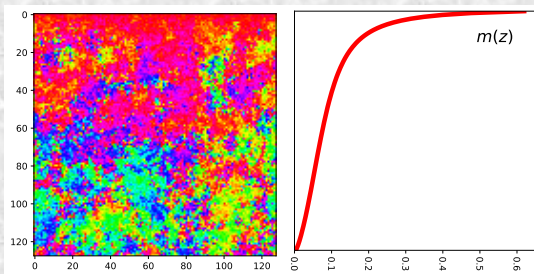


Advances in the boundary $O(N)$ universality class



Francesco Parisen Toldin



RWTHAACHEN
UNIVERSITY

New perspectives on Quantum Field Theory with Boundaries
Impurities, and Defects
Nordita, Stockholm, 4 August 2023

Boundary critical phenomena

In the vicinity of a second-order phase transitions:

- Power-laws, universality
- Renormalization Group:

$$K \rightarrow \mathcal{R}(K) \rightarrow \mathcal{R}(\mathcal{R}(K)) \dots$$

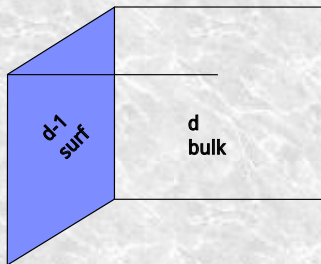
- Real systems: surfaces
- RG: bulk vs surface couplings

Cardy book (1996)

$$\begin{array}{l} \nearrow K_{\text{surf}}^{*(1)} \\ K_{\text{bulk}}^* \rightarrow K_{\text{surf}}^{*(2)} \\ \searrow \dots \end{array}$$

→ Rich bulk-surface phase diagram

→ Surface UC determine the critical Casimir force [Fisher, de Gennes \(1978\)](#)



Reviews: [Binder \(1983\)](#); [Diehl \(1986\)](#)

Unconventional Surface Critical Behavior Induced by a Quantum Phase Transition from the Two-Dimensional Affleck-Kennedy-Lieb-Tasaki Phase to a Néel-Ordered Phase

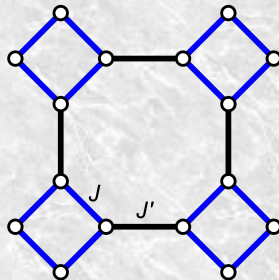
Long Zhang¹ and Fa Wang^{1,2}

¹*International Center for Quantum Materials, School of Physics, Peking University, Beijing 100871, China*

²*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

(Received 30 November 2016; revised manuscript received 13 January 2017; published 21 February 2017)

- Bulk: quantum phase transition in the $O(3)$ UC in $d = 2 + 1$
- Unexpected boundary exponents



A renewed interest: quantum spin models

PHYSICAL REVIEW LETTERS 120, 235701 (2018)

Engineering Surface Critical Behavior of (2+1)-Dimensional O(3) Quantum Critical Points

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²Kavli Institute for Theoretical Science and CAS Center for Excellence in Quantum Information and Quantum Physics, University of Chinese Academy of Sciences, Beijing 100049, China

SciPost

Continuous Néel-VBS quantum phase transition in non-local one-dimensional systems with SO(3) symmetry

Chao-Ming Jian¹, Yichen Xu², Xiao-Chuan Wu² and Cenke Xu²

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²Department of Physics, University of California, Santa Barbara, California 93106, USA

Letter

Spin versus bond correlations along dangling edges of quantum critical magnets

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(Received 2 November 2020; revised 11 December 2020)

Dangling edge spins of two-dimensional quantum critical magnets provide large contributions to the spin Hall conductivity.

Bulk and surface critical behavior of a quantum Heisenberg antiferromagnet on two-dimensional coupled diagonal ladders

Zhe Wang,¹ Fan Zhang,¹ and Wenan Guo^{1,2,†}

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(Received 29 September 2022; published 10 October 2022)

Conformal Boundary Conditions of Symmetry-Enriched Quantum Critical Spin Chains

Xue-Jia Yu^{1,2}, Rui-Zhen Huang^{2,3,4}, Hong-Hao Song^{2,3,4}, Limei Xu,^{1,2,3} Chengxiang Ding,⁵ and Long Zhang^{2,3,4}

PHYSICAL REVIEW B 106, 214409 (2022)

Persistent corner spin mode at the quantum critical point of a plaquette Heisenberg model

Yining Xu,¹ Chen Peng,² Zijian Xiong,^{3,4,5} and Long Zhang^{2,3,4}

PHYSICAL REVIEW B 98, 140403(R) (2018)

Rapid Communications

Nonordinary edge criticality of two-dimensional quantum critical magnets

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PHYSICAL REVIEW B 103, 024412 (2021)

Surface critical behavior of coupled Haldane chains

Wenjing Zhu,¹ Chengxiang Ding,² Long Zhang,¹ and Wenan Guo^{1,2,3,4}

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SciPost Physics

Submission

2, China

Special Transition and Extraordinary Phase on the Surface of a Two-Dimensional Quantum Heisenberg Antiferromagnet

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¹School of Microelectronics & Data Science, Anhui University of Technology, 243002, China

²Department of Physics, Beijing Normal University, Beijing 100875, China

³Institute for Theoretical Science, Beijing 100049, China

Quantum correspondence of special and extraordinary-log criticality: Villain's bridge

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PHYSICAL REVIEW B 106, 224502 (2022)

Quantum extraordinary-log universality of boundary critical behavior

Yanan Sun and Jian-Ping Lv^{1,2}

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A renewed interest: Conformal Field Theory



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The bootstrap program for boundary CFT_d

Pedro Liendo,^a Leonardo Rastelli^b and Balt C. van Rees

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Bootstrap equations for $\mathcal{N} = 4$ SYM with defects

Pedro Liendo^a and Carlo Meneghelli^b

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PUBLISHED: December 2, 2019



An analytic approach to BCFT_d

Dalimil Mazáč,^{a,b} Leonardo Rastelli^b and Xinan Zhou^{b,c}

^aSimons Center for Geometry and Physics, Stony Brook University,
Stony Brook, NY 11794, U.S.A.
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^cPrinceton University, Princeton, Princeton, NJ 08542, U.S.A.



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PUBLISHED: May 7, 2015

Boundary and interface CFTs from the conformal bootstrap

Ferdinando Gliozzi,^{a,b} Pedro Liendo,^c Marco Meineri^{d,e} and Antonio Rago^f

^aUniversità di Torino, ^bINFN, ^cUniversità di Padova, ^dUniversità di Pisa, ^eUniversità di Firenze, ^fUniversità di Bari



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Defects in conformal field theory

Billo,^a Vasco Gonçalves,^{b,c} Edoardo Lauria^d and Marco Meineri^{d,f}

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Radial coordinates for defect CFTs

The extraordinary boundary transition in the 3d $O(N)$ model via conformal bootstrap

Jaychandran Padayasi¹, Abhijith Krishna² and ...

IOP Publishing
J. Phys. A: Math. Theor. 53 (2020) 493002 (51pp)
<https://doi.org/10.1088/1751-8121/ab020e>

Topical Review

Boundary and defect CFT: open problems and applications

N Andrei¹, A Bissi², M Buican³, J Cardy^{4,5}, P Dorey⁶,
N Drukker⁷, J Erdmenger⁸, D Friedan⁹, D Fursaev¹⁰,
S Gubser¹¹, S Hellmuth¹², S Komatsu¹³, S Lee¹⁴, S Li¹⁵, S Minwalla¹⁶,
S Pufu¹⁷, S Sahoo¹⁸, S S. Pufu¹⁹, S S. Pufu²⁰, S S. Pufu²¹, S S. Pufu²²,
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S S. Pufu⁹⁸, S S. Pufu⁹⁹, S S. Pufu¹⁰⁰

JHEP05 (2015) 03

JHEP05 (2015) 03

Ed
SciPost
JHEP05 (2015) 03

Reexamining the boundary $O(N)$ universality class

PHYSICAL REVIEW LETTERS **126**, 135701 (2021)

Boundary Critical Behavior of the Three-Dimensional Heisenberg Universality Class

Francesco Parisen Toldin^{*}

Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

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[SciPost Phys. 12, 131 \(2022\)](#)

Boundary criticality of the $O(N)$ model in $d = 3$ critically revisited

Max A. Metlitski

Department of Physics, Massachusetts Institute of Technology,
Cambridge, MA 02139, USA

PHYSICAL REVIEW LETTERS **128**, 215701 (2022)

Boundary Criticality of the 3D $O(N)$ Model: From Normal to Extraordinary

Francesco Parisen Toldin^{*}

Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

Max A. Metlitski[†]

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Outline

- ① Surface critical behavior of the 3D $O(N)$ model
- ② $O(N)$: from Normal to Extraordinary
- ③ Line defect

① Surface critical behavior of the 3D $O(N)$ model

② $O(N)$: from Normal to Extraordinary

③ Line defect

The 3D $O(N)$ universality class

- Simplest model of classical magnets
Liquid-vapor, uniaxial ferromagnets ($N = 1$), superfluid ^4He ($N = 2$), isotropic magnets ($N = 3$), chiral transition QCD ($N = 4$), ...

Pelissetto, Vicari (2002)

- Numerous investigations: field-theory, high-T expansion, MC, ...
- Conformal Bootstrap in $d = 3$
Rigorous results, very precise

- CB with boundaries
Recent developments

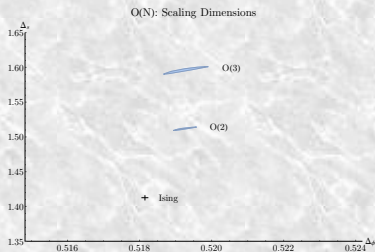
McAvity, Osborn (1995)

Liendo, Rastelli, van Rees (2013)

Gliozzi, Liendo, Menieri, Rago (2015)

Billó, Gonçalves, Lauria, Menieri (2016)

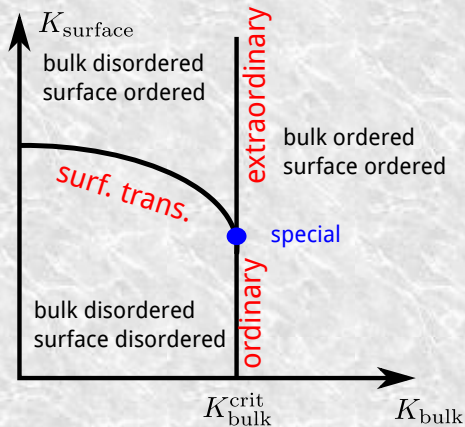
Padayasi, Krishnan, Metlitski, Gruzberg, Meineri (2022)



Kos, Poland, Simmons-Duffin, Vichi, (2016)

O(N) phase diagram $d = 3$: $N=1$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surf. trans.: usual $d = 2$ phase transition
- Ordinary: new surf. exponent

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}}$$

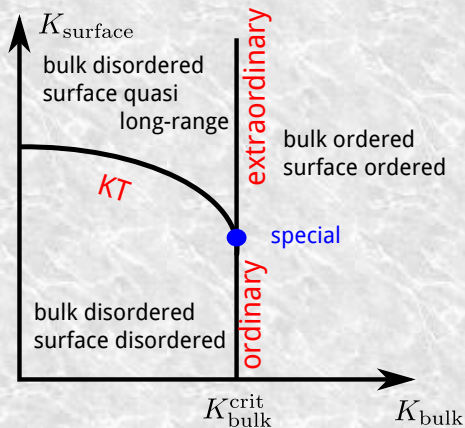
$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.2751(6)$$

Hasenbusch (2011)

- Special: multicritical point, relevant surface exponent + η_{\parallel}
- Extraordinary: ordered

O(N) phase diagram $d = 3$: $N=2$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surface transition: Berezinskii-Kosterlitz-Thouless
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.2286(25)$$

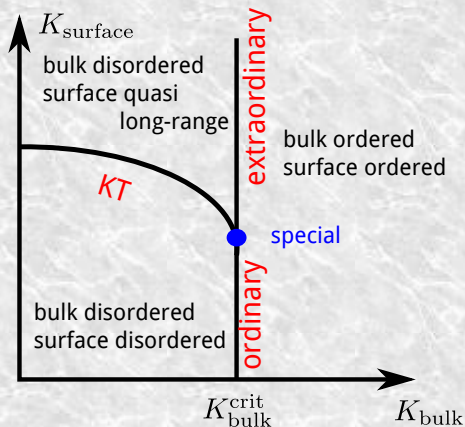
FPT (2013)

- Extraordinary: ordered?
first/second order?

Deng, Blöte, Nightingale (2005)

O(N) phase diagram $d = 3$: $N=2$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surface transition: Berezinskii-Kosterlitz-Thouless
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}}$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.2286(25)$$

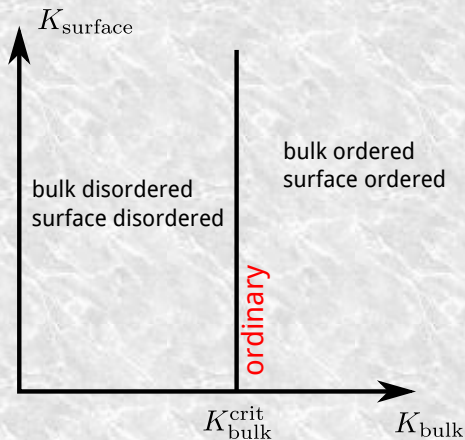
FPT (2013)

- Extraordinary: "extraordinary-log" phase

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0) \rangle \sim \frac{1}{(\log \mathbf{x})^q}$$

$O(N)$ phase diagram $d = 3$: $N=3$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- No surface transition
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.194(3)$$

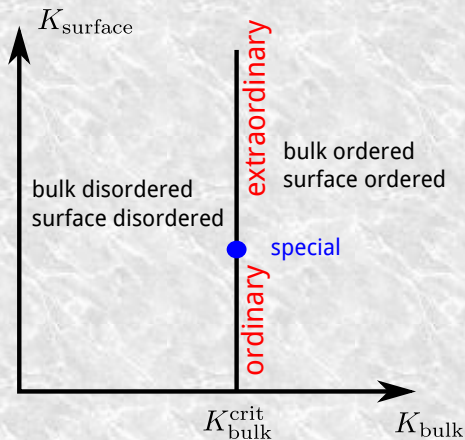
FPT (2013)

- Surface transition shifted to $K_{\text{surface}} \rightarrow \infty$
Only the ordinary UC

Krech (2000)

O(N) phase diagram $d = 3$: $N=3$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- No surface transition
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.194(3)$$

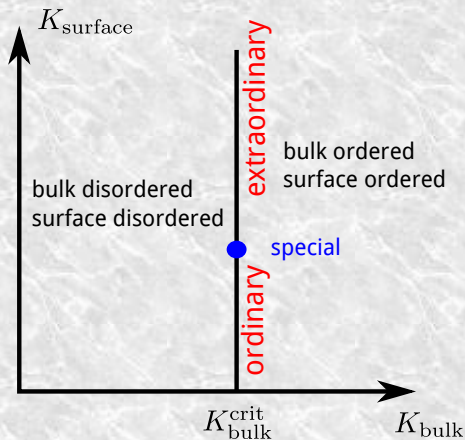
FPT (2013)

- “KT-like” special transition at finite K_{surface}

Deng, Blöte, Nightingale (2005)

O(N) phase diagram $d = 3$: $N=3$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- No surface transition
- Ordinary transition

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}},$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.194(3)$$

FPT (2013)

- Special: standard transition, relevant surface exponent + η_{\parallel}

$$\Delta_{\hat{s}} = 1.64(1) \quad \Delta_{\hat{\phi}} = 0.2635(10)$$

- Extraordinary-log phase
Correlations: power of a log
Logarithmic violation FSS

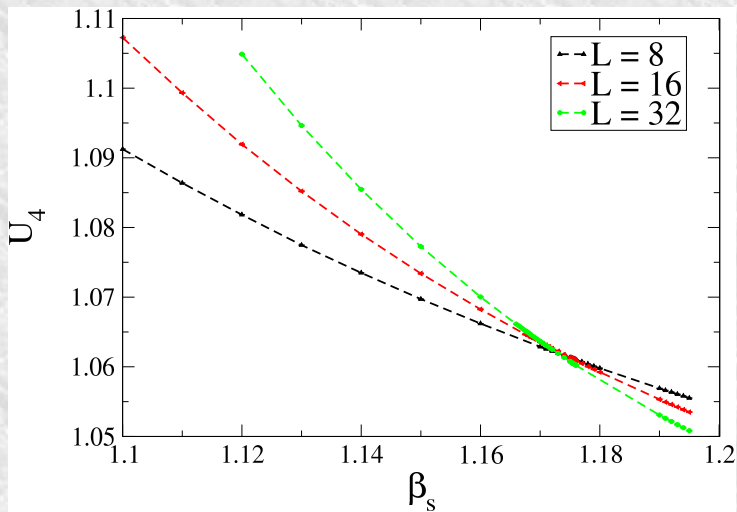
- ϕ^4 $N = 3$ lattice model, $L \times L \times L$ lattice, open b.c. z -direction

$$\begin{aligned}\mathcal{H} = & -\beta \sum_{\langle i j \rangle} \vec{\phi}_i \cdot \vec{\phi}_j - \beta_{s,\downarrow} \sum_{\langle i j \rangle_{s\downarrow}} \vec{\phi}_i \cdot \vec{\phi}_j \\ & - \beta_{s,\uparrow} \sum_{\langle i j \rangle_{s\uparrow}} \vec{\phi}_i \cdot \vec{\phi}_j + \sum_i \vec{\phi}_i^2 + \lambda(\vec{\phi}_i^2 - 1)^2\end{aligned}$$

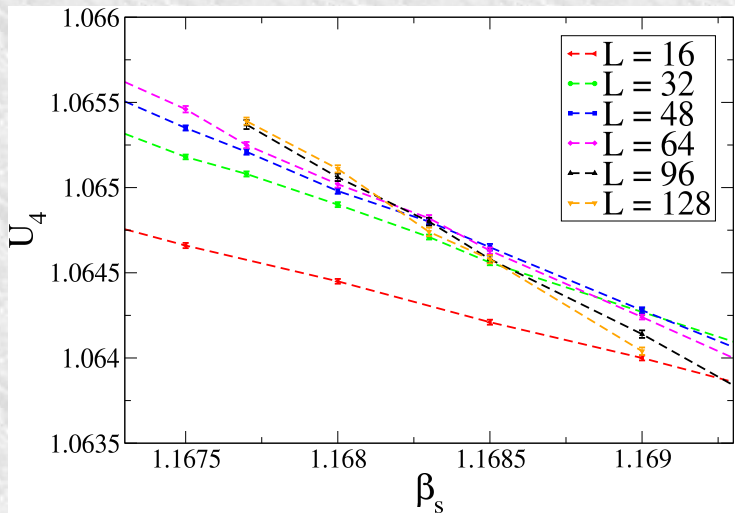
- Heisenberg UC; $\lambda = \lambda^* = 5.17(11)$ *improved* Hasenbusch (2020)
→ leading *bulk* scaling corrections suppressed
- MC simulations at $\lambda = \lambda^*$, $\beta = \beta_c(\lambda^*)$, $\beta_{s,\downarrow} = \beta_{s,\uparrow} = \beta_s$
- Scan the surface observables on β_s

Goal: surface transition?

Surface Binder ratio $U_4 = \langle M_S^4 \rangle / \langle M_S^2 \rangle^2$



Surface Binder ratio $U_4 = \langle M_S^4 \rangle / \langle M_S^2 \rangle^2$



Special surface transition

- Finite-Size Scaling analysis of U_4

Hasenbusch (1999); FPT, PRE 84, 025703R (2011); Review: FPT, PRE 105, 034137 (2022)

$$U_4 = f((\beta_s - \beta_{s,c})L^{y_{sp}})$$

- Special phase transition on the surface of a 3D Heisenberg UC

$$y_{sp} = 0.36(1) \quad \Delta_{\hat{s}} = 2 - y_{sp} = 1.64(1)$$

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0) \rangle_{\text{surf}} \sim \frac{1}{|\mathbf{x}|^{1+\eta_{\parallel}}} \quad \eta_{\parallel} = -0.473(2) \quad \Delta_{\hat{\phi}} = \frac{1 + \eta_{\parallel}}{2} = 0.2635(10)$$

Quantum spin models with unexpected boundary η_{\parallel} : accidentally close to the special transition (?)

- $\beta_s > \beta_{s,c}$: extraordinary phase

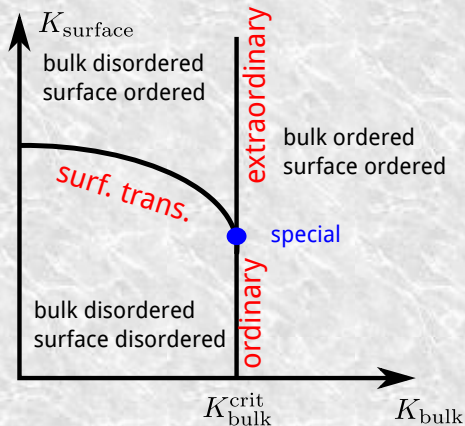
① Surface critical behavior of the 3D $O(N)$ model

② $O(N)$: from Normal to Extraordinary

③ Line defect

O(N) phase diagram $d = 3$: $N=1$

Bulk coupling K_{bulk} vs Surface coupling K_{surface}



- Surf. trans.: usual $d = 2$ phase transition
- Ordinary: new surf. exponent

$$\langle \phi(\mathbf{x})\phi(0) \rangle \sim \frac{1}{|\mathbf{x}|^{2\Delta_{\hat{\phi}}}}$$

$$\Delta_{\hat{\phi}} = (1 + \eta_{\parallel})/2 = 1.2751(6)$$

Hasenbusch (2011)

- Special: multicritical point, relevant surface exponent + η_{\parallel}
- Extraordinary: ordered

$O(N)$ Normal Universality Class

- 3D $O(N)$ model + surface field \vec{h}
→ ordered surface
- Ising UC: binary critical mixture $A + B$
 $\phi \sim c_A - c_{A,c}$
- Adsorption preference on surfaces

Fluid Interface Tensions near Critical End Points

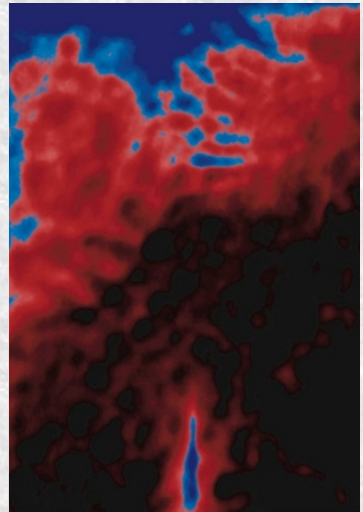
Michael E. Fisher and Paul J. Upton^(a)

PRL 65, 3405 (1990)

describes criticality of wall free energies at the so-called extraordinary surface transition^{1,7} (which, however, in the laboratory is more normal than the “ordinary” transition).

- Ising: normal = extraordinary

Diehl (1994)



Hertlein, Helden, Gambassi, Dietrich, Bechinger (2008)

From: Balibar, Nature News&Views (2008)

Critical adsorption

Adsorption layer governed by $\sim \xi$

$$m(z) = a|t|^\beta P_\pm(z/\xi), \quad t = \frac{T - T_c}{T_c}$$

$$P_+(\tilde{z}_+ \rightarrow \infty) \sim e^{-\tilde{z}_+}$$

$$P_-(\tilde{z}_- \rightarrow \infty) - 1 \sim e^{-\tilde{z}_-}$$

$$P_\pm(\tilde{z}_\pm \rightarrow 0) = c_\pm \tilde{z}_\pm^{-\beta/\nu}$$

MC: $c_+ = 0.844(6)$ FPT, S. Dietrich, JSTAT P11003 (2010)

FT: $c_+ = 0.94(4)$ Flöter, Dietrich (1995)

Exp.: $c_+ = 1.60(42)$ $c_+ = 0.77(19)$

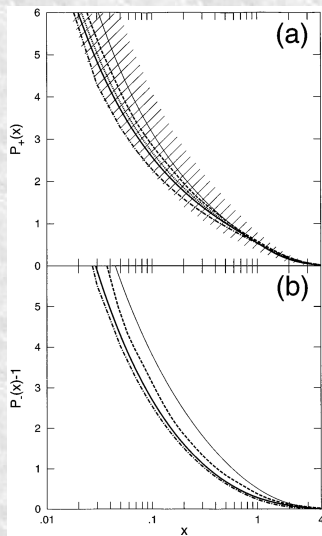
$c_+ = 1.14(29)$ $c_+ = 0.91(26)$

$c_+ = 1.05(9)$ $c_+ = 1.02(10)$

$c_+ = 1.25(9)$ $c_+ = 0.84(15)$

Flöter, Dietrich (1995)

$c_+ = 0.787^{+0.009}_{-0.015}$ Carpenter, Law, Smith (1999)



Carpenter, Law, Smith (1999)
Review: Law, Prog. Surf. Sc. (2001)

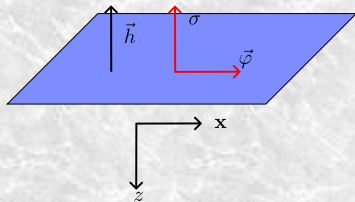
Normal Universality Class: $O(N > 1)$ model

- $O(N)$ -symmetric bulk, at criticality

- Introduce a surface field \vec{h}
→ ordered surface

Broken $O(N) \rightarrow O(N-1)$

$$\vec{\phi} = (\vec{\varphi}, \sigma) \quad \vec{\varphi} \perp \vec{h} \quad \sigma \parallel \vec{h}$$



- Longitudinal vs transverse surface correlations

J. Phys. A: Math. Gen., Vol. 10, No. 11, 1977. Printed in Great Britain. © 1977

Critical behaviour of semi-infinite systems

A J Bray and M A Moore
Department of Theoretical Physics, The University, Manchester, M13 9PL, UK

$$\langle \sigma(\mathbf{x})\sigma(\mathbf{y}) \rangle_c \sim |\mathbf{x} - \mathbf{y}|^{-2d}$$

$$\langle \vec{\varphi}(\mathbf{x})\vec{\varphi}(\mathbf{y}) \rangle \sim |\mathbf{x} - \mathbf{y}|^{-(2d-2)}$$

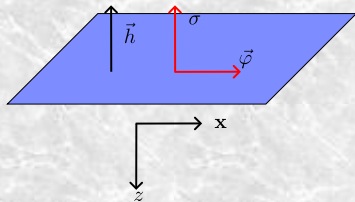
Normal Universality Class: $O(N > 1)$ model

- $O(N)$ -symmetric bulk, at criticality

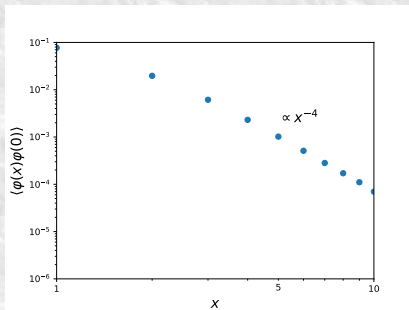
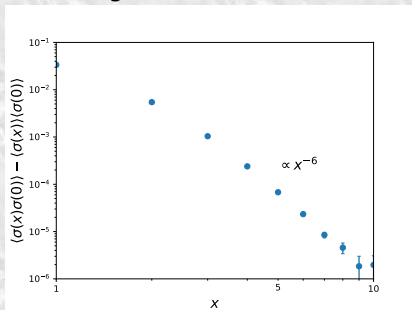
- Introduce a surface field \vec{h}
→ ordered surface

Broken $O(N) \rightarrow O(N-1)$

$$\vec{\phi} = (\vec{\varphi}, \sigma) \quad \vec{\varphi} \perp \vec{h} \quad \sigma \parallel \vec{h}$$



- Longitudinal vs transverse surface correlations

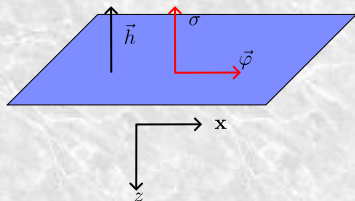


Boundary CFT for the Normal UC

- $O(N)$ broken to $O(N - 1)$
Boundary operators are representations of $O(N - 1)$
- Spectrum of the CFT:
 - Displacement operator D , $\Delta_D = d$. $O(N - 1)$ -scalar
 - Tilt operator t^i , $\Delta_t = d - 1$ $O(N - 1)$ -vector
- On the boundary, $\vec{\phi} = (\vec{\varphi}, \sigma)$

$$\sigma = \sum_{\text{bdy scalars}} \hat{\sigma}$$

$$\phi^i = \sum_{\text{bdy vectors}} \hat{\phi}^i$$



- Longitudinal/transverse correlation:
 D and t^i are the lowest operators

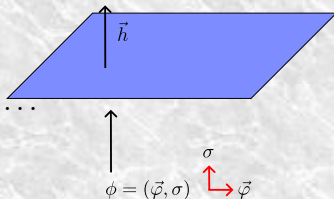
From Normal to Extraordinary

- Boundary Operator Product Expansion

$$z \rightarrow 0$$

$$\sigma(\mathbf{x}, z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_D (2z)^{3-\Delta_\phi} D(\mathbf{x}) + \dots$$

$$\varphi^i(\mathbf{x}, z) = b_t (2z)^{2-\Delta_\phi} t^i(\mathbf{x}) + \dots$$



- Universal parameter α [Metlitski \(2020\)](#)

$$\alpha \equiv \frac{\pi}{2} \left(\frac{a_\sigma}{4\pi b_t} \right)^2 - \frac{N-2}{2\pi}$$

- When $\alpha > 0$: extraordinary-log phase

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0) \rangle \sim \frac{1}{(\log \mathbf{x})^q} \quad q = \frac{N-1}{2\pi\alpha}$$

$\alpha(N=2) > 0$: extraordinary-log phase [Hu, Deng, Lv \(2021\)](#)

- Large- N : $\alpha < 0$: extraordinary-log phase survives for $N < N_c$
- Generalized to plane defect: always $\alpha > 0$ [Krishnan, Metlitski \(2023\)](#)

Extraordinary-log phase

- Log-power two-point function

$$\langle \vec{\phi}(\mathbf{x}) \cdot \vec{\phi}(0) \rangle \sim \frac{1}{(\log \mathbf{x})^q} \quad q = \frac{N-1}{2\pi\alpha}$$

- Logarithmic violation of FSS

$$U_4 - 1 \propto 1/(\ln L)^2$$

$$(\xi/L)^2 \simeq A + \alpha/2 \ln L$$

$$\Upsilon L \simeq A + 4\alpha/N \ln L$$

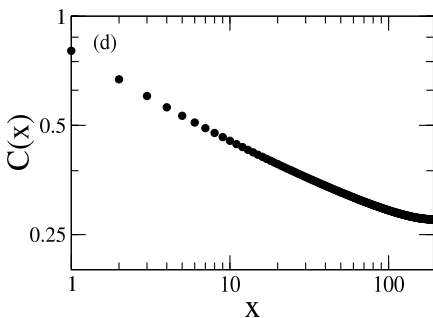
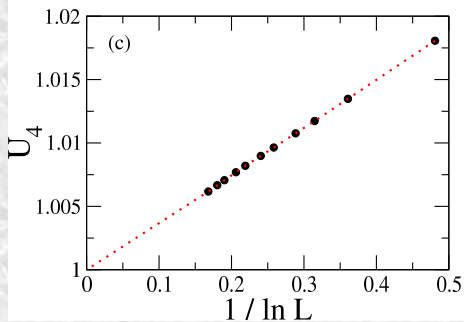
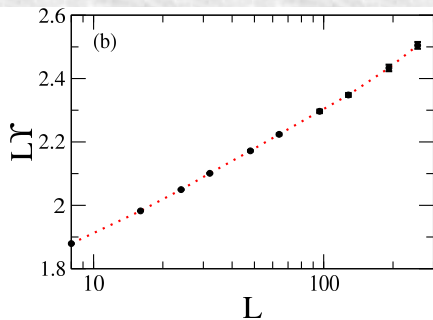
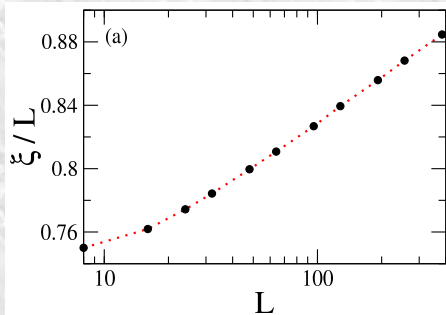
$U_4 = \langle M_s^4 \rangle / \langle M_s^2 \rangle^2$: Binder ratio

ξ : finite-size correlation length

Υ : helicity modulus (spin stiffness)

- On a standard fixed point: $U_4, \xi/L, \Upsilon L \sim \text{const}$

Extraordinary phase MC $N = 3, \beta_s = 1.5$



From Normal to Extraordinary

- Boundary Operator Product Expansion $z \rightarrow 0$

$$\sigma(\mathbf{x}, z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_D (2z)^{3-\Delta_\phi} D(\mathbf{x}) + \dots$$

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- Universal parameter α [Metlitski \(2020\)](#)

$$\alpha \equiv \frac{\pi}{2} \left(\frac{a_\sigma}{4\pi b_t} \right)^2 - \frac{N-2}{2\pi}$$

When $\alpha > 0$: extraordinary-log phase $N < N_c$, $N_c \geq 2$

Goal: determine a_σ , b_t for $N = 2, 3$

MC simulations: normal UC

- ϕ^4 N -components lattice model, $L \times L \times L$ lattice, open b.c. z-dir.

$$\mathcal{H} = -\beta \sum_{\langle i j \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \sum_i [\vec{\phi}_i^2 + \lambda(\vec{\phi}_i^2 - 1)^2] \\ - \beta_s \sum_{\langle i j \rangle_s} \vec{\phi}_i \cdot \vec{\phi}_j - \vec{h}_s \cdot \sum_{i \in S} \vec{\phi}_i$$

- $O(N)$ UC: $\lambda = \lambda^*$ improved
→ leading *bulk* scaling corrections suppressed
- MC simulations at $\lambda = \lambda^*$, $\beta = \beta_c(\lambda^*)$, $\beta_{s,\downarrow} = \beta_{s,\uparrow} = \beta$, $h_s > 0$.

Lattice to CFT

- Expand lattice observables in terms of CFT fields

$$\text{Bulk: } \phi_{\text{lat}}^i \propto \phi^i + \dots$$

$$\text{Surface: } \sigma_{\text{lat}} \propto \sigma + \dots, \quad \varphi_{\text{lat}}^i(x) \propto t^i + \dots$$

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- Fits of MC two-point functions \rightarrow fix normalization

$$\langle \phi^i(0) \phi^j(x) \rangle = \delta^{ij} |x|^{-2\Delta_\phi} \quad \langle t^i(0) t^j(x) \rangle = \delta^{ij} |x|^{-4}$$

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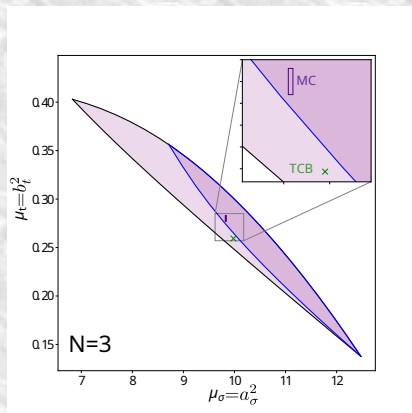
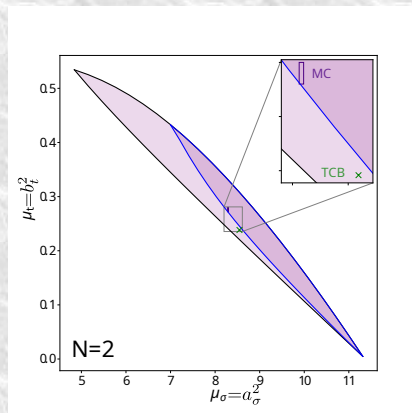
- From the Boundary OPE

$$\langle \sigma(z) \rangle \simeq \frac{a_\sigma}{(2(z+z_0))^{\Delta_\phi}} \left[1 + B \left(\frac{z+z_0}{L} \right)^3 \right]$$

$$\langle \vec{\varphi}(0, z) \vec{\varphi}(0, 0) \rangle \simeq \frac{16(N-1)b_t}{(2(z+z_0))^{2+\Delta_\phi}} \left[1 + B_\varphi \left(\frac{z+z_0}{L} \right)^3 + C(z+z_0)^{-2} \right]$$

- Fits of MC data \rightarrow universal BOPE a_σ, b_t

Comparison with Conformal Bootstrap



TCB = Truncated Conformal Bootstrap

Padayasi, Krishnan, Metlitski, Gruzberg, Meineri, SciPost (2022)

Summary $O(N)$ Normal UC

Normal
expl. symmetry-breaking
 a_σ, b_t BOPE



Extraordinary-log
no symmetry-breaking
Log. behavior $\alpha(a_\sigma, b_t)$

Summary $O(N)$ Normal UC

Normal
expl. symmetry-breaking
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Extraordinary-log
no symmetry-breaking
Log. behavior $\alpha(a_\sigma, b_t)$

α

N	MC_{normal}	$MC_{\text{extraordinary}}$	Truncated CB^a
2	0.300(5)	0.27(2) ^b	0.3567
3	0.190(4)	0.15(2)*	0.2236

^a Padayasi, Krishnan, Metlitski, Gruzberg, Meineri (2022)

^b Hu, Deng, Lv (2021)

① Surface critical behavior of the 3D $O(N)$ model

② $O(N)$: from Normal to Extraordinary

③ Line defect

Other boundaries: line defect

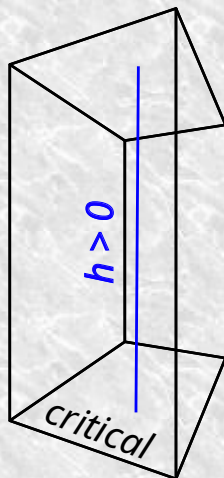
- Line defects in critical models

Hanke (2000); Vasilyev, Eisenriegler, Dietrich (2013);
Eisenriegler, Burkhardt (2016); Cuomo, Komargodski, Mezei (2022);
Cuomo, Komargodski, Mezei, Raviv-Moshe (2022);
Gimenez-Grau, Lauria, Liendo, van Vliet (2022); Gimenez-Grau (2022);
Giombi, Helfenberger, Khanchandani (2022); Pannell, Stergiou (2023);
Bianchi, Bonomi, de Sabbata (2023);
Aharony, Cuomo, Komargodski, Mezei, Raviv-Moshe (2023)

...

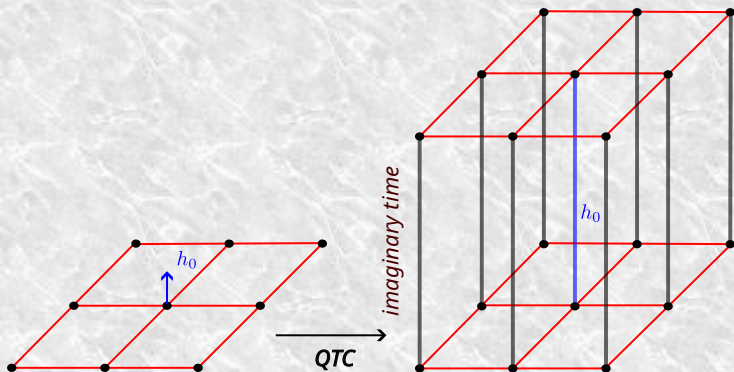
- $O(N)$ model, symmetry-breaking field on line defect

- $N = 1$: critical adsorption on elongated colloids FPT, Assaad, Wessel, 2017



Pinning-field approach

- Local ordering field, coupled to the order parameter Assaad, Herbut, 2013
- Symmetry-breaking: order parameter $\neq 0$ in finite V
Enhanced numerical stability
- Extrapolate the order parameter $V \rightarrow \infty$



Critical Adsorption on Defects in Ising Magnets and Binary Alloys

Andreas Hanke*

For $N = 1$ (Ising), via an “interpolation” of ε -expansion

$$\langle \phi(x)\phi(y) \rangle_c \sim |x - y|^{-2\Delta_\sigma}, \quad \Delta_\sigma \simeq 1.385(25)$$

Lattice model $N = 1$

Classical Blume-Capel model

$$\mathcal{H} = -K \sum_{\langle i j \rangle} S_i S_j + D \sum_i S_i^2 - h_0 \sum_{i \in \text{line}} S_i$$
$$S_i = -1, 0, 1$$

- Continuous phase transition in Ising UC
- Suppressed scaling corrections at $D = 0.655(20)$ Hasenbusch, 2010
⇒ improved model

Goal: determine Δ_σ

Monte Carlo determination of Δ_σ

- We simulate an improved model in the Ising UC

$$\mathcal{H} = -K \sum_{\langle i j \rangle} S_i S_j + D \sum_i S_i^2 - h_0 \sum_{i \in \text{line}} S_i, \quad S_i = -1, 0, 1$$

We take $h_0 = \infty \rightarrow$ fixed spins

- Finite-Size Scaling analysis of local magnetization and susceptibility

Δ_σ	Method
1.52(6)	MC
1.385(25)	FT Hanke (2000)
1.55(14)	ε -exp. Cuomo, Komargodski, Mezei (2022)
1.542	Large- N Cuomo, Komargodski, Mezei (2022)

Summary & Outlook

- A renewed interest in boundary critical phenomena
Advances in boundary conformal field theory
Boundary exponents, universal Boundary OPE coefficients

- Reexamination of the classical 3D $O(N)$ surface critical behavior
New extraordinary-log UC \leftrightarrow Normal UC

Summary & Outlook

- Many other geometries are of current interest:
 - Line defects
 - Plane defects [Krishnan, Metlitski \(2023\)](#); [Giombi, Liu \(2023\)](#)
 - Anisotropies on the boundary [Diehl, Eisenriegler \(1984\)](#); [Trépanier \(2023\)](#)
 - ...

- Quantum critical models

Acknowledgments



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Fagher Assaad (Würzburg)

References

- Critical exponents of the $O(N)$ ordinary universality class
FPT, Phys. Rev. B 108, L020404 (2023)
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Thank you!