Line Defects in Fermionic CFTs

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Based on 2211.11073 with S. Giombi and H. Khanchandani

Outline

- Intro/Motivation
- Line defects in fermionic CFTs
    1. Show the existence of a nontrivial defect IR fixed point
    2. Calculate DCFT data
    3. Check consistency with the g-theorem

Introduction

Conformal defect definition

A *conformal defect* is a non-local observable that preserves a subgroup of the conformal group. In a d-dimensional space with a p-dimensional defect, the full conformal group is broken to

$$SO(d+1,1) \to SO(p+1,1) \times SO(d-p)$$

Special cases

- Line defect: $p = 1$
  - Infinite straight line, circular defect, Wilson lines, ...
- Boundary or interface theory: $p = d - 1$

General properties of conformal defects

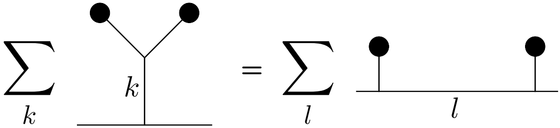
- Local excitations emerge in the presence of the line defect
- Can have an RG flow along the defect while the bulk remains at the same critical point
- Displacement operator present due to broken translational invariance perpendicular to defect

$$\partial_\mu T^{\mu i}(x) = D^i(\tau)\delta^{d-1}(\mathbf{x})$$

  - For p-dimensional defect, displacement operator has protected dimension $p+1$

General properties of conformal defects

- Bootstrap program for defect CFT based on crossing symmetry between bulk channel and defect channel decomposition



- Several developments in recent years [Liendo, Rastelli, van Rees '12, Billó, Goncalves, Lauria, Meineri '16, Lauria, Meineri, Trevisani '17 and '18, Lemos, Liendo, Meineri, Sarkar '17]

Gross-Neveu universality class

- Gross-Neveu (GN) with N_f Dirac fermions, each with $c_d = 2^{\lfloor d/2 \rfloor}$ components

$$S = -\int d^d x \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + \frac{g}{2} \left(\bar{\Psi}_i \Psi^i \right)^2 \right)$$

  - $U(N_f)$ symmetry

- Gross-Neveu-Yukawa (GNY)

$$S = \int d^d x \left(\frac{(\partial_\mu s)^2}{2} - \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + g_1 s \bar{\Psi}_i \Psi^i \right) + \frac{g_2}{24} s^4 \right)$$

  - "UV-complete" form of GN

Gross-Neveu universality class

- Universality: Same CFT describes

$$\begin{cases} \text{GN UV fixed point at } d = 2+\epsilon \\ \text{GNY IR fixed point at } d = 4-\epsilon \end{cases}$$

- Admits large N expansion for general $d \in (2,4)$
  - Starting with GN description, use Hubbard-Stratonovich auxiliary field σ to trade four-fermi interaction for $\sigma\bar{\Psi}_i\Psi^i$

$$S = -\int d^dx \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + \frac{1}{\sqrt{N}} \sigma \bar{\Psi}_i \Psi^i - \frac{\sigma^2}{2gN} \right)$$

- Physical relevance: GNY-type model proposed to describe semi-metal to insulator transition in Hubbard model [Herbut 0606195] [Assaad, Herbut 1304.6340]

Computing defect IR fixed point

Setting up the line defect

The infinite straight line defect we consider is realized by integrating an operator along a line

$$S_{\text{defect}} = S_{\text{CFT}} + h \int_{-\infty}^{\infty} d\tau O(\tau, \mathbf{x})$$

- If O has scaling dimension slightly less than 1, h is weakly relevant so we can hope to find a flow from the UV ($h = 0$) to an IR theory ($h = h_*$)
- Similar setup considered in [Allais, Sachdev 1406.3022][Cuomo, Komargodski, Mezei 2112.10634], with localized magnetic field in $O(N)$

Line defect in Gross-Neveu

Gross-Neveu at large N

$$S = -\int d^d x \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + \frac{1}{\sqrt{N}} \sigma \bar{\Psi}_i \Psi^i \right)$$

$$\Delta_\sigma = 1 + \frac{f_1(d)}{N} + O(1/N^2)$$

$f_1(d) < 0$ for $d \in (2, 4)$

Because $f_1(d)$ is negative, Δ_σ is slightly less than 1, so the following term is weakly relevant

$$h \int d\tau \sigma(\tau, \mathbf{x} = 0)$$

Line defect in Gross-Neveu-Yukawa

Gross-Neveu-Yukawa model

$$S = \int d^d x \left(\frac{(\partial_\mu s)^2}{2} - \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + g_1 s \bar{\Psi}_i \Psi^i \right) + \frac{g_2}{24} s^4 \right)$$

$$\Delta_s = 1 - \frac{3\epsilon}{N+6} + O(\epsilon^2)$$

Δ_s is slightly less than 1, so the following term is weakly relevant

$$h \int d\tau s(\tau, \mathbf{x} = 0)$$

Defect fixed points

- Use minimal subtraction scheme to renormalize defect coupling. E.g., for GNY

$$h_0 = M^{\epsilon/2}\left(h + \frac{\delta h}{\epsilon} + \dots\right)$$

- Find δh by requiring $\langle s(x)\rangle$ or $\langle \sigma(x)\rangle$ to be finite
- Find β_h as usual by requiring h_0 to be independent of renormalization scale M

$$M\frac{\partial h_0}{\partial M} = 0 = \# + \#\beta_h + \#\beta_{g_1} + \#\beta_{g_2}$$

  - β_{g_1} and β_{g_2} known from ordinary GNY, unchanged by presence of defect

Defect fixed points

- In both GN and GNY the defect fixed point h_* is not perturbatively small but $O(1)$
- Defect fixed points in GNY in $d = 4 - \epsilon$

$$h_*^2 = \begin{cases} 0 & \text{UV DCFT} \\ \frac{108}{6-N+\sqrt{N^2+132N+36}} + O(\epsilon) & \text{IR DCFT} \end{cases}$$

- Defect fixed points in GN at large N

$$h_*^2 = \begin{cases} 0 & \text{UV DCFT} \\ -\frac{2^{d+5}\pi^{\frac{3}{2}(d-3)}(d-3)(d-2)(1-\cos(\pi d))\Gamma(\frac{d-1}{2})\Gamma(d)}{\Gamma\left(\frac{d}{2}-1\right)^3\Gamma\left(\frac{d}{2}\right)^3 d\left((d-3)H_{\frac{d}{2}-2}+(3-d)H_{d-4}-1\right)} + O(\frac{1}{N}) & \text{IR DCFT} \end{cases}$$

Extracting DCFT data

- Automatically get one-point function coefficients since our beta-function calculation involved computing $\langle s(x) \rangle$ and $\langle \sigma(x) \rangle$

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$$\langle s(x)\rangle = \frac{\sqrt{\mathcal{N}_s}a_s}{|\mathbf{x}|^{\Delta_s}}, \qquad a_s^2 = \frac{27}{6-N+\sqrt{N^2+132N+36}}$$

$$\langle\sigma(x)\rangle = \frac{\sqrt{\mathcal{N}_\sigma}a_\sigma}{|\mathbf{x}|^{1}}, \quad a_\sigma^2 = -\frac{(d-3)(d-1)}{(d-2)d\left((d-3)H_{\frac{d}{2}-2}+(3-d)H_{d-4}-1\right)}$$

with $\mathcal{N}_s$, $\mathcal{N}_\sigma$ the two-point function normalizations of s and σ

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with $\mathcal{N}_s$, $\mathcal{N}_\sigma$ the two-point function normalizations of s and σ

- Since s is identified with σ (up to normalization) their 1-point function coefficients coincide in the overlapping regime of validity:

$$a_s^2 \xrightarrow{\text{Large N}} 3/8 + O(1/N)$$

$$a_\sigma^2 \xrightarrow{d=4-\epsilon} 3/8 + O(\epsilon)$$

Summary of defect scaling dimensions

	GNY $d=4-\epsilon$	Large N
Leading defect scalar	$1+\frac{6}{(N+6)}\epsilon$	$1-\frac{2^{d+2}(d-1)\sin\left(\frac{\pi d}{2}\right)\Gamma\left(\frac{d-1}{2}\right)}{Nd(d-2)\pi^{3/2}\Gamma\left(\frac{d}{2}-1\right)}$
Transverse spin l defect scalars	$1+l\ +\frac{6(1-l)}{(N+6)(1+2l)}\epsilon$	$1+l+O(\frac{1}{N})$
$U(N)$ fundamental defect fermions	$\frac{3}{2}+l\ +O(\epsilon)$	$\frac{d-1}{2}+l\ +O(\frac{1}{N})$

- Lowest twist operators for a given transverse spin l
- l is quantum number under $SO(d-1)$ = group of rotations around the defect
- Scalars in symmetric traceless representation of $SO(d-1)$
- Fermions in spin $l+1/2$ representation of $Spin(d-1)$

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Leading defect scalar

- Stress-tensor localized on defect (away from fixed point) is related to beta function of defect coupling:

$$T_D(\tau) = \beta_h \hat{s}(\tau)$$

- Differentiate both sides w.r.t. renormalization scale M. Note dimension of T_D fixed to 1

$$\Delta_{\hat{s}} = 1 + \frac{\partial \beta_h}{\partial h}$$

- This gives

$$\Delta(\hat{s}) = 1 + \frac{6}{(N+6)}\epsilon \xrightarrow{\text{Large N}} 1 + \frac{6}{N}\epsilon$$
$$\Delta(\hat{\sigma}) = 1 - \frac{2^{d+2}(d-1)\sin\left(\frac{\pi d}{2}\right)\Gamma\left(\frac{d-1}{2}\right)}{Nd(d-2)\pi^{\frac{3}{2}}\Gamma\left(\frac{d}{2}-1\right)} \xrightarrow{d=4-\epsilon} 1 + \frac{6}{N}\epsilon$$

  - $\hat{s}$ is irrelevant in IR DCFT. No other candidate for relevant operator on defect $\implies$ IR DCFT is stable

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Classical dimensions of defect operators

- Map theory to $H^2 \times S^{d-2}$
- Expand in eigenfunctions and perform a Kaluza-Klein reduction on S^{d-2} to obtain a tower of operators on H^2
- Use AdS/CFT dictionary to read off dimensions of defect scalars and fermions

Map to $H^2 \times S^{d-2}$

Weyl rescaling to go from flat space to $H^2 \times S^{d-2}$

$$ds^2 = \rho^2 \left(\frac{d\rho^2 + d\tau^2}{\rho^2} + ds^2_{S^{d-2}} \right) = \rho^2 ds^2_{H^2 \times S^{d-2}}$$

Map to $H^2 \times S^{d-2}$ (GNY)

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Recall GNY action with N_f fermions, each with c_d components

$$S_{GNY} = \int d^d x \left(\frac{(\partial_\mu s)^2}{2} - \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + g_1 s \bar{\Psi}_i \Psi^i \right) + \frac{g_2}{24} s^4 \right) + h_0 \int d\tau s(\tau, \mathbf{0})$$

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$$S_{GNY} = \int_{H^2 \times S^{d-2}} \frac{d\tau d\rho}{\rho^2} d^{d-2}\Omega \left(\frac{(\nabla_\mu s)^2}{2} + \frac{(d-2)(d-4)}{8} s^2 - \left(\bar{\Psi}_i \gamma \cdot \nabla \Psi^i + g_{1,0} s \bar{\Psi}_i \Psi^i \right) + \frac{g_{2,0}}{24} s^4 \right) + h_0 \int d\tau s(\tau, \mathbf{0})$$

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Representation of gamma matrices in $H^2 \times S^{d-2}$

$$\gamma^1 = \sigma^1 \otimes I, \quad \gamma^2 = \sigma^2 \otimes I, \quad \gamma^i = \sigma^3 \otimes \Gamma^i$$

Pauli matrix

c_{d-2}-dimensional identity

Decompose into eigenfunctions of Laplacian and Dirac operator on $H^2 \times S^{d-2}$

Laplacian and Dirac operators decompose as [Camporesi, Higuchi 9505009, ...]

$$(\nabla^2)_{H^2 \times S^{d-2}} = \nabla^2_{H^2} + \nabla^2_{S^{d-2}}$$
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\end{aligned}
$$

With eigenfunctions

$$
\begin{aligned}
(\nabla^2)_{S^{d-2}} Y_{lm} &= -l(l+d-3)Y_{lm} \\
(\not\nabla)_{S^{d-2}} \chi^{\pm}_{lm} &= \pm i \left(l + \frac{d}{2} - 1\right) \chi^{\pm}_{lm}
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Scalar and fermion decompose as

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s = \sum_{l,m} t_{lm}(\rho, \tau) Y_{lm}
$$

$$
\Psi = \sum_{l,m} \left(\psi^{+}_{lm} \otimes \chi^{+}_{lm} + \psi^{-}_{lm} \otimes \chi^{-}_{lm} \right)
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Decompose into eigenfunctions of Laplacian and Dirac operator on $H^2 \times S^{d-2}$ (GNY)

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$\downarrow$ Normalization: $\int_{S^2} Y^*_{l,m} Y_{l',m'} = \delta_{ll'}\delta_{mm'}, \qquad \int_{S^2} \chi^{\pm}{}_{lm} \chi^{\pm}_{l'm'} = \delta_{ll'}\delta_{mm'}$

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$$\boxed{\begin{aligned} \hat{\Delta}^f_l &= \frac{1}{2} + |m_f| \\ &= \frac{d-1}{2} + l \end{aligned}} \qquad \boxed{\begin{aligned} \hat{\Delta}^s_l &= \frac{1}{2} \pm \sqrt{\frac{1}{4} + m_s^2} \\ &= \frac{d}{2} - 1 + l \end{aligned}}$$

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$$S_{GN} = -\int\limits_{H^2 \times S^{d-2}} d^d x \sqrt{g} \left(\bar{\Psi}_i \gamma \cdot \partial \Psi^i + \frac{1}{\sqrt{N}} \sigma \bar{\Psi}_i \Psi^i \right) + h \int d\tau \sqrt{g} \sigma(\tau, \mathbf{x} = 0)$$

Decompose into eigenfunctions of Laplacian and Dirac operator on $H^2 \times S^{d-2}$ (GN)

Scalar and fernion decompose as $\begin{cases} s = \sum_{l,m} t_{lm}(\rho,\tau) Y_{lm} \\ \Psi = \sum_{l,m} \left(\psi^+_{lm} \otimes \chi^+_{lm} + \psi^-_{lm} \otimes \chi^-_{lm}\right) \end{cases}$

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$$\boxed{\begin{aligned} \hat{\Delta}^f_l &= \tfrac{1}{2} + |m_f| \\ &= \tfrac{d-1}{2} + l \end{aligned}}$$

Summary of defect scaling dimensions

	GNY $d = 4 - \epsilon$	Large N
Leading defect scalar	$1 + \frac{6}{(N+6)}\epsilon$	$1 - \frac{2^{d+2}(d-1)\sin\left(\frac{\pi d}{2}\right)\Gamma\left(\frac{d-1}{2}\right)}{N d(d-2)\pi^{3/2}\Gamma\left(\frac{d}{2}-1\right)}$
Transverse spin l defect scalars	$1 + l + \frac{6(1-l)}{(N+6)(1+2l)}\epsilon$	$1 + l \ \ + O(\frac{1}{N})$
$U(N)$ fundamental defect fermions	$\frac{3}{2} + l + O(\epsilon)$	$\frac{d-1}{2} + l + O(\frac{1}{N})$

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Spinning defect scalars in large N

- Look at defect operators induced by σ in defect channel OPE
- Two-point function of σ is

$$\langle \sigma(x_1)\sigma(x_2)\rangle = \underbrace{\frac{h_*^2 \mathcal{N}_\sigma^2 \pi^2}{|\mathbf{x}_1||\mathbf{x}_2|}}_{\text{from defect identity}} + \frac{\mathcal{N}_\sigma}{x_{12}^2}$$

- From [Liendo, Linke, Schomerus 1903.05222]: should have tower of operators with dimensions $1 + l + 2m$ with l the transverse spin and m the degeneracy per transverse spin:

$$\frac{1}{x_{12}^2} = \frac{1}{|\mathbf{x}_1||\mathbf{x}_2|} \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} b_{m,l}^2 \hat{f}_{1+l+2m,l}$$

- At $d = 4$, $b_{m,l}^2 = 0$ unless $m = 0 \implies$ have a tower of operators with dimensions $1 + l$ with degeneracy 0

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Anomalous dimensions using equations of motion (GNY)

Lesson from [Rychkov, Tan 1505.00963]: can extract anomalous dimensions using the following

(a) conformal symmetry constrains form of correlators

(b) operators satisfy an equation of motion at the WF fixed point

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- The bulk-boundary two-point function in GNY is constrained to take the form

$$\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle = \frac{(\mathbf{x}\cdot\mathbf{w})^l}{|\mathbf{x}|^{\Delta_s-\hat{\Delta}_l^s+l}(\mathbf{x}^2+(\tau-\tau')^2)^{\hat{\Delta}_l^s}}$$

where $\mathbf{w}$ is a null auxiliary vector in embedding space, τ in direction of defect, $\mathbf{x}$ in transverse direction

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- Applying bulk laplacian gives

$$\nabla^2\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle = \left[\frac{2\hat{\Delta}_l^s\,(2\Delta_s-d+2)}{\mathbf{x}^2+(\tau-\tau')^2} - \frac{\left(\Delta_s-\hat{\Delta}_l^s+l\right)\left(d-3+l-\Delta_s+\hat{\Delta}_l^s\right)}{\mathbf{x}^2}\right]\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle$$

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$$S_{GNY} = \int d^d x \sqrt{g} \left(\frac{(\nabla_\mu s)^2}{2} - \left(\bar{\Psi}_i \gamma \cdot \nabla \Psi^i + g_1 s \bar{\Psi}_i \Psi^i \right) + \frac{g_2}{24} s^4 \right) + h \int d\tau s(\tau, \mathbf{0})$$

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Applying bulk laplacian to bulk-boundary two-point function gives two diagrams at $O(\epsilon)$:

$$\begin{aligned}
\nabla^2 \langle s(x) \hat{s}_l(\tau', \mathbf{w}) \rangle &= \langle \left(\frac{g_2}{6} s^3 + g_1 \bar{\Psi}_i \Psi^i \right) \hat{s}_l(\tau', \mathbf{w}) \rangle \\
&= \frac{g_2}{2} \langle s^2(x) \rangle \langle s(x) \hat{s}_l(\tau', \mathbf{w}) \rangle \\
&\quad - g_1^2 \int d^d x_1 \langle \bar{\Psi}_i \Psi^i(x) \bar{\Psi}_j \Psi^j(x_1) \rangle \langle s(x_1) \hat{s}_l(\tau', \mathbf{w}) \rangle
\end{aligned}$$

Anomalous dimensions using equations of motion

Compare each side

$$\nabla^2\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle = \Bigg[\frac{2\hat{\Delta}_l^s\left(2\Delta_s - d + 2\right)}{\mathbf{x}^2 + (\tau-\tau')^2} - \frac{\left(\Delta_s - \hat{\Delta}_l^s + l\right)\left(d - 3 + l - \Delta_s + \hat{\Delta}_l^s\right)}{\mathbf{x}^2}\Bigg]\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle$$

$$\nabla^2\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle = \frac{g_2}{2}\langle s^2(x)\rangle\langle s(x)\hat{s}_l(\tau',\mathbf{w})\rangle - g_1^2\int d^dx_1\langle\bar{\Psi}_i\Psi^i(x)\bar{\Psi}_j\Psi^j(x_1)\rangle\langle s(x_1)\hat{s}_l(\tau',\mathbf{w})\rangle$$

The $O(\epsilon)$ terms in the top line are contained in the anomalous dimensions while the $O(\epsilon)$ terms in the bottom line are contained in the bulk coupling constants

- We can use free theory propagators in the integrals

Anomalous dimensions using equations of motion

After the dust settles we have

$$\hat{\gamma}_l^s = \gamma_s + \frac{9\epsilon}{(N+6)(1+2l)}$$

$$\hat{\Delta}_l^s = \Delta_s + l + \frac{9\epsilon}{(N+6)(1+2l)} = 1 + l + \frac{6(1-l)}{(N+6)(1+2l)}\epsilon.$$

- Anomalous dimensions of defect and bulk operators match as $l \to \infty$, consistent with [Lemos, Liendo, Meineri, Sarkar 1712.08185]

Same technique can be used to find anomalous dimensions of defect operators in $O(N)$ with localized magnetic field

- Anomalous dimensions of defect scalars with transverse spin 0 and 1 computed in [Cuomo, Komargodski, Mezei 2112.10634]. Can extend this to generic transverse spin l using equation of motion

Consistency with the g-theorem

- Consider a conformally equivalent setup: a circular line defect of radius R
- It was proven in [Cuomo, Komargodski, Raviv-Moshe 2108.01117] that the following decreases monotonically under an RG flow localized on the defect

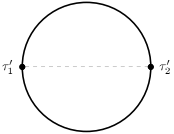
$$s = \left(1 - R\frac{\partial}{\partial R}\right)\log g$$
$$= \log g \qquad \text{at fixed points}$$

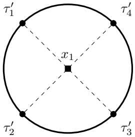
where g is the expectation of the circular defect

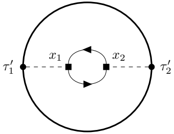
$$g = \langle e^{-h\int d\tau s}\rangle$$
$$\log g = \log(Z^{\text{bulk+defect}}/Z^{\text{bulk}})$$

Consistency with the g-theorem

- In GNY we computed $\log g$ to first order in ϵ at $d = 4 - \epsilon$

$\log g =$ 

$+$ 

$+$ 

and found consistency with the g-theorem

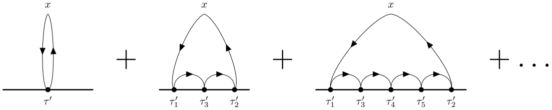
$$\log g\Big|_{h=h_*} - \log g\Big|_{h=0} = -\frac{81\epsilon}{2(N+6)\left(6 - N + \sqrt{N^2 + 132N + 36}\right)} < 0$$

Summary

- Found a defect IR fixed point using both ϵ expansion and large N techniques
- Computed various DCFT data and saw consistency in overlapping regime of validity for ϵ expansion and large N
- Checked consistency with the g-theorem

Future directions

- GN model in $d = 2 + \epsilon$ perturbed by a fermion bilinear $h \int d\tau \bar{\Psi}\Psi(\tau, \mathbf{0})$
  - Already infinite diagrams for free theory

$\langle \bar{\Psi}\Psi(x) \rangle =$ 

- Test predictions using Monte-Carlo, similar to [Toldin, Assaad, Wessel 1607.04270] where they determine scaling dimensions of the defect operators for the pinning field defect in the Ising CFT in $d = 3$

Thank you!