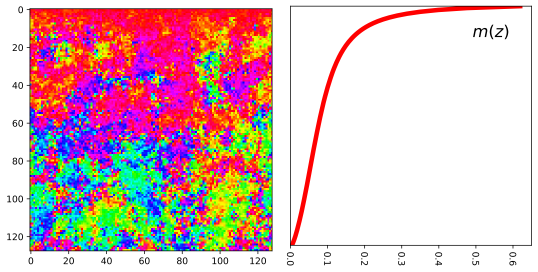
Advances in the boundary O(N) universality class



Francesco Parisen Toldin

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RWTH AACHEN UNIVERSITY

New perspectives on Quantum Field Theory with Boundaries Impurities, and Defects

Nordita, Stockholm, 4 August 2023

Boundary critical phenomena

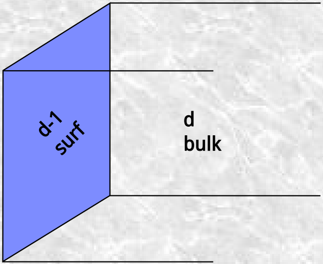
In the vicinity of a second-order phase transitions:

- Power-laws, universality
- Renormalization Group:

$$K \to \mathcal{R}(K) \to \mathcal{R}(\mathcal{R}(K)) \dots$$

- Real systems: surfaces
- RG: bulk vs surface couplings
  Cardy book (1996)

$$K^*_{\mathrm{bulk}} \begin{array}{l} \nearrow K^{*(1)}_{\mathrm{surf}} \\ \to K^{*(2)}_{\mathrm{surf}} \\ \searrow \dots \end{array}$$



Reviews: Binder (1983); Diehl (1986)

$\to$Rich bulk-surface phase diagram

$\to$Surface UC determine the critical Casimir force Fisher, de Gennes (1978)

A renewed interest: quantum spin models

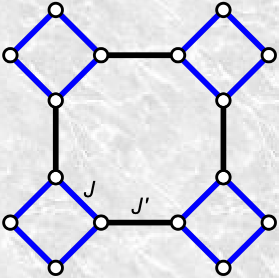
PRL **118,** 087201 (2017) PHYSICAL REVIEW LETTERS week ending 24 FEBRUARY 2017

Unconventional Surface Critical Behavior Induced by a Quantum Phase Transition from the Two-Dimensional Affleck-Kennedy-Lieb-Tasaki Phase to a Néel-Ordered Phase

Long Zhang[1] and Fa Wang[1,2]
[1]*International Center for Quantum Materials, School of Physics, Peking University, Beijing 100871, China*
[2]*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

(Received 30 November 2016; revised manuscript received 13 January 2017; published 21 February 2017)

- Bulk: quantum phase transition in the O(3) UC in $d = 2 + 1$
- Unexpected boundary exponents



A renewed interest: quantum spin models

A renewed interest: Conformal Field Theory

Reexamining the boundary O(N) universality class

PHYSICAL REVIEW LETTERS **126,** 135701 (2021)

Boundary Critical Behavior of the Three-Dimensional Heisenberg Universality Class

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SciPost Select SciPost Phys. 12, 131 (2022)

Boundary criticality of the $O(N)$ model in $d = 3$ critically revisited

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PHYSICAL REVIEW LETTERS **128,** 215701 (2022)

Boundary Criticality of the 3D O(N) Model: From Normal to Extraordinary

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Max A. Metlitski†
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Outline

1. Surface critical behavior of the 3D O(N) model

2. $O(N)$: from Normal to Extraordinary

3. Line defect

1. Surface critical behavior of the 3D O(N) model
2. $O(N)$: from Normal to Extraordinary
3. Line defect

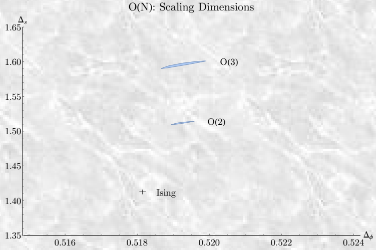
The 3D O(N) universality class

- Simplest model of classical magnets
  Liquid-vapor, uniaxial ferromagnets ($N = 1$), superfluid ^{4}He ($N = 2$), isotropic magnets ($N = 3$), chiral transition QCD ($N = 4$), ...

  Pelissetto, Vicari (2002)

- Numerous investigations: field-theory, high-T expansion, MC, ...
- Conformal Bootstrap in $d = 3$
  Rigorous results, very precise

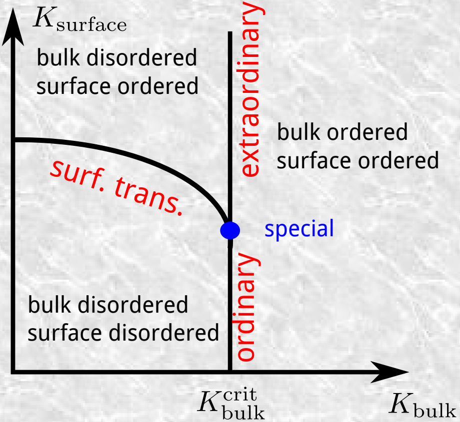
- CB with boundaries
  Recent developments
  McAvity, Osborn (1995)
  Liendo, Rastelli, van Rees (2013)
  Gliozzi, Liendo, Menieri, Rago (2015)
  Billó, Gonçalves, Lauria, Menieri (2016)
  Padayasi, Krishnan, Metlitski, Gruzberg, Meineri (2022)



Kos, Poland, Simmons-Duffin, Vichi, (2016)

O(N) phase diagram $d = 3$: N=1

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surf. trans.: usual $d = 2$ phase transition
- Ordinary: new surf. exponent

$$\langle \phi(\mathbf{x})\phi(0)\rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}}$$

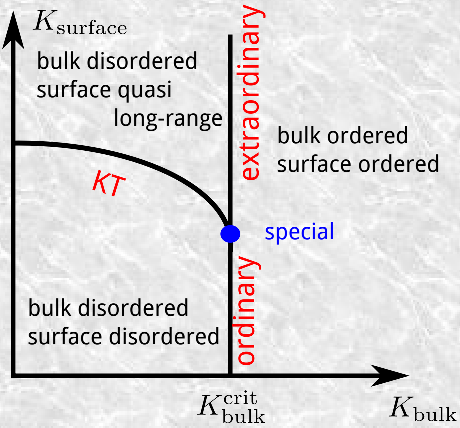
$$\Delta_{\hat{\phi}} = (1+\eta_{\|})/2 = 1.2751(6)$$

Hasenbusch (2011)

- Special: multicritical point, relevant surface exponent + $\eta_{\|}$
- Extraordinary: ordered

O(N) phase diagram $d = 3$: N=2

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surface transition: Berezinskii-Kosterlitz-Thouless
- Ordinary transition

$$\langle \phi(\mathbf{x}) \phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\|})/2 = 1.2286(25)$$

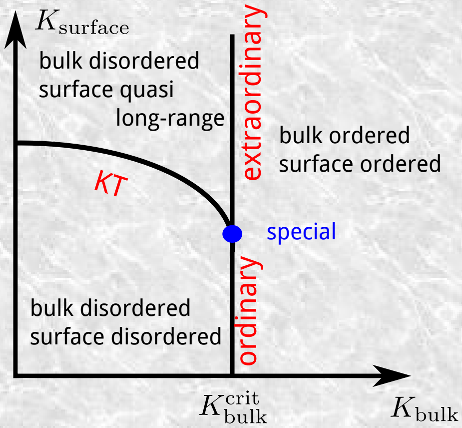
FPT (2013)

- Extraordinary: ordered? first/second order?

Deng, Blöte, Nightingale (2005)

O(N) phase diagram $d = 3$: N=2

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surface transition: Berezinskii-Kosterlitz-Thouless
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0)\rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1+\eta_{\parallel})/2 = 1.2286(25)$$

FPT (2013)

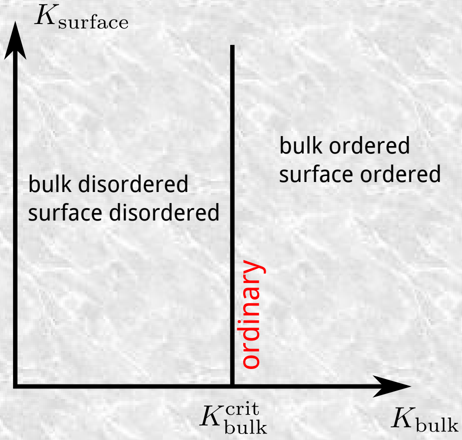
- Extraordinary: "extraordinary-log" phase

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0)\rangle \sim \frac{1}{(\log \mathbf{x})^{q}}$$

Metlitski (2020); Hu, Deng, Lv (2021)

O(N) phase diagram $d = 3$: N=3

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- No surface transition
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0)\rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.194(3)$$

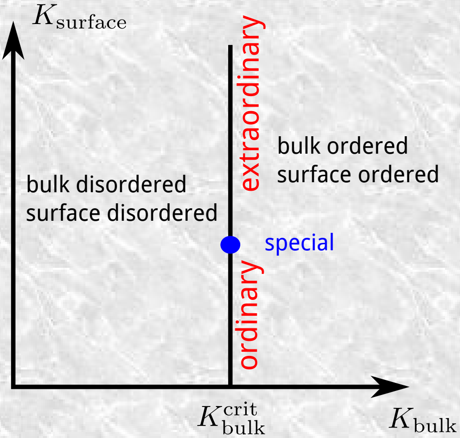
FPT (2013)

- Surface transition shifted to $K_{\text{surface}} \to \infty$
  Only the ordinary UC

Krech (2000)

O(N) phase diagram $d = 3$: N=3

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- No surface transition
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0)\rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1+\eta_{||})/2 = 1.194(3)$$

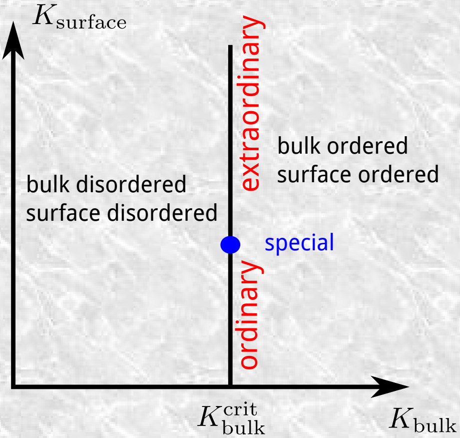
FPT (2013)

- "KT-like" special transition at finite K_{surface}

Deng, Blöte, Nightingale (2005)

O(N) phase diagram $d = 3$: N=3

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- No surface transition
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$$\langle \phi(\mathbf{x})\phi(0)\rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1+\eta_{\|})/2 = 1.194(3)$$

FPT (2013)

- Special: standard transition, relevant surface exponent + $\eta_{\|}$

$$\Delta_{\hat{s}} = 1.64(1) \quad \Delta_{\hat{\phi}} = 0.2635(10)$$

- Extraordinary-log phase
  Correlations: power of a log
  Logarithmic violation FSS

Model

- ϕ^4 $N = 3$ lattice model, $L \times L \times L$ lattice, open b.c. $z-$direction

$$\mathcal{H} = -\beta \sum_{\langle i\, j \rangle} \vec{\phi}_i \cdot \vec{\phi}_j - \beta_{s,\downarrow} \sum_{\langle i\, j \rangle_{s\downarrow}} \vec{\phi}_i \cdot \vec{\phi}_j - \beta_{s,\uparrow} \sum_{\langle i\, j \rangle_{s\uparrow}} \vec{\phi}_i \cdot \vec{\phi}_j + \sum_i \vec{\phi}_i^{\,2} + \lambda(\vec{\phi}_i^{\,2} - 1)^2$$

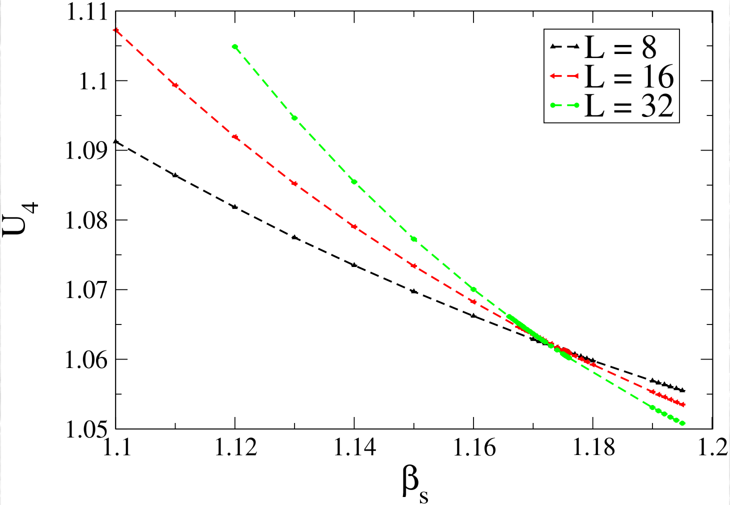
- Heisenberg UC; $\lambda = \lambda^* = 5.17(11)$ *improved* Hasenbusch (2020)
  $\rightarrow$ leading *bulk* scaling corrections suppressed

- MC simulations at $\lambda = \lambda^*$, $\beta = \beta_c(\lambda^*)$, $\beta_{s,\downarrow} = \beta_{s,\uparrow} = \beta_s$

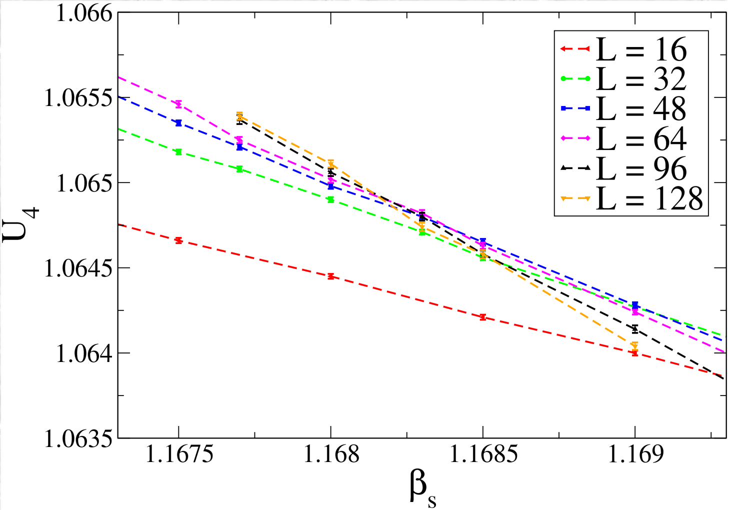
- Scan the surface observables on β_s

Goal: surface transition?

Surface Binder ratio $U_4 = \langle M_s^4 \rangle / \langle M_s^2 \rangle^2$



Surface Binder ratio $U_4 = \langle M_s^4 \rangle / \langle M_s^2 \rangle^2$



Special surface transition

- Finite-Size Scaling analysis of U_4

  Hasenbusch (1999); FPT, PRE 84, 025703R (2011); Review: FPT, PRE 105, 034137 (2022)

$$U_4 = f((\beta_s - \beta_{s,c})L^{y_{sp}})$$

- Special phase transition on the surface of a 3D Heisenberg UC

$$y_{\rm sp} = 0.36(1) \qquad \Delta_{\hat{s}} = 2 - y_{\rm sp} = 1.64(1)$$

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0) \rangle_{\rm surf} \sim \frac{1}{|\mathbf{x}|^{1+\eta_\parallel}} \quad \eta_\parallel = -0.473(2) \;\; \Delta_{\hat{\phi}} = \frac{1+\eta_\parallel}{2} = 0.2635(10)$$

  Quantum spin models with unexpected boundary $\eta_\parallel$: accidentally close to the special transition (?)

- $\beta_s > \beta_{s,c}$: extraordinary phase

1. Surface critical behavior of the 3D O(N) model
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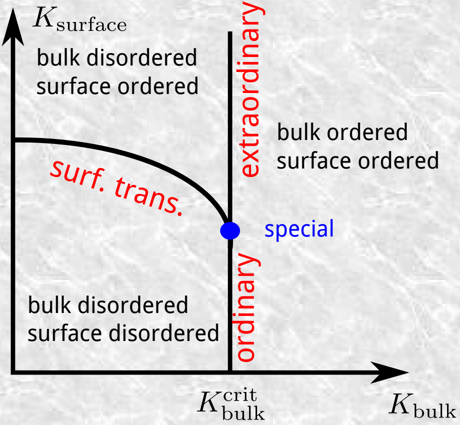
O(N) phase diagram $d = 3$: N=1

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



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$$\Delta_{\hat{\phi}} = (1 + \eta_{||})/2 = 1.2751(6)$$

Hasenbusch (2011)

- Special: multicritical point, relevant surface exponent + $\eta_{||}$
- Extraordinary: ordered

O(N) Normal Universality Class

- 3D O(N) model + surface field $\vec{h}$
  $\rightarrow$ ordered surface

- Ising UC: binary critical mixture $A + B$
  $\phi \sim c_A - c_{A,c}$

- Adsorption preference on surfaces

Fluid Interface Tensions near Critical End Points

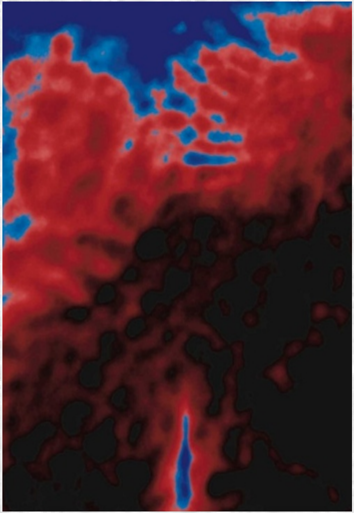
Michael E. Fisher and Paul J. Upton[a]

PRL 65, 3405 (1990)

describes criticality of wall free energies at the so-called extraordinary surface transition[1,7] (which, however, in the laboratory is more normal than the "ordinary" transition).

- Ising: normal = extraordinary

Diehl (1994)



Hertlein, Helden, Gambassi, Dietrich, Bechinger (2008)

From: Balibar, Nature News&Views (2008)

Critical adsorption

Adsorption layer governed by $\sim \xi$

$$m(z) = a|t|^{\beta} P_{\pm}(z/\xi), \; t = \frac{T - T_c}{T_c}$$

$$P_+(\tilde{z}_+ \to \infty) \sim e^{-\tilde{z}_+}$$

$$P_-(\tilde{z}_- \to \infty) - 1 \sim e^{-\tilde{z}_-}$$

$$P_{\pm}(\tilde{z}_{\pm} \to 0) = c_{\pm} \tilde{z}^{-\beta/\nu}$$

MC: $c_+ = 0.844(6)$ FPT, S. Dietrich, JSTAT P11003 (2010)

FT: $c_+ = 0.94(4)$ Flöter, Dietrich (1995)

Exp.: $c_+ = 1.60(42)$ $c_+ = 0.77(19)$

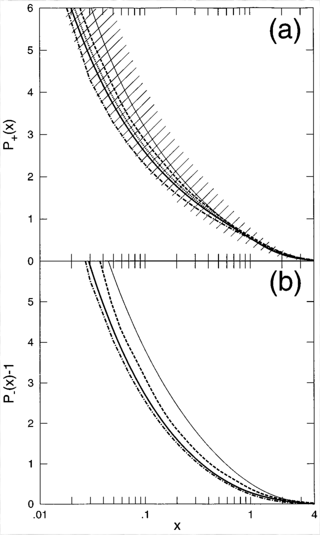
$c_+ = 1.14(29)$ $c_+ = 0.91(26)$

$c_+ = 1.05(9)$ $c_+ = 1.02(10)$

$c_+ = 1.25(9)$ $c_+ = 0.84(15)$

Flöter, Dietrich (1995)

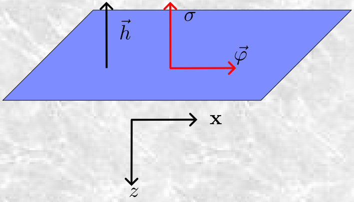
$c_+ = 0.787^{+0.009}_{-0.015}$ Carpenter, Law, Smith (1999)



Carpenter, Law, Smith (1999)
Review: Law, Prog. Surf. Sc. (2001)

Normal Universality Class: $O(N > 1)$ model

- $O(N)$-symmetric bulk, at criticality



- Introduce a surface field $\vec{h}$
  $\rightarrow$ ordered surface
  Broken $O(N) \rightarrow O(N-1)$
  $$\vec{\phi} = (\vec{\varphi}, \sigma) \qquad \vec{\varphi} \perp \vec{h} \qquad \sigma \parallel \vec{h}$$

- Longitudinal vs transverse surface correlations

J. Phys. A: Math. Gen., Vol. 10, No. 11, 1977. Printed in Great Britain. © 1977

Critical behaviour of semi-infinite systems

A J Bray and M A Moore
Department of Theoretical Physics, The University, Manchester, M13 9PL, UK

$$\langle \sigma(\mathbf{x})\sigma(\mathbf{y})\rangle_c \sim |\mathbf{x}-\mathbf{y}|^{-2d}$$
$$\langle \vec{\varphi}(\mathbf{x})\vec{\varphi}(\mathbf{y})\rangle \sim |\mathbf{x}-\mathbf{y}|^{-(2d-2)}$$

Normal Universality Class: $O(N > 1)$ model

- $O(N)$-symmetric bulk, at criticality
- Introduce a surface field $\vec{h}$
  $\rightarrow$ ordered surface
  Broken $O(N) \rightarrow O(N-1)$
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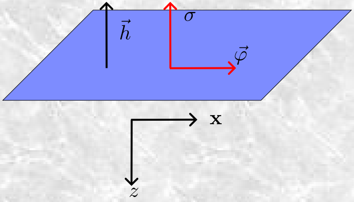
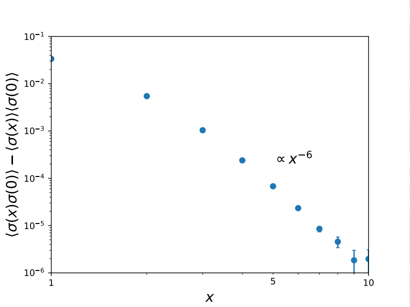
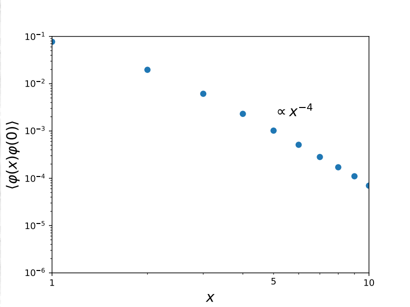


- Longitudinal vs transverse surface correlations





Boundary CFT for the Normal UC

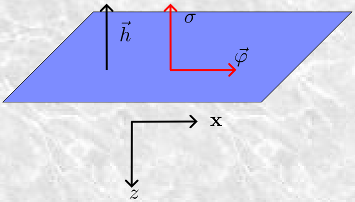
- $O(N)$ broken to $O(N-1)$
  Boundary operators are representations of $O(N-1)$

- Spectrum of the CFT:
  - Displacement operator D, $\Delta_D = d$. $O(N-1)$-scalar
  - Tilt operator t^i, $\Delta_t = d-1$ $O(N-1)$-vector

- On the boundary, $\vec{\phi} = (\vec{\varphi}, \sigma)$

$$\sigma = \sum_{\text{bdy scalars}} \hat{O}$$

$$\phi^i = \sum_{\text{bdy vectors}} \hat{O}^i$$

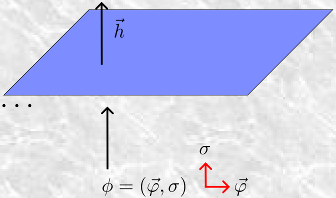


- Longitudinal/transverse correlation:
  D and t^i are the lowest operators

From Normal to Extraordinary

- Boundary Operator Product Expansion

  $z \to 0$



$$\sigma(\mathbf{x}, z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_{\mathrm{D}}(2z)^{3-\Delta_\phi}\mathrm{D}(\mathbf{x}) + \dots$$

$$\varphi^i(\mathbf{x}, z) = b_{\mathrm{t}}(2z)^{2-\Delta_\phi}\mathrm{t}^i(\mathbf{x}) + \dots$$

- Universal parameter α Metlitski (2020)

$$\alpha \equiv \frac{\pi}{2}\left(\frac{a_\sigma}{4\pi b_{\mathrm{t}}}\right)^2 - \frac{N-2}{2\pi}$$

- When $\alpha > 0$: extraordinary-log phase

$$\langle\vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0)\rangle \sim \frac{1}{(\log \mathbf{x})^q} \qquad q = \frac{N-1}{2\pi\alpha}$$

  $\alpha(N = 2) > 0$: extraordinary-log phase Hu, Deng, Lv (2021)

- Large-N: $\alpha < 0$: extraordinary-log phase survives for $N < N_c$
- Generalized to plane defect: always $\alpha > 0$ Krishnan, Metlitski (2023)

Extraordinary-log phase

- Log-power two-point function

$$\langle\vec{\phi}(\mathbf{x})\cdot\vec{\phi}(0)\rangle \sim \frac{1}{(\log \mathbf{x})^q} \qquad q = \frac{N-1}{2\pi\alpha}$$

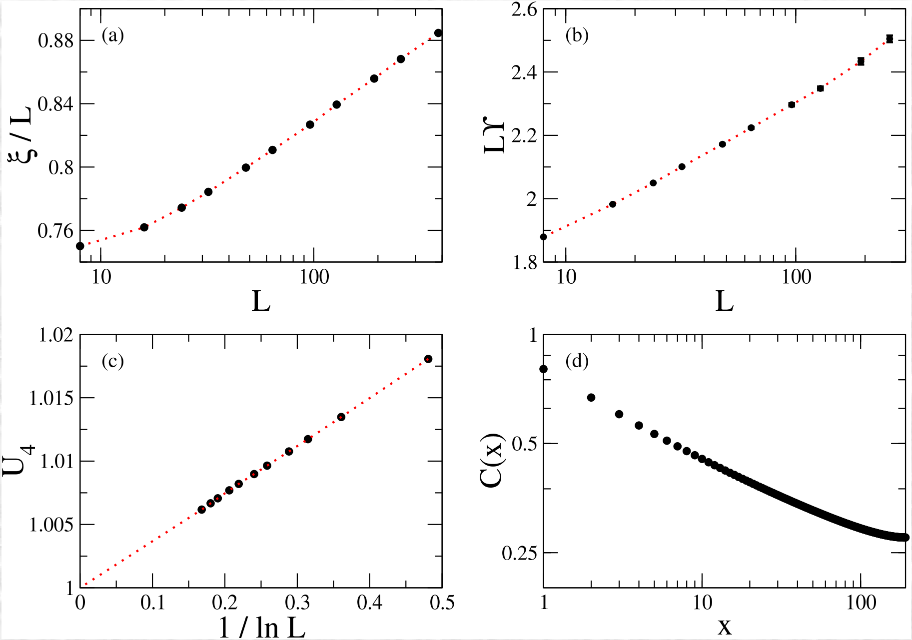
- Logarithmic violation of FSS

$$U_4 - 1 \propto 1/(\ln L)^2$$
$$(\xi/L)^2 \simeq A + \alpha/2 \ln L$$
$$\Upsilon L \simeq A + 4\alpha/N \ln L$$

$U_4 = \langle M_s^4\rangle/\langle M_s^2\rangle^2$: Binder ratio
ξ: finite-size correlation length
Υ: helicity modulus (spin stiffness)

- On a standard fixed point: U_4, ξ/L, $\Upsilon L \sim$ const

Extraordinary phase MC $N = 3$, $\beta_s = 1.5$



From Normal to Extraordinary

- Boundary Operator Product Expansion $z \to 0$

$$\sigma(\mathbf{x},z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_{\mathrm{D}}(2z)^{3-\Delta_\phi}\mathrm{D}(\mathbf{x}) + \dots$$

$$\varphi^i(\mathbf{x},z) = b_{\mathrm{t}}(2z)^{2-\Delta_\phi}\mathrm{t}^i(\mathbf{x}) + \dots$$

- Universal parameter α Metlitski (2020)

$$\alpha \equiv \frac{\pi}{2}\left(\frac{a_\sigma}{4\pi b_{\mathrm{t}}}\right)^2 - \frac{N-2}{2\pi}$$

When $\alpha > 0$: extraordinary-log phase $N < N_c$, $N_c \geq 2$

Goal: determine a_σ, b_t for $N = 2, 3$

MC simulations: normal UC

- ϕ^4 $N-$components lattice model, $L \times L \times L$ lattice, open b.c. $z-$dir.

$$\mathcal{H} = -\beta \sum_{\langle i\, j \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \sum_i [\vec{\phi}_i^2 + \lambda(\vec{\phi}_i^2 - 1)^2]$$

$$- \beta_s \sum_{\langle i\, j \rangle_s} \vec{\phi}_i \cdot \vec{\phi}_j - \vec{h}_s \cdot \sum_{i \in s} \vec{\phi}_i$$

- $O(N)$ UC: $\lambda = \lambda^*$ *improved*
  $\rightarrow$ leading *bulk* scaling corrections suppressed

- MC simulations at $\lambda = \lambda^*$, $\beta = \beta_c(\lambda^*)$, $\beta_{s,\downarrow} = \beta_{s,\uparrow} = \beta$, $h_s > 0$.

Lattice to CFT

- Expand lattice observables in terms of CFT fields

$$\text{Bulk: } \phi^i_{\mathrm{lat}} \propto \phi^i + \dots$$

$$\text{Surface: } \sigma_{\mathrm{lat}} \propto \sigma + \dots, \qquad \varphi^i_{\mathrm{lat}}(x) \propto t^i + \dots$$

Lattice to CFT

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$$\text{Surface: } \sigma_{\rm lat} \propto \sigma + \dots, \qquad \varphi^i_{\rm lat}(x) \propto t^i + \dots$$

- Fits of MC two-point functions → fix normalization

$$\langle \phi^i(0)\phi^j(x)\rangle = \delta^{ij}|x|^{-2\Delta_\phi} \qquad \langle t^i(0)t^j(x)\rangle = \delta^{ij}|x|^{-4}$$

Lattice to CFT

- Expand lattice observables in terms of CFT fields

$$\text{Bulk: } \phi^i_{\rm lat} \propto \phi^i + \dots$$

$$\text{Surface: } \sigma_{\rm lat} \propto \sigma + \dots, \qquad \varphi^i_{\rm lat}(x) \propto t^i + \dots$$

- Fits of MC two-point functions → fix normalization

$$\langle \phi^i(0)\phi^j(x)\rangle = \delta^{ij}|x|^{-2\Delta_\phi} \qquad \langle t^i(0)t^j(x)\rangle = \delta^{ij}|x|^{-4}$$

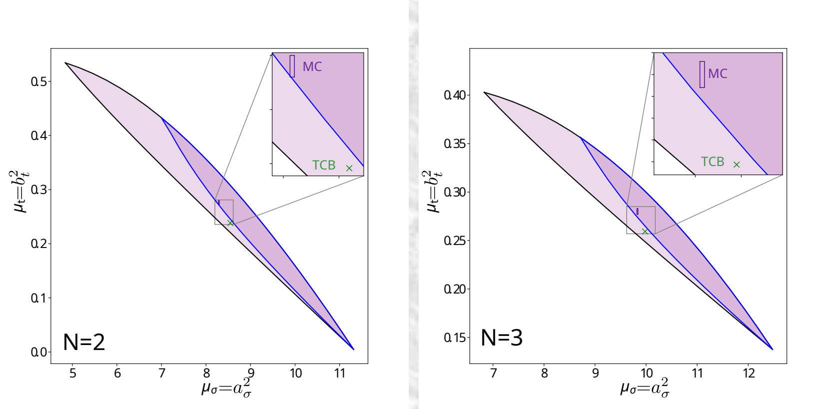
- From the Boundary OPE

$$\langle \sigma(z)\rangle \simeq \frac{a_\sigma}{(2(z+z_0))^{\Delta_\phi}}\left[1 + B\left(\frac{z+z_0}{L}\right)^3\right]$$

$$\langle \vec{\varphi}(0,z)\vec{\varphi}(0,0)\rangle \simeq \frac{16(N-1)b_{\rm t}}{(2(z+z_0))^{2+\Delta_\phi}}\left[1 + B_\varphi\left(\frac{z+z_0}{L}\right)^3 + C(z+z_0)^{-2}\right]$$

- Fits of MC data → universal BOPE a_σ, $b_{\rm t}$

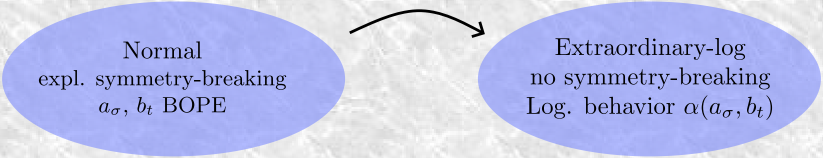
Comparison with Conformal Bootstrap



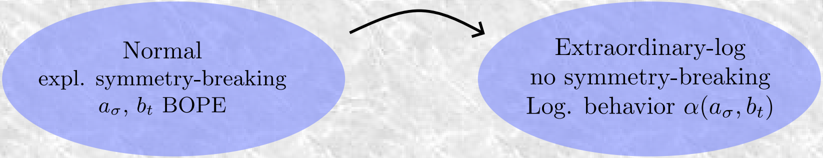
TCB = Truncated Conformal Bootstrap

Padayasi, Krishnan, Metlitski, Gruzberg, Meineri, SciPost (2022)

Summary $O(N)$ Normal UC



Summary O(N) Normal UC



α

N	MC$_{normal}$	MC$_{extraordinary}$	Truncated CB[a]
2	0.300(5)	0.27(2)[b]	0.3567
3	0.190(4)	0.15(2)*	0.2236

[a] Padayasi, Krishnan, Metlitski, Gruzberg, Meineri (2022)

[b] Hu, Deng, Lv (2021)

1. Surface critical behavior of the 3D O(N) model
2. $O(N)$: from Normal to Extraordinary
3. Line defect

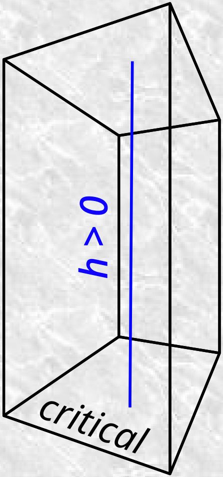
Other boundaries: line defect

- Line defects in critical models

  Hanke (2000); Vasilyev, Eisenriegler, Dietrich (2013);
  Eisenriegler, Burkhardt (2016); Cuomo, Komargodski, Mezei (2022);
  Cuomo, Komargodski, Mezei, Raviv-Moshe (2022);
  Gimenez-Grau, Lauria, Liendo, van Vliet (2022); Gimenez-Grau (2022);
  Giombi, Helfenberger, Khanchandani (2022); Pannell, Stergiou (2023);
  Bianchi, Bonomi, de Sabbata (2023);
  Aharony, Cuomo, Komargodski, Mezei, Raviv-Moshe (2023)
  ...

- $O(N)$ model, symmetry-breaking field on line defect

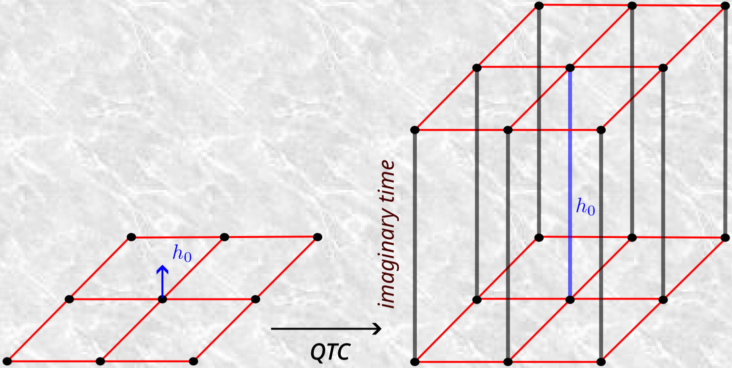
- $N = 1$: critical adsorption on elongated colloids FPT, Assaad, Wessel, 2017



Pinning-field approach

- Local ordering field, coupled to the order parameter Assaad, Herbut, 2013
- Symmetry-breaking: order parameter $\neq 0$ in finite V

  Enhanced numerical stability
- Extrapolate the order parameter $V \to \infty$



Correlations along the defect

VOLUME 84, NUMBER 10 PHYSICAL REVIEW LETTERS 6 MARCH 2000

Critical Adsorption on Defects in Ising Magnets and Binary Alloys

Andreas Hanke*

For $N = 1$ (Ising), via an "interpolation" of $\varepsilon-$expansion

$$\langle \phi(x)\phi(y)\rangle_c \sim |x-y|^{-2\Delta_\sigma}, \qquad \Delta_\sigma \simeq 1.385(25)$$

Lattice model $N = 1$

Classical Blume-Capel model

$$\mathcal{H} = -K \sum_{\langle i\, j \rangle} S_i S_j + D \sum_i S_i^2 - h_0 \sum_{i \in line} S_i$$

$$S_i = -1, 0, 1$$

- Continuous phase transition in Ising UC
- Suppressed scaling corrections at $D = 0.655(20)$ Hasenbusch, 2010
  $\Rightarrow$ improved model

Goal: determine Δ_σ

Monte Carlo determination of Δ_σ

- We simulate an improved model in the Ising UC

$$\mathcal{H} = -K \sum_{\langle i\,j \rangle} S_i S_j + D \sum_i S_i^2 - h_0 \sum_{i \in line} S_i, \qquad S_i = -1, 0, 1$$

We take $h_0 = \infty \to$ fixed spins

- Finite-Size Scaling analysis of local magnetization and susceptibility

Δ_σ	Method
1.52(6)	MC
1.385(25)	FT Hanke (2000)
1.55(14)	$\varepsilon-$exp. Cuomo, Komargodski, Mezei (2022)
1.542	Large$-N$ Cuomo, Komargodski, Mezei (2022)

Summary & Outlook

- A renewed interest in boundary critical phenomena
  Advances in boundary conformal field theory
  Boundary exponents, universal Boundary OPE coefficients

- Reexamination of the classical 3D $O(N)$ surface critical behavior
  New extraordinary-log UC $\leftrightarrow$ Normal UC

Summary & Outlook

- Many other geometries are of current interest:
  - Line defects
  - Plane defects Krishnan, Metlitski (2023); Giombi, Liu (2023)
  - Anisotropies on the boundary Diehl, Eisenriegler (1984); Trépanier (2023)
  - ...
- Quantum critical models

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References

- Critical exponents of the O(N) ordinary universality class
  FPT, Phys. Rev. B 108, L020404 (2023)

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  FPT, Phys. Rev. Lett. 126, 135701 (2021)

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- Line defect (a.k.a. pinning field)
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Thank you!