Color-dual Constraints on Gravitational EFT

Nic H Pavao

June 27, 2023

2203.03592, 2211.04441, 2307.XXXX

[with JJ Carrasco & Matt Lewandowski]

Amplifying Gravity at All Scales

Northwestern University

NORDITA
The Nordic Institute for Theoretical Physics

Insights of this Talk

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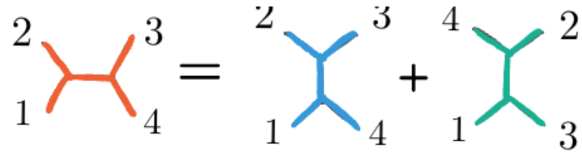
Intro to Amplitude-level structure

- Review of color-kinematics duality



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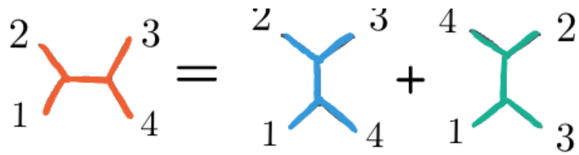
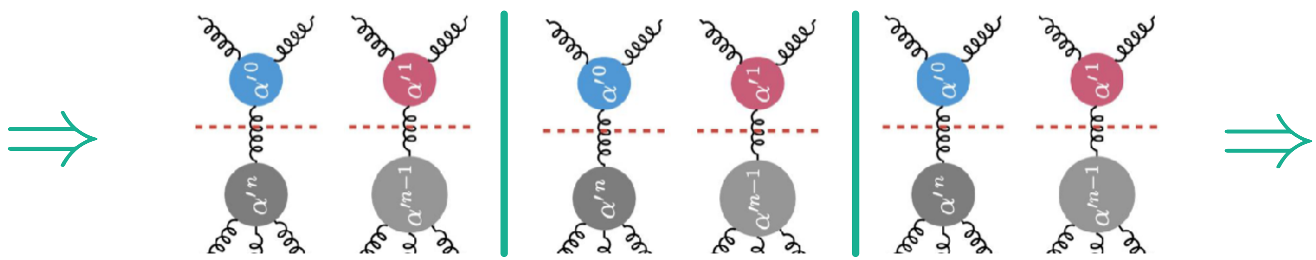


Color-dual constraints on EFT

- Bootstrap for gauge/gravity EFT operators
- Emergence of massive UV modes

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

Carrasco, NHP
2307.xxxxx



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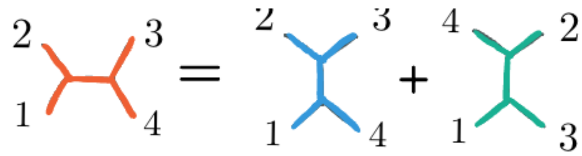

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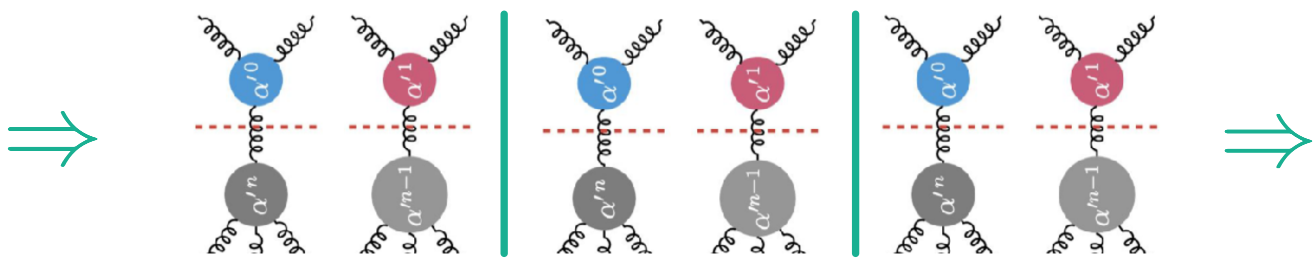
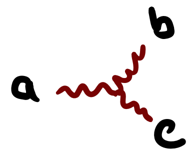
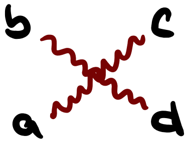


Applications to Gravitational EFT & Future directions

Amplitude-level structure

$$\mathcal{L}_{\text{int}}^{\text{YM}} = f^{abc}$$ 

$$+ f^{abe} f^{ecd}$$ 

$$f^{abc} = \text{tr}\left[T^a T^b T^c\right] - \text{tr}\left[T^a T^c T^b\right]$$

$$T^e_{ij} T^e_{kl} = \delta_{il}\delta_{jk} + \mathcal{O}\left(1/N_c\right)$$

non-planar

planar

$$\mathcal{A}_n^{\text{YM}} = \sum_{\sigma \in S^{n-1}} C(\sigma) A(1, \sigma_2, \ldots, \sigma_n)$$

Amplitude-level structure

$$\mathcal{A}_n^{\mathrm{YM}} = \sum_{\sigma \in S^{n-1}} C(\sigma) \underline{A(1, \sigma_2, ..., \sigma_n)}$$

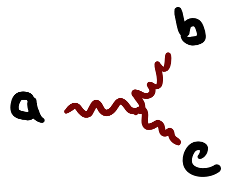
$$A\left(1, \alpha, n, \beta\right) = (-1)^{|\alpha|} \sum_{\sigma \in \alpha \sqcup\!\sqcup\, \beta^T} A\left(1, \sigma, n\right)$$

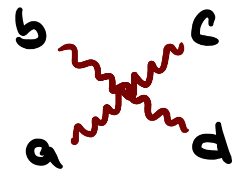
Kleiss-Kluijf (1989)

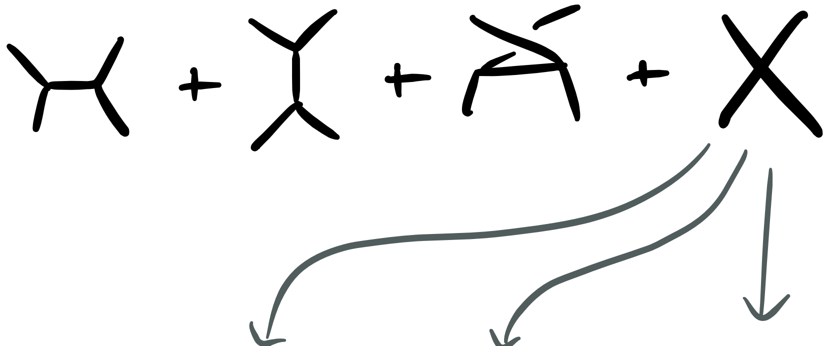
$$\sum_{i=2}^{n-1} k_1 \cdot (k_2 + \cdots + k_i) A(2, \ldots, i, 1, \ldots, n) = 0$$

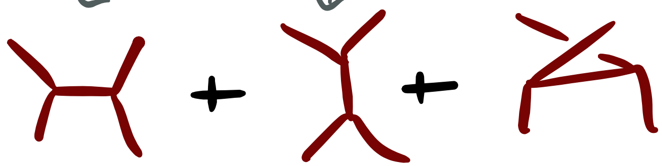
Bern- Carrasco-Johansson (2008)

Amplitude-level structure

$$\mathcal{L}_{\text{int}}^{\text{YM}} = f^{abc}$$ 

$$+ f^{abe} f^{ecd}$$ 

$$\mathcal{A}_4^{\text{YM}} =$$ 

$$\mathcal{A}_4^{\text{YM}} =$$ 

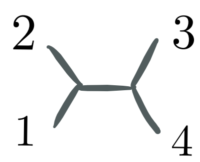
grows like φ³-theory, (2n-5)!!

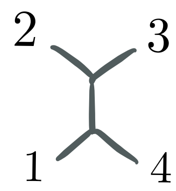
$$\mathcal{A} = \int d\Omega_l \sum_g \frac{1}{S_g} \frac{c_g \tilde{n}_g}{d_g}$$

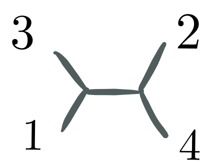
Amplitude-level structure

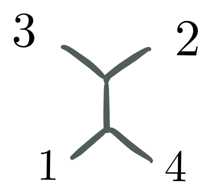
$$A\left(1,\alpha,n,\beta\right)=\left(-1\right)^{|\alpha|}\sum_{\sigma\in\alpha\,\text{Ш}\,\beta^{T}}A\left(1,\sigma,n\right)$$

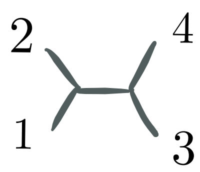
Kleiss-Kluijf (1989)

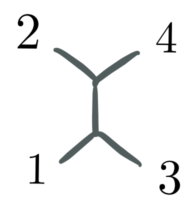
$A\left(1234\right)=$ 

\+ 

$A\left(1324\right)=$ 

\+ 

$A\left(1243\right)=$ 

\+ 

Amplitude-level structure

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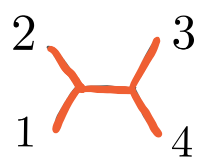
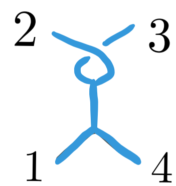
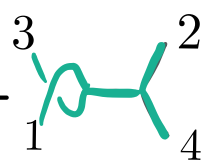
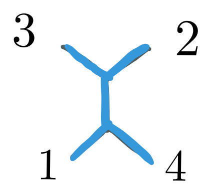
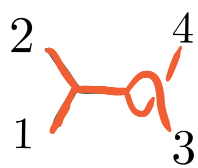
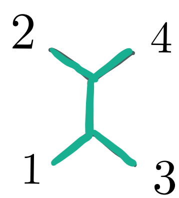
$A(1234) =$ 

$-$ 

$A(1324) = -$ 

$+$ 

$A(1243) = -$ 

$+$ 

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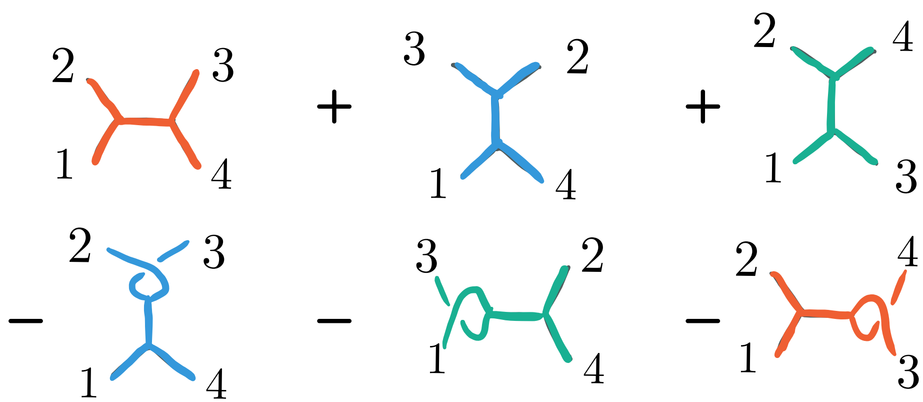
Kleiss-Kluijf (1989)

$$A\left(1234\right)+A\left(1324\right)+A\left(1243\right)=0$$



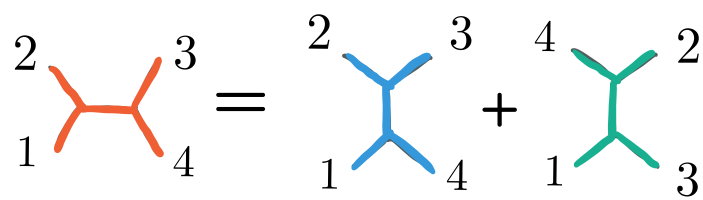
Amplitude-level structure

$$\sum_{i=2}^{n-1} k_1 \cdot (k_2 + \cdots + k_i) A(2, \ldots, i, 1, \ldots, n) = 0$$

Bern-Carrasco-Johansson (2008)

$$A(1234) = \frac{n_s}{s_{12}} + \frac{n_t}{s_{14}}$$

$$A(1324) = \frac{n_u}{s_{13}} - \frac{n_t}{s_{14}}$$



$$c_s = c_t + c_u$$

$$n_s = n_t + n_u$$

Amplitude-level structure

$$\sum_{i=2}^{n-1} k_1 \cdot (k_2 + \cdots + k_i) A(2, \ldots, i, 1, \ldots, n) = 0$$

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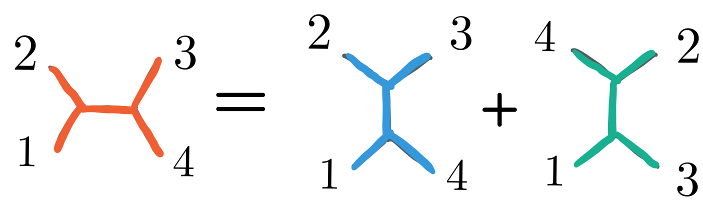


$$A\,(1324) = \frac{(n_s - n_t)}{s_{13}} - \frac{n_t}{s_{14}} = \frac{s_{12}}{s_{13}} A(1234)$$ ← D-dimensional

$$A\left(1^- 3^- 2^+ 4^+\right) = \frac{s_{12}}{s_{13}} A(1^- 2^+ 3^- 4^+)$$ ← D=4

Double-Copy Construction

When the partial amplitudes satisfy BCJ,
can replace c(g) with n(g) Bern, Carrasco, Johansson (2008)

$$c_i = c_j + c_k \quad \Leftrightarrow \quad n_i = n_j + n_k$$

Gauge theory for 1 SEK, Gravity for free!

$$\mathcal{A} = \int d\Omega_l \sum_g \frac{1}{S_g} \frac{c_g \tilde{n}_g}{d_g} \qquad \mathcal{M} = \int d\Omega_l \sum_g \frac{1}{S_g} \frac{n_g \tilde{n}_g}{d_g}$$

$$\epsilon^\mu \rightarrow k^\mu \quad \Leftrightarrow \quad \epsilon^\mu \epsilon^\nu \rightarrow k^\mu \epsilon^\nu + \epsilon^\mu k^\nu$$

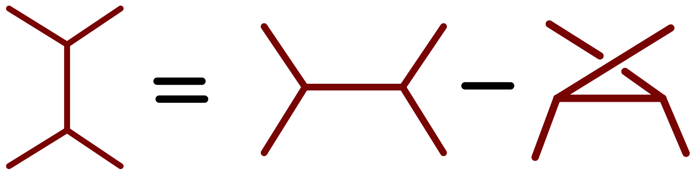
At tree level, can be performed
with KLT momentum kernel:

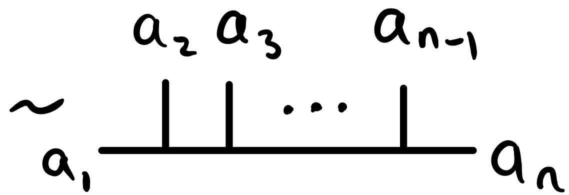
Kawai, Lewellen, Tye

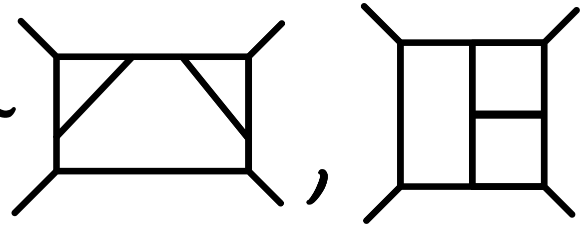
$$\mathcal{M}_n^{\mathrm{XY}} = \mathcal{A}_n^{\mathrm{X}} \otimes \mathcal{A}_n^{\mathrm{Y}}$$

Graphical Simplicity

Graphical organization
reveals hidden simplicity



$\mathcal{A}^{\text{tree}}_{n\text{-point}} \sim$ 

$\mathcal{A}^{\text{3-loop}}_{4\text{-point}} \sim$ 

Imposing this behavior
is equivalent to many
physical constraints

- gauge invariance
- soft theorems
- diffeo invariance

Carrasco, Rodina,
Cheung, Shen, Wen,...

Color-dual constraints on contact operators

Gauge/diffeo invariance and soft theorems

The traditional story

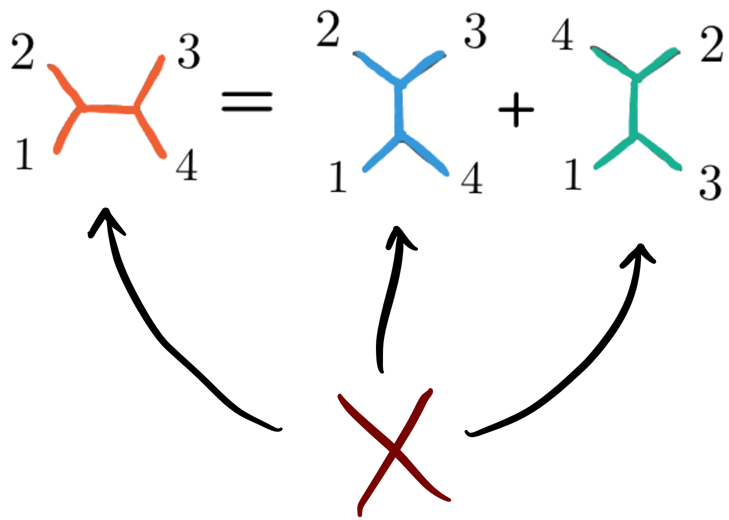
color-kinematics = gauge invariance

$$\mathcal{L} = (\partial A)^2 + g_1\, A^2(\partial A) + g_2\, A^4$$

The traditional story

color-kinematics = gauge invariance

$$\mathcal{L} = (\partial A)^2 + g_1 A^2(\partial A) + g_2 A^4 = -\frac{1}{4}F^2$$



$$\Rightarrow \quad g_2 \sim g_1^2$$

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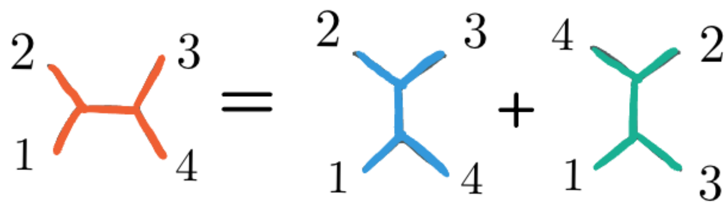
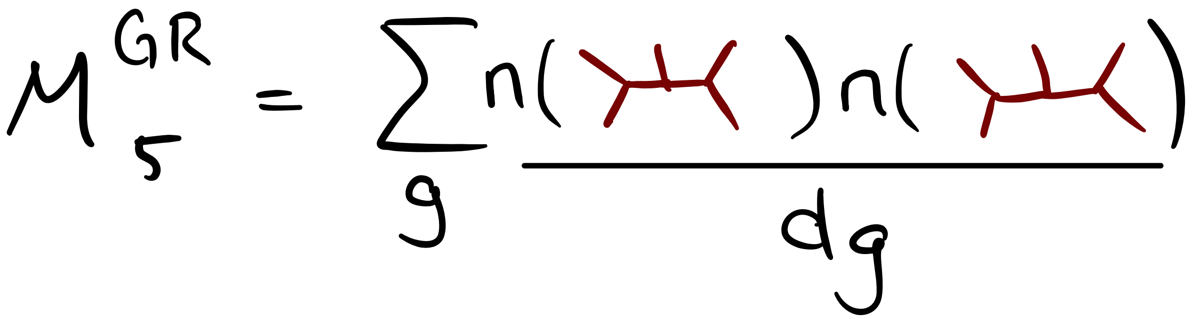


$$\Rightarrow \quad g_2 \sim g_1^2$$

color-kinematics + double copy = linear diffeo inv.



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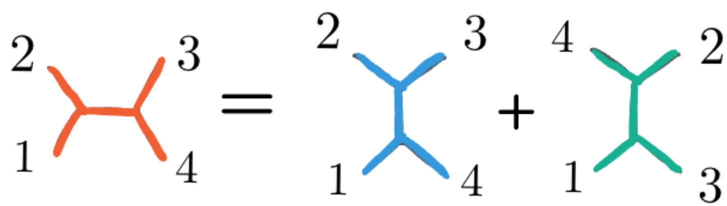

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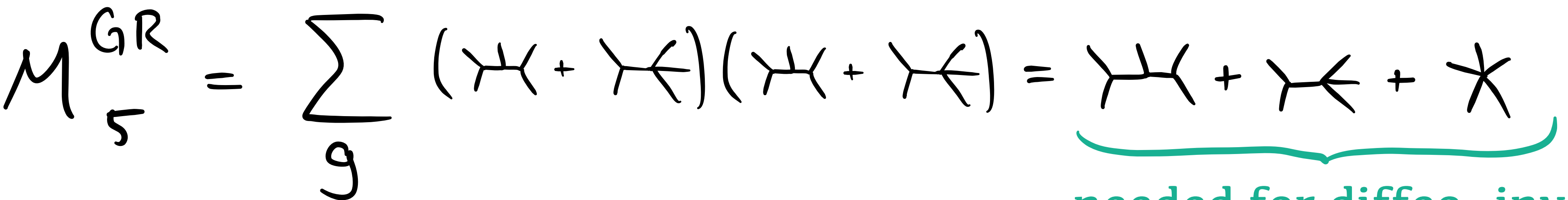
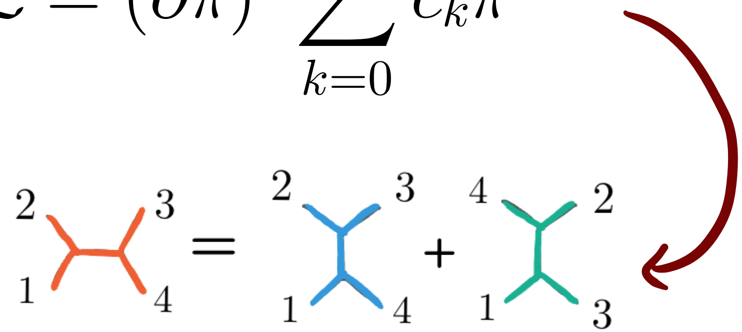


needed for diffeo. inv.

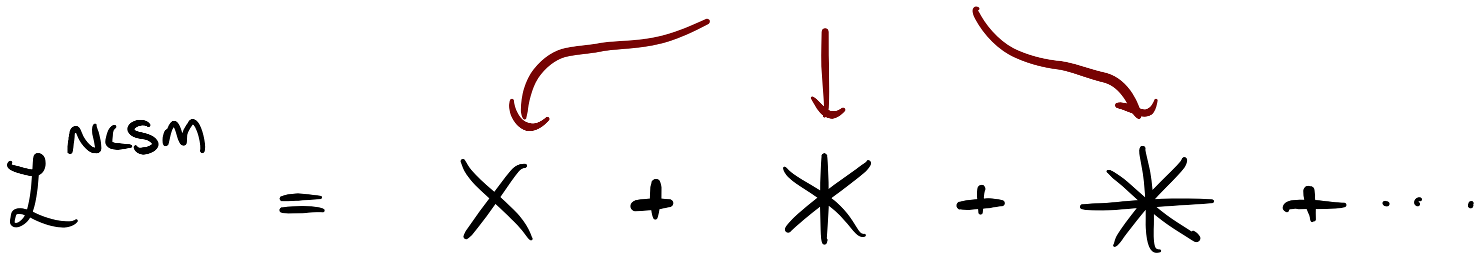
The traditional story

color-kinematics = soft bootstrap

$$\mathcal{L} = (\partial\pi)^2 \sum_{k=0} c_k \pi^{2k}$$



Carrasco, Rodina, Cheung, Shen, Wen,...



Resums to NLSM

$$\mathcal{L}^{\mathrm{NLSM}} = (\partial U)^\dagger (\partial U)\,, \quad U = e^{i\pi}$$

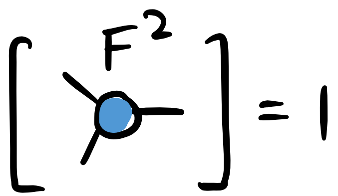
Color-dual constraints on higher-derivatives

Climbing towers to emergent massive modes

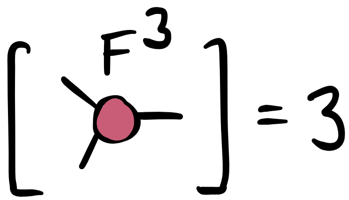
Higher-derivative constraints = tower

Novel contributions to pure vector

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 \quad +\alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$



$$A_3^{\text{YM}} = (\varepsilon_1 \varepsilon_2)(\varepsilon_3 k_1) + \text{cyc}$$



$$A_3^{F^3} = (\varepsilon_1 k_2)(\varepsilon_2 k_3)(\varepsilon_3 k_1)$$

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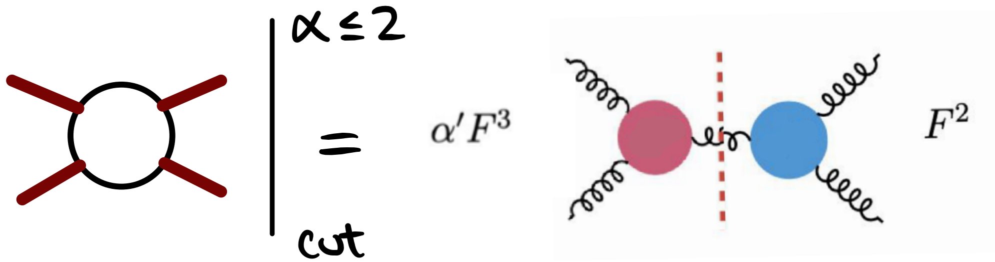
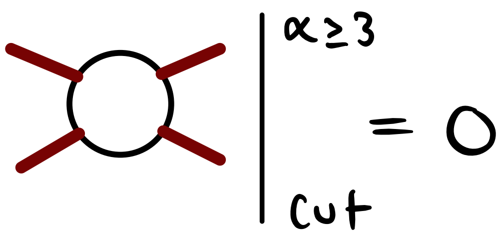
Broedel, Dixon

Carrasco, Rodina, Yin, Zekioglu





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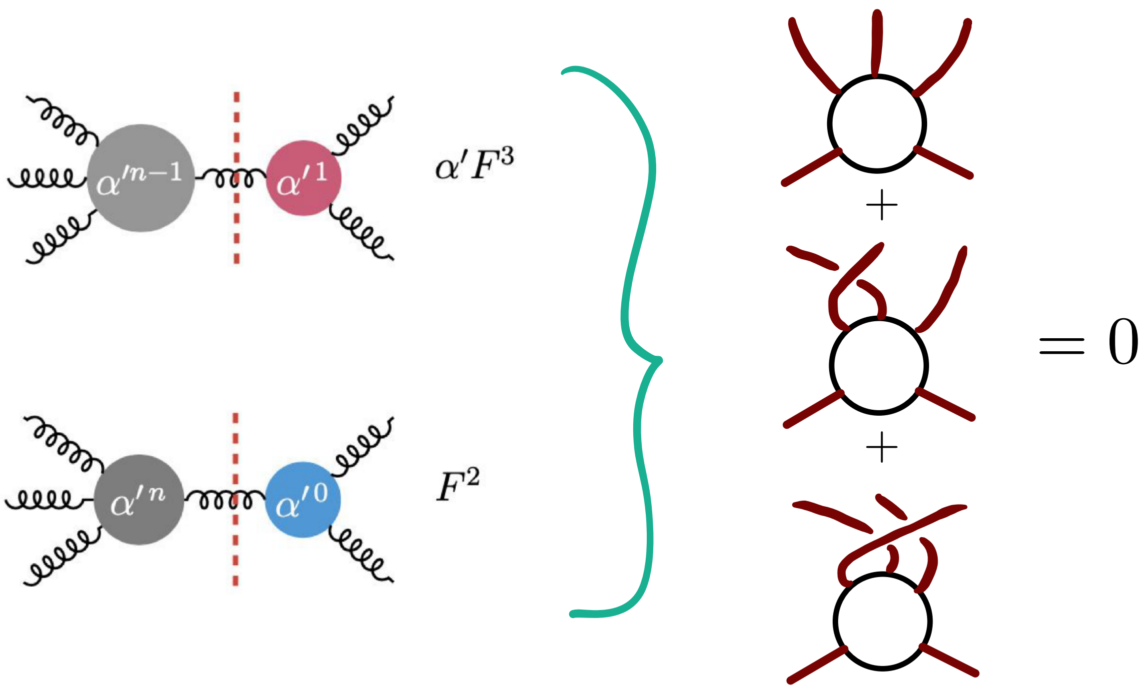

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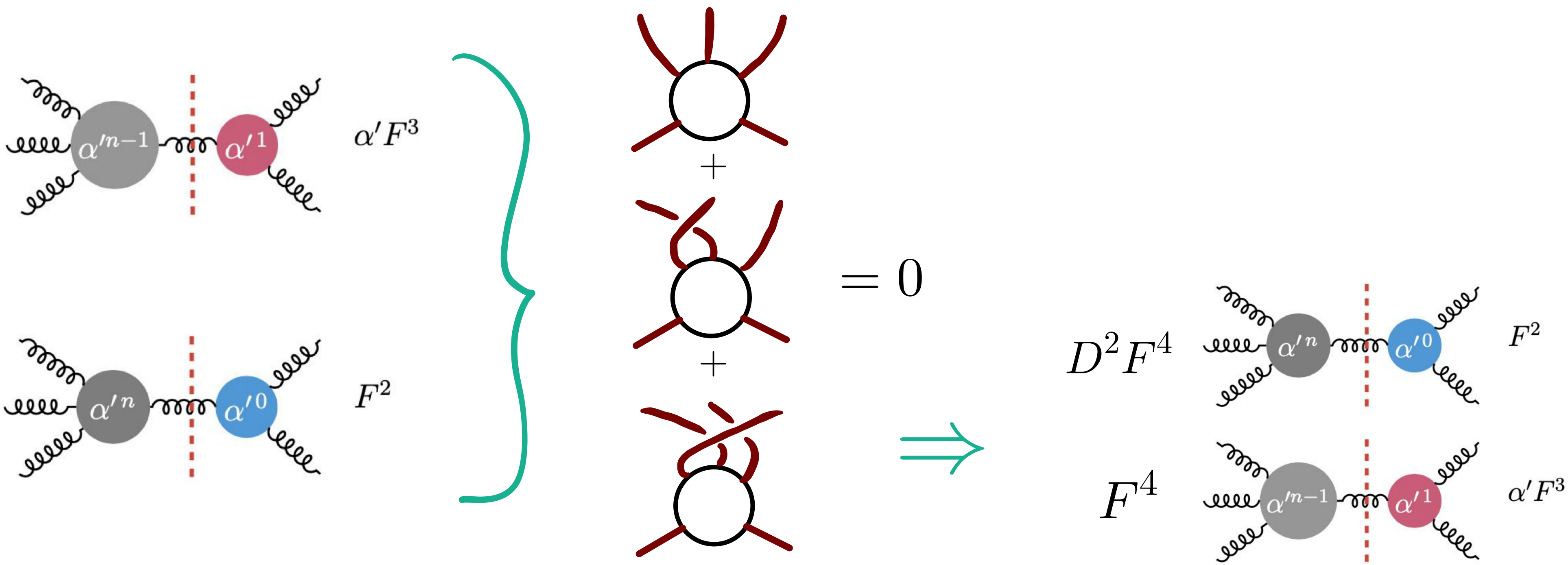

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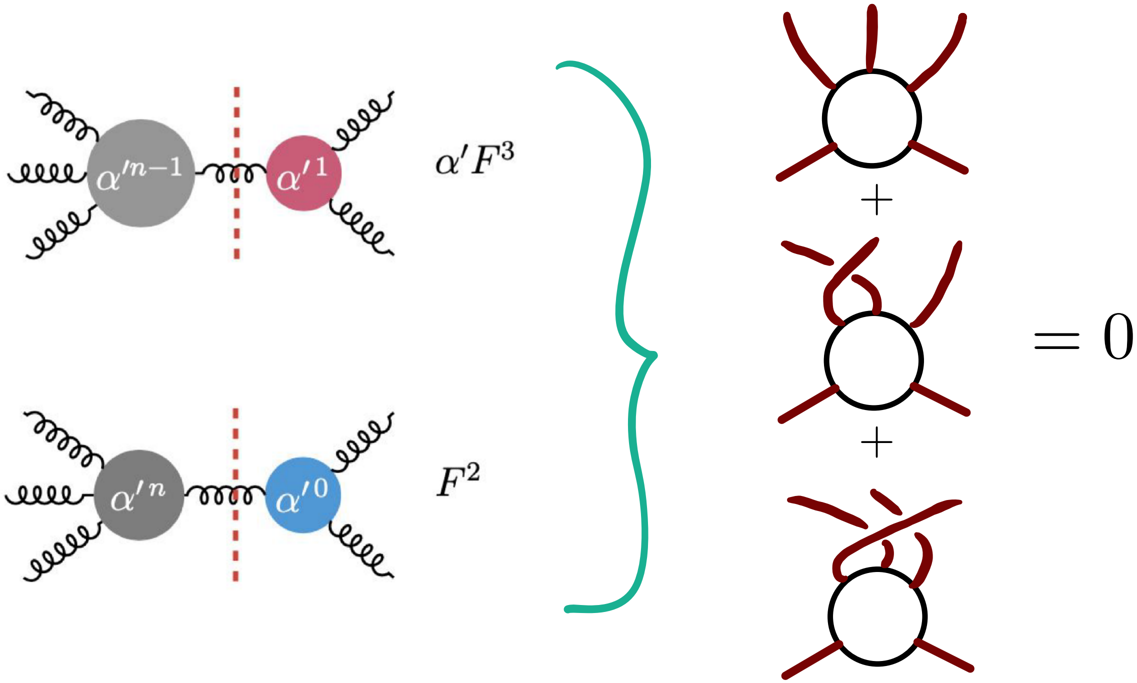

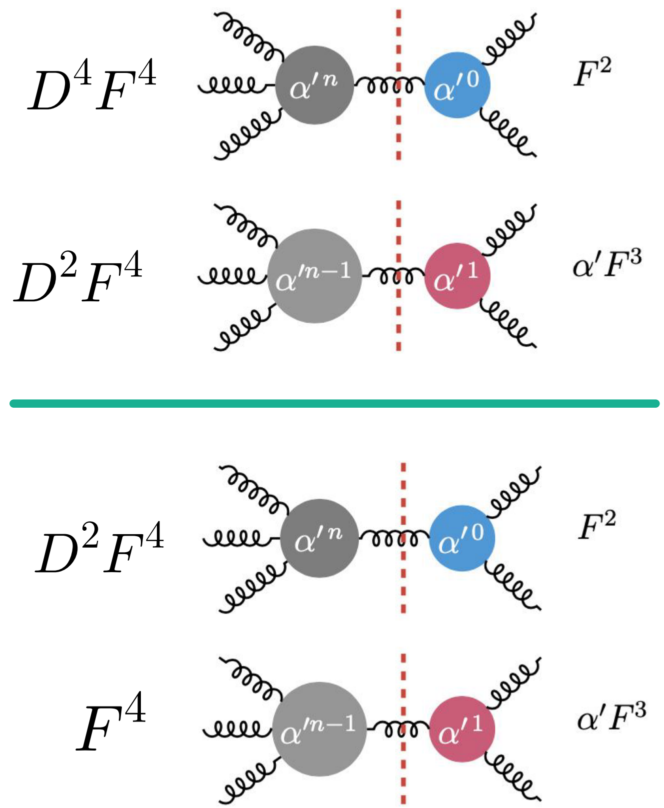

Higher-derivative constraints

Infinite tower to UV!

Novel contributions to pure vector

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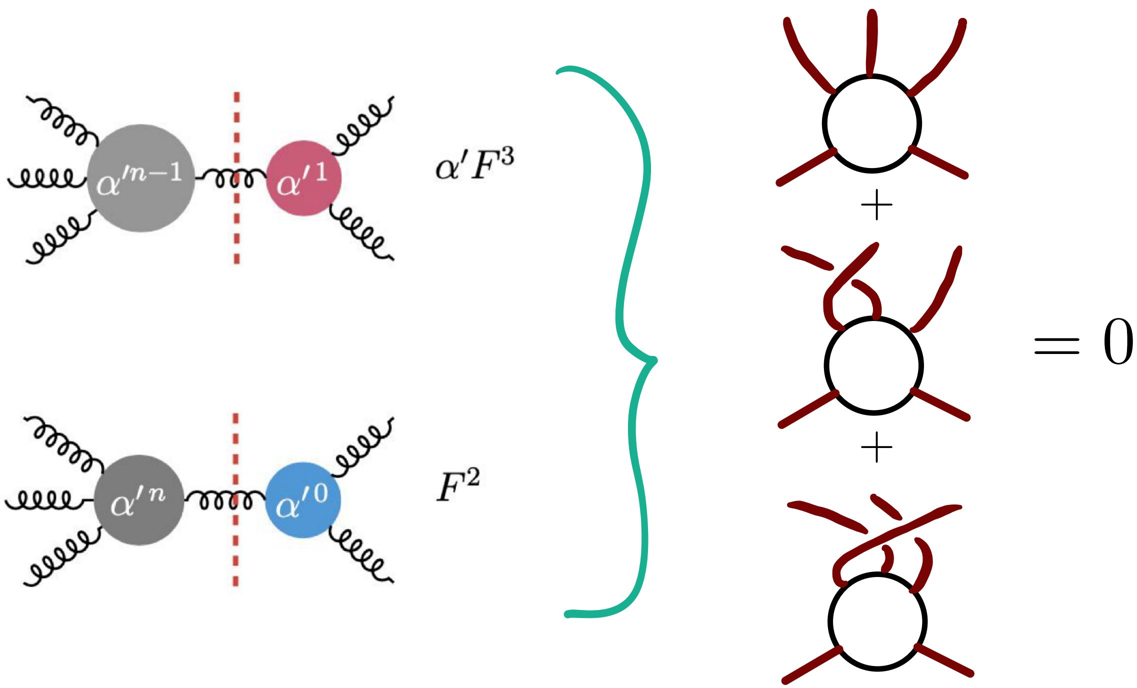
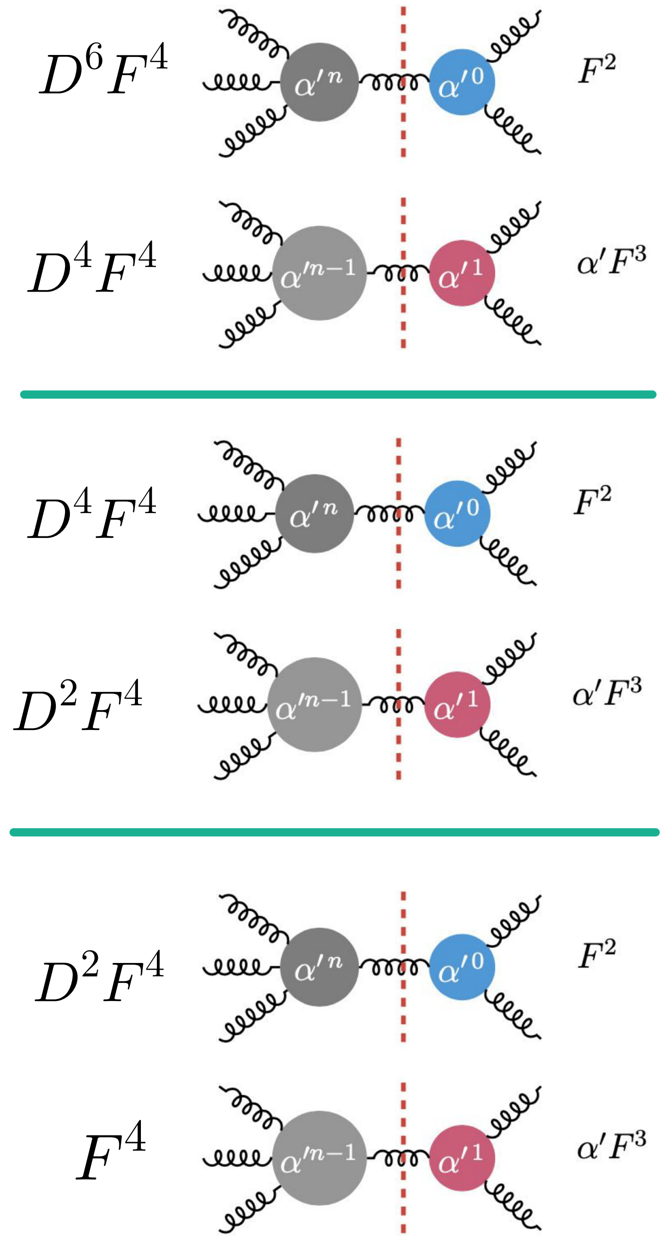
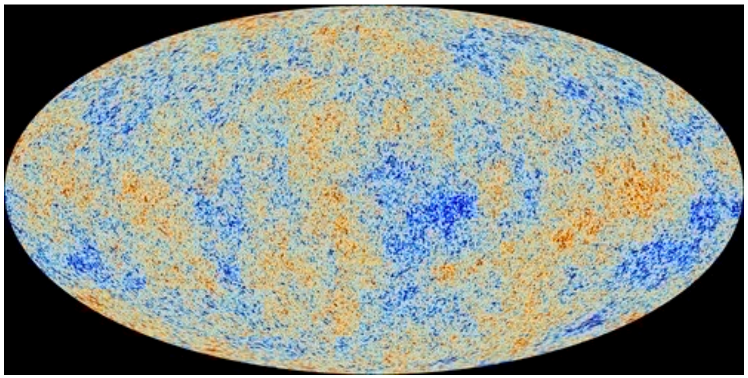


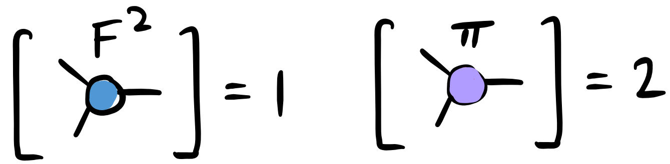


Are gauged-pions color-dual?

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

$$\mathcal{L}^{\text{cov.}\pi} = \frac{1}{2}\text{Tr}\left[\frac{D_\mu \pi\, D^\mu \pi}{\left(1-\lambda\pi^2\right)^2}\right] - \frac{1}{4}\text{Tr}\left[F^2\right]$$





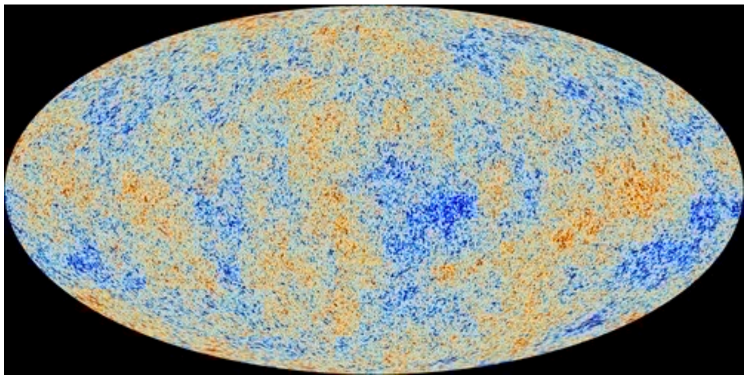
α- attractor inflation

$$\mathcal{M}^{\text{DBIVA}+\text{SG}} = \mathcal{A}^{\text{SYM}} \otimes \mathcal{A}^{\pi+\text{YM}}$$

Are gauged-pions color-dual?

Carrasco, Lewandowski, NHP
2203.03592, 2211.04441

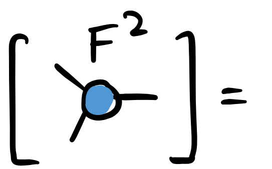
$$\mathcal{L}^{\text{cov.}\pi} = \frac{1}{2}\text{Tr}\left[\frac{D_\mu\pi\, D^\mu\pi}{(1-\lambda\pi^2)^2}\right] - \frac{1}{4}\text{Tr}\left[F^2\right] + \frac{\lambda}{g} F^3 + \cdots$$

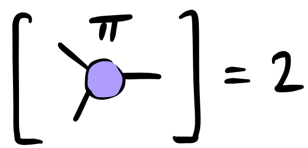


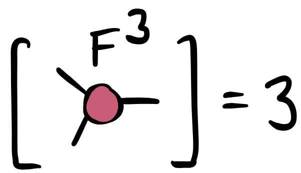
Need higher derivative tower

α- attractor inflation

$$\mathcal{M}^{\text{DBIVA}+\text{SG}} = \mathcal{A}^{\text{sYM}} \otimes \mathcal{A}^{\pi+\text{YM}}$$







Color-dual towers resum

Carrasco, Lewandowski, NHP
2203.03592, 2211.04431

same goes for scalar-sector

$$A_4^{(n)}(gggg) = u\left[\frac{(F_1F_2)(F_3F_4)}{s_{12}^2}(\lambda s_{12})^n + \text{cyc}(1,2,3)\right]$$

YM + F3 contacts

$$A_4^{(n)}(\pi\pi gg) = u\frac{(F_3F_4)}{s_{12}}(\lambda s_{12})^n$$

$$A_4^{(n)}(\pi\pi\pi\pi) = u\left[(\lambda s_{12})^{n-1} + \text{cyc}(1,2,3)\right]$$

$\mathcal{O}(\Lambda^n)$ \ $\lVert\pi\rVert = 2k$	$k=0$	$k=1$	$k=2$
$n=0$	YM 0	YMS 0	YMS 0
$n=1$	F^3 0	0	NLSM 0
$n=2$	F^4 0	0	0
$n=3$	D^2F^4 1	1	1
$n=4$	D^4F^4 1	1	1
$n=5$	D^6F^4 2	2	2

Color-dual towers resum

Carrasco, Lewandowski, NHP
2203.03592, 2211.04431

same goes for scalar-sector

$$A_4^{(n)}(gggg) = u\left[\frac{(F_1F_2)(F_3F_4)}{s_{12}^2}(\lambda s_{12})^n + \text{cyc}(1,2,3)\right]$$

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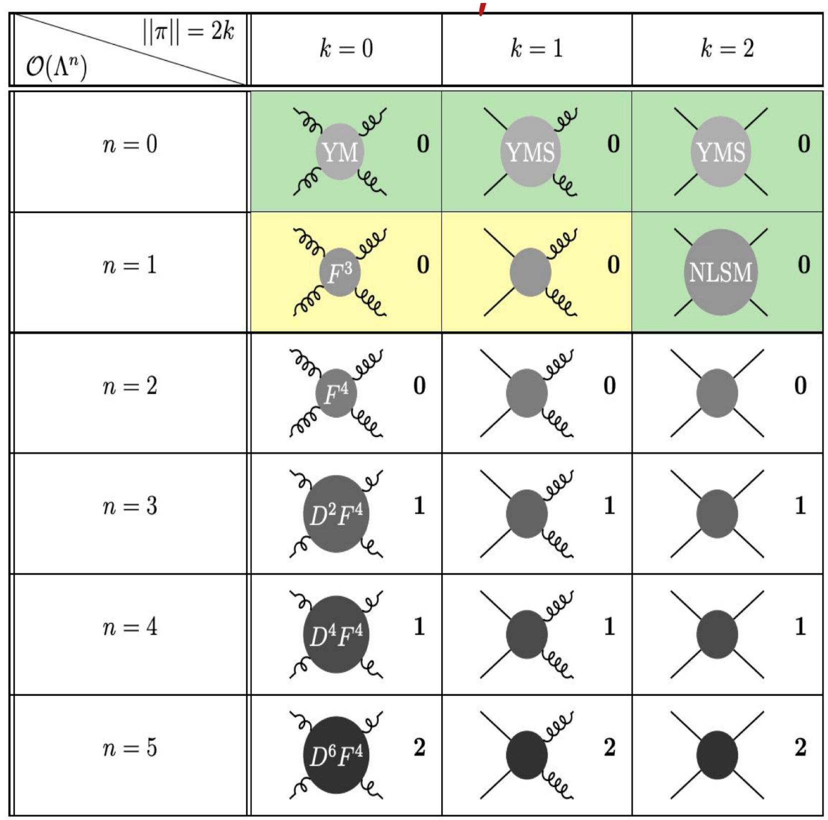


Resums to reveal a new massive residue in the UV!

$$m_{\text{UV}}^2 \sim \lambda^{-1}$$

$$A_{(1234)}^{\text{full}} \sim \sum_n A_{(1234)}^{(n)} \sim 1 + \lambda s_{12} + (\lambda s_{12})^2 \cdots$$

$$\sim \frac{\lambda}{s_{12} - m_{\text{UV}}^2} + \text{cyc}(1,2,3)$$

New massive particle residues

Higher derivative color-dual numerators encode massive residues

$$\mathcal{A} = \sum \frac{c_g n_g^{\mathrm{HD}}}{d_g} \sim \sum \frac{c_g n_g^{m}}{d_g - m^2}$$

$$\mathcal{L}^{\mathrm{YM}+F^3+\cdots} = -\frac{1}{4}F^2 + \frac{1}{3m^2}F^3 + \frac{1}{m^4}F^4 + \frac{1}{m^6}D^2F^4 + \frac{1}{m^8}D^4F^4 + \cdots$$

Resums to a color-dual dim-6 theory

$\Downarrow$ Setting remaining freedom to zero

$$\mathcal{L}^{\mathrm{YM}+DF^2} = (DF)^2 + (D\varphi)^2 - m^2(\varphi^2 + F^2) + \mathcal{L}_{\mathrm{int}}(\varphi, F)$$

Johansson, Nohle

How much freedom remains?

5-point constraints

$$\mathcal{L}^{\mathrm{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

$$\Rightarrow \quad \mathcal{L}^{DF^2+\mathrm{YM}+\mathrm{HD}}$$

$$\Downarrow$$

$$A_4^{DF^2+\mathrm{YM}+\mathrm{HD}} = A_4^{DF^2+\mathrm{YM}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

$$\sigma_k = s^k + t^k + u^k$$

More structure @ 6-point?

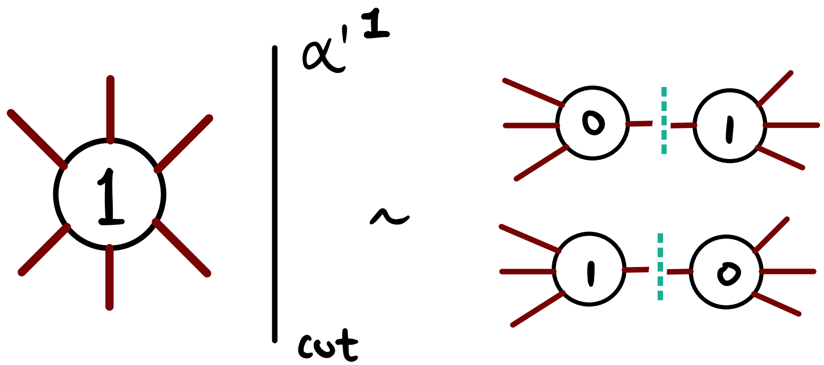
5-point constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4 \quad \Rightarrow \quad \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$

$$\Downarrow$$

$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

6-point constraints



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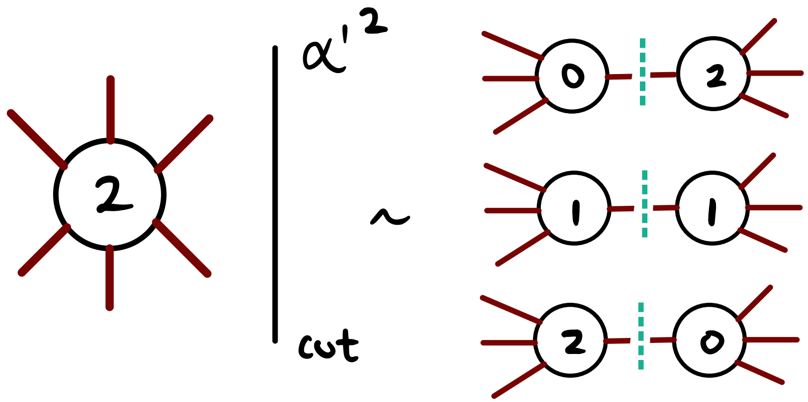
$$A_4^{DF^2+\mathrm{YM}+\mathrm{HD}} = A_4^{DF^2+\mathrm{YM}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

6-point constraints



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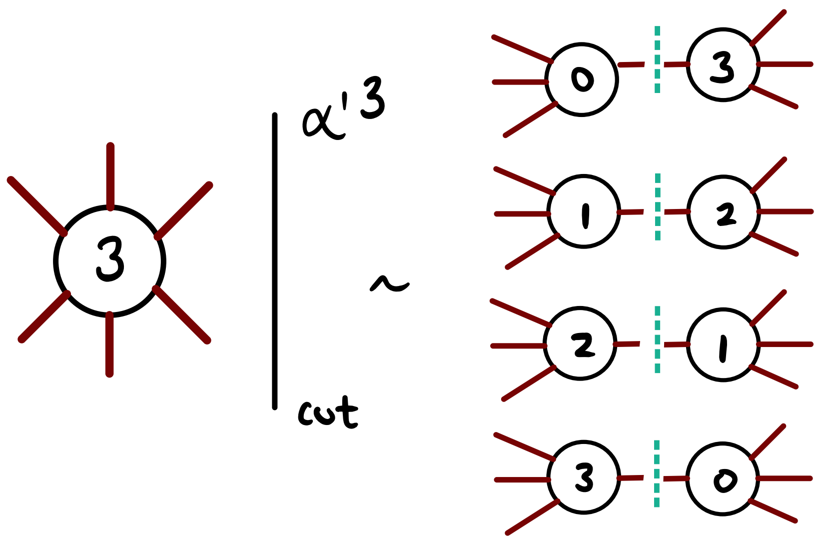
$$\mathcal{L}^{\mathrm{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

$$\Rightarrow \quad \mathcal{L}^{DF^2+\mathrm{YM}+\mathrm{HD}}$$

$$\Downarrow$$

$$A_4^{DF^2+\mathrm{YM}+\mathrm{HD}} = A_4^{DF^2+\mathrm{YM}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

6-point constraints



5-point vectors
>58K parameters
through $\mathcal{O}(\alpha^4)$

More structure @ 6-point?

5-point constraints

$$\mathcal{L}^{\text{YM}+F^3} = -\frac{1}{4}F^2 + \frac{\alpha'}{3}F^3 + \alpha'^2 F^4 + \alpha'^2 \sum_n c_{(n)} \alpha'^n D^{2n} F^4$$

$$\Rightarrow \quad \mathcal{L}^{DF^2+\text{YM}+\text{HD}}$$

$$\Downarrow$$

$$A_4^{DF^2+\text{YM}+\text{HD}} = A_4^{DF^2+\text{YM}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

Scalar theory examples: additional constraints!

6-point constraints

$$A_4^{\text{BAS}+\text{HD}} = A_4^{\text{BAS}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

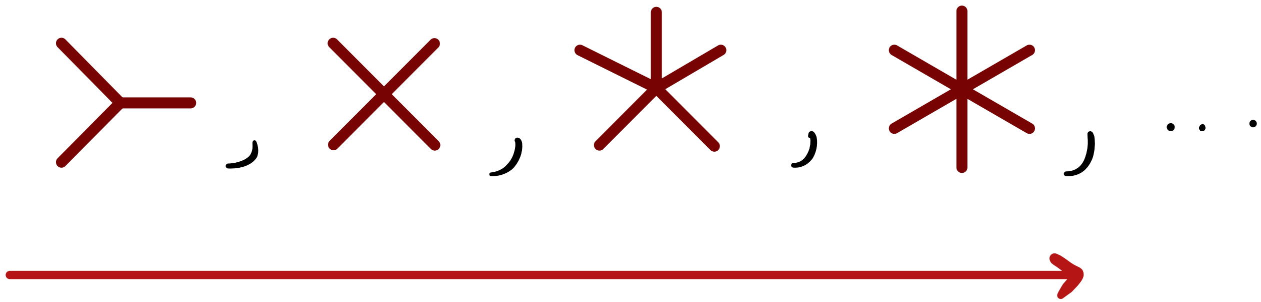
Chen, Elvang, Herderschee 2302.04895

$$A_4^{\chi\text{PT}} = A_4^{\text{NLSM}} \left(1 + \sum c_{(x,y)}\, \sigma_3^x \sigma_2^y\right)$$

Brown, Kampf, Oktem, Paranjape, Trnka 2305.05688

Takeaway: Color-dual + EFT

- Normally color-kinematics fixes OUT



Takeaway: Color-dual + EFT

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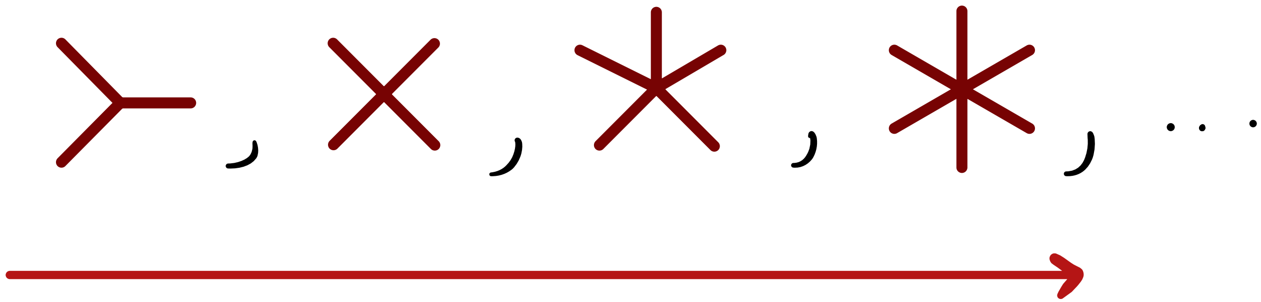
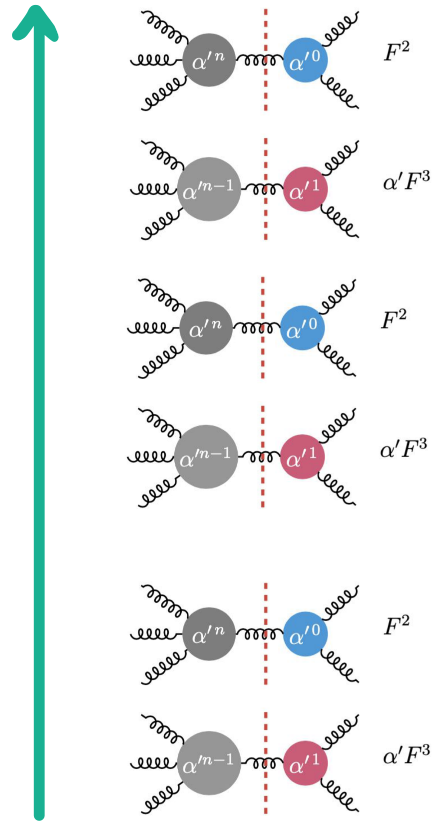
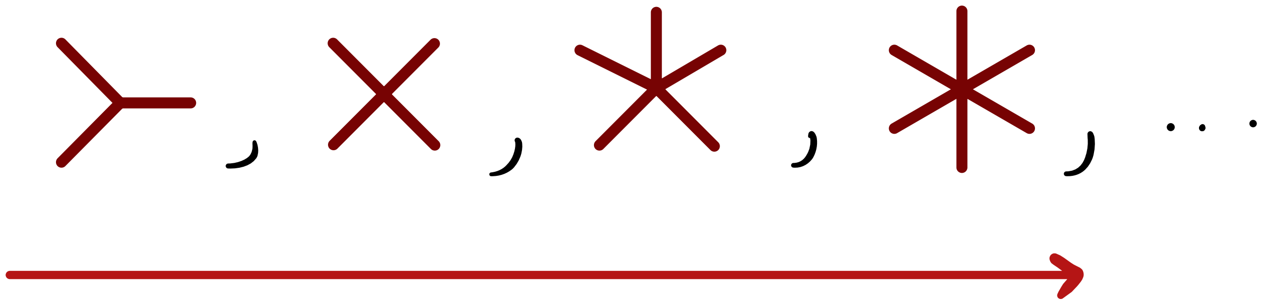


- But considering higher-derivatives, color-kinematics fixes operators UP

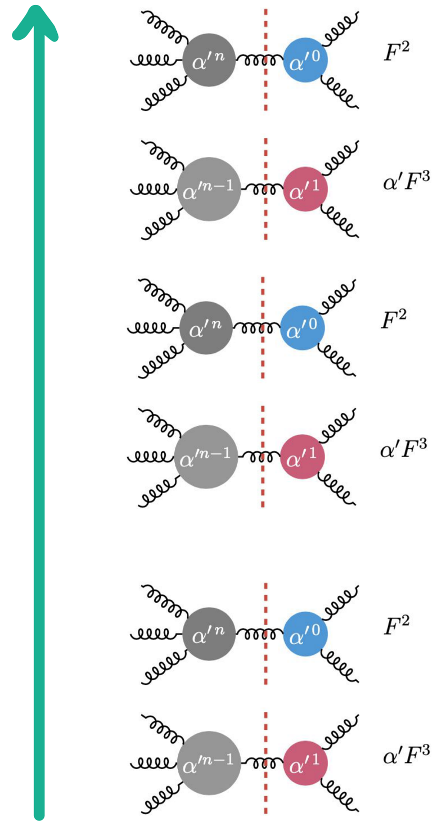


Takeaway: Color-dual + EFT

- Normally color-kinematics fixes OUT



- But considering higher-derivatives, color-kinematics fixes operators UP



- Introduces off-shell massive modes

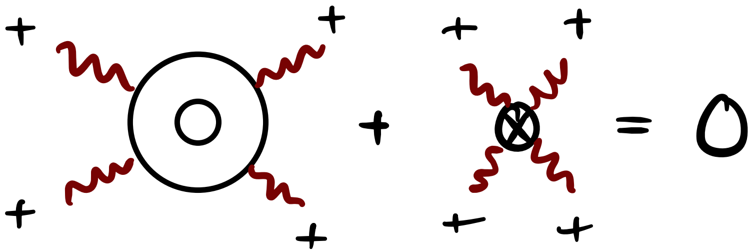
Significantly more rigid than envisioned

Modeling building implications for gravitational EFT

$$\mathcal{M}_n^{\mathrm{XY}} = \mathcal{A}_n^{\mathrm{X}} \otimes \mathcal{A}_n^{\mathrm{Y}}$$

App 1: UV completion of gravity?

$$\mathcal{M}^{\mathcal{N}=4\ \mathrm{SG}} = A^{\mathcal{N}=4\ \mathrm{sYM}} \otimes A^{\mathrm{YM}+F^3}$$



$$A_n^{\mathrm{N}^j\mathrm{MHV}} \otimes A_n^{\mathrm{N}^k\mathrm{MHV}} = 0 \quad j \neq k$$

@ tree-level

$$A^{1-\mathrm{loop}}_{(+,+,+,+)} = \frac{1}{3i}\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle} + \mathcal{O}(\epsilon)$$

anomalous @ 1-loop

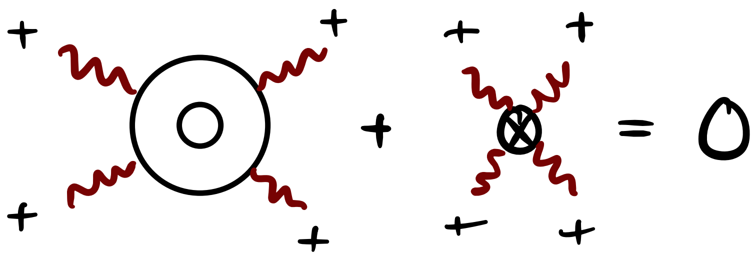
$$\mathcal{M}^{R^3} = A^{\mathrm{YM}+F^3} \otimes A^{\mathrm{YM}+F^3}$$

Bern, Carrasco, Edison, Kosower, Parra-Martinez, Roiban, Kallosh, and many others...

S-matrix anomalies source UV divergences

App 1: UV completion of gravity?

$$\mathcal{M}^{\mathcal{N}=4\ \mathrm{SG}} = A^{\mathcal{N}=4\ \mathrm{sYM}} \otimes A^{\mathrm{YM}+DF^2}$$



$$\mathcal{M}^{R^3} = A^{\mathrm{YM}+DF^2} \otimes A^{\mathrm{YM}+DF^2}$$

Einstein-Weyl conformal supergravities

The color-dual fates of F^3, R^3, and $\mathcal{N}=4$ supergravity

John Joseph M. Carrasco,[1,2] Matthew Lewandowski,[1] and Nicolas H. Pavao[1]

[1] *Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*
[2] *Institut de Physique Théorique, Universite Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France*

We find that the massless gauge theory of Yang-Mills deformed by a higher-derivative F^3 operator can not be tree-level color-dual without additional counterterms. The requirement of color-dual kinematics and consistent factorization between four- and five-points induces a tower of increasingly higher-dimensional operators. We find through explicit calculation that their amplitudes are consistent with the α' expansion of those generated by the $(DF)^2$ + YM theory, a known color-dual

Carrasco, Lewandowski, NHP
2203.03592

App 2: Cancel Born-Infeld anomaly?

Elvang, et al.

$$\mathcal{M}^{\mathrm{BI}} = A^{\mathrm{NLSM}} \otimes A^{\mathrm{YM}} \Rightarrow$$

$$\mathcal{M}^{\mathrm{BI,1\text{-}loop}}_{(++++)} \sim (s^4 + t^4 + u^4)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

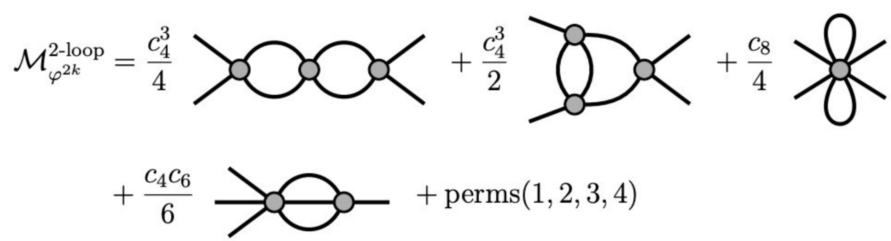
$$\mathcal{M}^{\mathrm{BI,2\text{-}loop}}_{(++++)} \sim (s^6 + t^6 + u^6)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{M}^{\mathrm{BI+HD}} = A^{\mathrm{NLSM}} \otimes A^{\mathrm{YM+HS??}}$$

anomaly cancellation

backup slides

Carrasco, NHP
2307.xxxx



App 3: Corrections to N=1 sYM in 10D?

Could emergent masses be a common feature?

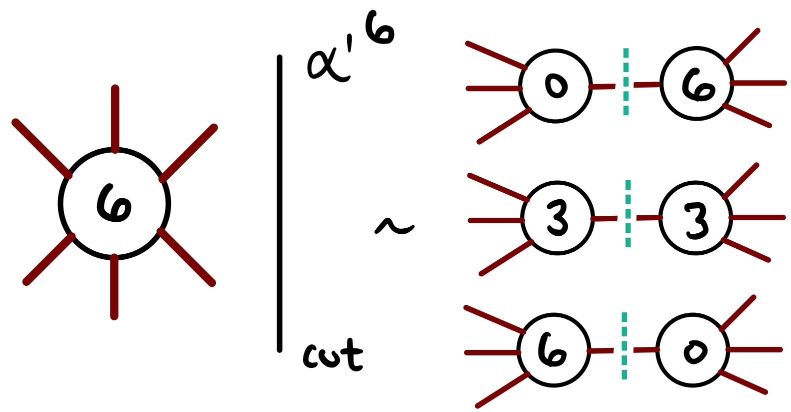
$$\mathcal{A} = \sum \frac{c_g n_g^{\text{HD}}}{d_g} \sim \sum \frac{c_g n_g^{m}}{d_g - m^2}$$

cancels 10D gauge anomaly

Color-dual SUSY higher-derivatives

$$\mathcal{L}^{\text{YM}} = -\frac{1}{4}F^2 + \left(D^2F^4 + F^5\right) + \dots$$

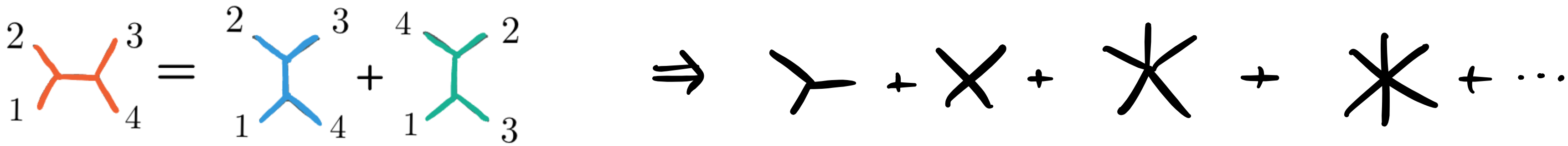
$$A_4^{\text{sYM+HD}} = A_4^{\text{sYM}}\left(1 + \sum c_{(x,y)}\,\sigma_3^x \sigma_2^y\right)$$



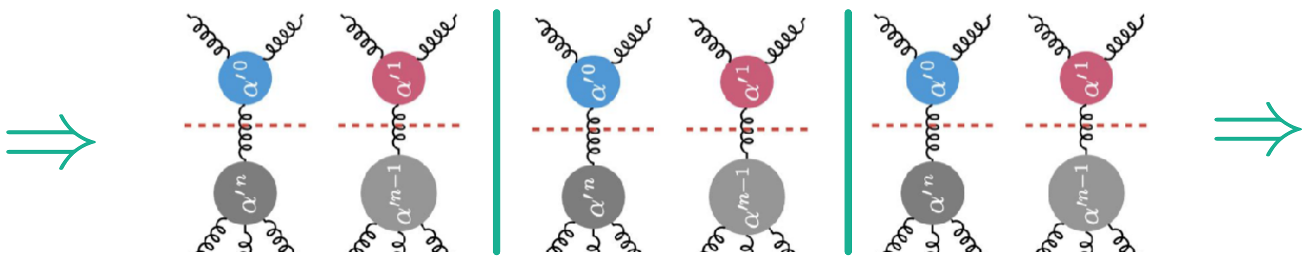
6-point constraints

Key Takeaways

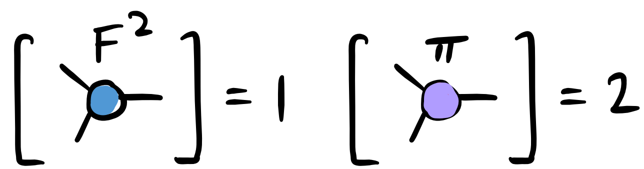
- Graphical organization reveals hidden simplicity
  - Can use this to bootstrap gauge/gravity EFT contacts



- Color-kinematic duality imposes unexpected EFT constraints
  - Higher derivative towers build massive residues



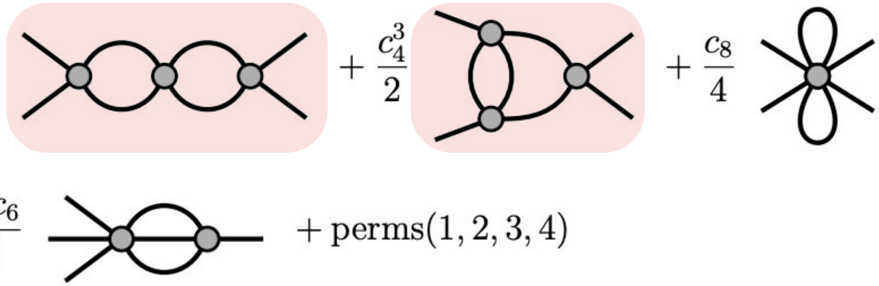
- Many future directions of discovery



Backup Slides

Two-loop Abelianized amplitudes

$$\mathcal{M}^{\text{2-loop}}_{\varphi^{2k}} = \frac{c_4^3}{4}\,[\text{diagram}] + \frac{c_4^3}{2}\,[\text{diagram}] + \frac{c_8}{4}\,[\text{diagram}] + \frac{c_4 c_6}{6}\,[\text{diagram}] + \text{perms}(1,2,3,4)$$



Carrasco, NHP
2307.xxxx

Fully D-dimensional integrals!

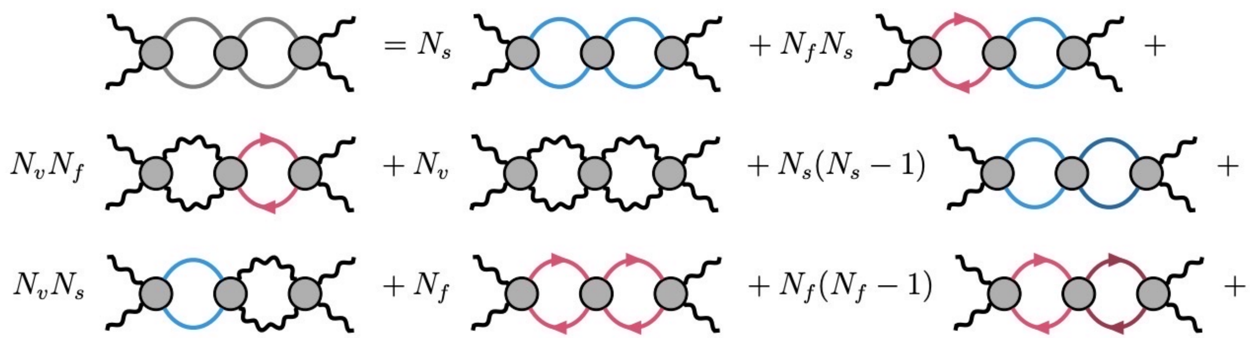
$$I_{3,x}^{\mu_1\ldots\mu_n} = \int \frac{d^D l}{(2\pi)^D} \frac{l^{\mu_1} l^{\mu_2} \ldots l^{\mu_n}}{l^2 (l+K_1)^{2x} (l+K_{12})^2} = \sum_{m+l+2k=n} a^x_{(m,l,k)} \mathcal{T}^{(m,l,k)}_{\text{tri}}$$

$$a^x_{(m,l,k)} = -\left[\frac{s_{12}}{D+2(m+l+k-1)}\right] a^x_{(m+1,l+1,k-1)}$$

$$a^x_{(m,l,0)} = -\left[\frac{D+2(m+l-2)}{D+2(m-2))}\right]\left[\frac{1}{s_{12}} a^{x-1}_{(m-1,l,0)} + a^x_{(m-1,l,0)}\right]$$

$$a_{(0,l,0)} = \frac{1}{s_{12}} a^{x-1}_{(0,l-1,0)}$$

Reproduces cancellations expected for supersymmetry



$$\Rightarrow \quad \mathcal{M}^{\mathcal{N}=1,4}_{(++++)} = 0$$

$$\mathcal{M}^{\text{BI,2-loop}}_{(++++)} \sim (s^6 + t^6 + u^6)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

Modeling building
with color-dual EFT

Part I

Building duality invariants

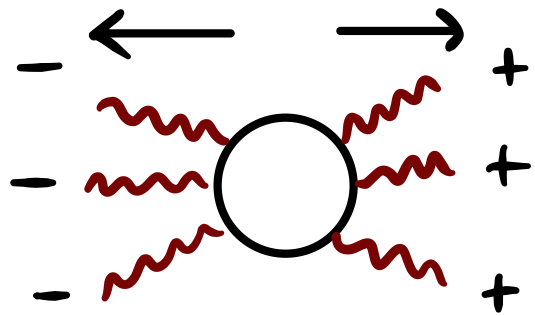
Universality in electromagnetic duality

Photon theories with helicity conservation

Equations of motion invariant under

$$F + iG \rightarrow e^{i\alpha}(F + iG)$$

$$G \equiv \frac{\partial \mathcal{L}}{\partial \tilde{F}}$$



$$n_{(-)} = n_{(+)}$$

Universality in electromagnetic duality

Photon theories with helicity conservation

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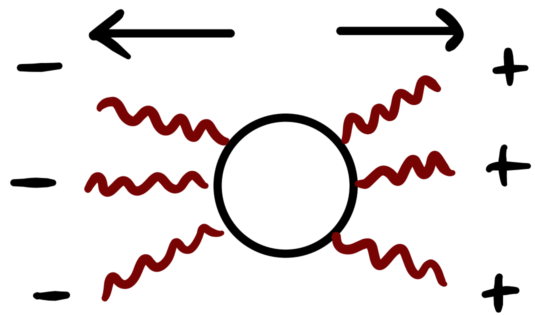

$$n_{(-)} = n_{(+)}$$

$$\mathcal{L}^{\text{EMf}} = \sqrt{-g}\left(R + \sum_I F_{I\mu\nu}F^{I\mu\nu}\right)$$

$$\mathcal{L}^{\text{BI}} = 1 - \sqrt{1 - F^2 + F^2(F\tilde{F})}$$

Universality in electromagnetic duality

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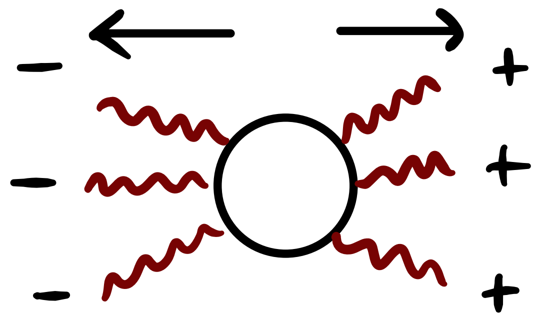

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$$\mathcal{M}^{\text{EMf}} = A^{\text{YMS}} \otimes A^{\text{YM}}$$

$$\mathcal{L}^{\text{BI}} = 1 - \sqrt{1 - F^2 + F^2(F\tilde{F})}$$

$$\mathcal{M}^{\text{BI}} = A^{\text{NLSM}} \otimes A^{\text{YM}}$$

Universality in electromagnetic duality

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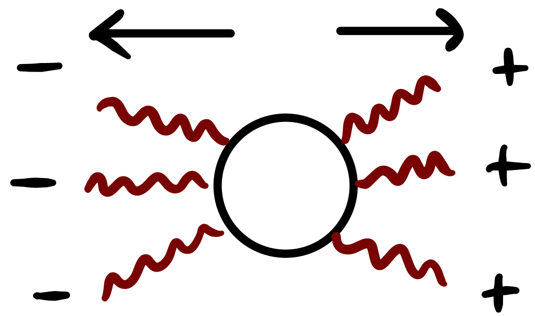


$$n_{(-)} = n_{(+)}$$

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2 \qquad \mathcal{M}^{\text{EMf}} = A^{\text{YMS}} \otimes A^{\text{YM}}$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi} \qquad \mathcal{M}^{\text{BI}} = A^{\text{NLSM}} \otimes A^{\text{YM}}$$

Universality in electromagnetic duality

A Color-Dual Puzzle:

$$A_n^{\mathrm{YMS}} \otimes A_n^{\mathrm{MHV}} = 0$$
$$n > 4$$

$$A_n^{\mathrm{NLSM}} \otimes A_n^{\mathrm{MHV}} = 0$$
$$n > 4$$

Universality in electromagnetic duality

A Color-Dual Puzzle:

$$\left[A_n^{\text{YMS}} \otimes A_n^{\text{MHV}} = 0 \qquad n > 4$$

$$A_n^{\text{NLSM}} \otimes A_n^{\text{MHV}} = 0 \qquad n > 4$$

These live in supergravity

R-symmetry:

$$A_n^{\text{N}^j\text{MHV}} \otimes A_n^{\text{N}^k\text{MHV}} = 0 \quad j \neq k$$

Universality in electromagnetic duality

A Color-Dual Puzzle:

$$\left[A_n^{\text{YMS}} \otimes A_n^{\text{MHV}} = 0\right.$$
$$n > 4$$

These live in supergravity

$$A_n^{\text{NLSM}} \otimes A_n^{\text{MHV}} = 0$$
$$n > 4$$

Why do pions filter non-vanishing Yang-Mills?

R-symmetry:

$$A_n^{\text{N}^j\text{MHV}} \otimes A_n^{\text{N}^k\text{MHV}} = 0 \quad j \neq k$$

Universality in electromagnetic duality

Answer:

NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

NLSM
=
YMS + HD

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

Universality in electromagnetic duality

Answer:

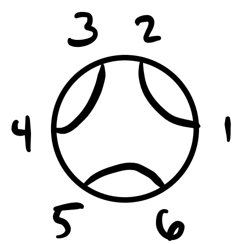
NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

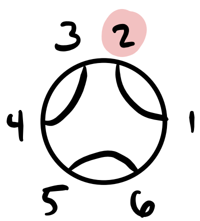
NLSM
=
YMS + HD

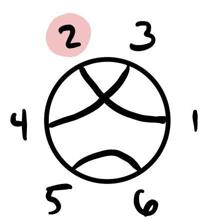
$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2 \qquad \mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

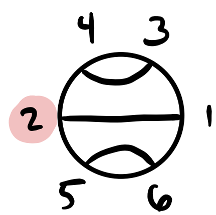
$$A_{(12)(34)(56)}^{\text{YMS}} =$$

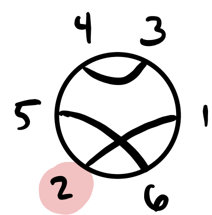


satisfies
BCJ relations

$k_2 k_1$


$+\, k_2 k_{13}$


$+\, k_2 k_{134}$


$+\, k_2 k_{1345}$


$= 0$

Universality in electromagnetic duality

Answer:

NHP
2210.12800

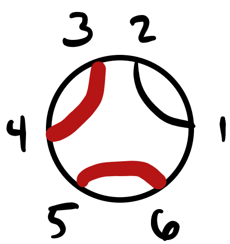
$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

NLSM
=
YMS + HD

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2$$

$$\mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

$$s_{34}s_{56}\, A_{(12)(34)(56)}^{\text{YMS}} =$$



Universality in electromagnetic duality

Answer:

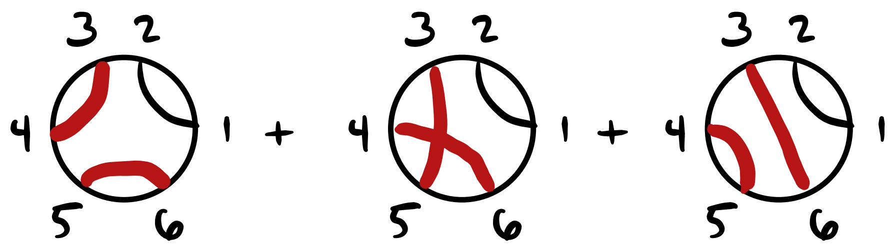
NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S_{(ij)^c}^{2|(k-1)}} s_{(\rho)} A_{(ij)(\rho)}^{\text{YMS}}.$$

NLSM
=
YMS + HD

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2 \qquad \mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

$$A_6^{\text{NLSM}} =$$



Universality in electromagnetic duality

Answer:

NHP
2210.12800

$$A_{2k}^{\text{NLSM}} = (-1)^{k-1} \sum_{\rho \in S^{2|(k-1)}_{(ij)^c}} s_{(\rho)} A^{\text{YMS}}_{(ij)(\rho)}.$$

NLSM
=
YMS + HD

$$\mathcal{L}^{\text{YMS}} = -\frac{1}{4}F^2 + (D\varphi^a)^2 + [\varphi^a, \varphi^b]^2 \qquad \mathcal{L}^{\text{NLSM}} = \text{Tr}[(\partial U)^\dagger (\partial U)] \quad U = e^{i\pi}$$

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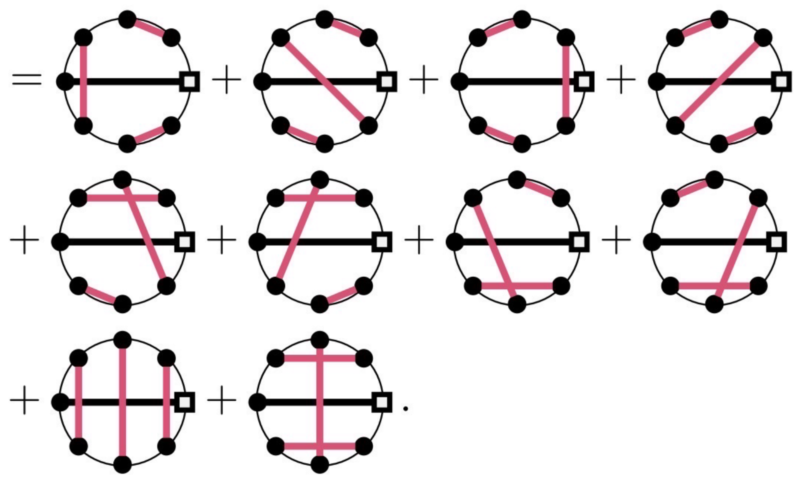
$|A_8^{\text{NLSM}}| =$



(39)

Universality in electromagnetic duality

$$A^{\mathrm{YM}} \equiv A^{\mathrm{YM}}(\{k_i k_j, \epsilon_i k_j, \epsilon_i \epsilon_j\})$$

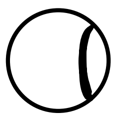
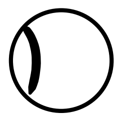
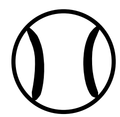
Idea: strip off polarization-products

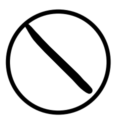
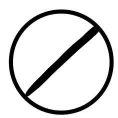
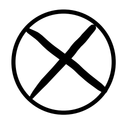
Reducible Amplitude Block Decomposition (RABID)

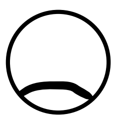
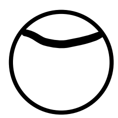
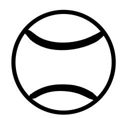
NHP
2210.12800

$$A^{\mathrm{YM}}_{(\sigma)} = \sum \epsilon_{(\rho)} \Delta^{(\rho)}_{(\sigma)}$$

$$\Delta^{(\rho)}_{(\sigma)}\left[\{k_i k_j, \epsilon_i kj\}\right]$$

$A_4^{\mathrm{YM}} = \epsilon_{(12)}$  $+\ \epsilon_{(34)}$  $+\ \epsilon_{(12)}\epsilon_{(34)}$ 

$\epsilon_{(13)}$  $+\ \epsilon_{(24)}$  $+\ \epsilon_{(13)}\epsilon_{(24)}$ 

$\epsilon_{(14)}$  $+\ \epsilon_{(23)}$  $+\ \epsilon_{(13)}\epsilon_{(24)}$ 

Universality in electromagnetic duality

$$A^{\rm YM} \equiv A^{\rm YM}(\{k_i k_j, \epsilon_i k_j, \epsilon_i \epsilon_j\})$$

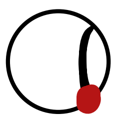
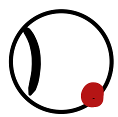
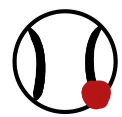
Idea: strip off polarization-products

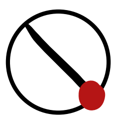
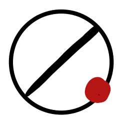
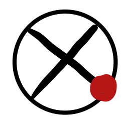
Reducible Amplitude Block Decomposition (RABID)

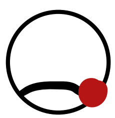
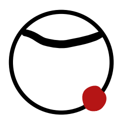
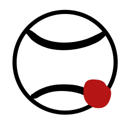
NHP
2210.12800

$$\boxed{A^{\rm YM}_{(\sigma)} = \sum \epsilon_{(\rho)} \Delta^{(\rho)}_{(\sigma)}}$$

$$\Delta^{(\rho)}_{(\sigma)}\left[\{k_i k_j, \epsilon_i k j\}\right]$$

$$A_4^{\rm YM}\Big|_{\epsilon_1 \to k_1} = k_1\epsilon_2$$  $$+ \epsilon_{(34)}$$  $$+ k_1\epsilon_2\epsilon_{(34)}$$ 

$$k_1\epsilon_3$$  $$+ \epsilon_{(24)}$$  $$+ k_1\epsilon_3\epsilon_{(24)}$$ 

$$k_1\epsilon_4$$  $$+ \epsilon_{(23)}$$  $$+ k_1\epsilon_4\epsilon_{(23)}$$ 

Universality in electromagnetic duality

$$A^{\text{YM}} \equiv A^{\text{YM}}(\{k_i k_j, \epsilon_i k_j, \epsilon_i \epsilon_j\})$$

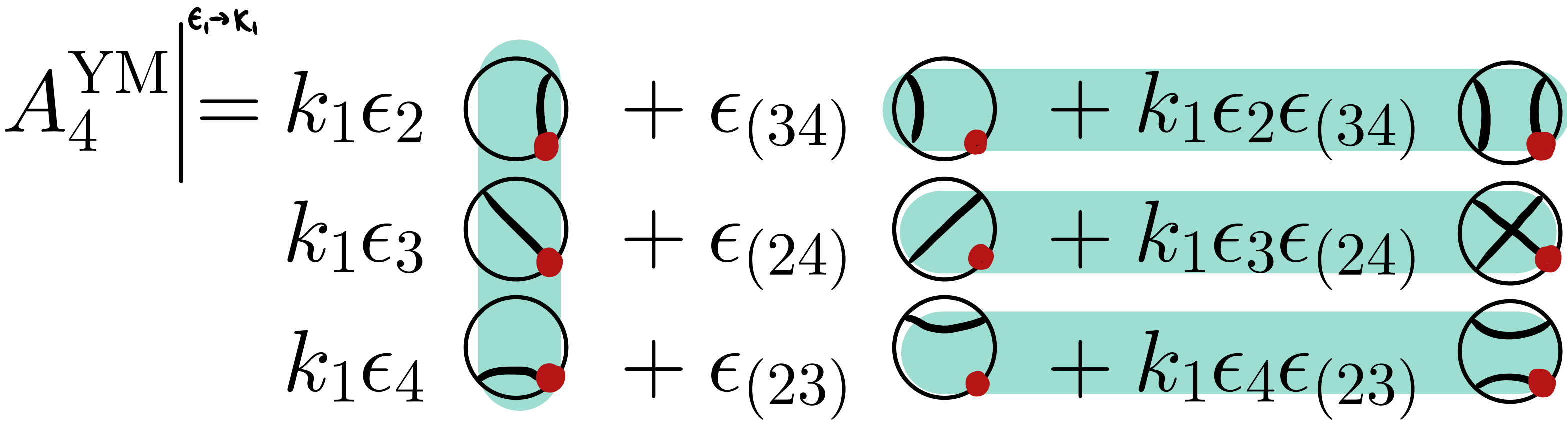
Idea: strip off polarization-products

Reducible Amplitude Block Decomposition (RABID)

NHP
2210.12800

$$A^{\text{YM}}_{(\sigma)} = \sum \epsilon_{(\rho)} \Delta^{(\rho)}_{(\sigma)}$$

$$\Delta^{(\rho)}_{(\sigma)} [\{k_i k_j, \epsilon_i kj\}]$$



Universality in electromagnetic duality

Reducible Amplitude Block Decomposition (RABID)

$$A_{(\sigma)}^{\mathrm{YM}} = \sum \epsilon_{(\rho)} \Delta_{(\sigma)}^{(\rho)}$$

$$\Delta_{(\sigma)}^{(\rho)}\left[\{k_i k_j, \epsilon_i k j\}\right]$$

Ward Identity between building blocks

$$\Delta_{(\sigma)}^{(\rho)}\Big|_{\epsilon_i \to k_i} = -\sum_{j \in \rho^c} k_i \epsilon_j \, \Delta_{(\sigma)}^{(\rho \cup (ij))}$$

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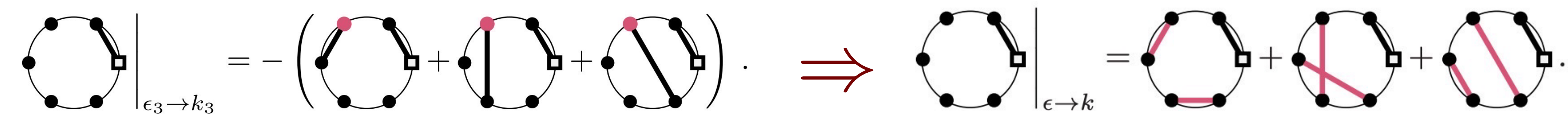
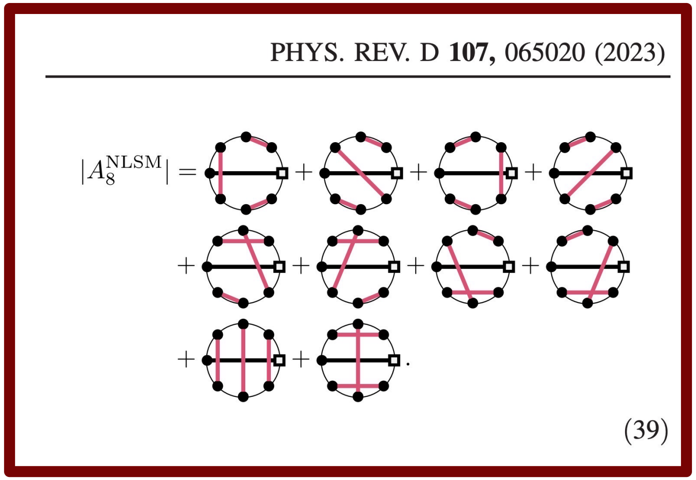
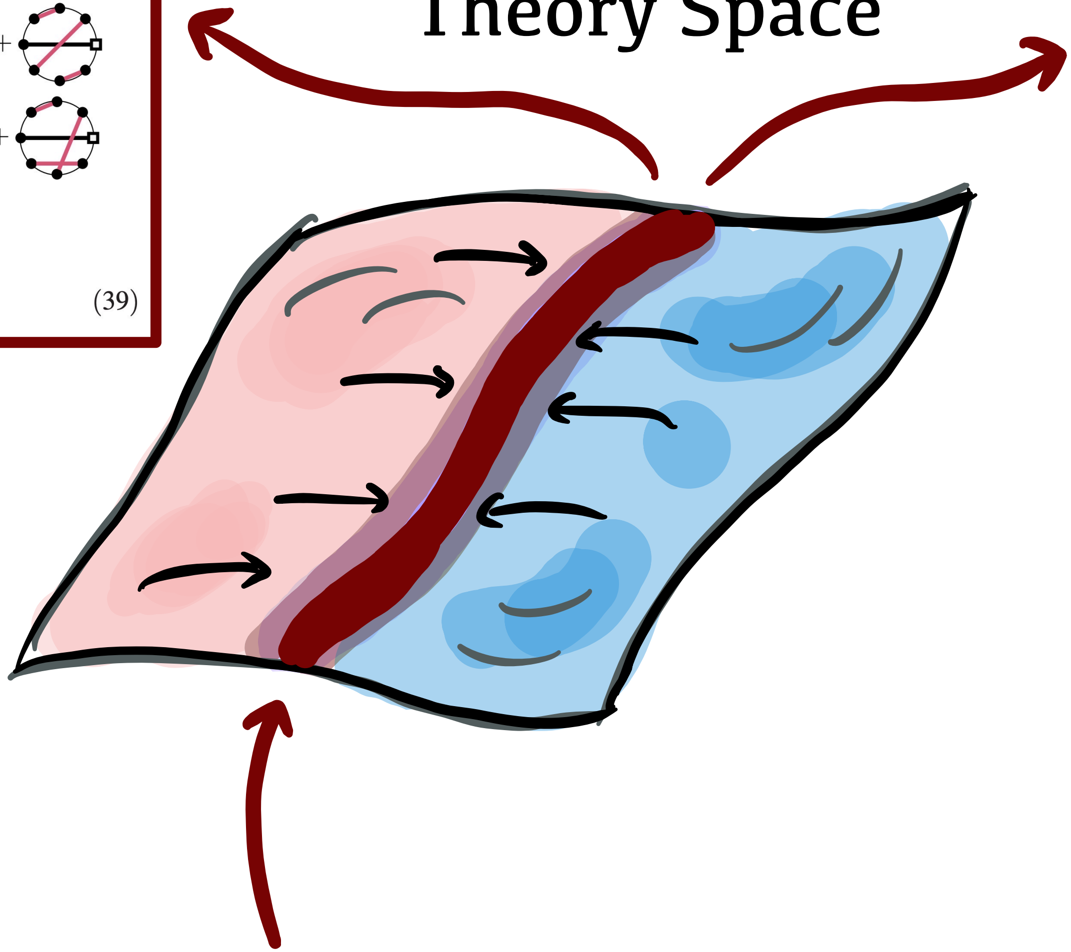


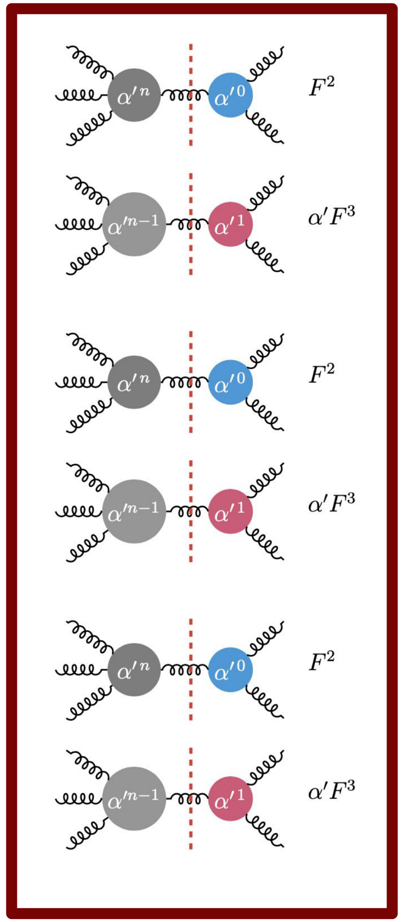
(24)

Modeling building with color-dual EFT



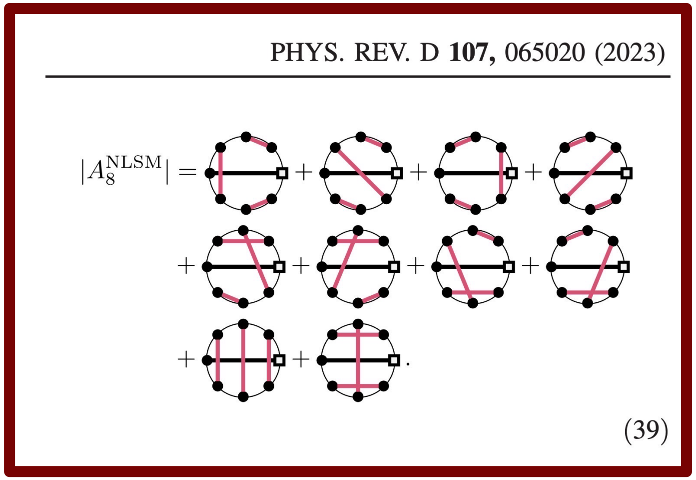
Effective Field Theory Space



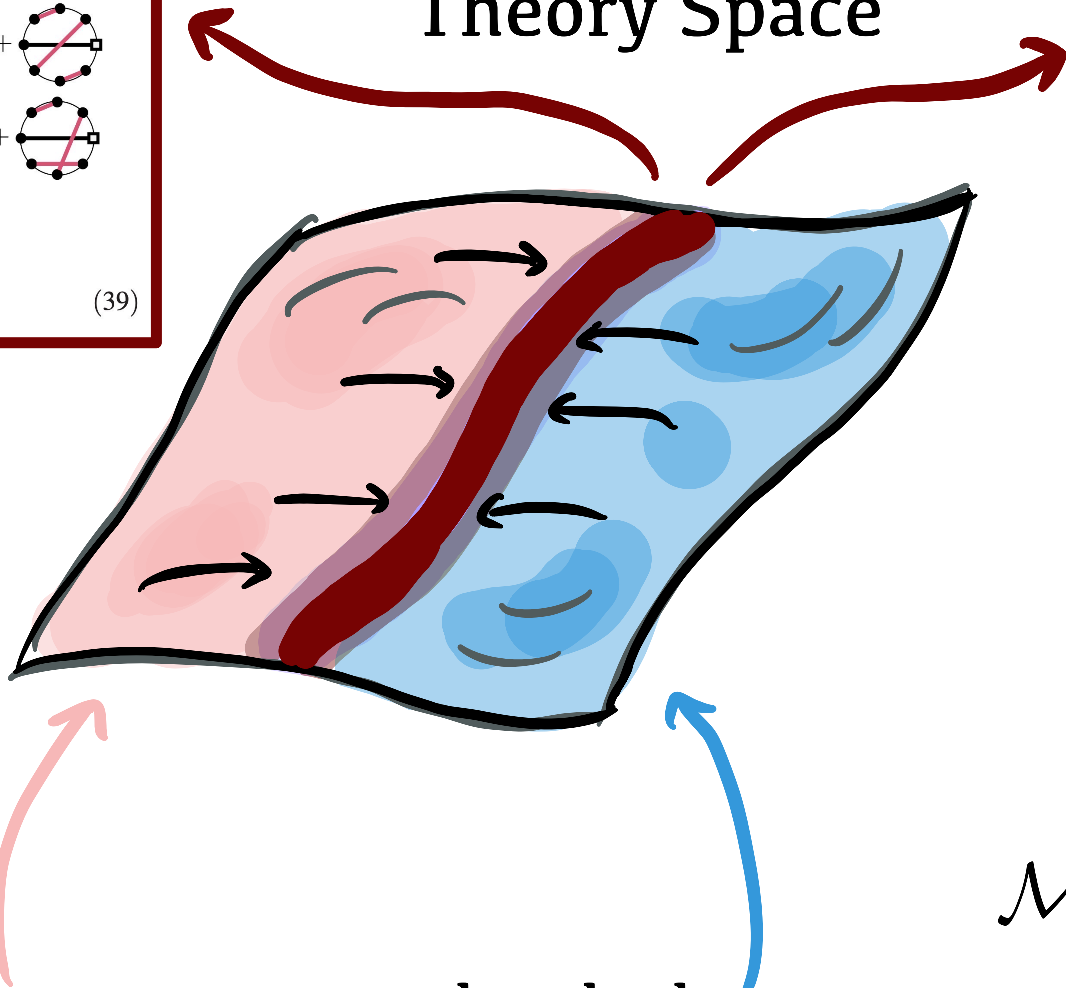


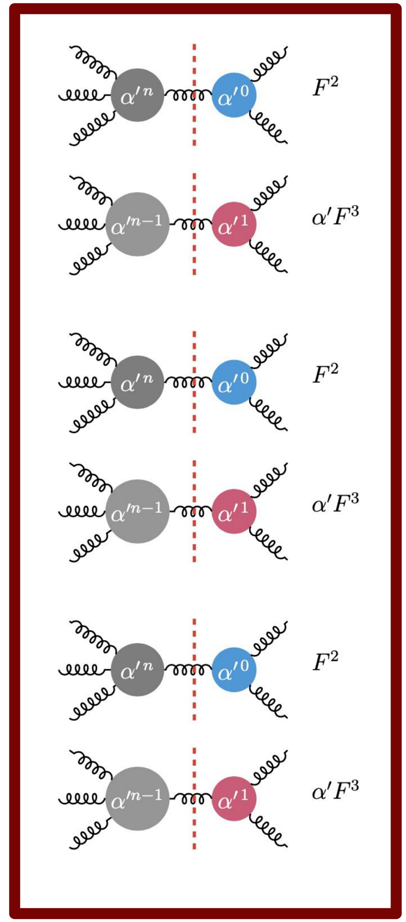
$$\sum_{\alpha} (k_1 \cdot k_\alpha) A(\alpha, 1, \beta, n) = 0$$

Modeling building with color-dual EFT



Effective Field Theory Space





$\mathcal{M}^{\mathcal{N}=4\ \rm SG}$

$\mathcal{M}^{R^3}$

can we use color-dual organization more generally?

$\mathcal{M}^{\rm BI+HD}$

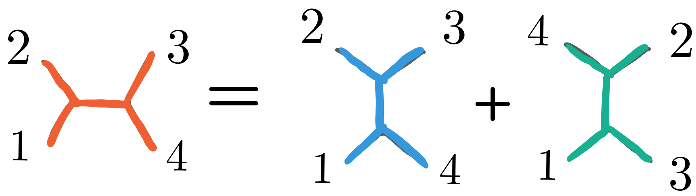
$\mathcal{M}^{\rm DBIVA+SG}_{\alpha\text{-attractors}}$

Modeling building
with color-dual EFT

Part II

stretching the capacity

Introducing symmetric structure

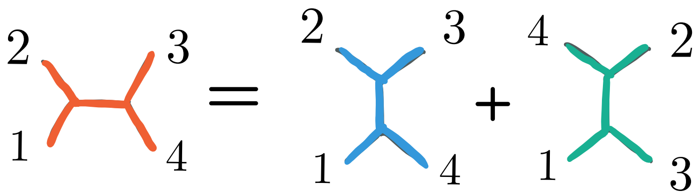


$\Leftrightarrow$

$$\sum_{\alpha}(k_1 \cdot k_\alpha)A(\alpha, 1, \beta, n) = 0$$

$$f^{12A}f^{A34} = \mathrm{Tr}\left[T^1T^2T^3T^4\right] - \mathrm{Tr}\left[T^1T^2T^4T^3\right] - \mathrm{Tr}\left[T^2T^1T^3T^4\right] + \mathrm{Tr}\left[T^2T^1T^4T^3\right]$$

Introducing symmetric structure



$$\Longleftrightarrow \quad \sum_{\alpha}(k_1 \cdot k_\alpha)A(\alpha, 1, \beta, n) = 0$$

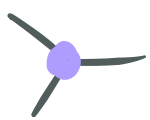
$$f^{12A}f^{A34} = \mathrm{Tr}\left[T^1T^2T^3T^4\right] - \mathrm{Tr}\left[T^1T^2T^4T^3\right] - \mathrm{Tr}\left[T^2T^1T^3T^4\right] + \mathrm{Tr}\left[T^2T^1T^4T^3\right]$$

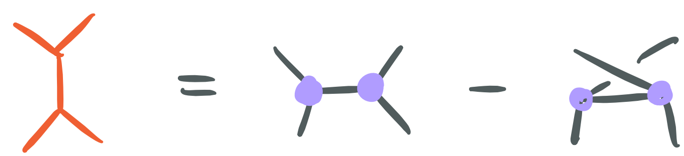
Could also introduce symmetric structure

Carrasco, NHP
2211.04431

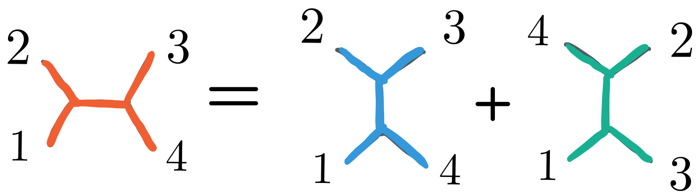
$$C_s^{ff} = C_t^{dd} - C_u^{dd}$$

$$d^{abc} = \mathrm{Tr}\left[\left\{T^a, T^b\right\}, T^c\right] \quad \Longrightarrow \quad f^{12A}f^{A34} = d^{14A}d^{A23} - d^{13A}d^{A24}$$





Introducing symmetric structure



$$\Leftrightarrow \quad \sum_{\alpha} (k_1 \cdot k_\alpha) A(\alpha, 1, \beta, n) = 0$$

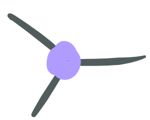
$$f^{12A} f^{A34} = \mathrm{Tr}\left[T^1 T^2 T^3 T^4\right] - \mathrm{Tr}\left[T^1 T^2 T^4 T^3\right] - \mathrm{Tr}\left[T^2 T^1 T^3 T^4\right] + \mathrm{Tr}\left[T^2 T^1 T^4 T^3\right]$$

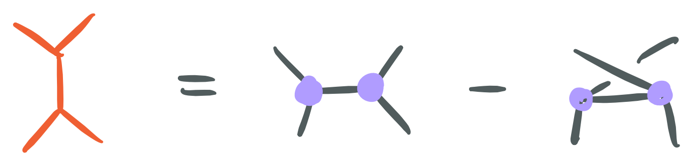
Could also introduce symmetric structure

Carrasco, NHP
2211.04431

$$C_s^{ff} = C_t^{dd} - C_u^{dd}$$

$$d^{abc} = \mathrm{Tr}\left[\left\{T^a, T^b\right\}, T^c\right] \quad \Rightarrow \quad f^{12A} f^{A34} = d^{14A} d^{A23} - d^{13A} d^{A24}$$





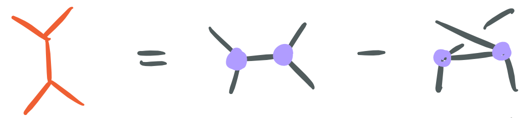
Decompose adjoint into symmetric structure

Introducing symmetric structure

Applying this to color-dual NLSM

Carrasco, NHP
2211.04431

$$A^{NLSM} = c_s^{ff}\, u + c_u^{ff}\, s$$



$$c_s^{ff} = c_t^{dd} - c_u^{dd}$$

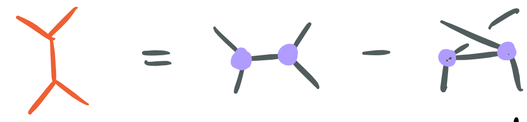
$$= \underbrace{c_s^{dd}\, s + c_t^{dd}\, t + c_u^{dd}\, u}_{\text{larger color basis}}$$

Introducing symmetric structure

Applying this to color-dual NLSM

Carrasco, NHP
2211.04431

$$A^{\mathrm{NLSM}} = c_s^{ff}\, u + c_u^{ff}\, s$$



$$c_s^{ff} = c_t^{dd} - c_u^{dd}$$

$$= \frac{c_s^{dd}\, s^2}{s} + \frac{c_t^{dd}\, t^2}{t} + \frac{c_u^{dd}\, u^2}{u}$$

NLSM is both an adjoint AND symmetric double-copy

$n_s^{ff} = t^2 - u^2$

$n_s^{dd} = s^2$

$$\mathcal{A}^{\mathrm{NLSM}} = \sum_g \frac{c_g^{\mathrm{ff}} n_g^{\mathrm{ff}}}{d_g} = \sum_g \frac{c_g^{\mathrm{dd}} n_g^{\mathrm{dd}}}{d_g}$$

* verified through 6-point tree

Double copy to Photon EFT

$$\mathcal{M}^{\text{BI,1-loop}}_{(++++)} \sim (s^4 + t^4 + u^4)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \Rightarrow n_s^{dd} = s^3\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{M}^{\text{BI+HD}} = \sum_g \frac{n_g^{\pi,\text{dd}} n_g^{\text{vec,dd}}}{d_g}$$

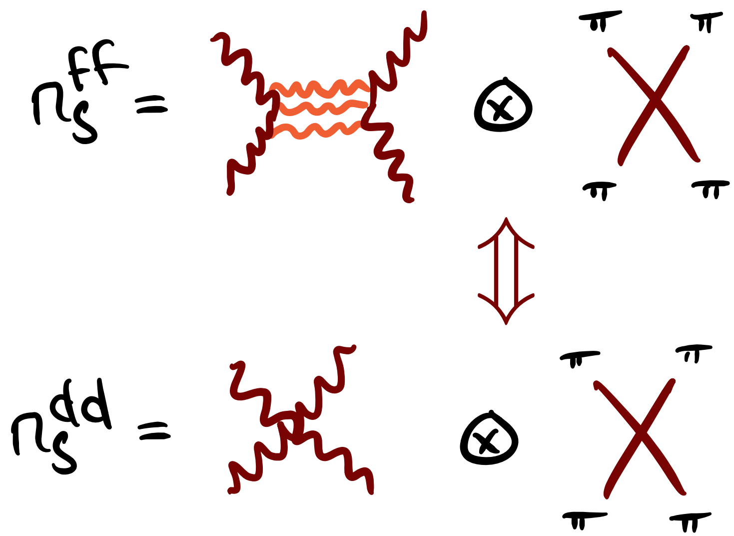
Carrasco, NHP
2211.04431

Double copy to Photon EFT

$$\mathcal{M}^{\text{BI,1-loop}}_{(++++)} \sim (s^4 + t^4 + u^4)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \quad \Rightarrow \quad n_s^{dd} = s^3 \frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{M}^{\text{BI+HD}} = \sum_g \frac{n_g^{\pi,\text{dd}} n_g^{\text{vec,dd}}}{d_g}$$

$$n_s^{\text{ff}} = (t^3 - u^3)\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$



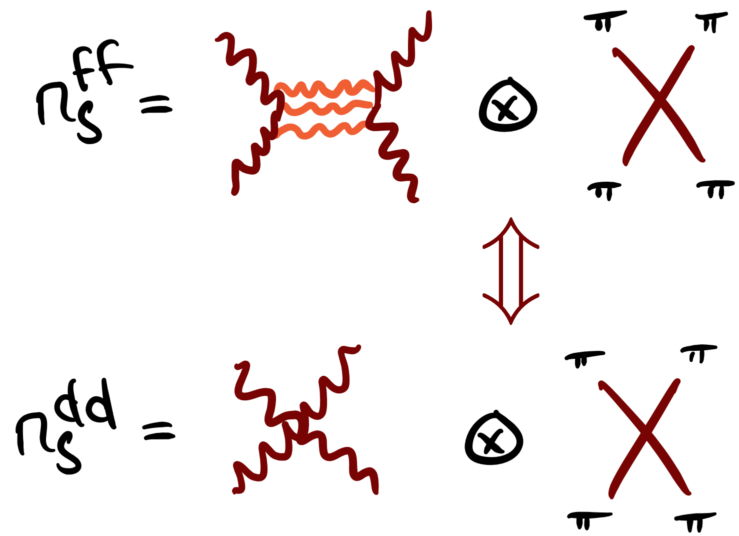
Carrasco, NHP
2211.04431

Double copy to Photon EFT

$$\mathcal{M}^{\text{BI,1-loop}}_{(++++)} \sim (s^4+t^4+u^4)\frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \Rightarrow \quad n_s^{dd} = s^3\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$

$$\mathcal{M}^{\text{BI+HD}} = \sum_g \frac{n_g^{\pi,\text{dd}} n_g^{\text{vec,dd}}}{d_g}$$

$$n_s^{\text{ff}} = (t^3-u^3)\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}$$



Symmetric structure enables additional freedom for EFT model building

$$\mathcal{A}_4^{\text{vec+HD}} = \sum_{g\in\Gamma^{(3)}}\sum_{x,y}\Big[a^{F^3}_{(x,y)}\sigma_3^x\sigma_2^y\frac{n_g^{\text{vec},F^3}c_g^{\text{ff}}}{d_g} + a^{F^2F^2}_{(x,y)}(n_g^{\text{dd},1})^x(n_g^{\text{dd},2})^y\frac{n_g^{\text{vec,dd},1}c_g^{\text{dd}}}{d_g} + a^{F^4}_{(x,y)}(n_g^{\text{dd},1})^x(n_g^{\text{dd},2})^y\frac{n_g^{\text{vec,dd},2}c_g^{\text{dd}}}{d_g}\Big]. \qquad (48)$$

$$n_s^{\text{vec,dd},1} \equiv (F_1F_2)\,(F_3F_4)$$

$$n_s^{\text{vec,dd},2} \equiv (F_1F_3F_2F_4)$$

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