Introduction to Flat Space Holography

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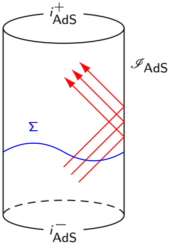
28th of June 2023

Motivations

- Holographic principle:

  Gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region.

- Explicit realization: AdS/CFT correspondence



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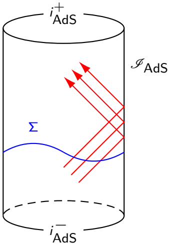
- Holographic principle:

  Gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region.

- Explicit realization: AdS/CFT correspondence

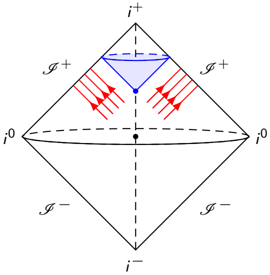
- Interesting observations:
  1. Asymptotic symmetries of AdS = conformal symmetries at the boundary
     $\Longrightarrow$ Powerful constraints implied for the dual theory.
  2. Closed gravitational system ("Gravity in a box")
     $\Longrightarrow$ Gravitational charges are conserved.



- How general is the holographic principle? Does it extend to asymptotically flat spacetimes?

Flat space holography program

(see *e.g.* [Susskind '99] [Polchinski '99] [Giddings '00] [de Boer-Solodukhin '03] [Arcioni-Dappiaggi '03] [Mann-Marolf '06] for early attempts).



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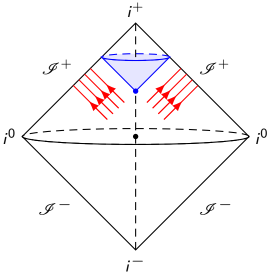
  1. Model for a large range of phenomena (from collider physics to astrophysics below the cosmological scale).
  2. Realistic model for the observation of gravitational waves.



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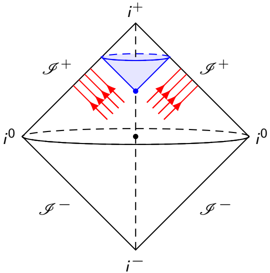
- Asymptotic symmetries: Bondi-van der Burg-Metzner-Sachs (BMS) group.
  [Bondi-van der Burg-Metzner '62] [Sachs '62]

  $\Longrightarrow$ Broadly studied in the literature
  (see *e.g.* [Newman-Unti '62] [Penrose '65] [Geroch '77] [Ashtekar-Streubel '81] [Barnich-Troessaert '10] [Strominger '13])



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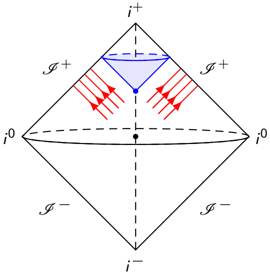
- Important obstructions to flat space holography:
  1. Null nature of $\mathscr{I}^+$ and $\mathscr{I}^-$.
  2. Radiation leaking through the conformal boundary.
     $\Longrightarrow$ Open gravitational system.
     $\Longrightarrow$ The BMS charges are not conserved.



Holographic nature of null infinity

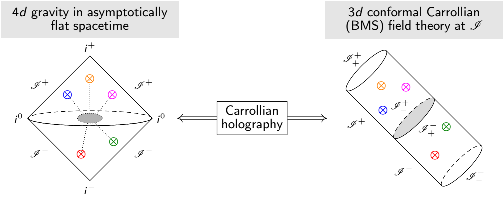
How to construct the holographic dual of asymptotically flat spacetime? (bottom-up approach)

- *Two distinct but complementary visions of* $\mathscr{I}^+$:

Picture 1: Carrollian holography	Picture 2: Celestial holography
$\mathscr{I}^+$ is seen as a boundary along which there is an evolution with respect to u	$\mathscr{I}^+$ is seen as a portion of Cauchy hypersurface pushed to infinity
Describe the dynamics of the system	Describe the state of the system
Flux-balance laws	Scattering problem between $\mathscr{I}^-$ and $\mathscr{I}^+$
Suggests a 4d bulk / 3d boundary *Carrollian holography*	Suggests a 4d bulk / 2d boundary *celestial holography*
Dual: 3d BMS field theory	Dual: 2d Celestial CFT

Carrollian holography

- *Carrollian holography*:



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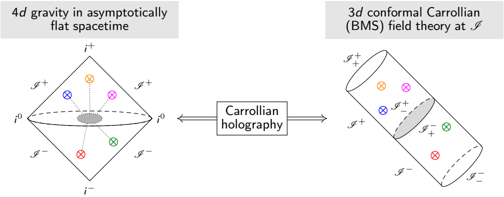

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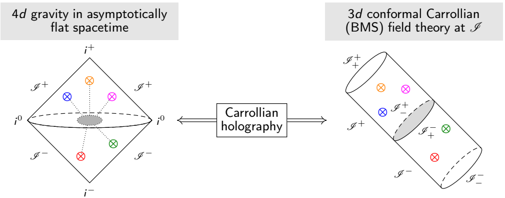

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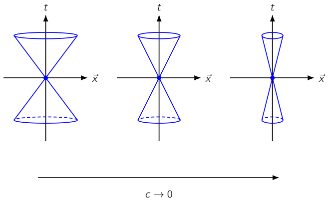
  $\implies$ Carrollian physics naturally induced at $\mathscr{I}$.



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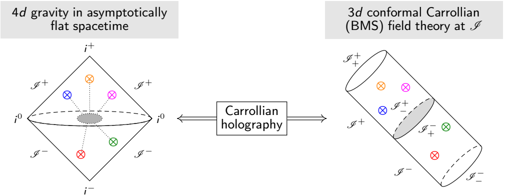

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  $\Longrightarrow$ Naturally arises from a flat limit procedure ($\Lambda \to 0$).

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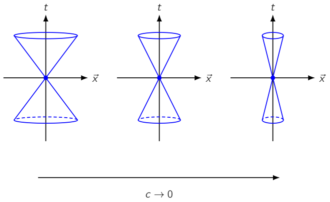

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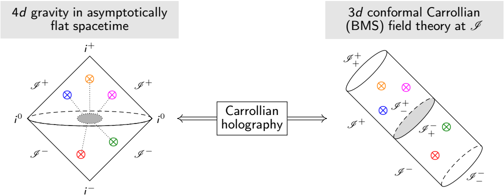

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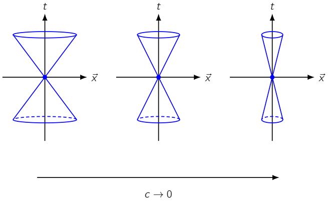
- **Pros:** Link with AdS/CFT, **Cons:** Few is known about quantum Carrollian CFTs.



Celestial holography

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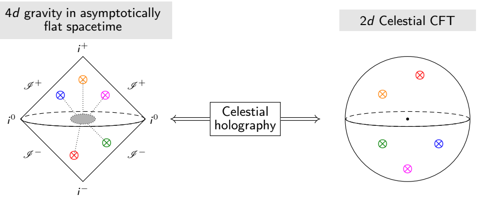

- $\mathcal{S}$-matrix elements in the bulk $\Longleftrightarrow$ Correlation functions in a 2d CFT

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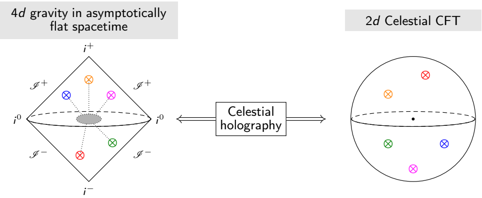

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- Massless scattering $\Longrightarrow$ Mellin transform: [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]

$$\langle \mathcal{O}_{\Delta_1,J_1}(z_1,\bar{z}_1)\dots\mathcal{O}_{\Delta_N,J_N}(z_N,\bar{z}_N)\rangle = \left(\prod_{i=1}^{N}\int_0^{+\infty} d\omega_i\, \omega_i^{\Delta_i-1}\right)\mathcal{A}(\{\omega_i\},\{z_i\},\{\bar{z}_i\})$$

  where the CCFT operators $\mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)$ are characterized by conformal dimension Δ_i and spin J_i.

- Conformal symmetries in CCFT induced by Lorentz transformations in the bulk: $\text{Conf}(S^2)\simeq$ Lorentz.

Celestial holography

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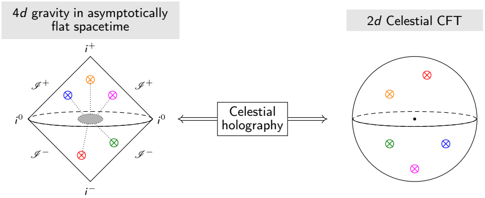

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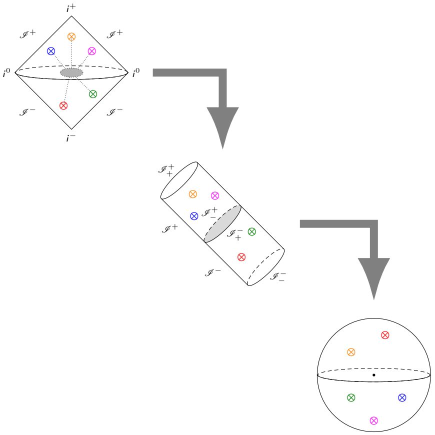
- **Pros:** 2d CFT techniques available, **Cons:** Link with AdS/CFT? Where is time?

Objectives

- **From the bulk...**
  Gravity in 4d asymptotically flat spacetime



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- **...to null infinity...**
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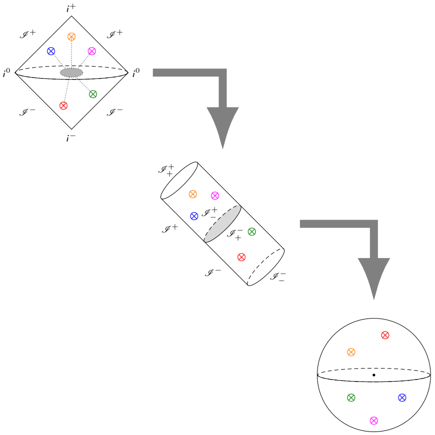

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- **...to null infinity...**
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- **...to the celestial sphere.**
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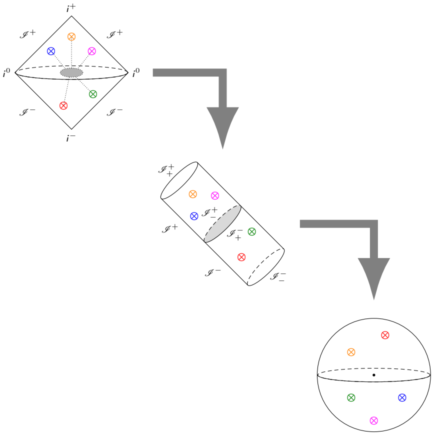

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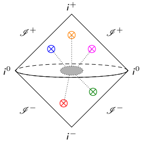

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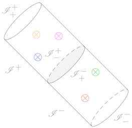
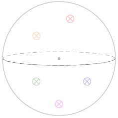


...to the celestial sphere.

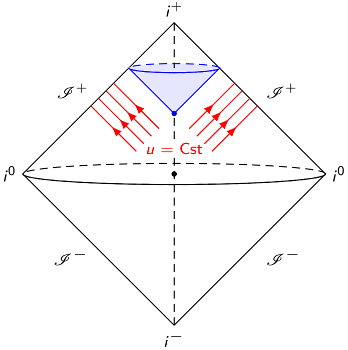


Solution space of 4d asymptotically flat spacetimes

- Asymptotically flat metric in Bondi coordinates to study $\mathscr{I}^+$: (u, r, x^A) where $x^A = (z, \bar{z})$ [Bondi-van der Burg-Metzner '62] [Sachs '62]:

$$
\begin{aligned}
ds^2 = & \left(\frac{2M}{r} + \mathcal{O}(r^{-2})\right) du^2 - 2\left(1 + \mathcal{O}(r^{-2})\right) dudr \\
& + \left(r^2 \mathring{q}_{AB} + r\, C_{AB} + \mathcal{O}(r^0)\right) dx^A dx^B \\
& + \left(\frac{1}{2}\partial_B C^B_A + \frac{2}{3r}(N_A + \frac{1}{4} C^B_A \partial_C C^C_B) + \mathcal{O}(r^{-2})\right) dudx^A .
\end{aligned}
$$

- Flat boundary metric: $\mathring{q}_{AB} dx^A dx^B = 2dzd\bar{z}$.
- Minkowski metric: $ds^2_{\text{Mink}} = -2dudr + 2r^2 dzd\bar{z}$.



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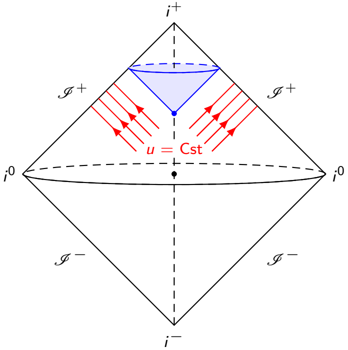

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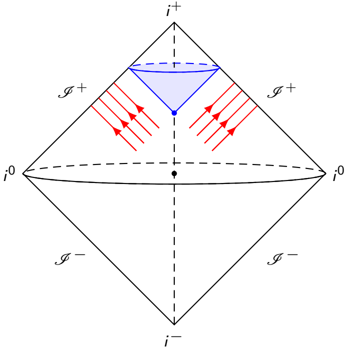
- Minkowski metric: $ds^2_{\text{Mink}} = -2dudr + 2r^2 dzd\bar{z}$.
- Subleading corrections in r with respect to Minkowski metric are obtained by solving the Einstein equations. They involve functions of (u, x^A):
  1. C_{AB} : asymptotic shear,
  2. $N_{AB} = \partial_u C_{AB}$: Bondi news (outgoing radiation),
  3. M : mass aspect,
  4. N_A : angular momentum aspect.
- Similar analysis at $\mathscr{I}^-$ in advanced Bondi coordinates (v, r, x^A).

- *Time evolution/constraint equations* on the mass and angular momentum aspects

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} \partial_A \partial_B N^{AB},$$

$$\partial_u N_A = \partial_A M + \frac{1}{16} \partial_A (N_{BC} C^{BC}) - \frac{1}{4} N^{BC} \partial_A C_{BC} - \frac{1}{4} \partial_B (C^{BC} N_{AC} - N^{BC} C_{AC}) - \frac{1}{4} \partial_B \partial^B \partial^C C_{AC} + \frac{1}{4} \partial_B \partial_A \partial_C C^{BC},$$

with $N_{AB} = \partial_u C_{AB}$ the Bondi news tensor.



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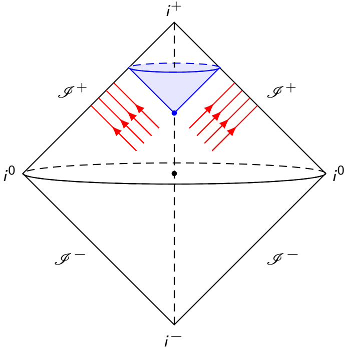
- *Bondi mass loss formula* [Trautman '58] [Bondi-van der Burg-Metzner '62]:

$$\partial_u \left[\int_{S^2_\infty} d^2 z\, M \right] = -\frac{1}{8} \int_{S^2_\infty} d^2 z\, N_{AB} N^{AB} \leq 0.$$

$\Longrightarrow$ The mass decreases in time due to the emission of gravitational waves.

$\Longrightarrow$ The analysis at $\mathscr{I}^+$ provides some information on the dynamics of the system.



Asymptotic symmetries

- Asymptotic symmetries = diffeomorphisms preserving Bondi gauge & asymptotic flatness, with non-trivial action at $\mathscr{I}^+$.

$$\lim_{r\to+\infty}(g_{\mu\nu}dx^\mu dx^\nu) = ds^2_{\text{Mink}}$$

- Restriction at $\mathscr{I}^+$ of the asymptotic symmetries:

$$\xi\Big|_{\mathscr{I}^+} = \bar{\xi}(\mathcal{Y},\mathcal{Y},\bar{\mathcal{Y}}) = \left[\mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

where

1. $\mathcal{T} = \mathcal{T}(z,\bar{z})$ is the supertranslation parameter;
2. $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar{z})$ are the superrotation parameters satisfying the conformal Killing equation.

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- Commutation relations:

$$[\bar{\xi}(\mathcal{T}_1,\mathcal{Y}_1,\bar{\mathcal{Y}}_1),\bar{\xi}(\mathcal{T}_2,\mathcal{Y}_2,\bar{\mathcal{Y}}_2)] = \bar{\xi}(\mathcal{T}_{12},\mathcal{Y}_{12},\bar{\mathcal{Y}}_{12}),$$

  with

$$\mathcal{T}_{12} = \mathcal{Y}_1\partial\mathcal{T}_2 - \frac{1}{2}\partial\mathcal{Y}_1\mathcal{T}_2 - (1\leftrightarrow 2) + \text{c.c.},\quad \mathcal{Y}_{12} = \mathcal{Y}_1\partial\mathcal{Y}_2 - (1\leftrightarrow 2),\quad \bar{\mathcal{Y}}_{12} = \bar{\mathcal{Y}}_1\bar{\partial}\bar{\mathcal{Y}}_2 - (1\leftrightarrow 2)$$

  where c.c. stands for complex conjugate terms $\Longrightarrow \mathfrak{bms}_4 \simeq \text{Lorentz} \loplus \mathfrak{supertranslations}$.

  [Bondi-van der Burg-Metzner '62] [Sachs '62]

  $\Longrightarrow$ Infinite-dimensional enhancement of Poincaré with "supertranslations".

- Extended BMS: $\boxed{\mathfrak{bms}_4^{\text{ext}} = (\text{Witt} \oplus \text{Witt}) \loplus \mathfrak{supertranslations}^*}$ [Barnich-Troessaert '10]

BMS surface charges

- At a cut $\mathcal{S}_u \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct "surface charges" associated with BMS symmetries using covariant phase space methods [Wald-Zoupas '99] [Barnich-Troessaert '10].

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- *BMS charges:*

  [Compère-Fiorucci-Ruzziconi '18] [Campiglia-Peraza '20] [Donnay-Ruzziconi '21] [Freidel-Pranzetti '21]

$$\bar{H}_\xi[g] = \frac{1}{8\pi G}\int_{\mathcal{S}_u} d^2 z\,[2\mathcal{T}\mathscr{M} + \mathcal{Y}\bar{\mathscr{N}} + \bar{\mathcal{Y}}\mathscr{N}]$$

$$\mathscr{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}}), \qquad \mathscr{N} = N_{\bar{z}} - u\bar{\partial}\mathscr{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz})\Big]$$
$$+\frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big].$$

- Remark: $\mathscr{M} = -\text{Re}\Psi_2^0$, $\mathscr{N} = -\Psi_1^0 + u\eth\Psi_2^0$ [Newman-Penrose '62] [Newman-Unti '62] [see Cynthia's talk].
- Consistency with the action of symmetries: (i) conserved when $N_{AB} = 0$, (ii) generate canonically the transformations on the radiative phase space, (iii) form a representation of BMS algebra at $\mathscr{I}^+_\pm$.

BMS surface charges

- At a cut $\mathcal{S}_u \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct "surface charges" associated with BMS symmetries using covariant phase space methods [Wald-Zoupas '99] [Barnich-Troessaert '10].
- *BMS charges:*

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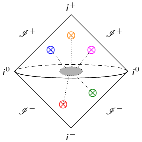
- Consistency with the action of symmetries: (i) conserved when $N_{AB} = 0$, (ii) generate canonically the transformations on the radiative phase space, (iii) form a representation of BMS algebra at $\mathscr{I}^+_\pm$.
- BMS flux-balance laws (*e.g.* Bondi mass loss formula):

$$\boxed{\frac{d}{du}\bar{H}_\xi[g] = \mathcal{F}_\xi[g] \neq 0\,, \quad \mathcal{F}_\xi[g]\Big|_{N_{AB}=0} = 0.}$$

$\Longrightarrow$ Important role in gravitational wave physics.

How to describe the BMS flux-blance laws from a Carrollian holography perspective?

From the bulk...



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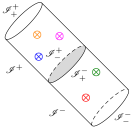
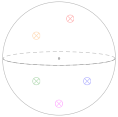


...to the celestial sphere.



Conformal Carroll $\simeq$ BMS

- Carrollian structure on $\mathscr{I}^+$ with coordinates $x^a = (u, z, \bar{z})$ [Geroch '77]:

$$(q_{ab}, n^c) \quad \text{with} \quad q_{ab} n^b = 0\,.$$

- Consistently with the Bondi metric, $q_{ab} dx^a dx^b = 0 du^2 + 2 dz d\bar{z}$ and $n^a \partial_a = \partial_u$.
- Conformal Carrollian symmetries are generated by vector fields $\bar{\xi} = \bar{\xi}^a \partial_a$ on $\mathscr{I}^+$ satisfying

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}\,, \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a\,,$$

  with $\alpha = \frac{1}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}})$.
- Solution:

$$\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right] \partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}\,.$$

  $\Longrightarrow$ Coincides with the restriction on $\mathscr{I}^+$ of the bulk BMS asymptotic Killing vectors.
  $\Longrightarrow$ The standard Lie bracket of these vector fields reproduces the $\mathfrak{bms}_4$ algebra.

- Isomorphism: $\boxed{\mathfrak{bms}_4 \simeq \text{Conformal Carroll algebra.}}$ [Duval-Gibbons-Horvathy '14]

Carrollian CFT

- Consider a 3d Carrollian CFT (theory exhibiting conformal Carroll/BMS symmetries).
$\Longrightarrow$ Putative holographic dual in Carrollian holography.
Coordinates: $x^a = (u, x^A)$, $x^A = (z, \bar{z})$. Carrollian structure: $ds^2 = 0\, du^2 + 2dzd\bar{z}$ and $n^a\partial_a = \partial_u$.
- Noether currents:

$$j^a_{\bar{\xi}} = \mathcal{C}^a{}_b \bar{\xi}^b, \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{C}^u{}_u & \mathcal{C}^u{}_B \\ \mathcal{C}^A{}_u & \mathcal{C}^A{}_B \end{bmatrix}.$$

$\Longrightarrow$ $\mathcal{C}^a{}_b$: Carrollian stress tensor; $\mathcal{C}^u{}_u$, $\mathcal{C}^u{}_B$, $\mathcal{C}^A{}_u$, $\mathcal{C}^A{}_B$: Carrollian momenta.
[Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [de Boer, Hartong, Obers, Sybesma, Vandoren '18] [Ciambelli-Marteau '18] [Donnay-Marteau '19] [Chandrasekaran-Flanagan-Shehzad-Speranza '21] [Freidel-Pranzetti '21] [Donnay-Herfray-Fiorucci-Ruzziconi '22]

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- Ward identities: $\boxed{\partial_a \langle j^a_{\bar{\xi}}(x)\rangle = 0}$

$$\begin{array}{rcccl} \textit{Carrollian translations} & : & \partial_b & \Rightarrow & \partial_a\langle \mathcal{C}^a{}_b\rangle = 0\,, \\ \textit{Carrollian rotation} & : & -z\partial + \bar{z}\bar{\partial} & \Rightarrow & \langle \mathcal{C}^z{}_z\rangle - \langle \mathcal{C}^{\bar{z}}{}_{\bar{z}}\rangle = 0\,, \\ \textit{Carrollian boosts} & : & \bar{x}^A\partial_u & \Rightarrow & \langle \mathcal{C}^A{}_u\rangle = 0\,, \\ \textit{Carrollian dilatation} & : & x^a\partial_a & \Rightarrow & \langle \mathcal{C}^a{}_a\rangle = 0\,, \end{array}$$

- Global conformal Carrollian symmetries ($\simeq$ 4d Poincaré symmetries) are enough to constrain $\mathcal{C}^a{}_b$.
  $\Longrightarrow$ No further constraints coming from supertranslations and superrotations.

Holographic correspondence

- Correspondence between boundary Carrollian momenta and bulk gravitational data at $\mathscr{I}^+$ [Donnay-Herfray-Fiorucci-Ruzziconi '22]:

$$\langle \mathcal{C}^u{}_u \rangle = \frac{\mathscr{M}}{4\pi G}\,, \quad \langle \mathcal{C}^A{}_B \rangle + \frac{1}{2}\delta^A{}_B \langle \mathcal{C}^u{}_u \rangle = 0\,,$$
$$\langle \mathcal{C}^u{}_A \rangle = \frac{1}{8\pi G}\left(\mathscr{N}_A + u\,\partial_A \mathscr{M}\right), \quad \mathscr{N}_A = (\bar{\mathscr{N}}, \mathscr{N})\,.$$

- Fixed by requiring compatibility between boundary Noether currents and bulk gravitational charges:

$$\bar{H}_\xi[g] = \int_{\mathcal{S}_u} d^2z\, \langle \mathcal{C}^u{}_a \rangle \bar{\xi}^a \stackrel{!}{=} \frac{1}{8\pi G}\int_{\mathcal{S}_u} d^2z\, [2\mathcal{T}\mathscr{M} + \mathcal{Y}\bar{\mathscr{N}} + \bar{\mathcal{Y}}\mathscr{N}]$$

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- Similar to the AdS/CFT dictionary where the holographic stress-energy tensor of the CFT is identified with some subleading order in the expansion of the bulk metric. [Balasubramanian-Kraus '99] [de Haro-Solodukhin-Skenderis '01]

$$ds^2 = \frac{\ell^2}{\rho^2}d\rho^2 + \frac{1}{\rho^2}\left(g^{(0)}_{ab} + \rho^2 g^{(2)}_{ab} + \rho^3 g^{(3)}_{ab} + \mathcal{O}(\rho^4)\right)dx^a dx^b, \qquad \langle T_{ab} \rangle = \frac{3}{16\pi G\ell} g^{(3)}_{ab}$$

Sourced Carrollian CFT

- **Problem:** assuming $\partial_a j^a_\xi = 0$, the BMS charges are conserved!
  $\Longrightarrow$ This is in contradiction with the flux-balance laws!

$$\frac{d}{du}\bar{H}_\xi[g] = \mathcal{F}_\xi[g] \neq 0\,, \quad \mathcal{F}_\xi[g]\Big|_{N_{AB}=0} = 0.$$

- **Solution:** couple the Carrollian CFT with external sources $\sigma(x)$ at the boundary!
  $\Longrightarrow$ Generically breaks the Noetherian symmetries.
  $\Longrightarrow$ The Carrollian currents are no longer conserved:

$$\partial_a j^a_K(x) = F_K(x), \qquad F_K(x)\Big|_{\sigma=0} = 0\,.$$

- External sources identified with the Bondi news: $\sigma_{AB} \sim N_{AB}$.
  $\Longrightarrow$ The sourced Ward identities reproduce the BMS flux-balance laws [Donnay-Fiorucci-Herfray-Ruzziconi '22].

In presence of radiation, the dual theory is a sourced Carrollian CFT

Comparison between AdS and flat

- Comparison between AdS/CFT and Carrollian holography:

AdS	Flat
Timelike boundary	Null boundary $\mathscr{I}^{\pm}$
Fefferman-Graham gauge	Bondi gauge
Conformal symmetries $\bar{\zeta}^a$	BMS symmetries $\bar{\xi}^a$
Relativistic stress tensor $T^a{}_b$	Carrollian stress tensor $\mathcal{C}^a{}_b$
Conserved charges: $\bar{H}_\xi = \int_{S^2} \langle T^t{}_a \rangle \bar{\zeta}^a, \quad \frac{d}{dt}\bar{H}_\xi = 0$	Flux-balance laws: $\bar{H}_\xi = \int_{S^2} \langle \mathcal{C}^u{}_a \rangle \bar{\xi}^a, \quad \frac{d}{du}\bar{H}_\xi = \mathcal{F}_\xi$
Closed system	Open system
Dual: 3d CFT	Dual: 3d sourced Carrollian CFT

- **Remark:** for non-radiative asymptotically flat spacetime, no external source is needed.
$\Longrightarrow$ Similar to AdS/CFT through $\Lambda \to 0$.
$\Longrightarrow$ Explains the success in 3d gravity
(entropy matching, entanglement entropy, effective action, correlation functions, Carroll anomaly ...).
[Barnich-Gomberoff-Gonzalez '12] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '13] [Bagchi-Fareghbal '12] [Detournay-Grumiller-Scholler-Simon '14] [Bagchi-Basu-Grumiller-Riegler '15] [Hartong '16] [Bagchi-Grumiller-Merbis '16] [Campoleoni-Ciambelli-Delfante-Marteau-Petropoulos-Ruzziconi '22]

Massless scattering in flat space

- Focus on the source sector (relevant for massless scattering in flat spacetime)

Can we encode the bulk $\mathcal{S}$-matrix into boundary Carrollian CFT correlators?

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Can we encode the bulk $\mathcal{S}$-matrix into boundary Carrollian CFT correlators?

- Consider a spin-s ($s = 0, 1, 2, \dots$) massless field in flat space:

$$\phi_I^{(s)}(X) = \frac{K_{(s)}}{16\pi^3} \sum_{\alpha=\pm} \int \omega \, d\omega \, d^2 w \left[a_\alpha^{(s)}(\omega, w, \bar{w}) \varepsilon_I^{*\alpha}(w, \bar{w}) \, e^{i\omega q^\mu X_\mu} + a_\alpha^{(s)}(\omega, w, \bar{w})^\dagger \varepsilon_I^\alpha(w, \bar{w}) \, e^{-i\omega q^\mu X_\mu} \right]$$

with $I = (\mu_1 \mu_2 \dots \mu_s)$ and

$$p^\mu(\omega, w, \bar{w}) = \omega \, q^\mu(w, \bar{w}), \quad q^\mu(w, \bar{w}) = \frac{1}{\sqrt{2}} \Big(1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w} \Big),$$

$$\varepsilon^\pm_{\mu_1 \dots \mu_s}(\vec{q}) = \varepsilon^\pm_{\mu_1}(\vec{q}) \varepsilon^\pm_{\mu_2}(\vec{q}) \dots \varepsilon^\pm_{\mu_s}(\vec{q}), \qquad \varepsilon^+_\mu(\vec{q}) = \partial_w q_\mu = \frac{1}{\sqrt{2}} \left(-\bar{w}, 1, -i, -\bar{w} \right), \qquad \varepsilon^-_\mu(\vec{q}) = [\varepsilon^+_\mu(\vec{q})]^*.$$

Amplitudes in position space

- Taking $r \to \infty$ (stationary phase approximation), we find the boundary values:

$$\bar{\phi}^{(s)}_{z\dots z}(u, z, \bar{z})^{\text{out}} = \lim_{r\to+\infty} \left(r^{1-s} \phi^{(s)\,\text{out}}_{z\dots z}(u, r, z, \bar{z}) \right)$$
$$= -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a^{(s)\,\text{out}}_{+}(\omega, z, \bar{z}) e^{-i\omega u} - a^{(s)\,\text{out}}_{-}(\omega, z, \bar{z})^\dagger e^{i\omega u} \right] \quad \text{at } \mathscr{I}^+,$$
$$\bar{\phi}^{(s)}_{z\dots z}(v, z, \bar{z})^{\text{in}} = -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a^{(s)\,\text{in}}_{+}(\omega, z, \bar{z}) e^{-i\omega v} - a^{(s)\,\text{in}}_{-}(\omega, z, \bar{z})^\dagger e^{i\omega v} \right] \quad \text{at } \mathscr{I}^-.$$

$\Longrightarrow$ Insertions for a massless scattering between $\mathscr{I}^-$ (in) and $\mathscr{I}^+$ (out).

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$\Longrightarrow$ Insertions for a massless scattering between $\mathscr{I}^-$ (in) and $\mathscr{I}^+$ (out).

- Scattering amplitudes in position space:

$$\begin{aligned}&\langle 0|\bar{\phi}^{(s)}_{I_1}(x_1)^{\text{out}}\dots\bar{\phi}^{(s)}_{I_n}(x_n)^{\text{out}}\bar{\phi}^{(s)}_{I_{n+1}}(x_{n+1})^{\text{in}\,\dagger}\dots\bar{\phi}^{(s)}_{I_N}(x_N)^{\text{in}\,\dagger}|0\rangle\\ &= \frac{1}{(2\pi)^N}\prod_{k=1}^{n}\int_0^{+\infty} d\omega_k\, e^{-i\omega_k u_k}\prod_{\ell=n+1}^{N}\int_0^{+\infty} d\omega_\ell\, e^{i\omega_\ell v_\ell}\mathcal{A}_N(\{\omega_i\},\{z_i\},\{\bar{z}_i\}).\end{aligned}$$

where $x_i^{\text{out/in}} = (u_i/v_i, z_i, \bar{z}_i)$.

Holographic identification

- Conformal Carrollian primary field:

$$\delta_{\bar{\xi}} \sigma_{(k,\bar{k})} = \left[\left(\mathcal{T} + \frac{u}{2} (\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}) \right) \partial_u + \mathcal{Y} \partial + \bar{\mathcal{Y}} \bar{\partial} + k \, \partial \mathcal{Y} + \bar{k} \, \bar{\partial} \bar{\mathcal{Y}} \right] \sigma_{(k,\bar{k})}, \qquad (k, \bar{k}): \text{ Carrollian weights.}$$

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$$\sigma^{\text{out}}_{(k,\bar{k})}(u, z, \bar{z}) = \bar{\phi}^{(s)\,\text{out}}_{l}(u, z, \bar{z}), \qquad \sigma^{\text{in}}_{(k,\bar{k})}(v, z, \bar{z}) = \bar{\phi}^{(s)\,\text{in}}_{l}(v, z, \bar{z})^\dagger$$

$\Longrightarrow$ This implies $k = \frac{1 \pm J}{2}$, $\bar{k} = \frac{1 \mp J}{2}$.

$\Longrightarrow$ For gravity ($s = 2$), $\sigma(u, z, \bar{z}) \equiv C_{zz}(u, z, \bar{z})$ ($J = 2$ and $(k, \bar{k}) = (\frac{3}{2}, -\frac{1}{2})$).

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- Carrollian correlators = scattering amplitudes in position space:

$$\boxed{\begin{gathered} \langle \sigma^{\text{out}}_{(k_1,\bar{k}_1)}(x_1) \dots \sigma^{\text{out}}_{(k_n,\bar{k}_n)}(x_n) \sigma^{\text{in}}_{(k_{n+1},\bar{k}_{n+1})}(x_{n+1}) \dots \sigma^{\text{in}}_{(k_N,\bar{k}_N)}(x_N) \rangle \\ = \\ \langle 0 | \bar{\phi}^{(s)}_{l_1}(x_1)^{\text{out}} \dots \bar{\phi}^{(s)}_{l_n}(x_n)^{\text{out}} \bar{\phi}^{(s)}_{l_{n+1}}(x_{n+1})^{\text{in}\,\dagger} \dots \bar{\phi}^{(s)}_{l_N}(x_N)^{\text{in}\,\dagger} | 0 \rangle . \end{gathered}}$$

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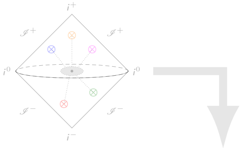
- Consistent with the Ward identities of the 3d Carrollian CFT.

$\Longrightarrow$ The low-point correlation functions are completely fixed by the conformal Carrollian symmetries.

$\Longrightarrow$ The source sector is controlled by an electric Carrollian CFT. [Bagchi-Banerjee-Basu-Dutta '22]

From the bulk...



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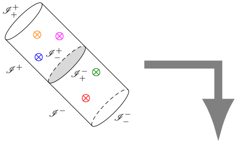
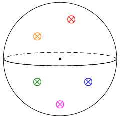
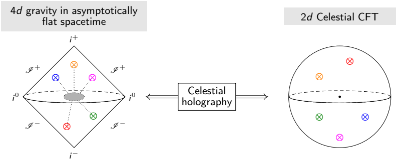


...to the celestial sphere.



Celestial holography



- Scattering of massless particles:
  [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]

$$\langle \mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{\Delta_N, J_N}(z_N, \bar{z}_N) \rangle$$

$$= \left(\prod_{i=1}^{N} \int_0^{+\infty} d\omega_i \, \omega_i^{\Delta_i - 1} \right) \mathcal{A}_N(\{\omega_i\}, \{z_i\}, \{\bar{z}_i\})$$

  where the CCFT operators $\mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i)$ are characterized by conformal dimension Δ_i and spin J_i.

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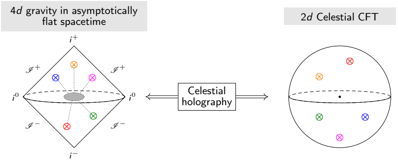

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- Change of basis: plane waves (energy eigenstates) $\Longrightarrow$ conformal primary wave functions (boost eigenstates).
- Transformation under BMS symmetries:

$$\delta_\xi \mathcal{O}^{\text{out/in}}_{(\Delta,J)}(z,\bar{z}) = \mp i\mathcal{T}(z,\bar{z})\mathcal{O}^{\text{out/in}}_{(\Delta+1,J)}(z,\bar{z}) + \left(\mathcal{Y}\partial_z + \bar{\mathcal{Y}}\bar{\partial} + h\,\partial\mathcal{Y} + \bar{h}\,\bar{\partial}\bar{\mathcal{Y}}\right)\mathcal{O}^{\text{out/in}}_{(\Delta,J)}(z,\bar{z})$$

  where $h = \frac{\Delta+J}{2}$ and $\bar{h} = \frac{\Delta-J}{2}$.
  $\Longrightarrow$ Makes the conformal properties of the scattering manifest.

Celestial encoding of soft theorems and collinear limits

- Soft (graviton) theorems for amplitudes ($\omega \to 0$):

$$\mathcal{A}_{N+1}(\omega q; p_1; \dots; p_N) = \left(\frac{1}{\omega} S^{(0)} + S^{(1)} \right) \mathcal{A}_N(p_1; \dots; p_N) + \mathcal{O}(\omega)$$

where $S^{(0)}$: leading soft factor [Weinberg '65], $S^{(1)}$: subleading soft factor [Cachazo-Strominger '14].

- Equivalent to BMS Ward identities for the scattering $[\bar{H}_{\xi}, \mathcal{S}] = 0$. [Strominger '13] [He-Lysov-Mitra-Strominger '14] [Kapec-Lysov-Pasterski-Strominger '14]
- After Mellin transform, become Ward identities in the celestial CFT: [Kapec-Mitra-Raclariu-Strominger '17] [Donnay-Puhm-Strominger '18]

$$\left\langle P(z,\bar{z}) \prod_{i=1}^{N} \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i) \right\rangle + \sum_{q=1}^{N} \frac{1}{z - z_q} \left\langle \dots \mathcal{O}_{\Delta_q + 1, J_q}(z_q, \bar{z}_q) \dots \right\rangle = 0 \quad \text{(leading soft theorem)} \quad P(z,\bar{z}) : \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$\left\langle T(z) \prod_{i=1}^{N} \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i) \right\rangle + \sum_{q=1}^{N} \left[\frac{\partial_q}{z - z_q} + \frac{h_q}{(z - z_q)^2} \right] \left\langle \prod_{i=1}^{N} \mathcal{O}_{\Delta_i, J_i}(z_i, \bar{z}_i) \right\rangle = 0 \quad \text{(subleading soft theorem)} \quad T(z) : (2, 0)$$

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- Collinear limit of amplitudes ($p_i \approx \alpha P$, $p_j \approx (1 - \alpha) P$): [Bern-Del Duca-Schmidt '98] [Bern-Dixon-Perelstein-Rozowsky '99]

$$\mathcal{A}_{N+1}(\dots p_i, p_j \dots) \approx \text{Split}(p_i, p_j) \mathcal{A}_N(\dots P \dots)$$

where, for instance, $\text{Split}^{+2}_{+2,+2}(p_i, p_j) = -\frac{\kappa}{2\alpha(1-\alpha)} \frac{[ij]}{\langle ij \rangle}$.

- Gives the OPEs in the celestial CFT: [Pate-Raclariu-Strominger-Yuan '19] [Fotopoulos-Stieberger-Taylor-Zhu '19]

$$\mathcal{O}_{\Delta_i, +2}(z_i, \bar{z}_i) \mathcal{O}_{\Delta_j, +2}(z_j, \bar{z}_j) \sim -\frac{\kappa}{2} \frac{\bar{z}_{ij}}{z_{ij}} B(\Delta_i - 1, \Delta_j - 1) \mathcal{O}_{\Delta_i + \Delta_j, +2}(z_j, \bar{z}_j) \qquad (\kappa = \sqrt{32\pi G})$$

A few facts about celestial CFT...

- Not only BMS symmetries, but a far bigger symmetry algebra $w_{1+\infty}$.

  $\Longrightarrow$ Extracted from the OPE structure in the celestial CFT [see Walker's talk].

  $\Longrightarrow$ Might lead to important constraints on the scattering in flat space.

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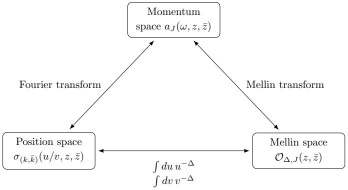
- The central charge vanishes. [Fotopoulos-Stieberger-Taylor-Zhu '19] [Donnay-Ruzziconi '21] [Banerjee-Pasterski '22]
  $\Longrightarrow$ The celestial CFT is a logarithmic CFT. [Grumiller-Fiorucci-Ruzziconi '23]

Relation between Carrollian and celestial CFT

How to relate Carrollian and celestial holography?



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- Relation between (quasi) conformal Carrollian primary operators and CCFT operators [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\left.\begin{aligned} \mathcal{O}^{out}_{\Delta_i,J_i}(z_i,\bar{z}_i) &= i^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} du_i\, u_i^{-\Delta_i}\, \sigma^{out}_{(k_i,\bar{k}_i)}(u_i,z_i,\bar{z}_i)\,, \\ \mathcal{O}^{in}_{\Delta_j,J_j}(z_j,\bar{z}_j) &= i^{\Delta_j}\Gamma[\Delta_j]\int_{-\infty}^{+\infty} dv_j\, v_j^{-\Delta_j}\, \sigma^{in}_{(k_j,\bar{k}_j)}(v_j,z_j,\bar{z}_j)\,. \end{aligned}\right\} \text{(Fourier + Mellin transforms)}$$

$\Longrightarrow$ Exchange between time and conformal dimension.
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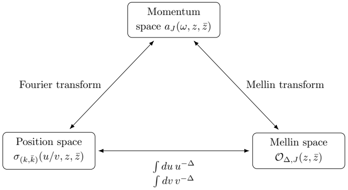

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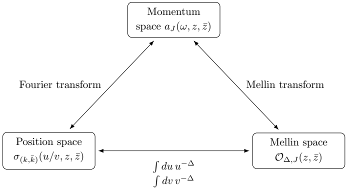

- Correlation functions ($N = m + n$):

$$\left\langle \prod_{i=1}^{m} \mathcal{O}^{out}_{\Delta_i,J_i}(z_i,\bar{z}_i) \prod_{j=1}^{n} \mathcal{O}^{in}_{\Delta_j,J_j}(z_j,\bar{z}_j) \right\rangle$$

$$= \left(\prod_{i=1}^{m} i^{\Delta_i}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} du_i\, u_i^{-\Delta_i}\right)\left(\prod_{j=1}^{n} i^{\Delta_j}\Gamma[\Delta_j]\int_{-\infty}^{+\infty} dv_j\, v_j^{-\Delta_j}\right)\langle \underbrace{\sigma^{out}_{(k_1,\bar{k}_1)}(x_1)\dots\sigma^{out}_{(k_m,\bar{k}_m)}(x_m)}_{\text{Insertions at } \mathscr{I}^+}\, \underbrace{\sigma^{in}_{(k_1,\bar{k}_1)}(x_1)\dots\sigma^{in}_{(k_n,\bar{k}_n)}(x_n)}_{\text{Insertions at } \mathscr{I}^-}\rangle\,.$$

- Ward identities in the sourced Carrollian CFT (encoding the BMS flux balance laws) are mapped on the celestial CFT Ward identities (encoding the leading and subleading soft theorems). [Donnay-Fiorucci-Herfray-Ruzziconi '22]

Summary and Perspectives

- Flat space holography different from AdS holography, partially due to the null nature of the boundary.
- Two complementary approaches to flat space holography:

$$\boxed{\text{Carrollian holography}} \Longleftrightarrow \boxed{\text{Celestial holography}}$$

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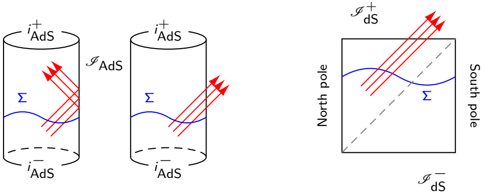
- What can we learn about gravity and scattering amplitudes in flat spacetime?
- Relation with (A)dS/CFT correspondence?
  - $\Longrightarrow$ In the bulk, the flat limit works provided one starts with leaky boundary conditions. [Compère-Fiorucci-Ruzziconi '19] [Fiorucci-Ruzziconi '21]
  - $\Longrightarrow$ Obtain the Carrollian CFT in the ultra-relativistic limit of the CFT?



Thank you!

INSTITUTE FOR THEORETICAL PHYSICS

AUSTRIAN SCIENCE FUND

Antipodal gluing

- Where does the Carrollian CFT live?
- Glue $\mathscr{I}^+$ and $\mathscr{I}^-$ by identifying antipodally $\mathscr{I}^+_-$ with $\mathscr{I}^-_+$:

$$\boxed{\hat{\mathscr{I}} = \mathscr{I}^- \sqcup \mathscr{I}^+}$$

[Donnay-Fiorucci-Herfray-Ruzziconi '22]

- Intrinsically, the gluing surface Σ_0 is distinguished by a vanishing n^a.
- Carrollian momenta and symmetry generators smoothly defined on $\hat{\mathscr{I}}$.

$\Longrightarrow$ Geometric implementation of the antipodal matching.
[Strominger '13] [Troessaert '17] [Henneaux-Troessaert '18] [Prabhu '19] [Capone-Nguyen-Parisini '22]