



MRC
Biostatistics
Unit



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Identifiability

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Structural identification

- seeking a parameter θ_0
- model that links data x and parameter θ to sample likelihood

$$\ell_n(x; \theta)$$

- or other objective function
- assume no noise: sample likelihood converged to true likelihood:

$$\ell_n(x; \theta) \xrightarrow{P} \ell(\theta)$$

Definition:

- θ_0 is structurally globally identified if and only if $\ell(\theta)$ is uniquely maximised at θ_0
- meaning $\ell(\theta)$ is globally concave, and its unique maximum is at parameter we seek θ_0 .

Examples of failures: when $\ell(\theta)$

- is uniquely maximised but at wrong parameter value $\theta^* \neq \theta_0$
- has multiple modes
- contains parameters that enter only as a product
- is flat

Local identification

- $\ell(\theta)$ has a local maximum at θ_0 (local peak)

Practical identification

- the parameter θ_0 is identifiable (no structural un-identifiability)
- but available data not informative enough

An example: early phase in an epidemic

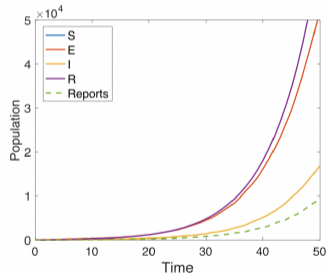
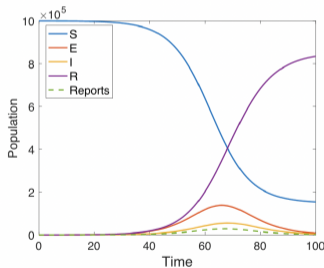
- An SEIR model

$$\dot{S} = -\beta I \frac{S}{N},$$

$$\dot{E} = \beta I \frac{S}{N} - \alpha E,$$

$$\dot{I} = \alpha E - \gamma I,$$

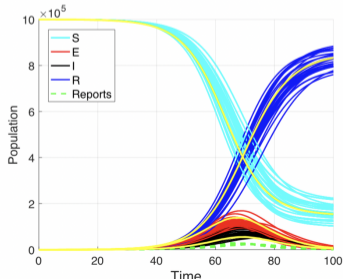
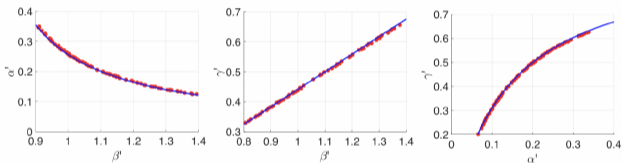
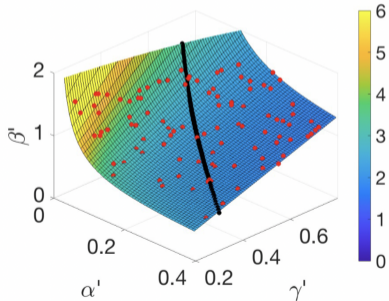
$$\dot{R} = \gamma I,$$



Evidence 2

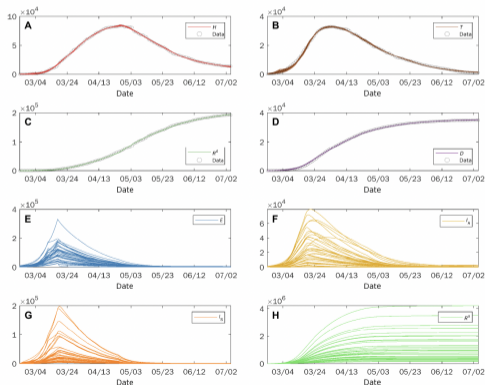
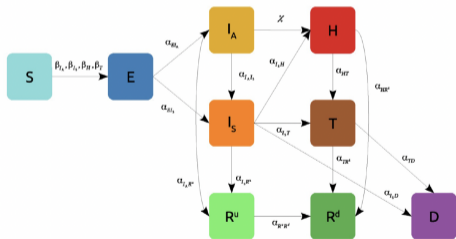
- The unidentifiability manifold

$$(\alpha' - \alpha)(\gamma' - \gamma - (\beta' - \beta)) + (\alpha/c - \beta)(\alpha' - \alpha) + \beta c(\gamma' - \gamma) - \alpha(\beta' - \beta) = 0,$$

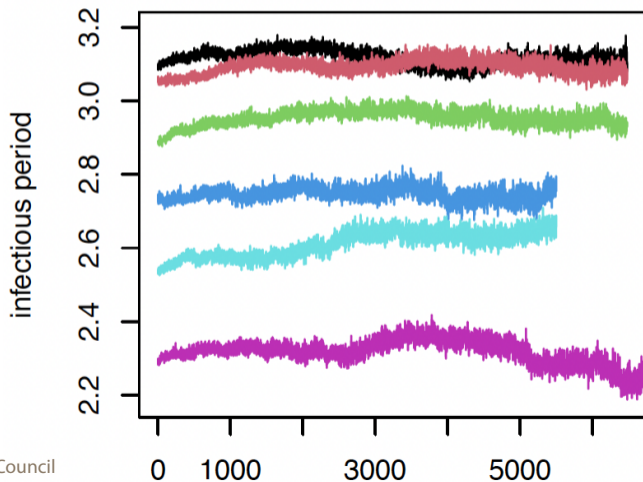


Lack of practical identifiability may hamper reliable predictions in COVID-19 epidemic models

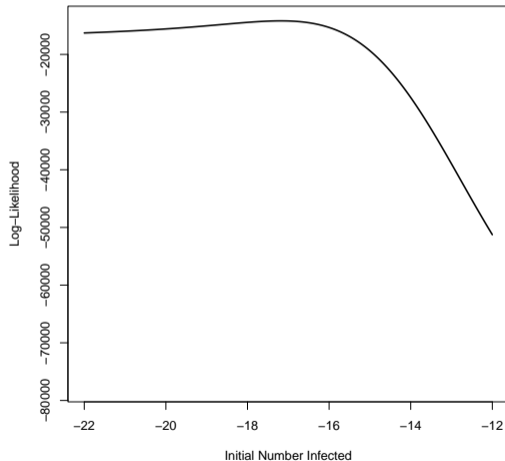
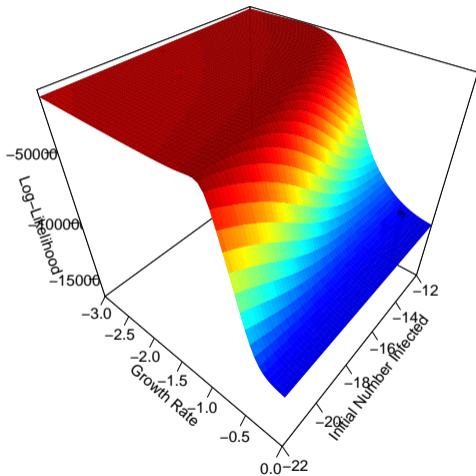
Luca Gallo^{1,2}, Mattia Frasca^{3,4*}, Vito Latora^{1,2,5,6}, Giovanni Russo⁷



Evidence from our Bayesian model

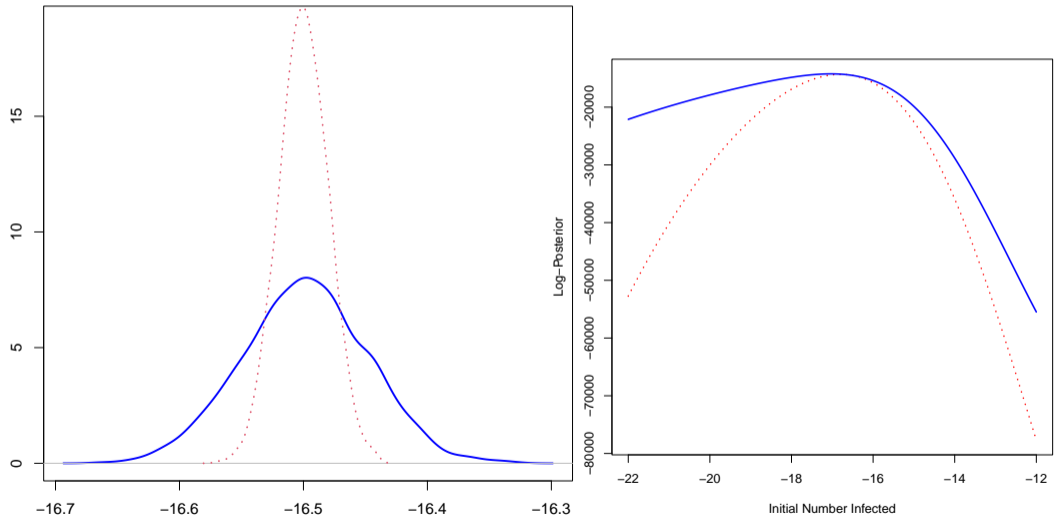


Unidentifiable parameters ψ, I_0

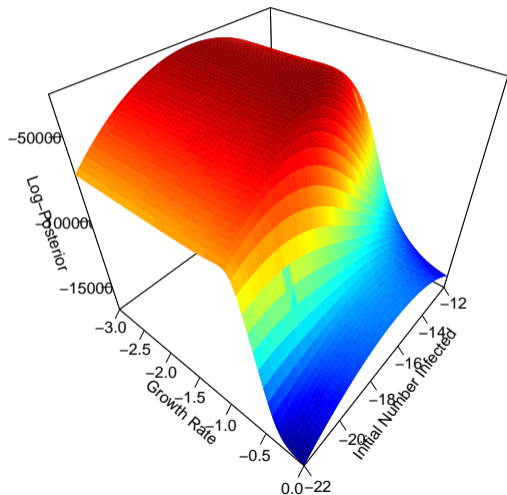
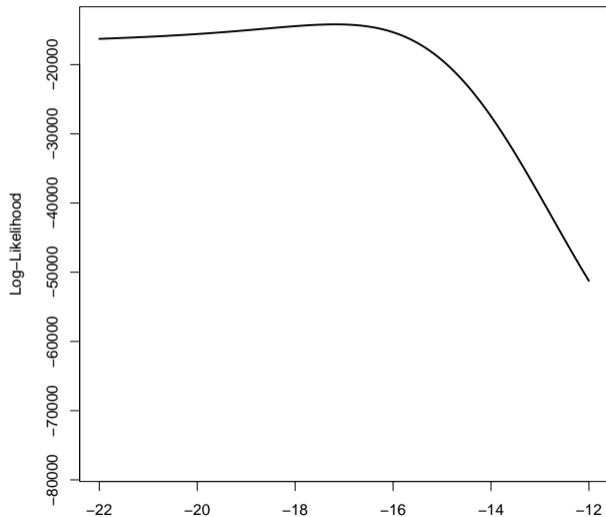


Unidentifiable parameters ψ, I_0

Priors



Unidentifiable parameters ψ, I_0



- Do you have experience of it? If so, in what form?
- How did you detected it?
- Is it always a problem? When it is not a problem?
- Did you try and resolve it? How?