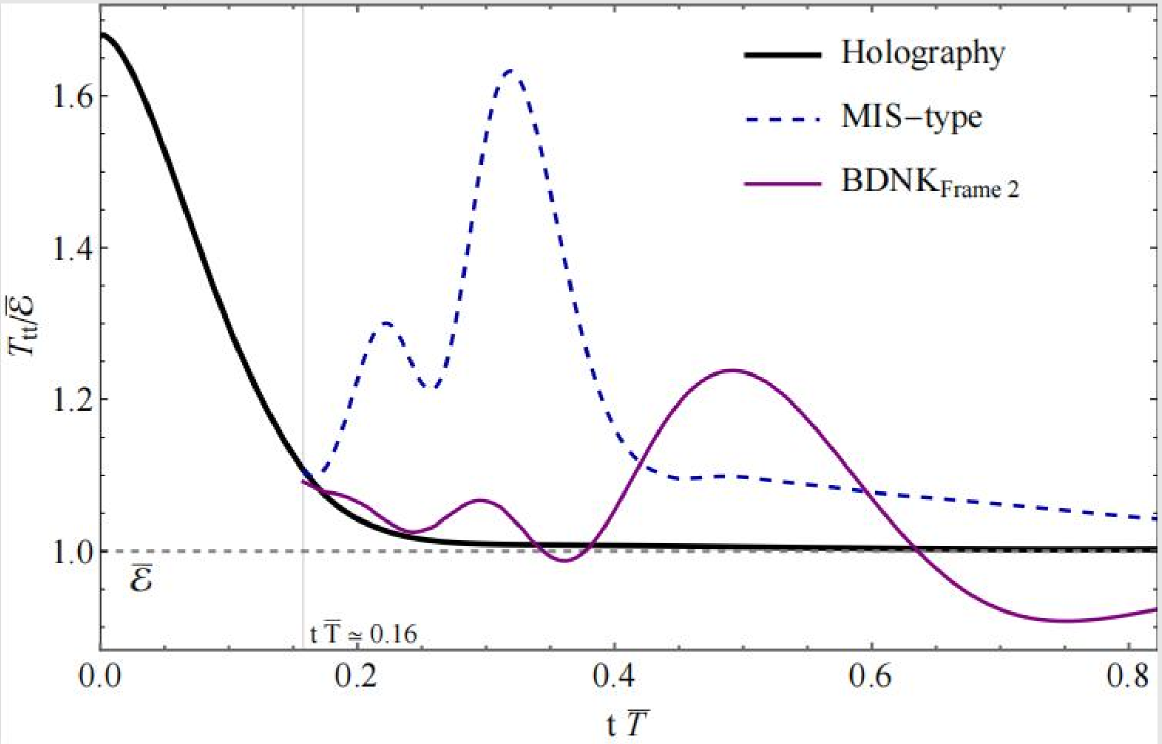
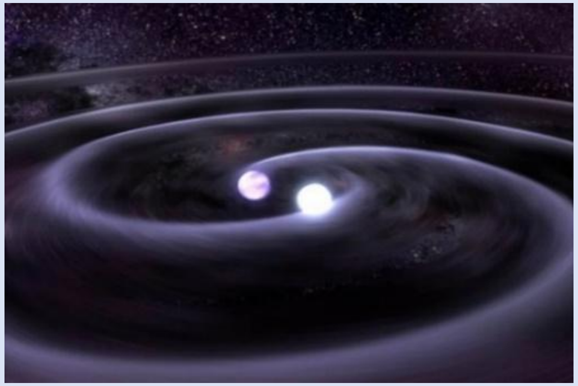


Bantilan, Bea, Figueras '22

Motivation: astrophysical systems

Neutron star mergers



State of the art	• Ideal hydrodynamics
Beyond state of the art	• **Viscous** hydrodynamics

Shibata et al'20
Chabanov, Rezzolla, Rischke '21

→ More realistic scenarios, closer to astrophysical systems.

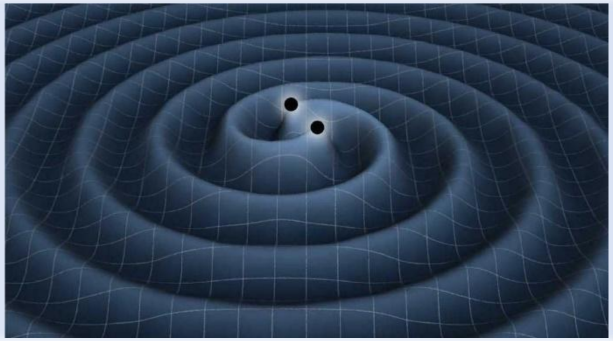
→ New era of precision gravitational waves (LISA, …)

Motivation: astrophysical systems

Motivation: astrophysical systems

Black hole mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \Rightarrow R_{\mu\nu} = 0$$



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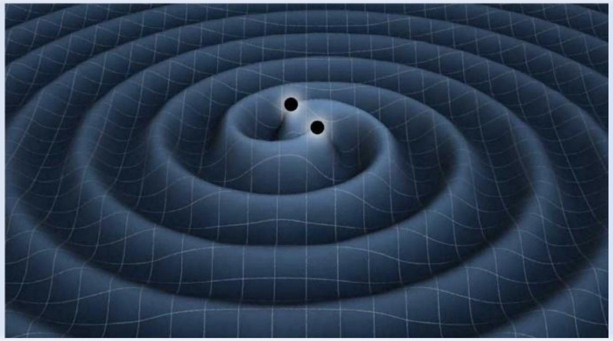
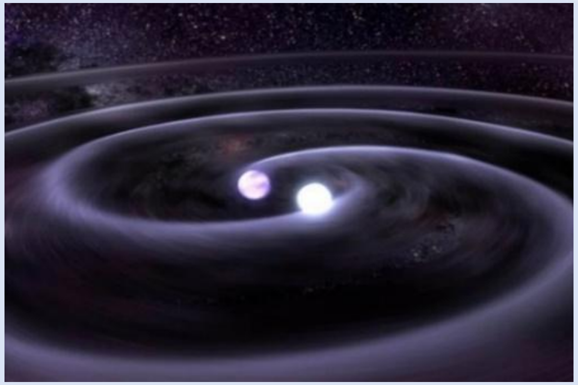


Neutron star mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

Matter must be specified



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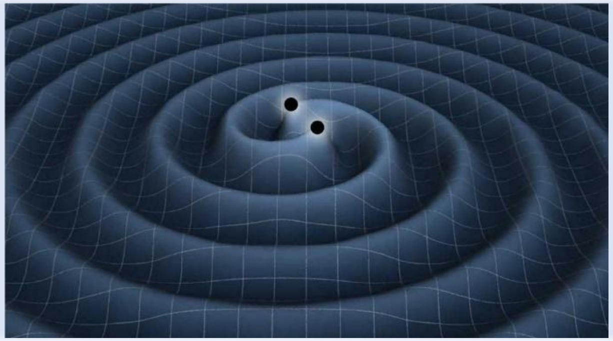

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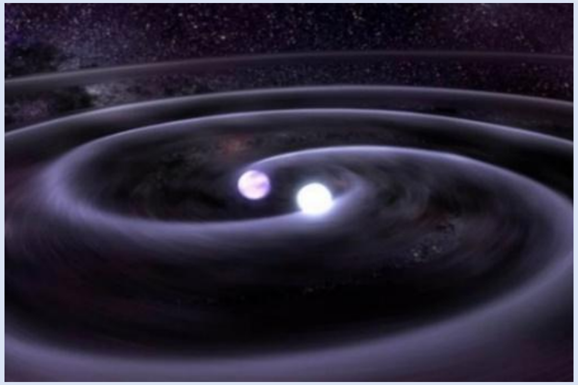


Ideally: solve gravity coupled to QCD

$\longrightarrow$ At the moment not feasible

BDNK vs MIS

- Why do we need another formulation of viscous hydrodynamics?

- Non linear

Nonlinea

Fábio S. Be

[1] *Escola de Ciências*

[2]

- The in ... evolution
- Constr

BDNK $a_2 > 1\,, \quad a_1 > \frac{4\,a_2}{a_2 - 1}\,.$

MIS

$$
\begin{aligned}
&(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0\\
&\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,\\
&\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_a + \Lambda_d) \geq 0, \quad a \neq d,\\
&\varepsilon + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0, \quad a \neq d\\
&\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d + \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d]\\
&+\frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} + (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,\\
&\varepsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d]\\
&-\frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,
\end{aligned}
$$

Do not depend on evolved variables

Causality ensured all along the evolution!

→ **BDNK might be good alternative to MIS!**

Evolve BDNK with realistic heavy-ion initial data.

Work in progress...

Exploring th

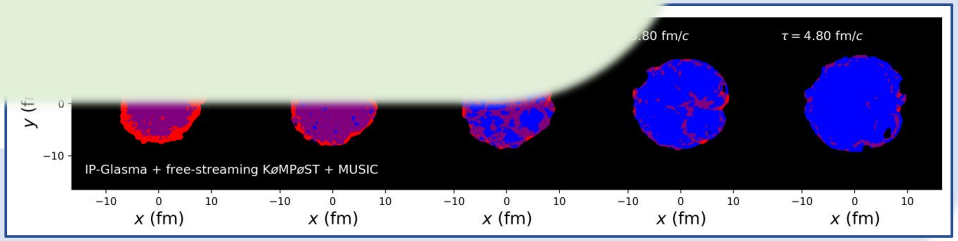
[2] *Depart*

[3] *RII*

Christopher Plu

[2] *Department of Physics, Kent State University, Kent, OH 44242, USA*

(Dated: March 31, 2021)



→ **Significant causality violations!**