Celestial Holography on backgrounds

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Based on 2209.02724 with Stephan Stieberger and Tomasz Taylor,
2302.12830 with Tomasz Taylor,
and work in progress with Tim Adamo and Wei Bu

Motivation

- Celestial Holography: S-matrix in 4D asymptotically flat spacetime is described by a putative celestial CFT living on the celestial sphere

4D Lorentz symmetry $\mathrm{SO}(3,1)$ ⟷ 2D conformal symmetry $\mathrm{SL}(2,C)$ on the celestial sphere

We will focus on amplitudes of massless particles at tree-level

- Bottom-up approach: From 4D scattering amplitudes to celestial amplitudes (CCFT correlators)

$$\left\langle \prod_{n=1}^{N} O^{a_n}_{\Delta_n,J_n}(z_n,\bar{z}_n) \right\rangle = \prod_{n=1}^{N} \int_0^\infty d\omega_n \omega_n^{\Delta_n-1} \mathscr{M}(\omega_n, z_n, \bar{z}_n, J_n, a_n),$$

Pasterski, Shao, Strominger (2017)

What properties does the CCFT have?

- Operator product expansions (OPE) — *Fan, Fotopoulos, Taylor (2019)*; *Pate, Raclariu, Strominger, Yuan (2019)*
- Symmetries: translation symmetries, BMS, $w_{1+\infty}$ algebra — *Stieberger, Taylor (2018)*; *Adamo, Mason, Sharma (2019)*; *Guevara, Himwich, Pate, Strominger (2021)*; *Strominger (2021)*
- Conformal blocks — *Fan, Fotopoulos, Stieberger, Taylor, Zhu (2021)*; *Atanasov, Melton, Raclariu, Strominger (2021)*
- Differential equations — *Banerjee, Ghosh (2020)*; *Hu, Ren, Srikant, Volovich (2021)*

- Celestial amplitudes are overconstrained by translation invariance in 4D

Consider (1,3) signature: celestial sphere $z \in \mathbb{C}$

$$\langle \phi_{\Delta_1,-}\, \phi_{\Delta_2,-}\, \phi_{\Delta_3,+} \rangle = 0\,, \qquad \langle \phi_{\Delta_1,-}\, \phi_{\Delta_2,-}\, \phi_{\Delta_3,+} \phi_{\Delta_4,+} \rangle \sim \delta(x - \bar{x})$$

Conformal invariant cross ratio: $x = \dfrac{z_{12} z_{34}}{z_{13} z_{24}}$

In (2,2) signature, δ function singularities still appear in low-point celestial amplitudes

- Similar constraints persist in n-point celestial amplitudes *Mizera, Pasterski (2022)*

- Several attempts to resolve this issue:
  - Perform integral transforms on celestial amplitudes: Shadow transform or light transform
  - Consider scatterings on backgrounds

- Basic idea: Break translation invariance in controllable ways by introducing backgrounds

Couple the YM system to dilaton backgrounds

Fan, Fotopolous, Stieberger, Taylor, Zhu (2022)

Casali, Melton, Strominger (2022)

Banerjee, Mandal, Manu, Paul (2023)

Consider celestial scatterings in non-trivial backgrounds

de Gioia, Raclariu (2022)

Gonzo, McLoughlin, Puhm (2022)

- Resultant celestial amplitudes take the standard forms of low-point correlators in CFT

$$\langle O_1\, O_2\, O_3 \rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3} z_{23}^{h_2+h_3-h_1} z_{13}^{h_3+h_1-h_2}\, \bar{z}_{12}^{\bar{h}_1+\bar{h}_2-\bar{h}_3} \bar{z}_{23}^{\bar{h}_2+\bar{h}_3-\bar{h}_1} \bar{z}_{13}^{\bar{h}_3+\bar{h}_1-\bar{h}_2}}$$

determined by conformal symmetry $SL(2,\mathbb{C})$, up to a constant

✦ No δ function singularity appears

Basic questions

- Is it possible to relate the celestial amplitudes on non-trivial backgrounds to ordinary CFTs?

- What happens to the celestial chiral algebras in backgrounds?

Outline

Part I

- Celestial MHV gluon amplitudes in a dilaton background
- Liouville CFT: semiclassical limit, light operators
- From Liouville correlation functions to celestial MHV amplitudes in the presence of a spherical dilaton shockwave
- Celestial supersymmetry

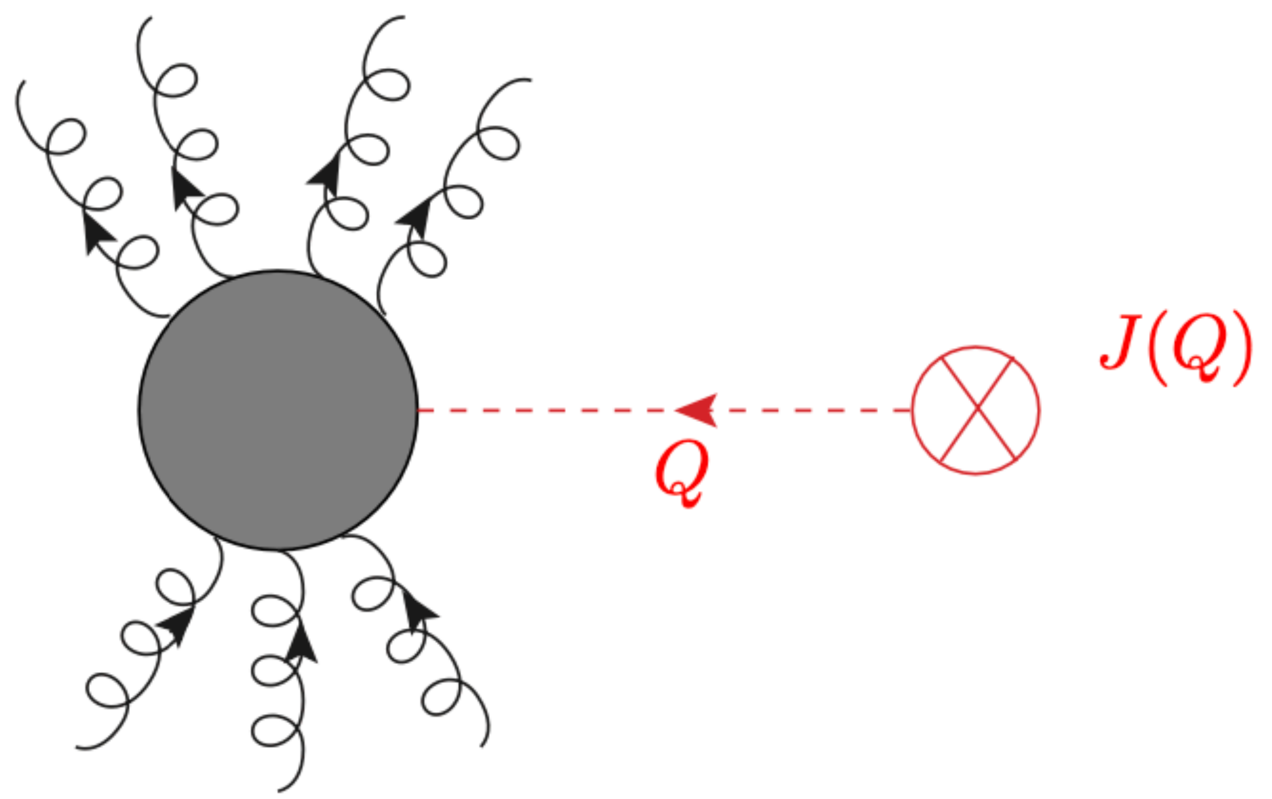
Part II

- MHV amplitudes on self-dual radiative backgrounds
- Celestial chiral algebras

Gluon scattering amplitudes in a dilaton background

Consider the four-dimensional Dilaton-Yang-Mills (DYM) theory

$$\mathscr{L} = \partial_\mu \Phi \partial^\mu \Phi^* - \frac{1}{2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\Phi\,\mathrm{tr}F_{-\mu\nu}F_-^{\mu\nu} - \frac{1}{2}\Phi^*\mathrm{tr}F_{+\mu\nu}F_+^{\mu\nu} + J^*\Phi + J\Phi^*$$



Tree-level maximally helicity violating (MHV) amplitudes with a dilaton

$$M(1^-,2^-,3^+,\cdots,N^+)_{\Phi(Q)} = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle N1\rangle}$$

Dixon, Glover, Khoze (2004)

$$\sum p_{in} - \sum p_{out} = Q$$

- ✦ Same as the Parke-Taylor formula for MHV amplitudes without the dilaton
- ✦ Φ couples to the MHV sector and projects out the $\overline{\text{MHV}}$ sector

The source J_Φ is connected to gluons via the propagator $\dfrac{1}{Q^2}$

Celestial MHV amplitudes in the dilaton background

These amplitudes are then converted into two-dimensional correlators of primary fields with dimensions Δ_i by performing Mellin transforms with respect to the light-cone energies

$$\mathcal{M}_N(z_1, \bar{z}_1, \ldots, z_N, \bar{z}_N \,|\, \Delta_1, \ldots, \Delta_N) = \mathcal{J}_N(z_i)\, \mathcal{S}(z_i, \bar{z}_i) \ ,$$

Fan, Fotopolous, Stieberger, Taylor, Zhu (2022)

where

$$\mathcal{J}_N(z_i) = \sum_{\pi \in S_{N-2}} f^{a_1 a_{\pi(2)} x_1} f^{x_1 a_{\pi(3)} x_2} \ldots f^{x_{N-3} a_{\pi(N-1)} a_N} \frac{z_{12}^4}{z_{1\pi(2)} z_{\pi(2)\pi(3)} \cdots z_{N1}}$$

Parke-Taylor formula

is the holomorphic soft factor, with $(a_1, a_2, \ldots a_N)$ labeling the gluon group indices.

The Mellin transforms are contained in the scalar part

$$\mathcal{S}_N(z_i, \bar{z}_i) = \int d^4X \int_{\omega_i \geq 0} J_\Phi(X) \frac{e^{iX\cdot Q}}{Q^2} d\omega_1\, \omega_1^{\Delta_1} d\omega_2\, \omega_2^{\Delta_2} \prod_{k\geq 3}^{N} d\omega_k\, \omega_k^{\Delta_k - 2},$$

We will identify the background dilaton source $J_\Phi(X)$ that yields $\mathcal{S}(z_i, \bar{z}_i)$ in exactly the same form as the Liouville correlator in the semiclassical limit

Liouville CFT

It is a 2D CFT that appeared in many places including

String theory *Polyakov (1981)*

AGT correspondence *Alday, Gaiotto, Tachikawa (2009)*

Many aspects of Liouville CFT have been studied. It is one of the well-known CFTs besides minimal models and Wess-Zumino-Witten (WZW) models

The three-point correlation functions of Liouville primary operators are described by the DOZZ formula

Dorn, Otto (1994) *Zamolodchikov, Zamolodchikov (1995)*

- **Basics of Liouville CFT**

For Liouville CFT, the Lagrangian density is given by

$$\mathscr{L} = \frac{1}{\pi}\frac{\partial\phi}{\partial z}\frac{\partial\phi}{\partial\bar{z}} + \mu e^{2b\phi},$$

Zamolodchikov, Zamolodchikov (1995)

where b is the dimensionless Liouville coupling constant and the scale parameter μ is usually called the cosmological constant.

Central Charge, Primary fields and correlation functions in Liouville CFT

The background charge at infinity $|z| \to \infty$: $Q = b + \frac{1}{b}$

is related to the central charge by $c = 1 + 6Q^2$

The primary fields of LFT are the exponential operators

$$V_\alpha(z, \bar{z}) = e^{2\alpha\phi(z,\bar{z})},$$

which are scalar fields with conformal weights

$$h(\alpha) = \bar{h}(\alpha) = \alpha(Q - \alpha)$$

We focus on the correlation functions of exponential operators

In general, Liouville correlation functions are hard to compute by using path integral method

There is, however, one notable exception, for a special configuration of the exponents,when

$$\sum_i \alpha_i = Q - \frac{m}{b} - nb$$

In this case, the Liouville correlation function can be computed by Dotsenko and Fateev Coulomb gas integral

We are interested in the case of single integrals that appear when $n = 0$ and $m = 1$.

With the exponents parametrized as

$$\alpha_i = \sigma_i b \ , \qquad \sum_i \sigma_i = 1$$

the correlation functions are

$$\left\langle \prod_i V_{\sigma_i b}(z_i, \bar{z}_i) \right\rangle \sim \prod_{i<j} | z_i - z_j |^{-4\sigma_i \sigma_j b^2} \int d^2 z \prod_i | z_i - z |^{-4\sigma_i}$$

Note that the dimensions of the Liouville primary fields are

$$d_i = h(\sigma_i b) + \bar{h}(\sigma_i b) = 2\sigma_i + 2b^2 \sigma_i (1 - \sigma_i)$$

To make contact with the primary fields in CCFT

$$\Delta_i = n_i + i\lambda_i$$

We will take the limit $b \to 0$, which corresponds to the semiclassical limit of Liouville CFT

Semiclassical limit of Liouville CFT and Light operators

Recall that the central charge $c = 1 + 6Q^2 = 1 + 6\left(b + \frac{1}{b}\right)^2$

The semiclassical limit of Liouville CFT corresponds to $b \to 0$, the central charge $c \to \infty$

The operators with the exponents scaling as $\sigma_i b$ when $b \to 0$ are called the "light" operators

One can show that the correlation functions of light operators are determined by the solutions of the Liouville equation which describe metrics on the sphere

Zamolodchikov, Zamolodchikov (1995)

The correlation functions of light operators that we are interested in are

$$\left\langle \prod_i V_{\sigma_i b}(z_i, \bar{z}_i) \right\rangle \sim \int d^2 z \prod_i |z_i - z|^{-4\sigma_i} \,, \quad \longleftarrow \quad \text{Denoted as Liouville integrals}$$

with constraint $\sum_i \sigma_i = 1$

From Liouville integral to Celestial MHV amplitudes

Our goal is to rewrite the Liouville integrals as Mellin transforms.

we are interested in MHV amplitudes in the helicity configurations (– – + + . . .). It is natural to fulfill the condition by

$$\sigma_1 = \frac{1+i\lambda_1}{2}, \ \sigma_2 = \frac{1+i\lambda_2}{2}, \quad \sigma_k = \frac{i\lambda_k}{2} \quad (k \geq 3)$$

with $\displaystyle\sum_i^N \lambda_i = 0$

The Liouville integrals become

$$I_N(z_1, \bar{z}_1, \ldots, z_N, \bar{z}_N) = \int d^2z \, |\, z_1 - z \,|^{-2(1+i\lambda_1)} \, |\, z_2 - z \,|^{-2(1+i\lambda_2)} \, |\, z_3 - z \,|^{-2i\lambda_3} \ldots$$

By using the embedding space formalism, one can express the two-dimensional conformal integrals as the integrals on a “Poincaré section” of a four-dimensional embedding space $\mathbb{R}_{3,1}$

Simmons-Duffin (2012)

- **Embedding coordinates**

$$X = (X^+, X^-, X^1, X^2)$$

and the Lorentzian inner product is

$$X \cdot Y = \frac{1}{2}(X^+Y^- + X^-Y^+) - X^1Y^1 - X^2Y^2$$

On the null-cone $X \cdot X = 0$,

$$X = \left(X^+, \frac{|z|^2}{X^+}, \frac{z+\bar{z}}{2}, \frac{z-\bar{z}}{2i}\right), \quad z \in \mathbb{C}$$

The Poincaré section is constructed by quotienting the null-cone by the rescaling $X \sim \rho X$

One can impose the gauge fixing condition $X^+ = 1$

The action of $SO(3,1)$ on $\mathbb{R}^{3,1}$ is inherited as $SL(2,\mathbb{C})$ transformations of the complex coordinates z. Note that on the Poincaré section,

$$|z_1 - z_2|^2 = 2X_1 \cdot X_2$$

By using the embedding coordinates, one can rewrite the Liouville integrals as

$$I_N = C_N \int d^4X \delta(X \cdot X)\, \delta(X^+ - 1)\, \theta(X^+ + X^-) \int_{\omega_i \geq 0} e^{iX \cdot (\sum_{i=1}^N \omega_i X_i)}\, d\omega_1\, \omega_1^{i\lambda_1} d\omega_2\, \omega_2^{i\lambda_2} \prod_{k \geq 3}^{N} d\omega_k\, \omega_k^{i\lambda_k - 1}$$

where $$C_N = -\frac{1}{2\,\Gamma(1 + i\lambda_1)\Gamma(1 + i\lambda_2)\prod_{k\geq 3}^N \Gamma(i\lambda_k)}$$

Note that $$\omega_j X_j = \omega_j \left(1, |z_j|^2, \frac{z_j + \bar{z}_j}{2}, \frac{z_j - \bar{z}_j}{2i}\right) = P_j$$ Same expression as the momenta of outgoing massless particles

Total momentum $$Q = \sum_i \omega_i X_i = \sum_i P_i$$

Recall that we need to impose the constraint $\sum_i^N \lambda_i = 0$

We find $$\delta\left(\sum_i \lambda_i\right) I_N \sim \int_{\omega_i \geq 0} \left(\frac{2\pi}{Q^2}\right) d\omega_1\, \omega_1^{i\lambda_1} d\omega_2\, \omega_2^{i\lambda_2} \prod_{k\geq 3}^{N} d\omega_k\, \omega_k^{i\lambda_k - 1}$$ ★

Recall that the celestial MHV amplitudes in a dilaton background:

$$\mathcal{M}_N(z_1, \bar{z}_1, \ldots, z_N, \bar{z}_N \,|\, \Delta_1, \ldots, \Delta_N) = \mathcal{J}_N(z_n)\, \mathcal{S}(z_n, \bar{z}_n) \ ,$$

where the scalar part

$$\mathcal{S}_N(z_i, \bar{z}_i) = \int d^4X \int_{\omega_i \geq 0} J_\Phi(X) \frac{e^{iX\cdot Q}}{Q^2} d\omega_1\, \omega_1^{\Delta_1} d\omega_2\, \omega_2^{\Delta_2} \prod_{k\geq 3}^{N} d\omega_k\, \omega_k^{\Delta_k - 2} \,.$$

We can match Eq.(★) with $\mathcal{S}_N(z_i, \bar{z}_i)$ by choosing

$$\Delta_1 = i\lambda_1,\ \Delta_2 = i\lambda_2,\ \ \Delta_k = 1 + i\lambda_k \ \ (k \geq 3) \ ,$$

provided that we identify the dilaton source as

$$\mathcal{J}_\Phi(X) = 2\pi\delta^{(4)}(X)$$

The solution of the equation $\Box\, \Phi_0(X) = 2\pi\delta^{(4)}(X)$:

$$\Phi_0(X) = \Phi_0(r, t) = -\frac{1}{2r}\delta(r - t)\, \theta(t)$$

Representing a spherical shockwave, retarded, singular at the light cone

As a consistency check, we computed Liouville integrals for $N = 3$ and $N = 4$ by using our method. They agree with the results reported previously in the literature.

- **Recall the holomorphic soft factor**

The negative helicity -1 gluons can be associated with the holomorphic operators $\hat{J}^a(z)$ in the adjoint representation of the gauge group, with chiral weights $(h = -1, \bar{h} = 0)$,

while the positive helicity +1 gluons to dimension +1 holomorphic $(h = 1, \bar{h} = 0)$ WZW currents $J^a(z)$.

Costello, Paquette (2022)

Starting from $\langle \hat{J}^{a_1}(z_1)\hat{J}^{a_2}(z_2)\rangle = \delta^{a_1 a_2} z_{12}^2\,,$

one obtains *Parke-Taylor formula*

$$\langle \hat{J}^{a_1}(z_1)\hat{J}^{a_2}(z_2)J^{a_3}(z_3)\cdots J^{a_N}(z_N)\rangle = \sum_{\pi\in S_{N-2}} f^{a_1 a_{\pi(2)} x_1} f^{x_1 a_{\pi(3)} x_2} \cdots f^{x_{N-3} a_{\pi(N-1)} a_N} \frac{z_{12}^4}{z_{1\pi(2)} z_{\pi(2)\pi(3)} \cdots z_{N1}} = \mathcal{J}_N(z_i)$$

Vertex operators of tree-level Celestial MHV amplitudes

We have shown that celestial MHV amplitudes in a spherical dilaton shockwave background

$$\mathcal{M}_N(z_1,\bar z_1,\ldots,z_N,\bar z_N\,|\,\Delta_1,\ldots,\Delta_N) = \mathcal{J}_N(z_n)\,\mathcal{S}(z_n,\bar z_n)$$

We are in a position to connect the tree-level celestial MHV amplitudes to the semiclassical limit of Liouville theory. We introduce the following operators:

$$\mathcal{O}_\lambda^{-a}(z,\bar z) = \Gamma(1+i\lambda)\,\widehat{J}^a(z)\,e^{(1+i\lambda)b\phi(z,\bar z)}\,, \qquad \mathcal{O}_\lambda^{+a}(z,\bar z) = \Gamma(i\lambda)\,J^a(z)\,e^{i\lambda b\phi(z,\bar z)}$$

and consider the limit of $b \to 0$.

In this limit, the dimension of $\mathcal{O}_\lambda^{-a}$ becomes $\Delta_- = i\lambda$ while the dimension of $\mathcal{O}_\lambda^{+a}$ is $\Delta_+ = 1 + i\lambda$

$$4\pi\delta\Big(\sum_{i=1}^N \lambda_i\Big)\Big\langle \mathcal{O}_{\lambda_1}^{-a_1}(z_1,\bar z_1)\,\mathcal{O}_{\lambda_2}^{-a_2}(z_2,\bar z_2)\,\mathcal{O}_{\lambda_3}^{+a_3}(z_3,\bar z_3)\cdots\mathcal{O}_{\lambda_N}^{+a_N}(z_N,\bar z_N)\Big\rangle = \mathcal{M}_N(z_1,\bar z_1,\ldots,z_N,\bar z_N\,|\,i\lambda_1,i\lambda_2,1+i\lambda_3,\ldots,1+i\lambda_N)$$

- These celestial MHV amplitudes satisfy Banerjee-Ghosh (BG) equations for $N=3$ and $N=4$

Banerjee, Ghosh (2020) *Hu, Ren, Srikant, Volovich (2021)*

$$\mathcal{M}_3(z_1,\bar z_1;z_2,\bar z_2;z_3,\bar z_3\,|\,i\lambda_1,i\lambda_2,1+i\lambda_3) = 2\pi\delta\Big(\sum_{i_1}^3 \lambda_i\Big) f^{a_1a_2a_3}\frac{z_{12}^3}{z_{13}z_{23}}\Gamma(-i\lambda_1)\Gamma(-i\lambda_2)\Gamma(1-i\lambda_3)(z_{12}\bar z_{12})^{i\lambda_3-1}(z_{23}\bar z_{23})^{i\lambda_1}(z_{13}\bar z_{13})^{i\lambda_2}$$

Celestial OPE and chiral algebra

- The OPEs of the CCFT operators can be computed by using the OPEs of current operators and the known OPEs of (light) Liouville operators

$$j^a(z_1)j^b(z_2) \sim \frac{f^{abc}}{z_{12}} j^c(z_2)\,.$$

$$\Gamma(\Delta_1 - 1)e^{(\Delta_1-1)b\phi(z_1,\bar{z}_1)}\Gamma(\Delta_2 - 1)e^{(\Delta_2-1)b\phi(z_2,\bar{z}_2)} = B(\Delta_1 - 1,\Delta_2 - 1)\Gamma(\Delta_1 + \Delta_2 - 2)e^{(\Delta_1+\Delta_2-2)b\phi(z_2,\bar{z}_2)}\,.$$

$$\Rightarrow \quad O^a_{\Delta_1,+1}(z_1,\bar{z}_1)O^b_{\Delta_2,+1}(z_2,\bar{z}_2) \sim B(\Delta_1 - 1,\Delta_2 - 1)\frac{f^{abc}}{z_{12}}O^c_{\Delta_1+\Delta_2-1,+1}(z_2,\bar{z}_2)\,,$$

- In the large central charge limit, all $\bar{L}_{-1}$ descendants are included in the conformal block. This leads to the same celestial chiral algebra (s-algebra) found in the flat background.

$$\left[\, S^{p,a}_m, S^{q,a}_n \right] = -\,i f^{abc}\, S^{p+q-1,c}_{m+n}$$

Celestial Supersymmetry

- Supersymmetry has not played much role in celestial holography
- SuperBMS in four dimensions does not imply supersymmetry in two dimensions

Fotopoulos, Stieberger, Taylor, Zhu (2020)

> Spacetime supersymmetry algebra includes supertranslations, which are genuinely nonholomorphic on a celestial sphere, while the two-dimensional superconformal algebras have factorized holomorphic and antiholomorphic parts.

- Consider supersymmetric Yang-Mills theory coupled to dilatons. *Taylor, Zhu (2023)*
- We show that in the presence of point-like dilaton sources, the CCFT operators associated with the gauge supermultiplet acquire a simple, factorized form. They factorize into the holomorphic (super)current part and the exponential “light” operators of Liouville CFT
- The current sector exhibits (1,0) supersymmetry, implementing spacetime supersymmetry in CCFT

MHV amplitudes on self-dual radiative backgrounds

- Explicit amplitude formulae are known

MHV gluon amplitudes on SD radiative backgrounds *Adamo, Mason, Sharma (2020)*

MHV graviton amplitudes on SD radiative backgrounds *Adamo, Mason, Sharma (2022)*

- They are obtained by using the twistor sigma model

- Turning on these SD radiative backgrounds corresponds to certain deformations of the complex structure on twistor space

MHV gluon amplitudes on SD radiative backgrounds

In the case where the SD radiative background is Cartan-valued

$$M_n = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n-1n\rangle\langle n1\rangle}\int d^4x \,\exp\left[\sum_{i=1}^{n}(ik_i\cdot x + e_i\, g(x,\kappa_i))\right]$$

- Simplest example of the SD background: self-dual plane wave (SDPW), where

$$g(x,\lambda) = -\, i\frac{\langle o\lambda\rangle}{\langle \iota\lambda\rangle}\mathscr{F}(x^-)\,,$$

$n_{\alpha\dot\alpha} = \iota_\alpha\tilde\iota_{\dot\alpha}$ specifies the direction of the plane wave and $\mathscr{F}(x^-)$ is related to the wave profile

- Collinear limits:

$$\lim_{i\to j} M_n(1^-,2^-,\ldots i^+, j^+,\ldots n^+) = \frac{\omega_P}{\omega_i\omega_j}\frac{1}{z_{ij}}M_{n-1}(1^-,2^-,\ldots P^+,\ldots n^+)$$

Holomorphic singularities are the same as the ones in the flat space.

MHV graviton amplitudes on SD radiative backgrounds

- Some properties:
- In the SD radiative backgrounds, translation symmetries are (partially) broken
- The square spinor gets dressed, while the angle spinor is unchanged

- 3-point MHV amplitude in SDPW background

$$\mathscr{M}_3 = \delta^3_{+,\perp}\left(\sum_{j=1}^{3} k_j\right)\int dx^- e^{\mathrm{i}F_3}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2} \quad = \int d^4x\sqrt{g}\ \exp\left(\mathrm{i}\sum_{j=1}^{3} F^{\dot\alpha}(x,\kappa_j)\,\tilde\kappa_{j\,\dot\alpha}\right)\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

MHV graviton amplitudes on SD radiative backgrounds

- 4-point MHV amplitude in SDPW background

$$\mathscr{M}_4 = \delta^3_{+,\perp}\left(\sum_{j=1}^{4} k_j\right)\int dx^- e^{\mathrm{i}F_4}\frac{\langle 12\rangle^7[[34]]}{\langle 12\rangle\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle} \quad = \int d^4x\sqrt{g}\;\exp\left(\mathrm{i}\sum_{j=1}^{4}F^{\dot\alpha}(x,\kappa_j)\,\tilde\kappa_{j\,\dot\alpha}\right)\frac{\langle 12\rangle^7[[34]]}{\langle 12\rangle\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle}$$

In the SDPW background, the square spinor gets dressed

$$\tilde K_{\dot\alpha} = \tilde\kappa_{\dot\alpha} - \frac{\langle o\kappa\rangle[\tilde\iota\kappa]\,f(x^-)}{\langle\iota\kappa\rangle}\,\tilde\iota_{\dot\alpha}$$

In the collinear limit $4\to 3$:

$$\lim_{4\to 3}[[34]] = [34]\,, \qquad \lim_{4\to 3}\left(F^{\dot\alpha}(x,\kappa_3)\,\tilde\kappa_{3\,\dot\alpha} + F^{\dot\alpha}(x,\kappa_4)\,\tilde\kappa_{4\,\dot\alpha}\right) = F^{\dot\alpha}(x,\kappa_P)\tilde\kappa_{P\,\dot\alpha}$$

Then

$$\lim_{4\to 3}\mathscr{M}_4(1^{--},2^{--},3^{++},4^{++}) = \frac{\omega_P^2}{\omega_3\omega_4}\,\frac{\bar z_{34}}{z_{34}}\,\mathscr{M}_3(1^{--},2^{--},P^{++})$$

Holomorphic singularities are the same as the ones in the flat space. This is also true for higher-point amplitudes

Celestial chiral algebras on SD radiative backgrounds

In the collinear limit, if we focus on the lead terms in z_{ij}, all the $\bar{z}_{ij}$ expansions take the same form as the formulae in flat space

- Celestial soft gluon algebra and $w_{1+\infty}$ algebra are undeformed

$$\left[S_m^{p,a}, S_n^{q,a}\right] = -\, i f^{abc}\, S_{m+n}^{p+q-1,c}$$

$$[w_m^p, w_n^q] \;=\; [m(q-1) - n(p-1)]\, w_{m+n}^{p+q-2}$$

- By using vertex operators in twistor sigma model, one can reach the same conclusions
- It would be interesting to see if we can construct a twistor string theory for these amplitudes, compared to the flat space case where the twistor string theory for the amplitudes in $\mathcal{N} = 8$ supergravity is known *Skinner (2013)*

Summary

Part I

- Celestial MHV gluon amplitudes in a dilaton background
- Connect Liouville correlation functions to celestial MHV gluon amplitudes in the presence of a spherical dilaton shockwave
- Celestial supersymmetry appears in the current sector

Part II

- MHV amplitudes on self-dual radiative backgrounds
- Celestial chiral algebras are undeformed

Outlook

- CCFT stress tensor in the context of part I
- Is there a celestial Liouville theory for graviton amplitudes?

Thank you!