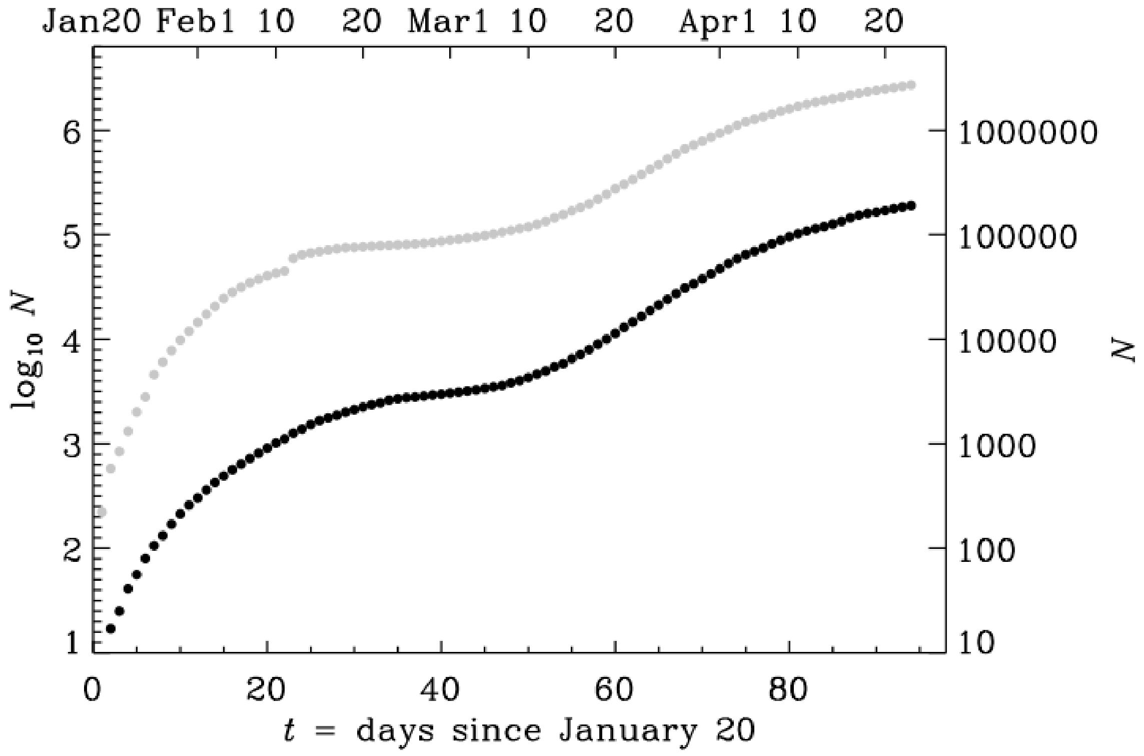
Spatially extended SIR models & quadratic growth

Axel Brandenburg (Nordita)

- Exponential growth for covid?
  - Not obvious
  - Containment efforts (Jan 20)?
- # of deaths tracks # cases
  - No delay?
  - Quadratic growth?



Total number of cases and death worldwide (dominated by China in the beginning)

Brandenburg (2020, Infect Disease Modelling 5, 681)
Brandenburg (2023, J Phys A: Math Theory 56, 044002)

Exponential vs quadratic growth

Examples

t	0	1	2	3	4	5
2^t	1	2	4	8	16	32
t^2	0	1	4	9	16	25

t	0	1	2	3	4	5	6	7	8	9	10	11
2^t	1	2	4	8	16	32	64	128	256	512	1024	2048
t^2	0	1	4	9	16	25	36	49	64	81	100	121

double each time

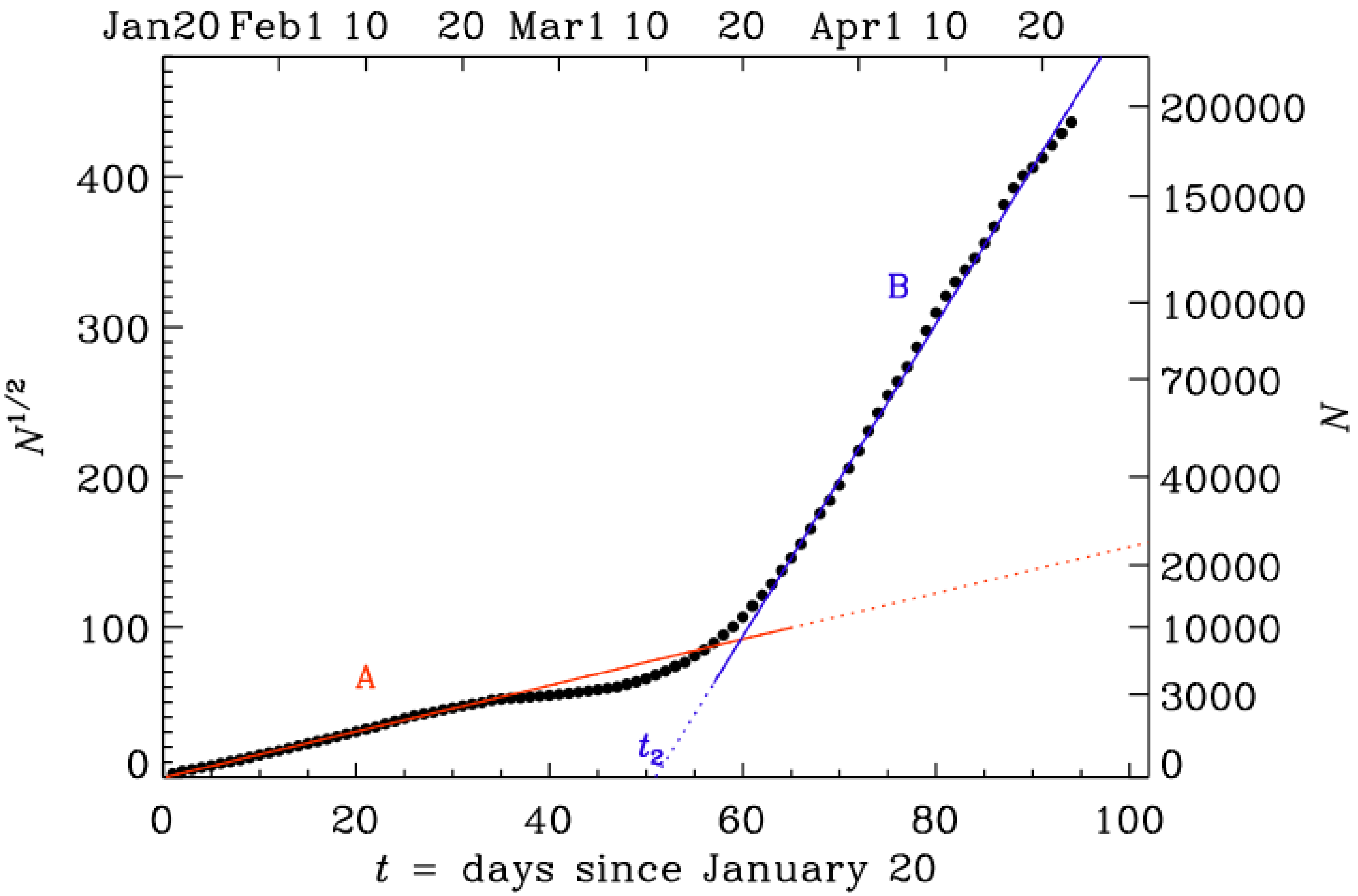
add $2N^{1/2}$ each time

To display:

- Exponential growth? → $\ln N$ proportional to t
- Quadratic growth? → $N^{1/2}$ proportional to t

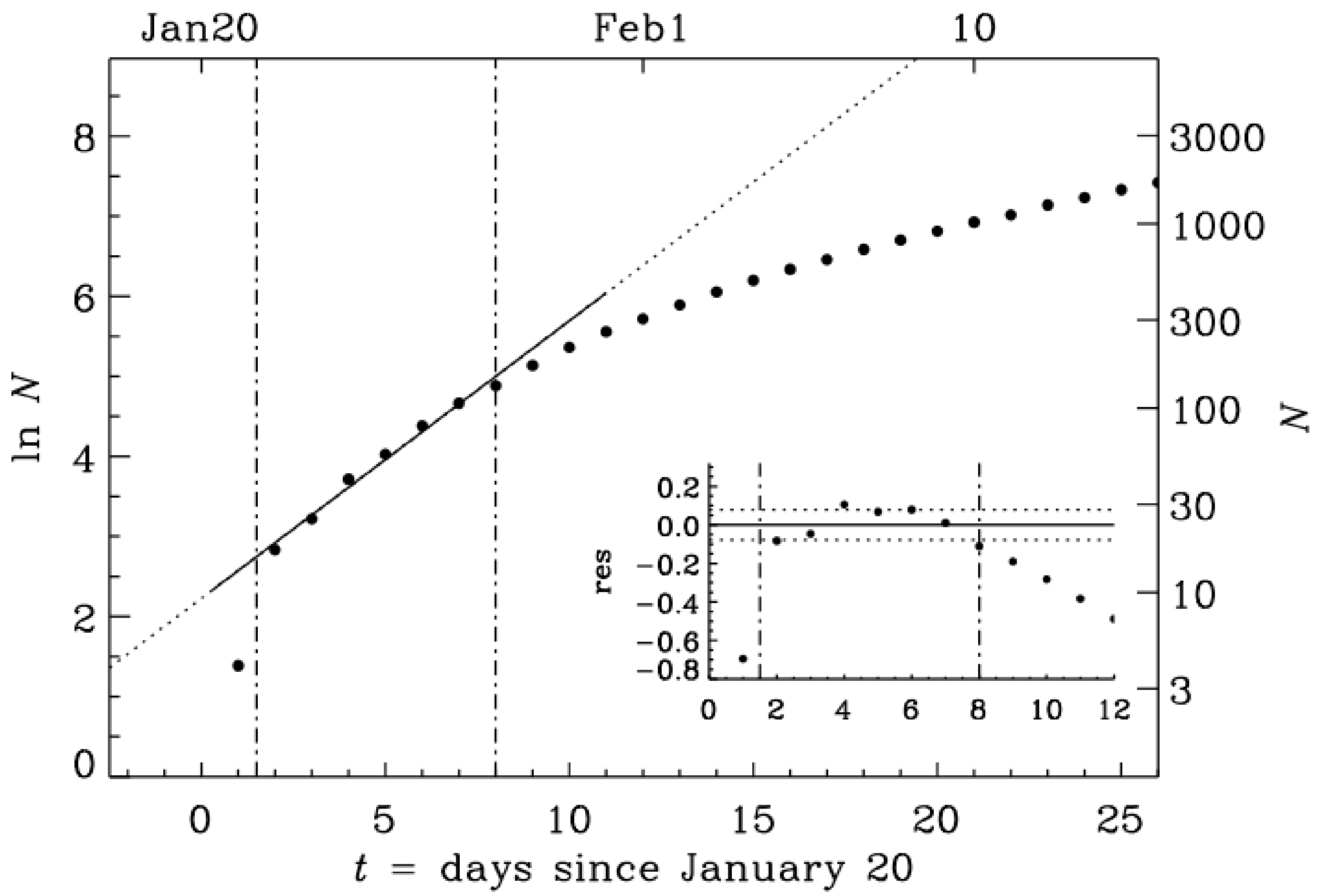
Piecewise quadratic growth

- Fit for first part
  - (t/0.7 days)2 excellent to fake data!
- Quadratic growth?
  - Not predicted
  - Generic reason?



in the 2014–15 Ebola epidemic in West Africa (Santermans et al., 2016). However, that the growth turns out to be quadratic to high accuracy already since January 20 is rather surprising and has not previously been predicted by any of the recently developed models of the COVID-19 epidemic (Chen et al., 2020; Liang, Xu, & Fu, 2020; Zhou, Liu, & Yang, 2020). It is therefore important to verify the credibility of the officially released data; see Robertson et al. (2019) for similar concerns in another context.

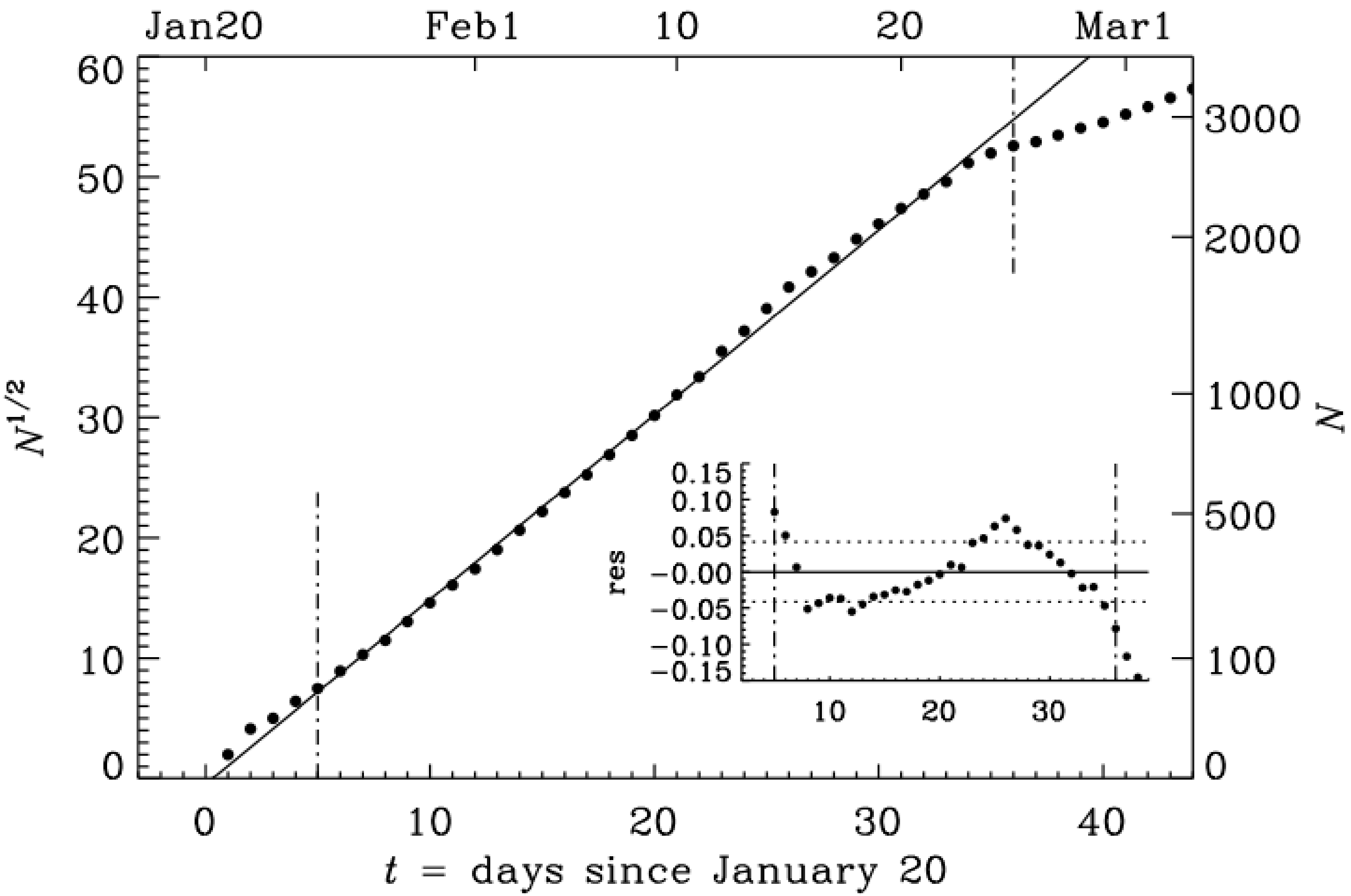
Early exponential growth?



Residual?

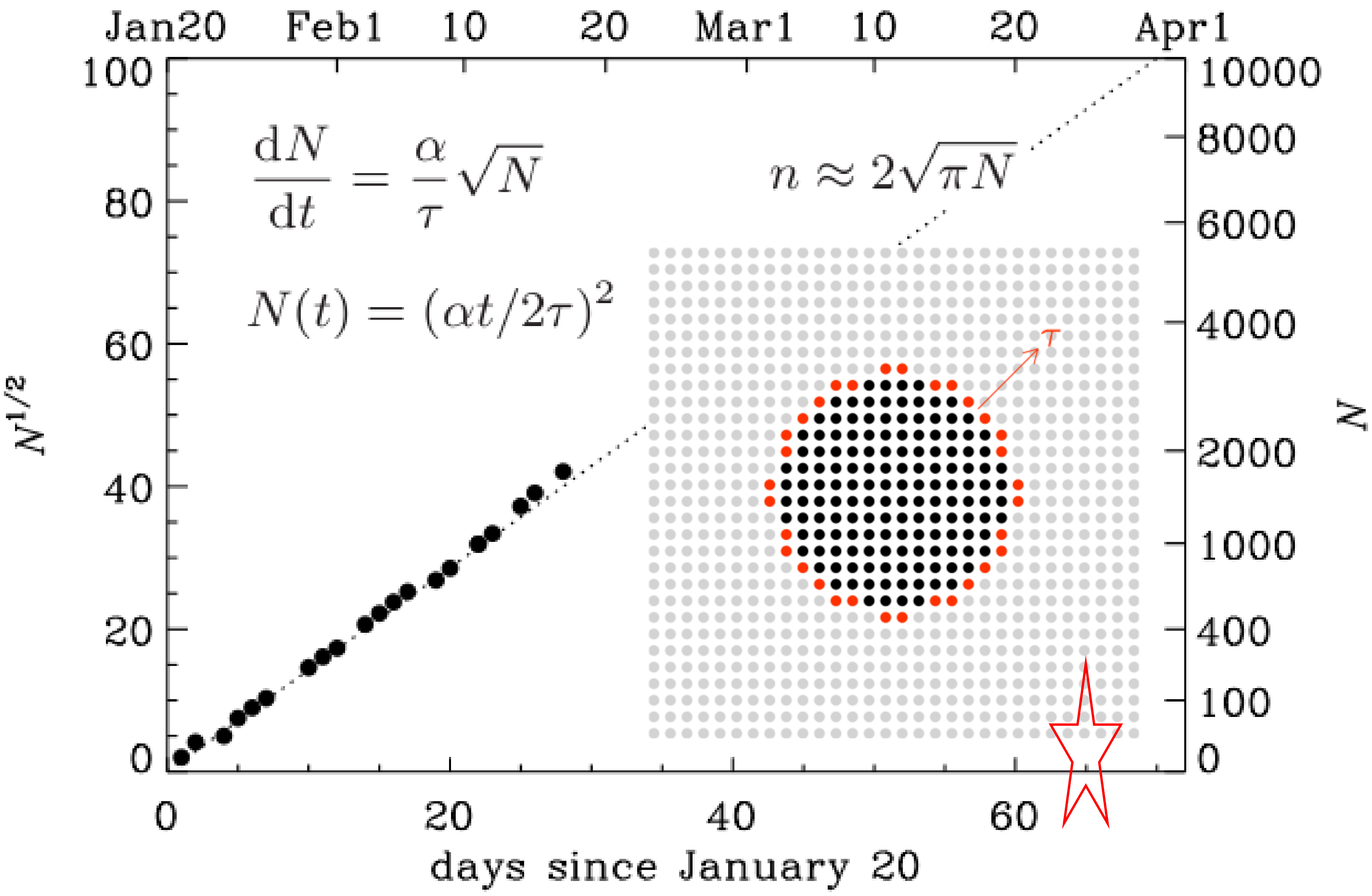
$$\text{res} = [N(t)/N_{\text{fit}}(t) - 1] \qquad N_{\text{fit}}(t) = \exp[(t - t_0)/\tau] \quad \text{(exponential fit)},$$

Comparison: quadratic growth (first part)

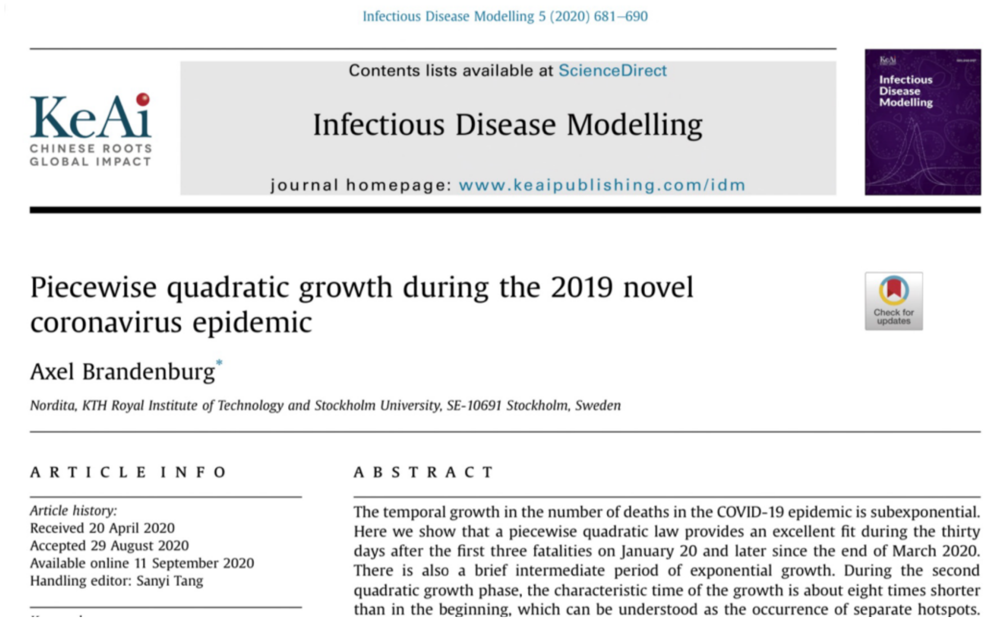


Residual $\text{res} = [N(t)/N_{\text{fit}}(t) - 1]$ $N_{\text{fit}}(t) = [(t - t_0)/\tau]^2$

Peripheral growth



Quadratic growth also for the second part



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Piecewise quadratic growth during the 2019 novel coronavirus epidemic

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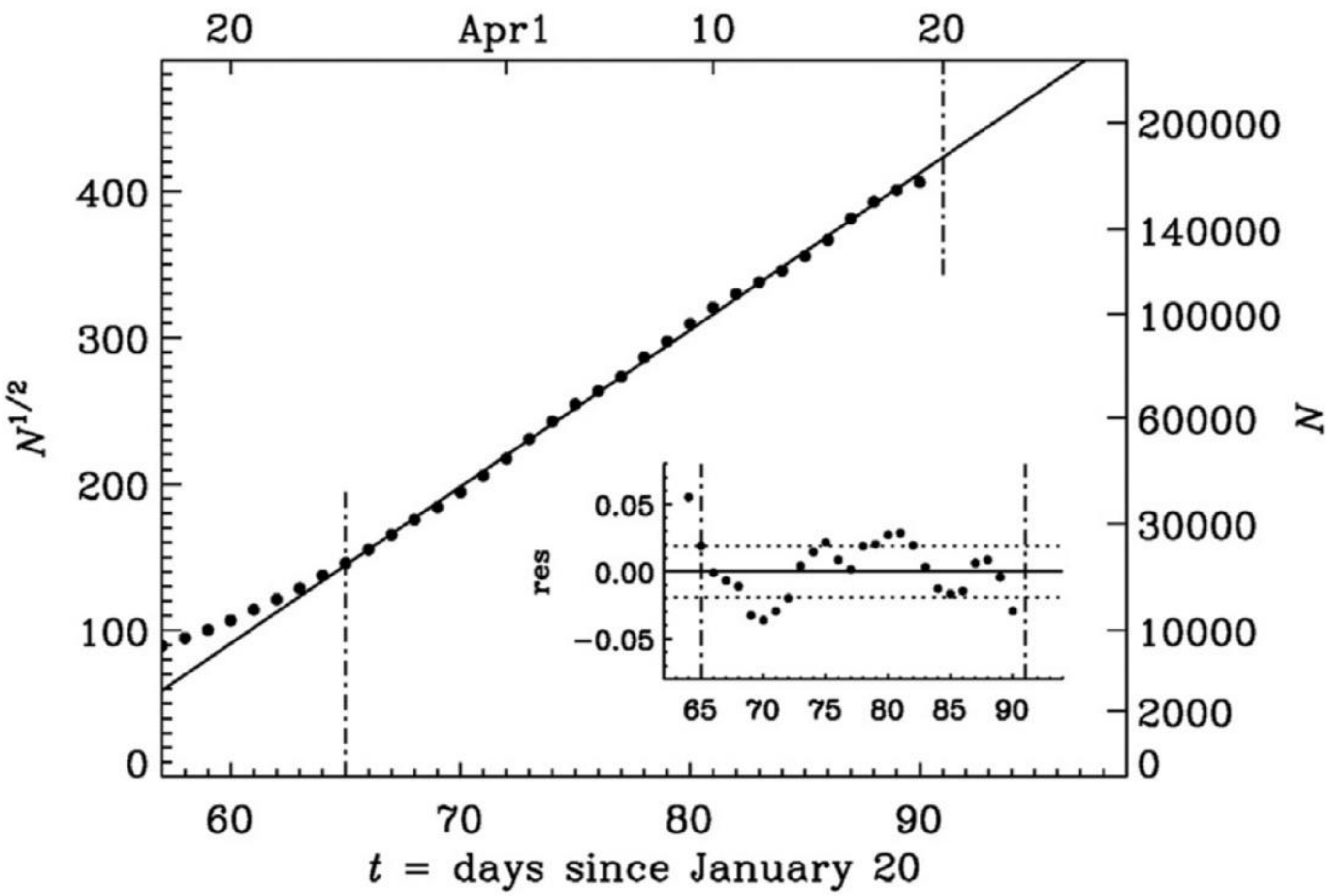
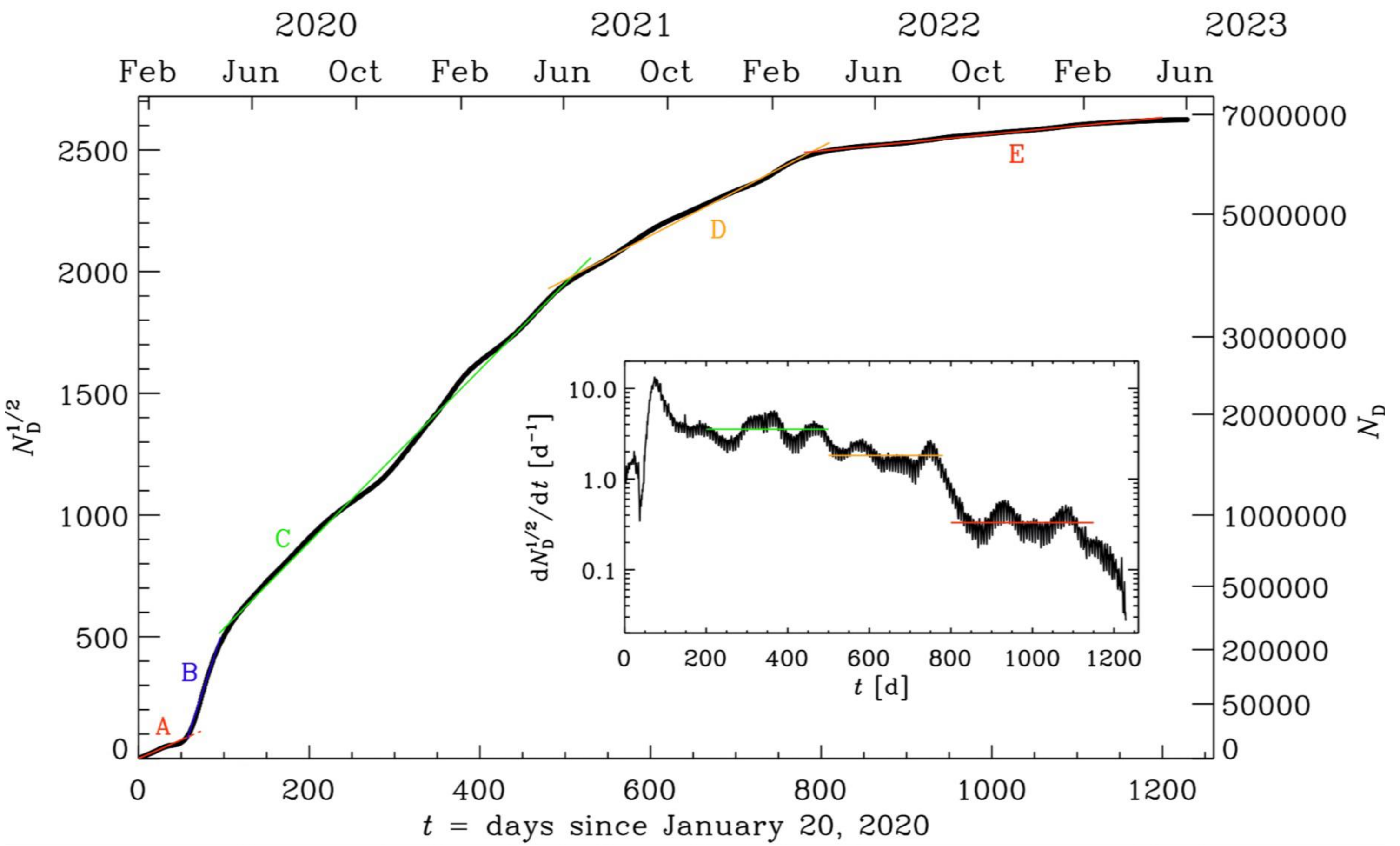
ABSTRACT

The temporal growth in the number of deaths in the COVID-19 epidemic is subexponential. Here we show that a piecewise quadratic law provides an excellent fit during the thirty days after the first three fatalities on January 20 and later since the end of March 2020. There is also a brief intermediate period of exponential growth. During the second quadratic growth phase, the characteristic time of the growth is about eight times shorter than in the beginning, which can be understood as the occurrence of separate hotspots.



Residual $\mathrm{res} = [N(t)/N_{\mathrm{fit}}(t) - 1]$ $N_{\mathrm{fit}}(t) = [(t - t_0)/\tau]^2$

Quadratic growth still persists today



Interval	$1/\tau_D$ [d^{-1}]	τ_D [d]
A	1.54	0.65
B	10.7	0.09
C	3.55	0.28
D	1.82	0.55
E	0.41	2.5

- Time constant 7 times shorter for B
- Time constants similar for A and D
  - Reason for this?

SIR model with spatial extent

$$\frac{\partial S}{\partial t} = -\lambda S I,$$

$$\frac{\partial I}{\partial t} = \lambda S I - \mu I + \kappa \nabla^2 I,$$

$$\frac{\partial R}{\partial t} = \mu I.$$

- Usually just exponential growth + saturation
  - Now also spreading (depends on λ and κ)
  - Constant front speed $c = 2\sqrt{\lambda\kappa}$ (Murray, 2003)
  - width $\sqrt{\kappa/\lambda} = 10^{-3}$.

Originally by Noble (1974, Nature 250, 726)

Fig. 1 Approximate chronology of the black death, 1347 to 1350 (From *The Black Death* by William L. Langer. Copyright © 1964 by Scientific American, Inc. All rights reserved.)

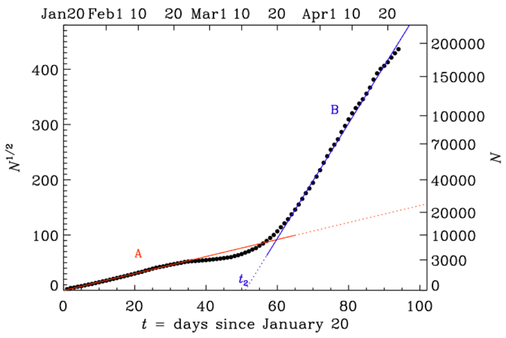
1350

1349

1348

At the time of the Black Death (1347 AD) the population density U of Europe was about 50 mile^{-2}. To try to get a rough idea of what D should be, we note that communication was such that one might expect minor news or gossip to diffuse a distance on the order of 100 miles in a year, that is $D \sim 10^4$ mile^2 yr^{-1} (ref. 4). The area swept out in ambulation at a nominal 1 mile h^{-1} slow walk, assuming $\langle fb \rangle \sim 0.5$ foot (corresponding to a 5-foot flea-hop times a 10% average transmission probability) is 0.4 mile^2 yr^{-1} and so $KU \sim 20$ yr^{-1}. Assuming a mortality rate of $\mu \sim 15$ yr^{-1} (corresponding to a 2-week infectious period) we expect a velocity of propagation of 200–400 mile yr^{-1}. This velocity is in substantial agreement with that which can be estimated from Fig. 1. (All the numbers given above were educated guesses, but the resulting speed is insensitive to their values since $v \sim \sqrt{KUD}$. My object in making these guesses was primarily to indicate the reasonableness of the model and its predictions.)

- 1000 km^2/yr



Piecewise quadratic observed previously

International Journal of Astrobiology 3 (3): 209–219 (2004) Printed in the United Kingdom
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How long can left and right handed life forms coexist?

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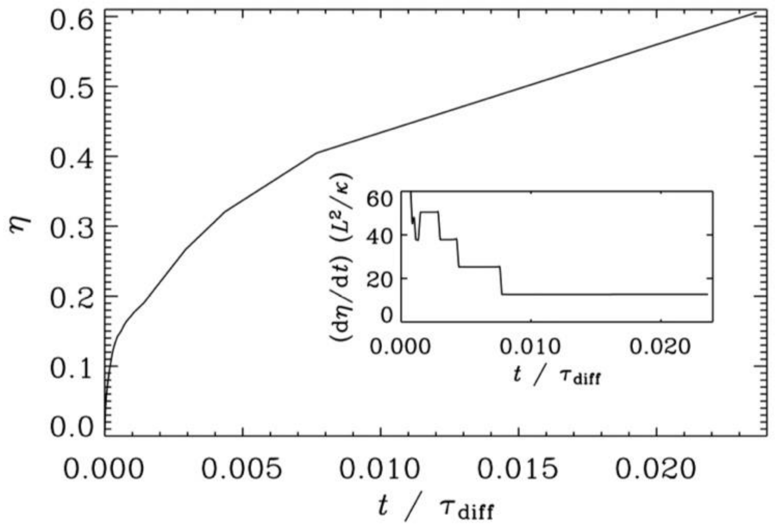
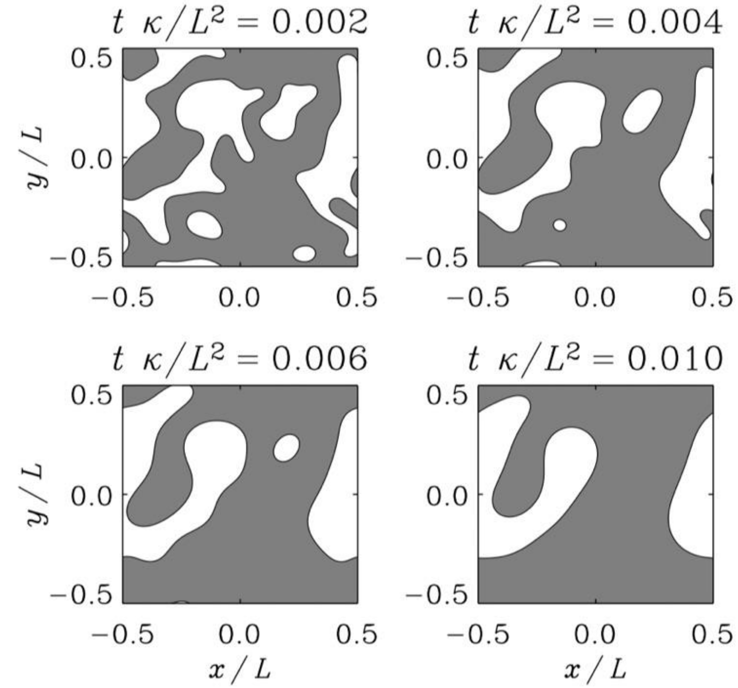
Abstract: Reaction-diffusion equations based on a polymerization model are solved to simulate the spreading of hypothetic left and right-handed life forms on the Earth's surface. The equations exhibit front-like behavior as is familiar from the theory of the spreading of epidemics. It is shown that the relevant time scale for achieving global homochirality is not, however, the time scale of front propagation, but the much longer global diffusion time. The process can be sped up by turbulence and large scale flows. It is speculated that, if the deep layers of the early ocean were sufficiently quiescent, there may have been the possibility of competing early life forms with opposite handedness.

Received 3 April 2004, accepted 31 July 2004

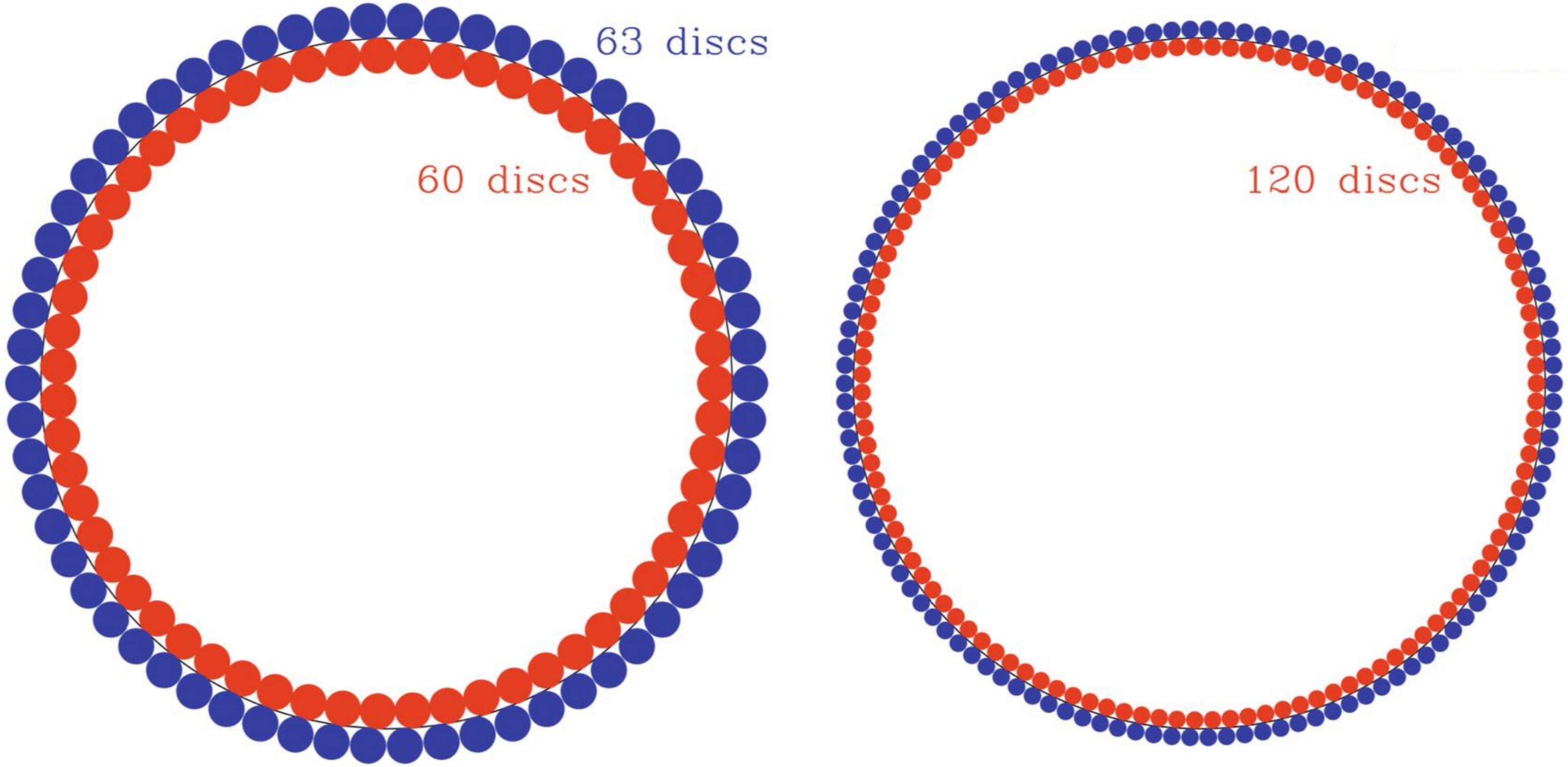
Key words: exobiology, homochirality, origin of life.

- But here the other way around



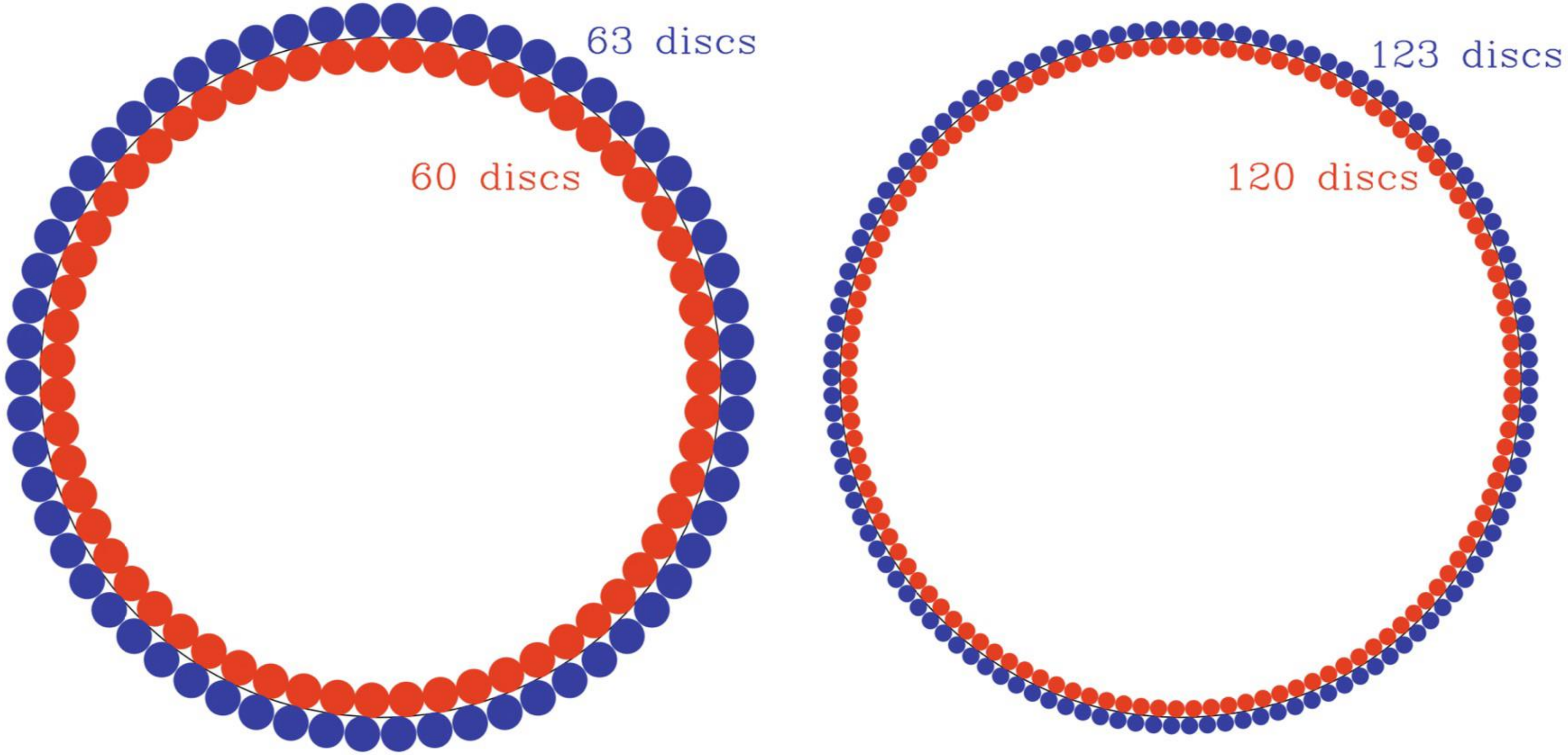


Excess on outer periphery?



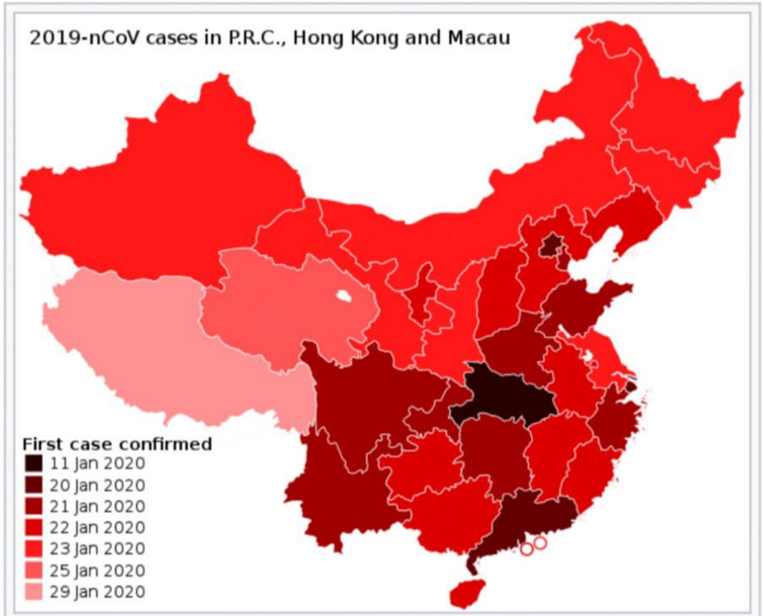
- 123 discs?
- 126 discs?

Excess on outer periphery?

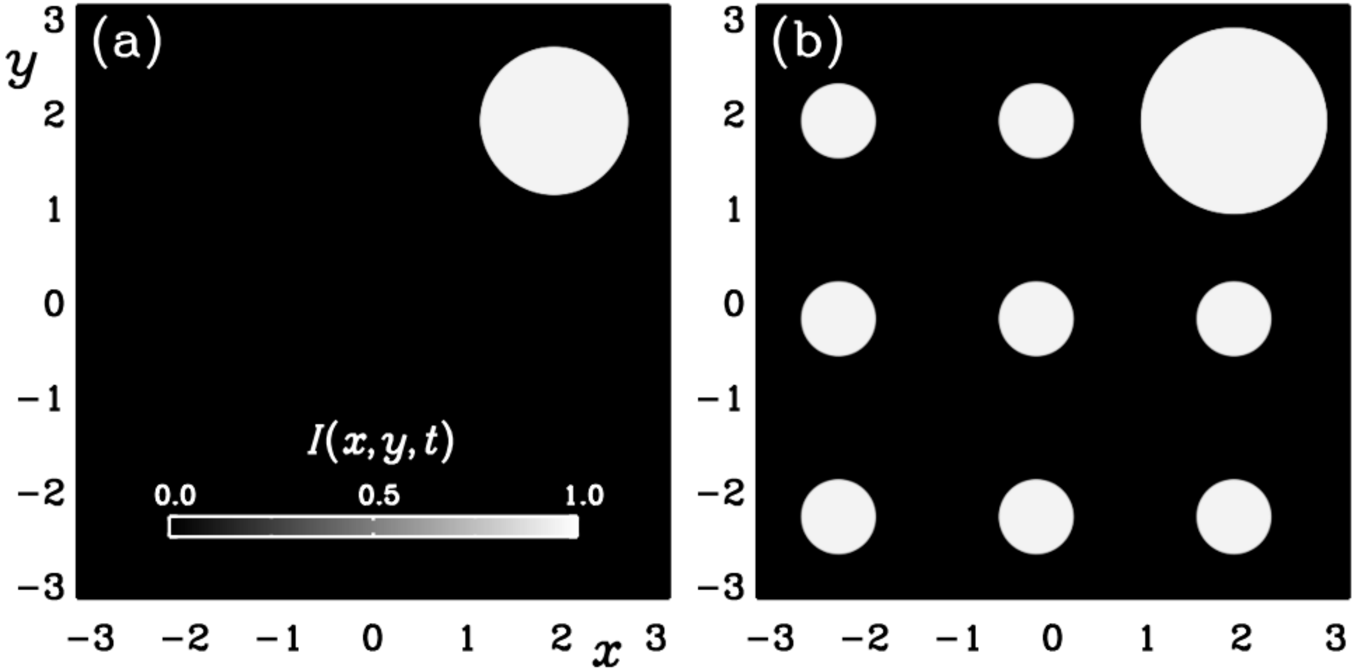


- 123 discs?
- 126 discs?

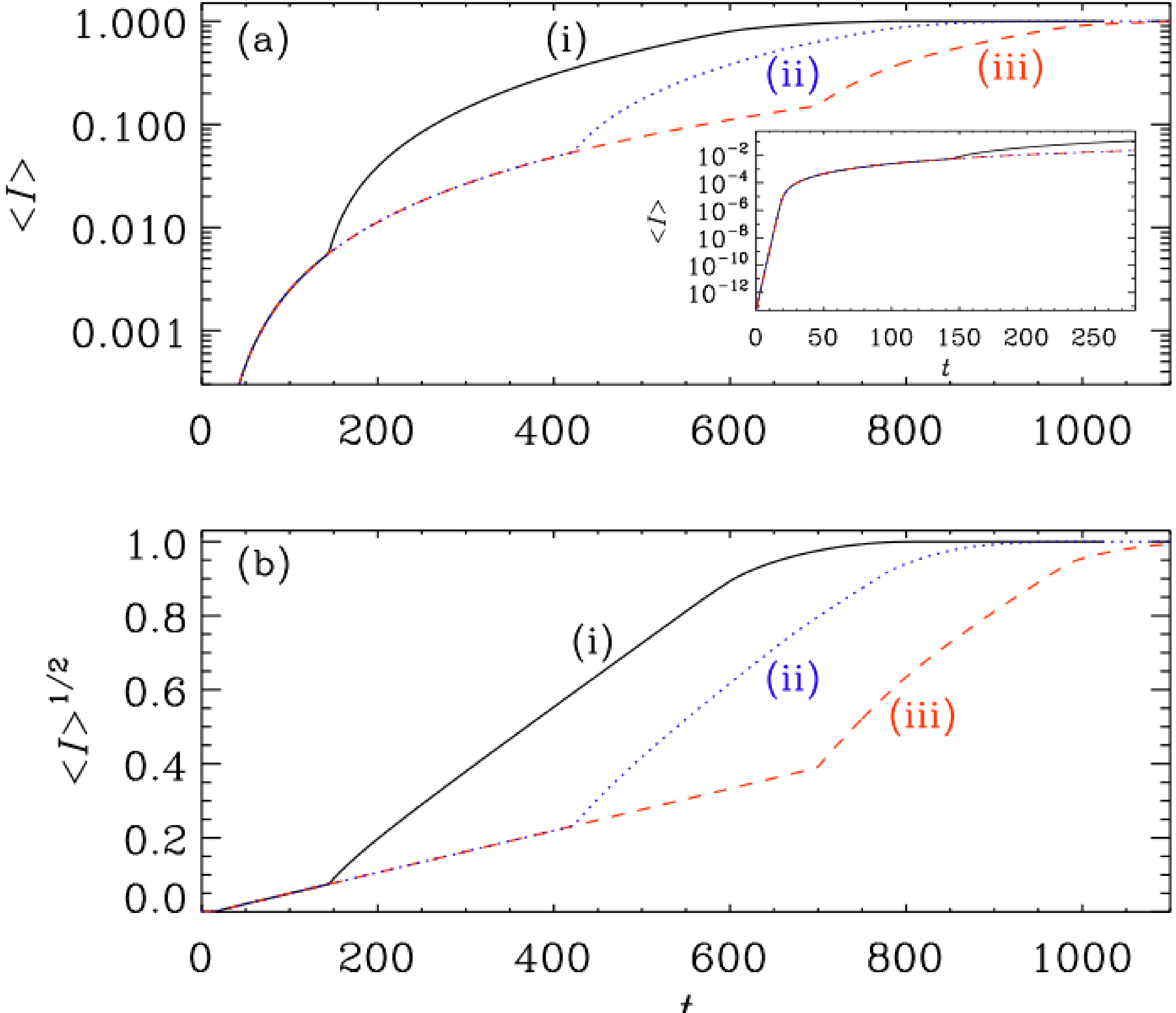
SIR model with spatial extent: multiple spreading centers



Date when first case in each first-level administration was reported



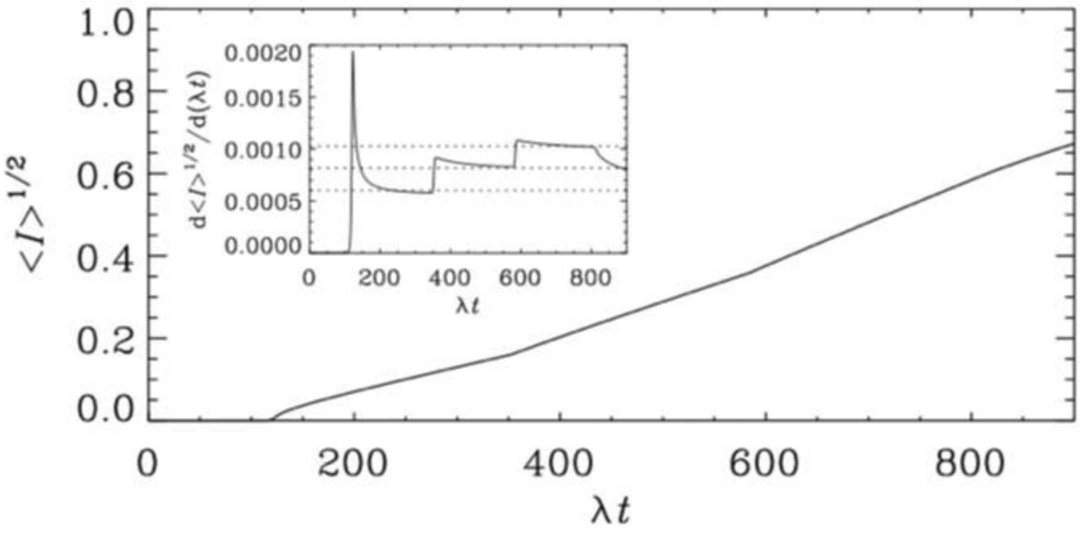
- **Other 8 hotspots** with much smaller initial *I*.
- Growth faster because of larger surface area

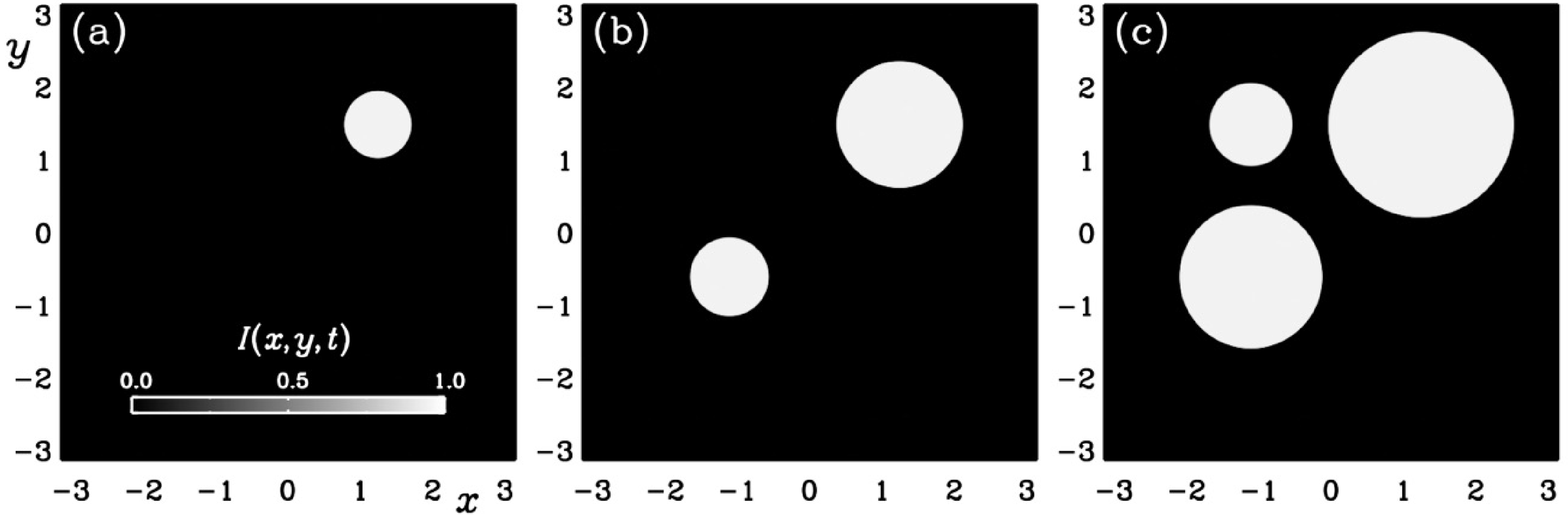


Piecewise quadratic growth perfectly confirmed

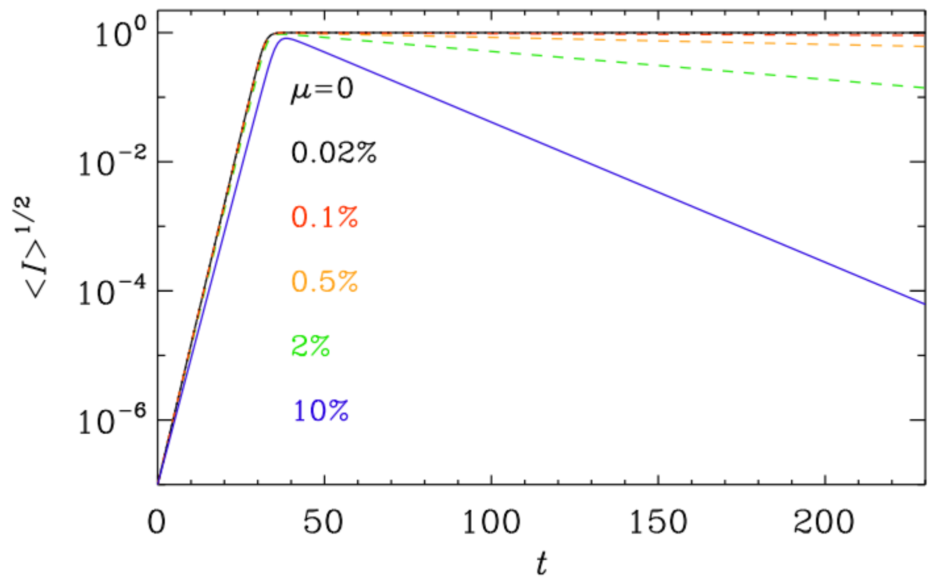
Piecewise quadratic: depends on # of structures

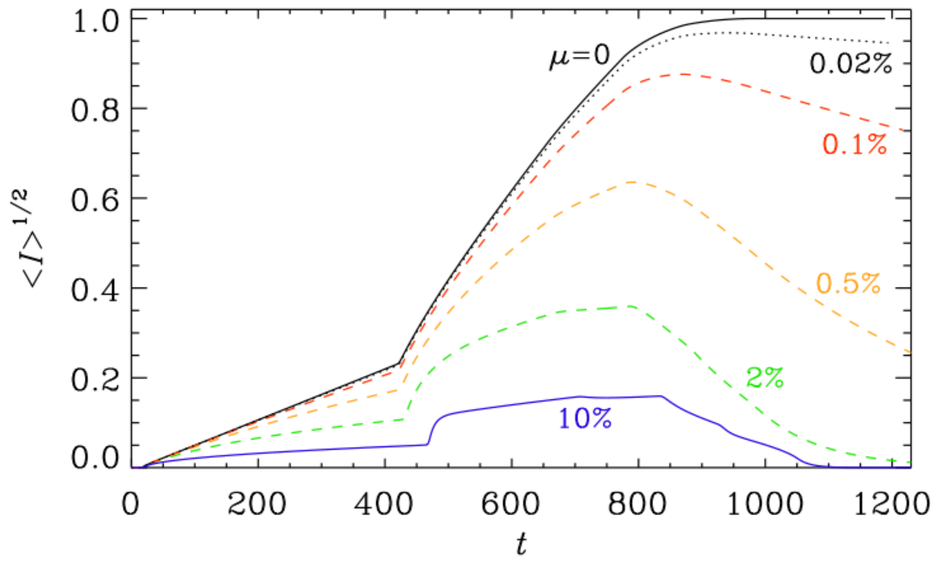
- Overshoot (spike)?
- Offset?
  - Not captured by simple peripheral growth model



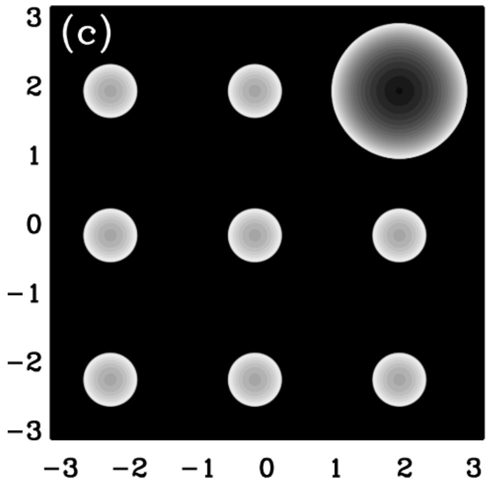


Difference: without/with spatial extent

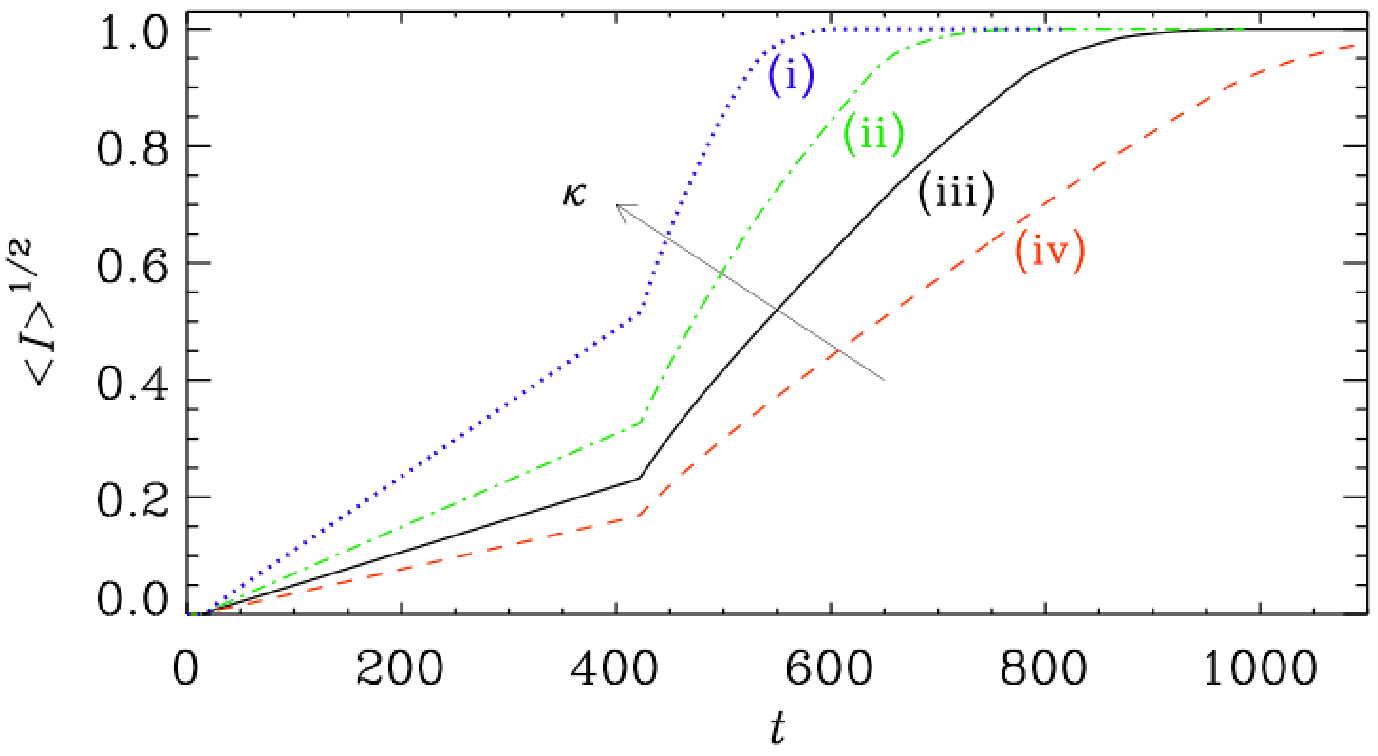




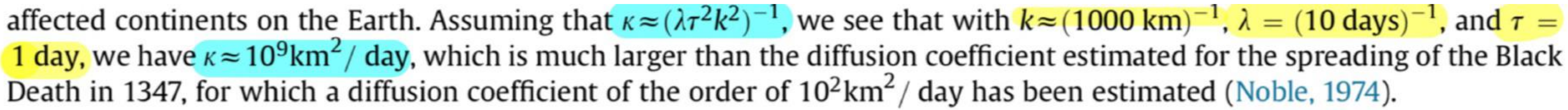
- Saturation very quickly
- Decline in 0D
- But further spreading in 2D



Speed depends on κ (diffusivity)

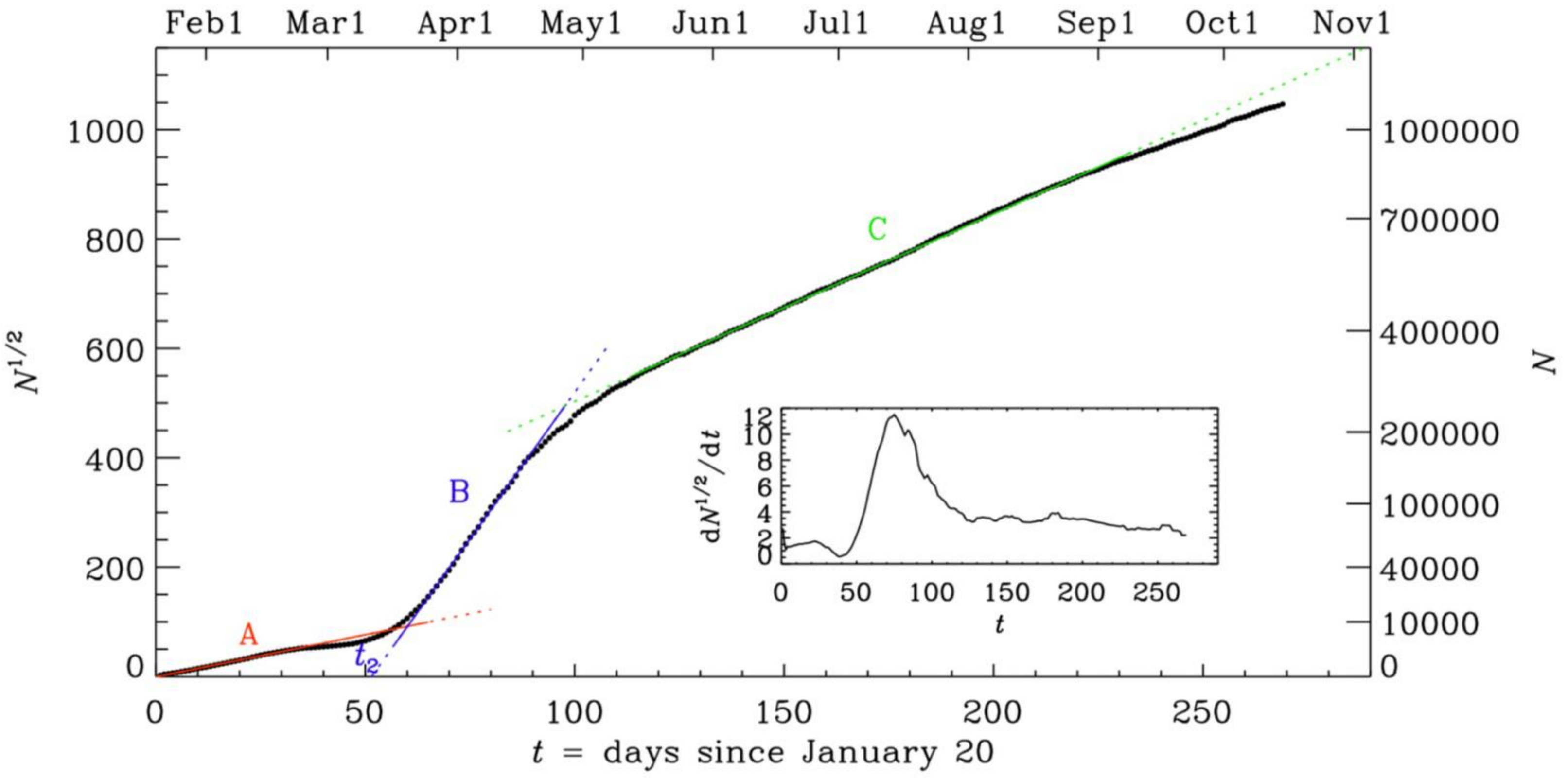


- Is an empirically determined quantity
- Comes out as huge → to be checked
- Spatially variable κ → intermediate decline?



affected continents on the Earth. Assuming that $\kappa \approx (\lambda\tau^2 k^2)^{-1}$, we see that with $k \approx (1000\ \text{km})^{-1}$, $\lambda = (10\ \text{days})^{-1}$, and $\tau = 1$ day, we have $\kappa \approx 10^9 \text{km}^2/\text{day}$, which is much larger than the diffusion coefficient estimated for the spreading of the Black Death in 1347, for which a diffusion coefficient of the order of $10^2 \text{km}^2/\text{day}$ has been estimated (Noble, 1974).

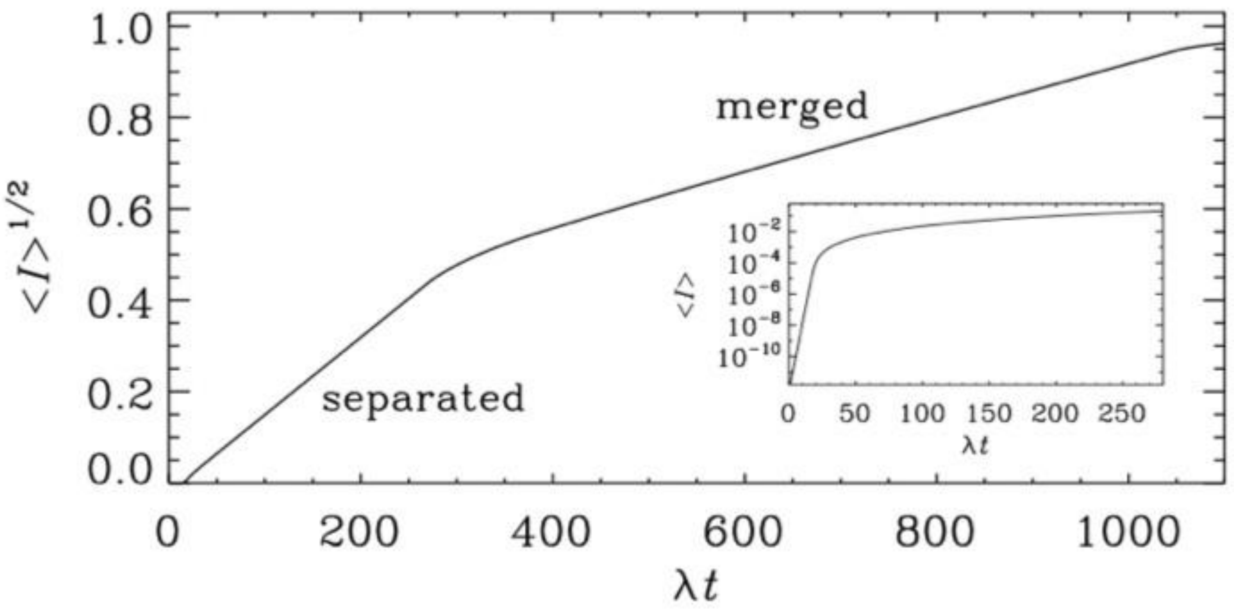
Late decrease in slope: how to model this?

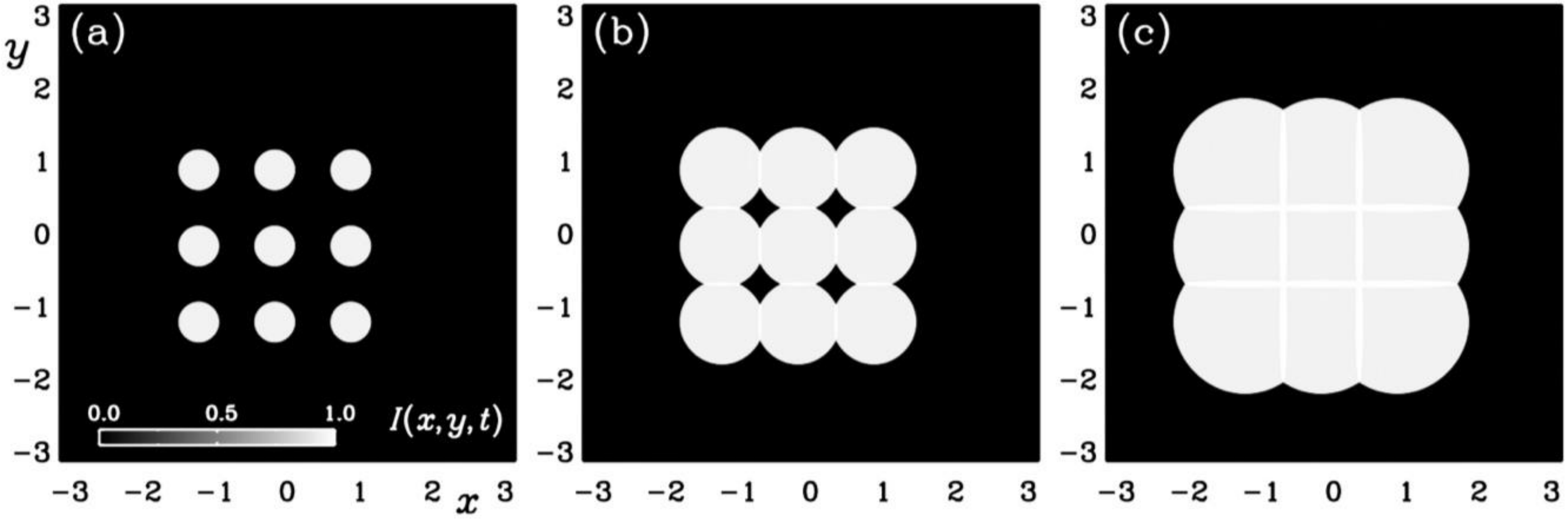


- Finally consequence of containment?

Modeling late decrease

- Natural consequence of merging hotspots
- Slope close to that at the beginning



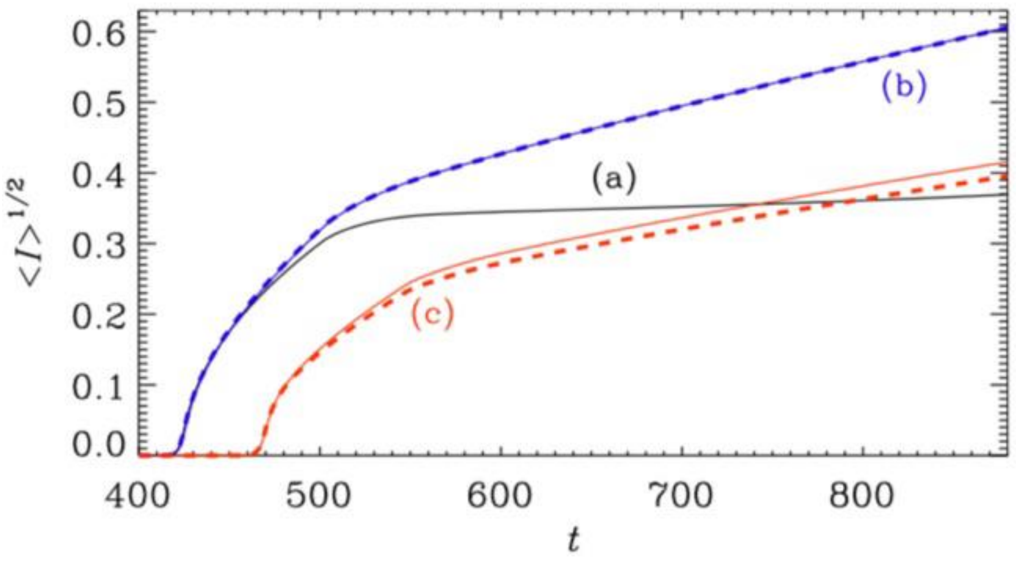


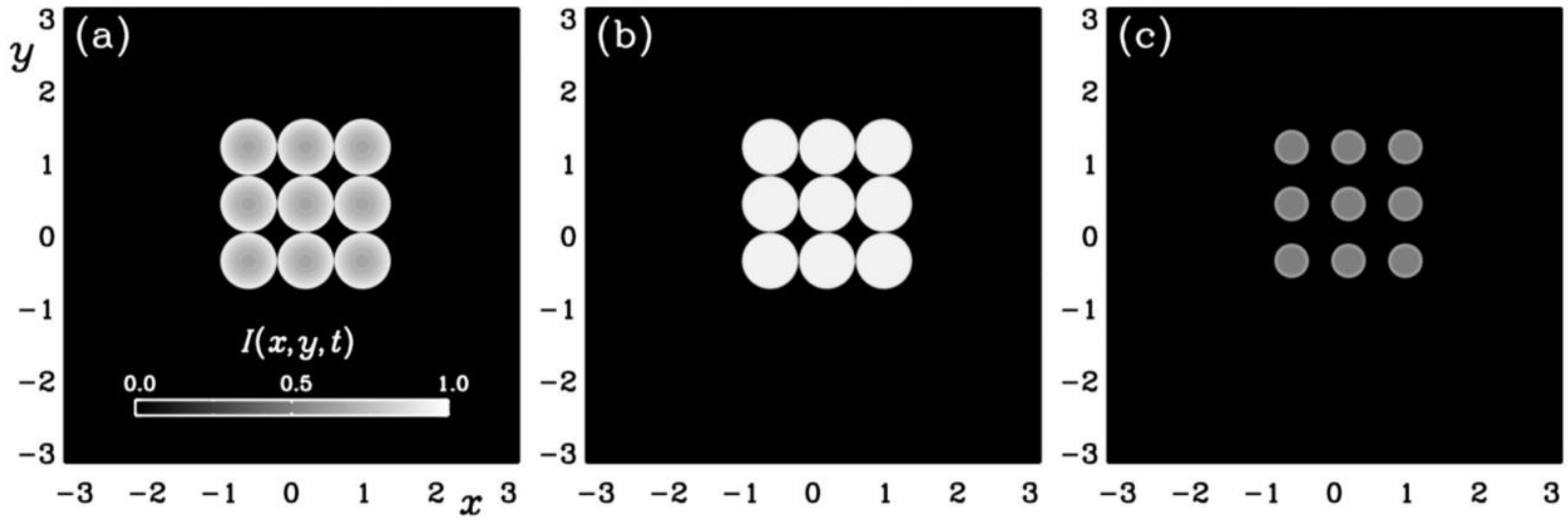
Models with reinfections

(a) No reinfection, low recovery

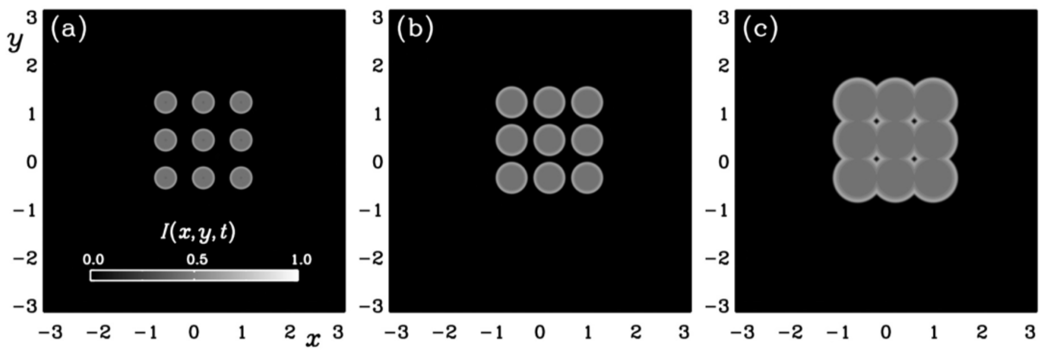
(b) Same with reinfection

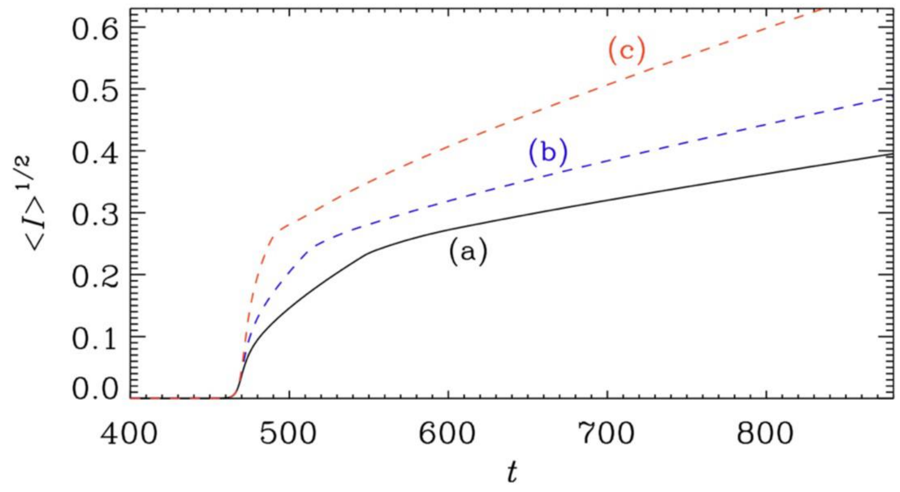
(c) Same as (b), but with larger recovery





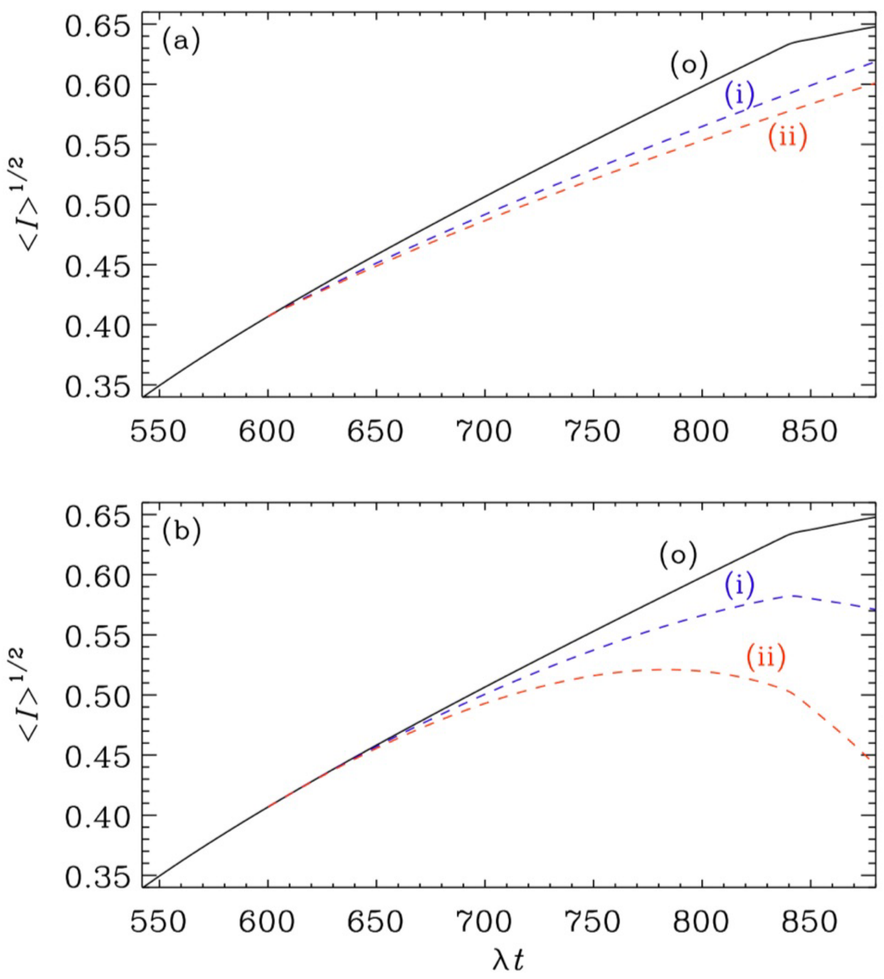
Increase of slope modeled by increasing κ





- Is an empirically determined quantity
- Comes out as huge → to be checked
- Spatially variable κ → intermediate decline?

Model decrease as decrease of reinfection rate (make γ' decrease smoothly)



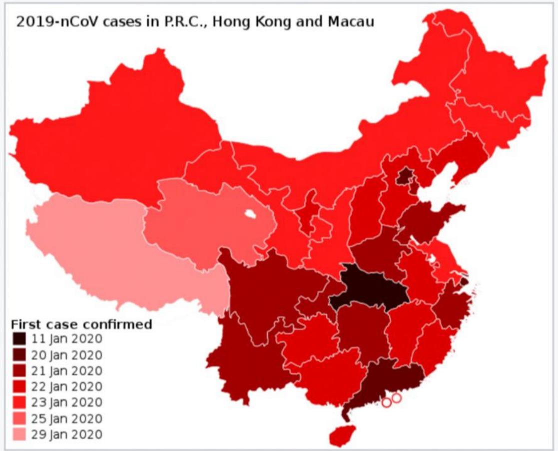
$$\gamma' = \gamma_0' \Theta(t;\, t_1, t_2)$$

$$\Theta(t) = \max\left\{0,\ 1 - \left[\frac{\max(0,\ t - t_1)}{t_2 - t_1}\right]^2\right\}^2$$

- Is an empirically determined quantity

Conclusions

- Piecewise quadratic growth observed
  - Explained by peripheral growth
  - Second phase explained several hotspots
  - Third phase explained by merging
- Useful descriptive model
- Implications?
  - Near-saturation locally
  - Subsequent spreading
- Can this be substantiated?



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