Magnonics: the quantum entanglement in antiferromagnetic materials

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1. OVERVIEW

1) Hierarchy of magnon entanglement in antiferromagnets:

Continuous variable entanglement between magnon modes in Heisenberg antiferromagnets with Dzyaloshinskii-Moriya (DM) interaction is examined. Different bosonic modes are identified, which allows to establish a hierarchy of magnon entanglement.

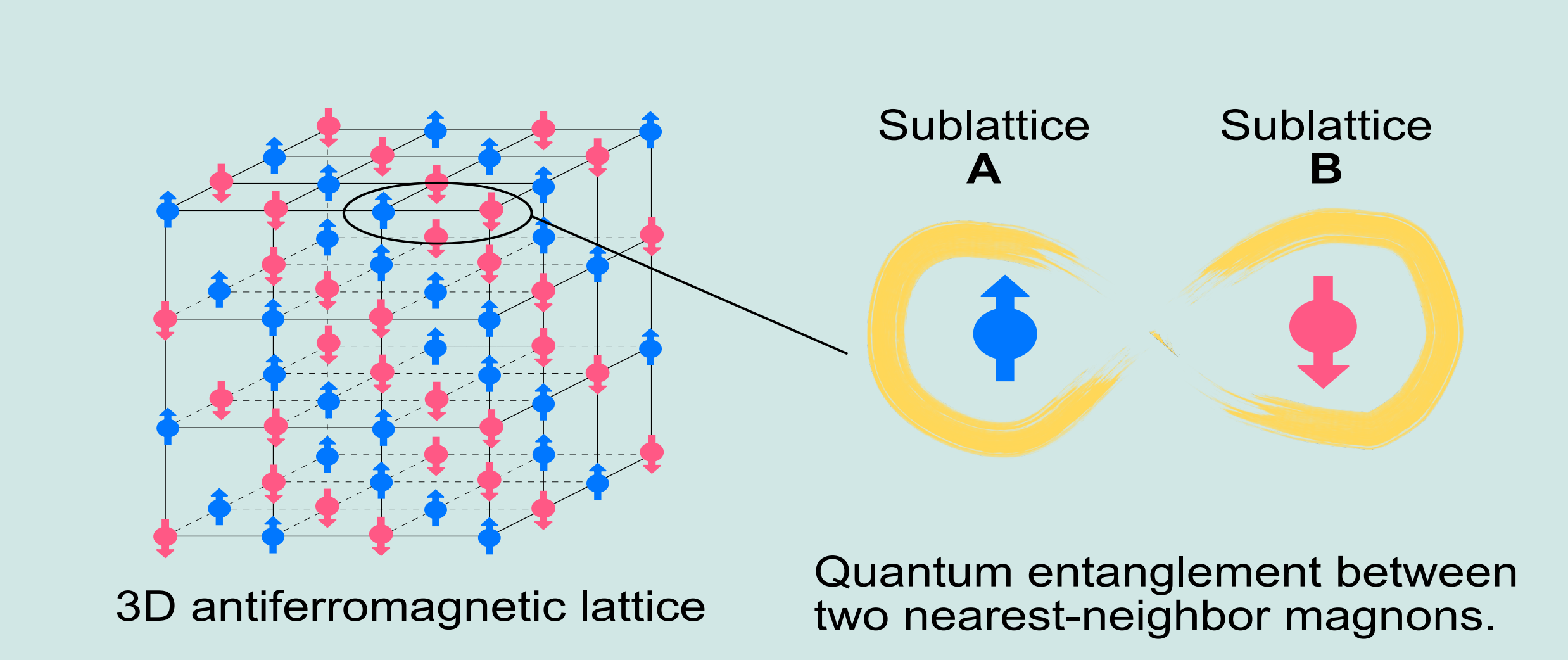
2) Magnon-magnon entanglement and its detection in a microwave cavity:

In AFM, two different magnon mode lead to experimentally detectable bipartite continuous variable magnon-magnon entanglement. The entanglement can be fully characterized via a single squeezing parameter, or, equivalently, entanglement parameter.

3) Room-temperature magnon-magnon quantum entanglement:

The magnon-magnon bipartite entanglement can stay active when the system temperature is over room-temperature, even over 300 K.

2. INTRODUCTION

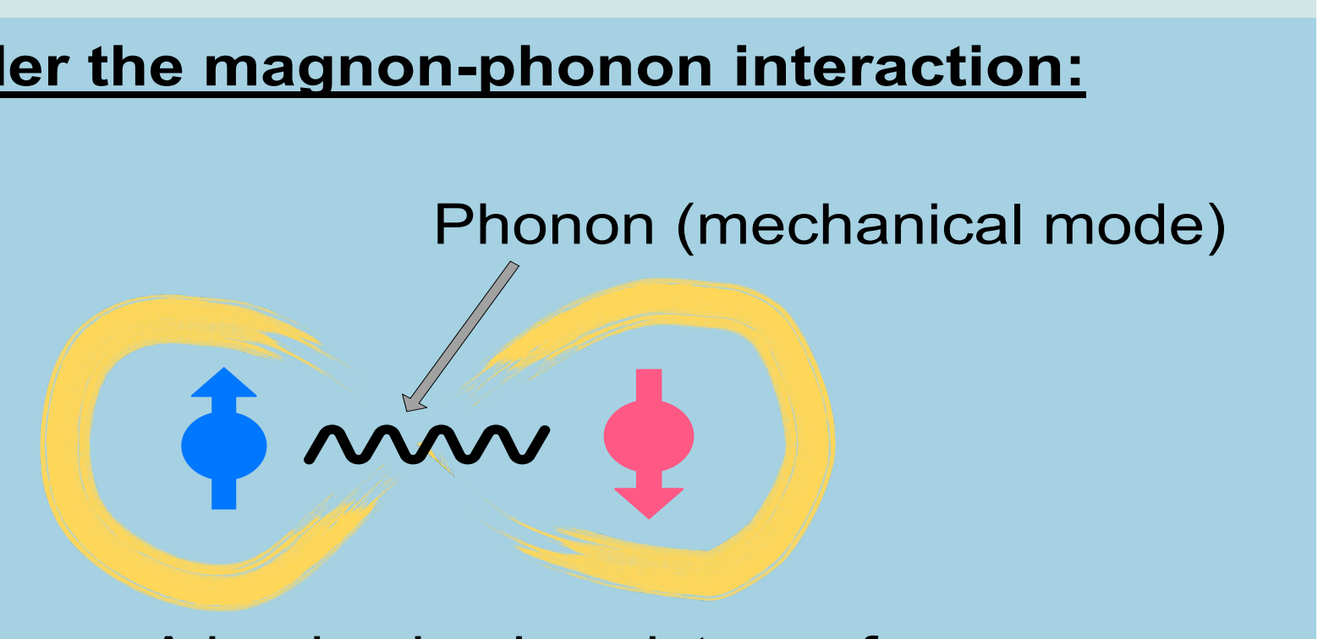


3D antiferromagnetic lattice

Quantum entanglement between two nearest-neighbor magnons.

Once we consider the magnon-phonon interaction:

How will the **magnon-magnon entanglement** change when we induce magnon-phonon interaction?



A basic physics picture of magnon-magnon-phonon system.

The spin Hamiltonian:

$$H_0 = J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j,\quad J>0,$$

bosonic operators Hamiltonian:

$$H_0 = \sum_{\mathbf{k}}\epsilon_{\mathbf{k}}(\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}+\beta^\dagger_{\mathbf{k}}\beta_{\mathbf{k}})$$

with Dzyaloshinskii-Moriya interaction:

$$H = \sum_{\mathbf{k}}\epsilon_{\mathbf{k}}(\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}+\beta^\dagger_{\mathbf{k}}\beta_{\mathbf{k}}) +izDS\sum_{\mathbf{k}}(\gamma_{\mathbf{k}}\alpha_{\mathbf{k}}\beta_{\mathbf{k}}-\gamma_{-\mathbf{k}}\alpha^\dagger_{\mathbf{k}}\beta^\dagger_{\mathbf{k}}),$$

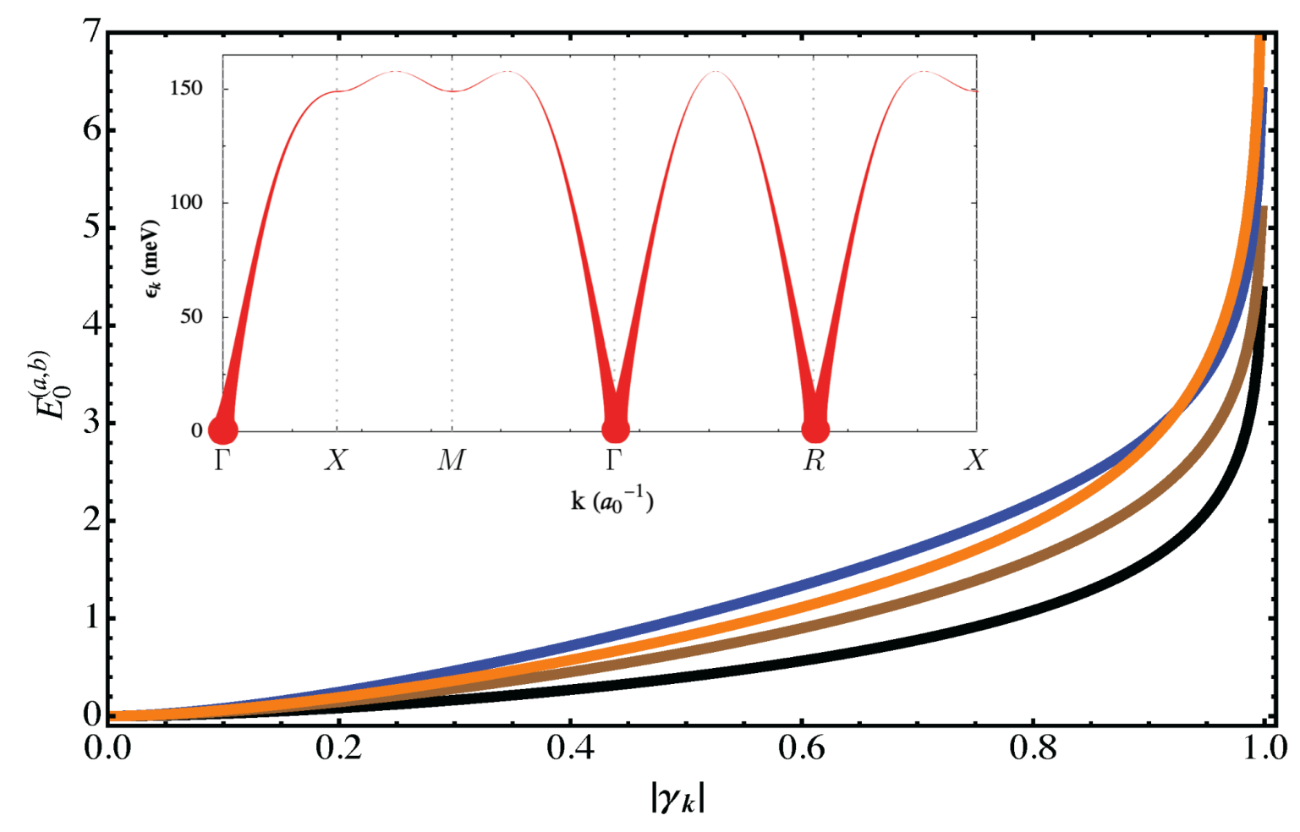
with magnon-photon interaction:

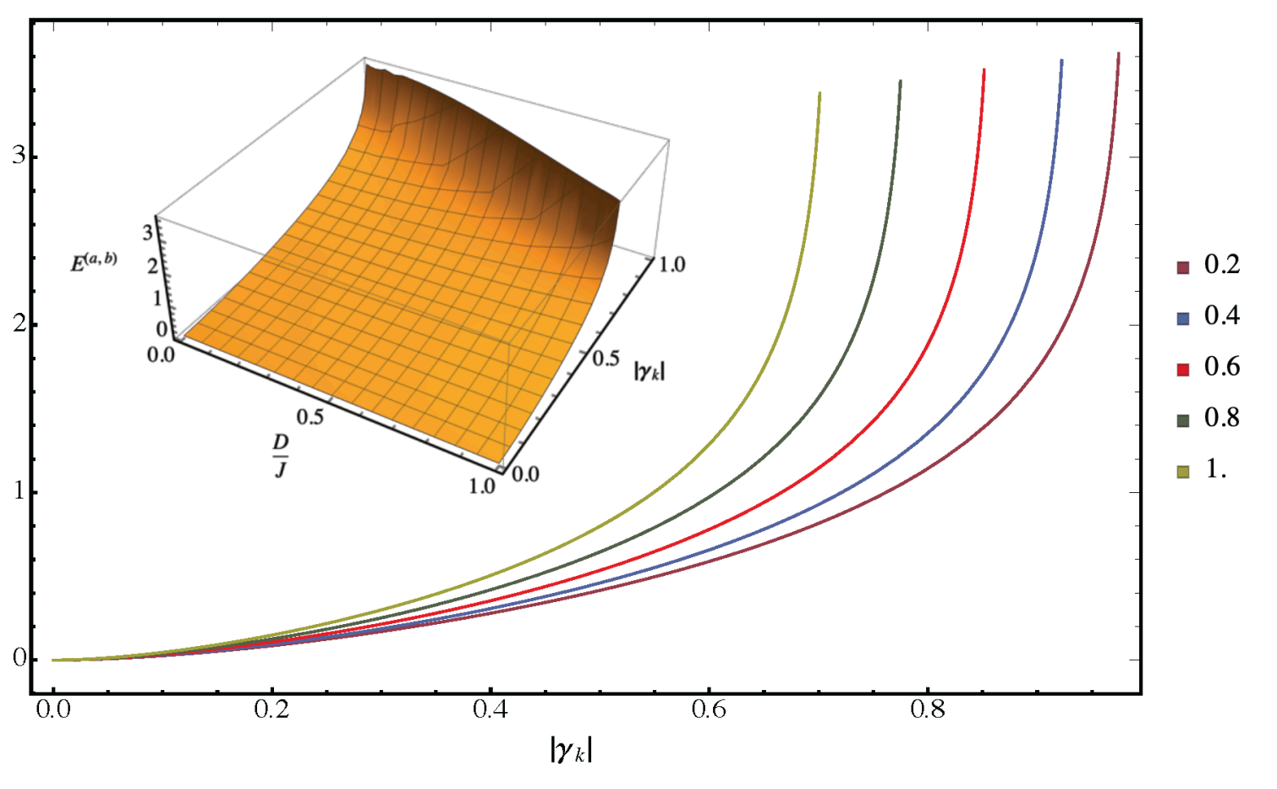
$$\begin{aligned} H_{\mathrm{mp}} &= \sum_{\mathbf{k}} H_{\mathrm{mp};\mathbf{k}},\\ H_{\mathrm{mp};\mathbf{k}} &= \omega_{\alpha_{\mathbf{k}}}\alpha^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}+\omega_{\beta_{-\mathbf{k}}}\beta^\dagger_{-\mathbf{k}}\beta_{-\mathbf{k}},+\omega(c^\dagger_{\mathbf{k}}c_{\mathbf{k}}+d^\dagger_{-\mathbf{k}}d_{-\mathbf{k}})\\ &+(\Delta_{\mathbf{k}}d^\dagger_{-\mathbf{k}}\beta_{-\mathbf{k}}+\Delta^*_{\mathbf{k}}d_{-\mathbf{k}}\beta^\dagger_{-\mathbf{k}})\\ &-(\Delta_{\mathbf{k}}c^\dagger_{\mathbf{k}}\alpha_{\mathbf{k}}+\Delta^*_{\mathbf{k}}c_{\mathbf{k}}\alpha^\dagger_{\mathbf{k}}), \end{aligned} \tag{12}$$

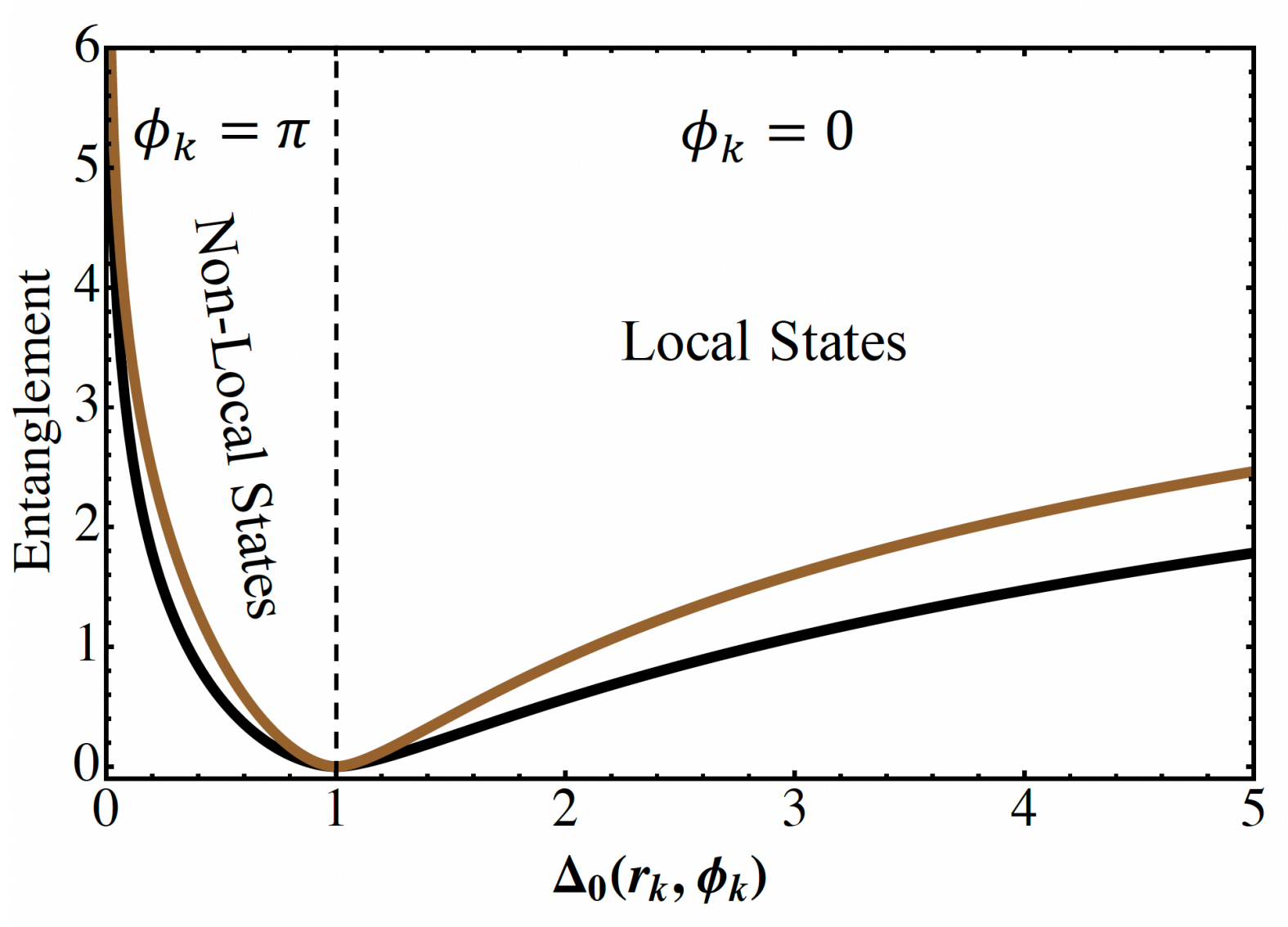
with magnon-phonon interaction:

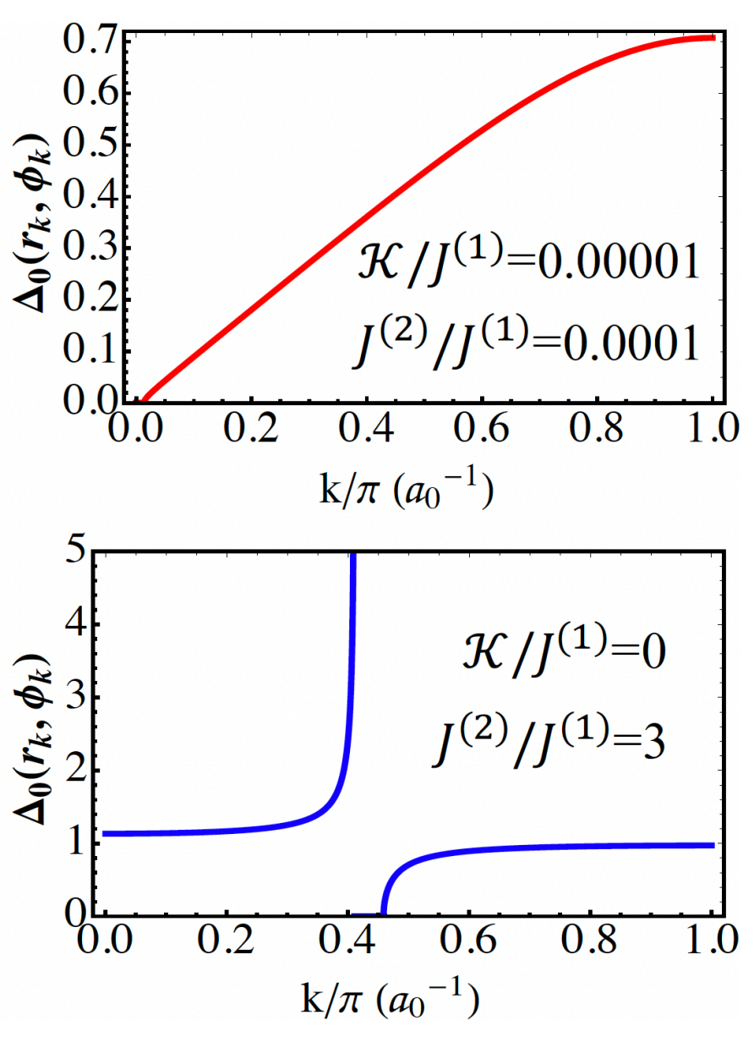
$$H_{\text{m-ph}} = \left[g_{ac}\hat{a}^\dagger\hat{a}+g_{bc}\hat{b}^\dagger\hat{b}\right]\left[\frac{\hat{c}+\hat{c}^\dagger}{\sqrt{2}}\right] + \omega_a\hat{a}^\dagger\hat{a}+\omega_b\hat{b}^\dagger\hat{b}+\omega_c\hat{c}^\dagger\hat{c}$$

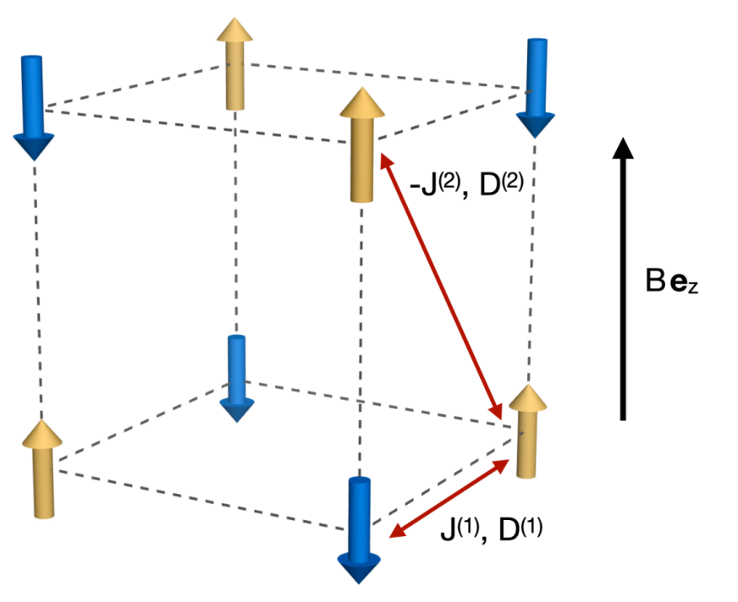
3. MAGNON-MAGNON QUANTUM ENTANGLEMENT IN ANTIFERROMAGNETS



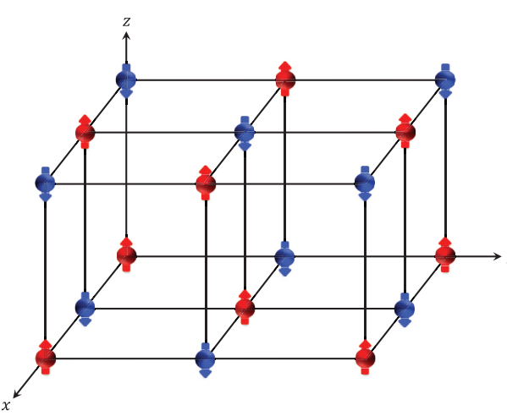






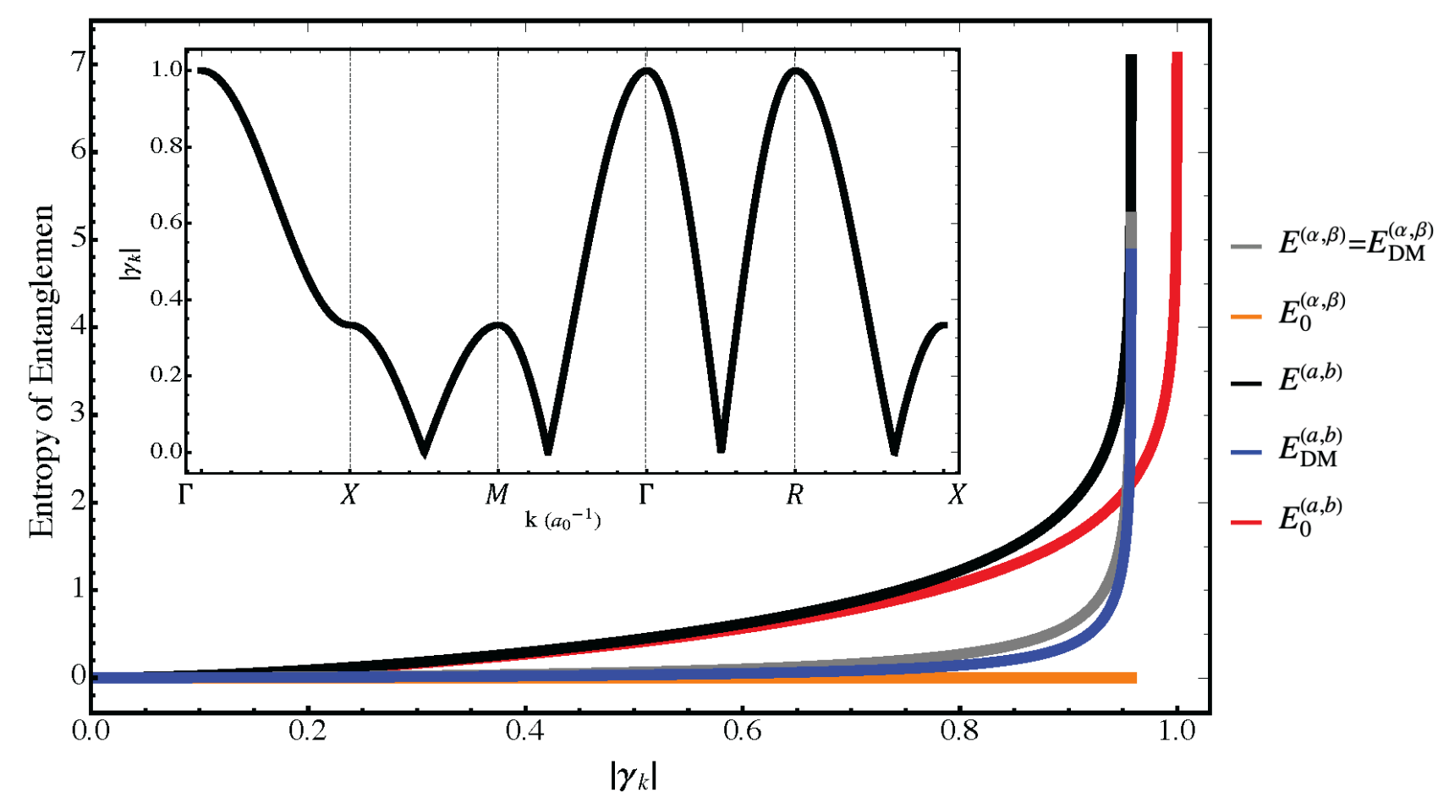


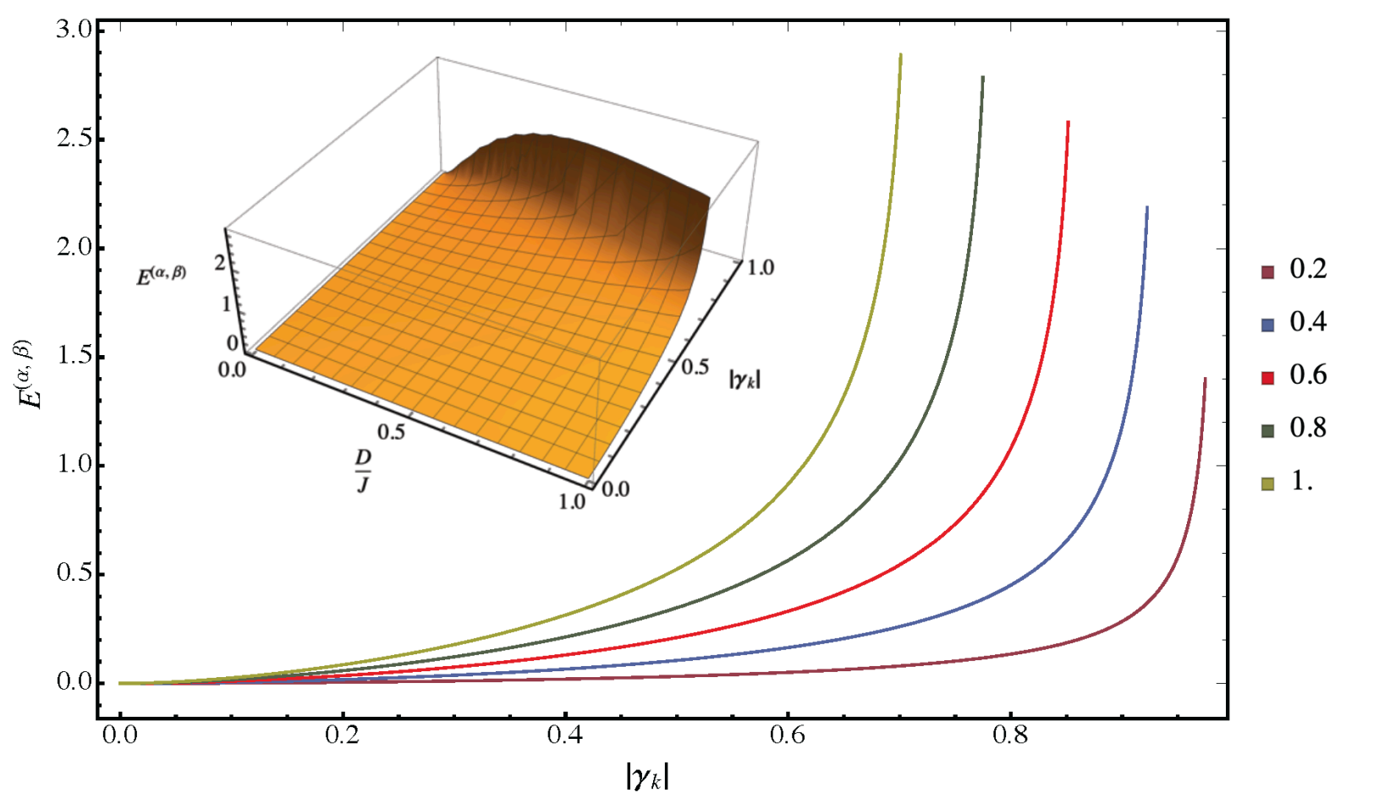
with Dzyaloshinskii-Moriya interaction D/J

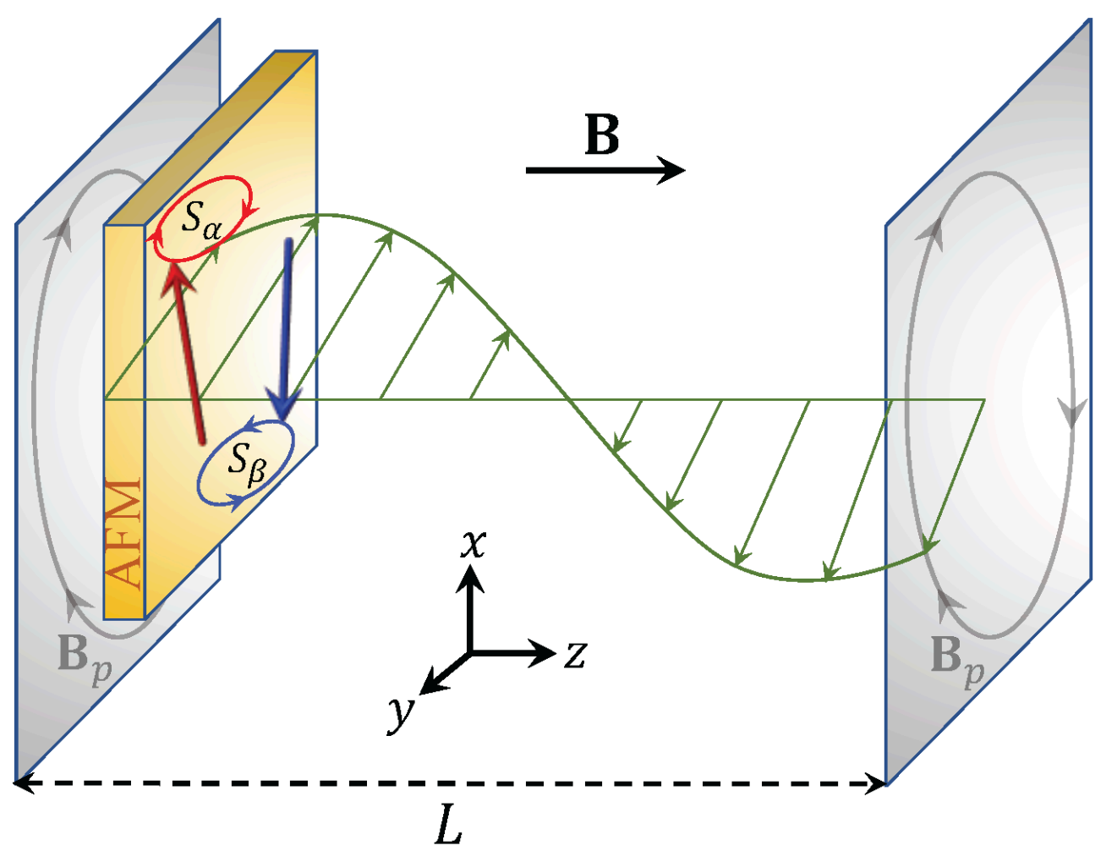


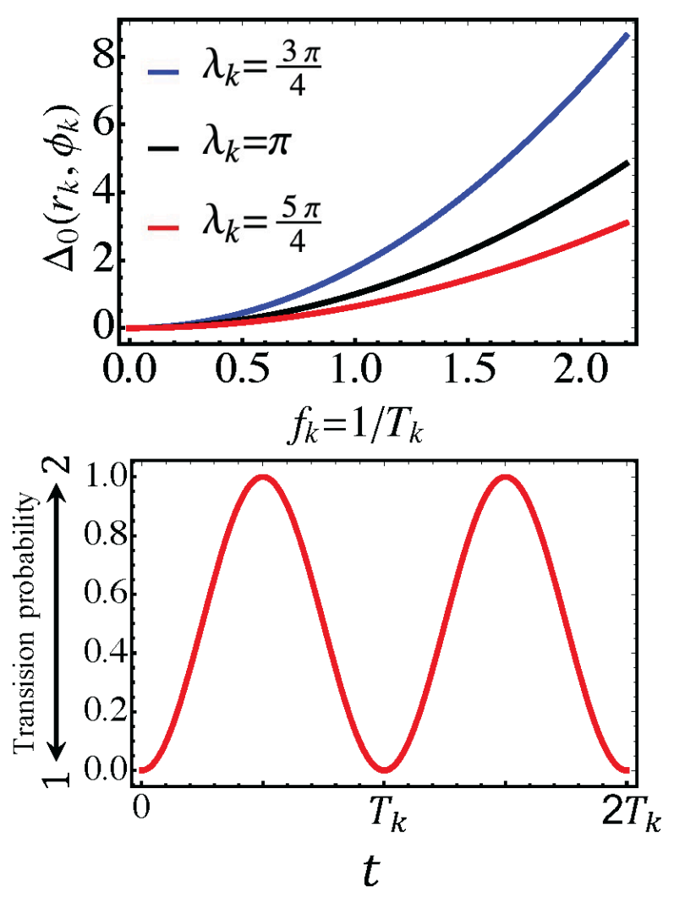
[Measure]

The *von Neumann entropy* of the reduced density matrix of one of the two subsystems.









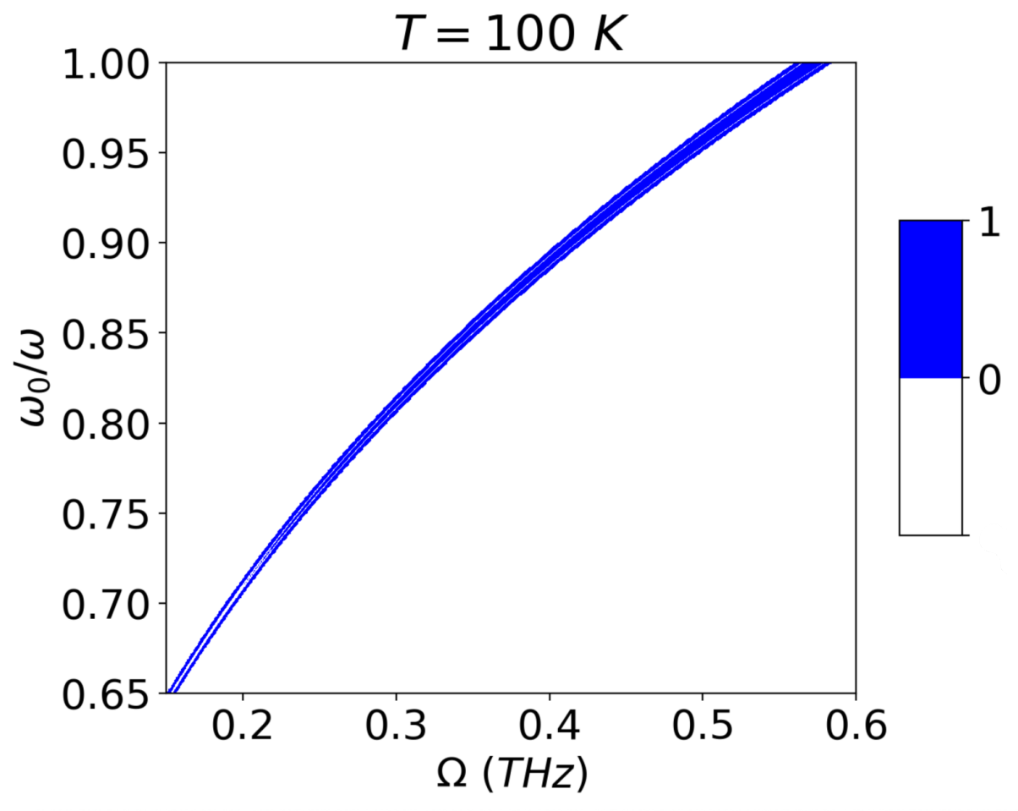
[Measure]

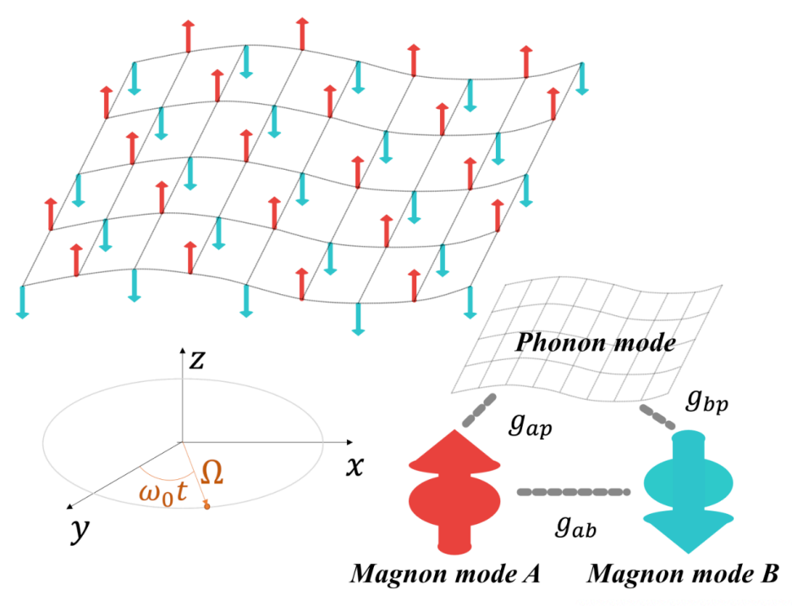
Einstein, Podolsky and Rosen (EPR) nonlocality.

$$\Delta(\psi)=\frac{1}{2}[\mathrm{Var}_\psi(X^A_{\mathbf{k}}+X^B_{\mathbf{k}})+\mathrm{Var}_\psi(P^A_{\mathbf{k}}-P^B_{\mathbf{k}})],$$

where $X^A_{\mathbf{k}}=\frac{a_{\mathbf{k}}+a^\dagger_{\mathbf{k}}}{\sqrt{2}}$ $\left(X^B_{\mathbf{k}}=\frac{b_{\mathbf{k}}+b^\dagger_{\mathbf{k}}}{\sqrt{2}}\right)$ and $P^A_{\mathbf{k}}=\frac{a_{\mathbf{k}}-a^\dagger_{\mathbf{k}}}{i\sqrt{2}}$ $\left(P^B_{\mathbf{k}}=\frac{b_{\mathbf{k}}-b^\dagger_{\mathbf{k}}}{i\sqrt{2}}\right)$ are assumed to be the dimensionless position and momentum quadratures for the $a_{\mathbf{k}}(b_{\mathbf{k}})$ mode

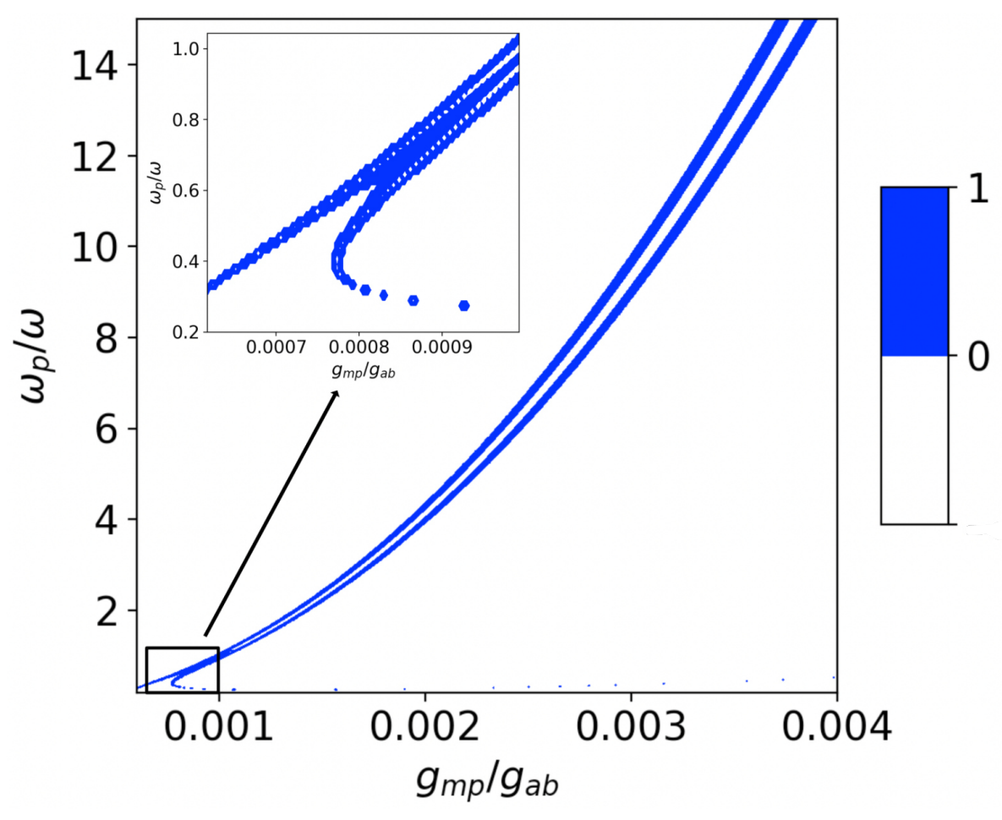
3. ROOM-TEMPERATURE ENTANGLEMENT





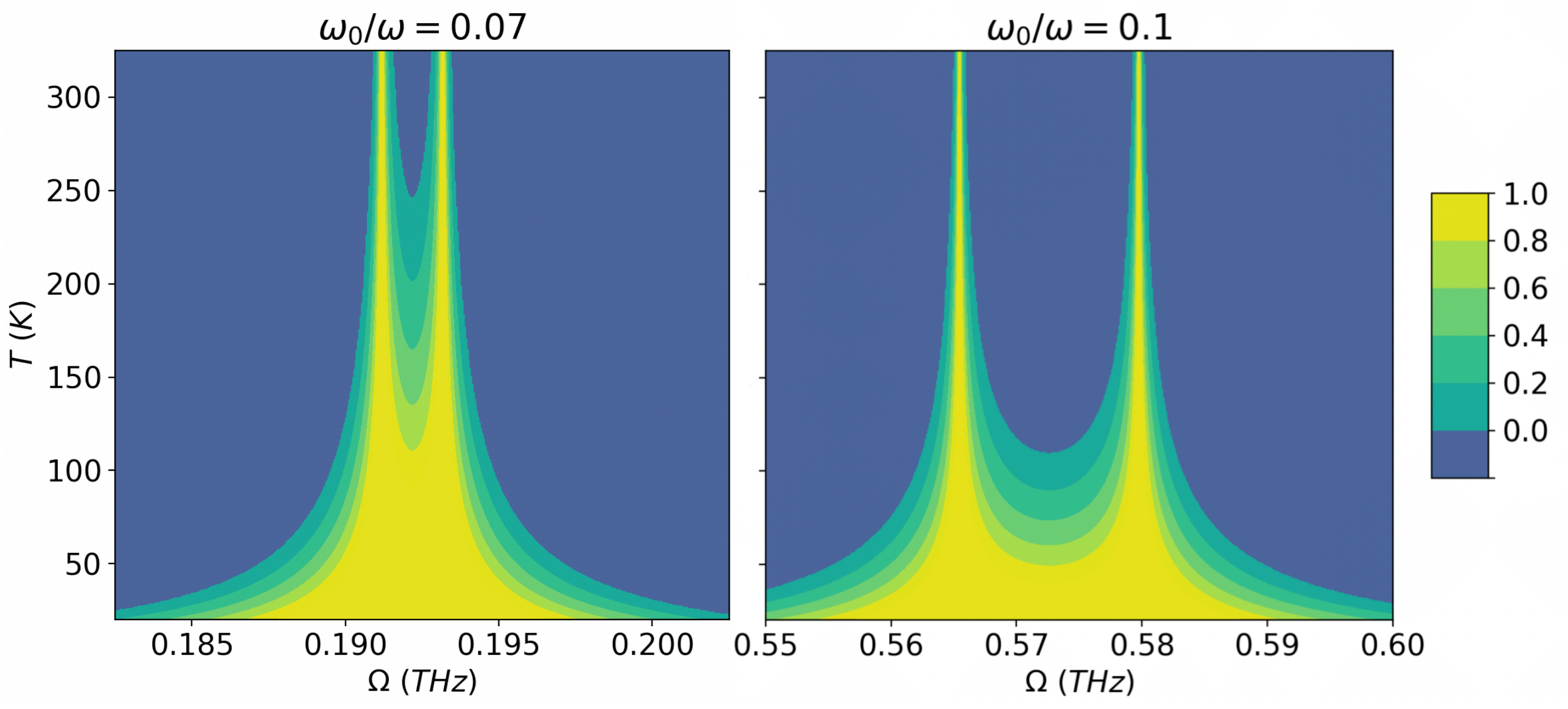
The Heisenbeg - Langevin equations:

$$\begin{aligned} \dot{\hat{a}} &= -(i\Delta_a+\kappa_a)\hat{a}-ig^*_{ab}\hat{b}^\dagger-ig_{ap}\hat{a}\hat{q}+\Omega+\sqrt{2\kappa_a}\hat{a}^{in},\\ \dot{\hat{b}} &= -(i\Delta_b+\kappa_b)\hat{b}-ig^*_{ab}\hat{a}^\dagger-ig_{bp}\hat{b}\hat{q}+\Omega+\sqrt{2\kappa_b}\hat{b}^{in},\\ \dot{\hat{q}} &= \omega_p\hat{p},\\ \dot{\hat{p}} &= -\omega_p\hat{q}-g_{ap}\hat{a}^\dagger\hat{a}-g_{bp}\hat{b}^\dagger\hat{b}-\gamma_p\hat{p}+\xi, \end{aligned}$$



[Measure]

Logarithmic negativity - bipartite subsystems.



4. CONCLUSIONS

1) There naturally is a hierarchy of different types of two-mode magnon entanglement in eigenstates of an antiferromagnet, where each level of this hierarchy is specified by the geometry of the spin lattice and individual exchange couplings

2) We propose a new and feasible measurement setup based on light and matter interaction to observe the EPR function through measurement of the magnon-photon transition frequency.

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