Wilson Loops, AdS/CFT and M2-branes

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Defect CFT on Wilson loop: $AdS_5/CFT_4 \to AdS_2/CFT_1$

[Giombi, Roiban, AT 2017]

• gauge-invariant correlators of operators on WL:
"defect" CFT_1 "induced" from $\mathcal{N} = 4$ SYM

• $\frac{1}{2}$-BPS WL: example of AdS_2/CFT_1
QFT in AdS_2 defined by superstring action in $AdS_5 \times S^5$

• AdS/CFT map: SYM fields $\perp$ to line
$\leftrightarrow$ string coords as fields in AdS_2

• "open string" analog of
$\mathrm{Tr}(\Phi^n ... F^k ...) \leftrightarrow$ closed-string vertex ops

• e.g. 4-point correlators at strong coupling
from Witten diagrams in AdS_2
combined with OPE, connections with integrability, bootstrap, etc.

$\mathcal{N}=4$ SYM: large N, $\lambda = g_{\rm YM}^2 N$

Maldacena-Wilson loop operator

$$W = {\rm tr}\, P\, e^{\oint dt \left(i\dot{x}^\mu A_\mu + |\dot{x}|\theta^I \Phi^I\right)}$$

generic $x^\mu(t)$ closed loop, $\theta^I(t)$ unit 6-vector: "locally" susy

• max 16 susy – $\frac{1}{2}$ BPS: infinite straight line (or circle), θ^I=const

$x^0 = t \in (-\infty, \infty),\ \theta^I \Phi^I = \Phi_6$

$$W = {\rm tr} P e^{\int dt (iA_t + \Phi_6)}$$

• local $O_i(t)$ on WL: gauge inv correlator [Drukker, Kawamoto 06]

$$\Big\langle O_1(t_1) O_2(t_2) \cdots O_n(t_n) \Big\rangle$$

$$\equiv \langle {\rm tr} P \big[O_1(t_1)\, e^{\int dt (iA_t + \Phi_6)}\, O_2(t_2)\ \cdots\ O_n(t_n)\, e^{\int dt (iA_t + \Phi_6)} \big] \rangle$$

• operator insertions are equivalent to deformations of WL

[Drukker, Kawamoto:06; Cooke, Dekel, Drukker:17]

• $O(t)$: $OSp(4^*|4)$ reps with dim Δ and rep of "internal" $SO(3) \times SO(5)$

• correlators define "defect" CFT_1 on the line
[Drukker et al:06; Sakaguchi, Yoshida:07; Cooke et al:17]
determined by spectrum of dims and OPE coeffs

• $\Big\langle ... \Big\rangle$ correlators satisfy all usual properties of CFT

• "elementary excitations": short rep of $OSp(4^*|4)$
8 bosonic + 8 fermionic operators with protected Δ:
5 scalars: Φ^a $(\Delta = 1)$ that do not couple to WL;
3 "displacement operators": $\mathbb{F}_{ti} \equiv iF_{ti} + D_i\Phi_6$ $(\Delta = 2)$

• protected dims: exact 2-point functions in planar SYM

$$\Big\langle \Phi^a(t_1)\Phi^b(t_2) \Big\rangle = \delta^{ab}\frac{C_\Phi(\lambda)}{t_{12}^2}, \quad \Big\langle \mathbb{F}_{ti}(t_1)\mathbb{F}_{tj}(t_2) \Big\rangle = \delta_{ij}\frac{C_\mathbb{F}(\lambda)}{t_{12}^4}$$

$$C_\Phi(\lambda) = 2B(\lambda)\,, \quad C_\mathbb{F}(\lambda) = 12B(\lambda)\,, \quad B(\lambda) = \frac{\sqrt{\lambda}\, I_2(\sqrt{\lambda})}{4\pi^2\, I_1(\sqrt{\lambda})}$$

$B(\lambda)$ – Bremsstrahlung function [Correa, Henn, Maldacena, Sever:12]

• 3-point functions of elementary bosonic operators
vanish by $SO(3) \times SO(5)$ symmetry

• 4-point: $G(t_1, ..., t_2; \lambda)$ constrained by 1d conf symm

$\mathrm{AdS}_5 \times S^5$ string theory

• WL $\rightarrow$ open string minimal surfaces in AdS_5 ending on contour defining WL operator at the boundary

• $\frac{1}{2}$-BPS Wilson line or circle:
minimal surface= AdS_2 embedded in AdS_5

• fundamental open string stretched in AdS_5:
preserves same $OSp(4^*|4)$ as $\frac{1}{2}$-BPS WL
1d conf group $SO(2,1)$ realized as isometry of AdS_2

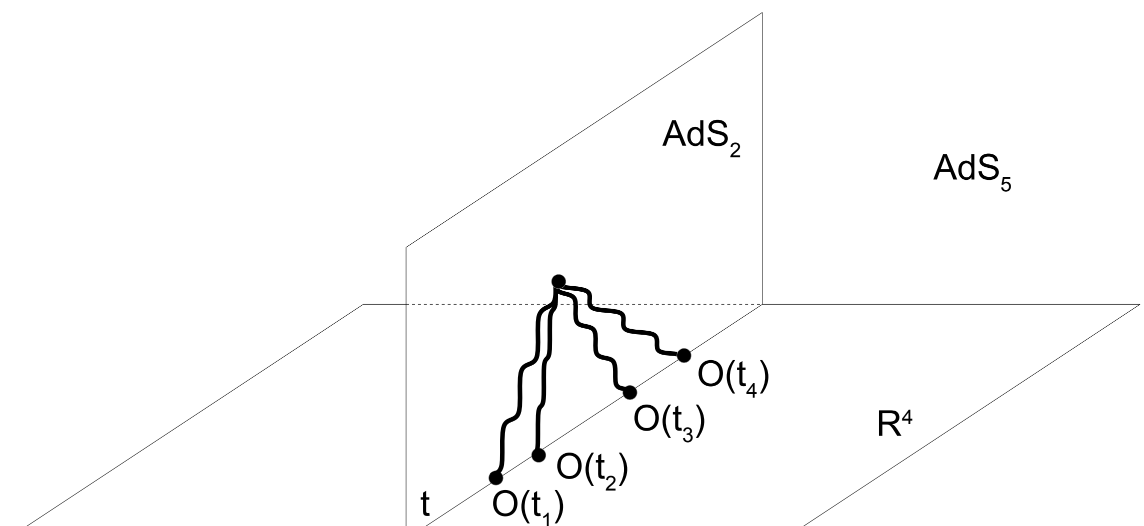
• expanding string action around AdS_2 surface:
AdS_2 multiplet of fluctuations transverse to string:
5 ($m^2 = 0$) scalars y^a in S^5; 3 ($m^2 = 2$) scalars x^i in AdS_5;
8 ($m^2 = 1$) fermions [Drukker, Gross, AT:00]

• identify 8+8 fields in AdS_2 with elementary CFT_1 insertions
[Sakaguchi, Yoshida:07; Faraggi, Pando Zayas:11; Fiol et al:13]

• AdS/CFT: add open-string sector (strings end at bndry) $\to$
gauge-inv operators = WL with insertions of local operators
• insertions of ops with protected dims
dual to "light" fields on AdS_2 string world-sheet
• $m^2 = \Delta(\Delta - d)$ for AdS_{d+1} scalar masses and CFT_d dims:
massless S^5 fields y^a dual to Φ^a in CFT_1 with $\Delta = 1$
massive AdS_5 fields x^i dual to $\mathbb{F}_{ti}$ with $\Delta = 2$

Strategy:
string action $\to$ interaction vertices for "light" AdS_2 fields
$\to$ tree-level Witten diagrams in AdS_2 $\to$ prediction for
4-point functions of protected ops on WL:
expansion parameter $\frac{1}{\sqrt{\lambda}}$: $\quad S = \frac{\sqrt{\lambda}}{2\pi} \int d^2\sigma \sqrt{h} \partial x \partial x + ...$
(cf. $\frac{1}{N^2}$ in AdS_5 sugra: $S = N^2 \int d^5x \sqrt{g} R + ...$)

• AdS_2 QFT: expect superstring action UV finite
$\rightarrow AdS_2/CFT_1$ duality should hold for any $T = \frac{\sqrt{\lambda}}{2\pi}$
• compute tree-level 4-point functions
use OPE to extract strong coupling corrections to dims
of "2-particle" ops built of 2 of protected insertions: $\Phi\partial_t^n\Phi$, etc.
compare with localization to YM_2, etc.



$AdS_5 \times S^5$ string in static gauge $\to$ AdS_2 QFT

bosonic part of superstring action in $AdS_5 \times S^5$ $\quad (T = \frac{\sqrt{\lambda}}{2\pi})$

$$S_B = \frac{1}{2} T \int d^2\sigma \sqrt{h}\, h^{\mu\nu} \Big[\frac{1}{z^2} \left(\partial_\mu x^r \partial_\nu x^r + \partial_\mu z \partial_\nu z \right) + \frac{\partial_\mu y^a \partial_\nu y^a}{(1 + \frac{1}{4} y^2)^2} \Big]$$

$\sigma^\mu = (t, s), \;\; r = (0, i) = (0, 1, 2, 3), \;\; a = 1, ..., 5$

minimal surface for straight Wilson line at Euclidean boundary

$$z = s \,, \qquad x^0 = t \,, \qquad\qquad x^i = 0 \,, \quad y^a = 0$$

induced metric is AdS_2: $\;\; g_{\mu\nu} d\sigma^\mu d\sigma^\nu = \frac{1}{s^2}(dt^2 + ds^2)$.

• Aim: study correlators of small fluctuations of "transverse"
2d fields (x^i, y^a) near AdS_2 minimal surface

$$ds^2_{AdS_5} = \frac{(1 + \frac{1}{4} x^2)^2}{(1 - \frac{1}{4} x^2)^2} ds^2_{AdS_2} + \frac{dx^i dx^i}{(1 - \frac{1}{4} x^2)^2} \,, \qquad ds^2_{AdS_2} = \frac{1}{z^2} (dx_0^2 + dz^2)$$

• Nambu action in static gauge $\quad S_B = T\int d^2\sigma\sqrt{h} = T\int d^2\sigma\ L_B$

$$h_{\mu\nu} = \frac{(1+\frac{1}{4}x^2)^2}{(1-\frac{1}{4}x^2)^2}\, g_{\mu\nu}(\sigma) + \frac{\partial_\mu x^i \partial_\nu x^i}{(1-\frac{1}{4}x^2)^2} + \frac{\partial_\mu y^a \partial_\nu y^a}{(1+\frac{1}{4}y^2)^2}\ , \qquad g_{\mu\nu} = \frac{1}{s^2}\delta_{\mu\nu}$$

• action of straight fundamental string in $\mathrm{AdS}_5 \times S^5$ along z:

2d theory of 3+5 scalars + 8 fermions in AdS_2

with $SO(2,1) \times [SO(3) \times SO(6)]$ symmetry

$$L_B = \ L_2 + L_{4x} + L_{2x,2y} + L_{4y} + ...$$

$$L_2 = \tfrac{1}{2} g^{\mu\nu}\partial_\mu x^i \partial_\nu x^i + x^i x^i + \tfrac{1}{2} g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a$$

$$L_{4x} = \ \tfrac{1}{8}(g^{\mu\nu}\partial_\mu x^i \partial_\nu x^i)^2 - \tfrac{1}{4}(g^{\mu\nu}\partial_\mu x^i \partial_\nu x^j)\ (g^{\rho\kappa}\partial_\rho x^i \partial_\kappa x^j)$$
$$+\tfrac{1}{4} x^i x^i (g^{\mu\nu}\partial_\mu x^j \partial_\nu x^j) + \tfrac{1}{2} x^i x^i\ x^j x^j$$

$$L_{2x,2y} = \tfrac{1}{4}(g^{\mu\nu}\partial_\mu x^i \partial_\nu x^i)(g^{\rho\kappa}\partial_\rho y^a \partial_\kappa y^a) - \tfrac{1}{2}(g^{\mu\nu}\partial_\mu x^i \partial_\nu y^a)(g^{\rho\kappa}\partial_\rho x^i \partial_\kappa y^a)$$

$$L_{4y} = \ -\tfrac{1}{4}(y^b y^b)(g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a) + \tfrac{1}{8}(g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a)^2$$
$$-\tfrac{1}{4}(g^{\mu\nu}\partial_\mu y^a \partial_\nu y^b)\ (g^{\rho\kappa}\partial_\rho y^a \partial_\kappa y^b)$$

x^i= 3$\times[m^2 = 2]$ and y^a= 5 $\times[m^2 = 0]$ + 8 fermions with $m^2 = 1$ in AdS_2

• 2d theory is UV finite, dual to CFT_1 for any $T = \frac{\sqrt{\lambda}}{2\pi}$

• bndry correlators $\Big\langle O(t_1)O(t_2)...O(t_n)\Big\rangle$ reproduced

by AdS_2 amplitudes of string sigma model (series in $\frac{1}{\sqrt{\lambda}}$)

$$\Big\langle O(t_1)O(t_2)...O(t_n)\Big\rangle_{\rm SYM} = \langle X(t_1)X(t_2)....X(t_n)\rangle_{{\rm AdS}_2}, \qquad X=(x,y)$$

e.g tree Witten diagrams with bulk-to-bndry props

• $y^a \to O \sim \Phi^a$ $(a=1,...,5)$ with $\Delta=1$

$x^i \to O \sim \mathbb{F}_{it}$ $(i=1,2,3)$ with $\Delta=2$

• L_B: no 3-vertices; 4-point tree-level correlators from 4-vertices

• 4-point function of $O_\Delta(t)$ restricted by $SO(2,1)$

$$\langle O_\Delta(t_1)O_\Delta(t_2)O_\Delta(t_3)O_\Delta(t_4)\rangle = \frac{1}{(t_{12}t_{34})^{2\Delta}}\,\mathcal{G}(\chi)\,, \qquad \chi=\frac{t_{12}t_{34}}{t_{13}t_{24}}$$

$$u \equiv \frac{t_{12}^2t_{34}^2}{t_{13}^2t_{24}^2}=\chi^2\,, \qquad v\equiv \frac{t_{14}^2t_{23}^2}{t_{13}^2t_{24}^2}=(1-\chi)^2$$

• $\mathcal{G}(\chi)$: OPE in conf blocks in $d=1$ [Dolan, Osborn:11]

$$\mathcal{G}(\chi)=\sum_h c_{\Delta,\Delta;h}\,\chi^h\,{}_2F_1(h,h,2h,\chi)$$

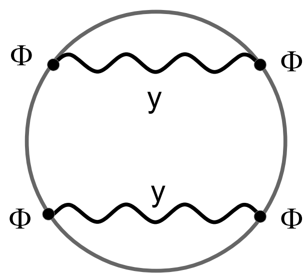
Example: 4-point function of S^5 fluctuations

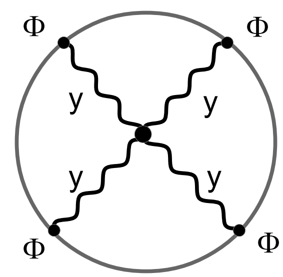
$$\left\langle \Phi^{a_1}(t_1)\Phi^{a_2}(t_2)\Phi^{a_3}(t_3)\Phi^{a_4}(t_4) \right\rangle$$

$$= \langle y^{a_1}(t_1) y^{a_2}(t_2) y^{a_3}(t_3) y^{a_4}(t_4) \rangle_{\mathrm{AdS}_2} = \frac{\left[C_\Phi(\lambda)\right]^2}{t_{12}^2 t_{34}^2} G^{a_1 a_2 a_3 a_4}(\chi)$$

$$\langle y^{a_1}(t_1) y^{a_2}(t_2) \rangle_{\mathrm{AdS}_2} = \left\langle \Phi^{a_1}(t_1)\Phi^{a_2}(t_2) \right\rangle = \delta^{a_1 a_2} \frac{C_\Phi(\lambda)}{t_{12}^2}$$

$G^{a_1 a_2 a_3 a_4}(\chi) = \delta^{a_1 a_2}\delta^{a_3 a_4} + O(\chi)$ – non-trivial function of λ





$$\Phi^{a_1a_2a_3a_4}_{\text{conn}} = \frac{(\mathcal{C}_{\Delta=1})^2}{t_{12}^2 t_{34}^2} G^{a_1a_2a_3a_4}_{(1)}(\chi)$$

$$G_S^{(1)}(\chi) = -\frac{2\left(\chi^4-4\chi^3+9\chi^2-10\chi+5\right)}{5(\chi-1)^2} + \frac{\chi^2\left(2\chi^4-11\chi^3+21\chi^2-20\chi+10\right)}{5(\chi-1)^3}\log|\chi| - \frac{2\chi^4-5\chi^3-5\chi+10}{5\chi}\log|1-\chi|\,,$$

$$G_T^{(1)}(\chi) = -\frac{\chi^2\left(2\chi^2-3\chi+3\right)}{2(\chi-1)^2} + \frac{\chi^4\left(\chi^2-3\chi+3\right)}{(\chi-1)^3}\log|\chi| - \chi^3\log|1-\chi|\,,$$

$$G_A^{(1)}(\chi) = \frac{\chi\left(-2\chi^3+5\chi^2-3\chi+2\right)}{2(\chi-1)^2} + \frac{\chi^3\left(\chi^3-4\chi^2+6\chi-4\right)}{(\chi-1)^3}\log|\chi| - (\chi^3-\chi^2-1)\log|1-\chi|$$

• allows to determine strong-coupling asymptotics of CFT_1 correlators

• Recent progress: combining with bootstrap and integrability
OPE coefficients of unprotected operators
[Niarchos, Papageorgakis et al 23;
Cavaglia, Gromov, Julius, Preti 22; Barrat, Lliendo, Peveri 22; Bliard 22;
Liendo, Meneghelli, Mitev 18; ...]

M2 branes and 2-defects: $AdS_7/CFT_6 \to AdS_3/CFT_2$

[Drukker, Giombi, Zhou, AT 2020]

analog AdS_2/CFT_1: defect AdS_3/CFT_2

- surface operators in 6d (2,0) theory at large N:
dual description as probe M2 ending on defect in $AdS_7 \times S^4$
- $\frac{1}{2}$-BPS operator: R^2 or S^2 conformal defect in CFT_6
- dual M2 brane has AdS_3 geometry: encodes 2d conf symm
defines defect CFT_2 for surface operator
- $\perp$ fluctuations of M2 brane: dual to protected ops on 2-surface
- M2 brane action: AdS_3 Witten diagrams $\to$ strong coupling expansion
of defect CFT correlators (+OPE, superconformal Ward identities, etc.)
- 1-loop M2 brane dets: conf anomaly of spherical defect at order N^0

• (2,0) 6d CFT on multiple M5-branes: $SU(N)$ generalization of (2,0) tensor multiplet: B_{mn} with $H_{mnk} = H^*_{mnk}$ + 5 real scalars Φ^I + 4 symplectic Majorana fermions

• in abelian theory: locally-susy surface operator (cf. M5's described by M2's ending on strings coupled to B)

$$W = \exp\Big(\int d^2x \big[i\, \tfrac{1}{2} \mathrm{e}^{\mu\nu} \partial_\mu X^m \partial_\nu X^n \, B_{mn}(X) + \sqrt{g(X)}\, \Phi_5(X) \big] \Big)$$

defect in (1,2) plane: $\exp \int d^2x \big[i\, B_{12}(X) + \Phi_5(X) \big]$

• $OSp(8^*|4)$ broken to $[OSp(4^*|2)]^2$: bosonic $SO(2,2) \times SO(4) \times SO(4)$

• ops on defect: short multiplet of 4 $\perp$ scalars Φ_a, $\Delta = 2$,
4 displacement ops $\mathrm{D}_i = \mathbb{H}_{12i} \equiv iH_{12i} + \partial_i \Phi_5$, $\Delta = 3$
8 fermions with $\Delta = 5/2$

• dual description of $\frac{1}{2}$-BPS surface operator:
probe M2-brane with 3-volume ending on 2-plane at bndry $\mathbb{R}^6$ of AdS_7 stretched along z of AdS_7 and localized at point in S^4

• action of M2: induced 3-geometry in static gauge is AdS_3
transverse fluctuations: 4 y^a (S^4 fluctuations) with $m^2 = 0$;
4 x^i (AdS_7 flucts $\perp$ 3-surface) with $m^2 = 3$; 8 fermions with $m^2 = \frac{9}{4}$
• correlators of these define 2d defect CFT
• $\mathrm{AdS}_3/\mathrm{CFT}_2$: dual boundary ops should have dims $\Delta = 2, 3$ and $\frac{5}{2}$
matching those of ops on the defect
• correlators of $X^I = (\mathrm{x}^i, y^a)$ in inverse eff tension expansion
$\mathrm{T}_2 = a^3 T_2 = \frac{2}{\pi} N$ (a = radius of AdS_7)
define large N limit of 6d correlators of $\mathcal{O}_I = (\mathbb{H}_{12i}, \Phi_a)$ on defect

$$\Big\langle \mathcal{O}(\vec{x}_1) \cdots \mathcal{O}(\vec{x}_n) \Big\rangle = \langle X(\vec{x}_1) \cdots X(\vec{x}_n) \rangle_{\mathrm{AdS}_3}$$

• cf. WL in $\mathcal{N} = 4$ SYM: there can also compute
weak-coupling expansion of correlators on gauge side
but not in (2,0) 6d theory lacking intrinsic definition; can mimic
in abelian 6d tensor multiplet [Gustavsson 04] $\langle \mathrm{D}^i(\vec{x}_1)\mathrm{D}^j(\vec{x}_2) \rangle \sim \frac{1}{|\vec{x}_{12}|^6}$

Membrane action in AdS$_7 \times$ S^4

M2-brane in AdS$_7 \times S^4$ dual to theory on N M5's

$$ds^2 = a^2 \left[ds^2_{\rm AdS_7} + \tfrac{1}{4} ds^2_{S^4} \right] , \qquad a^3 = 8\pi N \ell_P^3$$

$$F_4 = \pi^2 a^3 \Omega_4 \, , \qquad \int_{S^4} \Omega_4 = 1 \, , \qquad {\rm vol}(S^4) = \tfrac{8\pi^2}{3}$$

$$S = S_1 + S_2 \, : \qquad S_1 = T_2 \int d^3x \, \sqrt{\det h_{\mu\nu}} \, ,$$

$$h_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N G_{MN}(X)$$

$$S_2 = -i T_2 \int d^3x \, \tfrac{1}{3!} \epsilon^{\mu\nu\lambda} C_{MNK}(X) \, \partial_\mu X^M \partial_\nu X^N \partial_\lambda X^K$$

$$T_2 = (2\pi)^{2/3} (2\kappa^2_{11})^{-1/3} = \tfrac{1}{(2\pi)^2 \ell_P^3}$$

• M2 solution: ending on R^2 or S^2, induced AdS$_3$ geometry

(cf. M2 intersecting stack of M5 over 2-plane is $\frac{1}{2}$-BPS)

• effective tension (cf. $\mathrm{T}_1 = \frac{a^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$ for string in AdS$_5 \times S^5$)

$$\mathrm{T}_2 = a^3 T_2 = \frac{2}{\pi} N$$

• quantum M2 brane corrections: $\frac{1}{N}$ corrections in (2,0) CFT$_6$

• p-brane in AdS_{d+1} with world volume ending along a p-plane

$$ds^2_{d+1} = \frac{(1+\frac{1}{4}\mathrm{x}^2)^2}{(1-\frac{1}{4}\mathrm{x}^2)^2} ds^2_{p+1} + \frac{d\mathrm{x}^i d\mathrm{x}^i}{(1-\frac{1}{4}\mathrm{x}^2)^2}\,, \qquad ds^2_{p+1} = \frac{1}{\mathrm{z}^2}(d\mathrm{z}^2 + d\mathrm{x}^v d\mathrm{x}^v)$$

$v = 1, \cdots, p$ and $i = 1, \cdots, d-p$.
minimal surface ending on p-plane at the boundary

$$\mathrm{x}^v = x^v\,, \quad \mathrm{z} = z\,, \quad \mathrm{x}^i = 0\,, \quad ds^2_{p+1} = \frac{1}{z^2}(dz^2 + dx^v dx^v) \equiv g_{\mu\nu}(x) dx^\mu dx^\nu$$

• static gauge in the p-brane action in $\mathrm{AdS}_{d+1} \times S^n$

$$S_1 = \mathrm{T}_p \int d^{p+1}x \sqrt{\det\Big[\frac{(1+\frac{1}{4}\mathrm{x}^2)^2}{(1-\frac{1}{4}\mathrm{x}^2)^2} g_{\mu\nu} + \frac{\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^i}{(1-\frac{1}{4}\mathrm{x}^2)^2} + \frac{\partial_\mu y^a \partial_\nu y^a}{(1+y^2)^2}\Big]}$$

• expand in powers of fluctuations x^i and y^a

$$L = L_2 + L_{4\mathrm{x}} + L_{2\mathrm{x},2y} + L_{4y} + \dots$$
$$L_2 = \tfrac{1}{2}\big[g^{\mu\nu}\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^i + (p+1)\,\mathrm{x}^i \mathrm{x}^i\big] + \tfrac{1}{2} g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a$$

$$L_{4\mathrm{x}} = \tfrac{1}{8}(g^{\mu\nu}\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^i)^2 - \tfrac{1}{4}(g^{\mu\nu}\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^j)\,(g^{\rho\kappa}\partial_\rho \mathrm{x}^i \partial_\kappa \mathrm{x}^j)$$
$$+\tfrac{1}{4}p\,\mathrm{x}^i\mathrm{x}^i\,g^{\mu\nu}\partial_\mu \mathrm{x}^j \partial_\nu \mathrm{x}^j + \tfrac{1}{8}(p+1)^2 \mathrm{x}^i\mathrm{x}^i\,\mathrm{x}^j\mathrm{x}^j\,,$$
$$L_{2\mathrm{x},2y} = \tfrac{1}{4}(g^{\mu\nu}\partial_\mu \mathrm{x}^i \partial_\nu \mathrm{x}^i)\,(g^{\rho\kappa}\partial_\rho y^a \partial_\kappa y^a) - \tfrac{1}{2}(g^{\mu\nu}\partial_\mu \mathrm{x}^i \partial_\nu y^a)\,(g^{\rho\kappa}\partial_\rho \mathrm{x}^i \partial_\kappa y^a)$$
$$+\tfrac{1}{4}(p-1)\mathrm{x}^i\mathrm{x}^i\,g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a\,,$$
$$L_{4y} = -y^b y^b\,g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a + \tfrac{1}{8}(g^{\mu\nu}\partial_\mu y^a \partial_\nu y^a)^2 - \tfrac{1}{4}(g^{\mu\nu}\partial_\mu y^a \partial_\nu y^b)\,(g^{\rho\kappa}\partial_\rho y^a \partial_\kappa y^b)$$

string: $p=1,\ d=4,\ n=5$; here $p=2,\ d=6,\ n=4$

get 4 AdS_7 x^i with $m^2=3$ and S^4 y^a with $m^2=0$ in $g_{\mu\nu}=AdS_3$

- $AdS_7 \times S^4$ supermembrane action $\to 8\times$ 3d fermions with $m=\frac{3}{2}$
- WZ term in M2 action ($Y^AY^A=1,\ A=1,...,5;\ Y^5=\frac{1-y^2}{1+y^2},\ Y^a=\frac{\frac{1}{2}y^a}{1+y^2}$)

$$S_2 = -iT_2\int C_3 = -\frac{iN}{32\pi}\int d^4x\,\epsilon_{ABCDE}\,\epsilon^{\mu\nu\lambda\rho}\,Y^A\partial_\mu Y^B\partial_\nu Y^C\partial_\lambda Y^D\partial_\rho Y^E$$

- topological WZ term should not be renormalized

$$S_1 = \frac{N}{\pi}\int d^3x\,\sqrt{g}\,g^{\mu\nu}\partial_\mu y^a\partial_\nu y^a + \ldots$$
$$S_2 = -\frac{iN}{2\pi}\int d^3x\,\epsilon^{\mu\nu\lambda}\,\epsilon_{abcd}\,y^a\partial_\mu y^b\partial_\nu y^c\partial_\lambda y^d + ...$$

- use this action to compute correlation functions as in AdS_2/CFT_1 case
- recent progress: [Drukker, Probst, Trepanier 20; Meneghelli, Trepanier 22, ...]

1-loop M2 partition function → defect conformal anomaly

[Drukker, Giombi, Zhou, AT 2020]

example of quantum M2 brane computation (cf. below)

- fluctuation dets near AdS_3 M2 solution with S^2 as boundary

$$F_{1-\text{loop}} = -\log Z = \tfrac{1}{2}\left[4\log\det(-\nabla^2+3) + 4\log\det(-\nabla^2) - 8\log\det\Delta_{\frac{1}{2}}\right]$$

no bulk log UV div in 3d, $F_{1-\text{loop}}$ finite in analytic regularization

$$F_0^{(\Delta)} = \frac{1}{2}\log\det(-\nabla^2+m^2)\Big|_{m^2=\Delta(\Delta-2)} = -\frac{(\Delta-1)^3}{12\pi}\,\text{vol}(AdS_3)$$

$$F_{\frac{1}{2}}^{(\Delta)} = \frac{1}{2}\log\det(-\nabla^2+\tfrac{1}{4}R+m^2)\Big|_{m=\Delta-1} = -\frac{(\Delta-1)}{12\pi}\left[\Delta(\Delta-2)+\frac{1}{4}\right]\text{vol}(AdS_3)$$

- introduce IR cutoff R for AdS_3 with S^2 boundary:

$$\text{vol}(AdS_3) = -2\pi\log \text{R}$$

$$F_{1-\text{loop}} = 4F_0^{(\Delta=3)} + 4F_0^{(\Delta=2)} - 8F_{1/2}^{(\Delta=5/2)} = 3\log \text{R}$$

• tree-level + 1-loop eff action: $F = F_{\rm tree} + F_{1-\rm loop}$

$$F_{\rm tree} = {\rm T}_2 {\rm vol}({\rm AdS}_3) = -2\pi {\rm T}_2 \log {\rm R} = -4N \log {\rm R}$$

• coeff of log R is a conformal anomaly coefficient in defect CFT
[surface defects: in general 3 conf anom coeffs [Schwimmer, Theisen 08]
only one if S^2 defect]

$$F \equiv -{\rm b}\ \log {\rm R}\,, \qquad {\rm b} = 2\pi {\rm T}_2 - 3 + \mathcal{O}({\rm T}_2^{-1}) = 4N - 3 + \mathcal{O}(N^{-1})$$

• consistent with result of [Estes et al 18; Jensen et al 18] from
entanglement entropy for "bubbling" M5-M2 geometry with M2-branes
corresp to $\frac{1}{2}$-BPS surface defect operator in (2,0) theory in large $su(N)$ rep
• interpolating to case of single M2 suggests exact expression

$$b = (N-1)(4 + N^{-1}) = 4N - 3 - N^{-1}$$

Wilson loop in ABJM theory and quantum M2 brane

[Giombi, AT 23]

• use localization to check AdS/CFT
at non-planar level
special example of $\frac{1}{2}$ BPS Wilson loop
$\rightarrow$ learn about structure of string loops in AdS
• match quantum M2-brane correction in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$
and ABJM theory localization result for WL
1-loop M2: sum of ∞ set of string loops in $\mathrm{AdS}_4 \times \mathrm{CP}^3$

- existence of quantum supermembrane theory
open question: formally non-renormalizable
but semiclassical 1-loop computations are ok:
no log UV div at 1-loop in 3d
[Duff, Inami, Pope, Sezgin, Stelle 88; Bergshoeff, Sezgin, Townsend 88; Forste 99; Drukker, Giombi, Zhou, AT 20]

- evidence that semiclassical quantization of
M2 brane is under control and gives non-trivial
check of AdS_4/CFT_3 beyond planar limit

Plan:

- localization results for WL in SYM and ABJM
- matching leading order string theory results
- higher genus strong coupling terms $\sum_n c_n(\frac{g_s^2}{T})^n$:
  $\exp(c_1\frac{g_s^2}{T})$ in SYM and $(\sin\frac{2\pi}{k})^{-1}$ in ABJM
- $(\sin\frac{2\pi}{k})^{-1}$ as 1-loop M2 brane contribution
- generalizations

$\frac{1}{2}$ BPS circular WL in SYM and ABJM

• $\mathcal{N} = 4 \;\; SU(N)$ SYM: $\qquad \mathcal{W} = \mathrm{Tr}\, P e^{\int (iA+\Phi)}$

Localization → Gaussian matrix model: any N, g^2_{YM}

[Erickson, Semenoff, Zarembo 00; Drukker, Gross 01; Pestun 07]

$$\langle \mathcal{W} \rangle = e^{\frac{N-1}{8N} g^2_{\mathrm{YM}}} L^1_{N-1}(-\tfrac{1}{4} g^2_{\mathrm{YM}})$$

$$L^1_n(x) \equiv \frac{1}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$$

Large N, fixed $\lambda = N g^2_{\mathrm{YM}}$:

$$\langle \mathcal{W} \rangle = N \Big[\frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{48 N^2} I_2(\sqrt{\lambda}) + \ldots \Big]$$

$$\lambda \gg 1 : \qquad \langle \mathcal{W} \rangle = \frac{N}{\lambda^{3/4}} \sqrt{\tfrac{2}{\pi}}\, e^{\sqrt{\lambda}} + \ldots$$

• ABJM:
3d $\mathcal{N} = 6 \; U(N)_k \times U(N)_{-k}$ CS + bi-fund matter
[Aharony, Bergman, Jafferis, Maldacena 08]

low-energy limit of N M2's on $\mathbb{C}^4/\mathbb{Z}_k$
$z_i \to e^{\frac{2\pi i}{k}} z_i, \quad i = 1,2,3,4$
large N: dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

large k with $\lambda = \frac{N}{k}$=fixed:
perturbative type IIA string on $\mathrm{AdS}_4 \times \mathrm{CP}^3$

can define $\frac{1}{2}$ BPS circular WL

localization $\to$ matrix model: for any $N,\ k>2$

[Drukker, Marino, Putrov 10; Klemm, Marino, Schiereck, Sarouush 12]

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}} \frac{\mathrm{Ai}\left[(\frac{\pi^2}{2}k)^{1/3}\left(N-\frac{k}{24}-\frac{7}{3k}\right)\right]}{\mathrm{Ai}\left[(\frac{\pi^2}{2}k)^{1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right]}$$

• large N at fixed k ("M-theory regime"):

$$\mathrm{Ai}(x)\Big|_{x\gg 1} \simeq \frac{e^{-\frac{2}{3}x^{3/2}}}{2\sqrt{\pi}\,x^{1/4}} \sum_{n=0}^{\infty} \frac{(-\frac{3}{4})^n\,\Gamma(n+\frac{5}{6})\Gamma(n+\frac{1}{6})}{2\pi\, n!\, x^{3n/2}}$$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}\left[1-\frac{\pi\left(k^2+32\right)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}}+\mathcal{O}(\frac{1}{N})\right]$$

- $N, k \gg 1$, $\lambda = \frac{N}{k}$: 't Hooft expansion ("string theory regime"):

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi\lambda}{N}}\, e^{\pi\sqrt{2\lambda}}\Big[1 - \frac{\pi}{24\sqrt{2}}\frac{1}{\sqrt{\lambda}} + \mathcal{O}(\frac{1}{N})\Big]$$

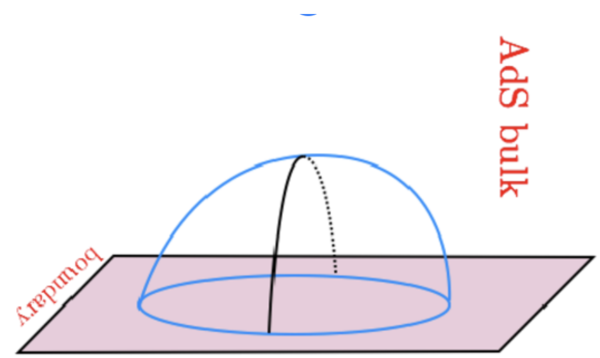
$$= \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}}\Big[1 + ...\Big]$$

- string in $\mathrm{AdS}_5 \times S^5$ and $\mathrm{AdS}_4 \times \mathrm{CP}^3$: g_s and $T = \frac{L^2_\mathrm{ads}}{2\pi\alpha'}$

$$\mathrm{SYM}: \quad g_\mathrm{s} = \frac{g^2_\mathrm{YM}}{4\pi} = \frac{\lambda}{4\pi N}\,, \qquad T = \frac{\sqrt{\lambda}}{2\pi}\,, \qquad \lambda = g^2_\mathrm{YM} N$$

$$\mathrm{ABJM}: \quad g_\mathrm{s} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad T = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \qquad \lambda = \frac{N}{k}$$

- $\langle \mathcal{W} \rangle$ as disk part f. of string near AdS_2 minimal surface



$$ds^2 = \frac{L^2_\mathrm{ads}}{z^2}(dr^2 + r^2 d\phi^2 + dx_s dx_s + dz^2), \quad r = \sqrt{1 - z^2}$$

$$\langle \mathcal{W} \rangle = Z_{\rm str} = \frac{1}{g_{\rm s}} Z_1 + \mathcal{O}(g_{\rm s})\,, \qquad Z_1 = \int [dx] ...\, e^{-T\int d^2\sigma\, \mathcal{L}}$$

$$\text{SYM}: \quad \langle \mathcal{W} \rangle = \sqrt{\frac{2}{\pi}} \frac{N}{\lambda^{3/4}}\, e^{\sqrt{\lambda}} + ... \; = \frac{1}{2\pi} \frac{\sqrt{T}}{g_{\rm s}} \; e^{2\pi T} + ...$$

$$\text{ABJM}: \quad \langle \mathcal{W} \rangle = \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}} + ... \; = \frac{1}{\sqrt{2\pi}} \frac{\sqrt{T}}{g_{\rm s}} \; e^{2\pi T} + ...$$

universal form at strong coupling [Giombi, AT 20]

$$\langle \mathcal{W} \rangle = {\rm c}_0 \frac{\sqrt{T}}{g_s} \; e^{2\pi T} \Big[1 + \mathcal{O}(T^{-1}) \Big] + \mathcal{O}(g_{\rm s})$$

$${\rm c}_0 = \frac{1}{(\sqrt{2\pi})^{d-3}}\,, \qquad d = 5,\ 4$$

reason: dual string theories in $\text{AdS}_n \times M^{10-n}$ are similar

- $e^{2\pi T} = e^{-T\text{vol}(\text{AdS}_2)}$, $\quad \text{vol}(\text{AdS}_2) = -2\pi$

[Berenstein, Corrado, Fischler, Maldacena 98]

- $\sqrt{T}$ from universal dependence of Z_1 on L_{ads}
- $\text{c}_0 = \frac{1}{(\sqrt{2\pi})^{d-3}} = \frac{1}{\sqrt{2\pi}}\,\bar{\text{c}}_0, \quad Z_1 \sim \bar{\text{c}}_0$

[Drukker, Gross, AT 00; Kruczenski, Tirziu 08; Buchbinder, AT 14; ...]

- extra $\frac{1}{\sqrt{2\pi}}$: sensitive to defn of GS

path integral measure; implicitly checked in
ratio of $\frac{1}{2}$ and $\frac{1}{4}$ BPS WL's [Medina-Rincon, Zarembo, AT 18]

- will be found directly in ABJM case

by quantum M2 brane computation

1-loop string partition function in $\mathrm{AdS}_d \times M^{10-d}$

near AdS_2 minimal surface

$$\log Z_1 = -\frac{1}{2}\log \frac{[\det(-\nabla^2+2)]^{d-2}\,[\det(-\nabla^2)]^{10-d}}{[\det(-\nabla^2+\frac{1}{2})]^{2d-2}\,[\det(-\nabla^2-\frac{1}{2})]^{10-2d}}$$

$$\log Z_1 = B_2 \log(L_{\rm ads}\,\Lambda) + \log \bar{c}_1\,, \qquad B_2 = \tfrac{1}{4\pi}\int d^2\sigma\,\sqrt{g}\,R^{(2)} = \chi$$

- $B_2 = \zeta_{\rm tot}(0) = \chi$: universal dependence on $L_{\rm ads}$

Λ should cancel against GS string measure: $\Lambda \to \frac{1}{\sqrt{\alpha'}}$

$$Z_1 \sim (\sqrt{T})^{\chi}\,, \qquad (Z_1)_{\rm disk} \sim \sqrt{T}\,, \qquad T = \frac{L_{\rm ads}^2}{2\pi\alpha'}$$

- from dets on disk ($d = 5,\ 4$): $\bar{c}_0 = \frac{1}{(\sqrt{2\pi})^{d-4}}$

Higher genus corrections: $\chi = 1 - 2h$

- disk with h handles: $g_s^{-1} \to g_s^{\chi}$, $\sqrt{T} \to (\sqrt{T})^{\chi}$
- thus prediction on string side:

$$\langle \mathcal{W} \rangle = e^{2\pi T} \sum_{h=0}^{\infty} c_h \left(\frac{g_s}{\sqrt{T}} \right)^{2h-1} \left[1 + \mathcal{O}(T^{-1}) \right]$$

remarkably is consistent with form of $\frac{1}{N}$ terms on gauge side

- SYM: $N \gg 1$, then $\lambda \gg 1$

$$\langle \mathcal{W} \rangle = e^{\frac{(N-1)\lambda}{8N^2}} L^1_{N-1}(-\tfrac{\lambda}{4N}) = e^{\sqrt{\lambda}} \sum_{h=0}^{\infty} \frac{\sqrt{2}}{96^h \sqrt{\pi}\, h!} \frac{\lambda^{\frac{3}{4}(2h-1)}}{N^{2h-1}} \left[1 + \mathcal{O}(\tfrac{1}{\sqrt{\lambda}}) \right]$$

- $\frac{g_s}{\sqrt{T}} \sim \frac{\lambda^{\frac{3}{4}}}{N}$ appears as expansion parameter
- from gauge theory: $c_h = \frac{1}{2\pi\, h!} \left(\frac{\pi}{12} \right)^h$, $c_0 = \frac{1}{2\pi}$

• large $T = \frac{\sqrt{\lambda}}{2\pi}$ terms at each order in $g_s = \frac{\lambda}{N}$: [Drukker, Gross]

$$\langle \mathcal{W} \rangle = W_1 \, e^{H} \left[1 + \mathcal{O}(T^{-1}) \right], \qquad W_1 = \frac{1}{2\pi} \frac{\sqrt{T}}{g_s} \, e^{2\pi T}$$

$$H \equiv \frac{\pi}{12} \frac{g_s^2}{T} = \frac{1}{96\pi} \frac{\lambda^{3/2}}{N^2}$$

conjectured interpretation: "handle operator"

• computing even 1-loop (torus) string correction is challenge

but will derive analog of $\exp(\frac{\pi}{12} \frac{g_s^2}{T})$ from quantum M2 in ABJM

$\frac{1}{N}$ expansion of $\frac{1}{2}$ BPS WL in ABJM

- in both $AdS_5 \times S^5$ and $AdS_4 \times CP^3$

universal form of expansion in small g_s, large T

$$\langle \mathcal{W} \rangle = e^{2\pi T} \frac{\sqrt{T}}{g_s} \Big(c_0 + \mathcal{O}(T^{-1}) + \frac{g_s^2}{T} \left[c_1 + \mathcal{O}(T^{-1}) \right] + ... \Big)$$

- ABJM: $\frac{g_s^2}{T} \sim \frac{\lambda^2}{N^2} = \frac{1}{k^2}$, corrections $T^{-1} \sim \frac{\sqrt{k}}{\sqrt{N}}$

exp of leading terms? no – from localization: $\frac{1}{\sin \frac{2\pi}{k}}$

$$\langle \mathcal{W} \rangle = \frac{1}{2 \sin \frac{2\pi}{k}} \, e^{\pi \sqrt{\frac{2N}{k}}} \left[1 + \mathcal{O}(\tfrac{1}{\sqrt{N}}) \right]$$

$$= \frac{1}{2 \sin \left(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} \right)} e^{2\pi T} \left[1 + O(T^{-1}) \right], \qquad \sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} = \frac{2\pi \lambda}{N} = \frac{2\pi}{k}$$

$$\frac{1}{2 \sin \left(\sqrt{\frac{\pi}{2}} \frac{g_s}{\sqrt{T}} \right)} = \frac{\sqrt{T}}{\sqrt{2\pi} g_s} \left[1 + \frac{\pi}{12} \frac{g_s^2}{T} + \frac{7\pi^2}{1440} \left(\frac{g_s^2}{T} \right)^2 + ... \right]$$

Main claim: $\frac{1}{\sin\frac{2\pi}{k}}$ comes from

1-loop M2 contribution on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

• large N, fixed k:

$\frac{1}{2}$ BPS WL described by M2-brane on $\mathrm{AdS}_2 \times S^1$

$$e^{-S_{\mathrm{M2}}} = e^{\pi\sqrt{\frac{2N}{k}}} \text{ from classical M2 action}$$

• 1-loop M2 correction $\to Z_1 = \frac{1}{\sin\frac{2\pi}{k}}$

• leading quantum M2 correction on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

describes large T terms at all orders in g_s

in IIA string theory on $\mathrm{AdS}_4 \times \mathrm{CP}^3$, i.e.

• highly non-trivial check of

$\mathrm{AdS}_4/\mathrm{CFT}_3$ duality to all orders in $\frac{1}{N}$

Review of basics

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left(R - \frac{1}{2 \cdot 4!} F_{mnk\ell} F^{mnk\ell} + \cdots \right)$$

M2 action in 11d background [Bergshoeff, Sezgin, Townsend 87]

$$S_{\rm M2} = T_2 \int d^3\sigma \left[\sqrt{-\det g_{mn}} + \hat{C}_3 \right]$$

$$g_{mn} = G_{MN}(x)\, \Pi_m^M \Pi_n^N + \ldots\,, \quad \hat{C}_3 = \tfrac{1}{6} \epsilon^{mnk}\, C_{MNK}(x)\, \Pi_m^M \Pi_n^N \Pi_k^K$$

$$\Pi_m^M = \partial_m x^M - i\bar{\theta}\Gamma^M \partial_m \theta\,, \qquad x^M = x^M(\sigma)$$

$$2\kappa_{11}^2 = (2\pi)^8\, \ell_P^9\,, \qquad T_2 = \left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{1/3} = \frac{1}{(2\pi)^2 \ell_P^3}$$

$$ds_{11}^2 = e^{-\frac{2}{3}\phi}\, ds_{10}^2 + e^{\frac{4}{3}\phi} (dx_{11} + e^{-\phi} A)^2, \qquad x_{11} \sim x_{11} + 2\pi \bar{R}_{11}$$

$$g_s = e^{\phi}; \quad R_{11} = g_s^{2/3} \bar{R}_{11}; \quad 2\kappa_{10}^2 = (2\pi)^7 g_s^2 \alpha'^4$$

- M2 wrapped on $x_{11} \to$ string [Duff, Howe, Inami, Stelle 87]

$$T_2 = \frac{1}{(2\pi)^2 \, \ell_P^3}, \qquad T_1 = 2\pi \, \bar{R}_{11} \, T_2 = \frac{1}{2\pi\alpha'}$$

- 11d M2-brane solution [Duff, Stelle 90] $\to \mathrm{AdS}_4 \times S^7$

$$ds_{11}^2 = \mathrm{L}^2 \Big(\frac{1}{4} ds^2_{\mathrm{AdS}_4} + ds^2_{S^7} \Big), \quad F_4 = dC_3 \sim \hat{\mathrm{N}} \, \epsilon_4,$$

$$\big(\frac{\mathrm{L}}{\ell_P}\big)^6 = 32\pi^2 N$$

- M2 on orbifold: $\quad \mathrm{AdS}_4 \times S^7/\mathbb{Z}_k, \qquad N \to Nk$

- S^7 as S^1 fibration over CP^3 and $\mathbb{Z}_k$ quotient

$$ds^2_{S^7} = ds^2_{\mathrm{CP}^3} + \frac{1}{k^2}(d\varphi + kA)^2\,, \qquad \varphi \equiv \varphi + 2\pi$$

$$ds^2_{\mathrm{CP}^3} = [\tfrac{\delta_{sr}}{1+|w|^2} - \tfrac{w_s \bar{w}_r}{(1+|w|^2)^2}] dw^s d\bar{w}^r$$

$$dA = i[\tfrac{\delta_{sr}}{1+|w|^2} - \tfrac{w_s \bar{w}_r}{(1+|w|^2)^2}] dw^r \wedge d\bar{w}^s$$

$$R_{11} = \frac{\mathrm{L}}{k}\,, \qquad \frac{\mathrm{L}}{\ell_P} = (32\pi^2 N k)^{1/6}$$

$$ds^2_{10} = L^2\Big(\frac{1}{4}ds^2_{AdS_4} + ds^2_{\mathrm{CP}^3}\Big), \qquad L = g_{\mathrm{s}}^{1/3}\,\mathrm{L}$$

$$g_{\mathrm{s}} = (\frac{\mathrm{L}}{k\,\ell_P})^{3/2} = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad \lambda = \frac{N}{k}$$

$$T = \frac{L^2_{\mathrm{ads}}}{2\pi\alpha'} = g_{\mathrm{s}}^{2/3}\frac{\mathrm{L}^2}{8\pi\alpha'} = \frac{\sqrt{\lambda}}{\sqrt{2}}\,, \qquad \frac{g_{\mathrm{s}}^2}{8\pi\,T} = \frac{\lambda^2}{N^2} = \frac{1}{k^2}$$

• M-theory expansion: $\frac{\mathrm{L}}{\ell_P} \gg 1$

$\mathrm{T}_2 \equiv T_2 \mathrm{L}^3 = \frac{1}{(2\pi)^2} \frac{\mathrm{L}^3}{\ell_P^3} = \frac{1}{\pi}\sqrt{2kN} \gg 1$

or large N for fixed k

• $\frac{1}{2}$ BPS WL: probe M2 brane intersecting AdS_4 boundary (multiple M2's) over line or circle

• compute M2 partition function for $\mathrm{T}_2 \gg 1$

compare to large N, fixed k expansion of $\langle \mathcal{W} \rangle$

• $\mathrm{AdS}_2 \times S^1$ M2 solution dual $\frac{1}{2}$-BPS Wilson loop:

wrapping S^1_φ of $S^7/\mathbb{Z}_k$ and AdS_2 of AdS_4

$S_{\mathrm{M2}} = \frac{1}{4}\mathrm{T}_2 \,\frac{2\pi}{k}\, \mathrm{vol}(\mathrm{AdS}_2) \ = -\pi\sqrt{\frac{2N}{k}}$

$e^{-S_{\mathrm{M2}}}$ matches leading factor in $\langle \mathcal{W} \rangle$

1-loop M2 brane partition function

• expand M2 action near $\mathrm{AdS}_2 \times S^1$ solution
fix 3d diff and κ-symm gauges $\to$ 8+8 3d fluctuations

• spectrum of fluctuations [Sakaguchi, Shin, Yoshida 10]
in static gauge: M2 coordinates
σ_1, σ_2 = AdS_2 directions; $\sigma_3 = \varphi$

• KK expansion of 3d fields in $\sigma_3 = (0, 2\pi)$:
tower $(n = 0, \pm 1, ...)$ of B+F 2d fields on AdS_2

• bosonic fluctuations in 2 $\perp$ AdS_4 directions:
tower of complex scalars η_n

$$m^2_{\eta_n} = \tfrac{1}{4}(kn-2)(kn-4)\,, \qquad n \in \mathbb{Z}$$

• fluctuations of CP^3 directions: 3 complex ζ^s_n $(s = 1, 2, 3)$

$$m^2_{\zeta^s_n} = \tfrac{1}{4}kn(kn+2)\,,$$

• fermions: tower of 8 two-component spinors

$$m_{\vartheta^a_n} = \tfrac{1}{2}kn \pm 1 \ (3{+}3 \text{ modes})\,, \quad m_{\vartheta^i_n} = \tfrac{1}{2}kn \ (2 \text{ modes})$$

• string limit $k \to \infty$: $n \neq 0$ modes decouple
$n = 0$: same as 2d string fluctuations around AdS_2
in IIA superstring on $AdS_4 \times CP^3$:
B: 2 of $m^2 = 2$; 6 of $m^2 = 0$;
F: 3+3 of $m = \pm 1$ and 2 of $m = 0$
• spectrum consistent with 2d susy: AdS_2 $\mathcal{N} = 1$ multiplets
scalar + Majorana fermion $m^2_{_B} = m_F(m_F - 1)$

- 1-loop M2 partition function on $AdS_2 \times S^1$: $\quad \mathrm{T}_2 = \mathrm{L}^3 T_2$

$$Z_{\rm M2} = Z_1 \, e^{-S_{\rm M2}} \Big[1 + \mathcal{O}\big(\frac{1}{\mathrm{T}_2}\big) \Big] \, , \qquad\qquad S_{M2} = -\frac{\pi}{k} \mathrm{T}_2$$

$$Z_1 = \prod_{n=-\infty}^{\infty} \mathcal{Z}_n \, , \qquad\qquad \mathcal{Z}_0 = \mathrm{AdS}_4 \times \mathrm{CP}^3 \text{ string on } \mathrm{AdS}_2$$

$$\mathcal{Z}_n = \frac{\Big[\det\Big(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}+1)^2\Big)\Big]^{\frac{3}{2}} \Big[\det\Big(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}-1)^2\Big)\Big]^{\frac{3}{2}} \det\Big(-\nabla^2 - \frac{1}{2} + (\frac{kn}{2}\Big)^2\Big)}{\det\Big(-\nabla^2 + \frac{1}{4}(kn-2)(kn-4)\Big) \Big[\det\Big(-\nabla^2 + \frac{1}{4}kn(kn+2)\Big)\Big]^3}$$

- compute dets by spectral zeta-function in AdS_2

[Drukker, Gross, AT 00; Buchbinder, AT 14]

$$\Gamma_1 = \tfrac{1}{2}\log\det(-\nabla^2 + m^2) = -\tfrac{1}{2}\zeta(0;m^2)\log\Lambda^2 - \tfrac{1}{2}\zeta'(0;m^2)$$

$$\zeta_B(0;m_B^2) = \frac{m_B^2}{2} + \frac{1}{6}$$

$$\zeta'_B(0;m_B^2) = -\tfrac{1}{12}(1+\log 2) - \int_0^{m_B^2+\frac{1}{4}} dx\, \psi(\sqrt{x}+\tfrac{1}{2})$$

$$\zeta_F(0;m_F) = -\frac{m_F^2}{2} + \frac{1}{12}$$

$$\zeta'_F(0;m_F) = -\tfrac{1}{6} + 2\log \mathrm{A} + |m_F| + \int_0^{m_F^2} dx\, \psi(\sqrt{x})$$

• cancellation of log UV ∞ in $\Gamma_1 = -\log Z_1$:

$$\zeta_{\rm tot}(0) = \tfrac{1}{2}\sum_{n\in\mathbb{Z}}\big(-2+4\big) = \sum_{n\in\mathbb{Z}} 1 = 1 + 2\zeta_R(0) = 0$$

contribution of all $n \neq 0$ massive KK modes
cancels log UV div of $AdS_4 \times CP^3$ string ($n = 0$)

• cancellation was to be expected:
no 1-loop log UV div in 3d theory
• Z_1 is thus finite:

$$\Gamma_1 = -\log Z_1 = -\tfrac{1}{2}\zeta'_{\rm tot}(0)\,, \qquad \zeta'_{\rm tot}(0) = \sum_{n=-\infty}^{\infty} \zeta'_{\rm tot}(0;n)$$

$$\zeta'_{\rm tot}(0;n) = 2\zeta'_{B}\big(0;\tfrac{1}{4}(kn-2)(kn-4)\big) + 6\zeta'_{B}\big(0;\tfrac{1}{4}kn(kn+2)\big) + 3\zeta'_F(0;\tfrac{kn}{2}+1) + 3\zeta'_F(0;\tfrac{kn}{2}-1) + 2\zeta'_F(0;\tfrac{kn}{2})$$

• combining B and F: remarkable simplifications
$n = 0$ string contribution =0 [Giombi, AT 20]; for $n > 0$:

$$\zeta'_{\rm tot}(0;n) + \zeta'_{\rm tot}(0;-n) = -2\log(\tfrac{k^2n^2}{4} - 1)\,, \quad k > 2$$

$$\Gamma_1 = \sum_{n=1}^{\infty} \log\big(\frac{k^2n^2}{4} - 1\big) = 2\sum_{n=1}^{\infty} \log\frac{kn}{2} + \sum_{n=1}^{\infty} \log\big(1 - \frac{4}{k^2n^2}\big)$$

$$\zeta_R(0) = -\tfrac{1}{2}, \qquad \zeta_R'(0) = -\tfrac{1}{2}\log(2\pi):$$

$$2\textstyle\sum_{n=1}^{\infty} \log\frac{kn}{2} = -\log\frac{k}{4\pi}$$

- Euler's relation: $\sin\pi x = \pi x \prod_{n=1}^{\infty}(1 - \frac{x^2}{n^2})$

$$\sum_{n=1}^{\infty} \log\big(1 - \tfrac{4}{k^2n^2}\big) = \log\Big[\prod_{n=1}^{\infty}\big(1 - \tfrac{4}{k^2n^2}\big)\Big] = \log\big(\frac{k}{2\pi}\sin\frac{2\pi}{k}\big)$$

- final result for $k > 2$ in precise agreement with localization

$$Z_1 = e^{-\Gamma_1} = \frac{1}{2\sin\frac{2\pi}{k}}$$

• final result $\Gamma_1 = \sum_{n=1}^{\infty} \log\left(\frac{k^2 n^2}{4} - 1\right)$
same as log of 1d determinant or contribution of loop of particle in inverted harmonic potential on circle $s \equiv s + 2\pi$

$$\Gamma = \tfrac{1}{2} \log \det{}' \left(-\tfrac{k^2}{4} \tfrac{d^2}{ds^2} - 1\right) = \sum_{n=1}^{\infty} \log\left(\tfrac{k^2}{4} n^2 - 1\right)$$

of all 2d fluctuation modes of M2 only 1 bosonic 1d mode survives after B-F cancellations
deeper reason? (cf. localization)
• note: Riemann ζ_R used above is standard way to define 1d determinants in QM path integral

• cases of $k = 1, 2$ require a separate treatment:

$$\Gamma_1^{k=1} = \log 4\,, \qquad Z_1^{k=1} = \tfrac{1}{4}$$

$$\Gamma_1^{k=2} = 0, \qquad Z_1^{k=2} = 1$$

localization result for $\langle \mathcal{W} \rangle$ is singular for $k = 1, 2$
may need reconsideration (cf. susy $\mathcal{N} = 6 \to 8$)

Generalizations and open problems

- $\frac{1}{\sqrt{N}}$ corrections: from higher M2 loops

expansion in effective M2 tension $\quad \mathrm{T}_2^{-1} = \frac{\pi}{\sqrt{2k}}\frac{1}{\sqrt{N}}$

$$\langle \mathcal{W} \rangle = \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\pi\sqrt{\frac{2N}{k}}}\Big[1 - \frac{\pi(k^2+32)}{24\sqrt{2}\,k^{3/2}}\frac{1}{\sqrt{N}} + \mathcal{O}\big(\frac{1}{N}\big)\Big]$$

$$= \frac{1}{2\sin\frac{2\pi}{k}}\, e^{\frac{\pi^2}{k}\mathrm{T}_2}\Big[1 - \frac{k^2+32}{24k}\frac{1}{\mathrm{T}_2} + \mathcal{O}\big(\frac{1}{\mathrm{T}_2^2}\big)\Big]$$

- $\frac{1}{\sqrt{N}} \sim$ 2-loop M2 contribution

UV finite despite apparent non-renormalizability?

• analogy: GS string in $AdS_5 \times S^5$ is formally non-renormalizable but 2-loop $\frac{1}{\sqrt{\lambda}}$ correction to cusp anom dim is finite: $E - S = f(\lambda) \log S$

$f(\lambda) = a_0\sqrt{\lambda} + a_1 + \frac{a_2}{\sqrt{\lambda}} + ...$

[Roiban, AT 07; Giombi, Ricci, Roiban, Vergu, AT 10]

matches $f(\lambda)$ in SYM [Basso, Korchemsky, Kotanski 07]

• similar 2-loop result in $AdS_4 \times CP^3$ string case also UV finite [Bianchi, Bianchi, Bres, Forini, Vescovi 14]

• conjecture: UV div cancel also at higher M2 loops

• GS in $AdS_5 \times S^5$ or $AdS_4 \times CP^3$ constrained by integrability; hidden symmetry also in M2 theory?

• generalization to $\frac{1}{6}$-BPS Wilson loop:
from localization [Klemm et al 12]

$$\langle W_{\frac{1}{6}} \rangle = \frac{i}{2 \sin \frac{2\pi}{k}} \sqrt{\frac{2N}{k}} \, e^{\pi \sqrt{\frac{2N}{k}}} (1 + \dots)$$

origin of $\sqrt{\frac{2N}{k}}$ as in string case [Drukker, Plefka, Young 08]
solution smeared over CP^1 in CP^3: 0-modes $(\sqrt{T})^2 \sim \sqrt{\lambda}$

M2: should also be smeared: $\mathrm{T}_2 \sim \sqrt{N}$
study fluctuations to get prefactor

• defect CFT defined by $\frac{1}{2}$-BPS WL
as in $\mathrm{AdS}_5 \times S^5$ or SYM case [Giombi, Roiban, AT 17]
or in $\mathrm{AdS}_7 \times S^4$ or (2,0) 6d case [Drukker, Giombi, Zhou, AT 20]
• studied in string $\mathrm{AdS}_4 \times \mathrm{CP}^3$ regime: $N, k \gg 1$, $\lambda = \frac{N}{k}$
[Bianchi, Bliard, Forini, Griguolo, Seminara 20;
Gorini, Griguolo, Guerrini, Penati, Seminara, Soresina 22]
• M-theory regime of large N, fixed k limit:
1d defect: M2 on $\mathrm{AdS}_2 \times S^1$
2d defect: M2 on AdS_3
• defect CFT interpretation of higher KK modes? correlators?
• conf anomaly of S^2 defect in 3d ABJM theory

$$\Gamma_{\text{tree+loop}} = -\mathrm{b}\,\mathrm{vol}(\mathrm{AdS}_3), \quad \mathrm{b} = b_0\sqrt{N} + b_1 + O(\tfrac{1}{\sqrt{N}})$$

$$b_0 = \pi\sqrt{\tfrac{2}{k}}, \quad b_1 = k\text{-independent number}$$

• Lesson: take quantum M2 brane seriously
use it to derive $1/N$ strong coupling corrections to
non-BPS observables not known
from localization or integrability
• example: cusp anom dim in ABJM
at strong coupling beyond planar limit [Giombi, AT]

$$f(\lambda, N) = \sqrt{2\lambda} - \frac{5}{2\pi}\log 2 + q_1(k) + \mathcal{O}(\frac{1}{\sqrt{\lambda}})$$

$$q_1 = \frac{2\pi}{3k^2} + \frac{2\pi^3}{45k^4} + ... = \frac{2\pi\lambda^2}{3N^2} + \frac{2\pi^3\lambda^4}{45N^4} + ...$$

• similar recent computation of M2 brane instanton
contribution to free energy of ABJM theory on S^3
[Beccaria, Giombi, AT 23]
generalizing string computation in $AdS_4 \times CP^3$
[Gautason, Puletti, van Muiden 23]