On Massive Higher Spins and their Interactions

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Why think about massive higher spins?

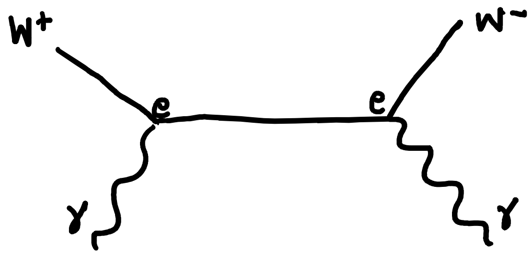
- They exist.

- They have become a useful tool for studying spinning black hole binary systems. e.g. Bern, Luna, Roiban, Shen, Zeng [2005.03071]; Chiodaroli, Johansson, Pichini [2107.14779]; Bern, Kosmopoulos, Luna, Roiban, Teng [2203.06202]; Febres Cordero, Kraus, Lin, Ruf, Zeng [2205.07357]; among many others!

- They, among other representations, are important for giving tree level crossing symmetric amplitudes good high energy behavior. Caron-Huot, Komargodski, Sever, Zhiboedev [1607.04253]

What is wrong with massive higher spins?

- Scattering amplitudes have bad high energy behavior



$$\frac{\eta_{\mu\nu} + p_\mu p_\nu / m^2}{p^2 + m^2}$$

- This results in non-renormalizeable loop divergences
- Known solution: demand a smooth $m \to 0$ limit

Ferrara, Porrati, Telegdi, Phys. Rev. D (1992); Cucchieri, Porrati, Deser [hep-th: 9408073] (1994)

What is wrong with massive higher spins?

- Interactions generically don't preserve the particle's DOF
  Fierz, Pauli, Proc. Roy. Soc. Lond. A. 173 (1939) 211

$$(\partial^2 - m^2)\,\phi_{\mu_1\ldots\mu_n} = 0$$
$$\partial^\lambda \phi_{\lambda\mu_2\ldots\mu_n} = 0$$
$$\phi^\lambda{}_{\lambda\mu_3\ldots\mu_n} = 0$$

$$\partial_\mu \to \nabla_\mu$$
$$[\nabla^2 - m^2, \nabla^\lambda] \neq 0$$

- Some gauge principle is needed to preserve the DOF while introducing interactions.

Lightning review of massless spin n fields

$\phi_{\mu_1 \cdots \mu_n}(x)$ $\qquad\qquad$ $\phi^{\lambda}{}_{\lambda}{}^{\omega}{}_{\omega \mu_5 \cdots \mu_n} = 0$

EOM: $\mathcal{F}_{\mu_1 \cdots \mu_n}(x) = \partial^2 \phi_{\mu_1 \cdots \mu_n} - n\, \partial_{(\mu_1} \partial^{\lambda} \phi_{\lambda \mu_2 \cdots \mu_n)} + \frac{n(n-1)}{2} \partial_{(\mu_1} \partial_{\mu_2} \phi^{\lambda}{}_{\lambda \mu_3 \cdots \mu_n)} = 0$

gauge symmetry: $\delta \phi_{\mu_1 \cdots \mu_n}(x) = n\, \partial_{(\mu_1} \epsilon_{\mu_2 \cdots \mu_n)}(x)$ $\quad$ with $\quad \epsilon^{\lambda}{}_{\lambda \mu_3 \cdots \mu_{n-1}} = 0$

$$\boxed{\delta \mathcal{F}_{\mu_1 \cdots \mu_n} = 0 \qquad\qquad \partial^{\lambda} \mathcal{F}_{\lambda \mu_1 \cdots \mu_{n-1}} - \tfrac{1}{2}(n-1)\, \partial_{(\mu_1} \mathcal{F}^{\lambda}{}_{\lambda \mu_2 \cdots \mu_{n-1})} = 0}$$

Dimensional reduction Aragone, Deser, Yang, Ann Phys. 179, 1, (1987)

- Starting with a $d+1$ dimensional massless field $\Phi_{\hat{\mu}_1 \cdots \hat{\mu}_n}(x, x_d)$

$$\Phi_{\hat{\mu}_1 \cdots \hat{\mu}_n}(x, x_d) = e^{i m x_d} \phi_{\hat{\mu}_1 \cdots \hat{\mu}_n}(x) + c.c. \qquad \hat{\mu} = 0, \ldots, d$$

- Decomposing into d dimensional components

$$\underbrace{\phi_{\mu_1 \cdots \mu_n}}_{\phi_n}, \quad \underbrace{\phi_{d\,\mu_1 \cdots \mu_{n-1}}}_{\phi_{n-1}}, \quad \underbrace{\phi_{dd\,\mu_1 \cdots \mu_{n-2}}}_{\phi_{n-2}}, \quad \underbrace{\phi_{ddd\,\mu_1 \cdots \mu_{n-3}}}_{\phi_{n-3}}, \quad \cdots, \quad \phi_{d \cdots d}$$

Massive gauge symmetry

- The massive theory has a gauge symmetry inherited from the massless theory, with gauge parameters ϵ_{n-1} , ϵ_{n-2}

$$\delta \phi_n = \partial \epsilon_{n-1} \qquad \delta \phi_{n-1} = \partial \epsilon_{n-2} + im\, \epsilon_{n-1}$$

$$\delta \phi_{n-2} = -\partial \epsilon'_{n-1} + 2im\, \epsilon_{n-2} \qquad \delta \phi_{n-3} = -\partial \epsilon'_{n-2} - 3im\, \epsilon'_{n-1}$$

Massive gauge invariance

$$\mathcal{F}_{\hat{\mu}_1 \cdots \hat{\mu}_n} \longrightarrow \mathcal{F}_n \,,\ \mathcal{F}_{n-1} \,,\ \mathcal{F}_{n-2} \,,\ \mathcal{F}_{n-3}$$

- Gauge invariance is guaranteed by the equations

$$\delta \mathcal{F}_{n-i} = 0$$

$$\partial \cdot \mathcal{F}_n - \frac{1}{2}\partial \mathcal{F}_n' - \frac{1}{2}\partial \mathcal{F}_{n-2} + im\,\mathcal{F}_{n-1} = 0$$

$$\partial \cdot \mathcal{F}_{n-1} - \frac{1}{2}\partial \mathcal{F}_{n-1}' - \frac{1}{2}\partial \mathcal{F}_{n-3} - \frac{1}{2} im\,\mathcal{F}_n' + \frac{1}{2} im\,\mathcal{F}_{n-2} = 0$$

Let's minimally couple this theory to electromagnetism/gravity

$$\partial_\mu \longrightarrow \nabla_\mu$$

- Gauge invariance is broken!

$$\delta \mathcal{F}_{n-i} \neq 0$$

$$\nabla \cdot \mathcal{F}_n - \tfrac{1}{2}\nabla \mathcal{F}_n' - \tfrac{1}{2}\nabla \mathcal{F}_{n-2} + im\,\mathcal{F}_{n-1} \neq 0$$

$$\nabla \cdot \mathcal{F}_{n-1} - \tfrac{1}{2}\nabla \mathcal{F}_{n-1}' - \tfrac{1}{2}\nabla \mathcal{F}_{n-3} - \tfrac{1}{2}\, im\,\mathcal{F}_n' + \tfrac{1}{2}\, im\,\mathcal{F}_{n-2} \neq 0$$

$$= (\text{stuff} \propto \nabla R) + m(\text{stuff} \propto R)$$

Restoring the gauge symmetry

- Must add non-minimal couplings

$$F_{n-i} = F_{n-i}^{(0)} + \Delta F_{n-i}^{(1)} + \frac{1}{m} \Delta F_{n-i}^{(2)} + \frac{1}{m^2} \Delta F_{n-i}^{(3)} + \ldots$$

- And correspondingly deform the gauge symmetry

$$\delta \phi_{n-i} = \delta \phi_{n-i}^{(0)} + \frac{1}{m} \delta \phi_{n-i}^{(1)} + \frac{1}{m^2} \delta \phi_{n-i}^{(2)} + \ldots$$

to cancel the gauge violations in F_{n-i} and the Bianchi identities

The first non-minimal couplings

- The $\mathcal{O}(m)$ gauge violation is <u>uniquely</u> cancelled by the following non-minimal corrections

electromagnetism

$$\Delta F^{(1)}_{\mu_1 \cdots \mu_{n-j}} = \underline{g}\, i e (n-j)\, F_{(\mu_1}{}^{\lambda}\, \phi_{\lambda \mu_2 \cdots \mu_{n-j})}$$

gravity

$$\Delta F^{(1)}_{\mu_1 \cdots \mu_{n-j}} = \underline{h} \left((n-j)(n-j-1)\, R^{\lambda}{}_{(\mu_1}{}^{\omega}{}_{\mu_2}\, \phi_{\lambda \omega \mu_3 \cdots \mu_{n-j})} - (n-j)\, R_{(\mu_1}{}^{\lambda}\, \phi_{\lambda \mu_2 \cdots \mu_{n-j})} \right)$$

$g = 1$ and $h = 1$

What is going on?

- Preferred values for good high energy behavior

Ferrara, Porrati, Telegdi, Phys. Rev. D (1992); Cucchieri, Porrati, Deser [hep-th: 9408073] (1994)

$$\underline{g=2} \quad , \quad h = 1$$

- Let's look at the gauge symmetry again

$$\delta \phi_n = \partial \epsilon_{n-1} \qquad \delta \phi_{n-1} = \partial \epsilon_{n-2} + im\, \epsilon_{n-1}$$

$$\delta \phi_{n-2} = -\partial \epsilon'_{n-1} + 2im\, \epsilon_{n-2} \qquad \delta \phi_{n-3} = -\partial \epsilon'_{n-2} - 3im\, \epsilon'_{n-1}$$

Instead consider

- Before taking the $m \to 0$ limit, gauge fix $\phi_{n-1}, \phi_{n-2} = 0$
- The ϕ_n, ϕ_{n-3} gauge fixed theory has a residual gauge symmetry

$$\delta \phi_n = \partial \epsilon_{n-1} \qquad \delta \phi_{n-3} = \frac{i}{2m} \partial (\partial \epsilon'_{n-1})' - 3im\, \epsilon'_{n-1}$$

provided that $\partial \partial \epsilon'_{n-1} - 2m^2 \epsilon_{n-1} = 0$

- Maintaining this gauge symmetry means

$$\Delta L \sim \phi_n \cdot J_n \longrightarrow \partial \cdot J_n \big|_{ST} = \mathcal{O}(m)$$

Summary

- We’ve found two gauge principles, which produce different interactions.

- Moral: There are as many ways to incorporate interactions with massive higher spins as there are independent gauge principles.

- What is the gauge principle that reproduces the minimal three point amplitude of Arkani-Hamed, Huang, Huang [1709.04891]

- This is in good hands Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov [2212.06120]

Thanks for listening!

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