

Color-dual Constraints on Gravitational EFT

Nicolas Pavao

June 27, 2023

2203.03592, 2211.04441, 2307.XXXX

[with JJ Carrasco & Matt Lewandowski]



Northwestern
University

Amplifying Gravity at All Scales



NORDITA
The Nordic Institute for Theoretical Physics

Insights of this Talk

Insights of this Talk

Intro to Amplitude-level **structure**

- Review of **color-kinematics** duality

$$\begin{array}{c} 2 \\ & \diagdown \\ & \text{---} \\ & \diagup \\ 1 & \diagdown \\ & \text{---} \\ & \diagup \\ 4 & 3 \end{array} = \begin{array}{c} 2 \\ & \diagdown \\ & \text{---} \\ & \diagup \\ 1 & \diagdown \\ & \text{---} \\ & \diagup \\ 4 & 3 \end{array} + \begin{array}{c} 4 \\ & \diagdown \\ & \text{---} \\ & \diagup \\ 1 & \diagdown \\ & \text{---} \\ & \diagup \\ 2 & 3 \end{array}$$

Insights of this Talk

Intro to Amplitude-level **structure**

- Review of **color-kinematics** duality

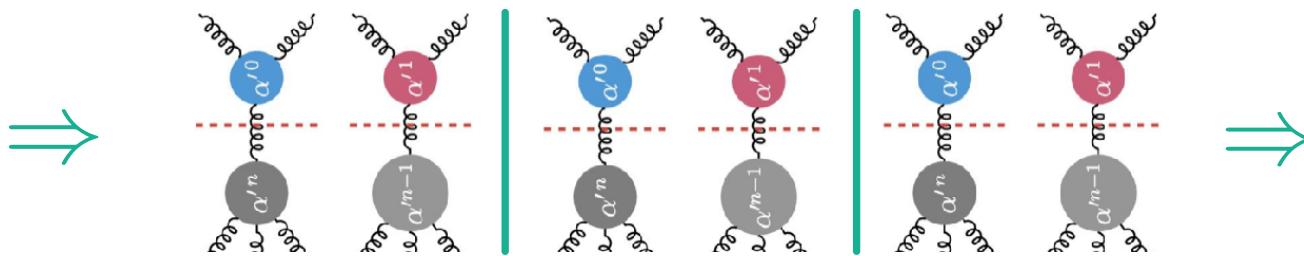
$$\begin{array}{c} 2 \\ & \diagdown \\ & \diagup \\ 1 & & 4 \\ & \diagup \\ 3 \end{array} = \begin{array}{c} 2 \\ & \diagdown \\ & \diagup \\ 1 & & 4 \\ & \diagup \\ 3 \end{array} + \begin{array}{c} 4 \\ & \diagdown \\ & \diagup \\ 1 & & 3 \\ & \diagup \\ 2 \end{array}$$

Color-dual **constraints** on EFT

- Bootstrap for gauge/gravity EFT operators
- Emergence of massive UV modes

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

Carrasco, NHP
2307.xxxxx



Insights of this Talk

Intro to Amplitude-level **structure**

- Review of color-kinematics duality

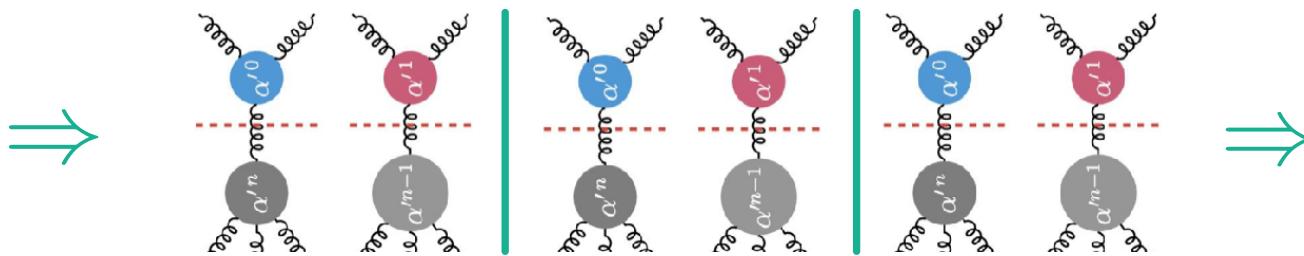
$$\begin{array}{c} 2 \\ & \diagdown \\ & \diagup \\ 1 & & 4 \\ & \diagup \\ 3 \end{array} = \begin{array}{c} 2 \\ & \diagdown \\ & \diagup \\ 1 & & 4 \\ & \diagup \\ 3 \end{array} + \begin{array}{c} 4 \\ & \diagdown \\ & \diagup \\ 1 & & 3 \\ & \diagup \\ 2 \end{array}$$

Color-dual **constraints** on EFT

- Bootstrap for gauge/gravity EFT operators
- Emergence of massive UV modes

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

Carrasco, NHP
2307.xxxxx



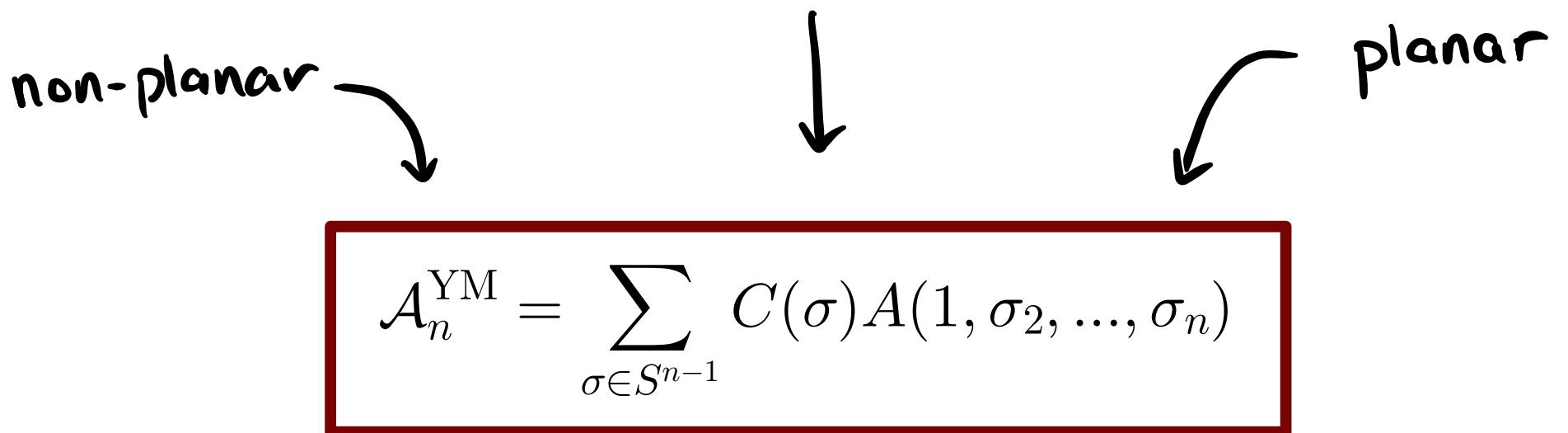
Applications to Gravitational EFT & Future directions

Amplitude-level structure

$$\mathcal{L}_{\text{int}}^{\text{YM}} = f^{abc} \begin{array}{c} \text{a} \\ \text{---} \\ \text{b} \\ \text{---} \\ \text{c} \end{array} + f^{abe} f^{ecd} \begin{array}{c} \text{b} \\ \text{---} \\ \text{a} \\ \text{---} \\ \text{d} \\ \text{---} \\ \text{c} \end{array}$$

$$f^{abc} = \text{tr} [T^a T^b T^c] - \text{tr} [T^a T^c T^b]$$

$$T_{ij}^e T_{kl}^e = \delta_{il} \delta_{jk} + \mathcal{O}(1/N_c)$$



Amplitude-level structure

$$\mathcal{A}_n^{\text{YM}} = \sum_{\sigma \in S^{n-1}} C(\sigma) A(1, \sigma_2, \dots, \sigma_n)$$

$$A(1, \alpha, n, \beta) = (-1)^{|\alpha|} \sum_{\sigma \in \alpha \sqcup \beta^T} A(1, \sigma, n)$$

Kleiss-Kluijf (1989)

$$\sum_{i=2}^{n-1} k_1 \cdot (k_2 + \cdots + k_i) A(2, \dots, i, 1, \dots, n) = 0$$

Bern- Carrasco-Johansson (2008)

Amplitude-level structure

$$\mathcal{L}_{\text{int}}^{\text{YM}} = f^{abc} \text{ (wavy line diagram)} + f^{abe} f^{ecd} \text{ (wavy line diagram)}$$

$$\mathcal{A}_4^{\text{YM}} = \text{ (tree-level Feynman diagrams)} + \text{ (loop Feynman diagrams)}$$

↓

$$\mathcal{A}_4^{\text{YM}} = \text{ (red tree-level Feynman diagrams)} + \text{ (red loop Feynman diagrams)}$$

grows like φ^3 -theory, (2n-5)!!

$$\mathcal{A} = \int d\Omega_l \sum_g \frac{1}{S_g} \frac{c_g \tilde{n}_g}{d_g}$$

Amplitude-level structure

$$A(1, \alpha, n, \beta) = (-1)^{|\alpha|} \sum_{\sigma \in \alpha \boxplus \beta^T} A(1, \sigma, n)$$

Kleiss-Kluijf (1989)

$$A(1234) = \begin{array}{c} 2 \\ \diagup \quad \diagdown \\ 1 & & 3 \\ & \diagdown \quad \diagup \\ & 4 \end{array} + \begin{array}{c} 2 \\ \diagup \quad \diagdown \\ 1 & & 3 \\ & \diagup \quad \diagdown \\ & 4 \end{array}$$

$$A(1324) = \begin{array}{c} 3 \\ \diagup \quad \diagdown \\ 1 & & 2 \\ & \diagdown \quad \diagup \\ & 4 \end{array} + \begin{array}{c} 3 \\ \diagup \quad \diagdown \\ 1 & & 2 \\ & \diagup \quad \diagdown \\ & 4 \end{array}$$

$$A(1243) = \begin{array}{c} 2 \\ \diagup \quad \diagdown \\ 1 & & 4 \\ & \diagdown \quad \diagup \\ & 3 \end{array} + \begin{array}{c} 2 \\ \diagup \quad \diagdown \\ 1 & & 4 \\ & \diagup \quad \diagdown \\ & 3 \end{array}$$

Amplitude-level structure

$$A(1, \alpha, n, \beta) = (-1)^{|\alpha|} \sum_{\sigma \in \alpha \boxplus \beta^T} A(1, \sigma, n)$$

Kleiss-Kluijf (1989)

$$A(1234) = \begin{array}{c} 2 \\ & \diagdown \\ 1 & & 3 \\ & \diagup \\ & 4 \end{array} - \begin{array}{c} 2 \\ & \diagup \\ 1 & & 3 \\ & \diagdown \\ & 4 \end{array}$$

$$A(1324) = - \begin{array}{c} 3 \\ & \diagup \\ 1 & & 2 \\ & \diagdown \\ & 4 \end{array} + \begin{array}{c} 3 \\ & \diagup \\ 1 & & 2 \\ & \diagdown \\ & 4 \end{array}$$

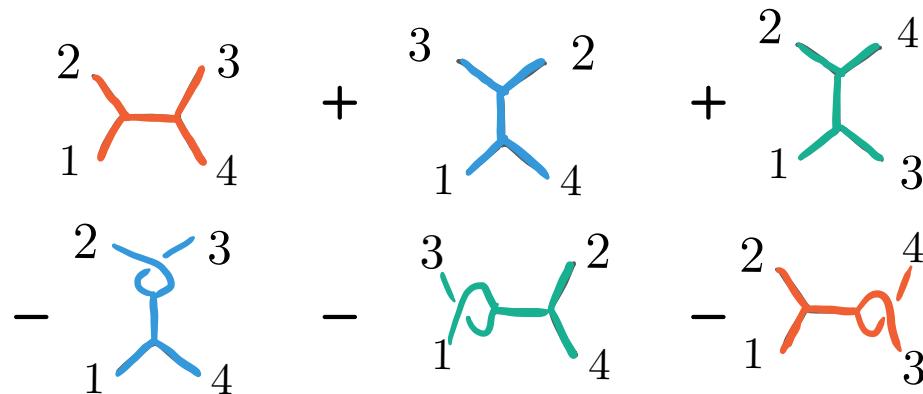
$$A(1243) = - \begin{array}{c} 2 \\ & \diagup \\ 1 & & 4 \\ & \diagdown \\ & 3 \end{array} + \begin{array}{c} 2 \\ & \diagup \\ 1 & & 4 \\ & \diagdown \\ & 3 \end{array}$$

Amplitude-level structure

$$A(1, \alpha, n, \beta) = (-1)^{|\alpha|} \sum_{\sigma \in \alpha \boxplus \beta^T} A(1, \sigma, n)$$

Kleiss-Kluijf (1989)

$$A(1234) + A(1324) + A(1243) = 0$$



Amplitude-level structure

$$\sum_{i=2}^{n-1} k_1 \cdot (k_2 + \cdots + k_i) A(2, \dots, i, 1, \dots, n) = 0$$

Bern-Carrasco-Johansson (2008)

$$A(1234) = \frac{n_s}{s_{12}} + \frac{n_t}{s_{14}}$$

$$A(1324) = \frac{n_u}{s_{13}} - \frac{n_t}{s_{14}}$$

$$\begin{array}{c} 2 \\ | \\ 1 \end{array} \begin{array}{c} 3 \\ | \\ 4 \end{array} = \begin{array}{c} 2 \\ | \\ 1 \end{array} \begin{array}{c} 3 \\ | \\ 4 \end{array} + \begin{array}{c} 4 \\ | \\ 1 \end{array} \begin{array}{c} 2 \\ | \\ 3 \end{array}$$

$$C_S = C_t + C_u$$

$$\pi_S = \pi_t + \pi_u$$

Amplitude-level structure

$$\sum_{i=2}^{n-1} k_1 \cdot (k_2 + \cdots + k_i) A(2, \dots, i, 1, \dots, n) = 0$$

Bern-Carrasco-Johansson (2008)

$$A(1234) = \frac{n_s}{s_{12}} + \frac{n_t}{s_{14}}$$

$$\begin{array}{c} 2 \\ | \\ 1 \end{array} \textcolor{red}{\diagup} \textcolor{red}{\diagdown} \begin{array}{c} 3 \\ | \\ 4 \end{array} = \begin{array}{c} 2 \\ | \\ 1 \end{array} \textcolor{blue}{\diagup} \textcolor{blue}{\diagdown} \begin{array}{c} 3 \\ | \\ 4 \end{array} + \begin{array}{c} 4 \\ | \\ 1 \end{array} \textcolor{teal}{\diagup} \textcolor{teal}{\diagdown} \begin{array}{c} 2 \\ | \\ 3 \end{array}$$

$$A(1324) = \frac{(n_s - n_t)}{s_{13}} - \frac{n_t}{s_{14}} = \frac{s_{12}}{s_{13}} A(1234) \leftarrow \text{D-dimensional}$$

$$A(1^-3^-2^+4^+) = \frac{s_{12}}{s_{13}} A(1^-2^+3^-4^+) \leftarrow D=4$$

Double-Copy Construction

When the partial amplitudes satisfy BCJ,
can replace $c(g)$ with $n(g)$ Bern, Carrasco, Johansson (2008)

$$c_i = c_j + c_k \quad \Leftrightarrow \quad n_i = n_j + n_k$$

Gauge theory for 1 SEK, Gravity for free!

$$\mathcal{A} = \int d\Omega_l \sum_g \frac{1}{S_g} \frac{c_g \tilde{n}_g}{d_g} \qquad \qquad \mathcal{M} = \int d\Omega_l \sum_g \frac{1}{S_g} \frac{n_g \tilde{n}_g}{d_g}$$

$$\epsilon^\mu \rightarrow k^\mu \quad \Leftrightarrow \quad \epsilon^\mu \epsilon^\nu \rightarrow k^\mu \epsilon^\nu + \epsilon^\mu k^\nu$$

At tree level, can be performed
with **KLT momentum kernel**:

Graphical Simplicity

Graphical organization
reveals hidden simplicity

$$\text{Diagram} = \text{Diagram} - \text{Diagram}$$

↓

Imposing this behavior
is equivalent to many
physical constraints

$$\mathcal{A}_{n\text{-point}}^{\text{tree}} \sim \begin{array}{c} a_2 \ a_3 \\ \vdots \quad \cdots \quad \vdots \\ a_1 \qquad \qquad \qquad a_{n-1} \qquad \qquad \qquad a_n \end{array}$$
$$\mathcal{A}_{4\text{-point}}^{\text{3-loop}} \sim \begin{array}{c} \text{Diagram}, \text{Diagram} \end{array}$$

- gauge invariance
- soft theorems
- diffeo invariance

Carrasco, Rodina,
Cheung, Shen, Wen, ...

Color-dual constraints on contact operators

*Gauge/diffeo invariance and
soft theorems*

The traditional story

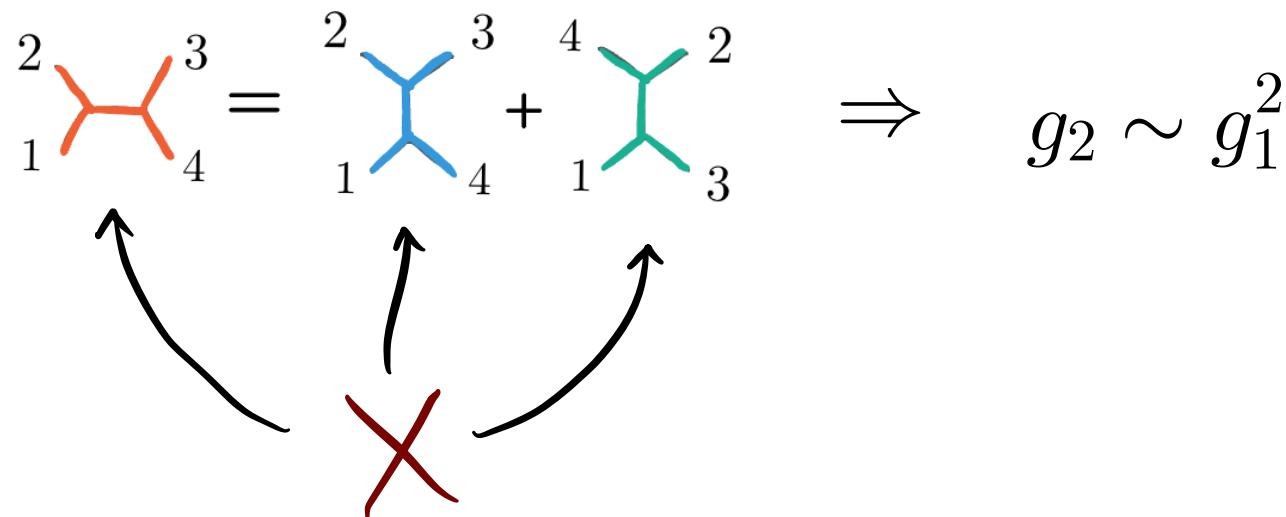
color-kinematics = gauge invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2(\partial A) + g_2 A^4$$

The traditional story

color-kinematics = gauge invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2(\partial A) + g_2 A^4 = -\frac{1}{4}F^2$$



The traditional story

color-kinematics = gauge invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2(\partial A) + g_2 A^4 = -\frac{1}{4} F^2$$

$$\begin{array}{c} 2 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ \diagup \\ 4 \end{array} = \begin{array}{c} 2 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ \diagup \\ 4 \end{array} + \begin{array}{c} 4 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 2 \\ \diagup \\ 3 \end{array} \Rightarrow g_2 \sim g_1^2$$

color-kinematics + double copy = linear diffeo inv.

$$M_5^{\text{GR}} = \sum_g \frac{n(\text{---})n(\text{---})}{dg}$$

The traditional story

color-kinematics = gauge invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2(\partial A) + g_2 A^4 = -\frac{1}{4}F^2$$

$$\begin{array}{c} 2 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ \diagup \\ 4 \end{array} = \begin{array}{c} 2 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ \diagup \\ 4 \end{array} + \begin{array}{c} 4 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 2 \\ \diagup \\ 3 \end{array} \Rightarrow g_2 \sim g_1^2$$

color-kinematics + double copy = linear diffeo inv.

$$M_5^{GR} = \sum_g (\text{---} + \text{---})(\text{---} + \text{---}) = \underbrace{\text{---} + \text{---} + \text{*}}_{\text{needed for diffeo. inv.}}$$

The traditional story

color-kinematics = soft bootstrap

$$\mathcal{L} = (\partial\pi)^2 \sum_{k=0} c_k \pi^{2k}$$

$$\begin{array}{c} 2 \\ & \diagdown \\ 1 & \text{---} & 3 \\ & \diagup \\ 4 \end{array} = \begin{array}{c} 2 \\ & \diagdown \\ 1 & \text{---} & 3 \\ & \diagup \\ 4 \end{array} + \begin{array}{c} 4 \\ & \diagdown \\ 1 & \text{---} & 2 \\ & \diagup \\ 3 \end{array}$$

Carrasco, Rodina,
Cheung, Shen, Wen, ...

$$\mathcal{L}^{\text{NLSM}} = \text{X} + \text{*} + \text{*} + \dots$$

Resums to NLSM

$$\mathcal{L}^{\text{NLSM}} = (\partial U)^\dagger (\partial U), \quad U = e^{i\pi}$$

Color-dual constraints on higher-derivatives

*Climbing towers to
emergent massive modes*

Higher-derivative constraints = tower

Novel contributions to pure vector

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

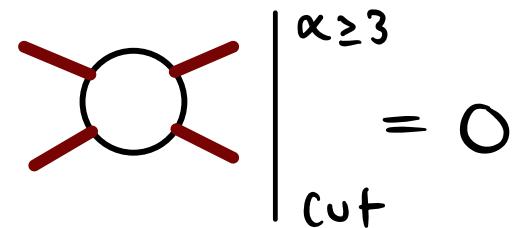
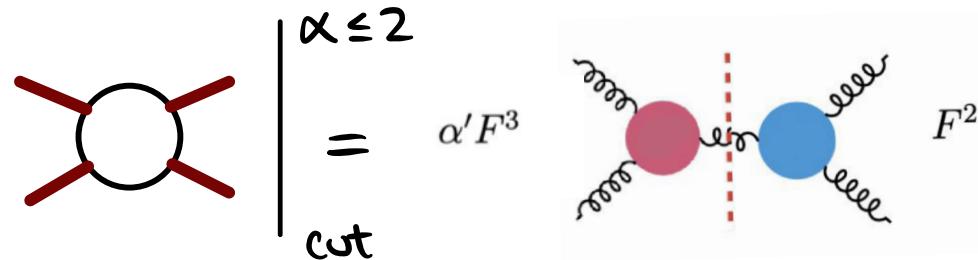
$$\left[\begin{array}{c} F^2 \\ \text{---} \\ \text{---} \end{array} \right] = 1 \quad A_3^{\text{YM}} = (\varepsilon_1 \varepsilon_2) (\varepsilon_3 k_1) + \text{cyc}$$

$$\left[\begin{array}{c} F^3 \\ \text{---} \\ \text{---} \end{array} \right] = 3 \quad A_3^{F^3} = (\varepsilon_1 k_2) (\varepsilon_2 k_3) (\varepsilon_3 k_1)$$

Higher-derivative constraints = tower

Novel contributions to pure vector

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 \quad \text{Broedel, Dixon}$$
$$+ \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \quad \text{Carrasco, Rodina, Yin, Zekioglu}$$



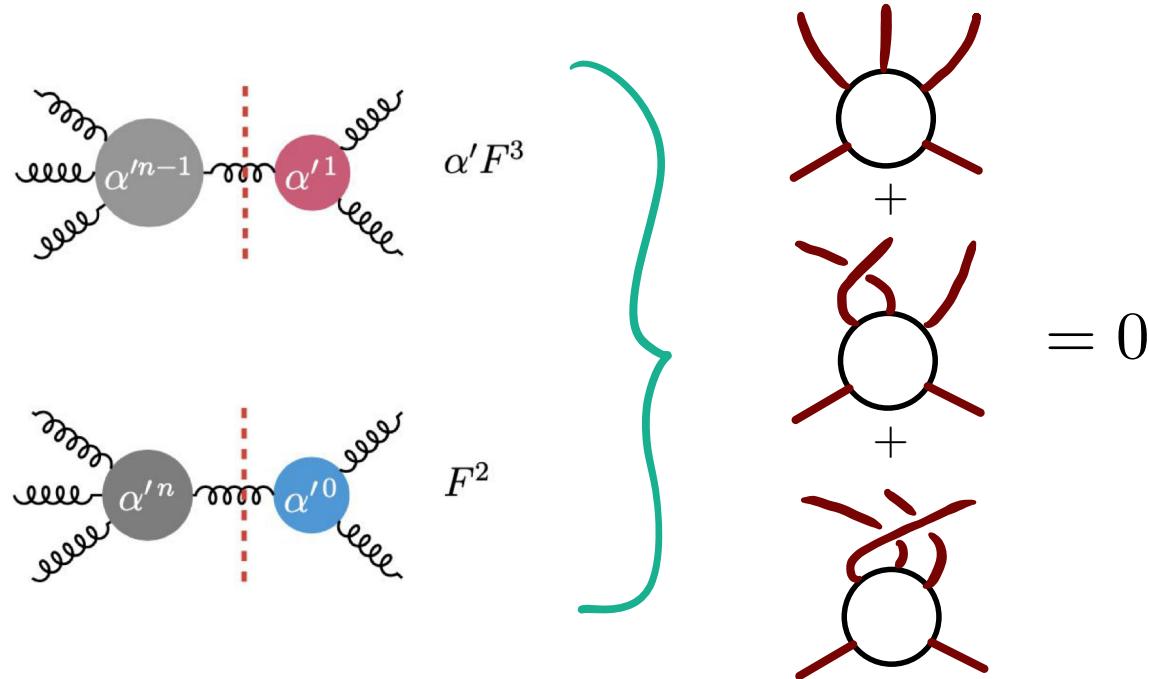
Higher-derivative constraints = tower

Novel contributions to pure vector

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4$$

$$+ \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441



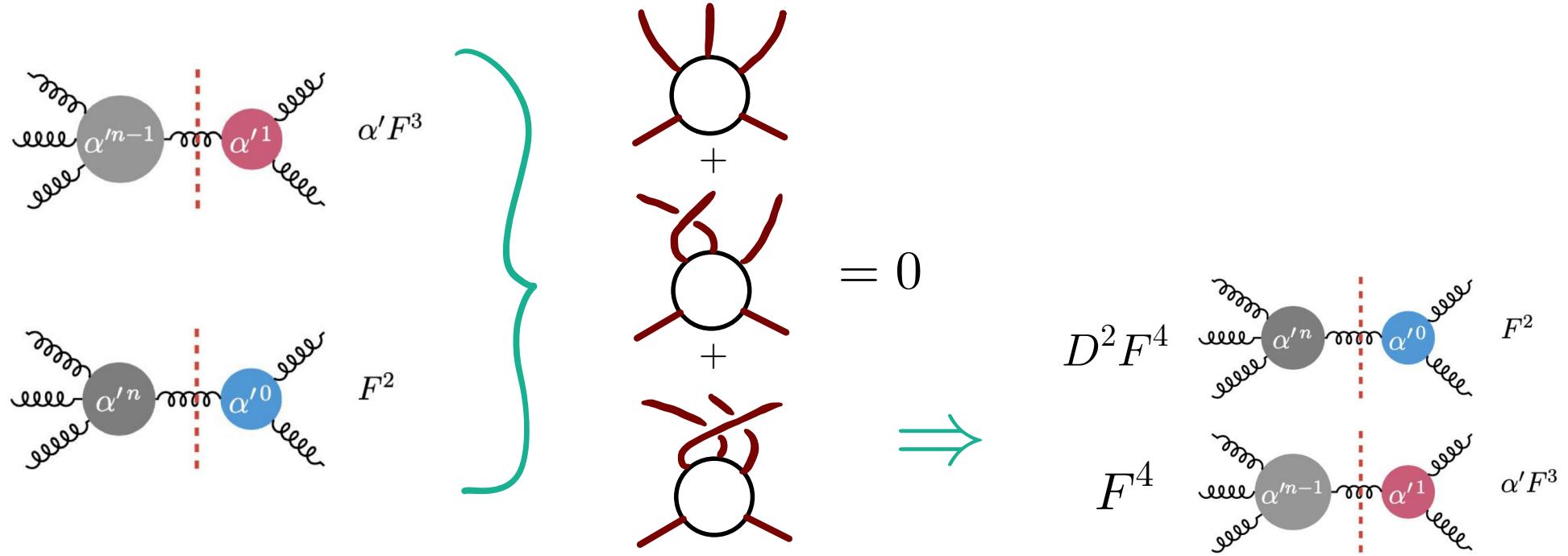
Higher-derivative constraints = tower

Novel contributions to pure vector

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4$$

$$+ \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441



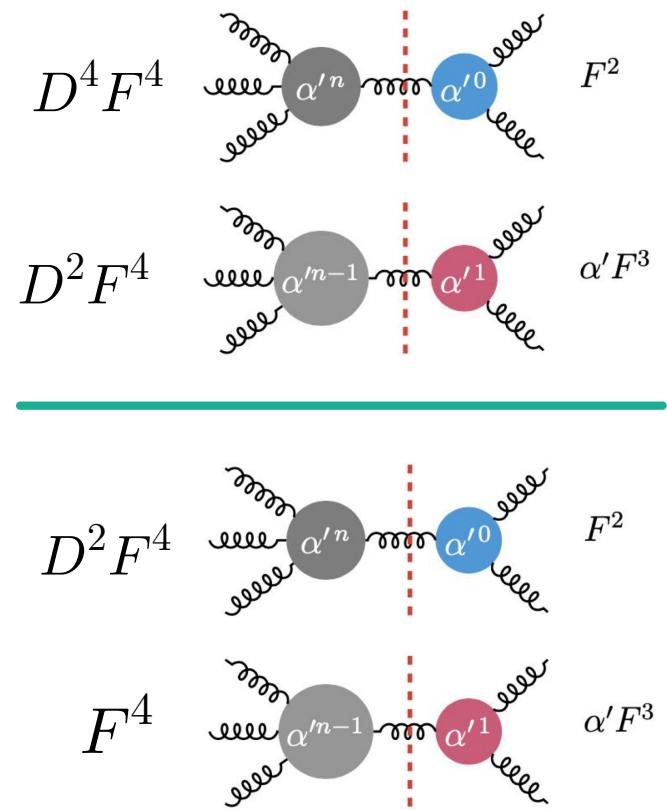
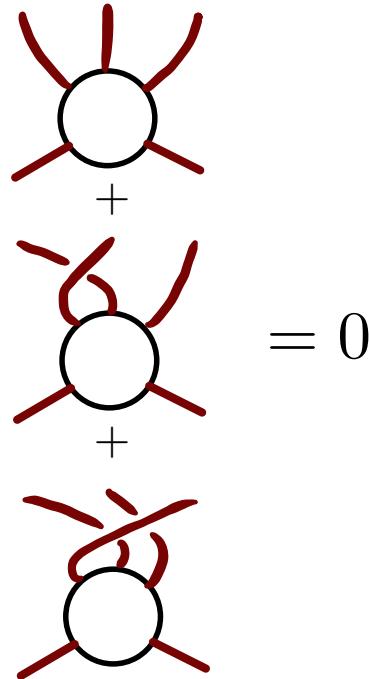
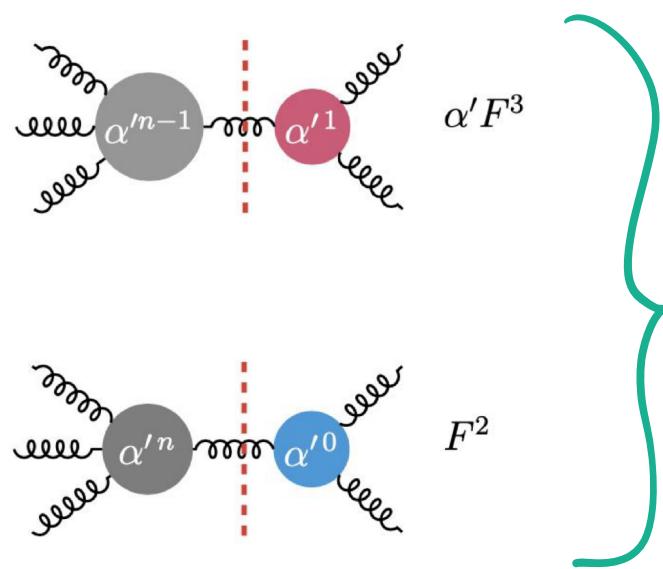
Higher-derivative constraints = tower

Novel contributions to pure vector

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4$$

$$+ \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441



Higher-derivative constraints

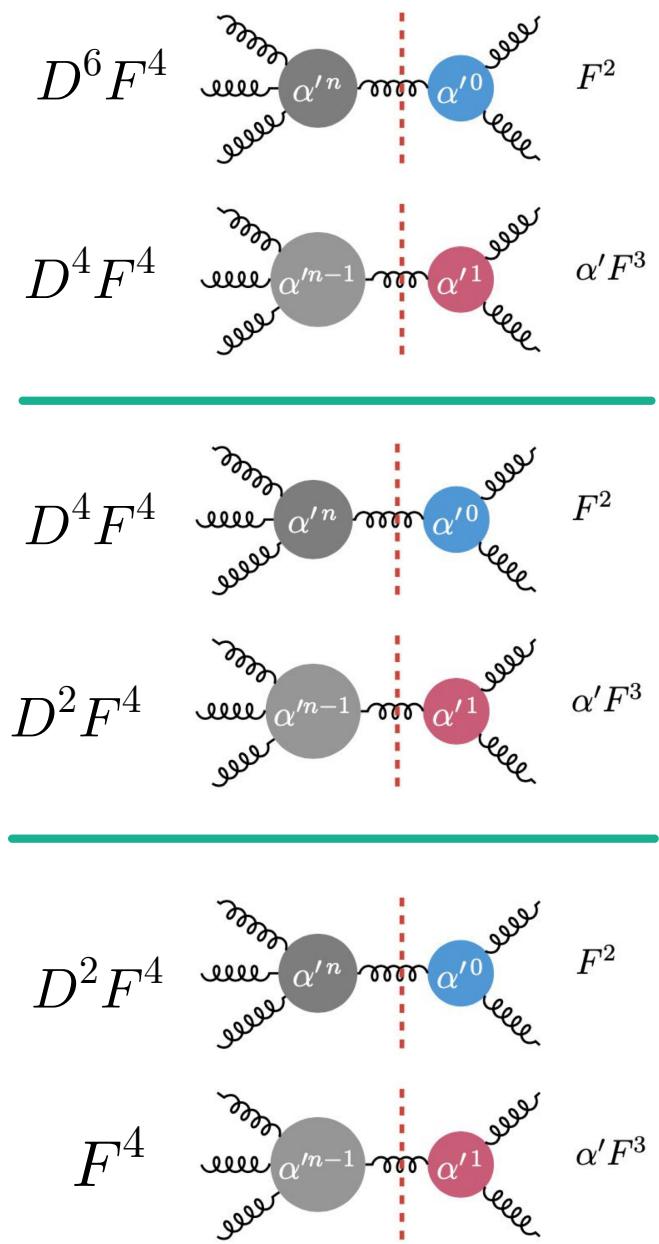
Infinite
tower to UV!

Novel contributions to pure vector

$$\begin{aligned}\mathcal{L}^{\text{YM}+F^3} = & -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 \\ & + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4\end{aligned}$$

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

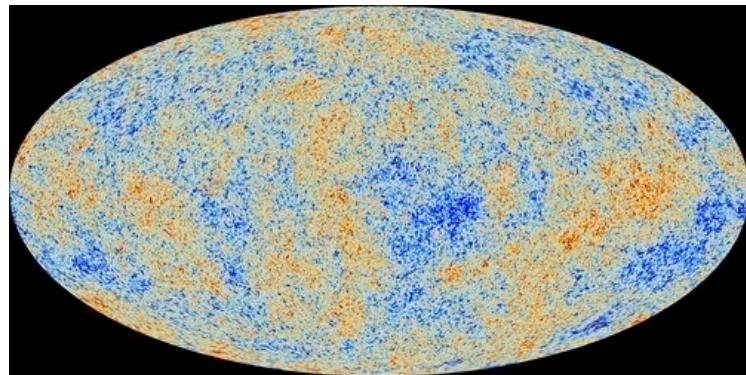
$$\left. \begin{array}{c} \text{Diagram 1: } \alpha' F^3 \\ \text{Diagram 2: } F^2 \end{array} \right\} = 0$$



Are gauged-pions color-dual?

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

$$\mathcal{L}^{\text{cov},\pi} = \frac{1}{2} \text{Tr} \left[\frac{D_\mu \pi D^\mu \pi}{(1 - \lambda \pi^2)^2} \right] - \frac{1}{4} \text{Tr} [F^2]$$



$$\left[\begin{array}{c} F^2 \\ \text{---} \end{array} \right] = 1 \quad \left[\begin{array}{c} \pi \\ \text{---} \end{array} \right] = 2$$

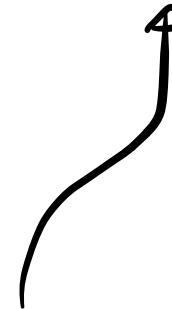
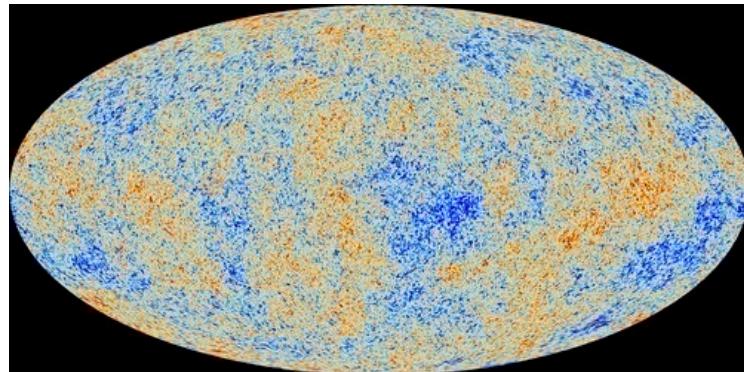
α - attractor inflation

$$M^{\text{DBI}VA+SG} = A^{\text{SYM}} \otimes A^{\pi+YM}$$

Are gauged-pions color-dual?

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

$$\mathcal{L}^{\text{cov},\pi} = \frac{1}{2} \text{Tr} \left[\frac{D_\mu \pi D^\mu \pi}{(1 - \lambda \pi^2)^2} \right] - \frac{1}{4} \text{Tr} [F^2] + \frac{\lambda}{g} F^3 + \dots$$



Need higher derivative tower

α - attractor inflation

$$\mathcal{M}^{\text{DBI}V\Lambda+\text{SG}} = A^{\text{SYM}} \otimes A^{\pi+\text{YM}}$$

$$\left[\begin{array}{c} F^2 \\ \text{---} \\ \text{---} \end{array} \right] = 1 \quad \left[\begin{array}{c} \pi \\ \text{---} \\ \text{---} \end{array} \right] = 2$$

$$\left[\begin{array}{c} F^3 \\ \text{---} \\ \text{---} \end{array} \right] = 3$$

Color-dual towers resum

Carrasco, Lewandowski, NHP
2203.03592, 2211.04431

same goes for scalar-sector

$$A_4^{(n)}(gggg) = u \left[\frac{(F_1 F_2)(F_3 F_4)}{s_{12}^2} (\lambda s_{12})^n + \text{cyc}(1, 2, 3) \right]$$

$$A_4^{(n)}(\pi\pi gg) = u \frac{(F_3 F_4)}{s_{12}} (\lambda s_{12})^n$$

$$A_4^{(n)}(\pi\pi\pi\pi) = u [(\lambda s_{12})^{n-1} + \text{cyc}(1, 2, 3)]$$

YM + F^3
contacts

$\mathcal{O}(\Lambda^n)$	$\ \pi\ = 2k$	$k=0$	$k=1$	$k=2$
$n=0$		0		0
$n=1$		0		0
$n=2$		0		0
$n=3$		1		1
$n=4$		1		1
$n=5$		2		2

Color-dual towers resum

Carrasco, Lewandowski, NHP
2203.03592, 2211.04431

same goes for scalar-sector

$$A_4^{(n)}(gggg) = u \left[\frac{(F_1 F_2)(F_3 F_4)}{s_{12}^2} (\lambda s_{12})^n + \text{cyc}(1, 2, 3) \right]$$

$$A_4^{(n)}(\pi\pi gg) = u \frac{(F_3 F_4)}{s_{12}} (\lambda s_{12})^n$$

$$A_4^{(n)}(\pi\pi\pi\pi) = u [(\lambda s_{12})^{n-1} + \text{cyc}(1, 2, 3)]$$

$\mathcal{O}(\Lambda^n)$	$\ \pi\ = 2k$	$k=0$	$k=1$	$k=2$
$n=0$		0		0
$n=1$		0		0
$n=2$		0		0
$n=3$		1		1
$n=4$		1		1
$n=5$		2		2

Resums to reveal a new massive residue in the UV!

$$m_{\text{UV}}^2 \sim \lambda^{-1}$$

$$\begin{aligned} A_{(1234)}^{\text{full}} &\sim \sum_n A_{(1234)}^{(n)} \sim 1 + \lambda s_{12} + (\lambda s_{12})^2 \dots \\ &\sim \frac{\lambda}{s_{12} - m_{\text{UV}}^2} + \text{cyc}(1, 2, 3) \end{aligned}$$

New massive particle residues

Higher derivative color-dual numerators encode massive residues

$$\mathcal{A} = \sum \frac{c_g n_g^{\text{HD}}}{d_g} \sim \sum \frac{c_g n_g^m}{d_g - m^2}$$

$$\mathcal{L}^{\text{YM}+F^3+\dots} = -\frac{1}{4}F^2 + \frac{1}{3m^2}F^3 + \frac{1}{m^4}F^4 + \frac{1}{m^6}D^2F^4 + \frac{1}{m^8}D^4F^4 + \dots$$

Resums to a color-dual dim-6 theory



Setting remaining freedom to zero

$$\mathcal{L}^{\text{YM}+DF^2} = (DF)^2 + (D\varphi)^2 - m^2(\varphi^2 + F^2) + \mathcal{L}_{\text{int}}(\varphi, F)$$

How much freedom remains?

5-point
constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \Rightarrow \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$


$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

$$\sigma_k = s^k + t^k + u^k$$

More structure @ 6-point?

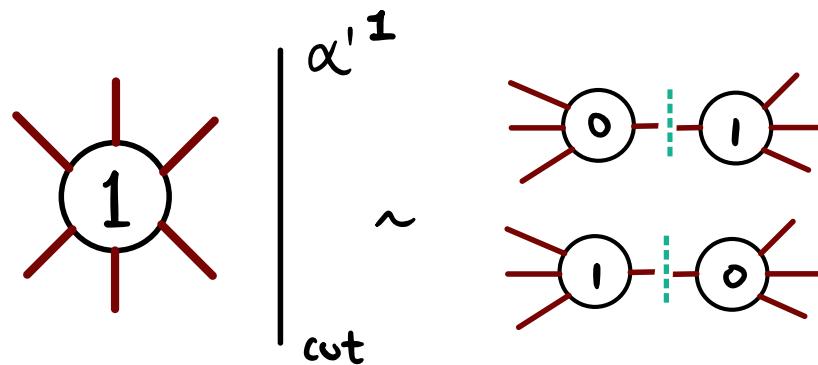
5-point
constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \Rightarrow \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$



$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

6-point
constraints



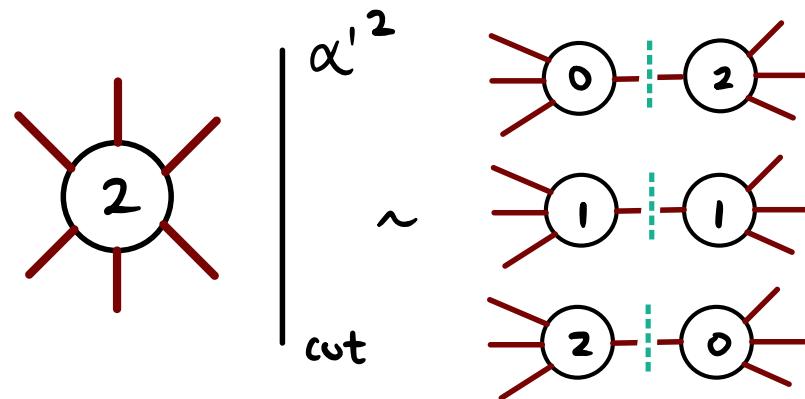
More structure @ 6-point?

5-point
constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \Rightarrow \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$

$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

6-point
constraints



More structure @ 6-point?

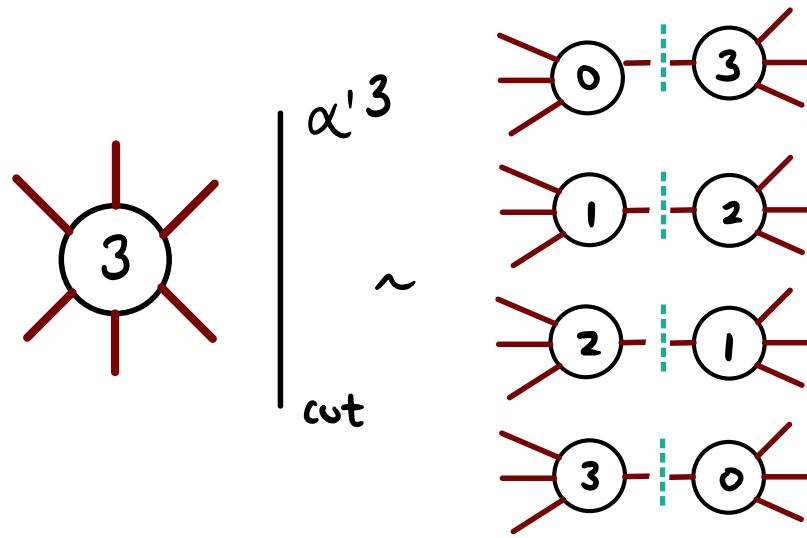
5-point
constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \Rightarrow \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$



$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

6-point
constraints



5-point vectors
>58K parameters
through $\mathcal{O}(\alpha^4)$

More structure @ 6-point?

5-point
constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \Rightarrow \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$

↓

$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

Scalar theory examples: additional constraints!

6-point
constraints

$$A_4^{\text{BAS}+\text{HD}} = A_4^{\text{BAS}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

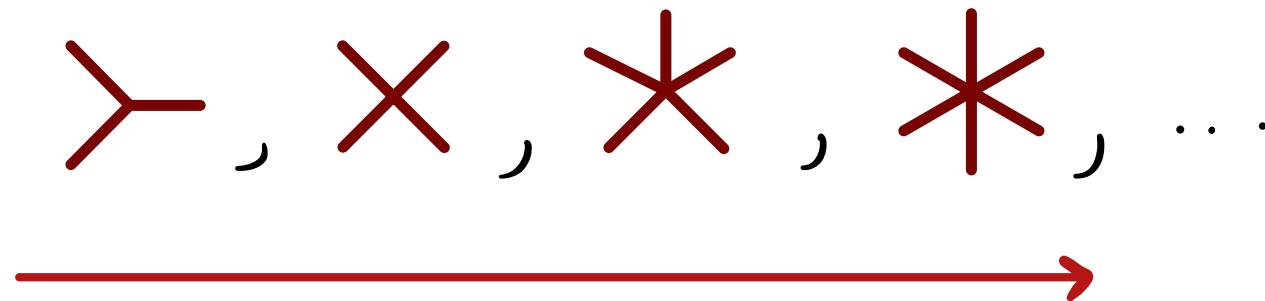
Chen, Elvang,
Herderschee
2302.04895

$$A_4^{\chi\text{PT}} = A_4^{\text{NLSM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$

Brown, Kampf, Oktem,
Paranjape, Trnka
2305.05688

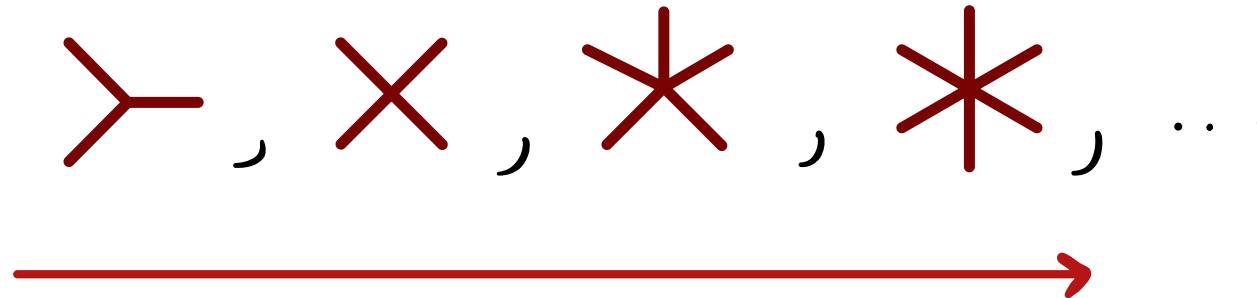
Takeaway: Color-dual + EFT

- Normally color-kinematics fixes **OUT**

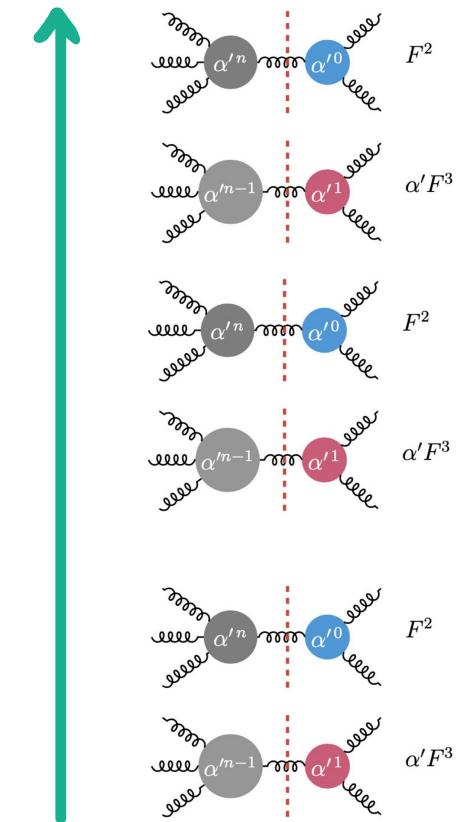


Takeaway: Color-dual + EFT

- Normally color-kinematics fixes **OUT**

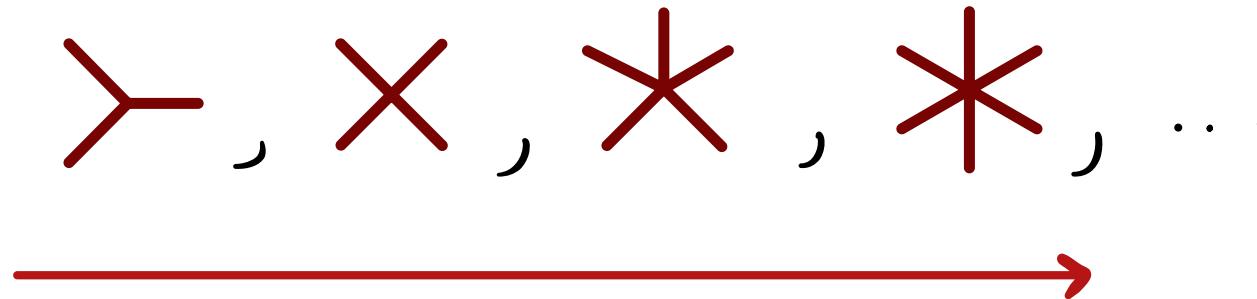


- But considering higher-derivatives,
color-kinematics fixes operators **UP**



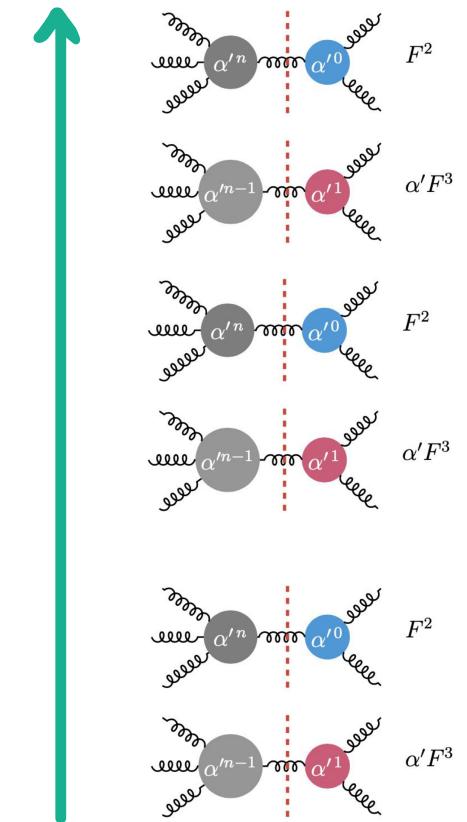
Takeaway: Color-dual + EFT

- Normally color-kinematics fixes **OUT**



- But considering higher-derivatives,
color-kinematics fixes operators **UP**
- Introduces off-shell **massive modes**

Significantly more **rigid** than envisioned

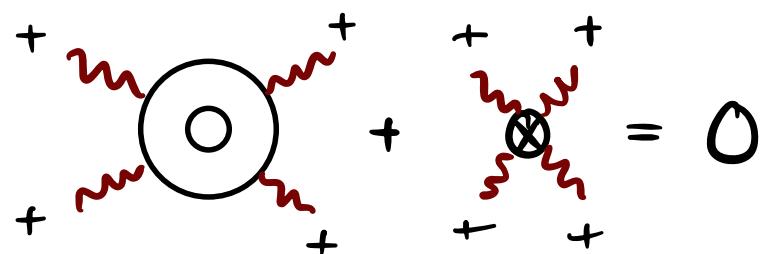


Modeling building implications for gravitational EFT

$$\mathcal{M}_n^{XY} = \mathcal{A}_n^X \otimes \mathcal{A}_n^Y$$

App 1: UV completion of gravity?

$$\mathcal{M}^{\mathcal{N}=4 \text{ SG}} = A^{\mathcal{N}=4 \text{ sYM}} \otimes A^{\text{YM}+F^3}$$



$$A_n^{\text{N}^j \text{MHV}} \otimes A_n^{\text{N}^k \text{MHV}} = 0 \quad j \neq k$$

@ tree-level

$$A_{(+,+,\dots,+)}^{\text{1-loop}} = \frac{1}{3i} \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} + \mathcal{O}(\epsilon)$$

anomalous @ 1-loop

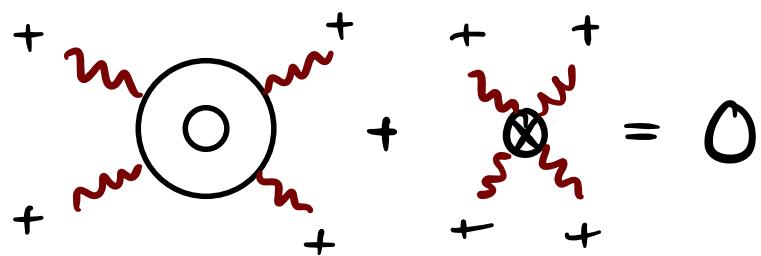
$$\mathcal{M}^{R^3} = A^{\text{YM}+F^3} \otimes A^{\text{YM}+F^3}$$

Bern, Carrasco, Edison, Kosower,
Parra-Martinez, Roiban, Kallosh,
and many others...

S-matrix anomalies source UV divergences

App 1: UV completion of gravity?

$$\mathcal{M}^{\mathcal{N}=4 \text{ SG}} = A^{\mathcal{N}=4 \text{ sYM}} \otimes A^{\text{YM}+DF^2}$$



Einstein-Weyl
conformal
supergravities

$$\mathcal{M}^{R^3} = A^{\text{YM}+DF^2} \otimes A^{\text{YM}+DF^2}$$

The color-dual fates of F^3 , R^3 , and $\mathcal{N} = 4$ supergravity

John Joseph M. Carrasco,^{1,2} Matthew Lewandowski,¹ and Nicolas H. Pavao¹

¹*Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*

²*Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France*

We find that the massless gauge theory of Yang-Mills deformed by a higher-derivative F^3 operator can not be tree-level color-dual without additional counterterms. The requirement of color-dual kinematics and consistent factorization between four- and five-points induces a tower of increasingly higher-dimensional operators. We find through explicit calculation that their amplitudes are consistent with the α' expansion of those generated by the $(DF)^2 + \text{YM}$ theory, a known color-dual

Carrasco, Lewandowski, NHP
2203.03592

App 2: Cancel Born-Infeld anomaly?

Elvang, et al.

$$\mathcal{M}^{\text{BI}} = A^{\text{NLSM}} \otimes A^{\text{YM}} \Rightarrow \begin{aligned} \mathcal{M}_{(++++)}^{\text{BI},1\text{-loop}} &\sim (s^4 + t^4 + u^4) \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} \\ \mathcal{M}_{(++++)}^{\text{BI},2\text{-loop}} &\sim (s^6 + t^6 + u^6) \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} \end{aligned}$$

$$\mathcal{M}^{\text{BI+HD}} = A^{\text{NLSM}} \otimes A^{\text{YM+HS??}}$$

anomaly cancellation

backup slides Carrasco, NHP
2307.xxxx

$$\mathcal{M}_{\varphi^{2k}}^{\text{2-loop}} = \frac{c_4^3}{4} \text{ (diagram)} + \frac{c_4^3}{2} \text{ (diagram)} + \frac{c_8}{4} \text{ (diagram)} + \frac{c_4 c_6}{6} \text{ (diagram)} + \text{perms}(1, 2, 3, 4)$$

App 3: Corrections to N=1 sYM in 10D?

Could emergent masses be a common feature?

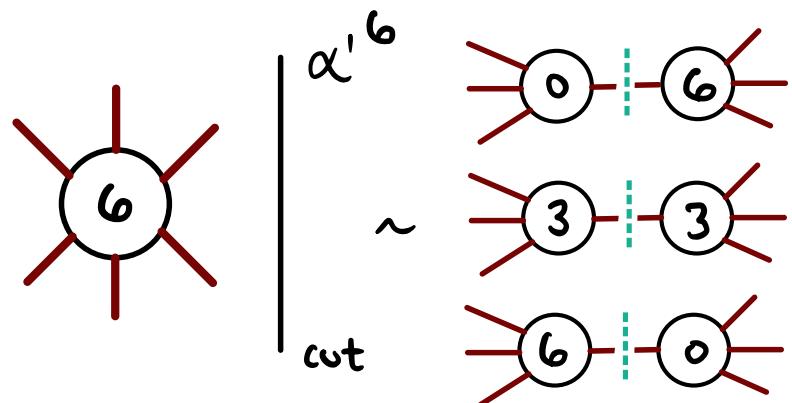
$$\mathcal{A} = \sum \frac{c_g n_g^{\text{HD}}}{d_g} \sim \sum \frac{c_g n_g^m}{d_g - m^2}$$

cancels 10D gauge anomaly

Color-dual SUSY higher-derivatives

$$\mathcal{L}^{\text{YM}} = -\frac{1}{4}F^2 + (D^2F^4 + F^5) + \dots$$

$$A_4^{\text{sYM+HD}} = A_4^{\text{sYM}} \left(1 + \sum c_{(x,y)} \sigma_3^x \sigma_2^y \right)$$



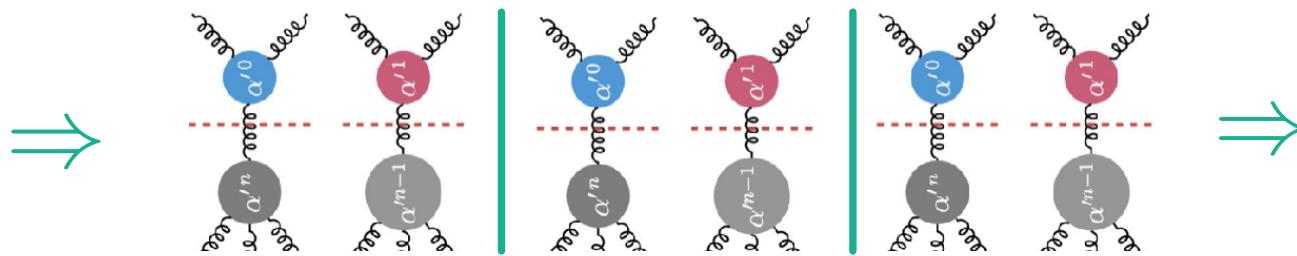
6-point constraints

Key Takeaways

- Graphical organization reveals hidden simplicity
 - Can use this to bootstrap gauge/gravity EFT contacts

$$\begin{array}{c} 2 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ \diagup \\ 4 \end{array} = \begin{array}{c} 2 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ \diagup \\ 4 \end{array} + \begin{array}{c} 4 \\ \diagdown \\ 1 \end{array} \text{---} \begin{array}{c} 2 \\ \diagup \\ 3 \end{array} \Rightarrow \begin{array}{c} \diagdown \\ \diagup \end{array} + \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagup \\ \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagdown \\ \diagdown \\ \diagup \end{array} + \dots$$

- Color-kinematic duality imposes unexpected **EFT constraints**
 - Higher derivative towers build **massive residues**



- Many **future directions** of discovery

$$\left[\text{---} \text{---} \text{---} \right] = 1 \quad \left[\text{---} \text{---} \text{---} \text{---} \right] = 2$$

Backup Slides

Two-loop Abelianized amplitudes

$$\mathcal{M}_{\varphi^{2k}}^{\text{2-loop}} = \frac{c_4^3}{4} \begin{array}{c} \text{Diagram 1} \end{array} + \frac{c_4^3}{2} \begin{array}{c} \text{Diagram 2} \end{array} + \frac{c_8}{4} \begin{array}{c} \text{Diagram 3} \end{array}$$

$$+ \frac{c_4 c_6}{6} \begin{array}{c} \text{Diagram 4} \end{array} + \text{perms}(1, 2, 3, 4)$$

Carrasco, NHP
2307.xxxx

Fully D-dimensional integrals!

$$I_{3,x}^{\mu_1 \dots \mu_n} = \int \frac{d^D l}{(2\pi)^D} \frac{l^{\mu_1} l^{\mu_2} \dots l^{\mu_n}}{l^2(l+K_1)^{2x}(l+K_{12})^2}$$

$$= \sum_{m+l+2k=n} a_{(m,l,k)}^x \mathcal{T}_{\text{tri}}^{(m,l,k)}$$

$$\left\{ \begin{array}{l} a_{(m,l,k)}^x = - \left[\frac{s_{12}}{D+2(m+l+k-1)} \right] a_{(m+1,l+1,k-1)}^x \\ a_{(m,l,0)}^x = - \left[\frac{D+2(m+l-2)}{D+2(m-2)} \right] \left[\frac{1}{s_{12}} a_{(m-1,l,0)}^{x-1} + a_{(m-1,l,0)}^x \right] \\ a_{(0,l,0)} = \frac{1}{s_{12}} a_{(0,l-1,0)}^{x-1} \end{array} \right.$$

Reproduces cancellations expected for supersymmetry

$$\begin{aligned}
 & \text{Diagram 1} = N_s \text{Diagram 2} + N_f N_s \text{Diagram 3} + \\
 & N_v N_f \text{Diagram 4} + N_v \text{Diagram 5} + N_s(N_s - 1) \text{Diagram 6} + \\
 & N_v N_s \text{Diagram 7} + N_f \text{Diagram 8} + N_f(N_f - 1) \text{Diagram 9} + \\
 & \Rightarrow \mathcal{M}_{(+++)}^{\mathcal{N}=1,4} = 0
 \end{aligned}$$

$$\mathcal{M}_{(+++)}^{\text{BI,2-loop}} \sim (s^6 + t^6 + u^6) \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle}$$

Modeling building with color-dual EFT

Part I

Building duality invariants

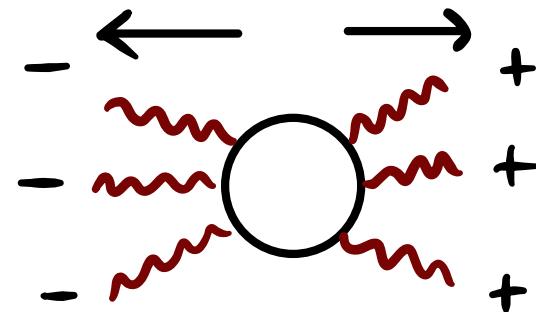
Universality in electromagnetic duality

Photon theories with helicity conservation

Equations of motion
invariant under

$$F + iG \rightarrow e^{i\alpha} (F + iG)$$

$$G \equiv \frac{\partial \mathcal{L}}{\partial \tilde{F}}$$



$$\underline{n_{(-)} = n_{(+)}}$$

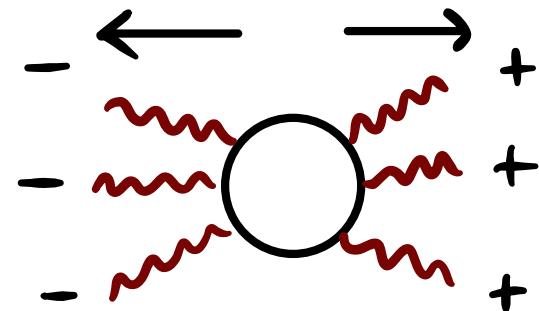
Universality in electromagnetic duality

Photon theories with helicity conservation

Equations of motion
invariant under

$$F + iG \rightarrow e^{i\alpha} (F + iG)$$

$$G \equiv \frac{\partial \mathcal{L}}{\partial \tilde{F}}$$



$$\underline{n_{(-)} = n_{(+)}}$$

$$\mathcal{L}^{\text{EMf}} = \sqrt{-g} \left(R + \sum_I F_{I\mu\nu} F^{I\mu\nu} \right)$$

$$\mathcal{L}^{\text{BI}} = 1 - \sqrt{1 - F^2 + F^2(F\tilde{F})}$$

Universality in electromagnetic duality

Photon theories with helicity conservation

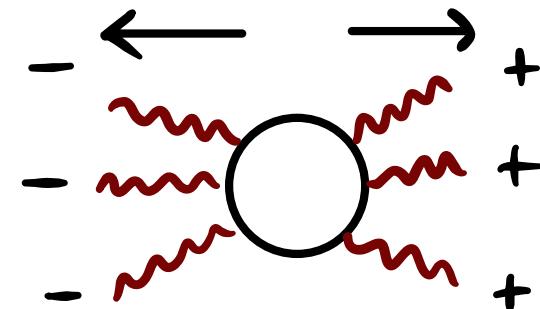
Equations of motion
invariant under

$$F + iG \rightarrow e^{i\alpha} (F + iG)$$

$$G \equiv \frac{\partial \mathcal{L}}{\partial \tilde{F}}$$

$$\mathcal{L}^{\text{EMf}} = \sqrt{-g} \left(R + \sum_I F_{I\mu\nu} F^{I\mu\nu} \right)$$

$$\mathcal{L}^{\text{BI}} = 1 - \sqrt{1 - F^2 + F^2(F\tilde{F})}$$



$$\underline{n(-) = n(+)}$$

$$\mathcal{M}^{\text{EMf}} = A^{\text{YMS}} \otimes A^{\text{YM}}$$

$$\mathcal{M}^{\text{BI}} = A^{\text{NLSM}} \otimes A^{\text{YM}}$$

Universality in electromagnetic duality

Photon theories with helicity conservation

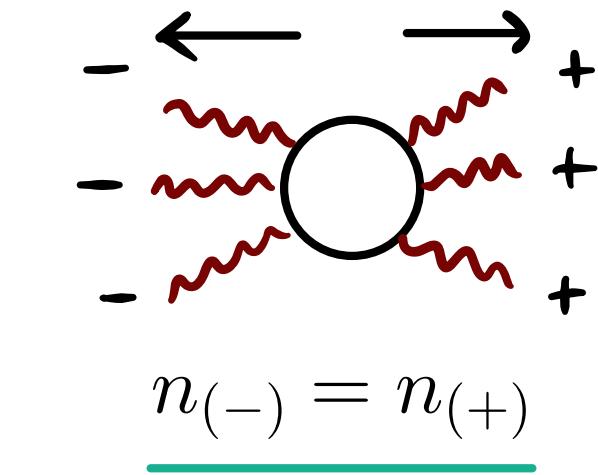
Equations of motion
invariant under

$$F + iG \rightarrow e^{i\alpha} (F + iG)$$

$$G \equiv \frac{\partial \mathcal{L}}{\partial \tilde{F}}$$

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$



$$\mathcal{M}^{\text{EMf}} = A^{\text{YMS}} \otimes A^{\text{YM}}$$

$$\mathcal{M}^{\text{BI}} = A^{\text{NLSM}} \otimes A^{\text{YM}}$$

Universality in electromagnetic duality

A Color-Dual Puzzle:

$$A_n^{\text{YMS}} \otimes A_n^{\text{MHV}} = 0$$
$$n > 4$$

$$A_n^{\text{NLSM}} \otimes A_n^{\text{MHV}} = 0$$
$$n > 4$$

Universality in electromagnetic duality

A Color-Dual Puzzle:

$$\left[A_n^{\text{YMS}} \otimes A_n^{\text{MHV}} = 0 \quad n > 4 \right] \qquad \qquad \left[A_n^{\text{NLSM}} \otimes A_n^{\text{MHV}} = 0 \quad n > 4 \right]$$

These live in
supergravity



R-symmetry:

$$A_n^{\text{N}^j\text{MHV}} \otimes A_n^{\text{N}^k\text{MHV}} = 0 \quad j \neq k$$

Universality in electromagnetic duality

A Color-Dual Puzzle:

$$[A_n^{\text{YMS}} \otimes A_n^{\text{MHV}} = 0 \quad n > 4]$$

$$A_n^{\text{NLSM}} \otimes A_n^{\text{MHV}} = 0 \quad n > 4$$

These live in
supergravity

R-symmetry:

$$A_n^{\text{N}^j\text{MHV}} \otimes A_n^{\text{N}^k\text{MHV}} = 0 \quad j \neq k$$

Why do pions filter
non-vanishing
Yang-Mills?

Universality in electromagnetic duality

Answer:

NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

$$\begin{matrix} \text{NLSM} \\ = \\ \text{YMS} + \text{HD} \end{matrix}$$

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

Universality in electromagnetic duality

Answer:

NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

$$\begin{aligned} \text{NLSM} \\ = \\ \text{YMS} + \text{HD} \end{aligned}$$

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

satisfies
BCJ relations

$$A_{(12)(34)(56)}^{\text{YMS}} = \text{Diagram}$$

$$k_2 k_1 \text{Diagram} + k_2 k_{13} \text{Diagram} + k_2 k_{134} \text{Diagram} + k_2 k_{1345} \text{Diagram} = 0$$

Universality in electromagnetic duality

Answer:

NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

NLSM
=
YMS + HD

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

$$s_{34}s_{56} A_{(12)(34)(56)}^{\text{YMS}} =$$


Universality in electromagnetic duality

Answer:

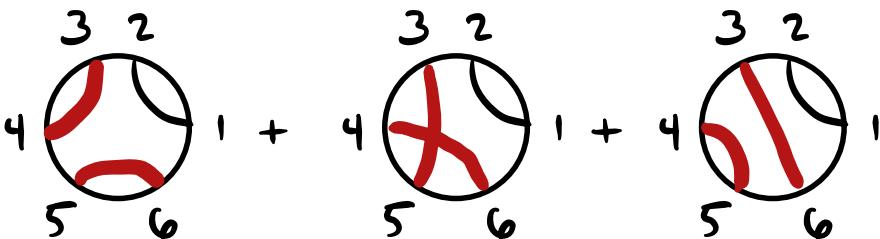
NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

$$\begin{matrix} \text{NLSM} \\ = \\ \text{YMS} + \text{HD} \end{matrix}$$

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

$$A_6^{\text{NLSM}} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3},$$


Universality in electromagnetic duality

Answer:

NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

NLSM
=
YMS + HD

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2 \quad \mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

PHYS. REV. D **107**, 065020 (2023)

$$|A_8^{\text{NLSM}}| = \begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \\ + \\ \text{Diagram 3} \\ + \\ \text{Diagram 4} \\ + \\ \text{Diagram 5} \\ + \\ \text{Diagram 6} \end{array}$$

(39)

Universality in electromagnetic duality

$$A^{\text{YM}} \equiv A^{\text{YM}}(\{k_i k_j, \epsilon_i k_j, \epsilon_i \epsilon_j\})$$

Idea: strip off polarization-products

Reducible Amplitude Block Decomposition (**RABID**)

NHP
2210.12800

$$A_{(\sigma)}^{\text{YM}} = \sum \epsilon_{(\rho)} \Delta_{(\sigma)}^{(\rho)}$$

$$\Delta_{(\sigma)}^{(\rho)} [\{k_i k_j, \epsilon_i k_j\}]$$

$$A_4^{\text{YM}} = \epsilon_{(12)} \begin{array}{c} \text{circle} \\ \diagup \diagdown \end{array} + \epsilon_{(34)} \begin{array}{c} \text{circle} \\ \diagup \diagup \end{array} + \epsilon_{(12)} \epsilon_{(34)} \begin{array}{c} \text{circle} \\ \diagup \diagup \\ \diagdown \diagdown \end{array}$$
$$\epsilon_{(13)} \begin{array}{c} \text{circle} \\ \diagup \diagdown \end{array} + \epsilon_{(24)} \begin{array}{c} \text{circle} \\ \diagup \diagup \end{array} + \epsilon_{(13)} \epsilon_{(24)} \begin{array}{c} \text{circle} \\ \diagup \diagup \\ \times \times \end{array}$$
$$\epsilon_{(14)} \begin{array}{c} \text{circle} \\ \diagup \diagdown \end{array} + \epsilon_{(23)} \begin{array}{c} \text{circle} \\ \diagup \diagup \end{array} + \epsilon_{(13)} \epsilon_{(24)} \begin{array}{c} \text{circle} \\ \diagup \diagup \\ \diagdown \diagdown \end{array}$$

Universality in electromagnetic duality

$$A^{\text{YM}} \equiv A^{\text{YM}}(\{k_i k_j, \epsilon_i k_j, \epsilon_i \epsilon_j\})$$

Idea: strip off polarization-products

Reducible Amplitude Block Decomposition (RABID)

NHP
2210.12800

$$A_{(\sigma)}^{\text{YM}} = \sum \epsilon_{(\rho)} \Delta_{(\sigma)}^{(\rho)}$$

$$\Delta_{(\sigma)}^{(\rho)} [\{k_i k_j, \epsilon_i k_j\}]$$

$$A_4^{\text{YM}} \Big| = k_1 \epsilon_2 \begin{array}{c} \text{circle} \\ \text{red dot} \end{array} + \epsilon_{(34)} \begin{array}{c} \text{circle} \\ \text{red dot} \end{array} + k_1 \epsilon_2 \epsilon_{(34)} \begin{array}{c} \text{circle} \\ \text{red dot} \end{array}$$
$$k_1 \epsilon_3 \begin{array}{c} \text{circle} \\ \text{black diagonal line} \\ \text{red dot} \end{array} + \epsilon_{(24)} \begin{array}{c} \text{circle} \\ \text{black diagonal line} \\ \text{red dot} \end{array} + k_1 \epsilon_3 \epsilon_{(24)} \begin{array}{c} \text{circle} \\ \text{X} \\ \text{red dot} \end{array}$$
$$k_1 \epsilon_4 \begin{array}{c} \text{circle} \\ \text{red dot} \\ \text{black curve} \end{array} + \epsilon_{(23)} \begin{array}{c} \text{circle} \\ \text{red dot} \\ \text{black curve} \end{array} + k_1 \epsilon_4 \epsilon_{(23)} \begin{array}{c} \text{circle} \\ \text{red dot} \\ \text{black curve} \end{array}$$

Universality in electromagnetic duality

$$A^{\text{YM}} \equiv A^{\text{YM}}(\{k_i k_j, \epsilon_i k_j, \epsilon_i \epsilon_j\})$$

Idea: strip off polarization-products

Reducible Amplitude Block Decomposition (RABID)

NHP
2210.12800

$$A_{(\sigma)}^{\text{YM}} = \sum \epsilon_{(\rho)} \Delta_{(\sigma)}^{(\rho)}$$

$$\Delta_{(\sigma)}^{(\rho)} [\{k_i k_j, \epsilon_i k_j\}]$$

$$A_4^{\text{YM}} \Big| = k_1 \epsilon_2 \begin{array}{c} \text{---} \\ \text{---} \end{array} + \epsilon_{(34)} \begin{array}{c} \text{---} \\ \text{---} \end{array} + k_1 \epsilon_2 \epsilon_{(34)} \begin{array}{c} \text{---} \\ \text{---} \end{array}$$
$$k_1 \epsilon_3 \begin{array}{c} \text{---} \\ \text{---} \end{array} + \epsilon_{(24)} \begin{array}{c} \text{---} \\ \text{---} \end{array} + k_1 \epsilon_3 \epsilon_{(24)} \begin{array}{c} \text{---} \\ \text{---} \end{array}$$
$$k_1 \epsilon_4 \begin{array}{c} \text{---} \\ \text{---} \end{array} + \epsilon_{(23)} \begin{array}{c} \text{---} \\ \text{---} \end{array} + k_1 \epsilon_4 \epsilon_{(23)} \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

Universality in electromagnetic duality

Reducible Amplitude Block Decomposition (RABID)

$$A_{(\sigma)}^{\text{YM}} = \sum \epsilon_{(\rho)} \Delta_{(\sigma)}^{(\rho)}$$
$$\Delta_{(\sigma)}^{(\rho)} [\{k_i k_j, \epsilon_i k_j\}]$$

Ward Identity between building blocks

$$\Delta_{(\sigma)}^{(\rho)}|_{\epsilon_i \rightarrow k_i} = - \sum_{j \in \rho^c} k_i \epsilon_j \Delta_{(\sigma)}^{(\rho \cup (ij))}$$

PHYS. REV. D 107, 065020 (2023)

$$\left. \begin{array}{c} \text{Diagram: A circle with four black dots and a square vertex at the top-right, with a curved line from the top-left dot to the square.} \\ |_{\epsilon_3 \rightarrow k_3} \end{array} \right) = - \left(\begin{array}{c} \text{Diagram: Circle with four dots, top-left dot pink, square at top-right, curved line from top-left to square.} \\ + \text{Diagram: Circle with four dots, top dot pink, square at top-right, curved line from top to square.} \\ + \text{Diagram: Circle with four dots, top-right dot pink, square at top-right, curved line from top-left to square.} \end{array} \right). \quad \Rightarrow \quad \left. \begin{array}{c} \text{Diagram: Circle with four dots, square at top-right, curved line from top-left to square.} \\ |_{\epsilon \rightarrow k} \end{array} \right) = \begin{array}{c} \text{Diagram: Circle with four dots, square at top-right, curved line from top-left to square, pink.} \\ + \text{Diagram: Circle with four dots, square at top-right, curved line from top to square, pink.} \\ + \text{Diagram: Circle with four dots, square at top-right, curved line from bottom-left to square, pink.} \end{array}.$$

(24)

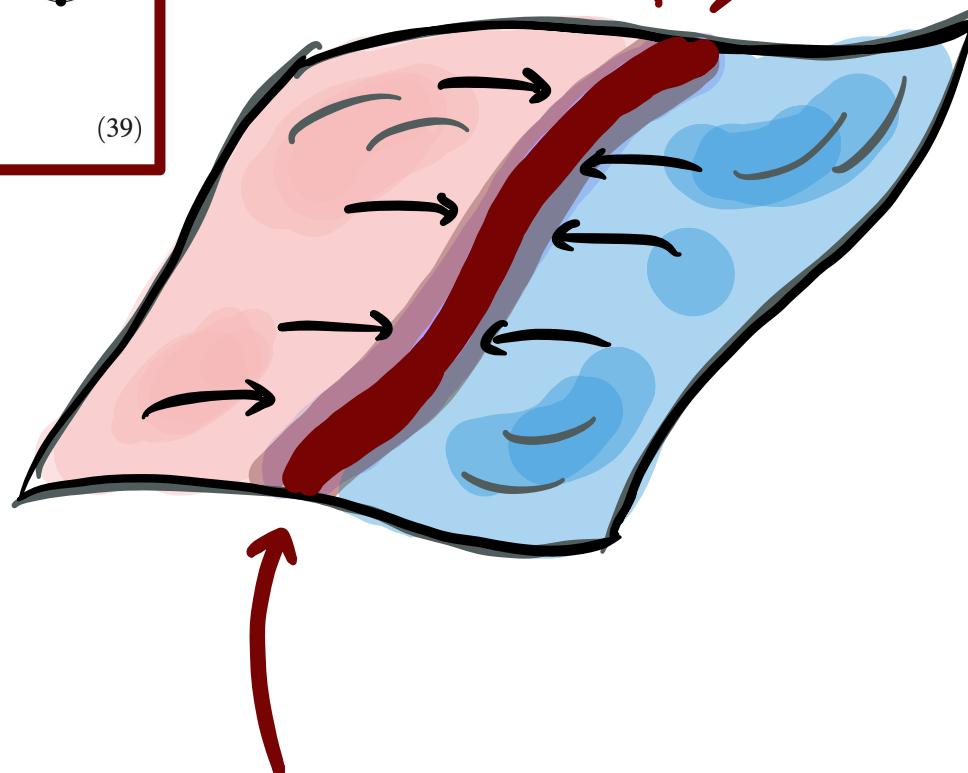
Modeling building with color-dual EFT

PHYS. REV. D 107, 065020 (2023)

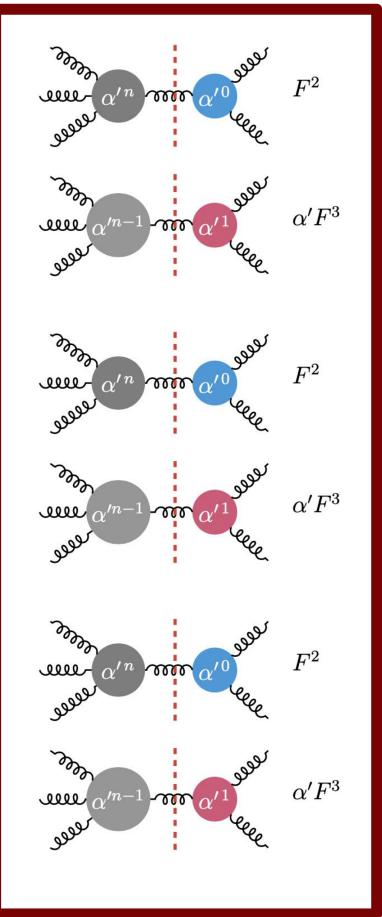
$$|A_8^{\text{NLSM}}| = \sum_{\alpha} \left(\alpha'^n \cdot \alpha'^0 + \alpha'^{n-1} \cdot \alpha'^1 \right) F^2 + \sum_{\alpha} \left(\alpha'^n \cdot \alpha'^0 + \alpha'^{n-1} \cdot \alpha'^1 \right) \alpha' F^3$$

(39)

Effective Field
Theory Space



$$\sum_{\alpha} (k_1 \cdot k_{\alpha}) A(\alpha, 1, \beta, n) = 0$$



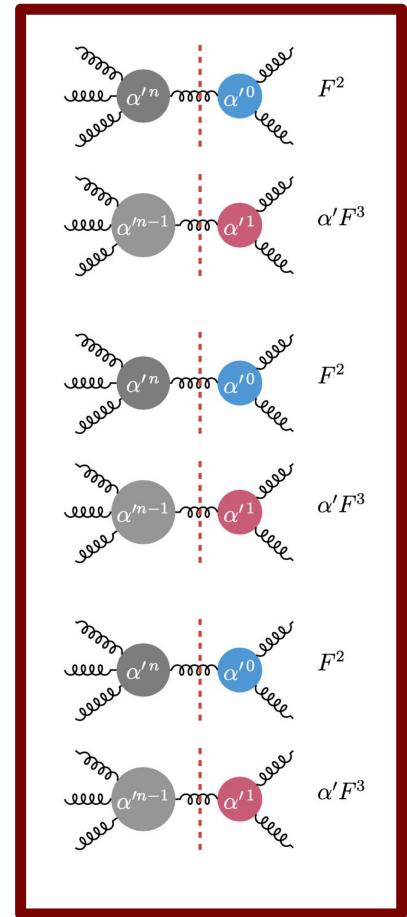
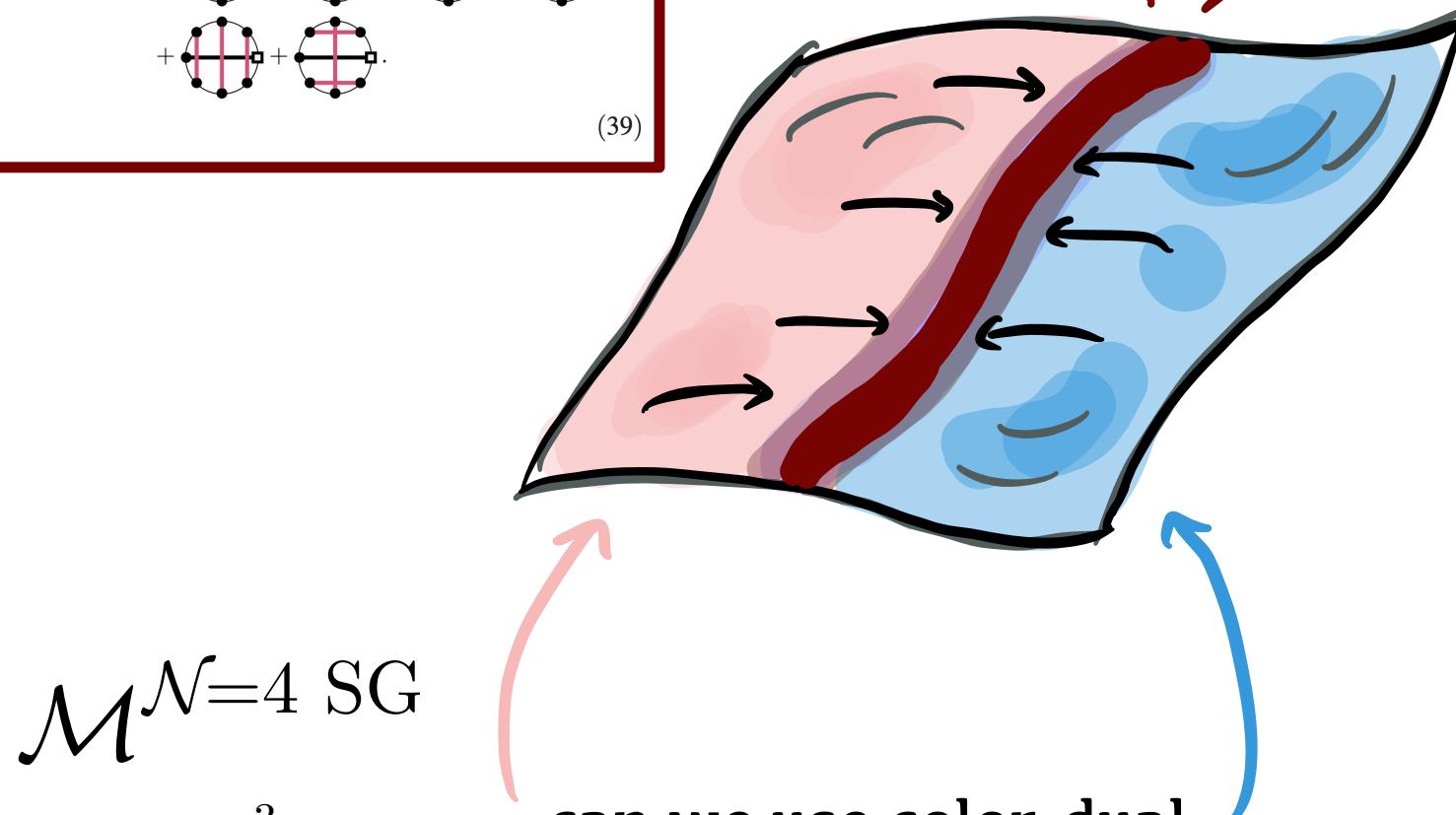
Modeling building with color-dual EFT

PHYS. REV. D 107, 065020 (2023)

$$|A_8^{\text{NLSM}}| = \sum_{\text{diagrams}} + \sum_{\text{diagrams}} + \sum_{\text{diagrams}} + \sum_{\text{diagrams}}$$

(39)

Effective Field
Theory Space



$\mathcal{M}^{\text{BI+HD}}$

$\mathcal{M}_{\alpha\text{-attractors}}^{\text{DBIVA+SG}}$

can we use color-dual
organization more generally?

Modeling building with color-dual EFT

Part II

stretching the capacity

Introducing symmetric structure

$$\begin{array}{c} 2 \\ \diagup \quad \diagdown \\ \textcolor{red}{\text{---}} \quad \textcolor{red}{\text{---}} \\ | \qquad | \\ 1 \qquad 4 \\ \text{---} \end{array} = \begin{array}{c} 2 \\ \diagup \quad \diagdown \\ \textcolor{blue}{\text{---}} \quad \textcolor{blue}{\text{---}} \\ | \qquad | \\ 1 \qquad 4 \\ \text{---} \end{array} + \begin{array}{c} 4 \\ \diagup \quad \diagdown \\ \textcolor{teal}{\text{---}} \quad \textcolor{teal}{\text{---}} \\ | \qquad | \\ 1 \qquad 3 \\ \text{---} \end{array} \iff \sum_{\alpha} (k_1 \cdot k_{\alpha}) A(\alpha, 1, \beta, n) = 0$$

$\textcolor{red}{\curvearrowright} \quad f^{12A} f^{A34} = \text{Tr} [T^1 T^2 T^3 T^4] - \text{Tr} [T^1 T^2 T^4 T^3] - \text{Tr} [T^2 T^1 T^3 T^4] + \text{Tr} [T^2 T^1 T^4 T^3]$

Introducing symmetric structure

$$\text{Diagram: } \begin{array}{c} 2 \\ | \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ | \\ 4 \end{array} = \begin{array}{c} 2 \\ | \\ 1 \end{array} \text{---} \begin{array}{c} 3 \\ | \\ 4 \end{array} + \begin{array}{c} 4 \\ | \\ 1 \end{array} \text{---} \begin{array}{c} 2 \\ | \\ 3 \end{array} \quad \Leftrightarrow \quad \sum_{\alpha} (k_1 \cdot k_{\alpha}) A(\alpha, 1, \beta, n) = 0$$

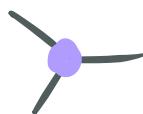
A red curved arrow points from the left side of the equation to the term $f^{12A} f^{A34}$.

$$f^{12A} f^{A34} = \text{Tr} [T^1 T^2 T^3 T^4] - \text{Tr} [T^1 T^2 T^4 T^3] - \text{Tr} [T^2 T^1 T^3 T^4] + \text{Tr} [T^2 T^1 T^4 T^3]$$

Could also introduce symmetric structure

Carrasco, NHP
2211.04431

$$d^{abc} = \text{Tr} [\{T^a, T^b\}, T^c] \quad \Rightarrow \quad f^{12A} f^{A34} = d^{14A} d^{A23} - d^{13A} d^{A24}$$



$$\text{Diagram: } \begin{array}{c} & & \\ & \diagup & \diagdown \\ & & \end{array} = \begin{array}{c} & & \\ & \diagup & \diagdown \\ & & \end{array} - \begin{array}{c} & & \\ & \diagdown & \diagup \\ & & \end{array}$$

Introducing symmetric structure

$$\begin{array}{c}
 \text{Diagram: } \\
 \begin{array}{ccc}
 \text{2} & & \text{3} \\
 \text{1} & \diagup & \diagdown \\
 & \text{---} & \\
 & \diagdown & \diagup \\
 \text{4} & & \text{3}
 \end{array}
 = \begin{array}{cc}
 \text{2} & \text{3} \\
 \text{1} & \diagup \\
 & \text{---} \\
 & \diagdown \\
 \text{4} &
 \end{array}
 + \begin{array}{cc}
 \text{4} & \text{2} \\
 \text{1} & \diagup \\
 & \text{---} \\
 & \diagdown \\
 \text{3} &
 \end{array}
 \end{array}
 \iff \sum_{\alpha} (k_1 \cdot k_{\alpha}) A(\alpha, 1, \beta, n) = 0$$

(Red curved arrow pointing from the first diagram to the equation)

$$f^{12A} f^{A34} = \text{Tr} [T^1 T^2 T^3 T^4] - \text{Tr} [T^1 T^2 T^4 T^3] - \text{Tr} [T^2 T^1 T^3 T^4] + \text{Tr} [T^2 T^1 T^4 T^3]$$

Could also introduce symmetric structure

Carrasco, NHP
2211.04431

$$d^{abc} = \text{Tr} [\{T^a, T^b\}, T^c] \quad \Rightarrow \quad f^{12A} f^{A34} = d^{14A} d^{A23} - d^{13A} d^{A24}$$

$$\begin{array}{c}
 \text{Diagram: } \\
 \begin{array}{c}
 \text{---} \\
 | \quad | \\
 \text{---} \\
 | \quad | \\
 \text{---}
 \end{array}
 = \begin{array}{c}
 \text{---} \\
 | \quad | \\
 \text{---} \\
 | \quad | \\
 \text{---}
 \end{array}
 - \begin{array}{c}
 \text{---} \\
 | \quad | \\
 \text{---} \\
 | \quad | \\
 \text{---}
 \end{array}
 \end{array}$$

(Red curved arrow pointing from the first diagram to the equation)

Decompose adjoint into symmetric structure

Introducing symmetric structure

Applying this to color-dual NLSM

Carrasco, NHP
2211.04431

$$A^{\text{NLSM}} = C_S^{\text{ff}} U + C_U^{\text{ff}} S \longrightarrow$$

$$\begin{aligned} \text{Y} &= \text{---} - \text{---} \\ C_S^{\text{ff}} &= C_t^{\text{dd}} - C_U^{\text{dd}} \end{aligned}$$

$$= C_S^{\text{dd}} S + C_t^{\text{dd}} t + C_U^{\text{dd}} U$$



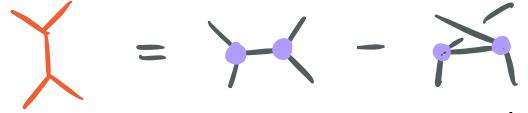
larger color basis

Introducing symmetric structure

Applying this to color-dual NLSM

Carrasco, NHP
2211.04431

$$A^{\text{NLSM}} = C_S^{\text{ff}} v + C_U^{\text{ff}} S \longrightarrow$$



$$C_S^{\text{ff}} = C_t^{\text{dd}} - C_U^{\text{dd}}$$

$$= \frac{C_S^{\text{dd}} S^2}{S} + \frac{C_t^{\text{dd}} t^2}{t} + \frac{C_U^{\text{dd}} v^2}{v}$$

NLSM is both an **adjoint** AND **symmetric** double-copy

$$n_S^{\text{ff}} = t^2 - v^2$$

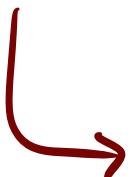
$$n_S^{\text{dd}} = S^2$$

$$A^{\text{NLSM}} = \sum_g \frac{c_g^{\text{ff}} n_g^{\text{ff}}}{d_g} = \sum_g \frac{c_g^{\text{dd}} n_g^{\text{dd}}}{d_g}$$

* verified
through
6-point tree

Double copy to Photon EFT

$$\mathcal{M}_{(++++)}^{\text{BI,1-loop}} \sim (s^4 + t^4 + u^4) \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} \quad \Rightarrow \quad n_s^{dd} = s^3 \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle}$$


$$\mathcal{M}^{\text{BI+HD}} = \sum_g \frac{n_g^{\pi,\text{dd}} n_g^{\text{vec,dd}}}{d_g}$$

Double copy to Photon EFT

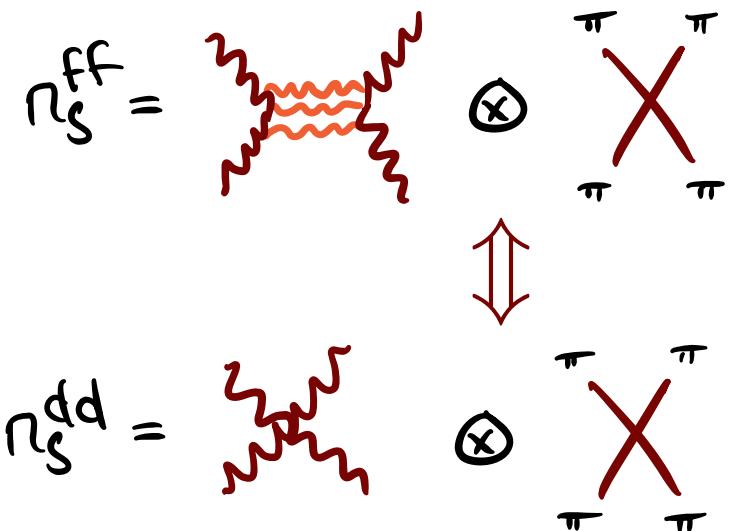
$$\mathcal{M}_{(++++)}^{\text{BI,1-loop}} \sim (s^4 + t^4 + u^4) \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle}$$



$$n_s^{dd} = s^3 \frac{[12] [34]}{\langle 12 \rangle \langle 34 \rangle}$$

 $\mathcal{M}^{\text{BI+HD}} = \sum_g \frac{n_g^{\pi, \text{dd}} n_g^{\text{vec, dd}}}{d_g}$

$$n_s^{\text{ff}} = (t^3 - u^3) \frac{[12] [34]}{\langle 12 \rangle \langle 34 \rangle}$$



Double copy to Photon EFT

$$\mathcal{M}_{(++++)}^{\text{BI,1-loop}} \sim (s^4 + t^4 + u^4) \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle}$$



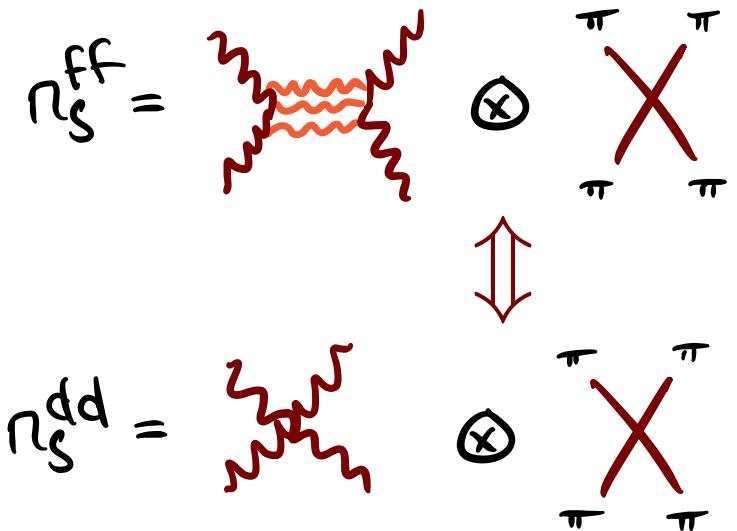
$$n_s^{dd} = s^3 \frac{[12] [34]}{\langle 12 \rangle \langle 34 \rangle}$$

$$\mathcal{M}^{\text{BI+HD}} = \sum_g \frac{n_g^{\pi,dd} n_g^{\text{vec,dd}}}{d_g}$$

$$n_s^{\text{ff}} = (t^3 - u^3) \frac{[12] [34]}{\langle 12 \rangle \langle 34 \rangle}$$

Symmetric structure enables additional freedom for EFT model building

$$\begin{aligned} \mathcal{A}_4^{\text{vec+HD}} = & \sum_{g \in \Gamma^{(3)}} \sum_{x,y} \left[a_{(x,y)}^{F^3} \sigma_3^x \sigma_2^y \frac{n_g^{\text{vec},F^3} c_g^{\text{ff}}}{d_g} \right. \\ & + a_{(x,y)}^{F^2 F^2} (n_g^{\text{dd},1})^x (n_g^{\text{dd},2})^y \frac{n_g^{\text{vec,dd},1} c_g^{\text{dd}}}{d_g} \\ & \left. + a_{(x,y)}^{F^4} (n_g^{\text{dd},1})^x (n_g^{\text{dd},2})^y \frac{n_g^{\text{vec,dd},2} c_g^{\text{dd}}}{d_g} \right]. \quad (48) \end{aligned}$$



$$n_s^{\text{vec,dd},1} \equiv (F_1 F_2) (F_3 F_4)$$

$$n_s^{\text{vec,dd},2} \equiv (F_1 F_3 F_2 F_4)$$

