Anomalous transport in driven periodic systems: distribution of the absolute negative mobility effect

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Abstract

In this study we consider a paradigmatic model of the nonlinear Brownian motion in a driven periodic system which exhibits the absolute negative mobility. So far research on this anomalous transport feature has been limited mostly to the single case studies due to the complexity of the multidimensional parameter space. Here we utilized the computational power of GPU supercomputers to analyze nearly 10^9 parameter regimes. This let us to draw qualitative conclusions on how the emergence of negative mobility depends on the system parameters and find the optimal parameter values for which it occurs most frequently.

Langevin equation

The dynamics of a Brownian particle can be described by a Langevin equation

$$M\dot{v} + \Gamma v = -\frac{\mathrm{d}U(x)}{\mathrm{d}x} + A\cos(\Omega t) + F + \sqrt{2\Gamma k_B T}\xi(t). \tag{1}$$

To reduce the number of quantities and make the analysis more general, we transform it to a dimensionless form. Depending on whether we are interested in the influence of inertia or the friction on the system, we use different scaling variants:

$$\dot{v}_1 + \gamma v_1 = -\frac{\mathrm{d}\hat{U}(\hat{x})}{\mathrm{d}\hat{x}} + a\cos(\omega_1 t_1) + f + \sqrt{2\gamma D}\,\hat{\xi}(t_1) \tag{2}$$

to study friction and

$$m\dot{v}_2 + v_2 = -\frac{\mathrm{d}\hat{U}(\hat{x})}{\mathrm{d}\hat{x}} + a\cos(\omega_2 t_2) + f + \sqrt{2D}\,\hat{\xi}(t_2) \tag{3}$$

to analyze inertia.

Negative mobility

The only perturbation that breaks the spatial symmetry of (1) is the constant force f. Therefore we can define the mobility $\mu(f)$ relating the average velocity of the particle $\langle v\rangle$ to the static force f

$$\langle v\rangle = \mu(f)\, f. \tag{4}$$

The negative mobility effect occurs when $\mu(f) < 0$ (see fig. 1).

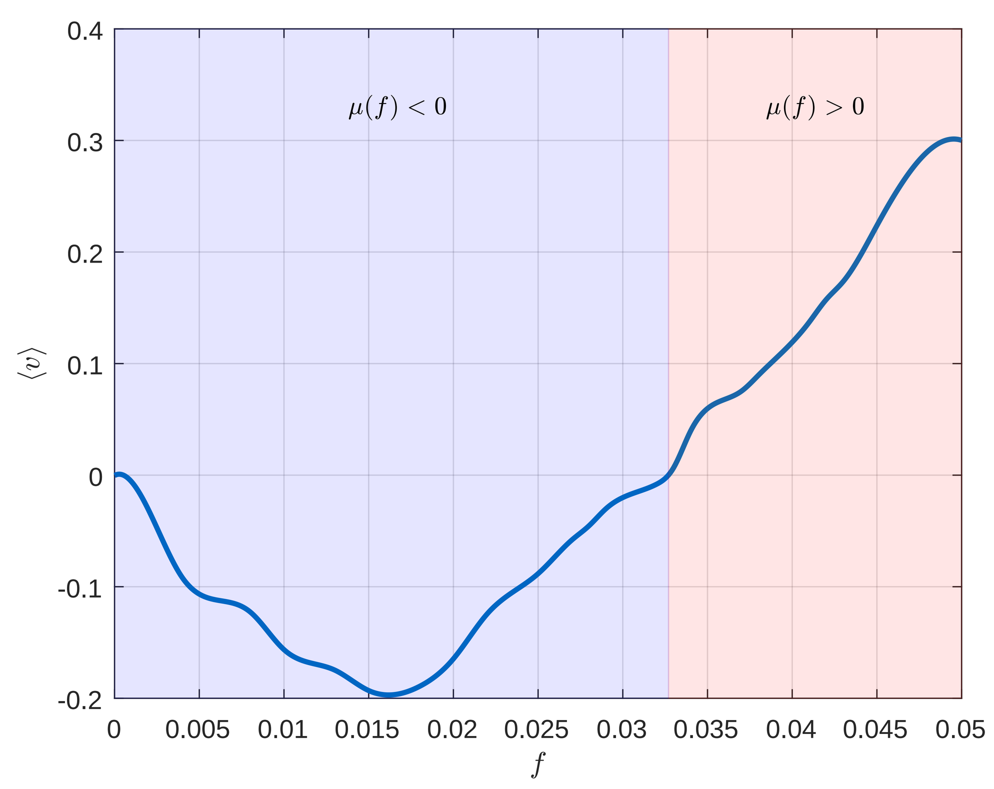
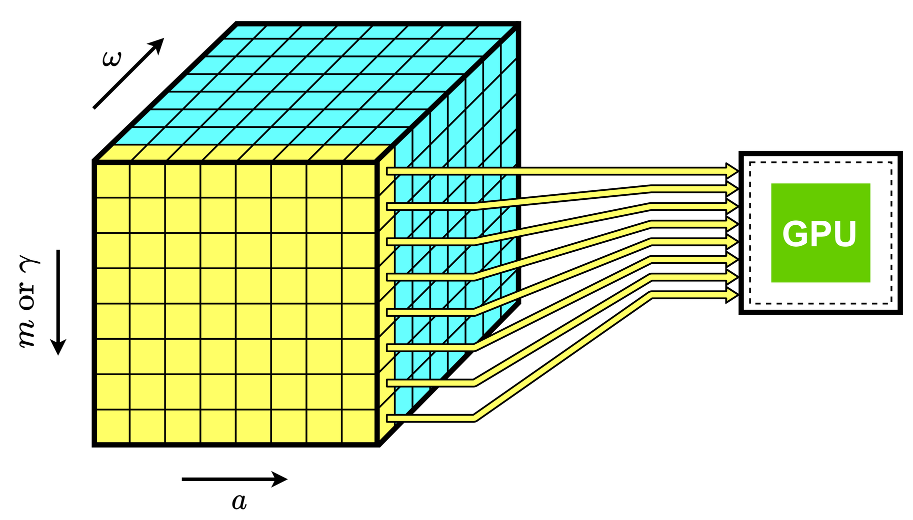


Fig. 1: The average velocity $\langle v\rangle$ versus the constant force f for a system exhibiting the negative mobility effect.

Simulation procedure

The Langevin equation cannot be solved analytically. To analyze its five-dimensional parameter space numerically, we harvested the GPU supercomputers. The simulation procedure was as follows: for each run we fixed the values of the static force f and thermal fluctuations intensity D and simulated a $400 \times 400 \times 400$ cuboid in $\{\gamma, a, \omega_1\}$ or $\{m, a, \omega_2\}$ subspace. The memory capacity allowed us to run one slice in $\{\gamma, a\}$ or $\{m, a\}$ subspace in parallel.



Conclusions

In this study we used the GPU supercomputers to analyze the distribution of the negative mobility effect in the parameter space of a Langevin equation. We found that this effect occurs most frequently for the value of friction coefficient $\gamma = 1.1749$. The corresponding optimal value of the constant force is $f = 0.5$. We also noticed that increasing the value of the static force f and the frequency of the external driving ω causes shift of the negative mobility regions towards lower values of dissipation γ. In most cases the thermal fluctuations influenced the anomalous transport behaviour destructively.

Distribution of negative mobility in $\{a, \omega\}$ subspace

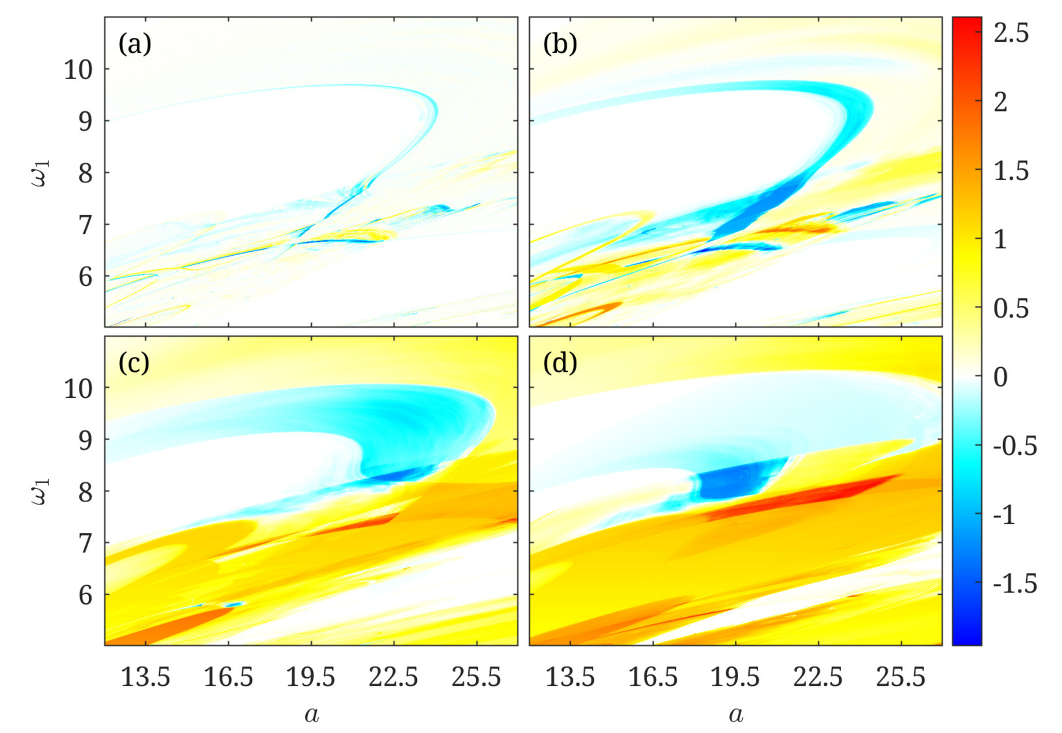


Fig. 3: The average velocity $\langle v\rangle$ versus amplitude a and frequency ω of the external driving for $\gamma = 1.1749$ and different values of f. Panel (a) $f = 0.01$, (b) $f = 0.1$, (c) $f = 0.5$, (d) $f = 1.0$.

Distribution of negative mobility in $\{\gamma, f\}$ subspace

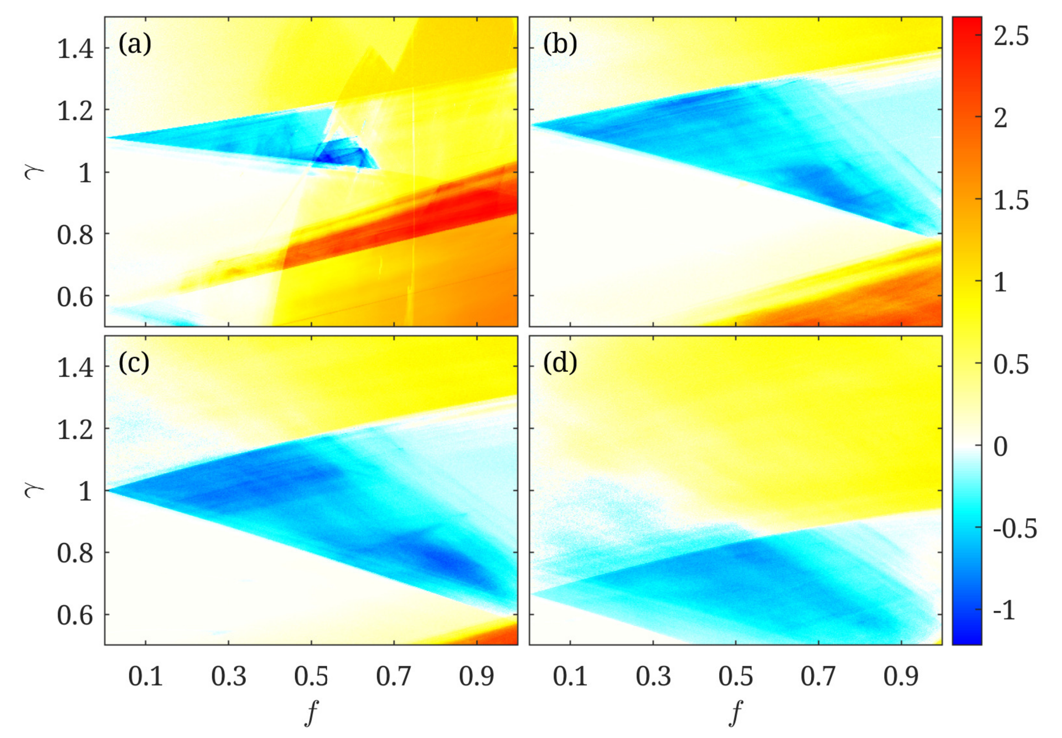


Fig. 4: The average velocity $\langle v\rangle$ versus friction coefficient γ and static force f for $a = 24.4375$ and different values of ω. Panel (a) $\omega = 8.6$, (b) $\omega = 9.3$, (c) $\omega = 10.0$, (d) $\omega = 10.6$.

Optimal values for the emergence of negative mobility

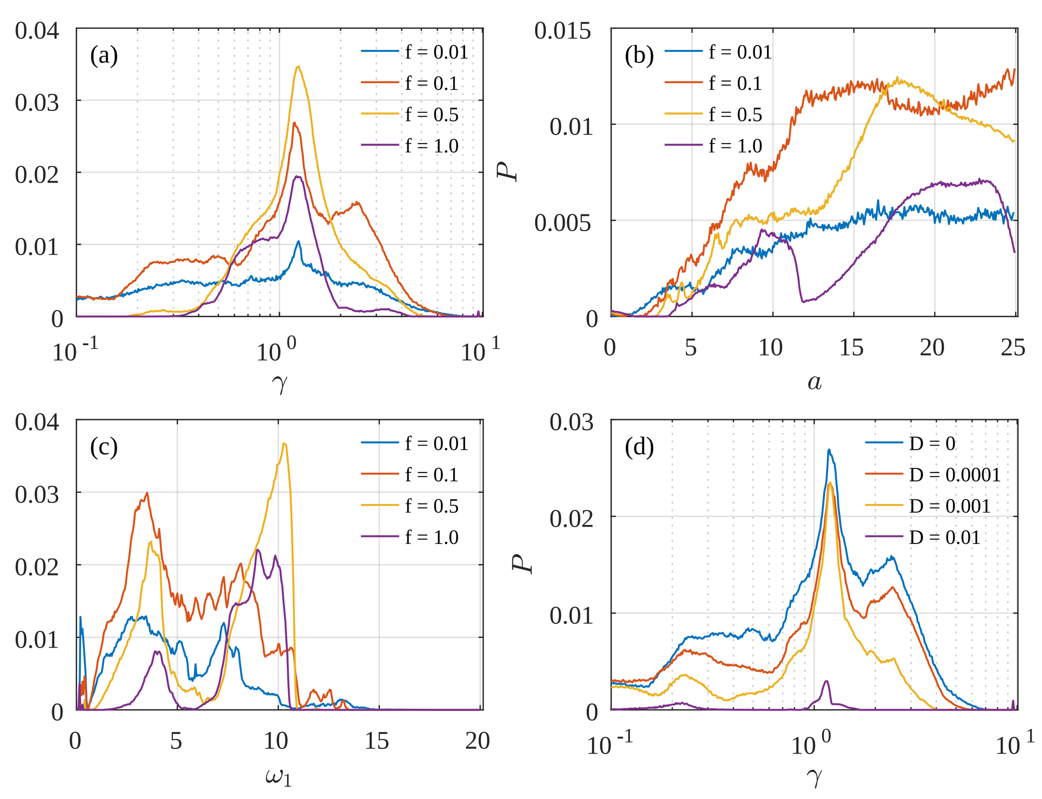


Fig. 5: Probability P for the emergence of absolute negative mobility as a function of (a) friction coefficient γ, (b) amplitude a and (c) frequency ω_1, for different values of constant force f. On (d) P is plotted versus γ for various thermal noise intensities D and $f = 0.1$.

References

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