Workshop: QFT with boundaries Impurities and Defects,
Nordita, August 1, 2023

Holographic interfaces, QFTs on AdS and Wormholes

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Bibliography

Ongoing work with:

Ahmad Ghodsi, Francesco Nitti, Christopher Rosen, to appear soon

Ahmad Ghodsi, Francesco Nitti, Valentin Noury

Published in ArXiv:2209.12094

Introduction

• We shall address the following problems:

♠ Holographic interfaces between two QFTs.

♠ The notion of “proximity” in QFT.

♠ The dynamics of QFTs on AdS space.

♠ Euclidean wormholes.

• As we shall see there are some connections between such problems.

QFT on AdS

- This problem was first seriously adressed by Callan and Wilczek in 1990.

- Their interest was in IR physics.

- Their motivation were the IR divergences that plagued QCD perturbation theory and which made perturbative calculations hard to control.

- The important property of AdS space for this purpose was that even massless fields, had propagators that vanished exponentially as large distances, like massive fields in flat space.

- The reason is that the Laplacian and other relevant operators have a gap in AdS.

- Critical systems are described by mean field theory above the upper critical dimension. But AdS acts as an infinite-dimensional space. Therefore critical fluctuations should be weak in any dimension.

• Generically speaking, AdS is expected to "quench" strong IR physics.

• An extra ingredient is that the QFT on AdS must realize the AdS symmetry that is like conformal invariance in one-less dimension.

Callan+Wilczek

• The structure of instantons is also expected to be different:

♠ In flat space, in QCD we expect to have an instanton liquid rather than a (dilute) instanton gas.

Witten

♠ Above the deconfinement phase transition, we expect an instanton gas instead.

• In AdS an instanton gas is generically expected.

Calln+Wilczek

• Chiral invariance for fermions is broken by boundary conditions.

• An important ingredient for QFT in AdS: boundary conditions.

A confining gauge theory on AdS_4

• There are two types of boundary conditions: electric (Dirichlet) and magnetic (Neumann)

Aharony+Marolf+Rangamani

♠ With electric: gluons are allowed in AdS, they are gapped, and there is an $SU(N)$ global symmetry at weak coupling. Only boundary currents possible.

♠ With magnetic: electric charges are not allowed in bulk, there are O(1) degrees of freedom, and there is confinement (imposed by the bcs).

• There are also many other boundary conditions associated to subgroups.

• For asymptotically free gauge theories with Dirichlet boundary conditions a confinement/deconfinement phase transition is expected

Aharony+Berkooz+Tong+Yankielowicz

♠ $\Lambda L_{Ads} \gg 1$ Confining phase.

♠ $\Lambda L_{AdS} \ll 1$ Deconfined phase.

• With magnetic boundary conditions one expects confinement at all scales, and a free energy of $O(1)$. This is a kind of trivial confinement as no electric charges are allowed in the bulk.

• So far, the only clear criterion for confinement is the order of magnitude of the free energy: $O(1)$ or $O(N^2)$ when $N \to \infty$.

• Wilson loops do not provide an easy criterion for confinement, as for large Wilson loops, the area and the perimeter scale the same way, in global coordinates.

• It is possible that subleading differences may tell the difference.

• But in Poincaré coordinates there are two classes of loops with different behavior for length and area.

• However QFT on AdS in different coordinates gives rise to a different quantum theory.

Conformal Theories on AdS

- The prime example, N=4 SYM was analyzed in some detail.

Gaiotto+Witten,Aharony+Marolf+Rangamani, Aharony+Berdichevsky+Berkooz+Shamir

- Boundary conditions on R^4_+ that preserve supersymmetry have been classified, and there are many.

Gaiotto+Witten

- Upon a conformal transformation the theory can be put on AdS_4 in Poincaré coordinates.

- Dirichlet bc generically involve non-trivial vevs for three of the six scalars.

- At weak coupling the theory is generically non-confining.

- But at strong coupling some boundary conditions induce confinement.

• For example, using S-duality, the $g >> 1$ theory with a Higgs condensate is mapped to a $g << 1$ theory with a magnetic condensate that should be confining.

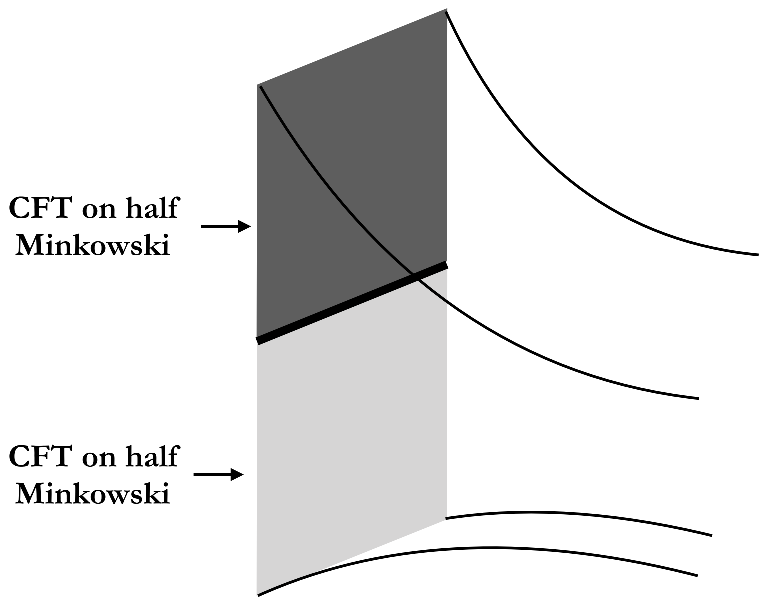
• In particular, S-duality interchanges (among others) Dirichlet and Neumann bc.

• With Neumann bc no order parameter exists that distinguishes a confining from a non-confining phase.

• Therefore, no sharp transition is expected in accordance with the large susy.

• The finite-temperature behavior is not understood.

(Holographic) Interfaces



• We may do a conformal transformation on each of the pieces to map it to AdS in Poincaré coordinates with the boundary at the interface.

• Clearly the two boundaries touch on the interface.

The holographic picture

• A natural metric anzatz for the ground state of a QFT$_d$ on AdS$_d$ is

$$ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu \qquad (1)$$

where $\zeta_{\mu\nu}$ the unit radius AdS$_d$ metric.

• The asymptotics of e^A near the boundary $u \to -\infty$ control the source for the radius of the AdS$_d$ slice metric.

• In the CFT case the bulk solution is global AdS$_{d+1}$ sliced with AdS$_d$ slices and

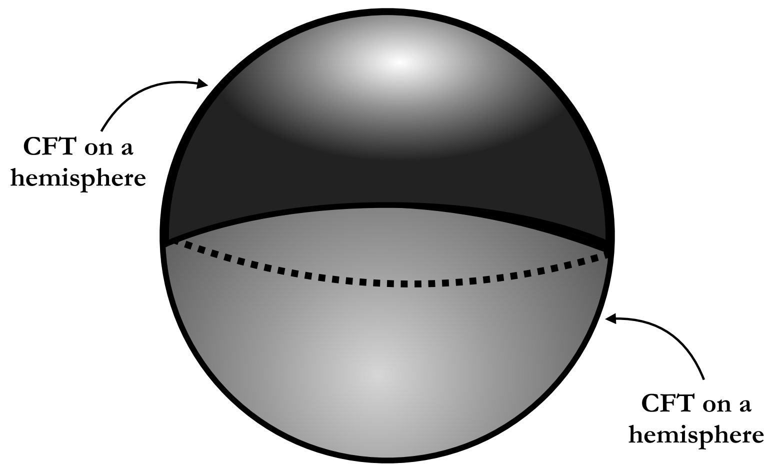
$$e^A = \cosh\frac{u}{\ell}$$

and $-\infty < u < +\infty$. This is a non-monotonic scale factor.

• The metric has two AdS boundaries. One, B_+, at $u = -\infty$ and another B_- at $u = +\infty$.

• The metric is locally AdS, and can be mapped to global AdS by a (large) diffeomorphism.

• In the Euclidean case, the two boundaries are isomorphic to $B^d_\pm$ and they intersect at the equator forming the single boundary S^{d-1} of AdS$_d$.



• Similar remarks hold for the Minkowski signature AdS space.

• Unlike the case of non-negative curvature slices, in the case of AdS slices, the negative curvature of the slice is responsible for the scale factor e^A NOT being monotonic.

• In the bulk AdS case, corresponding to a CFT on AdS$_d$, the gravitational solution is interpreted as two copies of the (same) CFT:

one on $B^+ \sim AdS_d$ and the other on $B_- \sim AdS_d$.

• The boundaries of B_+ and B_+ are common and are isomorphic to the equator of the S^{d-1}.

• Therefore the interpretation is of two copies of a CFT on AdS_d with common boundary and transparent boundary conditions on the common boundary

• However, we may turn on more fields and in general the two UV CFTs can be different.

Wormholes versus interfaces

• The general case with scalar operators (and RG flows) turned on and with the asymptotic metric a general negative constant curvature metric $\zeta_{\mu\nu}$ is still described by the ansatz

$$ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu \tag{2}$$

with

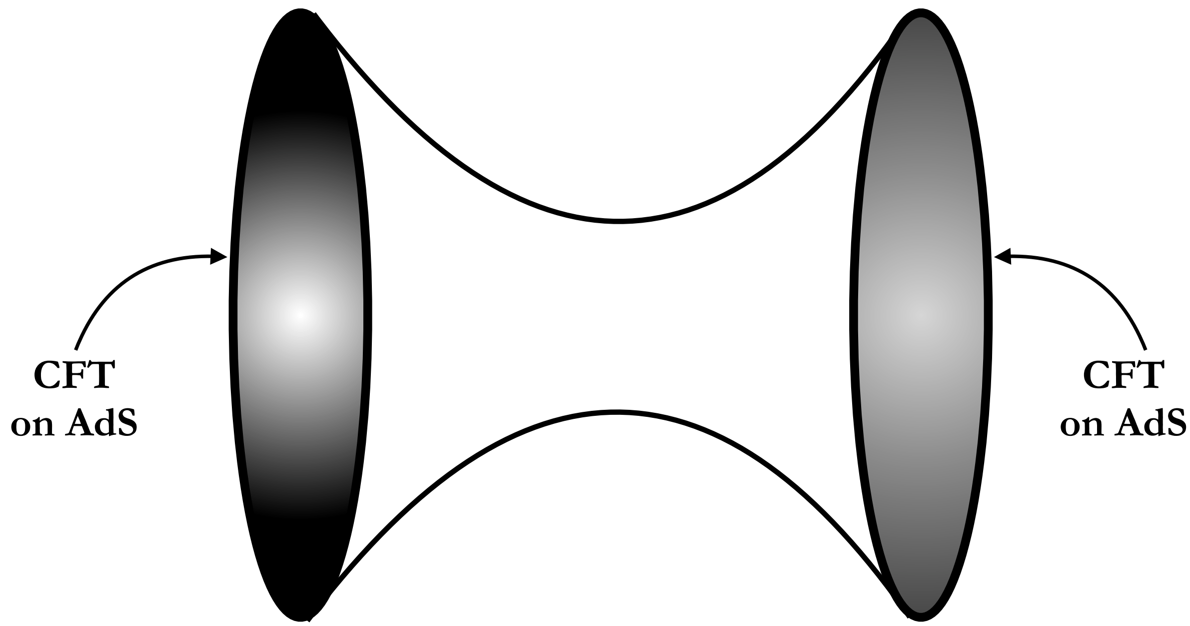
$$R(\zeta)_{\mu\nu} = -\frac{d-1}{L^2}\zeta_{\mu\nu}$$

while other fields, (like scalars) can change continuously between $-\infty < u < +\infty$.

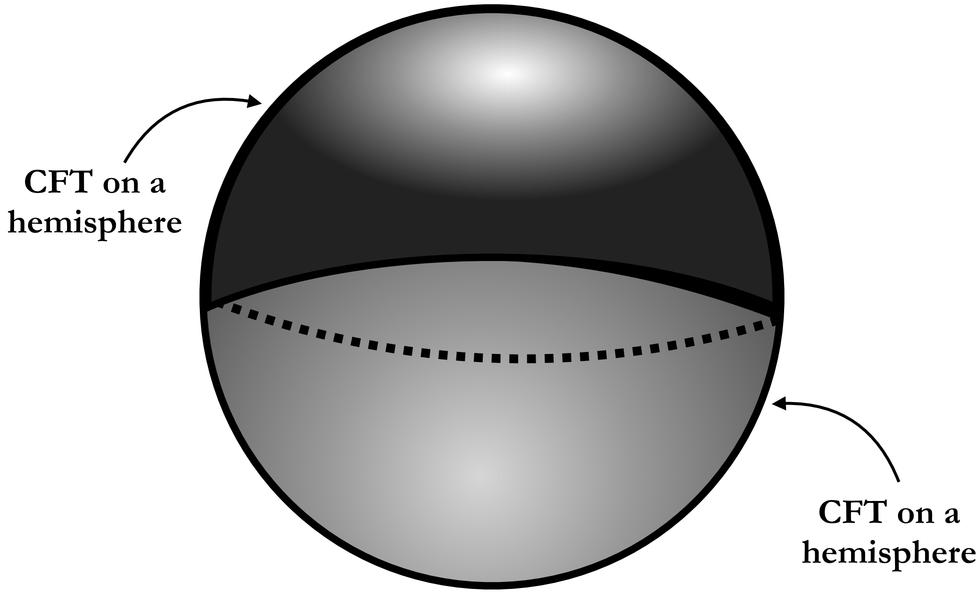
• Such a solution to the gravitational equations has always two boundaries at $B_\pm$ at $u = \pm\infty$.

• The interpretation of the solution depends however on the nature of the negative curvature Einstein manifold M_ζ with metric ζ.

• If the negative curvature manifold M_ζ is compact ($g > 2$ Riemann surface in $d = 2$ or Schottky manifolds in $d > 2$) then the solution describes an AdS$_d$-sliced wormhole.



• When the slices are full AdS$_d$ spaces in global coordinates then the dual describes two CFTs interacting via their common boundary



• In the conformal case $e^A = \cosh \frac{u-u_0}{\ell}$ and

$$\frac{R_L}{R_R} = e^{\frac{2u_0}{\ell}}$$

• When the slices are AdS in Poincaré coordinates the solutions describes a (Janus) interface between two CFTs.

Proximity in QFT

- The notion of "proximity" in Quantum Field Theory is an intuitive notion.

- One way to define it is in terms of the "landscape of QFTs".

- This is a space that is relatively easy to imagine and describe locally.

- But it is rather difficult to control globally.

- One possible definition of the notion of proximity among CFTs is : can QFT_1 and QFT_2 live in the same Hilbert space?

- If there is flow connecting CFT_1 to CFT_2 we can claim that the two theories can live in the same Hilbert space.

- Another was formulated by van Raamsdonk: the CFT masquerade, mostly relevant for CFT duals.

"When the states of CFT_1 can be approximated by CFT_2?"

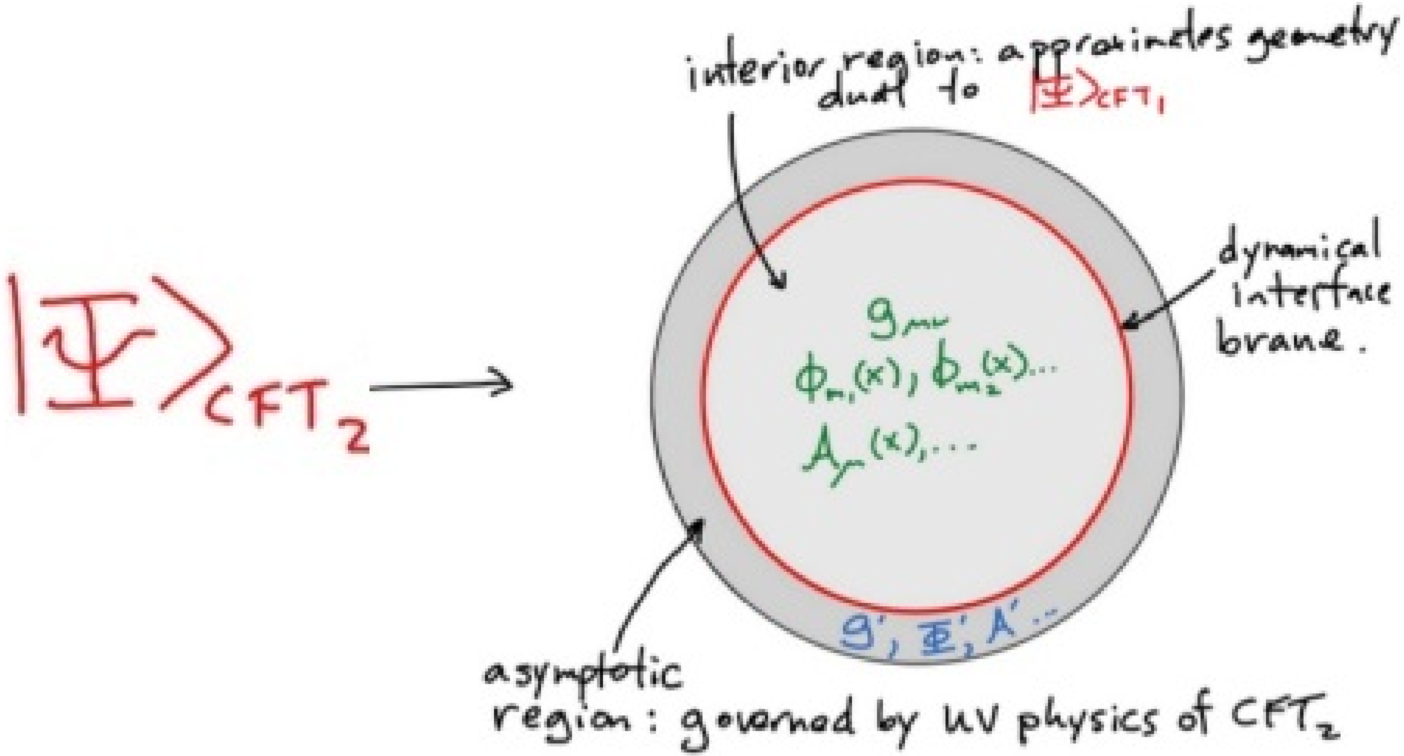
or

"Can a suitably chosen state of CFT_1, faithfully encode the space-time dual to a state of CFT_2?"

or

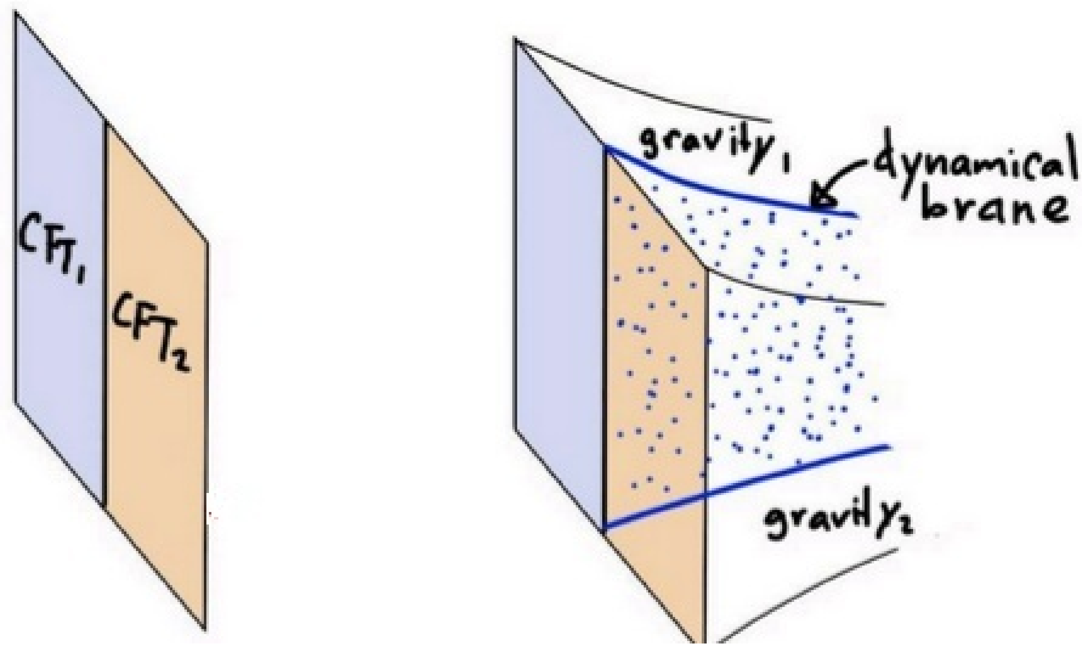
"Can two theories with different operator spectra describe the same bulk geometry?"

• Different semiclassical theories of gravity could be part of a larger non-perturbative theory.



• Van Raamsdonk gave simple solvable examples where the two CFTs are interfaced by a bulk brane.

• This notion is very close to the RG connection, as a continuous version of this setup is a holographic RG flow.



- Another example is theories that can share an inteface.

- They may be generating a bulk brane or

Takayanagi

- They may be like Janus interface geometries.

Bak+Gutperle+Hirano, + many others

The AdS-sliced RG flows

• We assume an Einstein-dilaton theory in order to simplify our explorative task.

$$S_{Bulk} = M_P^{d-1} \int du\, d^d x \sqrt{-g} \Big(R - \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi - V(\Phi) \Big) .$$

$$ds^2 = du^2 + e^{2A(u)} \zeta_{\mu\nu} dx^\mu dx^\nu , \qquad \Phi = \Phi(u)$$

• The metric ζ is any negative curvature Einstein metric.

$$R^{(\zeta)}_{\mu\nu} = \kappa \zeta_{\mu\nu} \,, \qquad R^{(\zeta)} = d\kappa \,, \qquad \kappa = -\frac{(d-1)}{\alpha^2} .$$

• κ can be rescaled by a shift in $A(u)$.

• The solution is characterized by the scalar field profile $\Phi(u)$ and by the scale factor $A(u)$, which are related via the bulk Einstein equations. • Note that for all constant curvature metrics, the equations are the same!

• We shall systematically study the solutions to these equations for $R^{(\zeta)} \sim \kappa < 0$.

- The regular solutions have generically two boundaries $B_\pm$ at $u = \pm\infty$.

- They are both conformal to M_ζ.

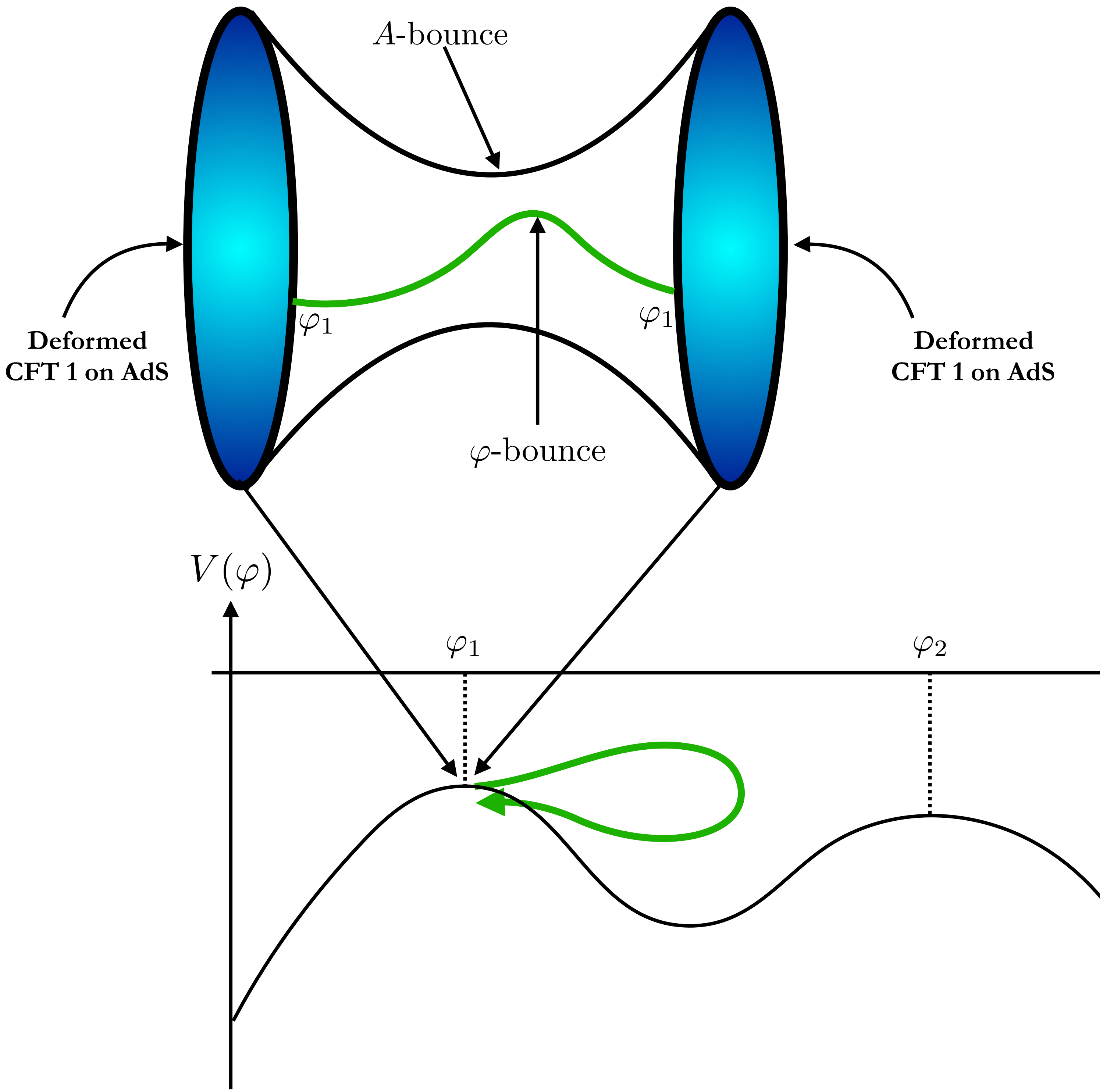
- The end-points are at extrema $\Phi_\pm$ of the bulk potential $V(\Phi)$.

- The associated CFTs are CFT_+ and CFT_-

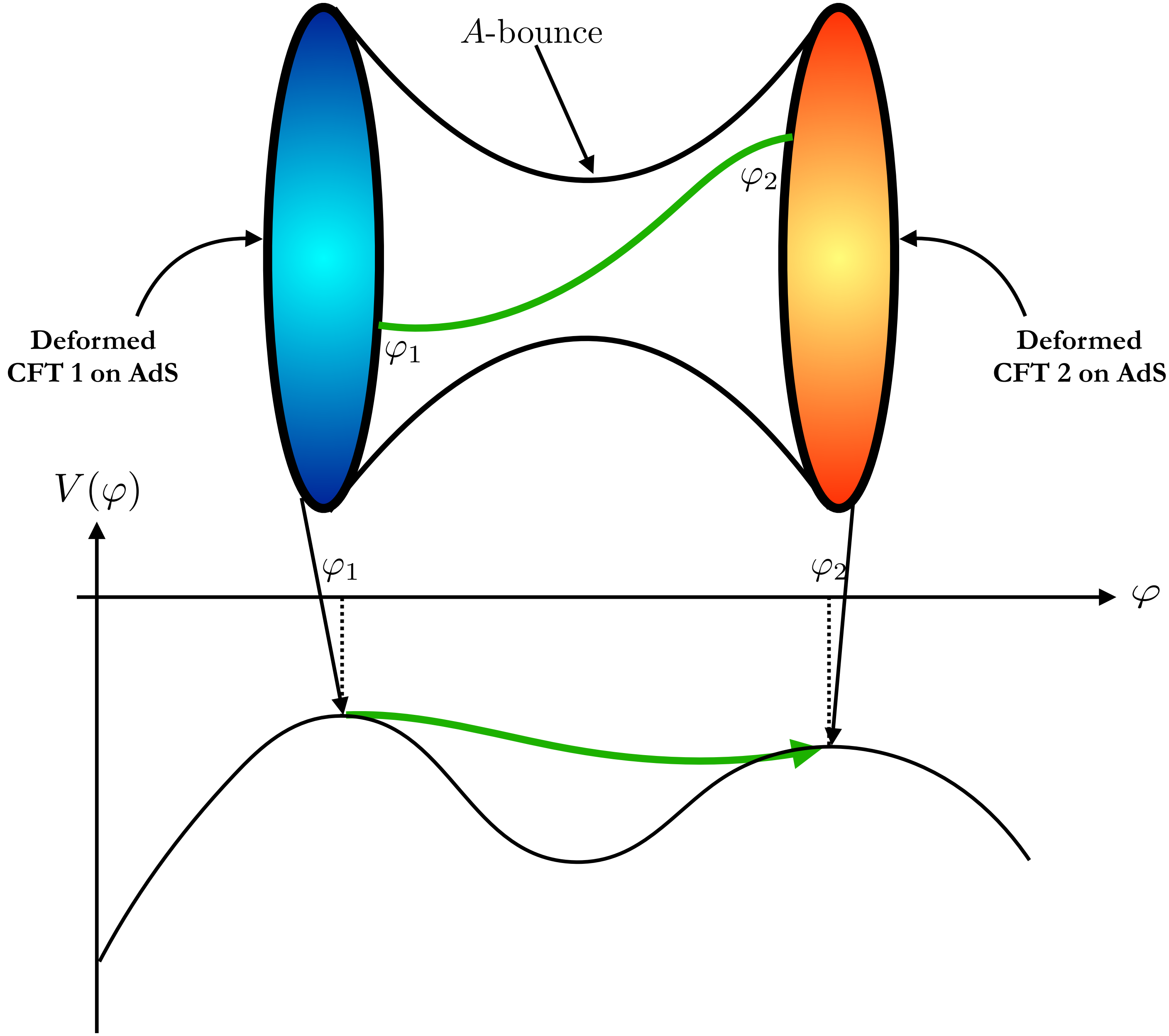
- Every solution (A, Φ) to these equations corresponds to:

♠ A wormhole solution if M_ζ is compact. It connects CFT_+ at B_+ to CFT_- at B_-.

♠ An interface solution if M_ζ is non-compact. The interface $B_+ \cup B_-$ is between CFT_+ and CFT_-.



A-bounce
φ_1
φ_1
Deformed
CFT 1 on AdS
Deformed
CFT 1 on AdS
φ-bounce
$V(\varphi)$
φ_1
φ_2


A-bounce
φ_2
φ_1
Deformed
CFT 1 on AdS
Deformed
CFT 2 on AdS
$V(\varphi)$
φ_1
φ_2
φ

The QFT couplings

• At each boundary, initial or final the metric asymptotes to M_ζ and the only parameter (source) is its curvature, $R_{i,f}$.
• The scalar will also have sources at the two boundaries:

$$\Phi(u) \to \Phi_-^{(i)} \quad , \quad u \to -\infty,$$

$$\Phi(u) \to \Phi_-^{(f)} \quad , \quad u \to +\infty,$$

• Therefore, we have four dimensionful couplings: $R_{i,f}$, $\Phi_-^{(i,f)}$.

• As the overall scale is irrelevant, the pair of theories is characterized by three dimensionless numbers which we take to be:

$$\mathcal{R}_i = \frac{R_i^{UV}}{\left(\Phi_-^{(i)}\right)^{2/\Delta_-^i}}, \quad \mathcal{R}_f = \frac{R_f^{UV}}{\left(\Phi_-^{(f)}\right)^{2/\Delta_-^f}}, \quad \xi = \frac{\left(\Phi_-^{(i)}\right)^{1/\Delta_-^i}}{\left(\Phi_-^{(f)}\right)^{1/\Delta_-^f}}$$

The first order formalism

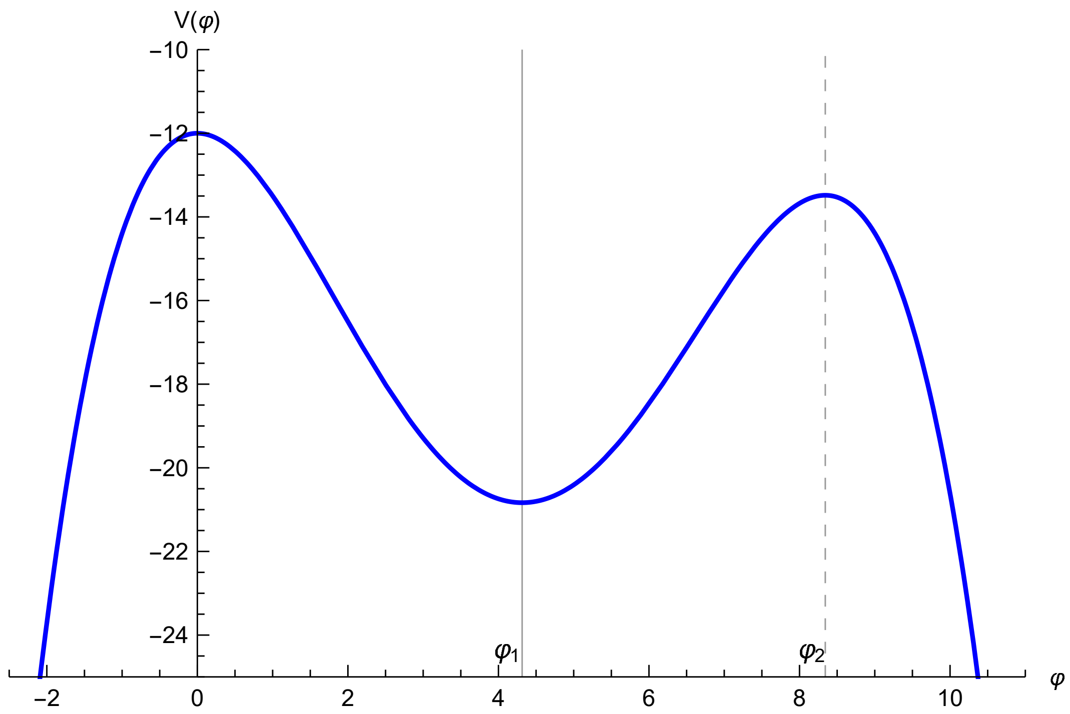
- We define the "superpotentials" (no supersymmetry)

$$\dot{A} \equiv -\frac{1}{2(d-1)} W(\Phi) \,,$$

$$\dot{\Phi} \equiv S(\Phi) \,,$$

Classifying the solutions, Part I

- We pick $d = 4$ and a generic quartic potential



- The left maximum is at $\Phi = 0$.

- The right maximum is at $\Phi_2 = 8.34$.

- The minimum is located at $\Phi_1 = 4.31$.

• "Technical" definitions:

♠ A-bounce is a point where $\dot{A} = 0 \to W = 0$. It always exists when the slice curvature is negative.

• Our solutions will have a single A-bounce. We shall denote its position by Φ_0.

♠ Φ-bounce is a point where $\dot{\Phi} = 0 \to S = 0$. It is a point where the first order equations break down but the second order equations do not.

• We always start our solution at an A-bounce at $\Phi = \Phi_0$ $(W(\Phi_0) = 0)$and we solve the first order equations

$$\frac{d}{2(d-1)}W^2 + (d-1)S^2 - dSW' + 2V = 0\,,$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0\,.$$

• We only need an extra "initial" condition: $S_0 \equiv \dot{\Phi}|_{\Phi=\Phi_0} \equiv S(\Phi_0)$.

• The two parameters $(\Phi_0, S_0) \in R^2$ are the complete initial data of the first order system.

• For each pair (Φ_0, S_0) there is a unique solution.

Reminder: asymptotics near extrema of the potential

• Solution START AND END (generically) at extrema of the potential.

• Near a maximum of the potential, there are two branches of solutions known as the $-$ and the $+$ branch.

♠ The $-$ branch contains the generic solutions that contain both source and vev.

♠ The $+$ branch contains only the special solution for which the source vanishes (relevant vev-driven flow).

• For both types of solutions above, the metric has an AdS boundary at the maximum.

• We denote these asymptotics as $Max_{\pm}$.

• Near a minimum of the potential we also have the $+$ and $-$ branches of solutions.

♠ The $-$ branch contains the generic solution.

• It does not exist for non-zero slice curvature. It exists only for flat slices and in that case it describes the IR-end of an RG flow.

♠ The $+$ branch contains the special solution. The bulk metric has an AdS boundary in this case

• The solution describes a UV fixed-point perturbed by the vev of an irrelevant operator.

• In principle it can exist for both flat and curved slices.

• We denote these asymptotics as $Min_{\pm}$.

• $Max_{\pm}$ and Min_{+} are associated to AdS boundaries and therefore to QFT UV fixed points.

• Min_{-}, to a shrinking slice geometry and therefore to an IR Fixed point.

• The $+$ branch solutions, as they contain less integration constants, exist only in fine-tuned cases.

• The Min_{-} solution does not exist, when the (dimensionless) curvature of the slice $\mathcal{R} \neq 0$.

Classification of complete flows

♠ $\mathcal{R} = 0$. All flows start and end at extrema of the potential.. They have a single AdS boundary.

• (Max_-, Min_-). This is the generic relevant flow driven by a relevant operator.

• (Max_+, Min_-). This is a flow driven by the vev of a relevant operator.

• (Min_+, Min_-). This is a flow driven by the vev of an irrelevant operator.

♠ $\mathcal{R} > 0$.

• In this case, although flows can start at extrema of the potential, (both maxima as $Max_\pm$ and minima as Min_+), they always end at intermediate points, not at extrema.

• The end is always an IR end-point where the slice volume vanishes.

♠ $\mathcal{R} < 0$.

• It is not possible for a flow to be regular and end at intermediate points (non-extrema of the potential), (there is no slicing of flat space with AdS slices).

• Therefore, all regular flows must start and end at extrema of the potential.

• As the asymptotic solution Min_- does not exist when $\mathcal{R} \neq 0$, we have in total the following 9 options, all of them having two AdS boundaries.

• (Max_-, Max_-), (Max_+, Max_+).

• (Max_-, Max_+) and its reverse (Max_+, Max_-).

• (Max_-, Min_+) and its reverse, (Min_+, Max_-).

• (Max_+, Min_+) and its reverse, (Min_+, Max_+)

• (Min_+, Min_+).

- As mentioned the Max_+, Min_+ asymptotics are fine-tuned (they have half the adjustable integration constants).

- Therefore the generic solutions will be of the (Max_-, Max_-) type.

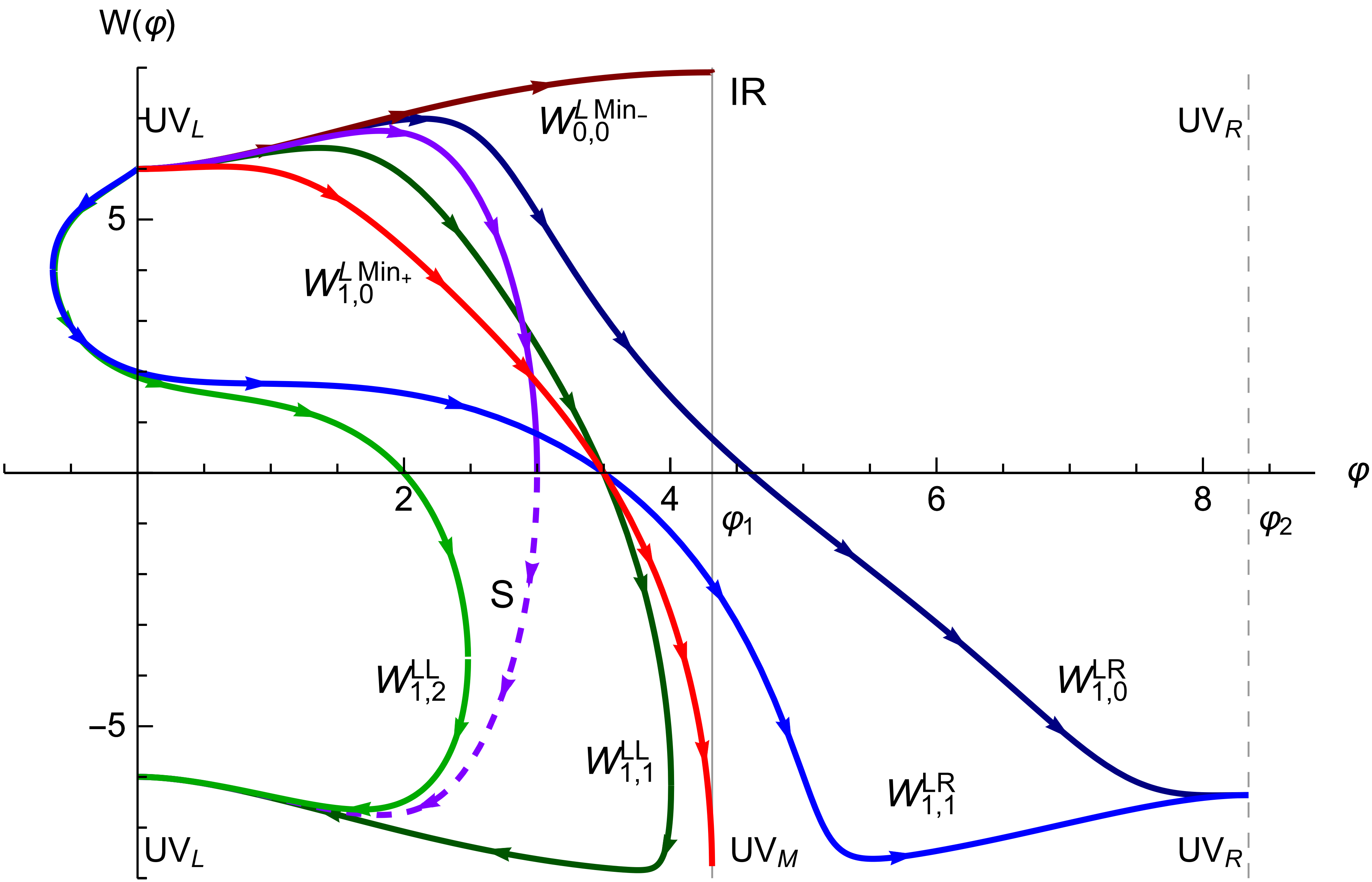
- Single fine-tuning of the potential or the integration constants is needed for the (Max_-, Max_+) and (Max_-, Min_+) solutions to exist.

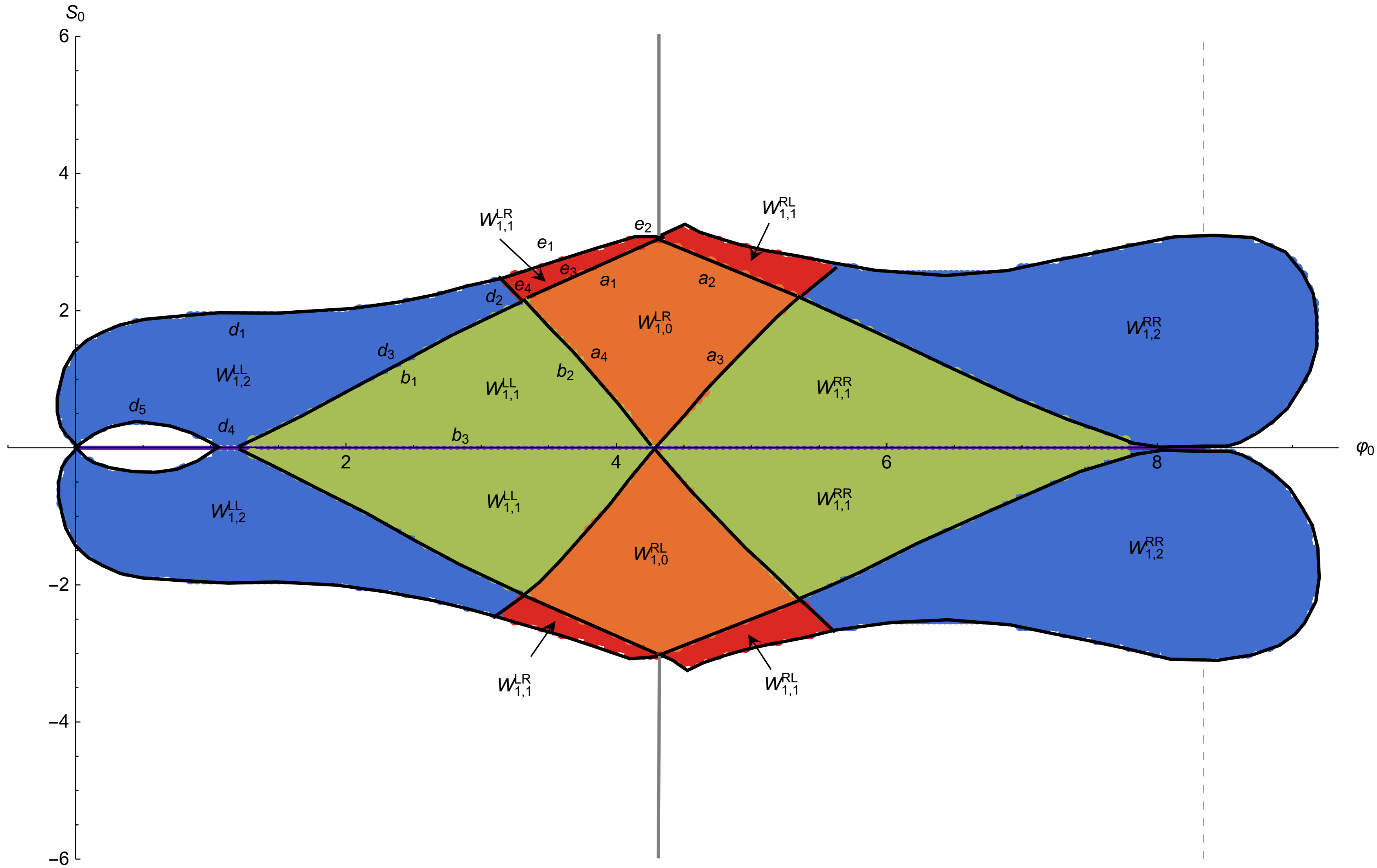
- Double fine-tuning is needed for (Max_+, Max_+), (Max_+, Min_+) and (Min_+, Min_+) to exist.

- We shall find examples of all types fine-tuned or not except the (Min_+, Min_+) solutions.

- We believe the reason is that we have a potential with only one minimum.

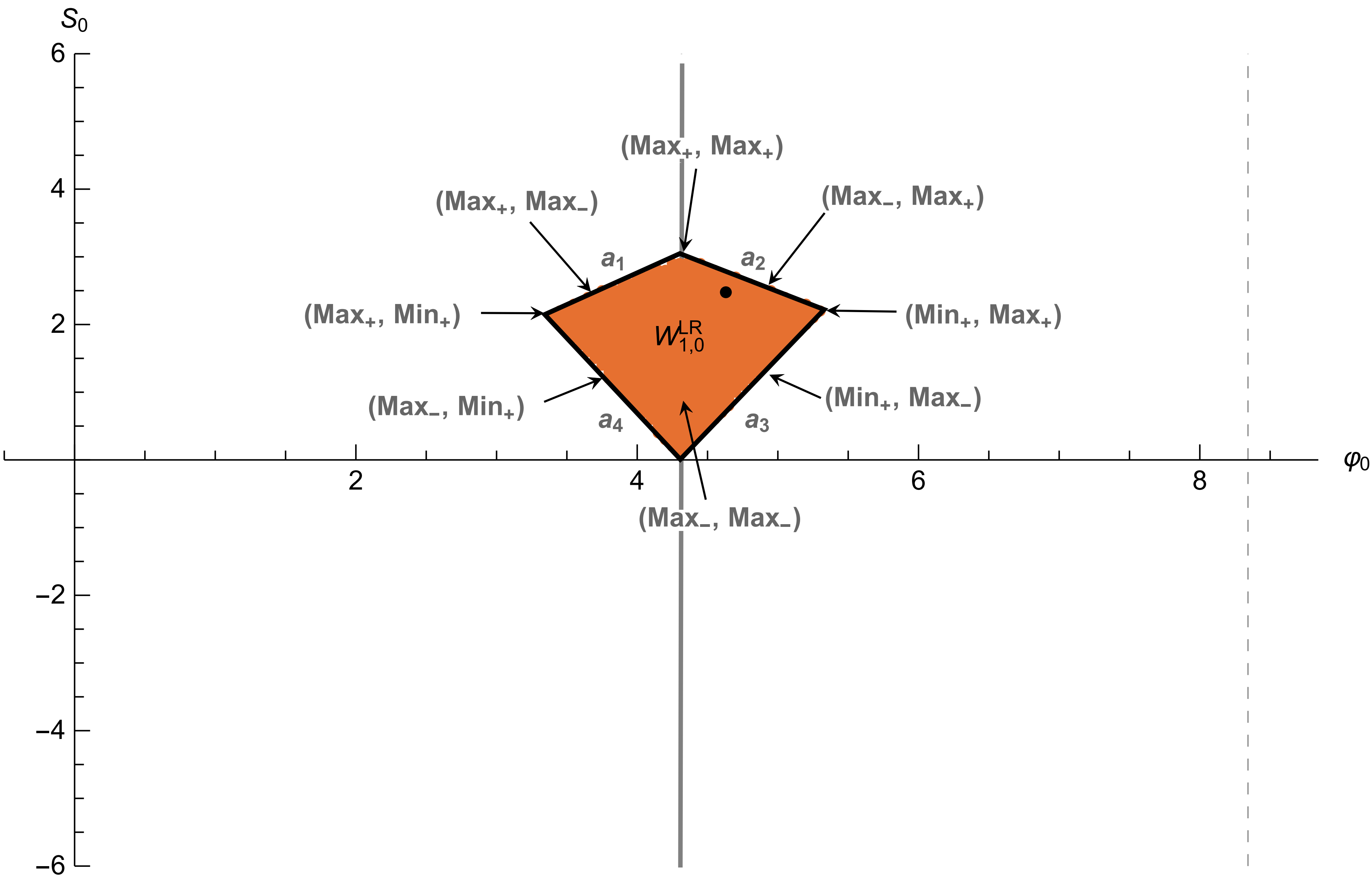
The space of solutions

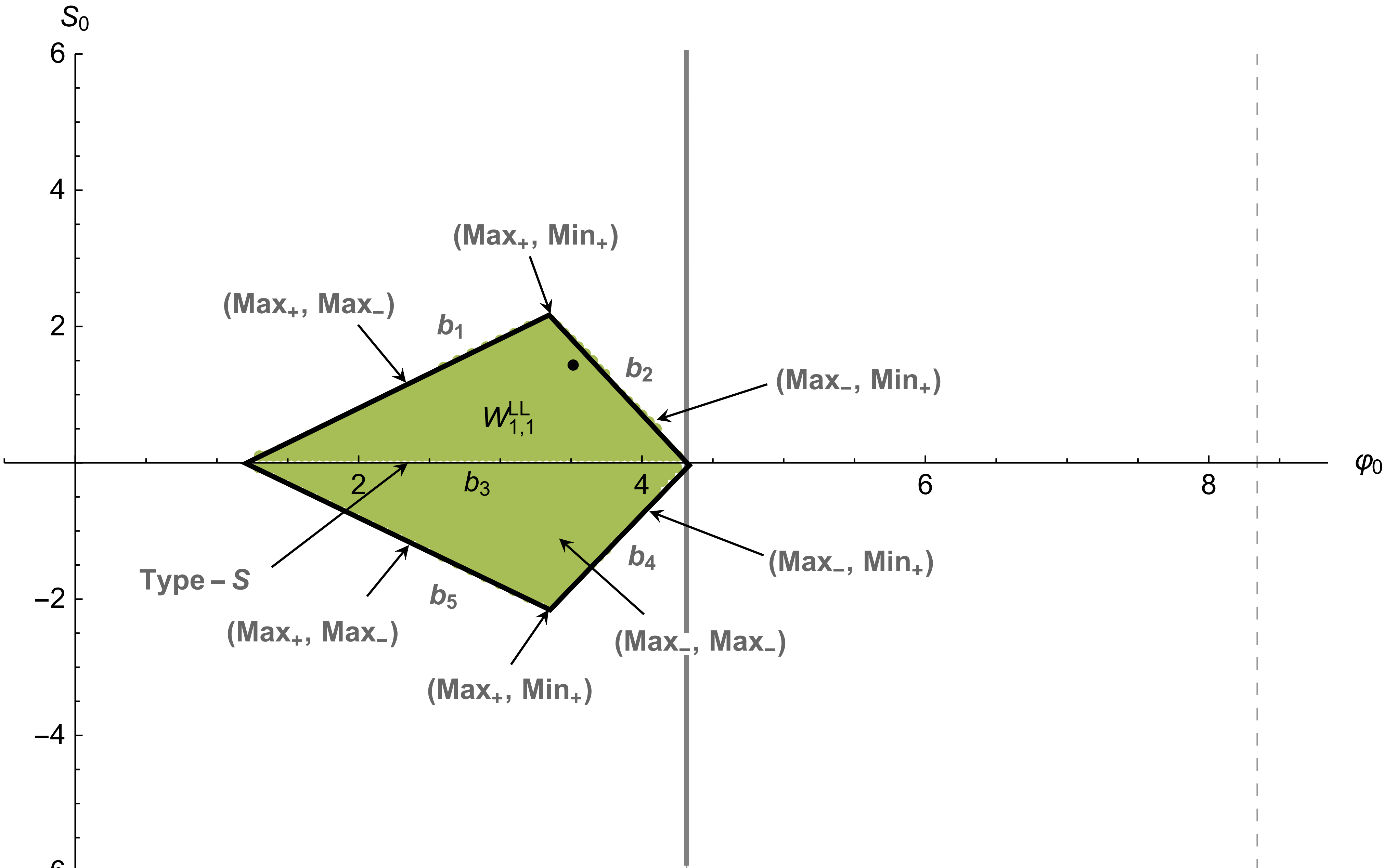




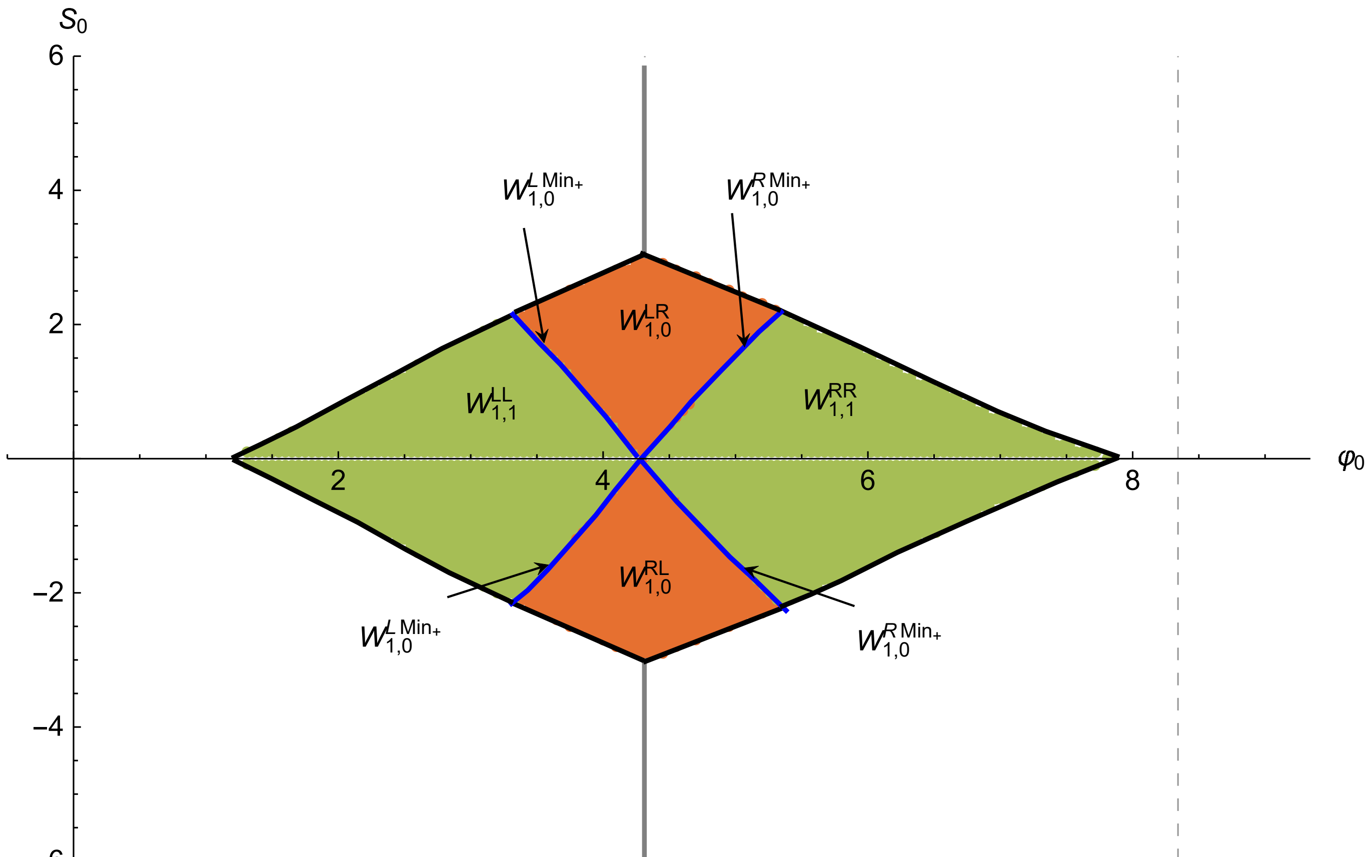
S_0
φ_0
$W^{LR}_{1,1}$
$W^{RL}_{1,1}$
$W^{LR}_{1,0}$
$W^{RL}_{1,0}$
$W^{LL}_{1,1}$
$W^{LL}_{1,2}$
$W^{RR}_{1,1}$
$W^{RR}_{1,2}$
a_1 a_2 a_3 a_4
b_1 b_2 b_3
d_1 d_2 d_3 d_4 d_5
e_1 e_2 e_3 e_4

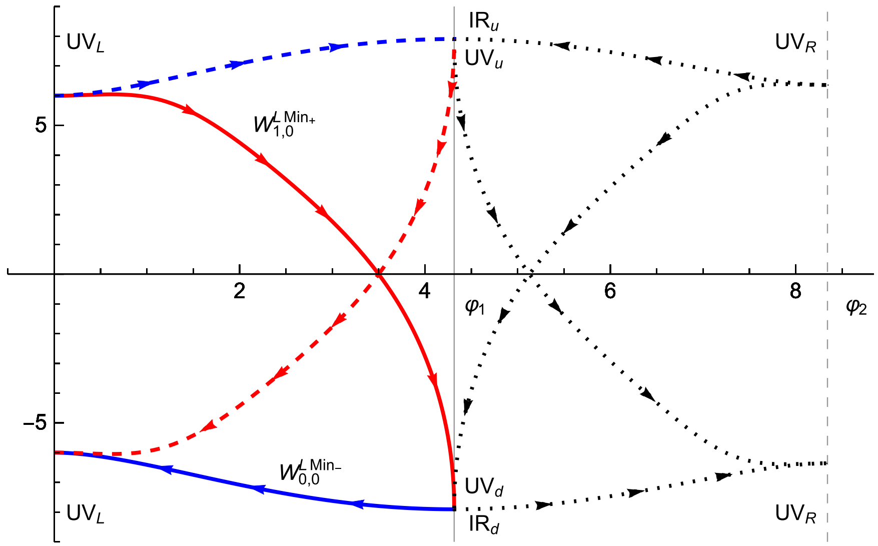
The region boundaries and tuned flows

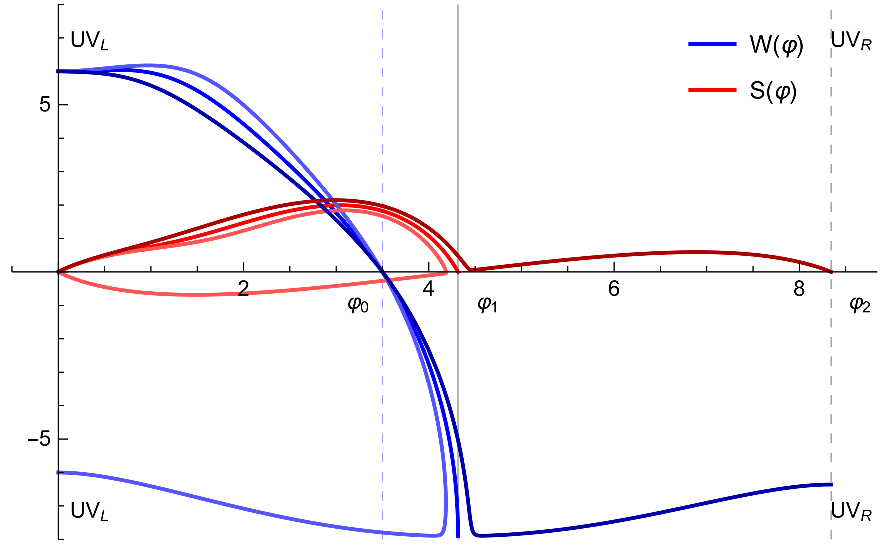



S_0
6
4
2
−2
−4
−6
2
4
6
8
φ_0
(Max₊, Min₊)
(Max₊, Max₋)
b_1
b_2
(Max₋, Min₊)
$W^{LL}_{1,1}$
b_3
Type − S
b_4
(Max₋, Min₊)
b_5
(Max₊, Max₋)
(Max₋, Max₋)
(Max₊, Min₊)

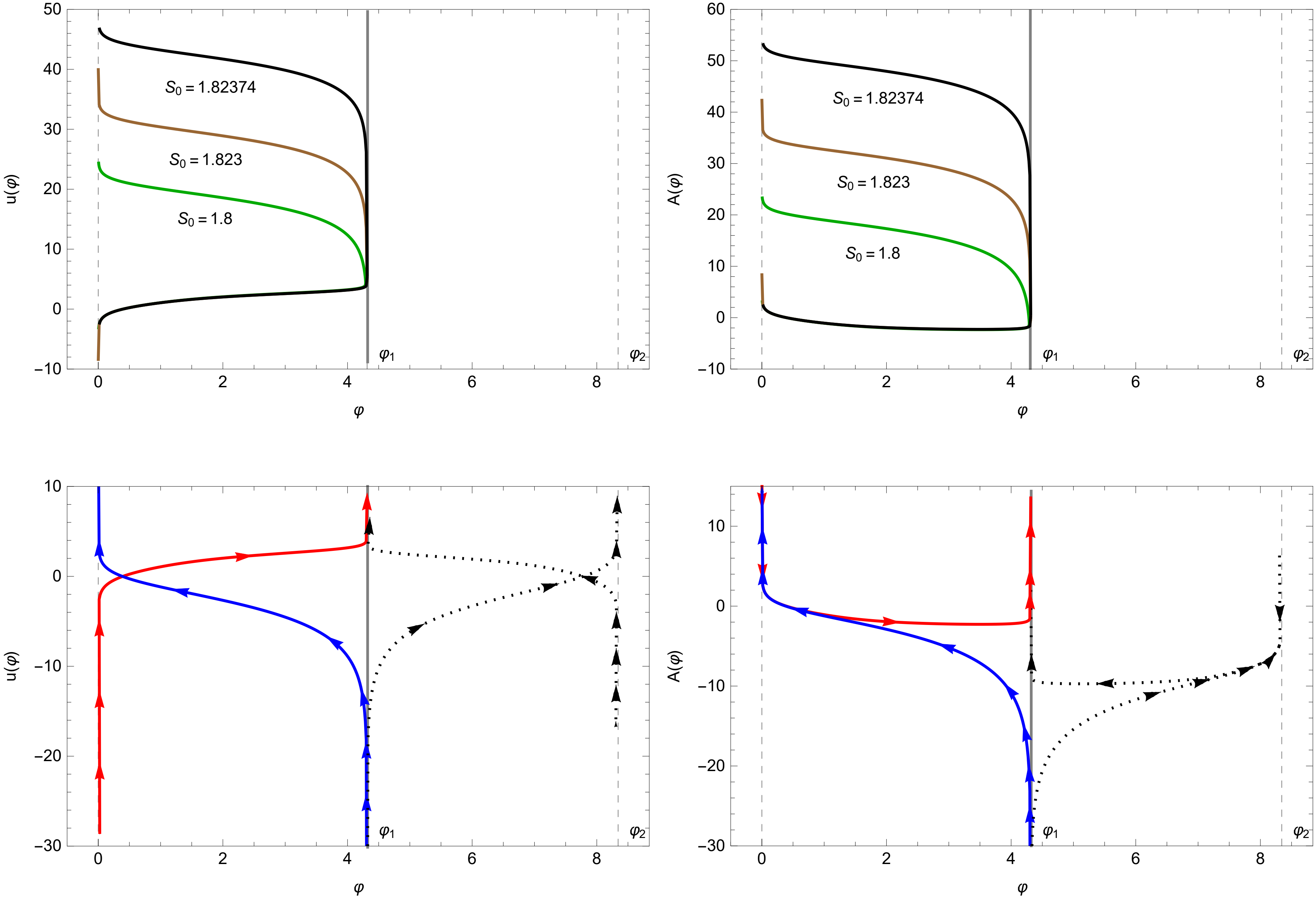
Flow fragmentation, walking and emergent boundaries







(a): An example of an RG flow between a maximum and a minimum. For the solid curves, (Max_-, Min_+) is a flow between a UV fixed point at maximum $\Phi = 0$ and another UV fixed point at the minimum $\Phi = \Phi_1$. For the (Max_-, Min_-) part of the solution, the minimum is an IR fixed point. The dashed curves show the flipped image of the solid curves. The black dotted curves are other possible RG flows with the same UV fixed points. (b): At a fixed Φ_0 when the value of S_0 is exactly on the border of type $W_{1,0}^{LR}$ and type $W_{1,1}^{LL}$, we have the $W_{1,0}^{LMin_+}$ branch solution (the middle flow). If we increase or decrease the value of S_0 we have the $W_{1,0}^{LR}$ or $W_{1,1}^{LL}$ solutions respectively.

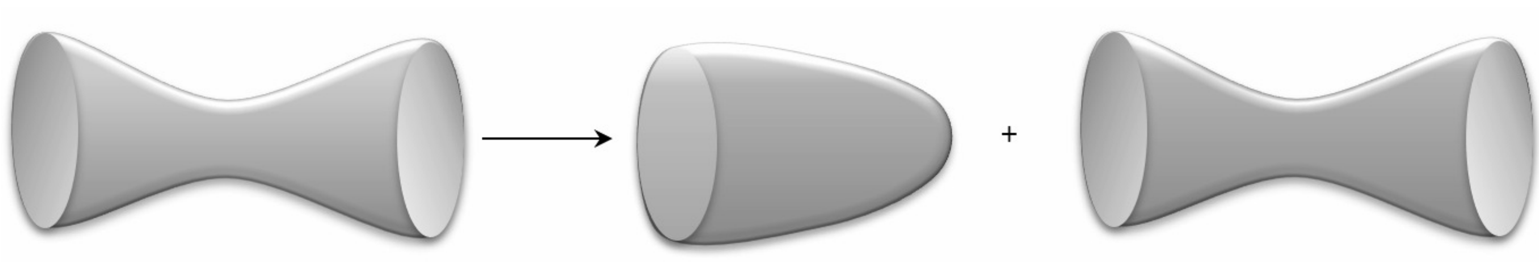


The behavior of the holographic coordinate and scale factor in terms of Φ for the $W_{1,0}^{LMin_+}$ and $W_{0,0}^{LMin_-}$ RG flows. The red curve belongs to $W_{1,0}^{LMin_+}$ and the blue to $W_{0,0}^{LMin_-}$.

• In this limiting region we have an explicit example of solution fragmentation.

• There are two phenomena visible in this example.

♠ Walking. This the phenomenon when an intermediate AdS region appears between the UV and IR, or between UV and UV as is the case here



♠ The emergence of a new boundary.

Single boundary solutions

• To obtain a single boundary, one can orbifold a symmetric solution.

Aharony+Marolf+Rangamani

• This can be done in the class of solutions we called S. They have $S_0 = 0$ and they are completely symmetric.

• We obtain the half space with $u \in (-\infty, u_0)$.

• We can interpret such solutions by inserting an end-of-the-world brane at u_0.

• But because $\dot{A} = \dot{\Phi} = 0$ at u_0, this brane is both tensionless and chargless.

• However, a look at correlators indicates that conformal invariance is broken (For AdS-sliced AdS).

• In the two boundary case, we have four possible two-point functions $\langle OO \rangle$: $G_{++}, G_{+-}, G_{-+}, G_{--}$

• The symmetric orbifold gives

$$G = G_{++} + G_{+-} = \frac{1}{2^\Delta}\left[\frac{1}{(\cosh L - 1)^\Delta} + \frac{1}{(\cosh L + 1)^\Delta}\right]$$

$$\cosh L = 1 + \frac{(z-z')^2 + |x-x'|^2}{zz'}$$

• The conformal correlator obtained from a Weyl transformation of flat space is the first piece only.

• This may be due to the fact that most boundary conditions break conformal invariance.

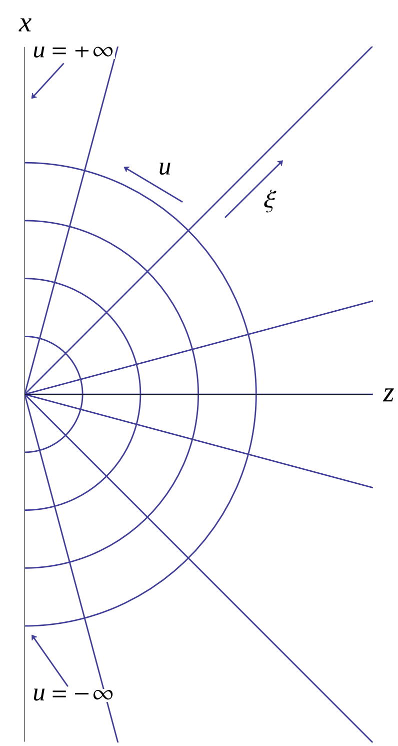
• If instead we insert a brane at $u = u_0$ and impose Dirichlet bc we obtain a similar result with a relative minus sign. (The orbibold corresponds to Neumann)

• Are there bc on the brane so that we obtain a conformal correlator?

• Yes, but they are generically non-local on the brane.

Interface correlators

- The picture of overlapping boundaries in AdS-sliced flows is "singular".

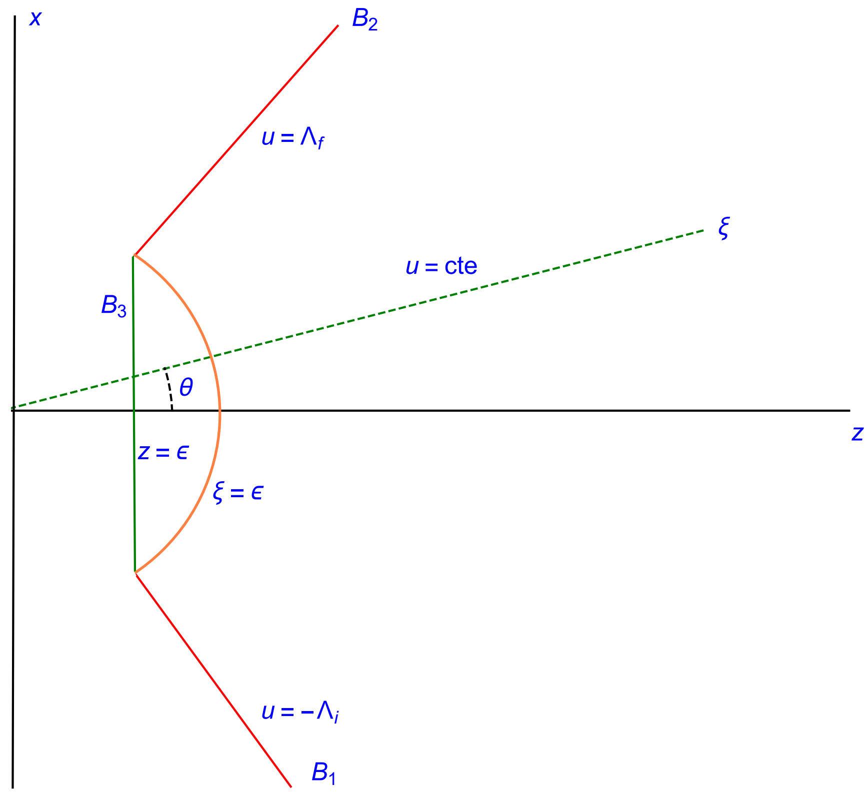


Relation between Poincaré coordinates (x, z) and AdS-slicing coordinates (ξ, u). Constant u curves are half straight lines all ending at the origin ($\xi \to 0^-$); Constant ξ curves are semicircle joining the two halves of the boundary at $u = \pm\infty$.

• The regular picture contains three boundaries:

♠ Two of them ($B_{1,2}$) are at $u = \pm\infty$.

♠ There is a third boundary, B_3, for all values of u that contains the boundaries of AdS slices.



• For a well-defined variational problem apart from the GH term on $B_{1,2,3}$ one needs to add the Hayward term at the two corners, $B_1 \cup B_3$ and $B_2 \cup B_3$.

$$S_H = \frac{1}{8\pi G_N} \int d^{d-1}x \sqrt{-h}\, \text{arccos}(n.\tilde{n})$$

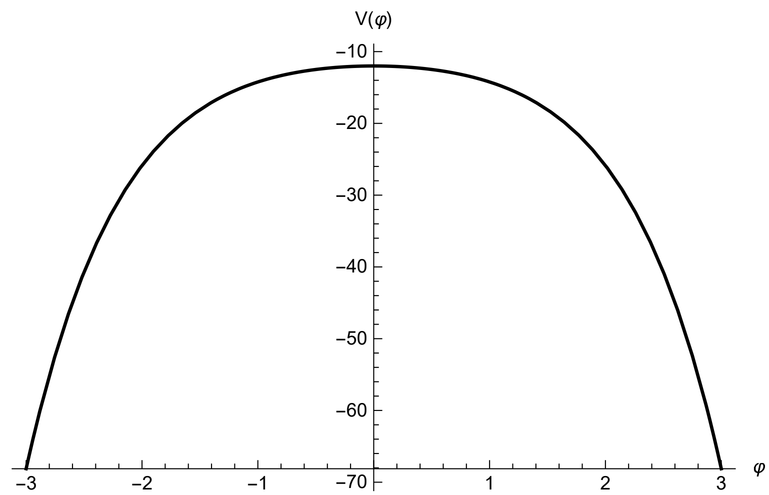
• Correlators of insertions at the $B_{1,2}$ boundaries are done the same way as in standard AdS.

• Calculating correlators on the interface is problematic.

• We could not find a universal form of counterterms on a shifted boundary that removes all divergences from interface correlators.

• This is an open problem.

Confining Theories on AdS



- The confining solutions are solutions where the scalar runs off to infinity.

- These are singular solutions (naked singularities)

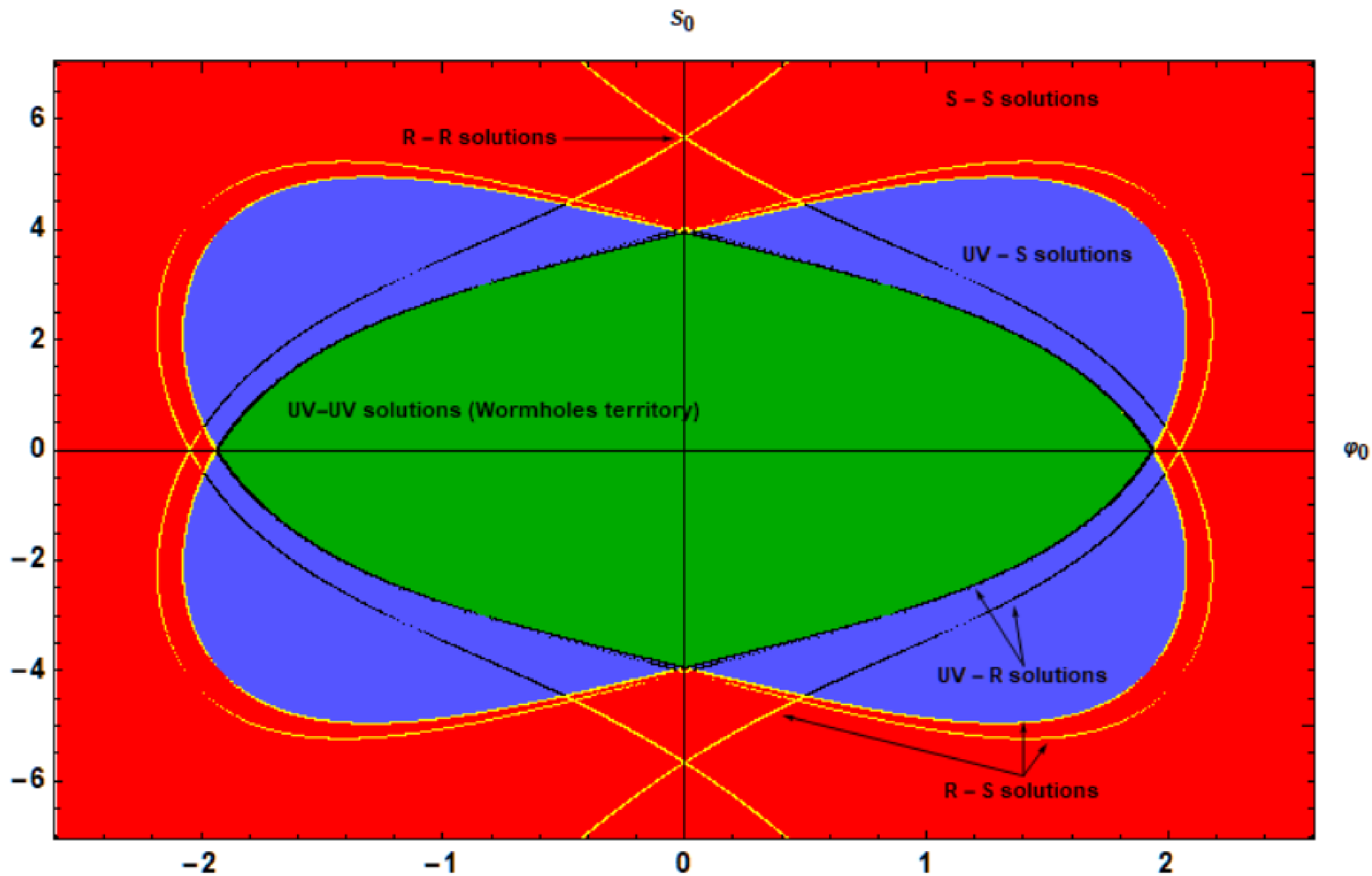
- But one of the one parameter family of solutions is "less" singular.

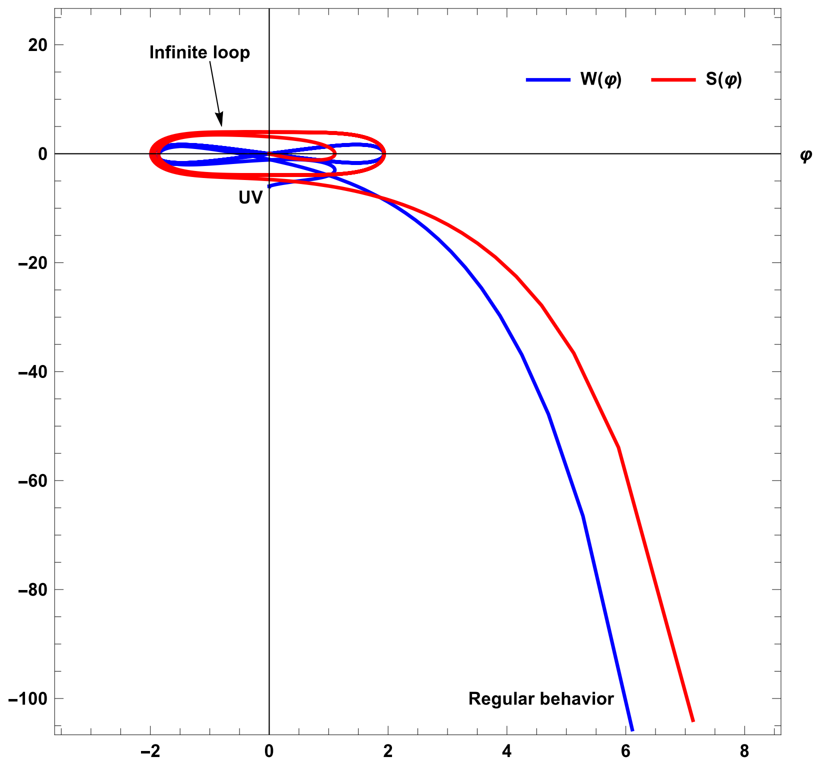
- This corresponds to a resolvable singularity and can be resolved by KK states.

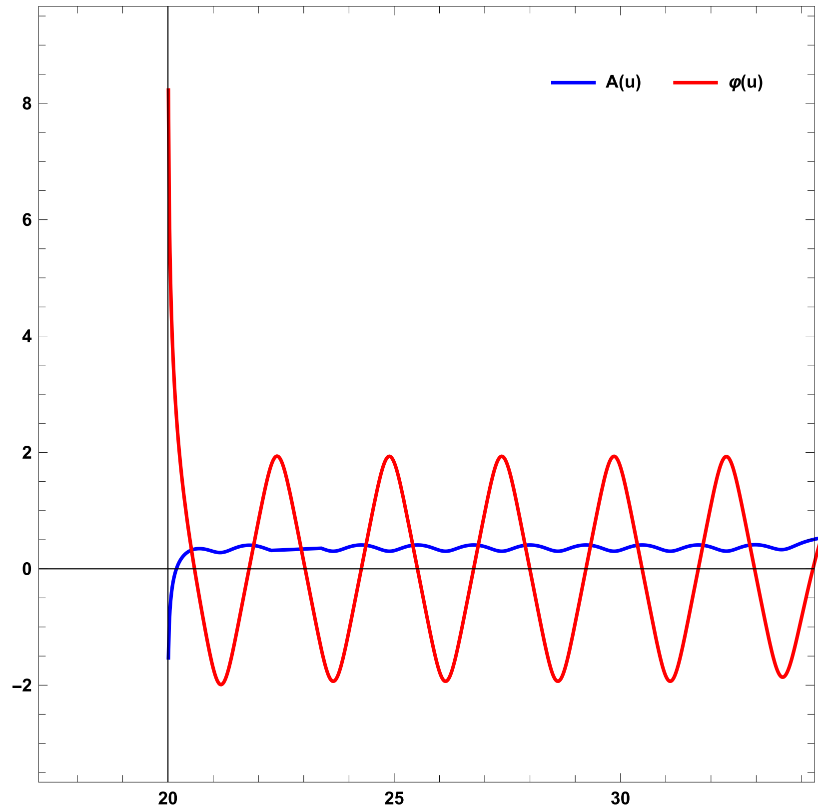
Gubser: the good, the bad and the naked

• Such solutions correspond to confining ground states.

• In the case of AdS slices new phenomena appear. Unlike non-confining theories , there is an infinite number of solutions with a single AdS boundary.







- The solution with no oscillations has the lowest free energy.

- Is there Efimov scaling here?

Conclusions

- We have studied RG flow solutions with slices that have constant negative curvature manifolds.

- Such solutions have generically two boundaries and can be interpreted as wormholes or interfaces.

- We have analysed in detail a simple example.

- The results suggested that proximity is close to RG Flow connection but can be more general.

- We found also exotic limiting cases where one obtains all possible exotic RG flows.

- Other phenomena found include flow (multi)-fragmentation, walking behavior, and the generation of new boundaries.

- We defined single boundary solutions via orbifolding and using a bulk end-of-the-world brane. All seem to give non-conformal correlators.

Open Ends

- The case of constructing a single holographic theory on AdS is still open.

- The exploration of larger regions of the scalar manifold is important in order to see how local the proximity of such solutions is in field space.

- On-going analysis suggests that for confining theories on AdS, one can find regular solutions with one boundary.

- The Wilson loops of QFTs on AdS are currently under study.

- The fate of the instanton gas in AdS can be studied with holographic methods.

- Entanglement in single theories as well as interfaces is interesting to compute and decipher.

THANK YOU!

The bulk Einstein Equations

• The solution is characterized by the scalar field profile $\Phi(u)$ and by the scale factor $A(u)$, which are related via the bulk Einstein equations.

$$2(d-1)\ddot{A} + \dot{\Phi}^2 + \frac{2}{d} e^{-2A} R^{(\zeta)} = 0$$

$$d(d-1)\dot{A}^2 - \frac{1}{2}\dot{\Phi}^2 + V - e^{-2A} R^{(\zeta)} = 0$$

$$\ddot{\Phi} + d\dot{A}\dot{\Phi} - V' = 0\,,$$

The first order formalism

- We define the "superpotentials" (no supersymmetry)

$$\dot{A} \equiv -\frac{1}{2(d-1)} W(\Phi)\,, \quad \dot{\Phi} \equiv S(\Phi)\,, \quad R^{(\zeta)} e^{-2A(u)} \equiv T(\Phi)\,.$$

- The equations of motion become

$$\frac{d}{2(d-1)} W^2 + (d-1) S^2 - d S W' + 2V = 0\,,$$

$$S S' - \frac{d}{2(d-1)} S W - V' = 0\,.$$

- Once a solution is found we can evaluate

$$T(\Phi) = \frac{d}{4(d-1)} W^2(\Phi) - \frac{S(\Phi)^2}{2} + V(\Phi)$$

The bulk integration constants vs QFT parameters

• When $R^\zeta > 0$ the flows describe spaces with a single boundary dual to a single QFT with a relevant coupling.

• The bulk equations have three (dimensionless) integration constants.

• One corresponds to the dimensionless curvature $\mathcal{R}$.

• The second corresponds to the (dimensionless) scalar vev. It must be tuned for regularity.

• The third is not physical as it can be removed by a radial translation.

♠ In the first order formalism the (W,S) equations have two integration constants: one is $\mathcal{R}$, and the second is the scalar vev. The scalar vev is tuned in terms of $\mathcal{R}$ regularity.

• Then T is determined uniquely and from it we determine $A(\Phi)$.

• The first order equation for Φ has one more integration constants.

• This integration constant is trivial and is not a parameter of the dual theory (it is the relevant scale).

• In total, in both cases there is a free arbitrary constant $\mathcal{R}$ and the second (vev) is a function of $\mathcal{R}$.

The bulk integration constants again

• The number of integration constants in the bulk equation is the same(3).

• Here, there is no regularity condition. The solutions are generically regular. Therefore, the scalar vev is an independent parameter and does not depend on $\mathcal{R}$.

• One constant is always redundant as usual.

• All parameters at the second boundary are determined from the solution, evolved from the first boundary.

• Overall our two-boundary solutions depend on two dimensionless independent parameters.

• This is one less from the three we would expect in the general case: $\mathcal{R}_{i,f}$ and ξ.

♠ We shall recover the extra missing parameter by generalizing our solutions later.

Classifying the solutions, II

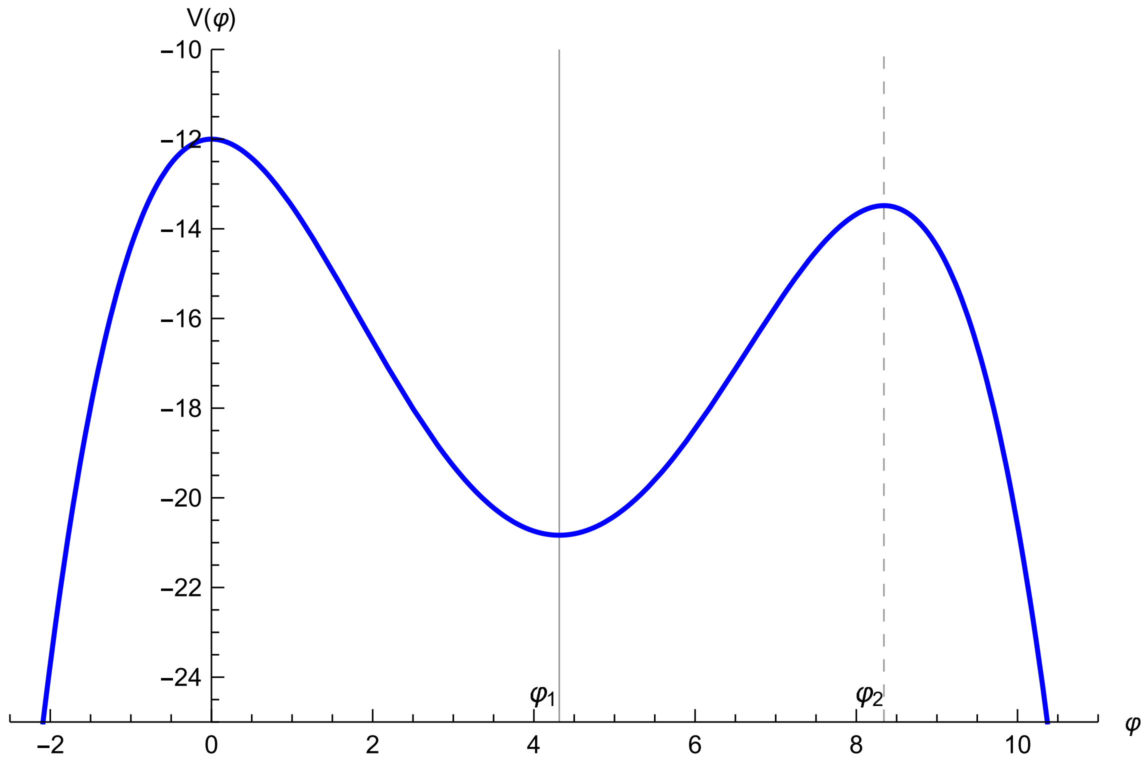
- We picked $d = 4$ and a generic quartic potential that we parametrized as

$$V(\Phi) = -\frac{12}{\ell_L^2} + \frac{\Delta_L(\Delta_L - 4)}{2\ell_L^2}\Phi^2 - \frac{(\Phi_1 + \Phi_2)\Delta_L(\Delta_L - 4)}{3\ell_L^2\Phi_1\Phi_2}\Phi^3 + \frac{\Delta_L(\Delta_L - 4)}{4\ell_L^2\Phi_1\Phi_2}\Phi^4,$$

where Φ_1 and Φ_2 are defined as

$$\Phi_1 = \frac{12\ell_R^2\sqrt{\ell_R^2 - \ell_L^2}\Delta_L(\Delta_L - 4)}{\sqrt{\ell_R^2\Delta_L(\Delta_L - 4) - \ell_L^2\Delta_R(\Delta_R - 4)}\left(\ell_R^2\Delta_L(\Delta_L - 4) + \ell_L^2\Delta_R(\Delta_R - 4)\right)}$$

$$\Phi_2 = \frac{12\sqrt{\ell_R^2 - \ell_L^2}}{\sqrt{\ell_R^2\Delta_L(\Delta_L - 4) - \ell_L^2\Delta_R(\Delta_R - 4)}}.$$



• The left maximum is at $\Phi = 0$. The AdS length is $\ell_L = 1$ and the scaling dimension $\Delta_L = 1.6$.

• The right maximum is at $\Phi = 8.34$. The AdS length is $\ell_R = 0.94$ and the scaling dimension $\Delta_R = 1.1$.

• The minimum is located at $\Phi_1 = 4.31$. It has $\Delta_+^{min} = 4.37$.

• "Technical" definitions:

♠ A-bounce is a point where $\dot{A} = 0 \to W = 0$. It always exists when the slice curvature is negative.

• Our solutions will have a single A-bounce. We shall denote its position by Φ_0.

♠ Φ-bounce is a point where $\dot{\Phi} = 0 \to S = 0$. It is a point where the first order equations break down but the second order equations do not.

♠ An IR-bounce is a point where both $\dot{A} = \dot{\Phi} = 0$.

• All bounces are defined AWAY from extremal points of V.

• We always start our solution at the (unique) A-bounce at $\Phi = \Phi_0$ and we solve the first order equations

$$\frac{d}{2(d-1)}W^2 + (d-1)S^2 - dSW' + 2V = 0\,,$$

$$SS' - \frac{d}{2(d-1)}SW - V' = 0\,.$$

• We only need an extra "initial" condition: $S_0 \equiv \dot{\Phi}|_{\Phi=\Phi_0} \equiv S(\Phi_0)$.

• The two parameters $(\Phi_0, S_0) \in R^2$ are the complete initial data of the first order system.

• For each pair (Φ_0, S_0) there is a unique solution.

• We then start solving the equations to the left and right of Φ_0 until we reach an AdS boundary on each side. Then our solution (W, S) is complete.

• We then solve the equations for Φ, A.

$$R^{(\zeta)}e^{-2A(u)} = \frac{d}{4(d-1)}W^2(\Phi) - \frac{S(\Phi)^2}{2} + V(\Phi) \quad , \quad \dot{\Phi} = S$$

Three parameter solutions

- So far our ansatz missed one dimensionless parameter

- To recover it we modify it to:

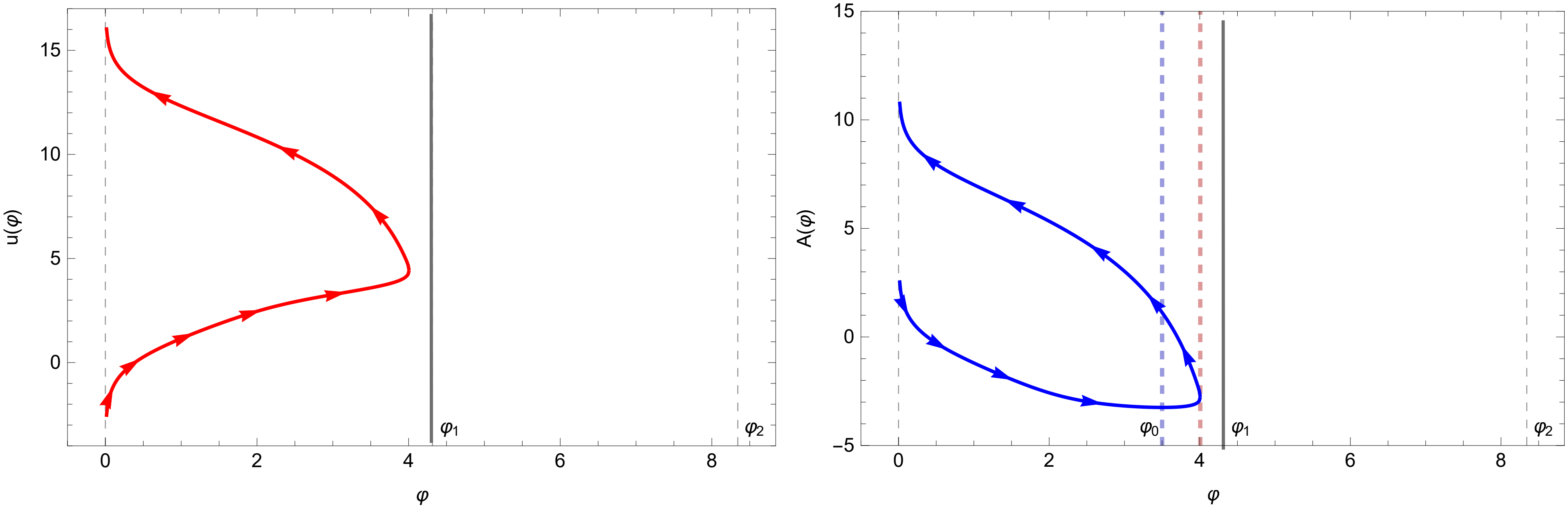
$$A = \begin{cases} \bar{A}(u) & u < u_* \\ \\ \bar{A}(\tilde{u} - \delta) & u_* + \delta < \tilde{u} < +\infty \end{cases},$$

$$\Phi = \begin{cases} \bar{\Phi}(u) & u < u_* \\ \\ \bar{\Phi}(\tilde{u} - \delta) & u_* + \delta < \tilde{u} < +\infty \end{cases},$$

- This satisfies the Israel conditions at $u = u_*$ and A, Φ and their derivatives are continuous.

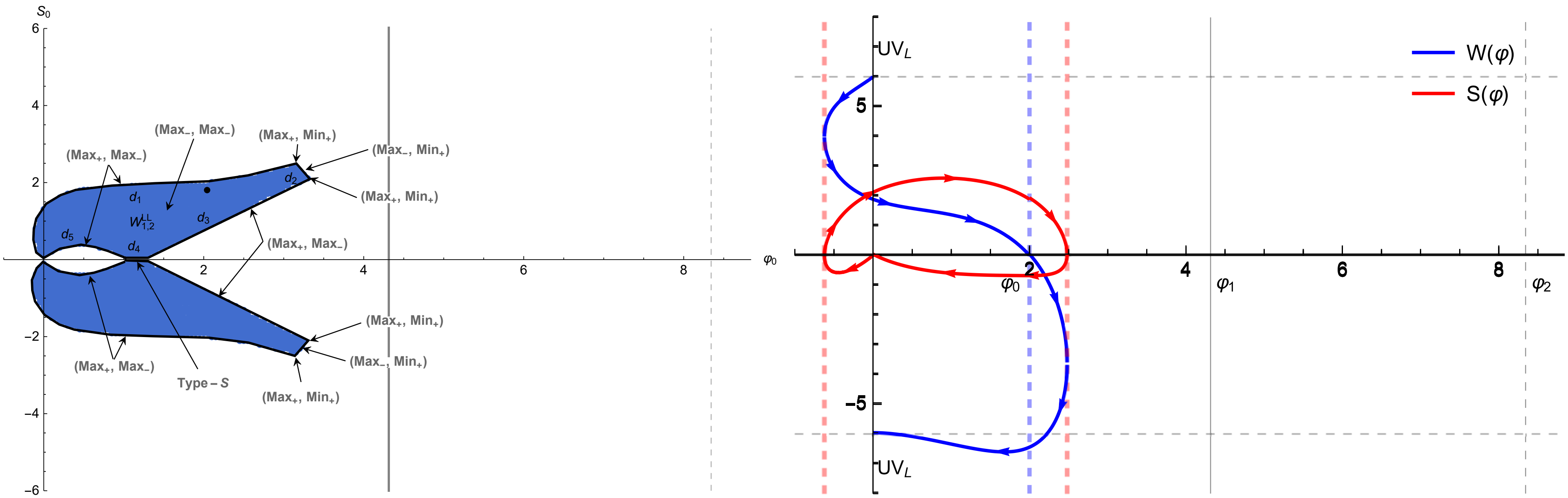
$$R_i^{UV} = \bar{R}_i^{UV}, \quad \Phi_-^i = \bar{\Phi}_-^i, \quad R_f^{UV} = e^{2\delta/\ell} \bar{R}_f^{UV}, \quad \Phi_-^f = e^{\delta \Delta_-^f/\ell} \bar{\Phi}_-^f$$

$W^{LL}_{1,1}$

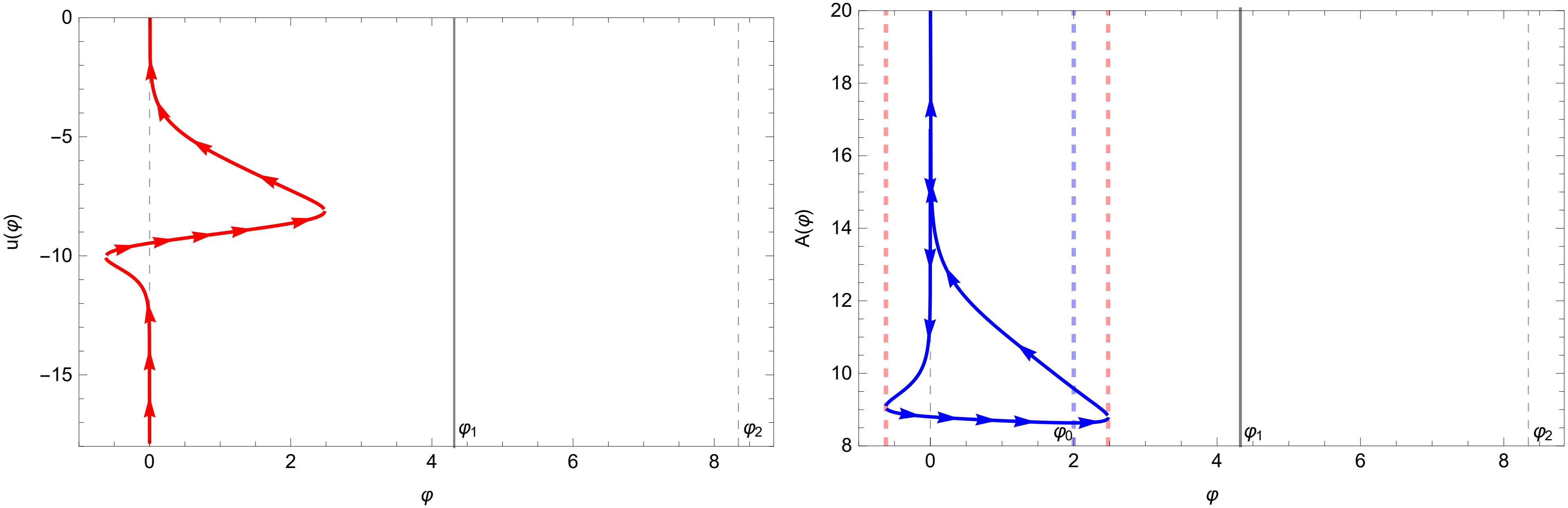


(a): The holographic coordinate at top UV_L tends to $-\infty$ and at bottom UV_L to $+\infty$.
(b): The scale factor has an A-bounce at $\Phi_0 = 3.5$ (blue dashed line) and a Φ-bounce at $\Phi = 4.0$ (red dashed line).

$W^{LL}_{1,2}$

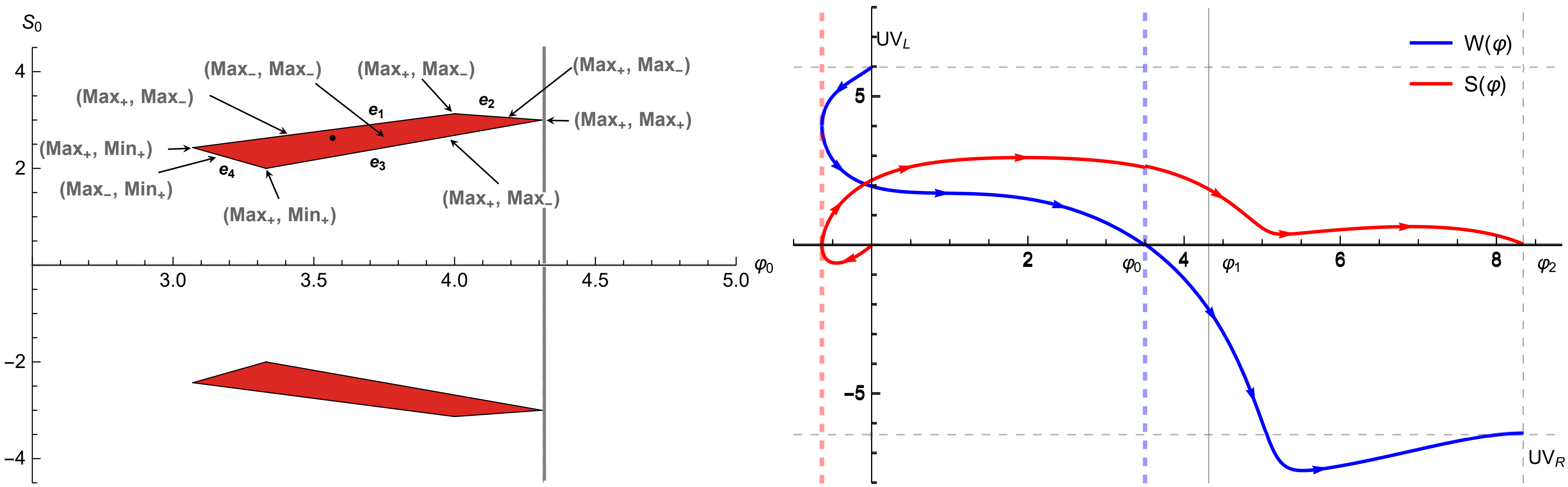


(a): The space of the $W^{LL}_{1,2}$ solutions is the upper blue region. The black dot represents the specific solutions of the diagram (b). The lower blue region corresponds to the solutions with an extra Φ-bounce near the bottom UV_L. (b): The blue and red curves for W, S, describe an RG flow that connects the UV_L fixed point to itself but after two Φ-bounces. The location of the Φ-bounces are indicated by red dashed lines.

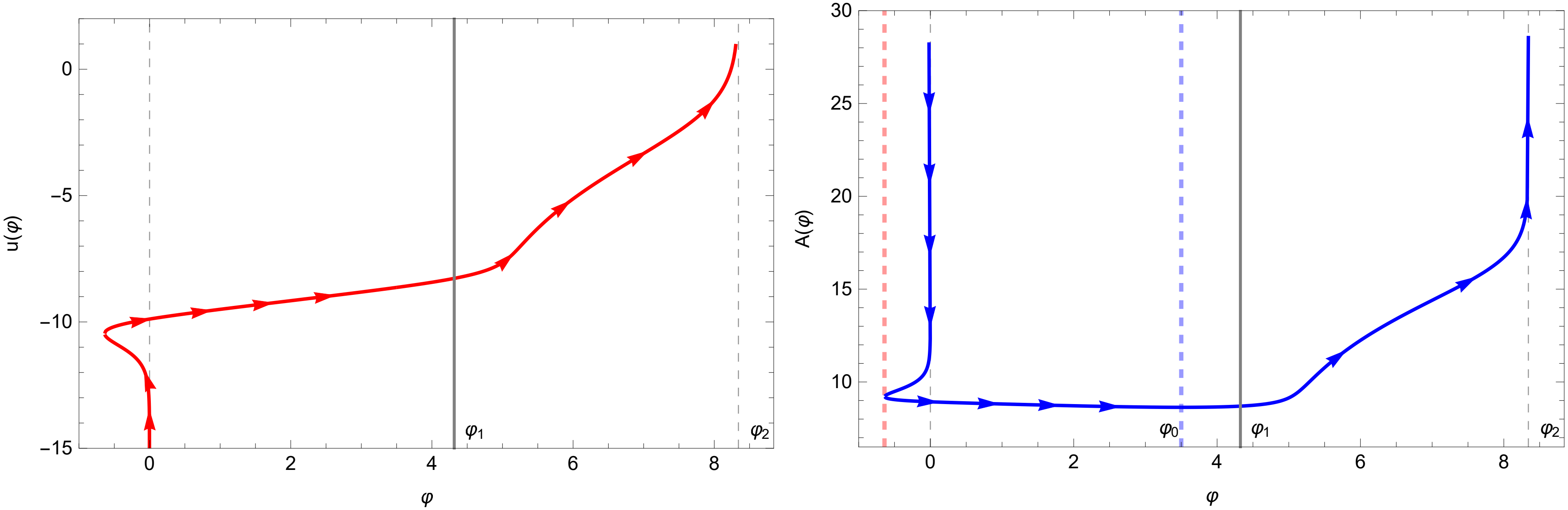


(a): The holographic coordinate at top UV_L boundary tends to $-\infty$ and for bottom UV_L to $+\infty$. (b): The scale factor has an A-bounce at $\Phi = 2.0$, the blue dashed line. The first Φ-bounce on the left occurs at $\Phi = -0.62$ and the second one at $\Phi = 2.48$, the red dashed lines.

$W^{LR}_{1,1}$

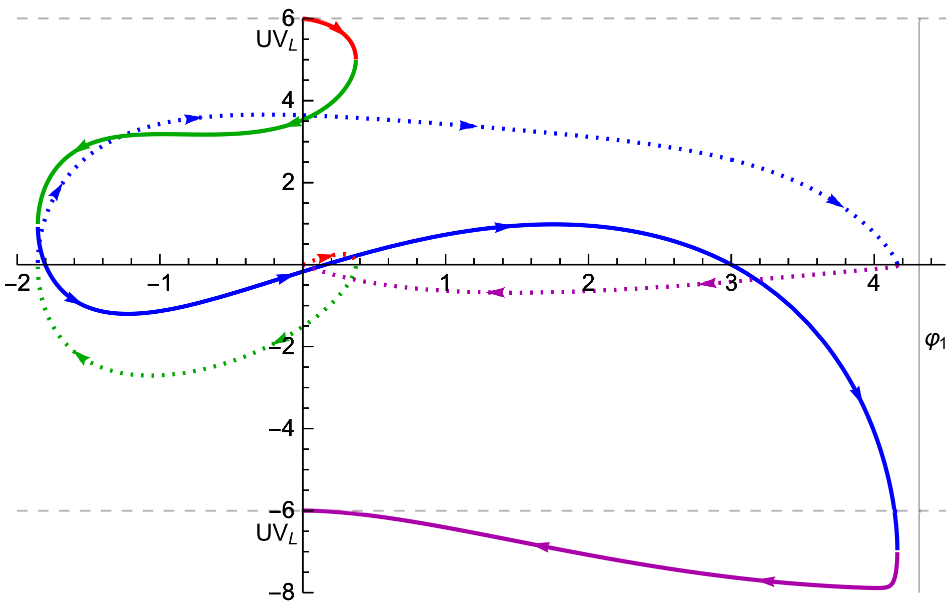


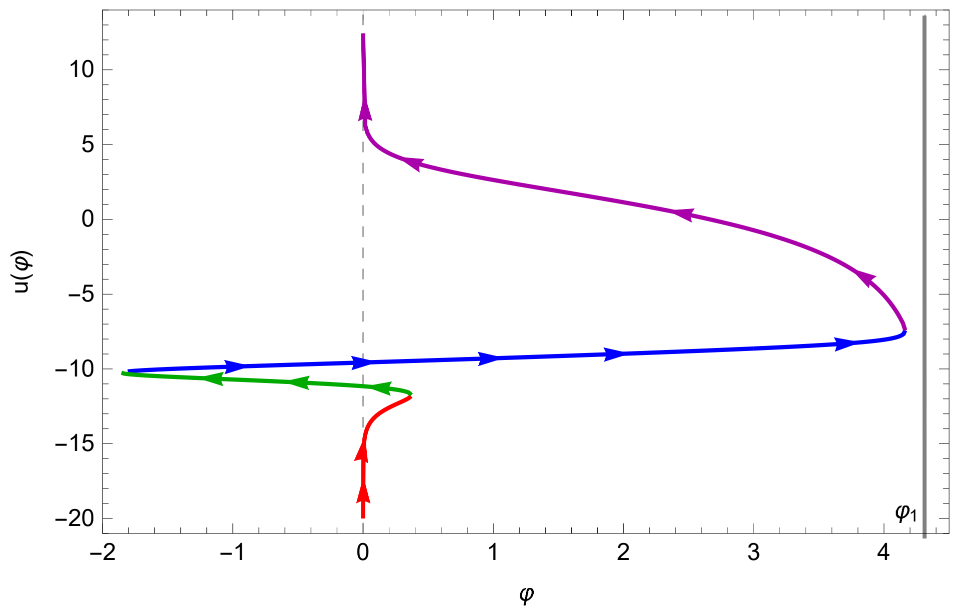
(a): A zoomed picture of the space of the $W^{LR}_{1,1}$ solutions. The black dot represents the RG flow in the diagram (b). (b): The RG flows of type $W^{LR}_{1,1}$ are between the UV_L boundary and UV_R. There is a Φ-bounce at $\Phi < 0$, the red dashed line. Notice that the red region at $S_0 < 0$ in figure (a) is the space of solutions with an extra Φ-bounce near UV_L but at $W < 0$.



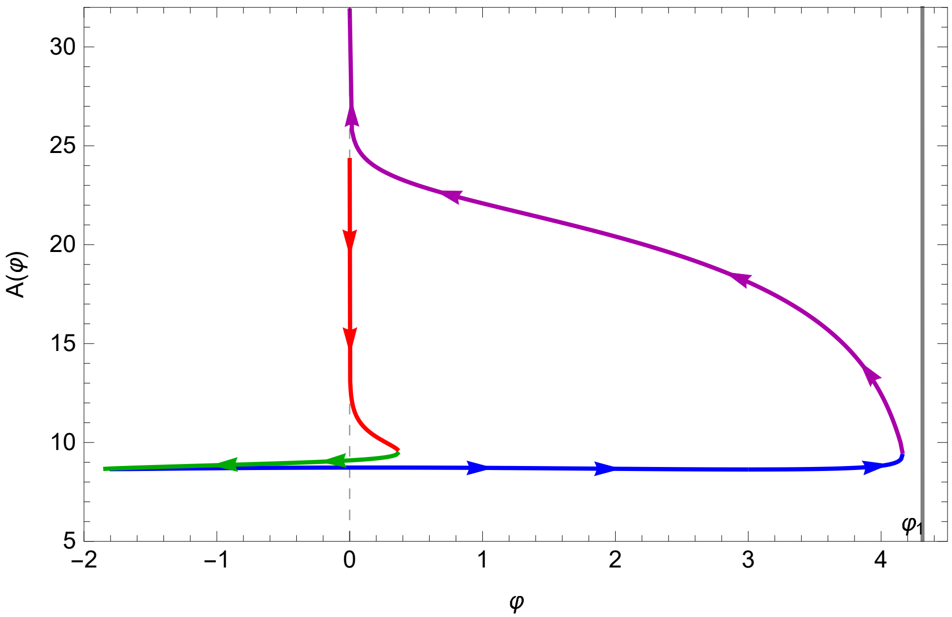
(a): The holographic coordinate at UV_L boundary tends to $-\infty$ and at UV_R to $+\infty$. (b): The scale factor has an A-bounce at $\Phi_0 = 3.5$, the blue dashed line. A Φ-bounce occurs at $\Phi = -0.64$, the red dashed line.

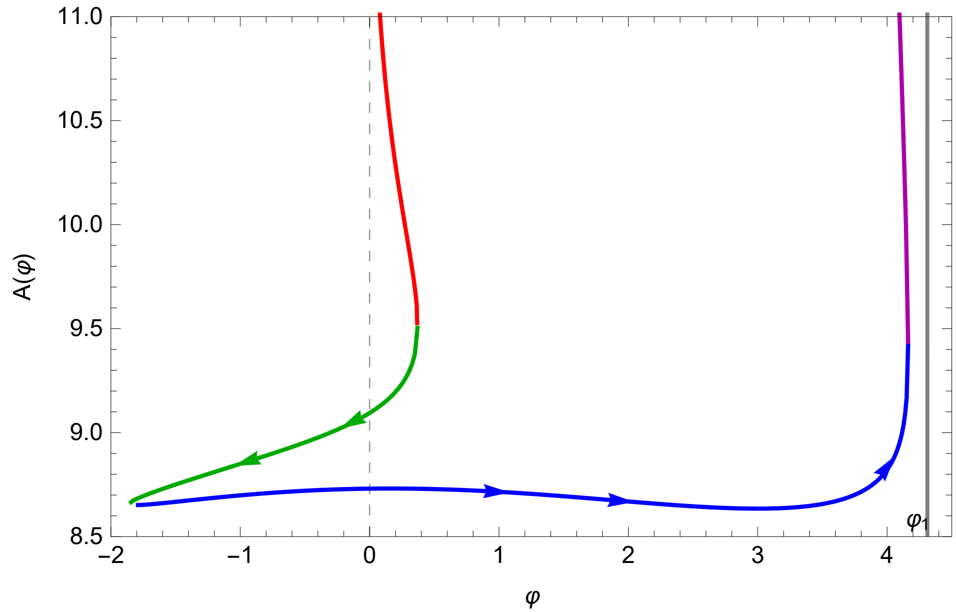
A (3,3) (A-bounce,Φ-bounce) solution



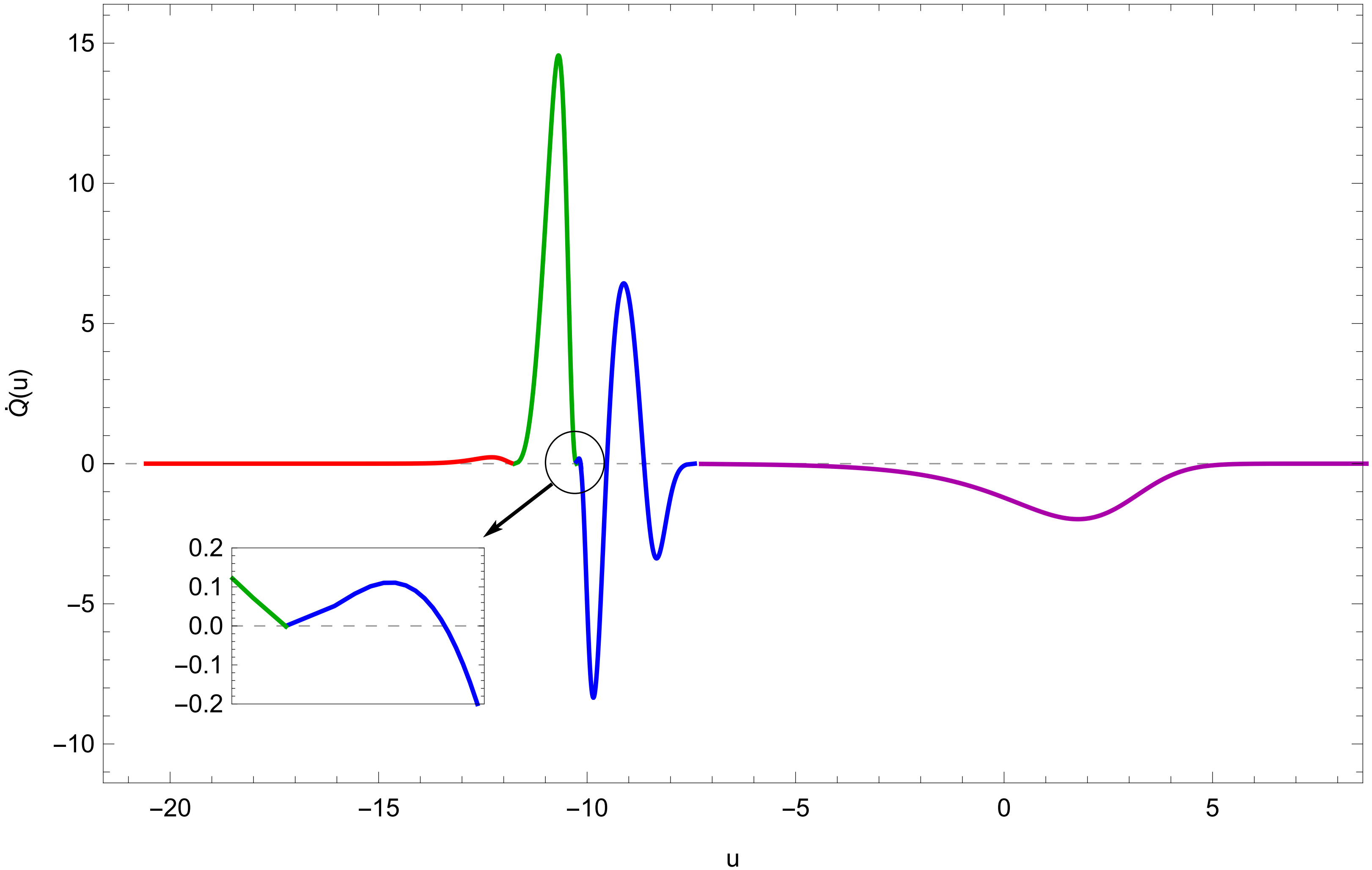


(a): An example of a multi-Φ-bounce solution, $W^{LL}_{3,3}$. The solid line is $W(\Phi)$ and dotted line is $S(\Phi)$. In this case an RG flow connects two UV boundaries on the left UV fixed point after three Φ-bounces. Unlike the previous cases the geometry here has three A-bounces.





(b)and (c) show the behavior of holographic coordinate and scale factor in terms of Φ. Figure (d) is the magnification of the bottom of figure (c). It shows that there are three A-bounces for this RG flow.

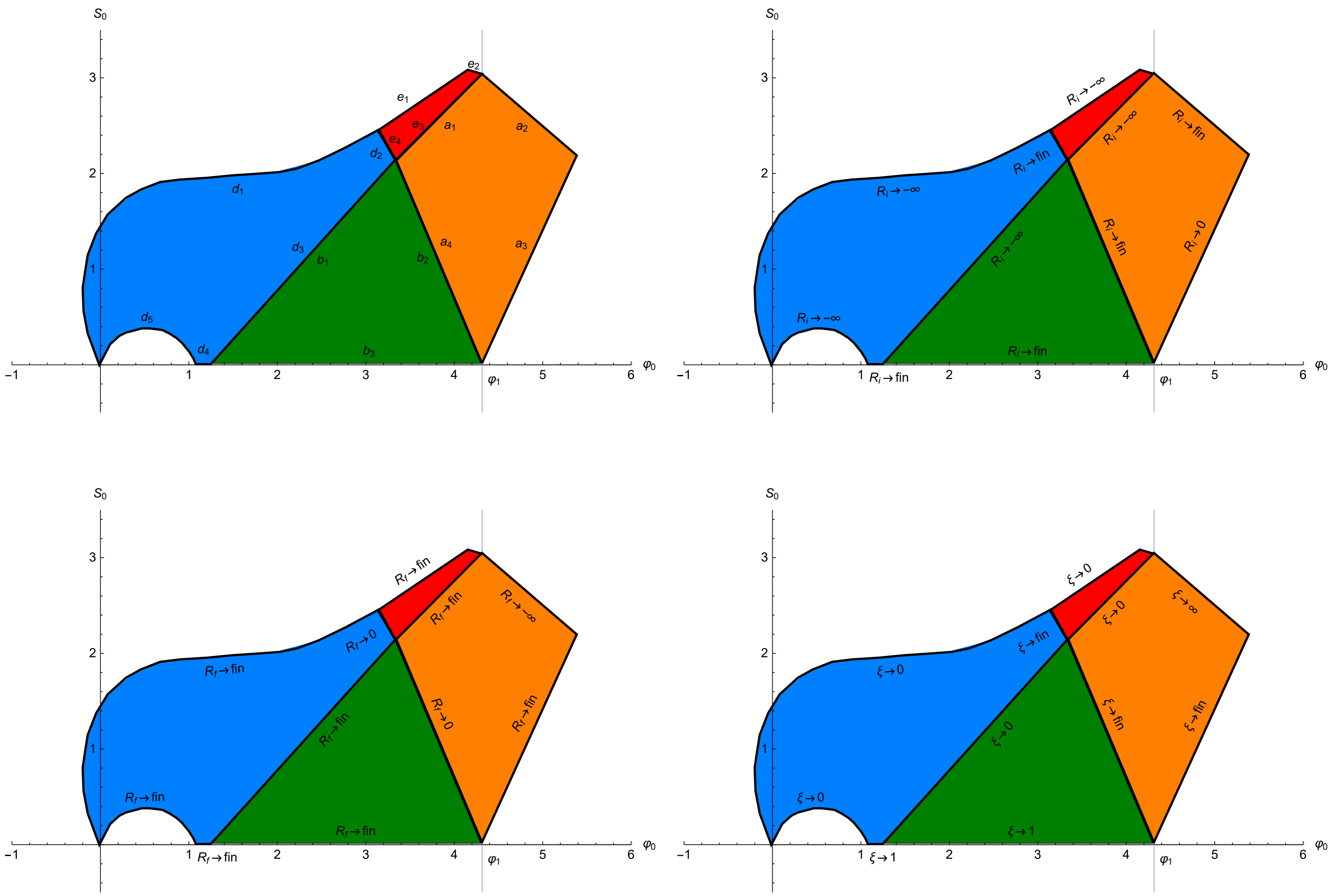


(e): The roots of $\dot{Q}$

$$Q(u) = \frac{1}{2}\dot{\Phi}^2 - V \geq 0\,, \qquad \dot{Q} = \frac{d}{2(d-1)} W S^2\,.$$

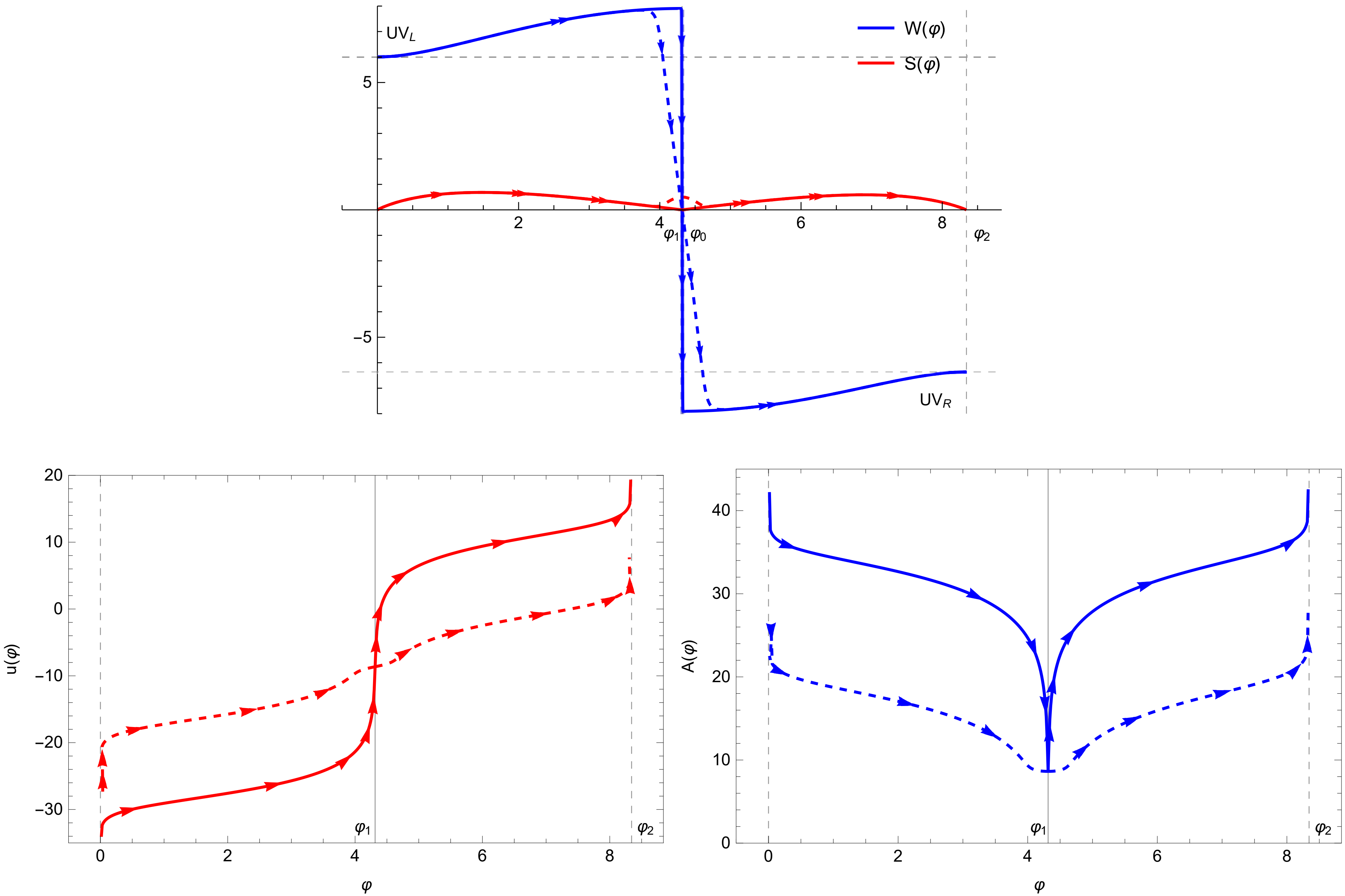
shows the location of Φ-bounces where the color of the graph is changed and location of A-bounces where the blue part of the curve crosses the u axis.

The behavior of relevant couplings



(a) Space of solution with its boundaries. (b) and (c): The behavior of $\mathcal{R}_i$ and $\mathcal{R}_f$ at boundaries. (d): The ratio of two relevant couplings, ξ, at boundaries.

The $a_3 \cup a_4$ solution: triple fragmentation



Along the fixed line $\Phi_0 = \Phi_1$ i.e. the minimum of the potential, if we decrease the value of S_0 down to zero, gradually the dashed curves in all figures above move toward the solid curves. In above curves the dashed curves have $S_0 = 0.5$ and the solid ones $S_0 = 0.01$.

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 0 minutes
- Introduction 1 minutes
- QFT on AdS 3 minutes
- A Confining Gauge Theory on AdS 6 minutes
- Conformal Theories on AdS 8 minutes
- (Holographic) Interfaces 9 minutes
- The holographic picture 12 minutes
- Wormholes vs interfaces 15 minutes
- Proximity in QFT 19 minutes
- The AdS-sliced RG Flows 24 minutes
- The QFT couplings 25 minutes

- The bulk Einstein equations 58 minutes
- The first order formalism 59 minutes
- The bulk integration constants in the two boundary case 60 minutes
- The bulk integration constants again 61 minutes
- Classifying the solutions, II 66 minutes
- Three parameter solutions 67 minutes
- $W^{LL}_{1,1}$ 68 minutes
- $W^{LL}_{1,2}$ 69 minutes
- $W^{LR}_{1,1}$ 70 minutes
- A (3,3) (A-bounce,Φ-bounce) solution 71 minutes
- The behavior of relevant couplings 72 minutes
- The $a_3 \cup a_4$ solution: triple fragmentation 73 minutes