Topological spin excitations in non-Hermitian spin chains with a generalized kernel polynomial algorithm

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Physical Background

Dynamics of open quantum systems

$H = H_s + H_e + H_{se}$

$t_e \ll t_r$ (Born-Markov)

$t_r \cdot \Delta \gg 1$ (Rotating wave)

t_e: environment correlation time

t_r: system relaxation time

Δ: system energy level spacing

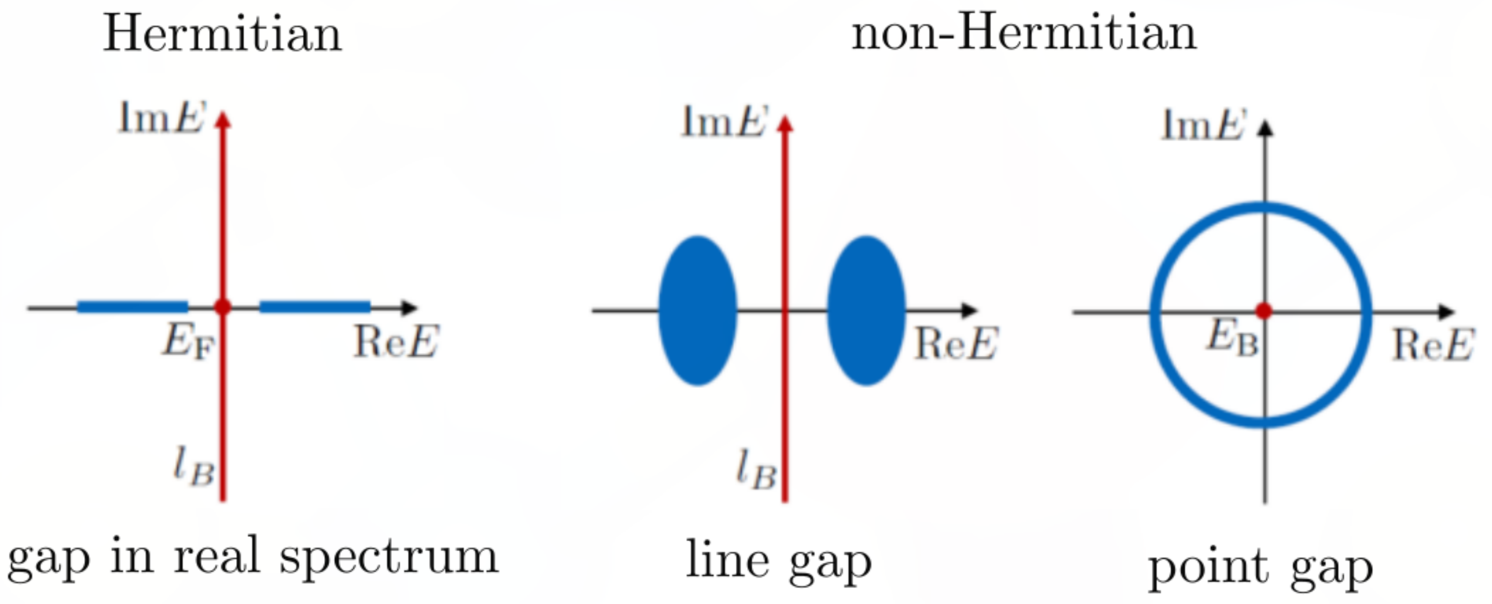
$$\frac{d}{dt}\rho_s = -i[\rho_s, H_s] + \sum_\alpha \left(L_\alpha \rho_s L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho_s\} \right) := \mathcal{L}[\rho_s]$$

vectorize ρ_s into $|\tilde{\rho}_s\rangle$: $\frac{d}{dt}|\tilde{\rho}_s\rangle = \tilde{\mathcal{L}}|\tilde{\rho}_s\rangle$

no quantum jump: $H_{\text{eff}} = H_s - \frac{i}{2}\sum_\alpha L_\alpha^\dagger L_\alpha$

Dynamics given by non-Hermitian matrices $\tilde{\mathcal{L}}$ and H_{eff}

(non-)Hermitian topology



Topology[1] — topological boundary modes — non-Hermitian skin effect

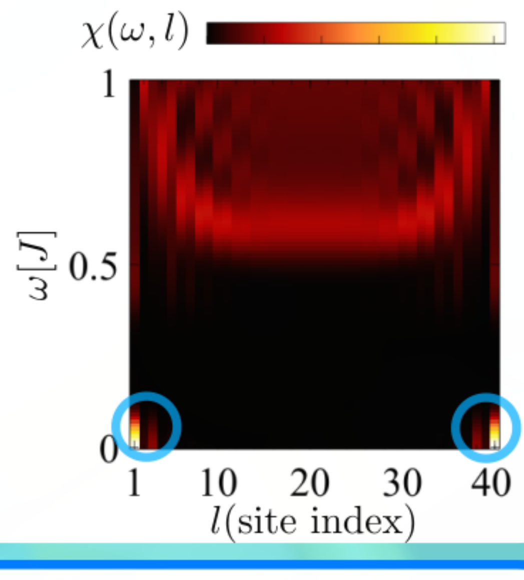
Challenging to characterize non-Hermitian many-body topology

Technical Background

Spectral function for Hermitian many-body topology

$$H = \frac{J}{2}\sum_{l=1}^{20}(S_{2l-1}^x S_{2l}^x + S_{2l-1}^y S_{2l}^y) + J\sum_{l=1}^{19}(S_{2l}^x S_{2l+1}^x + S_{2l}^y S_{2l+1}^y) + J\sum_{l=1}^{39} S_l^z S_{l+1}^z$$

$$\chi(\omega, l) = \langle \text{GS}|S_l^z \delta(\omega - H + E_0) S_l^z|\text{GS}\rangle$$



- computed efficiently with matrix product states and kernel polynomial method
- reveals topological boundary modes

Kernel polynomial method (KPM)[2]

- single variable function: $\rho(E) = \langle\psi_L|\delta(E - H)|\psi_R\rangle$
- Chebyshev expansion: $\rho(E) = \frac{1}{\pi\sqrt{1-E^2}}(\mu_0 + 2\sum_{n=1}^\infty \mu_n T_n(E))$

  $T_n(E) = \cos(n \arccos(E))$ $\quad T_{n+1}(x) = 2\omega T_n(x) - T_{n-1}(x)$
- recursively compute μ_n: $\mu_n = \langle\psi_L|T_n(H)|\psi_R\rangle = \langle\psi_L|v_n\rangle$

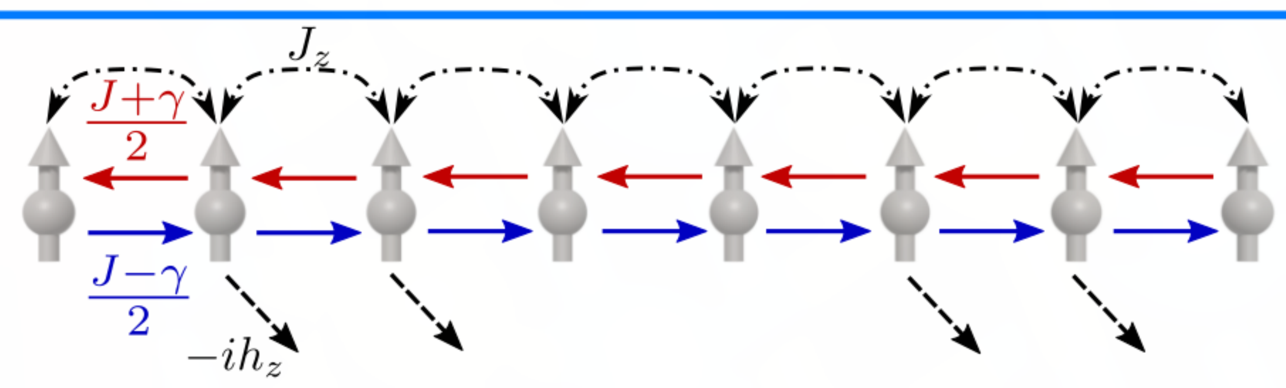
  $|v_0\rangle = |\psi_R\rangle \quad |v_{n+1}\rangle = 2H|v_n\rangle - |v_{n-1}\rangle$

  can be done with matrix product states represention for $|v_n\rangle$

Spectral function for non-Hermitian many-body topology?

Our findings

Model



$$H = \sum_{l=1}^{L-1}\left(\frac{J+\gamma}{2}S_l^+ S_{l+1}^- + \frac{J-\gamma}{2}S_l^- S_{l+1}^+ + J_z S_l^z S_{l+1}^z\right) + \sum_{l=1}^{L} ih_l^z S_l^z$$

$$h_l^z = -h_z \text{ if } l \mod 4 = 2,3$$

$J_z = \gamma = 0$: topological for $h_z > 0$ $\quad$ Topology survive for $J_z > 0$?

non-Hermitian Kernel polynomial method (NHKPM)

- double variable function: $f(\omega) = \langle\psi_L|\delta^2(\omega - H)|\psi_R\rangle$

  $$\delta^2(\omega - H) = \sum_n \delta(\Re(\omega - E_n))\delta(\Im(\omega - E_n))|\Psi_{\text{R},n}\rangle\langle\Psi_{\text{L},n}|$$
- no direct Chebyshev expansion, encounter with Hermitrization:

  $$\mathcal{H} = \begin{pmatrix} & \omega - H \\ \omega^* - H^\dagger & \end{pmatrix} \quad \text{s.t.} \quad f(\omega) = \tfrac{1}{\pi}\partial_{\omega^*}G(E=0)$$

  $$G(E) = \langle \text{L}|(E - \mathcal{H})^{-1}|\text{R}\rangle \quad |\text{L}\rangle = \begin{pmatrix} 0 \\ |\psi_\text{L}\rangle \end{pmatrix} \quad |\text{R}\rangle = \begin{pmatrix} |\psi_\text{R}\rangle \\ 0 \end{pmatrix}$$
- do Chebyshev expansion on $G(E)$ to efficiently compute $f(\omega)$[3]

Spectral function for non-Hermitian many-body topology

$$\mathcal{S}(\omega, l) = \langle \text{GS}_\text{L}|S_l^- \delta^2(\omega + E_{\text{GS}} - H)S_l^+|\text{GS}_\text{R}\rangle + \langle \text{GS}_\text{L}|S_l^+ \delta^2(\omega + E_{\text{GS}} - H)S_l^-|\text{GS}_\text{R}\rangle$$

- Faithfully revealing topological boundary states

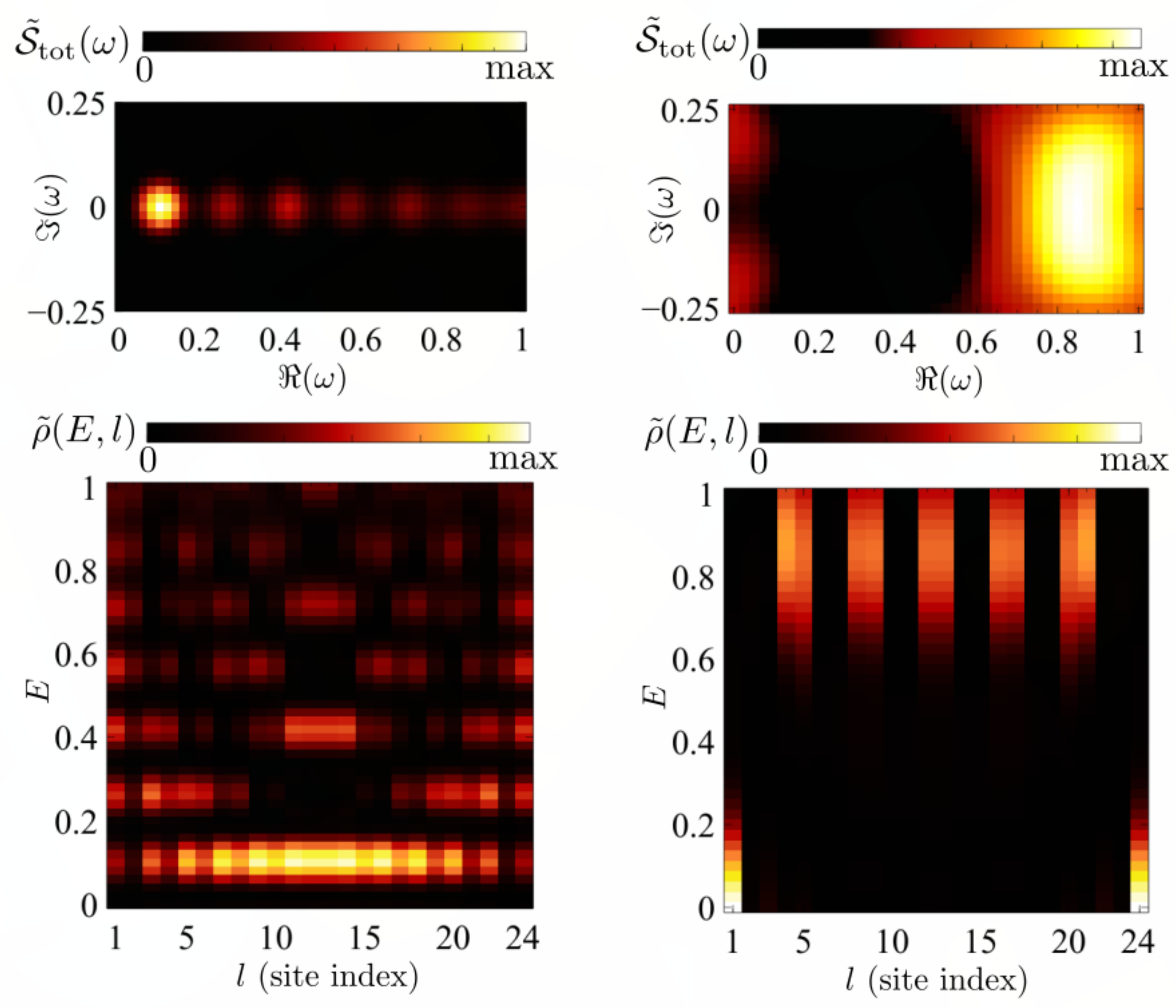
$L = 24$

$J_z = J/2, \gamma = 0$

$h_z = 0$, trivial $\quad$ $h_z = J$, topological

total spectrum: $\tilde{S}_{\text{tot}}(\omega) = \left|\sum_{l=1}^{L}\mathcal{S}(\omega, l)\right|$

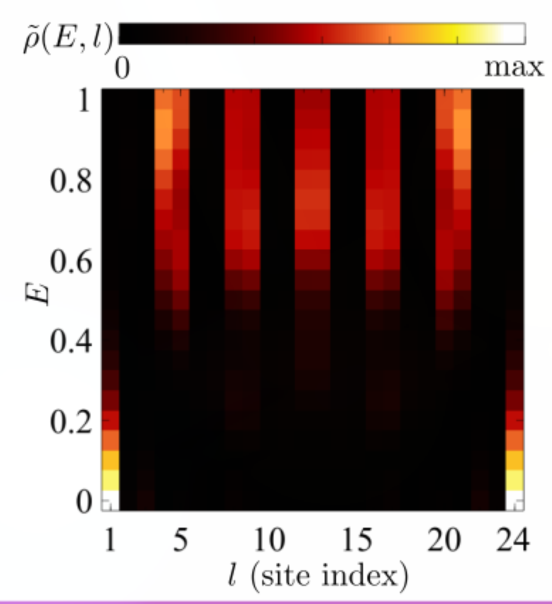
local spectrum: $\tilde{\rho}(E \in \mathbb{R}, l) = \left|\int \mathcal{S}(E + iy, l)dy\right|$



- Stable in the presence of non-Hermitian skin effect

$J_z = J/2, \gamma = 0.1J$

$h_z = J$, topological



Take home

We developed a generalized kernel polynoimial algorithm to efficiently compute spectral functions in non-Hermitian many-body systems, which is particularly useful for characterizing non-Hermitian many-body topology[4].

Bibliography

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[3] https://github.com/GUANGZECHEN/NHKPM.jl

[4] G. Chen, F. Song and J. L. Lado, arXiv:2208.06425