The (spectral harmonic) test-field method
characterizing emergent phenomena in MRI turbulence

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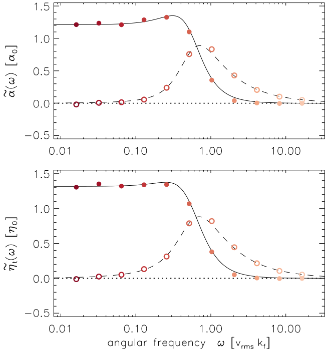

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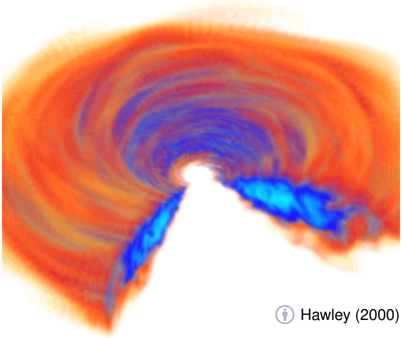
the engine: magnetorotational instability

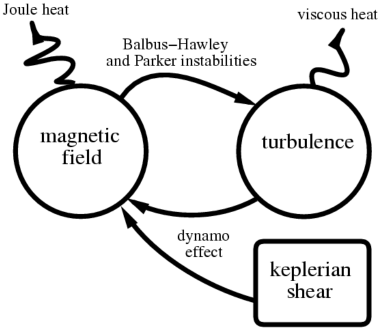


Hawley (2000)

- accretion luminosity via turbulent viscosity
- non-thermal coronal X-ray emission via B-fields
- weak magnetic fields destabilise shear flow MRI, Balbus & Hawley (1991)
- robust linear instability – problem solved ...

- ... twenty-five years later, saturation mechanism remains enigmatic
- attempts
  - ? linear theory
  - ? direct simulations
  - ? limit-cycle regime
  - ? parasitic instabilities
  - ? mean-field dynamo
  - ? transition into chaos

the key: survival of large-scale fields



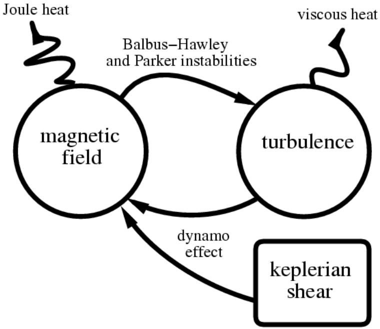
Brandenburg (1995)

- stratified shearing boxes have all ingredients for a classical strato-cyclonic dynamo
- large-scale dynamo is less likely Pm-dependent Brandenburg (2001)
- tall-enough (un-)stratified ZNF converged Davis, Stone & Pessah (2010), Shi, Stone & Huang (2016), Ryan et al. (2017), Coleman et al. (2017)
- (cyclic) dynamo already seen in unstratified ZNF case Lesur & Ogilvie (2008), Herault et al. (2013/15) Squire & Bhattacharjee (2015a/b)

- complementary route:

  study evolution of embedded poloidal flux (itself subject to turbulence)

the key: survival of large-scale fields



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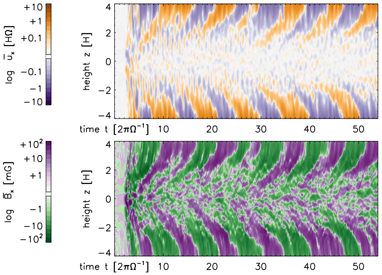
MRI dynamo vs. MHD winds

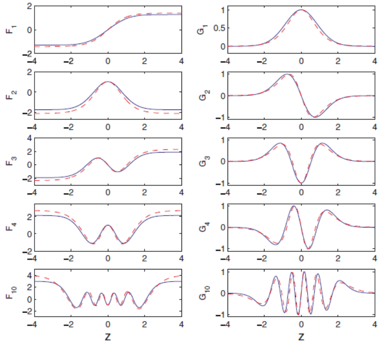
- butterfly diagram also in the velocity

$$\mathbf{V}^{\rm ch} = \mathbf{u_0}\,\mathbf{F}(\tfrac{\mathbf{z}}{\mathbf{H}})\,\mathrm{e}^{\mathbf{st}}\,[\mathbf{e_x}\ \cos\theta + \mathbf{e_y}\ \sin\theta]$$
$$\mathbf{B}^{\rm ch} = \mathbf{B_0}\,\mathbf{G}(\tfrac{\mathbf{z}}{\mathbf{H}})\,\mathrm{e}^{\mathbf{st}}\,[\mathbf{e_x}\ \sin\theta - \mathbf{e_y}\ \cos\theta]$$



→ model 'A1' from Gressel, Nelson & Turner (2011)

strong fields → low wave-number



weak fields → high wave-number

Latter, Fromang & Gressel (2010)

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topological constraints

- Dynamical quenching (new notation: $a = A'$, $b = B'$, $\ldots$)
  - non-linear effects in the EMF

$$\partial_t \bar{\mathcal{E}} = \overline{u \times (\partial_t b)} + \overline{(\partial_t u) \times b}$$

$$\rightarrow \qquad \alpha = \alpha_{\rm K} + \alpha_{\rm M} = -\tfrac{1}{3}\tau_{\rm K} \langle \omega \cdot u \rangle + \tfrac{1}{3}\tau_{\rm M} \langle j \cdot b \rangle / \rho$$

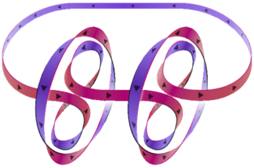
  - magnetic helicity evolution

$$\partial_t \langle \bar{A} \cdot \bar{B} \rangle = +2 \langle \bar{\mathcal{E}} \cdot \bar{B} \rangle - 2\eta \langle \bar{J} \cdot \bar{B} \rangle$$

$$\partial_t \langle a \cdot b \rangle = -2 \langle \bar{\mathcal{E}} \cdot \bar{B} \rangle - 2\eta \langle j \cdot b \rangle$$

  - time evolution for effective α effect

$$\partial_t \alpha \; = \; -2\eta_{\rm t}\, k_{\rm f}^2 \left(\frac{\alpha \langle \bar{B}^2 \rangle - \eta_{\rm t} \langle \bar{J} \cdot \bar{B} \rangle + \text{fluxes}}{B_{\rm eq}^2} + \frac{\alpha - \alpha_{\rm K}}{\eta_{\rm t}/\eta} \right)$$



Blackman (2014)

using $\alpha_{\rm K} = \text{const.}$, $\langle \bar{\mathcal{E}} \cdot \bar{B} \rangle = \langle \alpha \bar{B} \cdot \bar{B} \rangle - \langle \eta_{\rm t} \bar{J} \cdot \bar{B} \rangle$ and $\langle a \cdot b \rangle \simeq k_{\rm f}^{-2} \langle j \cdot b \rangle$

- Blackman & Field (2000) ▪ Vishniac & Cho (2001) ▪ Blackman & Brandenburg (2002) ▪ Vishniac & Shapovalov (2014) ▪ Squire & Bhattacharjee (2015a/b)

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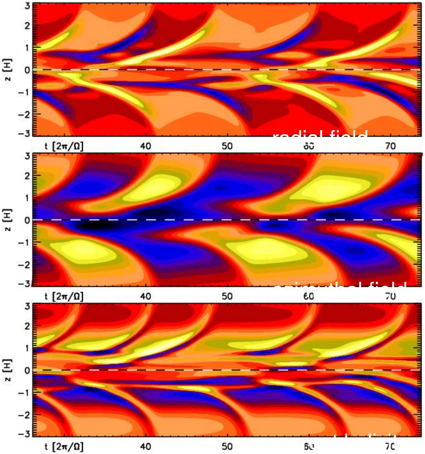
dynamically quenched mean-field model



- 1D “$\alpha_{\rm M}$” toy model / direct MHD simulation

Gressel (2010), MNRAS 405

- reproduces (at least decently) qualitative features:
  - asymmetry in B_R and B_ϕ
  - intermittent parity, chaotic features (Rm dependent)
  - frequency doubling in helicity (caused by phase shift)
- quantitative agreement difficult due to sensitive parameter dependencies
- note that MRI dynamo always appears quenched / dominated by fluctuations

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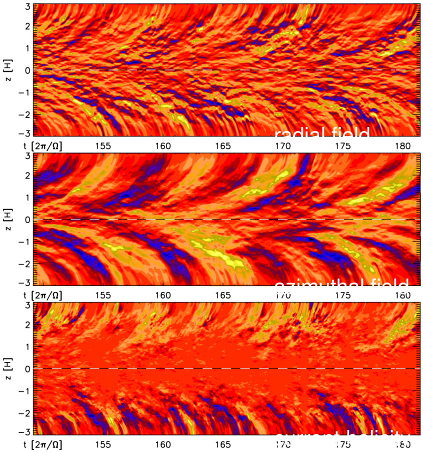

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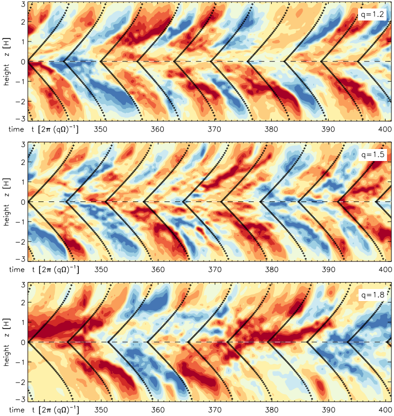
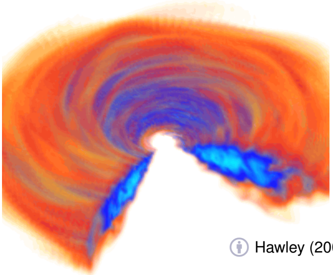
1D "α_M" toy model / direct MHD simulation

For the remainder of the talk, we will investigate the hypothesis that the coherent fields that are appearing in accretion disk turbulence are a consequence of a classical ***strato-cyclonic*** *mean-field dynamo.*

Gressel & Pessah (2015)





Hawley (2000)

emergent phenomena

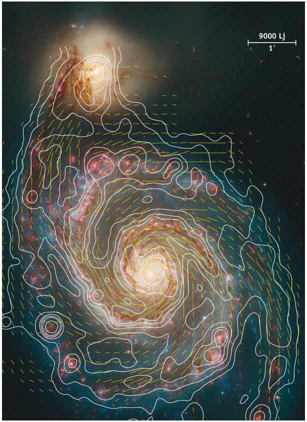
- The induction term:
  - $\nabla\times(\mathbf{V}\times\mathbf{B}) \equiv -\mathbf{B}(\nabla\cdot\mathbf{V})+(\mathbf{B}\cdot\nabla)\mathbf{V}-(\mathbf{V}\cdot\nabla)\mathbf{B}$

- Mean-field approach:
  - split into mean + fluctuation

    $\mathbf{V}=\overline{\mathbf{V}}+\mathbf{v}'$ and $\mathbf{B}=\overline{\mathbf{B}}+\mathbf{b}'$
  - mean-field equation with $\bar{\mathcal{E}}=\overline{\mathbf{v}'\times\mathbf{b}'}$

    $$\partial_t\overline{\mathbf{B}}=\nabla\times\left(\overline{\mathbf{V}}\times\overline{\mathbf{B}}+\bar{\mathcal{E}}-\eta_{\rm m}\nabla\times\overline{\mathbf{B}}\right)$$

- Parametrise small-scale effects $\bar{\mathcal{E}}$ as a functional of $\overline{\mathbf{V}}$, $\overline{\mathbf{B}}$, $\overline{f(\mathbf{v}')}$
  - typically $\bar{\mathcal{E}}_i=\alpha_{ij}\bar{B}_j-\tilde{\eta}_{ij}\,\varepsilon_{jkl}\partial_k\bar{B}_l$



A. Fletcher/R. Beck, SuW/HHT, STScI/AURA

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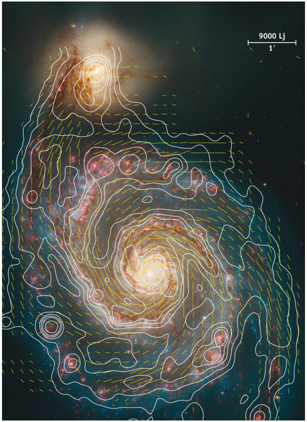

A. Fletcher/R. Beck, SuW/HHT, STScI/AURA

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- The induction term:
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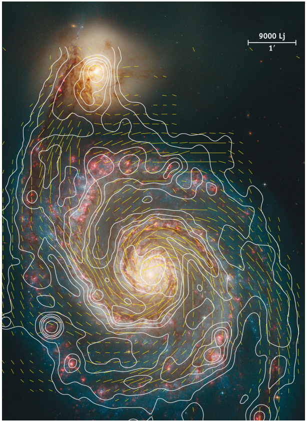

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the mean induction equation

- Splitting-up terms in the induction equation

$$\partial_t \overline{\overline{\mathbf{B}} + \mathbf{b}'} = \nabla \times \overline{\left[(\overline{\mathbf{V}} + \mathbf{v}') \times (\overline{\mathbf{B}} + \mathbf{b}')\right]} + \eta_{\rm m}\, \nabla^2\, \overline{\left[\overline{\mathbf{B}} + \mathbf{b}'\right]}$$

- Using Reynold's averaging rules
  - idempotence: $\overline{\overline{f}} = \overline{f}$
  - symmetric perturbations: $\overline{f'} = 0$
  - summation: $\overline{f + g} = \overline{f} + \overline{g}$
  - mixed product: $\overline{\overline{f} \times g'} = \overline{f} \times \overline{g'} = 0$

- Leads to the mean-field induction equation

$$\partial_t \overline{\mathbf{B}} = \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}}) + \nabla \times \bar{\mathcal{E}} + \eta_{\rm m}\, \nabla^2\, \overline{\mathbf{B}}$$

with turbulent EMF $\bar{\mathcal{E}} = \overline{\mathbf{v}' \times \mathbf{b}'}$ subsuming all small-scale effects

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motivating α/η-type closures

- Aim: find an evolution equation for the mean EMF

$$\bar{\mathcal{E}}(z,t) = \overline{\mathbf{v}'(\mathbf{z},\mathbf{t})\times\mathbf{b}'(\mathbf{z},\mathbf{t})} = \overline{\mathbf{v}'(\mathbf{z},\mathbf{t}) \times \int_{\tau=\mathbf{0}}^{\mathbf{t}} \partial_\tau \mathbf{b}'(\mathbf{z},\tau)\,\mathrm{d}\tau} \; + \; \ldots$$

- Compute magnetic field fluctuations:

$$\begin{aligned} \partial_t(\overline{\mathbf{B}}+\mathbf{b}') &= \nabla\times\left[(\overline{\mathbf{V}}+\mathbf{v}')\times(\overline{\mathbf{B}}+\mathbf{b}')\right] + \eta\,\nabla^2\,(\overline{\mathbf{B}}+\mathbf{b}') \\ -\qquad \partial_t\overline{\mathbf{B}} &= \nabla\times[\overline{\mathbf{V}}\times\overline{\mathbf{B}} + \overline{\mathbf{v}'\times\mathbf{b}'}] + \eta\,\nabla^2\,\overline{\mathbf{B}} \\ \hline \partial_t\mathbf{b}' &= \nabla\times\left[\,\overline{\mathbf{V}}\times\mathbf{b}' + \mathbf{v}'\times\mathbf{b}' - \overline{\mathbf{v}'\times\mathbf{b}'} + \mathbf{v}'\times\overline{\mathbf{B}} - \eta\nabla\times\mathbf{b}'\,\right] \end{aligned}$$

- third-order moments appear when substituted into $\bar{\mathcal{E}}(z,t)$
- ad-hoc parametrisation $\bar{\mathcal{E}}_i = \alpha_{ij}\bar{B}_j + \eta_{ijk}\partial_k\bar{B}_j$

1. context / theory
   - conceptual issues
   - dynamo theory in a nutshell

2. methods / results
   - the quasi-kinematic test-field method
   - results pertaining to MRI turbulence

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spectral harmonic TF method

- Ansatz for test-field inhomogeneity:

$$\overline{\mathcal{B}}_{(0)} = \cos(\omega t)\ \cos(k_z z)\,\hat{\mathbf{x}}\,, \qquad \overline{\mathcal{B}}_{(1)} = \cos(\omega t)\ \sin(k_z z)\,\hat{\mathbf{x}}\,,$$
$$\overline{\mathcal{B}}_{(2)} = \cos(\omega t)\ \cos(k_z z)\,\hat{\mathbf{y}}\,, \qquad \overline{\mathcal{B}}_{(3)} = \cos(\omega t)\ \sin(k_z z)\,\hat{\mathbf{y}}\,.$$

- Evolution equation for associated fluctuations:

$$\partial_t \mathcal{B}'_{(\nu)} = \nabla \times \left[\mathbf{v}' \times \overline{\mathcal{B}} + \bar{\mathbf{v}} \times \mathcal{B}'_{(\nu)} - \overline{\mathbf{v}' \times \mathcal{B}'_{(\nu)}} + \mathbf{v}' \times \mathcal{B}'_{(\nu)} - \eta_{\rm m} \nabla \times \mathcal{B}'_{(\nu)}\right]$$

- Then, using $\bar{\mathcal{E}}^{(\nu)} \equiv \overline{\mathbf{v}' \times \mathcal{B}'_{(\nu)}}$, the matrix inversion is trivial:

$$\begin{pmatrix} \tilde{\alpha}_{ij}(k_z,\omega) \\ k_z\ \tilde{\beta}_{ijz}(k_z,\omega) \end{pmatrix} = \mathrm{e}^{\mathrm{i}\omega \mathrm{t}} \begin{pmatrix} \cos(k_z z) & \sin(k_z z) \\ -\sin(k_z z) & \cos(k_z z) \end{pmatrix} \begin{pmatrix} \bar{\mathcal{E}}_i^{(2j-2)} \\ \bar{\mathcal{E}}_i^{(2j-1)} \end{pmatrix}$$

Gressel & Elstner (2020), MNRAS 494, 1180

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non-locality / finite “domain of dep.”

- classical local closure relation:

$$\bar{\mathcal{E}}_i(z,t) = \alpha_{ij}(z,t)\,\overline{B}_j(z,t) \;-\; \eta_{ij}(z,t)\,\varepsilon_{jzl}\,\partial_z\overline{B}_l(z,t)\,.$$

- non-local generalisation:

$$\bar{\mathcal{E}}_i(z) = \int \hat{\alpha}_{ij}(z,\zeta)\,\overline{B}_j(z-\zeta) \;-\; \hat{\eta}_{ij}(z,\zeta)\,\varepsilon_{jzl}\,\partial_z\overline{B}_l(z-\zeta)\,\mathrm{d}\zeta\,,$$

$$\tilde{\bar{\mathcal{E}}}_i(k_z) = \tilde{\alpha}_{ij}(k_z)\,\tilde{\overline{B}}_j(k_z) \;-\; \tilde{\eta}_{ij}(k_z)\,\mathrm{i}k_z\,\varepsilon_{jzl}\,\tilde{\overline{B}}_l(k_z)$$

- empiric *ad hoc* model for kernel function:

$$\tilde{\alpha}(k_z) = \frac{\alpha_0}{1+\left(k_z/k_{\rm c}^{(\alpha)}\right)^2}\,,\qquad \tilde{\eta}(k_z) = \frac{\eta_0}{1+\left(k_z/k_{\rm c}^{(\eta)}\right)^2}$$

Brandenburg, Rädler & Schrinner (2008), A&A 482, 739
Gressel & Pessah (2015), ApJ 810, 59

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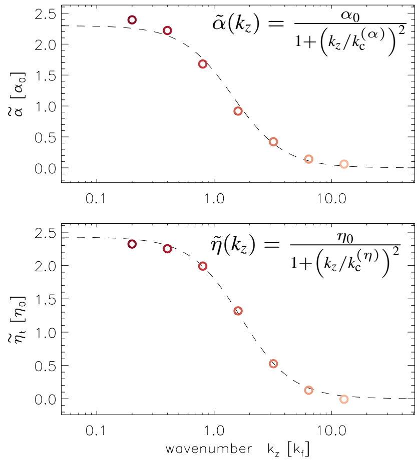
helical forcing (k-dependence)

- benchmark: $\rightarrow$ helically-forced hydro
  (i.e., strict kinematic limit)



$$\partial_t\,\rho + \nabla\cdot(\rho\mathbf{v}) = 0\,,$$
$$\partial_t(\,\rho\mathbf{v}\,) + \nabla\cdot(\,\rho\mathbf{v}\mathbf{v}\,) = -\nabla p + \nabla\cdot\tau + \mathbf{f}_{\rm hel}(\mathbf{k},t)$$

- amplitudes for the α effect (top) and turbulent diffusion, $\eta_{\rm T}$ (bottom)
- helically-forced isothermal turbulence with $k_{\rm f} = 5$, ${\rm Ma} \simeq 0.1$, and ${\rm Re} = {\rm Rm} \simeq 10$.
- dashed lines indicate Lorentzian fits with $k_{\rm c}^{(\alpha)} = 1.45$ and $k_{\rm c}^{(\eta)} = 1.70$

Gressel & Elstner (2020), MNRAS 494, 1180

Bbg, Rädler & Schrinner (2008), A&A 482, 739

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time-lag / “memory” effects

- classical local closure relation:

$$\bar{\mathcal{E}}_i(z,t) = \alpha_{ij}(z,t)\,\overline{B}_j(z,t) \;-\; \eta_{ij}(z,t)\,\varepsilon_{jzl}\,\partial_z\overline{B}_l(z,t)\,.$$

- non-instantaneous generalisation:

$$\bar{\mathcal{E}}_i(z,t) = \int \hat{\alpha}_{ij}(z,t')\,\overline{B}_j(z,t-t') - \hat{\eta}_{ij}(z,t')\,\varepsilon_{jzl}\,\partial_z\overline{B}_l(z,t-t')\,\mathrm{d}t'\,,$$

$$\tilde{\bar{\mathcal{E}}}_i(\omega) = \tilde{\alpha}_{ij}(\omega)\,\tilde{\overline{B}}_j(\omega) \;-\; \tilde{\eta}_{ij}(\omega)\,\mathrm{i}k_z\,\varepsilon_{jzl}\,\tilde{\overline{B}}_l(\omega)$$

- empiric *ad hoc* model for kernel function:

$$\tilde{\alpha}(\omega) = \alpha_0\,\frac{1-\mathrm{i}\,\omega\,\tau_\mathrm{c}^{(\alpha)}}{\left(1-\mathrm{i}\,\omega\,\tau_\mathrm{c}^{(\alpha)}\right)^2 + \left(\omega_0^{(\alpha)}\,\tau_\mathrm{c}^{(\alpha)}\right)^2}\,,$$

Hubbard & Brandenburg (2009), ApJ 706, 712 Gressel & Pessah (2022), ApJ 928, 118

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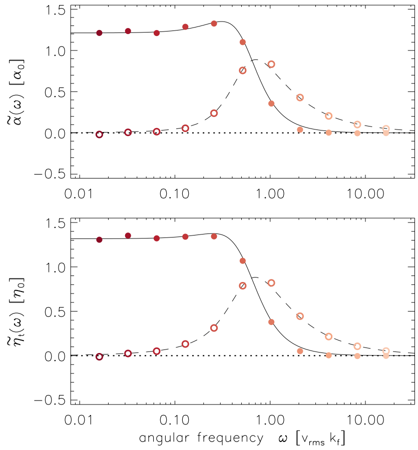
helical forcing (frequency dependence)



- for $k_{\rm f} \simeq 3$, $\mathrm{Ma} \simeq 0.1$, $\mathrm{Re} = \mathrm{Rm} \simeq 22$.
- solid and dashed lines show a simultaneous fit to the real and imaginary part, respectively

$$\Re = \frac{1 + (\omega^2 + \omega_0^2)\,\tau_c^2}{4\,\omega^2\tau_c^2 + (1 - (\omega^2 - \omega_0^2)\,\tau_c^2)^2}$$

$$\Im = \frac{1 + (\omega^2 - \omega_0^2)\,\tau_c^2}{4\,\omega^2\tau_c^2 + (1 - (\omega^2 - \omega_0^2)\,\tau_c^2)^2}\;\omega\,\tau_c$$

- slight tension with previous results...
  (but also different codes, Pencil vs. NIRVANA-III)

Gressel & Elstner (2020), MNRAS 494, 1180
Hubbard & Brandenburg (2009), ApJ 706, 712

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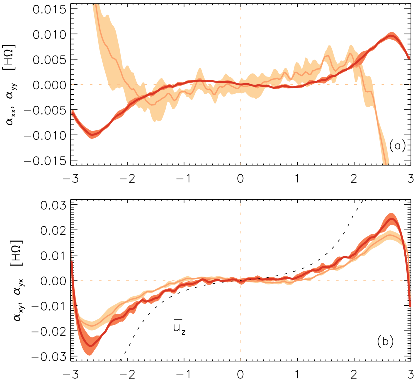
test-field α effect for MRI turbulence



Gressel & Pessah (2015), ApJ 810, 59

- new test-field results for weaker shear of $q = 1.2$
- pronounced negative α effect near midplane
  Brandenburg (1998),
  Rüdiger & Pipin (2000)
- as previously: off-diagonal tensor elements both positive (anisotropic pumping)

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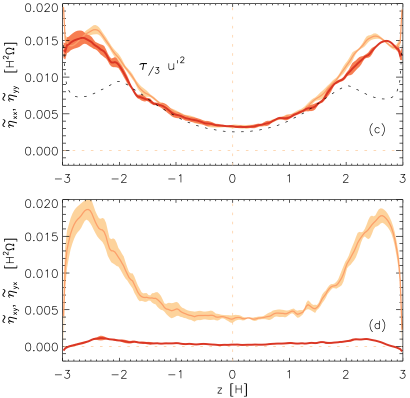
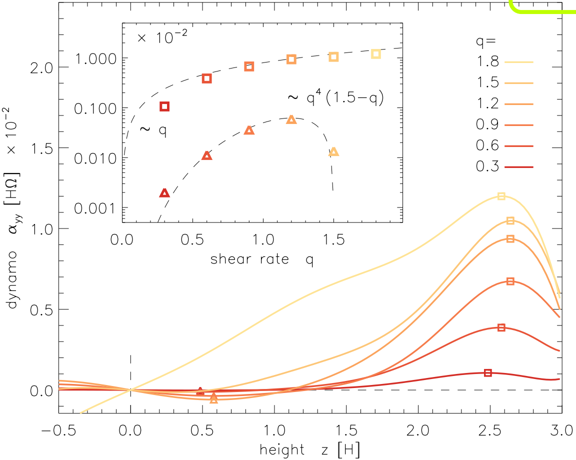
test-field turbulent η for MRI turbulence

- turbulent diffusion consistent with theory (i.e., for $z < 2\,H$ / high $\beta_{\rm P}$)
- off-diagonals both positive (i.e., no independent Rädler effect–dynamo)
- weak $\tilde{\eta}_{yx}$ responsible for butterfly diagram?!



Gressel & Pessah (2015), ApJ 810, 59

shear-rate dependence of coefficients



Rüdiger & Pipin (2000) Ziegler & Rüdiger (2001) Gressel & Pessah (2015), ApJ 810, 59

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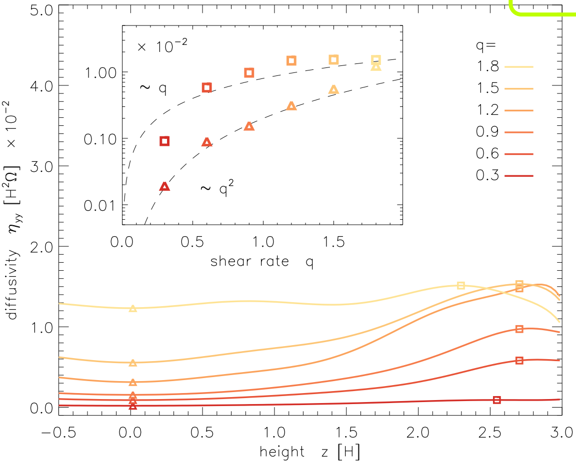

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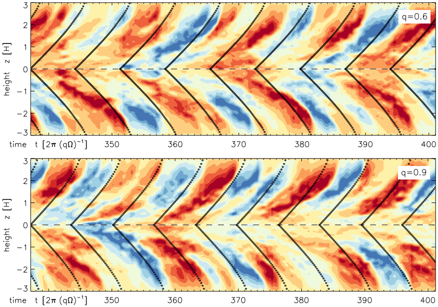
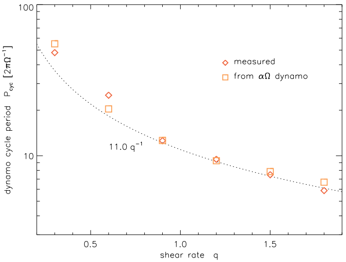
the dynamo cycle period

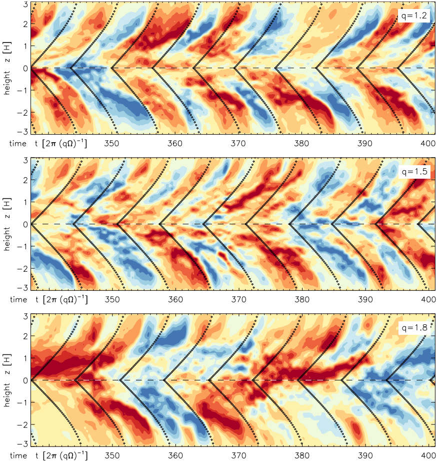


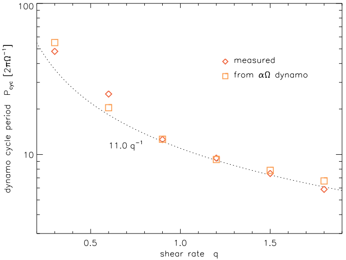
Gressel & Pessah (2015), ApJ 810, 59



- $\omega_{\rm cyc} \simeq \left|\frac{1}{2}\,\alpha_{yy}\,q\Omega\,k_z\right|^{1/2}$
- shear-rate dependence predicted well by $\alpha\Omega$ dispersion relation
- propagation direction still “wrong”

the dynamo cycle period



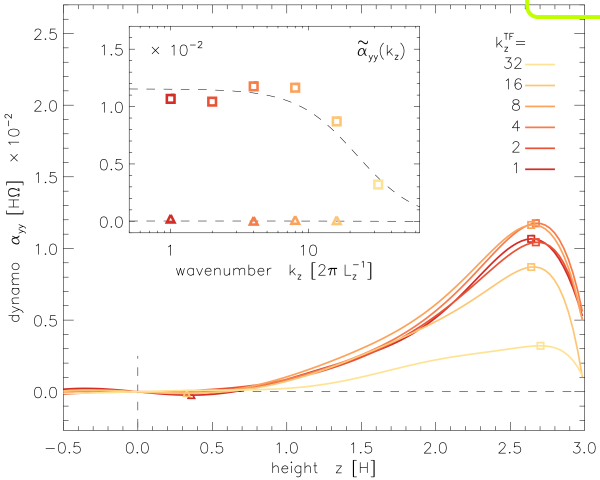


- $\omega_{\rm cyc} \simeq \left|\frac{1}{2}\,\alpha_{yy}\,q\Omega\,k_z\right|^{1/2}$
- shear-rate dependence predicted well by $\alpha\Omega$ dispersion relation
- propagation direction still “wrong”

Gressel & Pessah (2015), ApJ 810, 59

AIP

scale-dependence of coefficients



Gressel & Pessah (2015), ApJ 810, 59

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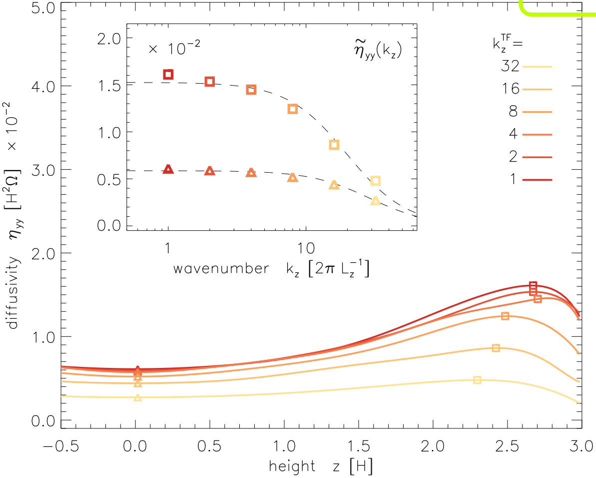
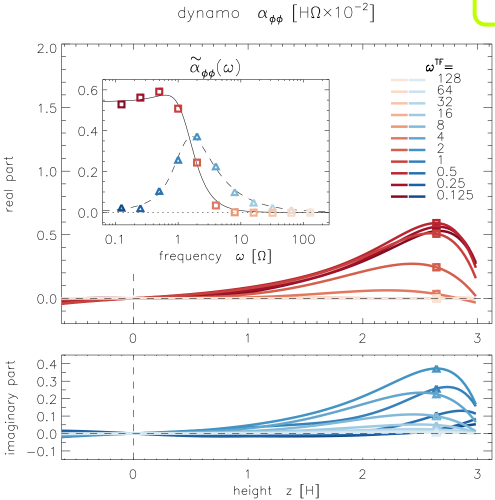


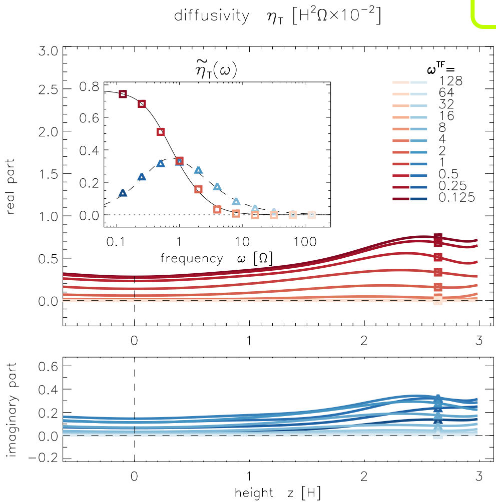
Gressel & Pessah (2015), ApJ 810, 59

time-lag-dependence of coefficients



Gressel & Pessah (2022), ApJ 928, 118

time-lag-dependence of coefficients



Gressel & Pessah (2022), ApJ 928, 118

summary of results

- Mean-field Dynamo in stratified MRI
  - test-field diagnostics are an extremely useful tool
  - “butterfly” diagram can be reproduced by simple toy models
  - precise origin of dynamo effect still unidentified ($\alpha\Omega$ vs. topological)
- Shear-rate dependence of the dynamo
  - dynamo cycle period well explained as function of shear-rate
  - yet, propagation direction still puzzling
- Non-local closure in space/time
  - established the scale-separation ratio of the MRI dynamo
  - promising non-local formulation including memory effects

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Thank you for your attention.