Jointly modelling the epidemics and the economy with an application to COVID-19

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My expertise

Econometrics and applied macroeconomics

- econometrics: branch of economics that focuses on developing statistical methods for economic problems (causality, taking models to data, identification, counterfactuals)
- main expertise: time series analysis with time-varying parameters - developing methods for change point detection, bootstrap for time series, policy analysis
- effectiveness of monetary and fiscal policy

Epidemiology (recently)

- *Disentangling the effect of measures, variants and vaccines on SARS-CoV-2 Infections in England: A dynamic intensity model (2022)*. The Econometrics Journal. (with Adriana-Cornea Madeira and Joao Madeira)
- *Age-specific transmission dynamics of SARS-CoV-2 during the first two years of the pandemic (2022)*. Working Paper (with Amir Alipoor, Ganna Rozhnova, Sen Pei and Jeffrey Shaman).

Outline of talk

- Literature review of epi-econ models in economics + econometrics
- Mainly presenting paper *Disease-economy trade-offs under alternative epidemic control strategies (2022*, Nature Communications) by Thomas Ash, Antonio Bento, Daniel Kaffine, Akhil Rao, Ana Bento

Contribution (preliminary)

- thoughts on extending the model in Ash et al (2022) to include undocumented infectives
- model is calibrated: ideas on how to bring this model to data.

Parallels econ-epi

Macroeconomics versus macro-scale epidemic modelling

- both are model based, and models are nonlinear
- one notable difference is macro models include expectations about the future
- mostly homogeneous models, heterogeneous agent models recently popular
- extensive use of Bayesian MCMC and Kalman filter or particle filter when unobservables are stochastic.

Microeconomics versus other epidemiology models

- random samples, randomized experiments
- one difference maybe: when randomized experiments not available, quasi-experiments
- identification via controls, across group variations, or external variables (called instruments).

Econ papers on COVID-19

Macroeconomic models with an epidemiology component

- Atkeson (2020, Working paper) used an SIR model to study cost of the COVID-19 epidemic for the United States
- Alvarez et al (2021, *American Economic Review: Insights*) and Eichenbaum, Rebelo, and Trabandt (2021, Review of Financial Studies) study optimal mitigation policies in simple economies with SIR disease dynamics
- Acemoglu et al. (2021, *American Economic Review: Insights*) study disease mitigation in environments with multiple ages and sectors
- Baqaee et al. (2020,WP) and Azzimonti et al. (2022, WP) study how the network structure of economic sectors and geography can be exploited in the design of optimal mitigation policies
- Boppart et al (2022, *Journal of Economic Dynamics and Control*) - integrated epi-econ assessment of vaccination
- Arellano et al (2023, Review of Economic Studies) study how debt relief can alleviate both economic and disease burden
- many more unpublished, as NBER WP and CEPR Special Issue on Covid Papers: `https://cepr.org/publications/covid-economics-papers`.

Econometric papers on estimating epi models

The Econometrics Journal
https://academic.oup.com/ectj/pages/virtual-issue-covid-19

- Cho (2021): impact of potential NPIs in Sweden with "synthetic" Sweden
- Stöye (2021): bounds of number of infections by bounding selectivity and accuracy of tests
- Hansen (2022): estimating relative contagiousness of variants
- Korolev (2022): show "reduced form" (regression based) estimation of NPIs are consistent with multiple outcomes, while modelling less prone to these identification issues

Journal of Econometrics https://www.sciencedirect.com/journal/journal-of-econometrics/vol/220/issue/1

- Chernozhukov et al (2021): causal impact of masks in US
- Hortacsu et al (2021): estimating ascertainment rate in a model with known epidemic
- Gourieroux and Jasiak (2021): identifying SIR models using time-varying transition probabilities
- Korolev (2021): estimating R_0 in an otherwise poorly identified model.

Econometric papers on estimating epi models

Arias et al (2023, American Economic Journal: Macroeconomics)

- study causal effect of NPIs
- careful SIRD model for Belgium, estimated on case, hospitalization, death and seroprevalence data
- obtain implied death and effective reproductive numbers series
- feed it into a vector autoregression model with stringency index and economic variable
- study the dynamic effect of a "shock" in stringency index on both the economy and the reproduction number and deaths
- problem: stringency index changes contacts, which changes the estimates of the effective reproduction number

Epi-econ model in Ash et al (2020)

Disease dynamics:

$$\begin{aligned}
S_{t+1} &= S_t - \tau \mathcal{C}_t S_t I_t \\
I_{t+1} &= I_t + \tau \mathcal{C}_t S_t I_t - (P^R + P^D) I_t \\
R_{t+1} &= R_t + P^R I_t \\
D_{t+1} &= D_t + P^D I_t \\
N_t &= 1.
\end{aligned}$$

- S, I, R, D are expressed in proportions of population
- $\mathcal{C}_t$ are average contacts of a susceptible individual with infectious individuals
- *which is a function of daily activities.*

Contact function

$$\mathcal{C}_t = \rho_o + \rho_c c_t^S c_t^I + \rho_\ell l_t^S l_t^I$$

- activities: consumption c, labor l and other o
- roughly corresponds to contact surveys where l is work and school contacts, o is unavoidable daily contacts, and c is the rest
- at time zero, no infectives, so:

$$\mathcal{C}_0 = \rho_o + \rho_c c_0^2 + \rho_l l_0^2 = \sum_{i=1}^{3} \mathcal{C}_{0,i}$$

- $\mathcal{C}_{0,i}$ approximated from contact surveys
- c_0, l_0 based on macroeconomic economic data (freely available)
- then ρ_o, ρ_c, ρ_l are inferred, and assumed constant over time
- however, consumption and labor of susceptibles and infectives $c_t^S, l_t^S, c_t^I, l_t^I$ will be chosen by individuals based on a maximization problem.

Econ model

- for each disease state $m \in \{S, I, R\}$
- individuals maximize a different utility function
- and choose consumption c_t and labor l_t subject to budget constraints.

Utility function $u(c_t, l_t)$ has to satisfy

- increasing in c_t, decreasing in l_t
- usually time not spent consuming is normalized to one, and can be used for labor l_t or leisure $1 - l_t$
- increasing in c_t and $1 - l_t$
- second derivative of utility function should be negative definite for consumption and leisure
- meaning that additional units of consumption and leisure give you less and less of additional utility.

Per period utility function

Common choice

$$u(c_t, l_t) = \frac{[c_t^{\alpha}(1 - l_t)^{1-\alpha}]^{1-\eta} - 1}{1 - \eta}$$

- $\alpha \in (0, 1)$: how much individuals value consumption relative to leisure
- η is the risk aversion parameter
- $\eta = 0$ means individuals are indifferent to uncertainties associated with risk, and $\eta > 0$ individuals try to avoid uncertainties associated with risk
- the form $c_t^{\alpha}(1 - l_t)^{1-\alpha}$ implies that the elasticity of substitution between consumption and leisure is one.

Utility function

Elasticity of substitution of x_2 wrt to x_1 for $f(x_1, x_2)$ is

$$\frac{d(x_2/x_1)}{x_2/x_1} / \frac{dMRS(x_1, x_2)}{MRS(x_1, x_2)}$$

where $MRS =$ the marginal rate of substitution of x_2 wrt x_1

$$MRS = \frac{\partial u/\partial x_2}{\partial u/\partial x_1}$$

- for $f(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$ elasticity of substitution between consumption and leisure is equal to one
- elasticity is zero for no substitutes
- positive for imperfect substitutes
- infinity for perfect substitutes.

Utility function

To allow for elasticity of substitution to be constant but different than one, Ash et al (2022) use the constant elasticity of substitution utility function

$$u(c_t, l_t) = \frac{[v(c_t, l_t)]^{1-\eta} - 1}{1-\eta}$$

$$v(c_t, l_t) = \left[\alpha^{\frac{1}{\sigma}} c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (1-l_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- σ is now the elasticity of substitution between consumption and leisure, and it is constant.

Economic decision

Probability of a susceptible being infected at time is

$$P_t^I = \tau \mathcal{C}_t I_t$$

Decision of a susceptible individual is to maximize utility today, caring about how this choice affects his consumption tomorrow :

$$U_t^S = \max_{c_t^S, l_t^S} \left\{ u(c_t, l_t) + \delta[(1 - P_t^I) U_{t+1}^S + P_t^I U_{t+1}^I] \right\}$$
$$U_t^m = U^m(S_t, I_t, R_t), m \in \{S, I\}$$

- δ is how much I care about utility tomorrow compared to today
- with probability P_t^I, I get next period the utility of an infectious individual, and with probability $(1 - P_t^I)$ that of a susceptible
- also known as Bellman equation, and solved usually backward in time recursively, or via value function iteration
- S_t, I_t, R_t, D_t are state variables, c_t, l_t are control variables.
- assumed perfect foresight on P_t^I
- Ash et al (2022) also consider poor forecasts of P_t^I

Economic decision

- susceptibles also have a budget constraint:

$$pc_t^S = w_t \phi^S l_t^S$$

- $\phi^S = 1$ is the productivity of a susceptible individual
- real wage per unit of effective (fully productive) labor is w_t
- real wage is just wage where inflation was removed; calibrated from macro data
- therefore, price $p = 1$
- assumed no saving and no borrowing
- so $c_t^S = w_t \, l_t^S$, and the only control variables to be chosen are l_t^S
- susceptibles care about being infected, and their decision is also constrained by the disease dynamics.

Economic decision

Infectives have a different maximization problem:

$$
\begin{aligned}
U_t^I &= \max_{c_t^S, l_t^S} \left\{ u(c_t, l_t) + \delta[(1 - P^R - P^D) U_{t+1}^I + P^R U_{t+1}^R + P^D U_{t+1}^D] \right\} \\
U_t^m &= U^m(S_t, I_t, R_t), m \in \{I, R\} \\
U_t^D &= \Omega
\end{aligned}
$$

subject to budget constraint:

$$
c_t^I = w_t \Phi^I l_t^I,
$$

and disease dynamics

- where productivity of infected is $\Phi^I < 1$ in Ash et al (2022), and calibrated based on share of asymptomatic/pre-symptomatic infected
- and P^R and P^D where the probabilities of recovered given infected and dying given infected
- $U_t^D = \Omega$ is the utility of dying (negative usually): how much you prefer to die next period than consume less this period
- in Ash et al (2022), it is calibrated based on value of statistical life.

Economic decision

Utility of recovered:

$$U_t^R = \max_{c_t^R, l_t^R} \left\{ u(c_t^R, l_t^R) + \delta U_{t+1}^R \right\}$$

subject to budget constraint:

$$c_t^R = w_t \, l_t^R.$$

- it can be shown that this is a **static decision** given disease dynamics

What is optimal for each individual is not optimal for everyone:

- infectives care only about their own utility
- they don't care about infecting others
- therefore, susceptibles, even the absence of any NPIs, have to reduced their consumption and labor if they care about maintaining some of their future consumption and labor
- hence, they also reduced their contacts
- not that susceptibles still do not care about utility of others, only of their own.

Social planner

- at beginning of the epidemic, we have many susceptible individuals, and they are the ones that have to "isolate" (reduce their contacts) because infectives may infect them
- this results in many infections but also large economic losses (less productivity when infective, less consumption)
- a benevolent government/social planner would care about everyone's utility
- a social planner would recognize this problem and would instead isolate infected individuals, letting the susceptibles work
- so number of infected individuals are reduced through coordinating the choice of labor and consumption for everyone.

Social Planner's Decision

$$\max_{\mathbf{c}_t, \mathbf{l}_t} \sum_{t=0}^{\infty} \left\{ S_t u(c_t^S, l_t^S) + I_t u(c_t^I, l_t^I) + R_t u(c_t^R, l_t^R) + D_t \Omega \right\}$$

subject to:

$$p c_t^m = w_t \Phi_t^m l_t^m, \qquad m \in \{S, I, R\}$$

and disease dynamics.

- any further reduction in dynamics in this model will have to come from willingness of social planner to add to the optimization above a penalty on the number of cases
- for example, because of potentially exceeding hospital capacity
- something which is not in the individual's objective function
- this problem can also be solved by writing a Bellman equation.

Solving the models

- call the first scenario where social planner does not intervene voluntary isolation
- and solving the social planner's problem targeted isolation
- Ash et al (2022) assumes everything is deterministic
- calibrate all parameters
- and solves the optimization problem via value function iteration.

Use of calibrated model

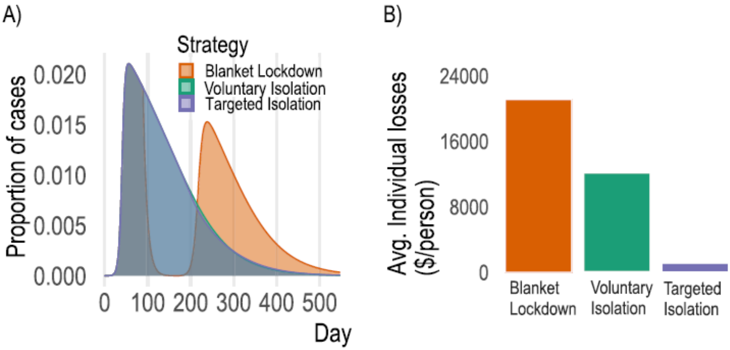
- social planner's problem (targeted isolation) can be solved in a similar fashion.

Total consumption equals total production (called gross domestic product GDP in the paper, but in reality close to consumption data since the model does not have investment and government purchases):

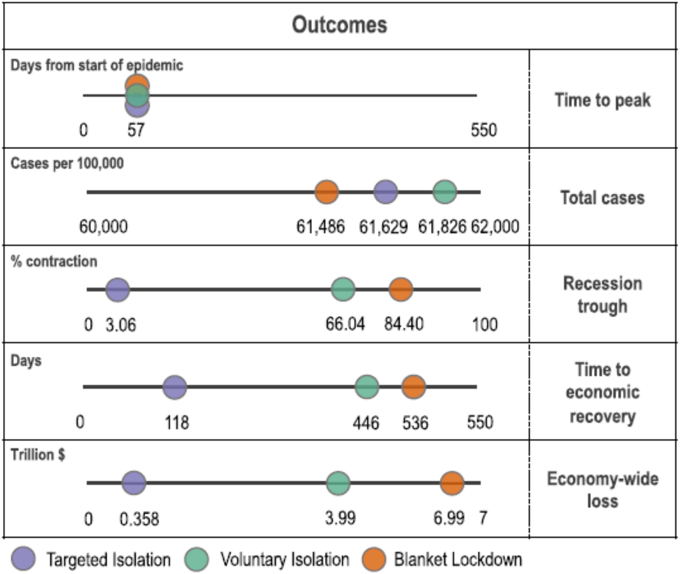
$$GDP_t = S_t c_t^S + I_t c_t^I + R_t c_t^R$$

- so we can calculate the difference in both infections and in GDP between the voluntary and the targeted isolation
- they also do so for a lockdown, where number of contacts is constraint based on measured lockdown contacts.

Ash et al (2022), Figure 2

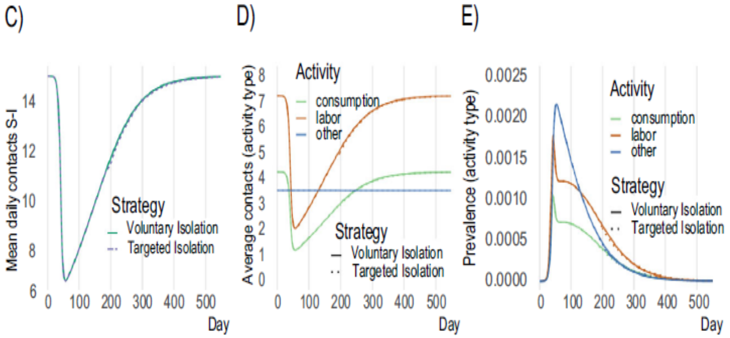


Ash et al (2022), Figure 2



Ash et al (2022), Figure 3

Overall contacts $\sim$ same with targeted and voluntary isolation, but are redistributed from infected to susceptible

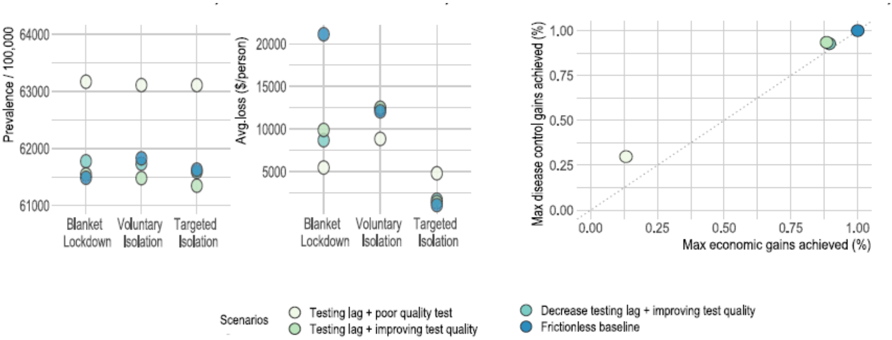


But due to infected isolating more, probability of contact between S and I is smaller, resulting in less infections.

$$P_t^I = \tau \mathcal{C}_t I_t$$

Ash et al (2022), Figure 4

Test delays (8 days, decrease over time) and/or poor quality tests (10% detection, increases over time)



Solving the model

- Recall that we only need to choose $l_t^m, m \in \{S, I\}$ and solve for function $U_t^m = U^m(S_t, I_t, R_t)$, everything else is in closed form
- assume U_T^m are the steady-state values, after pandemic passed, equal to those pre-pandemic U_0
- start with pre-pandemic consumption and leisure, and contacts
- use a numerical routine to choose l_t^m maximize $U(S_t, I_{t,} R_t)$ (fixed point iteration, inner loop to find U)
- feed this and c_t^m (which is proportional to labor l_t^m) into contact function to obtain new contacts $\mathcal{C}_t$
- new contacts imply new $S_{t+1}, I_{t+1}, R_{t+1}$, so feed it back to the loop
- similar techniques for solving the voluntary isolation (individuals) and targeted isolation (social planner) problem.

On grid value function iteration

Initialisation:

- We have grid on S, I, R: $G_S \in^{N_X}$, $G_I \in^{N_I}$, $G_R \in^{N_R}$. Let stacked grid be $G = (G_S, G_I, G_R) \in^N$ where $N = N_X + N_I + N_R$.
- We have initial guess of value function $V^0(S, I, R)$ for all S, I, R on grid. So, V^0 is $N \times 1$ vector. Since the Bellman equation is a well-behaved contraction mapping, this initial guess can be anything and usually convergence is guaranteed.

1. When entering step i of the inner loop, we have previous step's value function $V^{i-1}(S, I, R)$, which is a $N \times 1$ vector. Then, in step i, we calculate <u>for each</u> (S, I, R) in the grid G:

$$V^i(S, I, R) = \max_{c,l} u(c, l) + \beta V^{i-1}(S', I', R')$$

where

$$\begin{aligned} S' &= S - \tau C(A) IS \\ I' &= I + \tau C(A) IS - (P^R + P^D) I \\ R' &= R + P^R I \end{aligned}$$

2. Continue loop until V^i and V^{i-1} are close enough.

On grid value function iteration

- we obtain the converged value function $V^*(S, I, R)$ for all (S, I, R) in the grid, which is the $U(\cdot)$ function we seek
- more importantly, we obtain choices $c^*(S, I, R)$ and $l^*(S, I, R)$, which we can use in the actual model using initial values S_0, I_0, R_0
- there are stochastic extensions of this.

Estimation

- stochasticity with an exogenous variable governed by a stochastic process ξ.

General decision:

$$U(x|\mathcal{F}_t) = \max_x \{u(x, y, \xi_t) + \delta E[U(y)|\mathcal{F}_t\}$$

- when parameters are also stochastic,
- to avoid value function iteration, people typically linearize around the steady state
- then use a "rational expectation solution" to put the model in linear state space formulation
- and employ Kalman filtering technique
- linearizing around the stead state here doesn't make sense, at least not at the beginning of the epidemic

Thoughts?

Model extensions?

- undetected cases should be treated separately: these individuals will behave as susceptibles, but they will also be in general less infectious
- meaning they will both reduce their contacts (resulting in less work and therefore an economic loss), and not factor in the fact they may be infectious

The disease dynamics are given by:

$$
\begin{aligned}
S_{t+1} &= S_t - \tau \mathcal{C}_t I_t^d S_t - \mu \tau \mathcal{C}_t I_t^u S_t \\
I_{t+1}^d &= I_t^d + \tau \mathcal{C}_t I_t^d S_t + \mu \tau \mathcal{C}_t I_t^u S_t - (P^R + P^D) I_t^d \\
I_{t+1}^u &= I_t^u + (1-d) S_t (P_t^d + \mu P_t^u) - (P^R + P^D) I_t^u \\
R_{t+1} &= R_t + P^R (I_t^u + I_t^d) \\
D_{t+1} &= D_t + P^R (I_t^u + I_t^d)
\end{aligned}
$$

- d is case detection rate
- $0 < \mu < 1$ is relative infectiousness of undetected cases.

Extensions

Simplified utility:

$$v(c,l) = c^{\alpha}(1-l)^{1-\alpha}$$
$$u(c,l) = \frac{v(c,l)^{1-\eta} - 1}{1-\eta}$$

- decision of R_t, l_t^d decision as before
- and in both cases, static and in closed form.

Undocumented Infected Problem

- assume that an undocumented infective individual is unaware of their disease state and thus behaves in the same way as a susceptible
- the undocumented infected compartment is not observable to individuals
- it is reasonable to expect the central planner to estimate the number of undocumented infected people
- we assume that susceptible individuals do not estimate the number of undocumented infectives with whom they may come into contact.

True versus perceived infection probability

True infection probability is:

$$P_t^{true} = \tau \mathcal{C}_t I_t^d + \mu \tau \mathcal{C}_t I_t^u$$

Perceived infection probability is:

$$P_t = \tau \mathcal{C}_t I_t^d$$

- therefore, S, I^u reduce their consumption/labor less
- because they don't estimate well future losses
- economic benefits may initially seem large
- but documented infectives are less productive than before, so may result in more loss

Other limitations to address in original model?