Non-perturbative Cosmological Bootstrap: Källén–Lehmann representation

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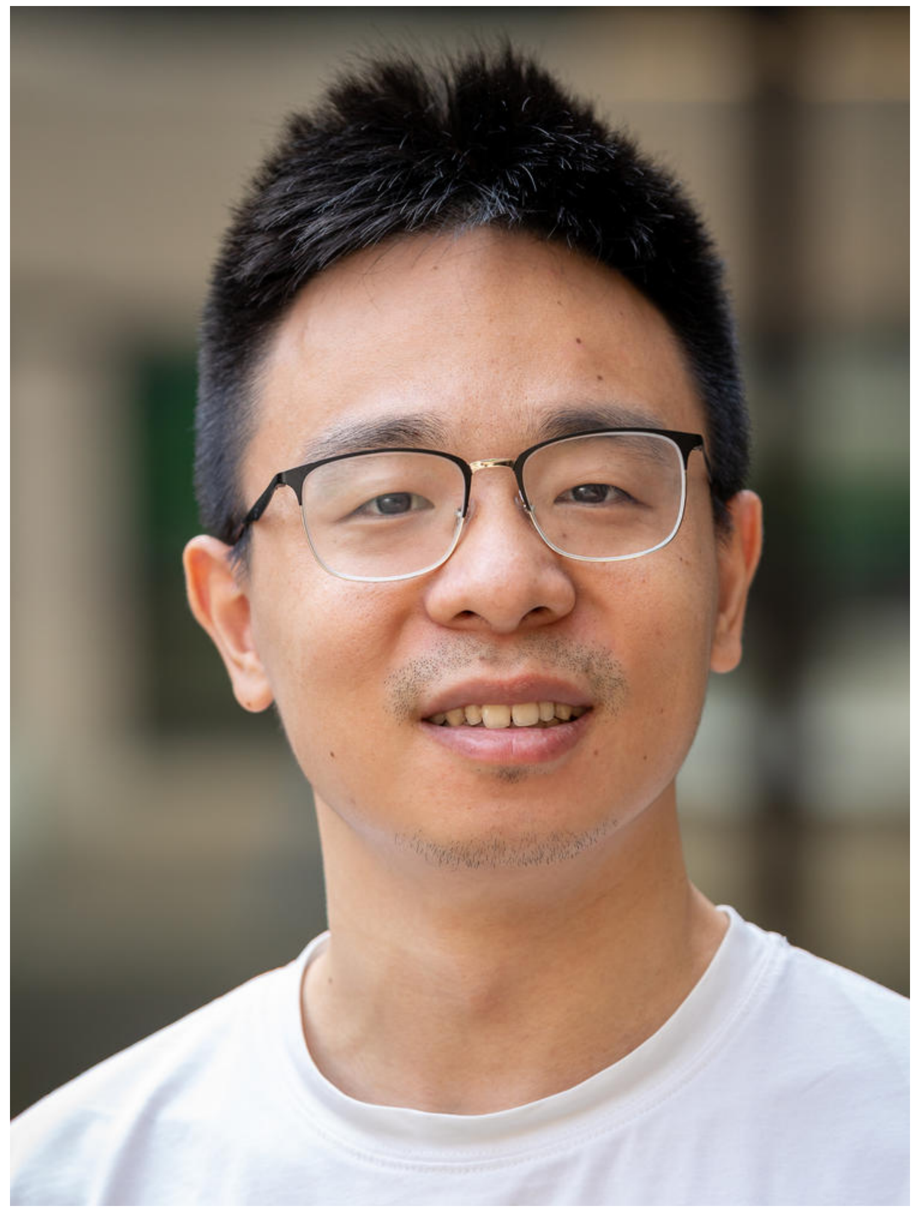
NORDITA

The Nordic Institute for Theoretical Physics

arXiv:2107.13871 Matthijs Hogervorst, João Penedones, KSV
arXiv:2306.00090: Manuel Loparco, João Penedones, KSV, Zimo Sun

Motivation/Future direction:

Non-perturbative study of QFT in dS_{d+1}

General Principles

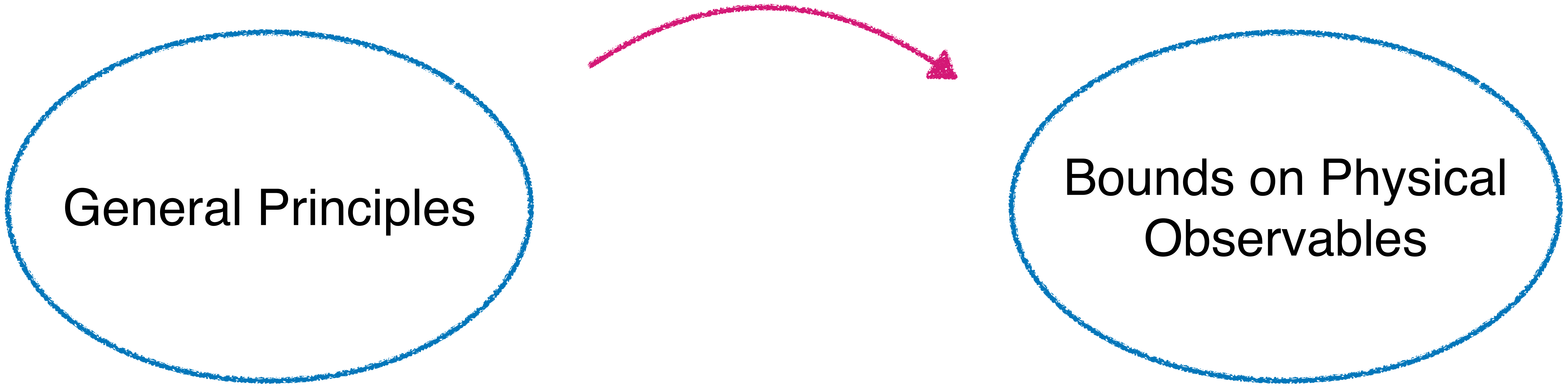
Bounds on Physical Observables

Cosmological bootstrap!

Non-perturbative $\supset$ Perturbative $\supset$ Free

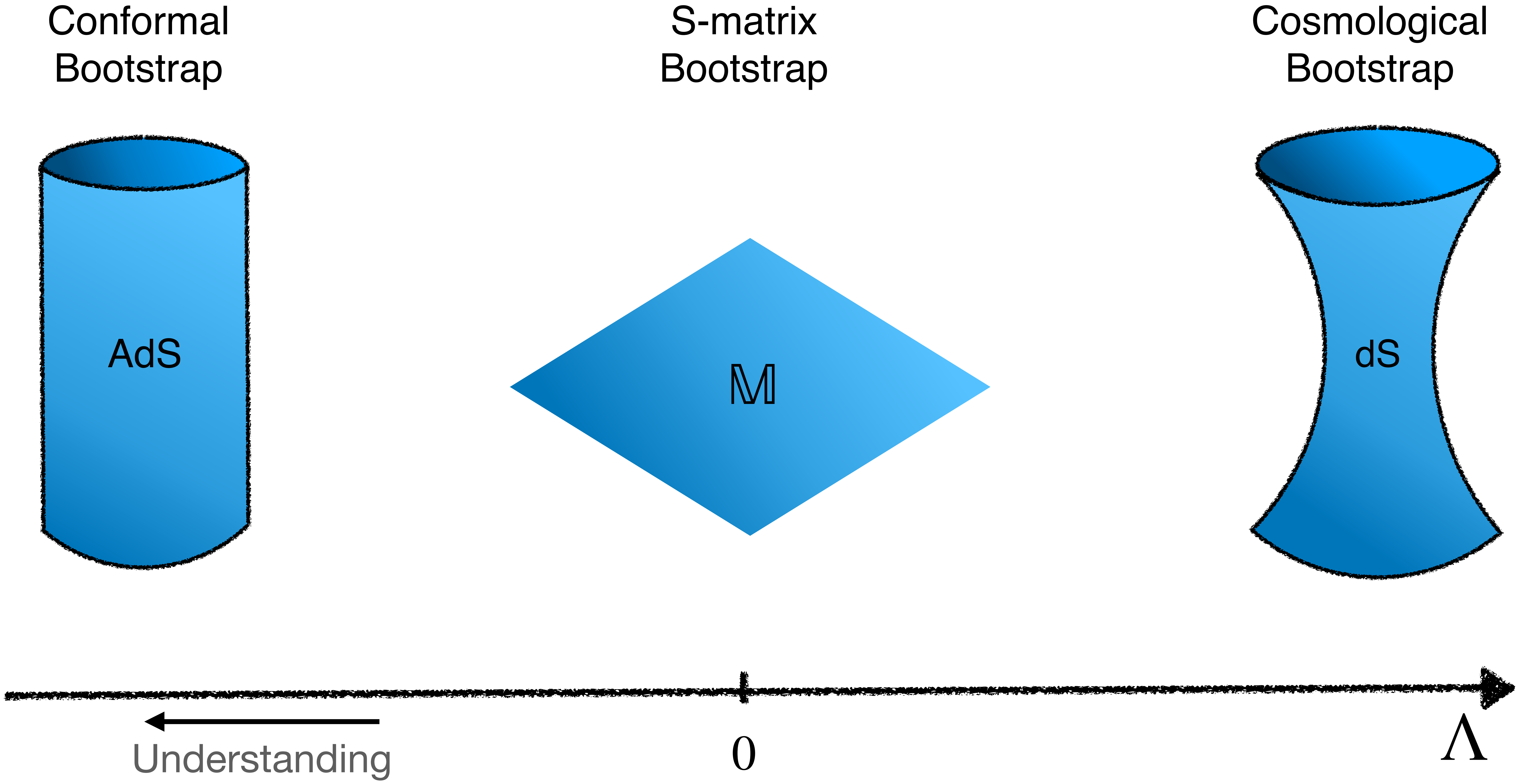
Motivation/Future direction:

Non-perturbative study of QFT in dS_{d+1}



Cosmological bootstrap!

Sorry, not sorry! No Feynman diagrams in this talk


Conformal
Bootstrap
S-matrix
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Cosmological
Bootstrap
AdS
M
dS
Understanding
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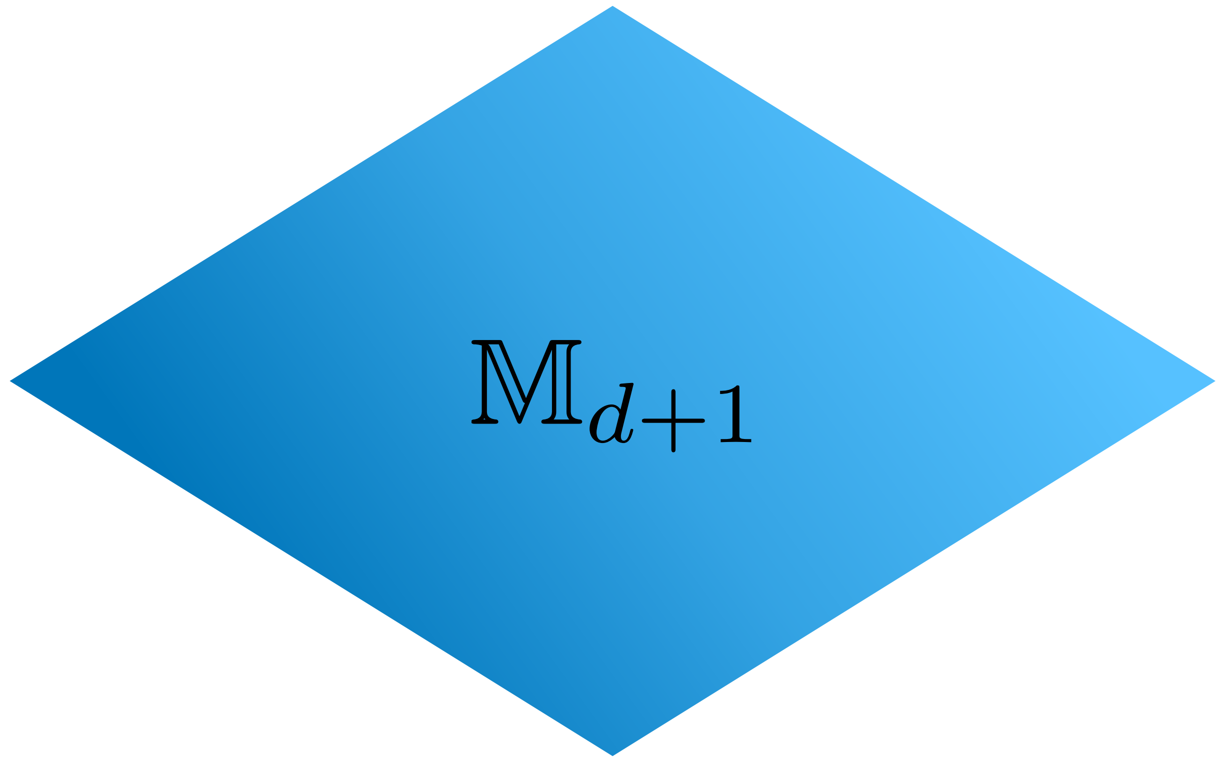
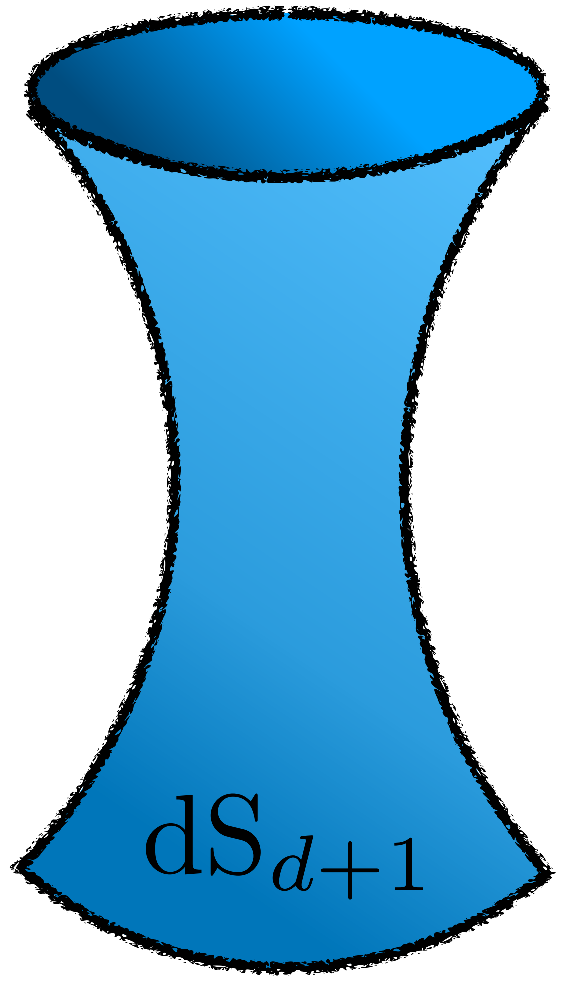
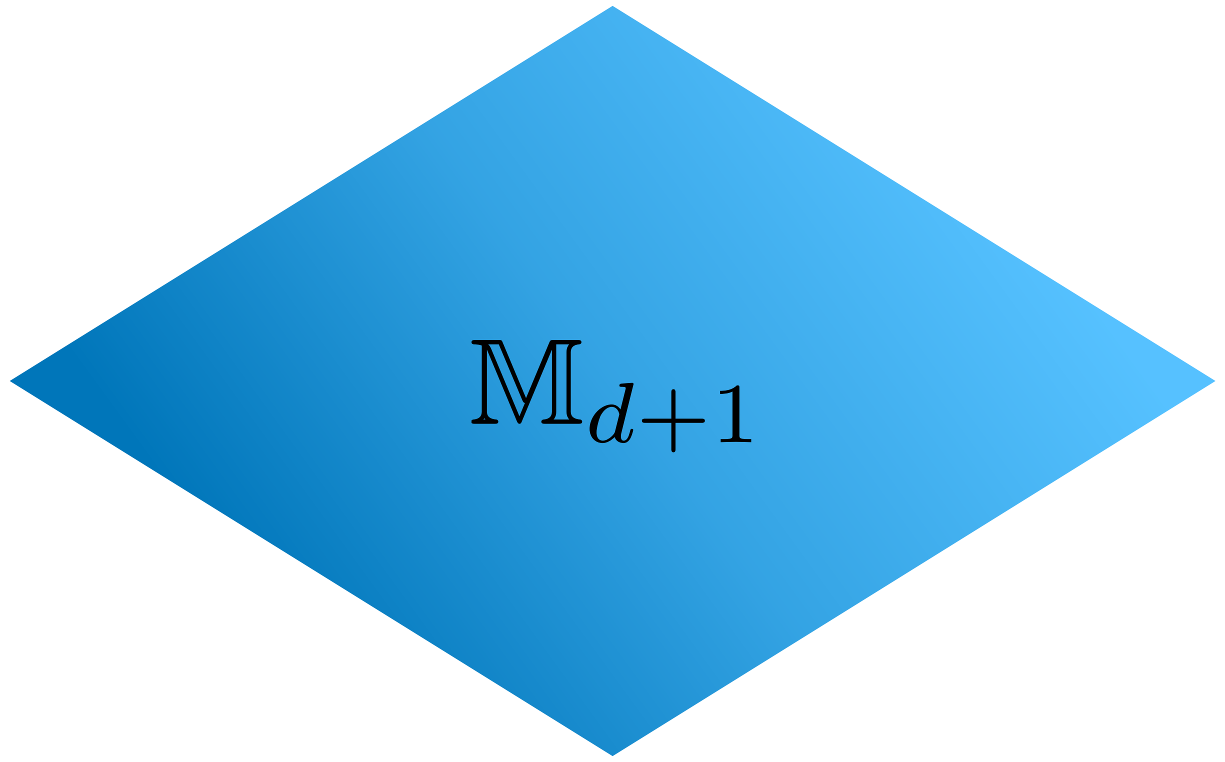
What is de Sitter?

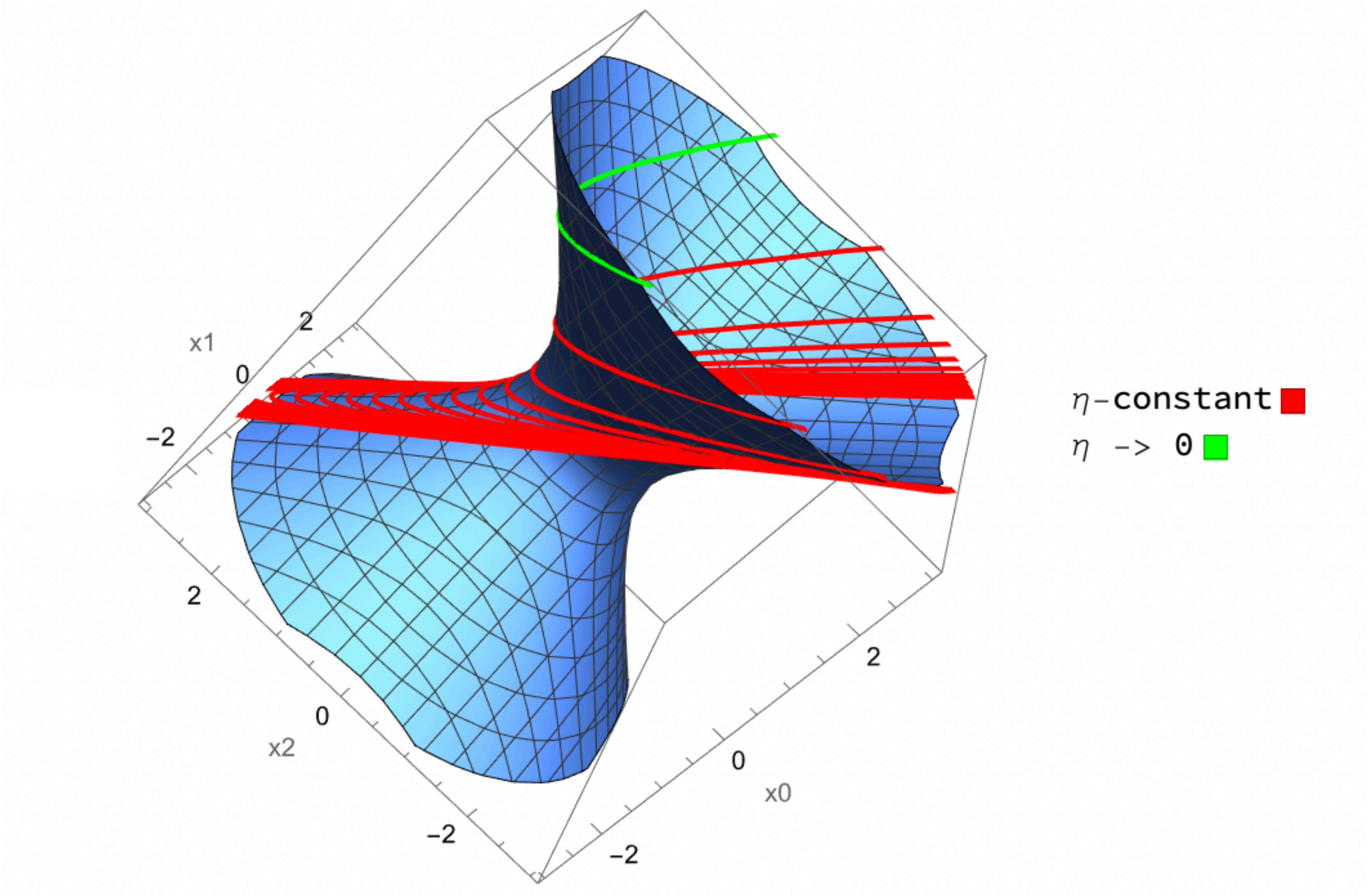




$$-Y_0^2 + Y_1^2 + \cdots + Y_d^2 = R^2$$

Group symmetry	Poincare group	Conformal group $SO(1, d+1)$
Isometries	$H \; , \; P_i \; , \; B_i \; , \; M_{ij}$	$D \; , \; P_i \; , \; K_i \; , \; M_{ij}$
Coordinate system	$ds^2 = -dt^2 + d\vec{x}^2$	$ds^2 = \dfrac{-d\eta^2 + d\vec{y}^2}{\eta^2}$ $\eta < 0 \; , \; \text{boundary} : \eta \to 0$





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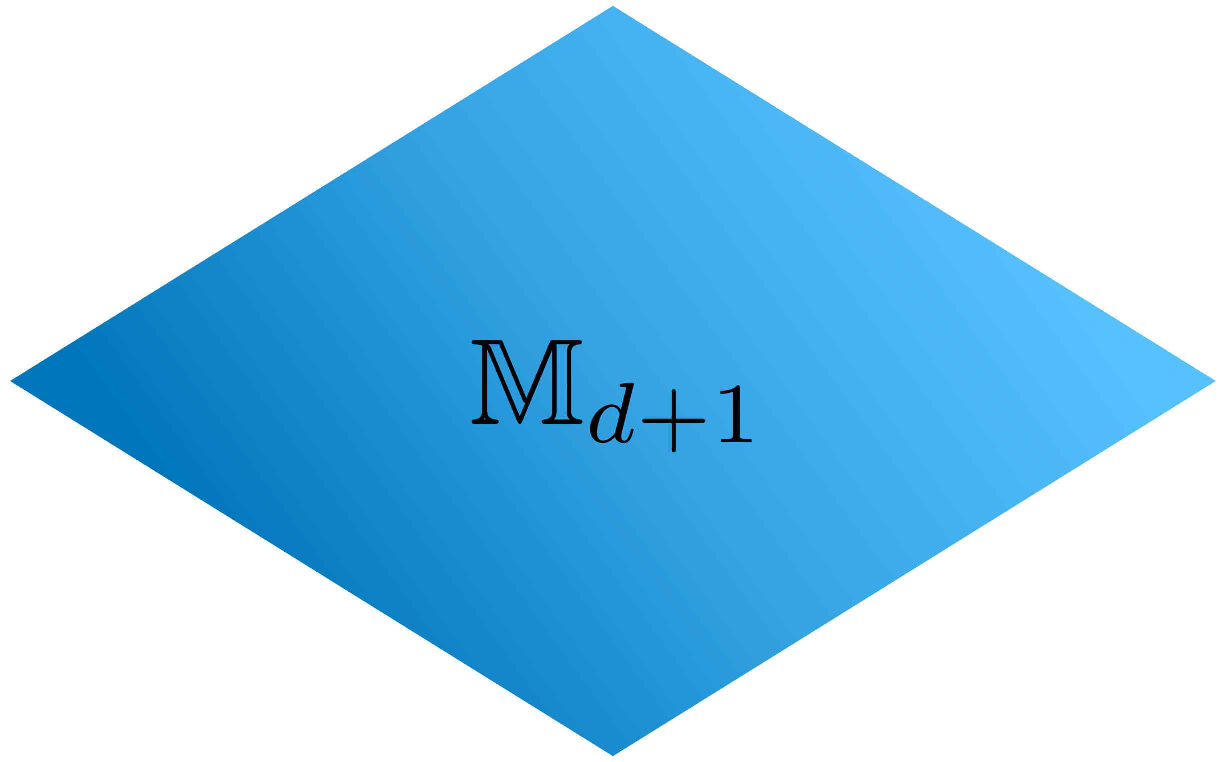
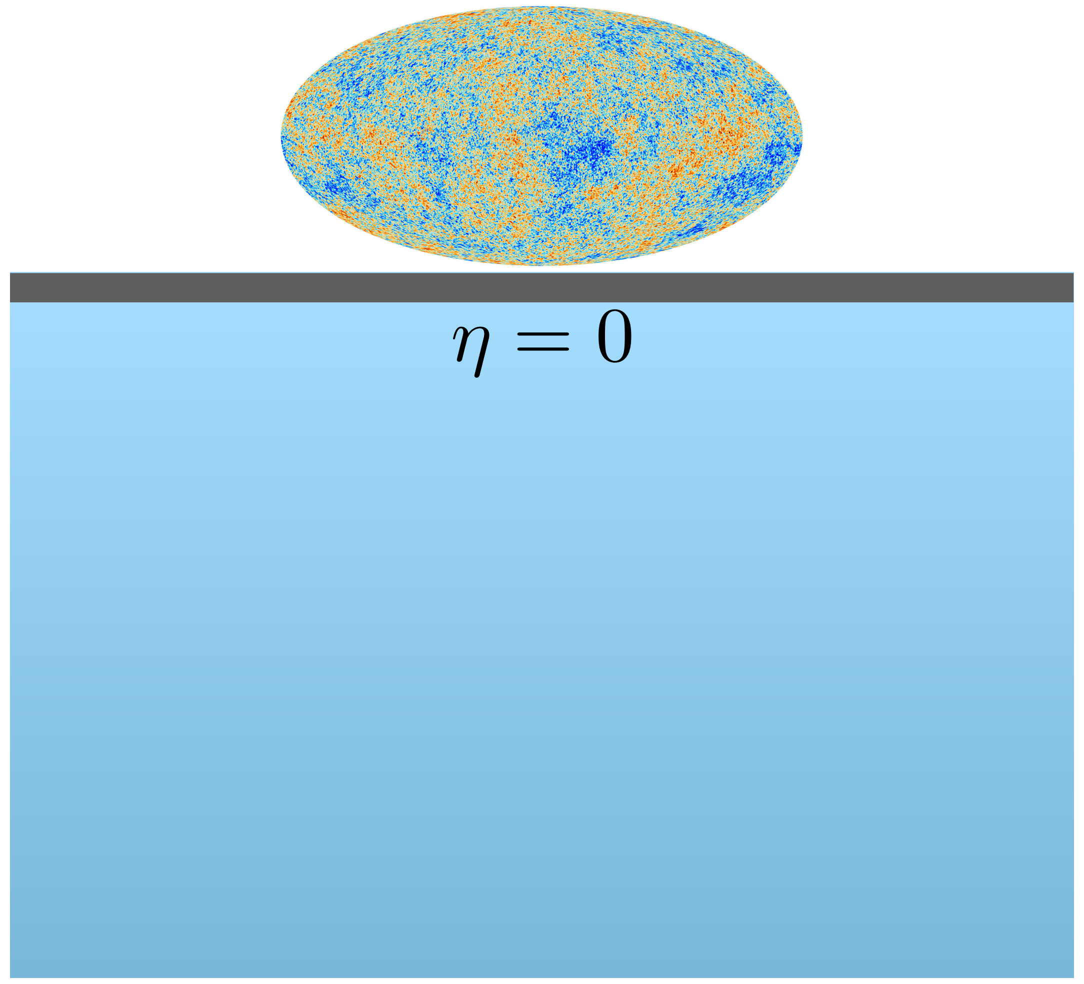
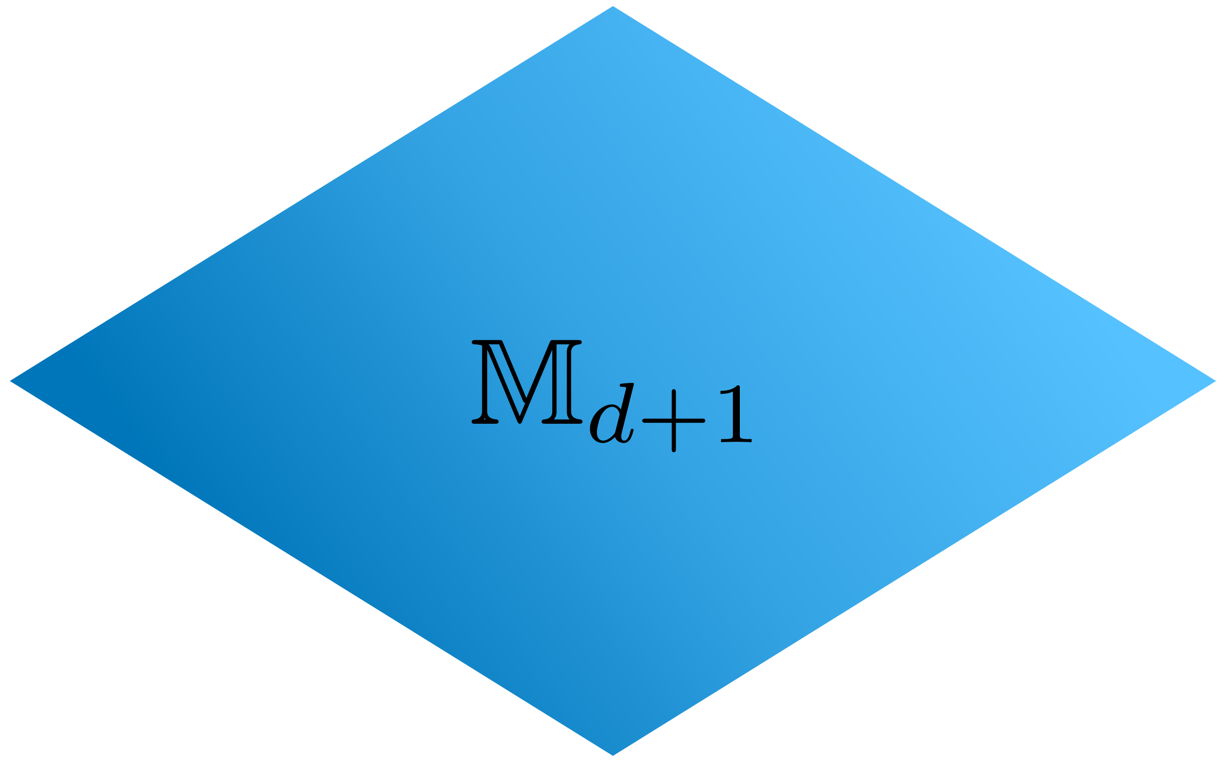
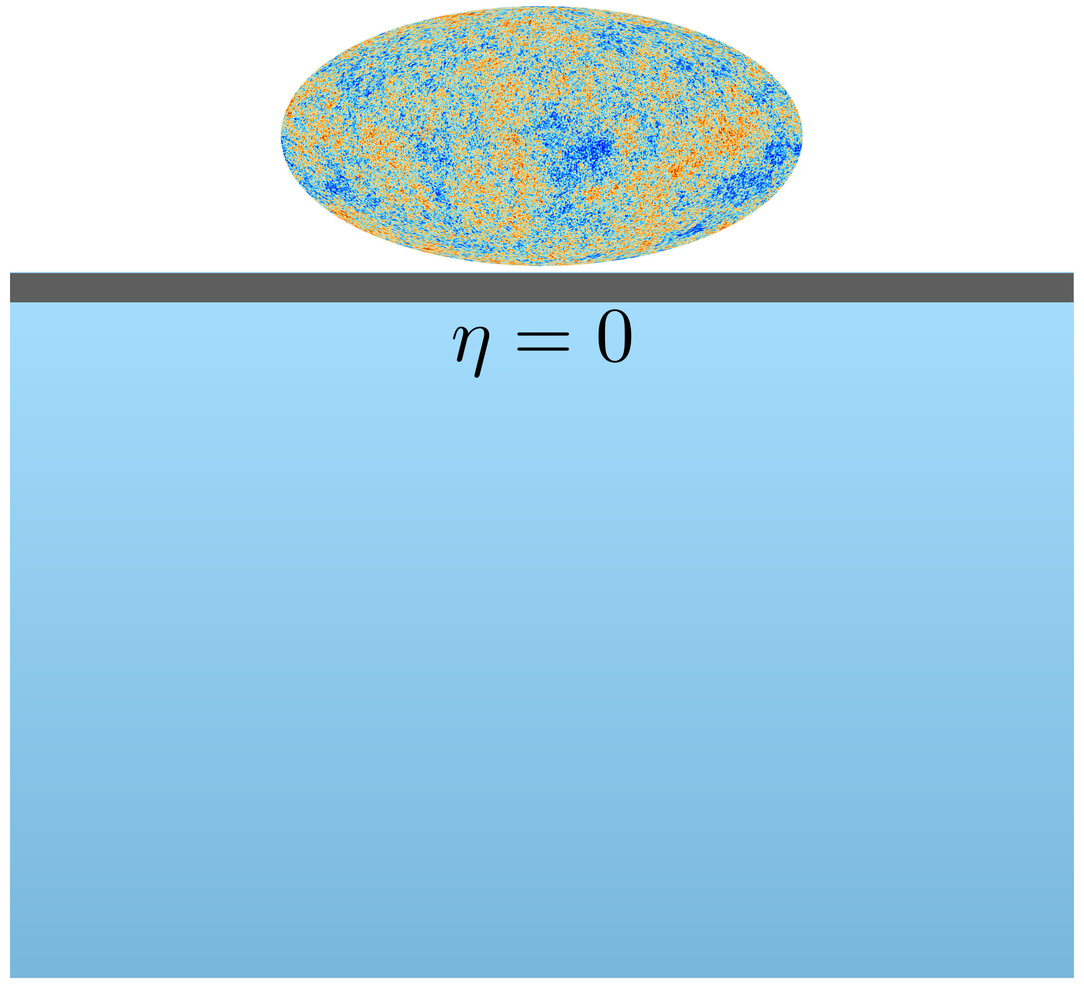




Hilbert space	$\vert m, \ell, \vec{k}\rangle$	$\vert \Delta, \ell, \vec{k}\rangle$
Unitary Irreps:	$m \geq 0,\ \ \ell \in \mathbb{Z}^+$	$\Delta = \dfrac{d}{2} + i\lambda,\ \ \ell \in \mathbb{Z}^+$





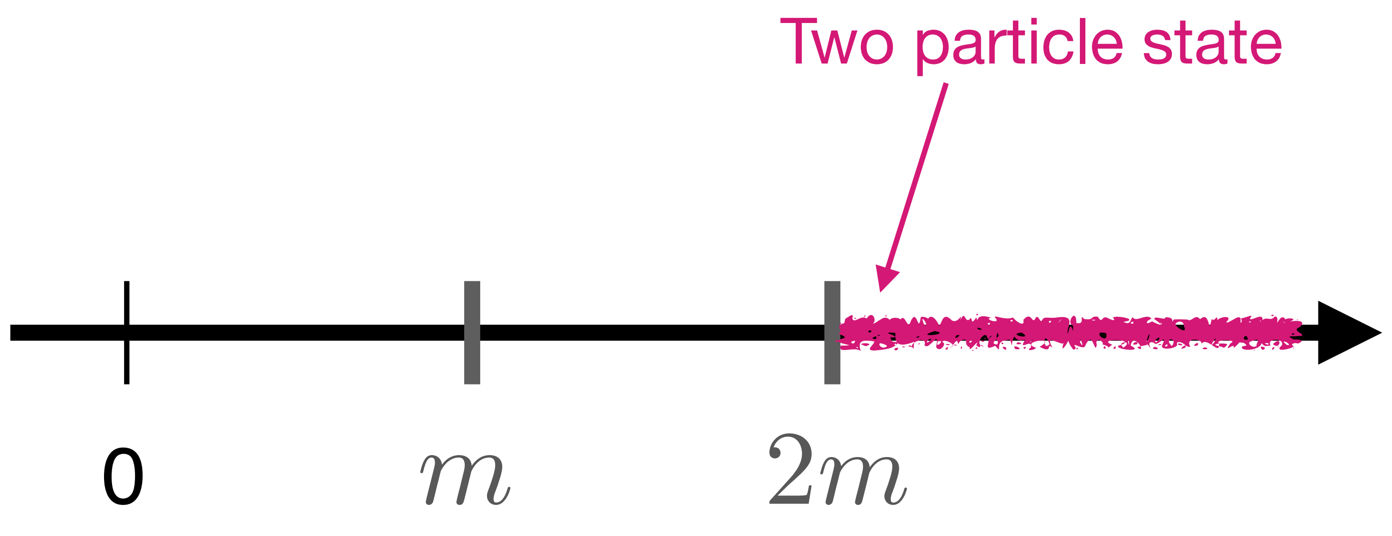
Hilbert space

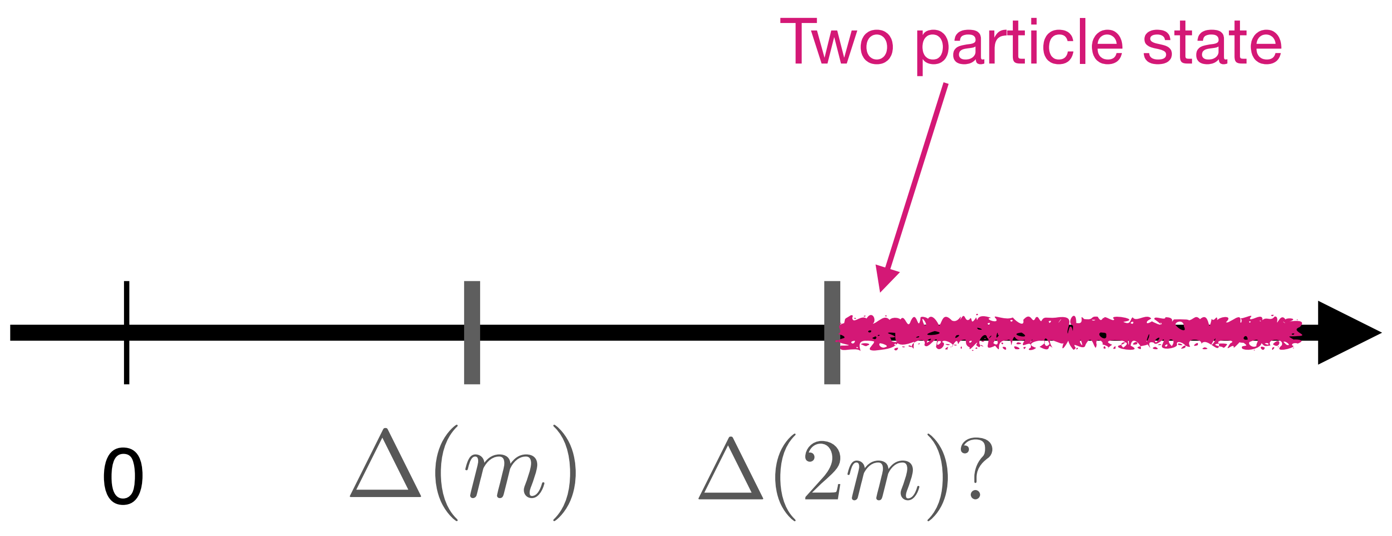
$$|m, \ell, \vec{k}\rangle \qquad |\Delta, \ell, \vec{k}\rangle$$

Unitary Irreps:

$$m \geq 0,\ \ \ell \in \mathbb{Z}^+ \qquad \Delta = \frac{d}{2} + i\lambda,\ \ \ell \in \mathbb{Z}^+$$

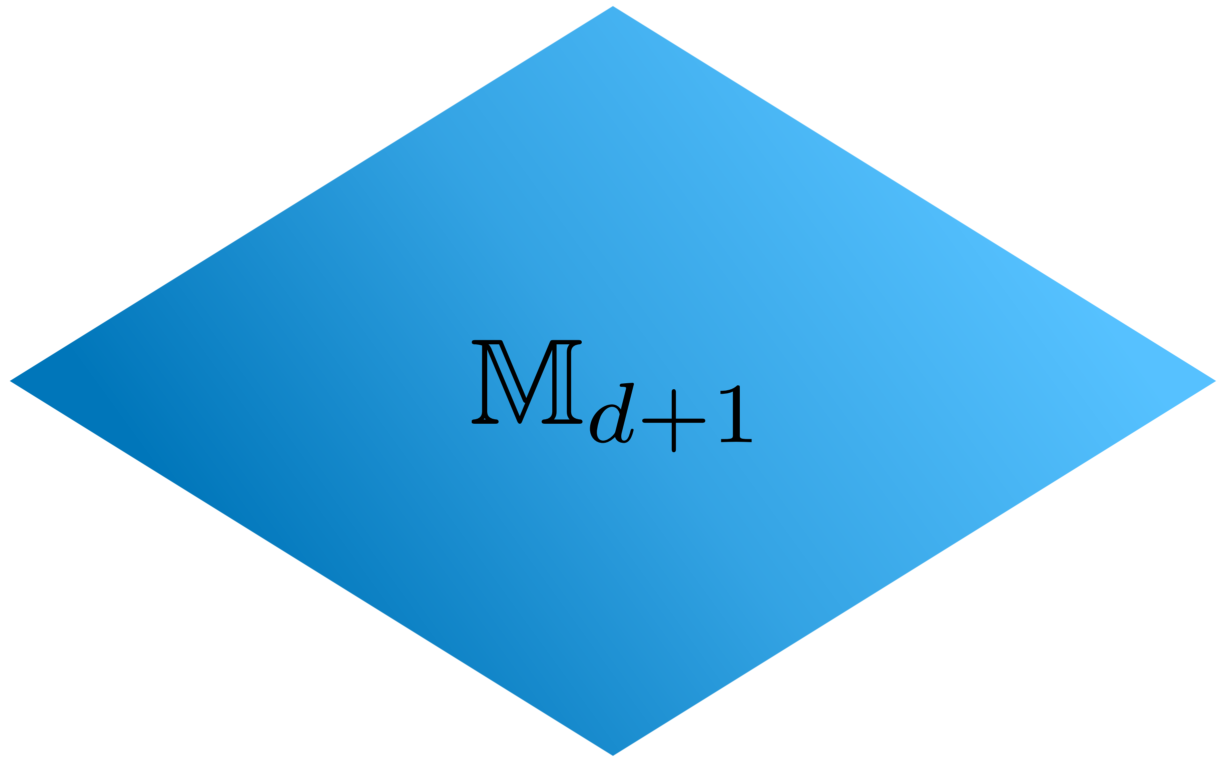
Example: Massive free scalar:

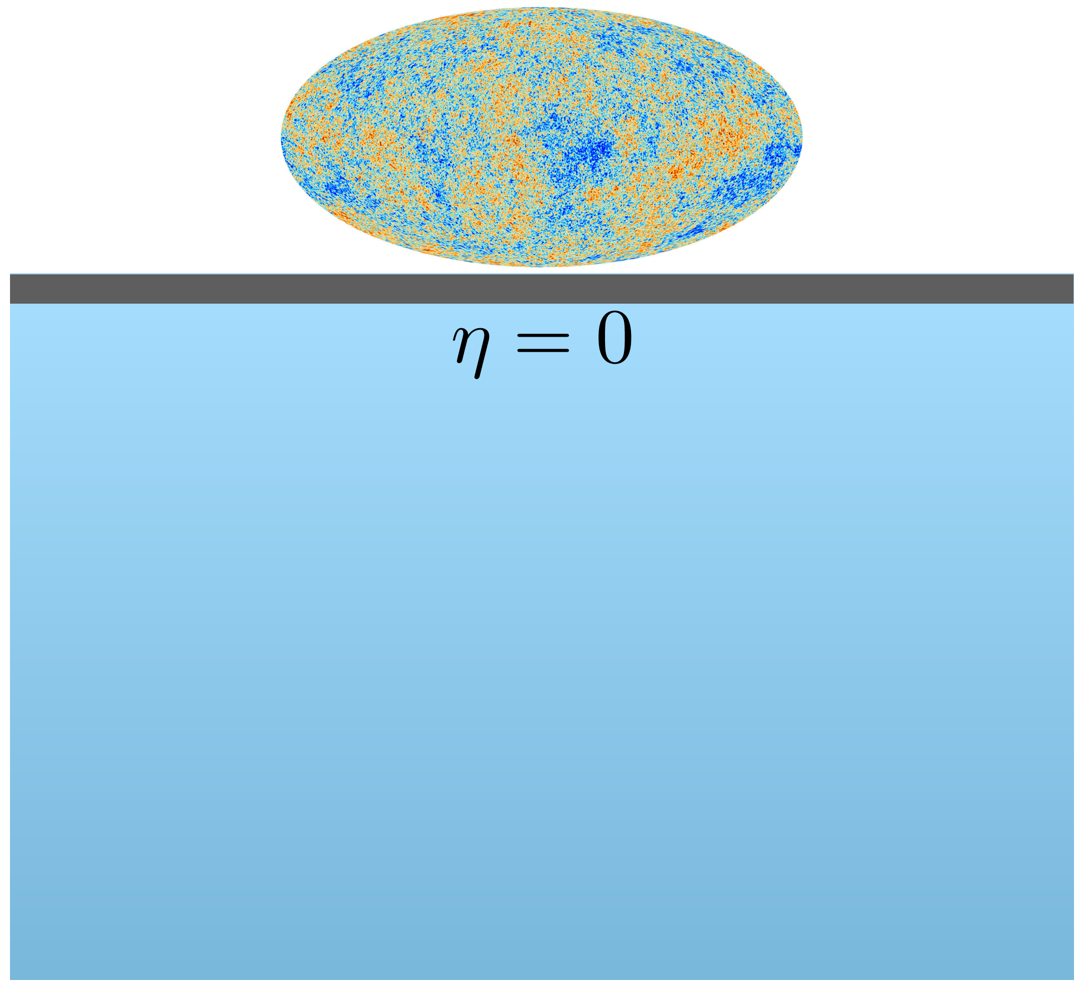




$$\Delta(d-\Delta) = m^2 R^2 \ : \text{free massive scalar}$$

$$\Delta(m) = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - m^2 R^2}$$





Hilbert space

$$|m, \ell, \vec{k}\rangle \qquad\qquad |\Delta, \ell, \vec{k}\rangle$$

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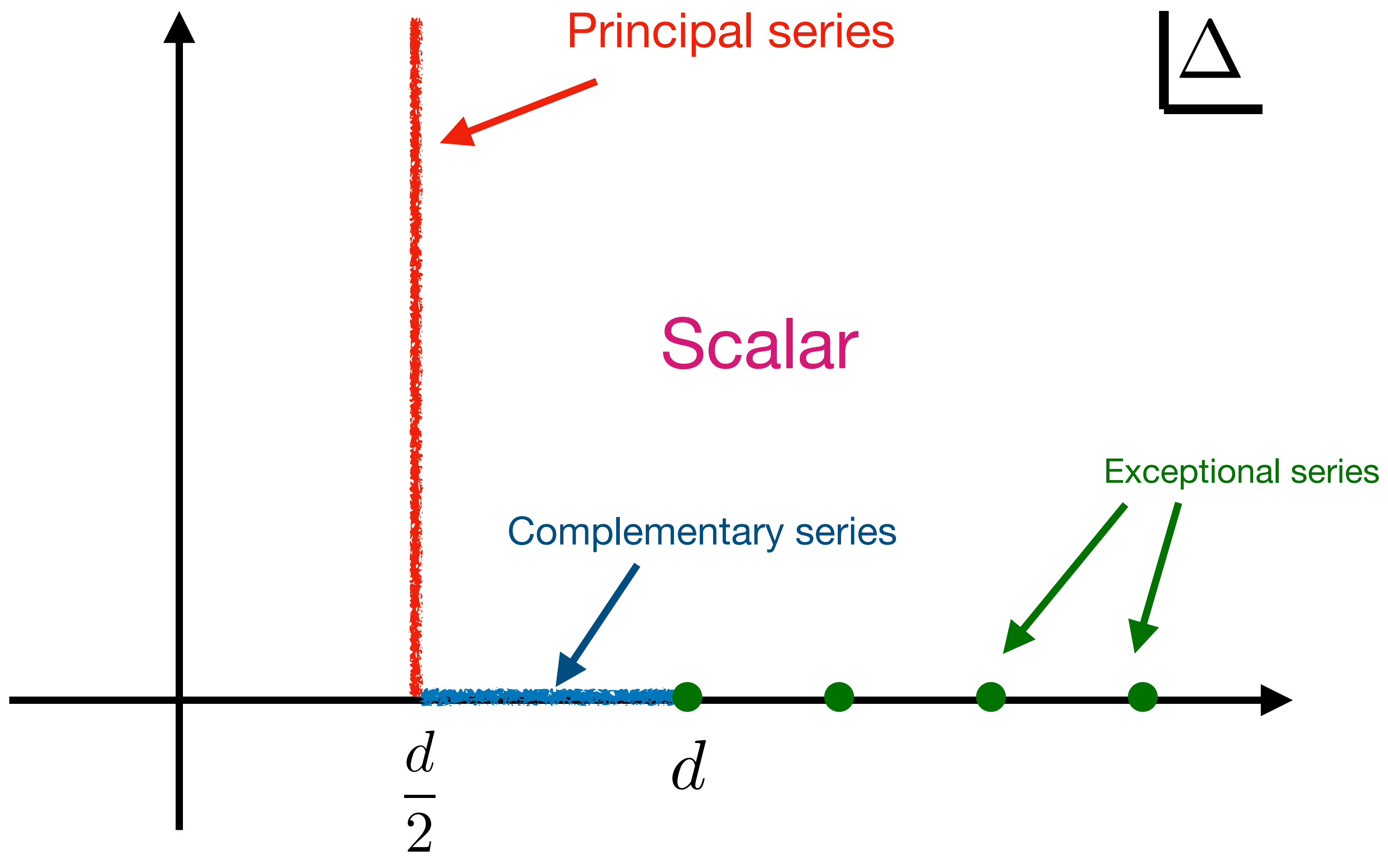
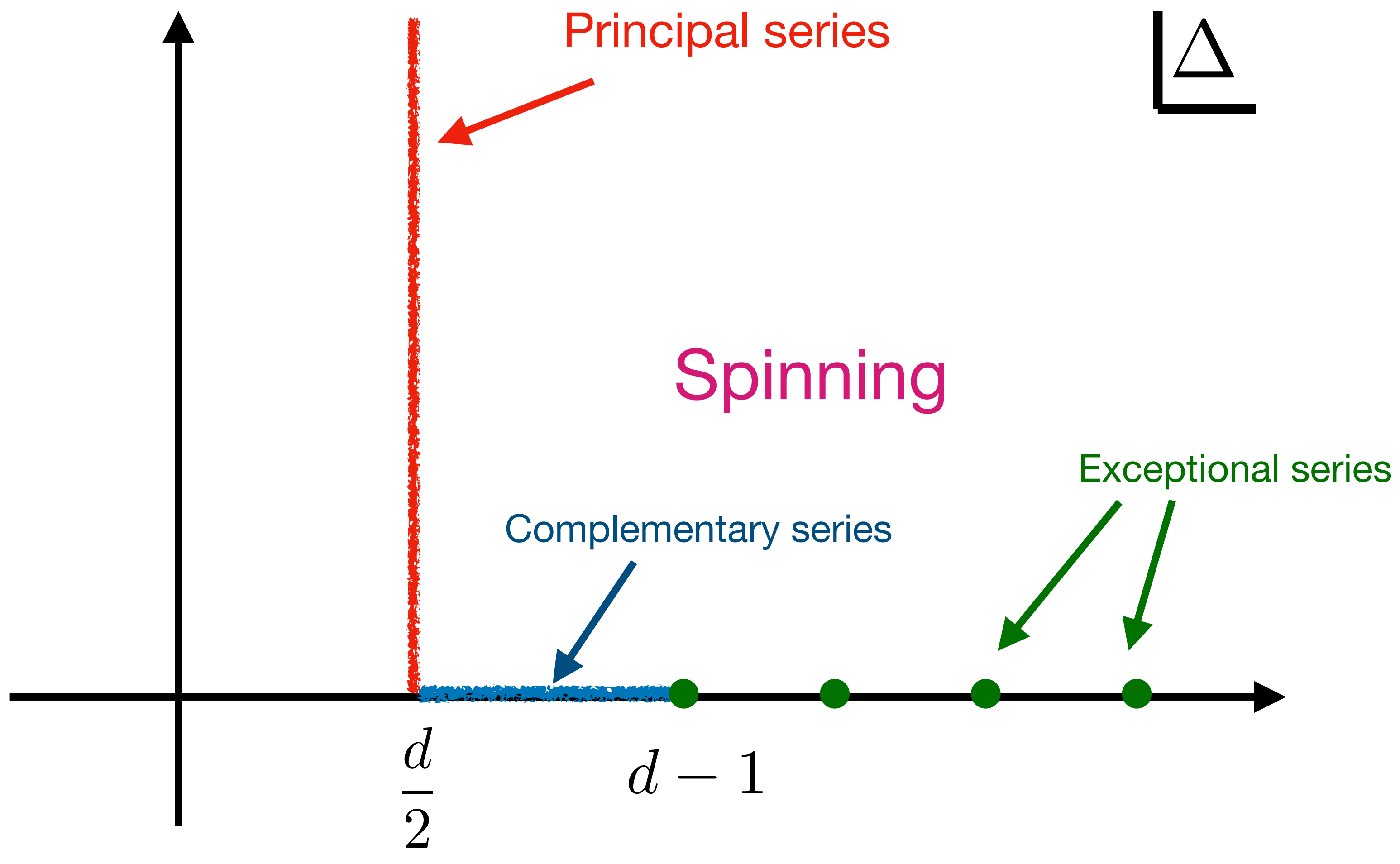




How do we study QFT in dS Non-perturbatively?

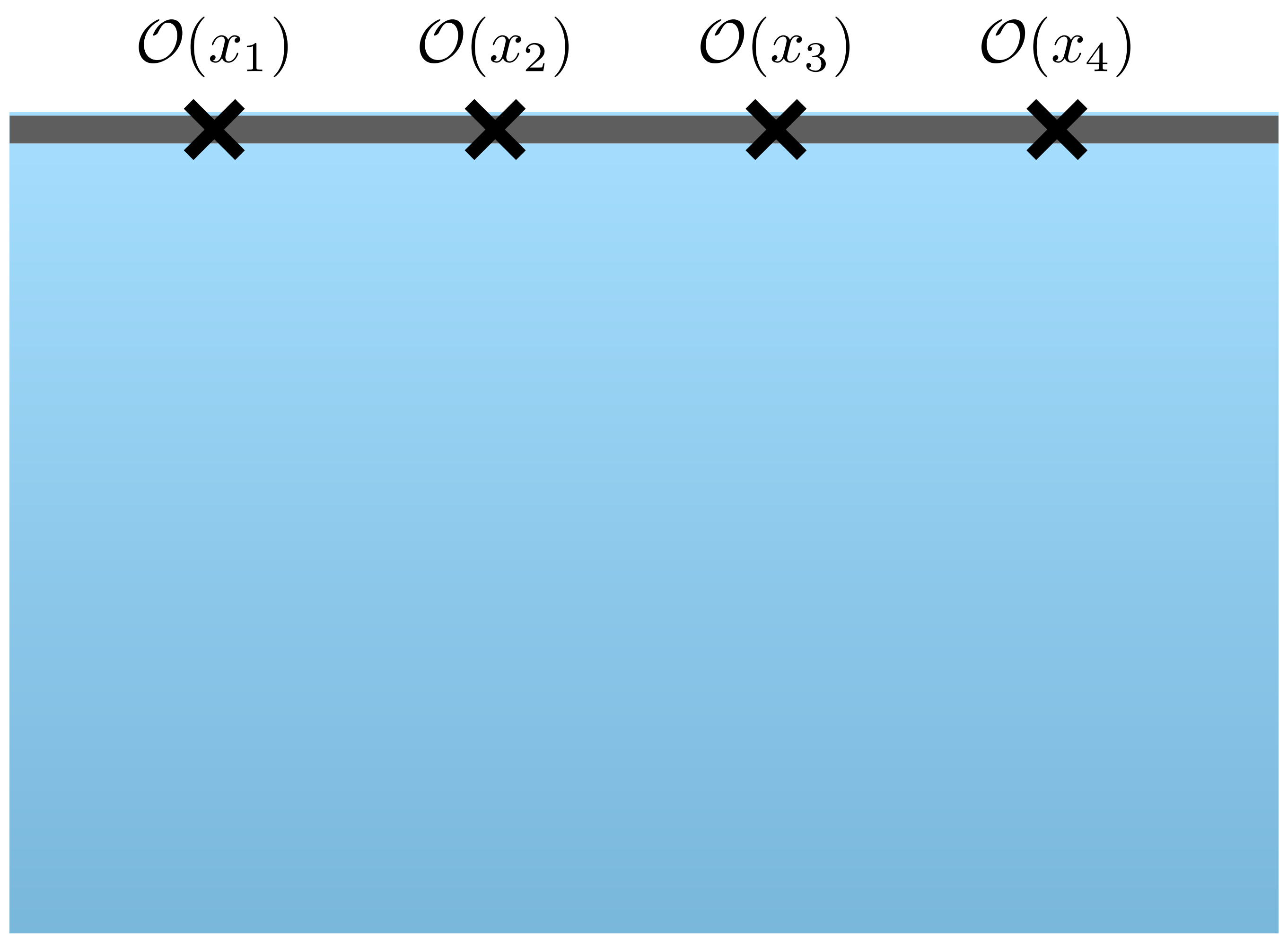
One way: (not the main focus of this talk)

Conformal boundary $\xrightarrow{?}$ Conformal bootstrap

Conformal Bootstrap vs Cosmological Bootstrap

- Conformal invariance
- Unitarity — Positivity
- Crossing symmetry

$$\langle \mathcal{O}(x_1) \ldots \mathcal{O}(x_4) \rangle_{\mathrm{CFT}} = \sum_{\Delta,\ell} \lambda^2_{\Delta,\ell} \, g_{\Delta,\ell}(z,\bar{z})$$



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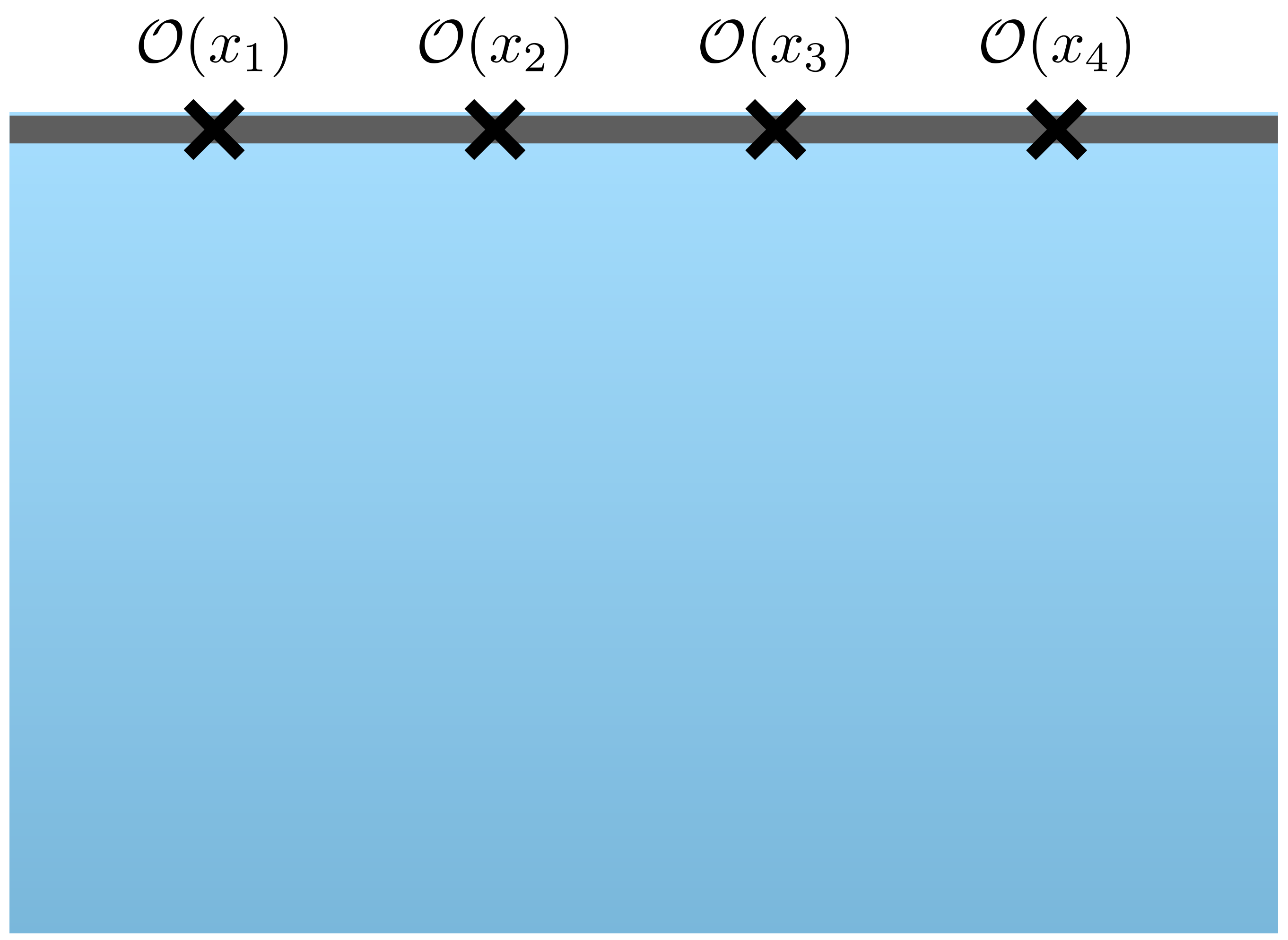

$$\langle \mathcal{O}(x_1)\ldots\mathcal{O}(x_4)\rangle_{\mathrm{CFT}} = \sum_{\Delta,\ell} \lambda^2_{\Delta,\ell}\, g_{\Delta,\ell}(z,\bar{z})$$

$$\langle \mathcal{O}(x_1)\ldots\mathcal{O}(x_4)\rangle_{\mathrm{dS}} = \sum_{\ell} \int_{\Delta} I_{\Delta,\ell}\, \Psi_{\Delta,\ell}(z,\bar{z})$$

Conformal Bootstrap vs Cosmological Bootstrap

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OPE

≥ 0

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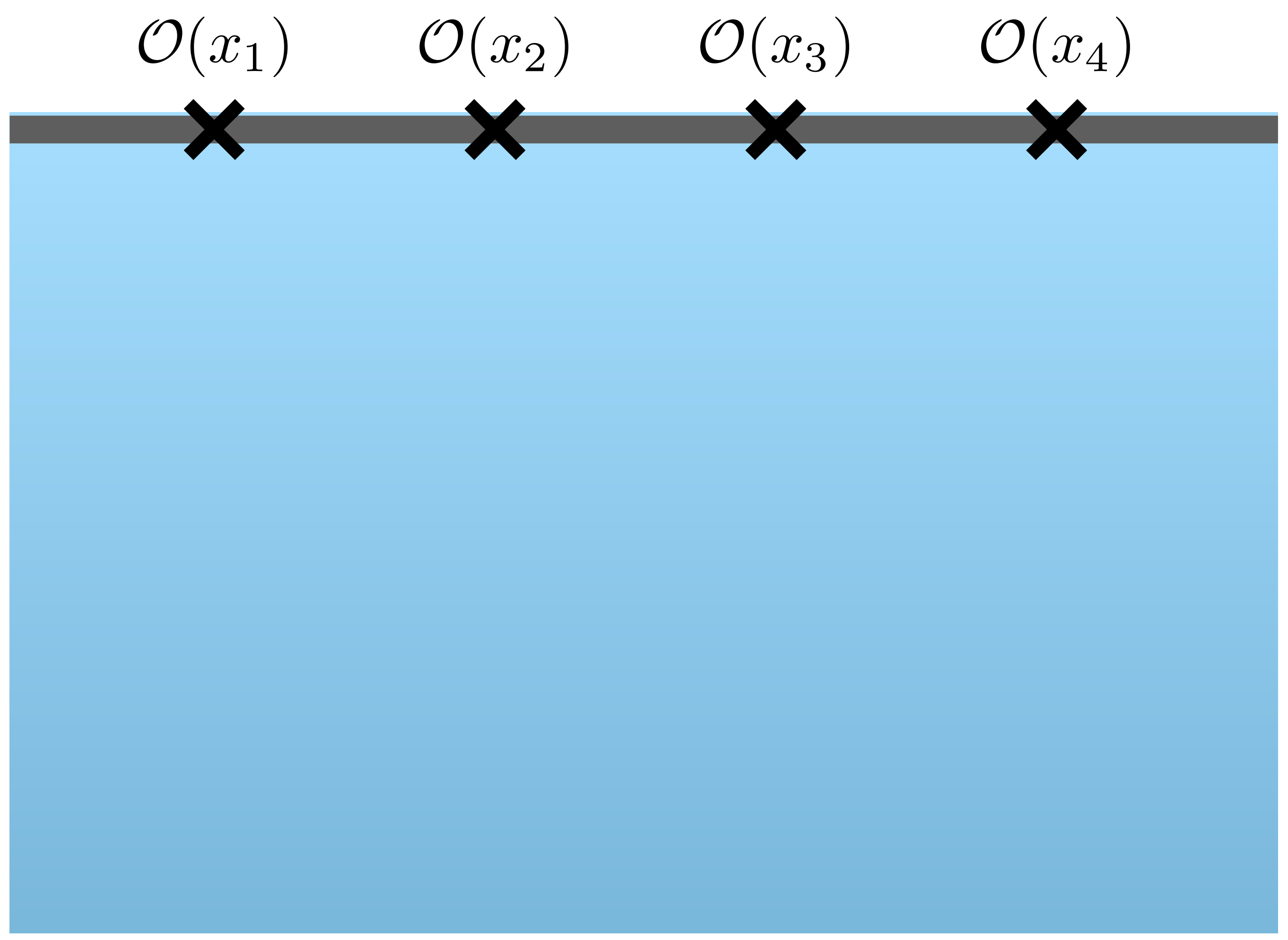
≥ 0

Boundary operators

Irreps



positivity + crossing $\longrightarrow$ bounds on $I_{\Delta,\ell}$: $0 < I_{\Delta,\ell} < \#$

2d de Sitter [2107.1387]

Outline:

- Källén–Lehmann(KL): Non-perturbative

Bulk two-point functions:

1. Do we have a KL decomposition for dS too?
2. Does unitarity imply positivity for dS?
3. Can we find boundary theory / boundary operators?
4. Can we invert the KL decomposition and find explicit expression for spectral densities?
5. Can we understand what controls the analytic properties of spectral densities?

Källén–Lehmann spectral decomposition

KL decomposition in Minkowski:

A spectral decomposition of the two-point function into a sum/integral over free propagators (kinematical functions)

$$\langle \phi(x_1)\phi(x_2)\rangle = \int d\mu^2 \rho(\mu^2)\, G_{\mathrm{free}}(x_{12}, \mu^2)$$

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- Non-perturbative!
- Symmetry fixes the x-dependence
- Unitarity $\longrightarrow$ Positive density

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A spectral decomposition of the two-point function into a sum/integral over free propagators (kinematical functions)

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- Symmetry fixes the x-dependence
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Is it useful? Yes! Some examples:

1. No higher derivative terms in the UV complete Lagrangian
2. Bounds on EFT coefficients

Sketch of the derivation in Minkowski

$$\mathbb{1} = \sum_n |\psi_n\rangle\langle\psi_n| = \int_{p,\mu} |p,\mu\rangle\langle p,\mu|$$

$|p,\mu\rangle$ single-particle state with mass μ

$$\langle\phi(x_1)\phi(x_2)\rangle$$

Sketch of the derivation in Minkowski

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$$\langle\phi(x_1)\phi(x_2)\rangle = \int_{p,\mu} \langle 0|\phi(x_1)|p,\mu\rangle\langle p,\mu|\phi(x_2)|0\rangle$$

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$$\langle 0|\phi(x)|p,\mu\rangle = C_\mu e^{ip.x}$$

Fixed by spacetime symmetry

Sketch of the derivation in Minkowski

$$\langle \phi(x_1)\phi(x_2)\rangle = \int_{p,\mu} \langle 0|\phi(x_1)|p,\mu\rangle\langle p,\mu|\phi(x_2)|0\rangle$$

$$= \int_\mu |C_\mu|^2 \int_p e^{ip.(x_1-x_2)}$$

$$= \int_0^\infty d\mu^2\, \underbrace{\rho(\mu^2)}_{\geq 0}\, G_{\text{free}}(x_{12},\mu)$$

Integration over momentum

Minkowski vs dS

$$\mathbb{1} = \sum_n |\psi_n\rangle\langle\psi_n| = \int_{p,\mu} |p,\mu\rangle\langle p,\mu|$$

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$$\mathbb{1} = \sum_n |\psi_n\rangle\langle\psi_n| = \int_{P,\Delta} |P,\Delta\rangle\langle P,\Delta|$$

$|P,\Delta\rangle$ Fourier trnasform of $|\vec{k},\Delta\rangle$

$$\langle\phi(x_1)\phi(x_2)\rangle = \int_{P,\Delta} \langle 0|\phi(Y_1)|P,\Delta\rangle\langle P,\Delta|\phi(Y_2)|0\rangle$$

Minkowski vs dS

$$\mathbb{1} = \sum_n |\psi_n\rangle\langle\psi_n| = \int_{p,\mu} |p,\mu\rangle\langle p,\mu|$$

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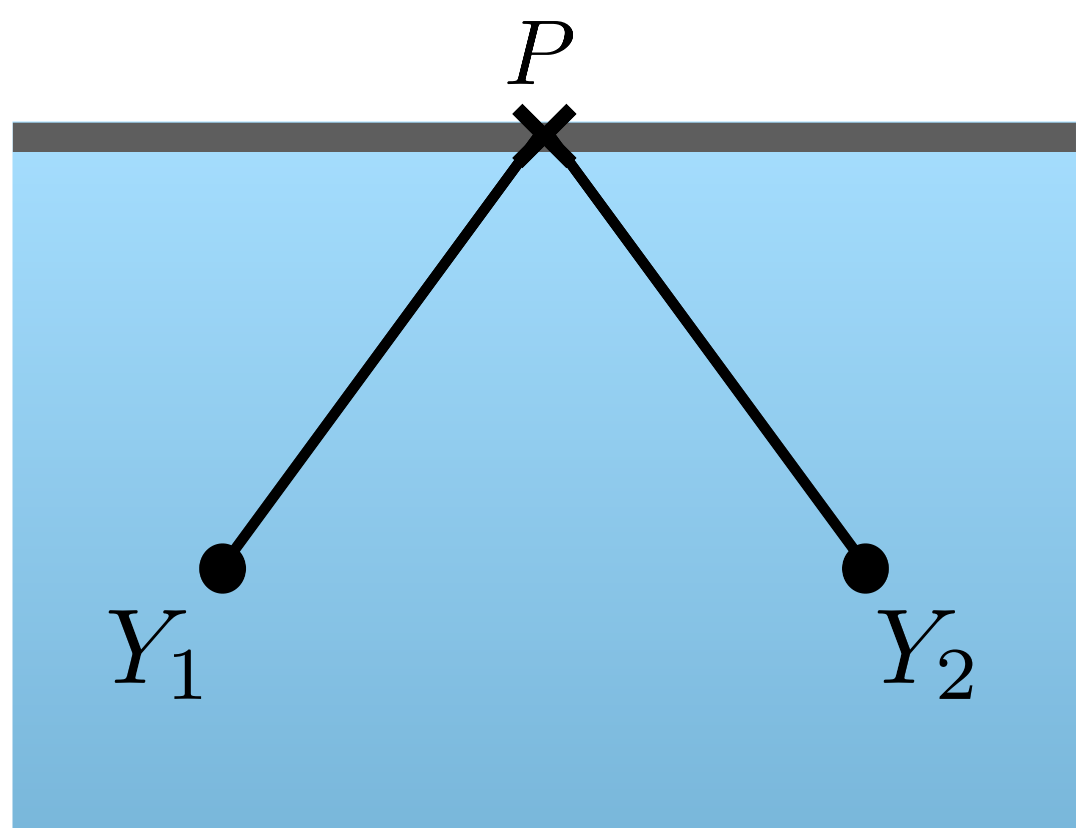
$$\langle 0|\phi(Y)|P,\Delta\rangle = c_{\mathcal{O}}\mathcal{K}(Y,P)$$



Minkowski vs dS

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$|p,\mu\rangle$ single-particle state with mass μ

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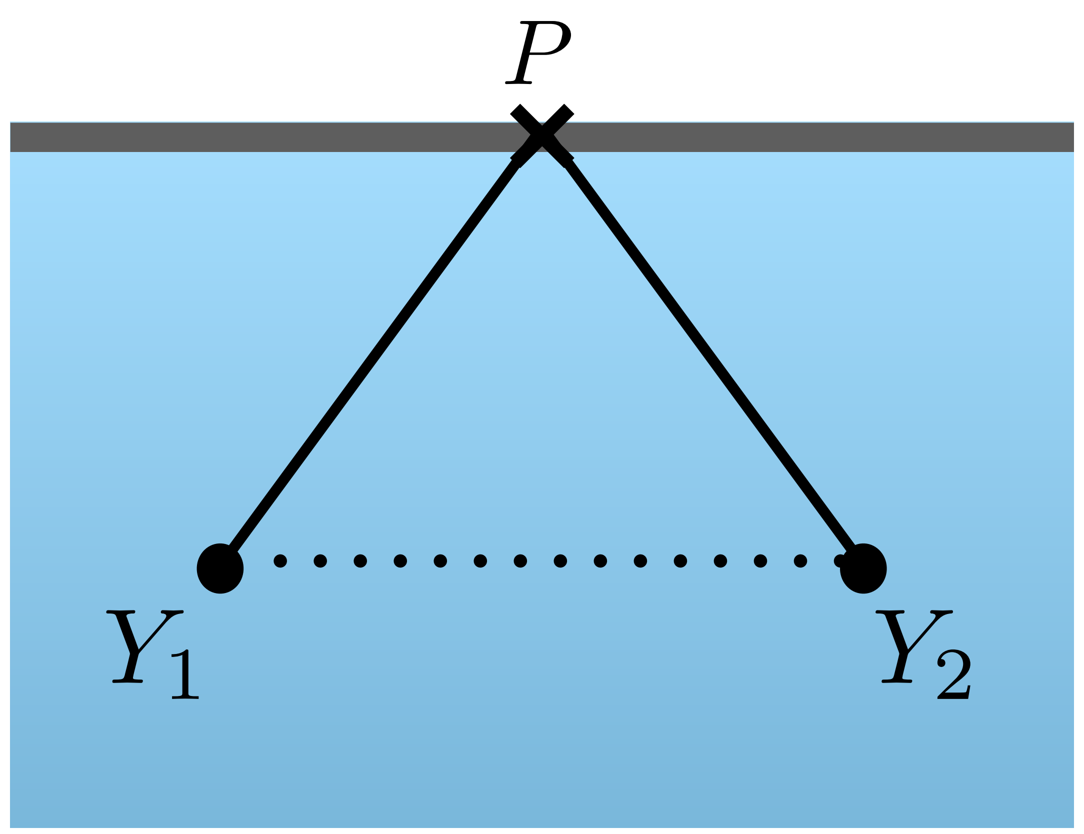


KL decomposition in Minkowski:

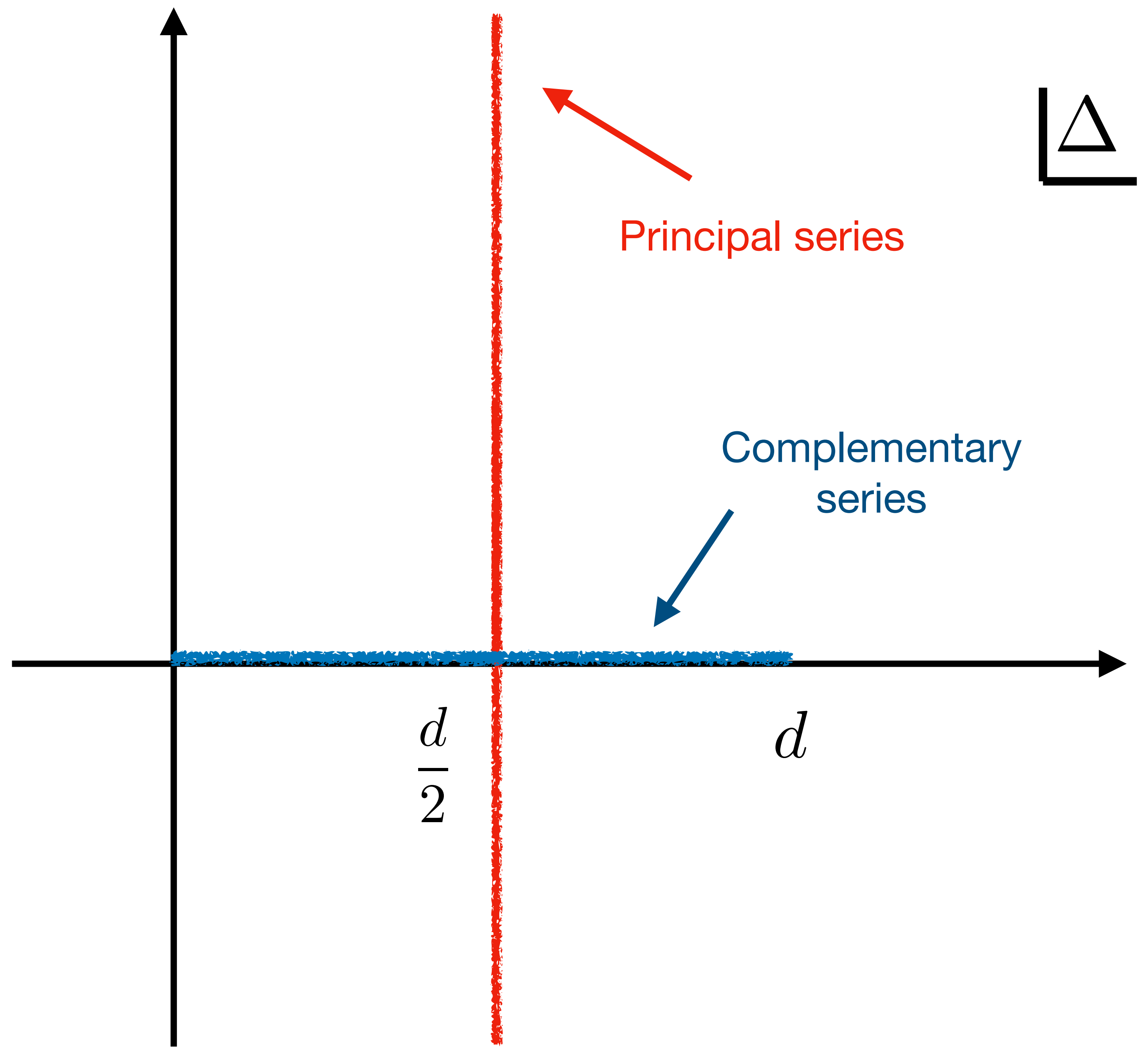
$$\langle \phi(x_1)\phi(x_2)\rangle = \int d\mu^2 \overset{\geq 0}{\rho(\mu^2)}\, G_{\text{free}}(x_{12}, \mu^2)$$

1. Do we have a KL decomposition for dS too? Yes
2. Does unitarity imply positivity for dS? Yes

KL decomposition in dS

$$\langle \phi(Y_1)\phi(Y_2)\rangle = \int_{\text{reps}} \overset{\geq 0}{\rho(\Delta)}\, G_{\text{free}}(Y_{12}, \Delta)$$

$$\langle T^{(J)}(Y_1)T^{(J)}(Y_2)\rangle = \sum_{\ell=0}^{J} \int_{\text{reps}} \overset{\geq 0}{\rho_\ell^{(J)}(\Delta)}\, \nabla_1^{J-\ell}\nabla_2^{J-\ell}\; G_\ell(Y_{12}, \Delta)$$



KL decomposition in Minkowski:

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Bessel

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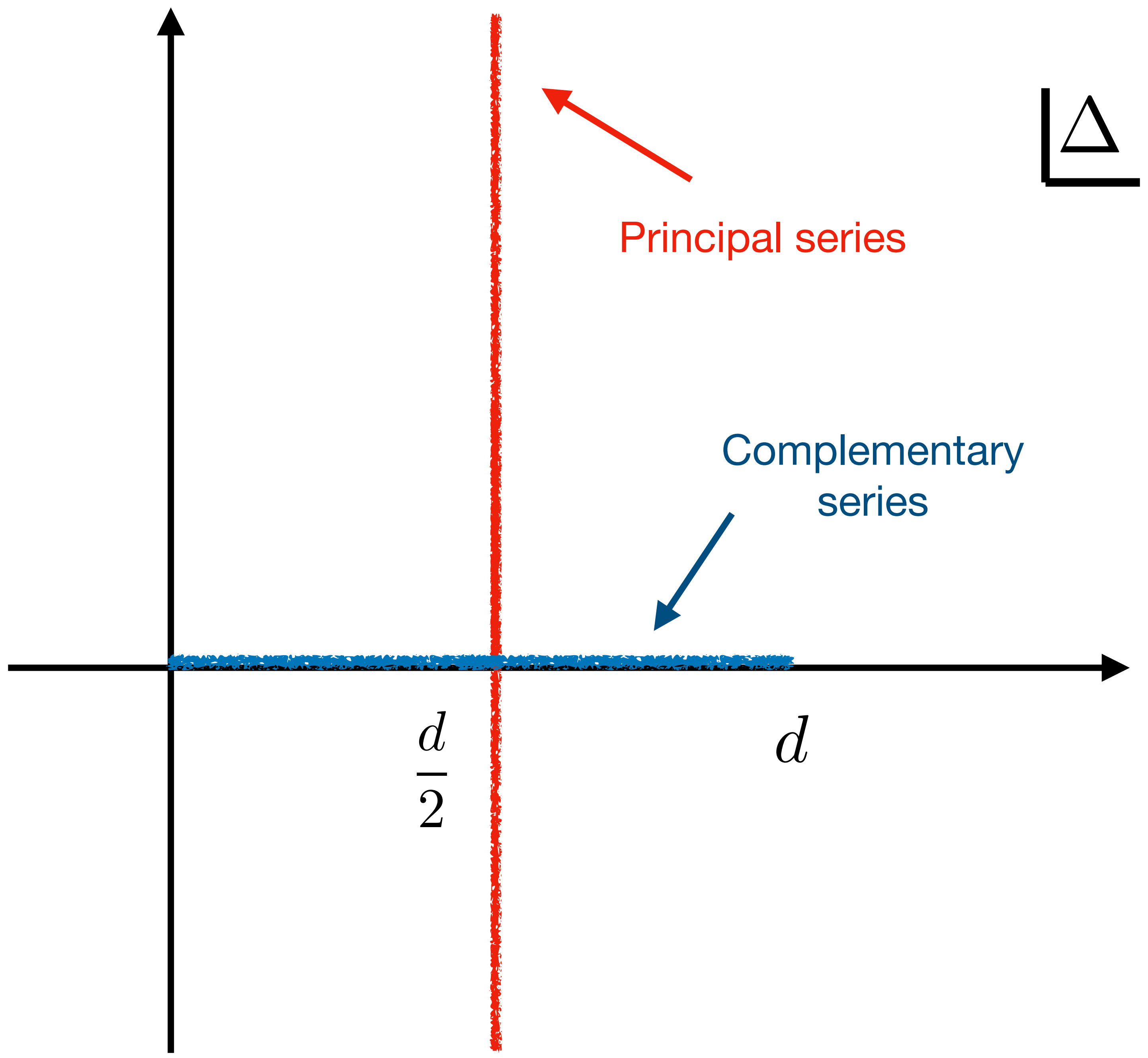
2. Does unitarity imply positivity for dS? Yes

KL decomposition in dS

$$\langle \phi(Y_1)\phi(Y_2)\rangle = \int_{\text{reps}} \overset{\geq 0}{\rho(\Delta)}\, G_{\text{free}}(Y_{12},\Delta)$$

$${}_2F_1(\Delta, d-\Delta, \frac{d+1}{2}, \frac{\sigma+1}{2})$$

$$\sigma = Y_1 \cdot Y_2 = \frac{\eta_1^2+\eta_2^2-\mathbf{y}_{12}^2}{2\eta_1\eta_2}$$

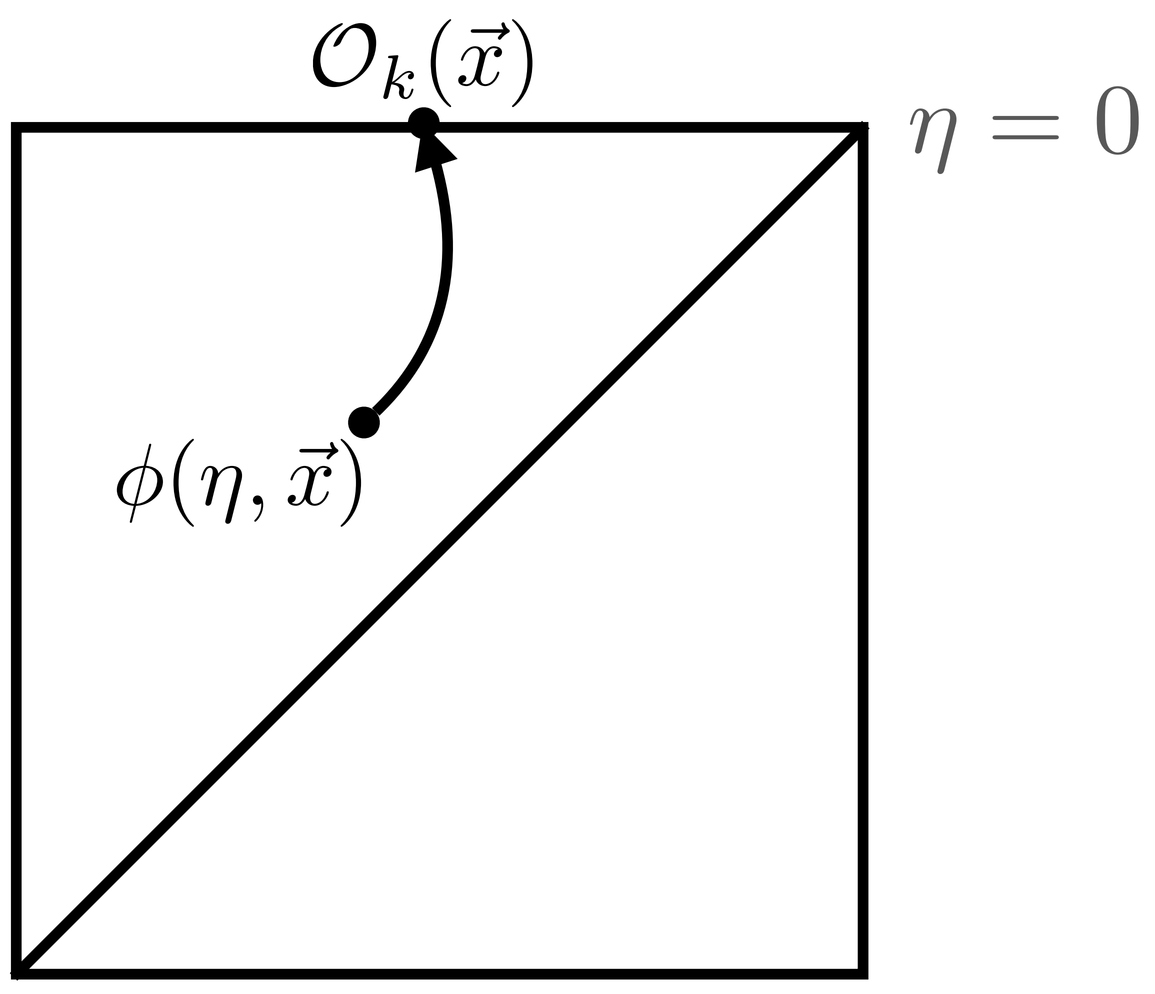


3. Can we find boundary theory operator content?

Spectral density and boundary operators:

$$\langle \phi(\eta, \vec{y}_1)\phi(\eta, \vec{y}_2)\rangle = \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta\, \rho(\Delta)\, G_{\text{free}}(\eta, \vec{y}_{12})$$

$\eta \to 0$



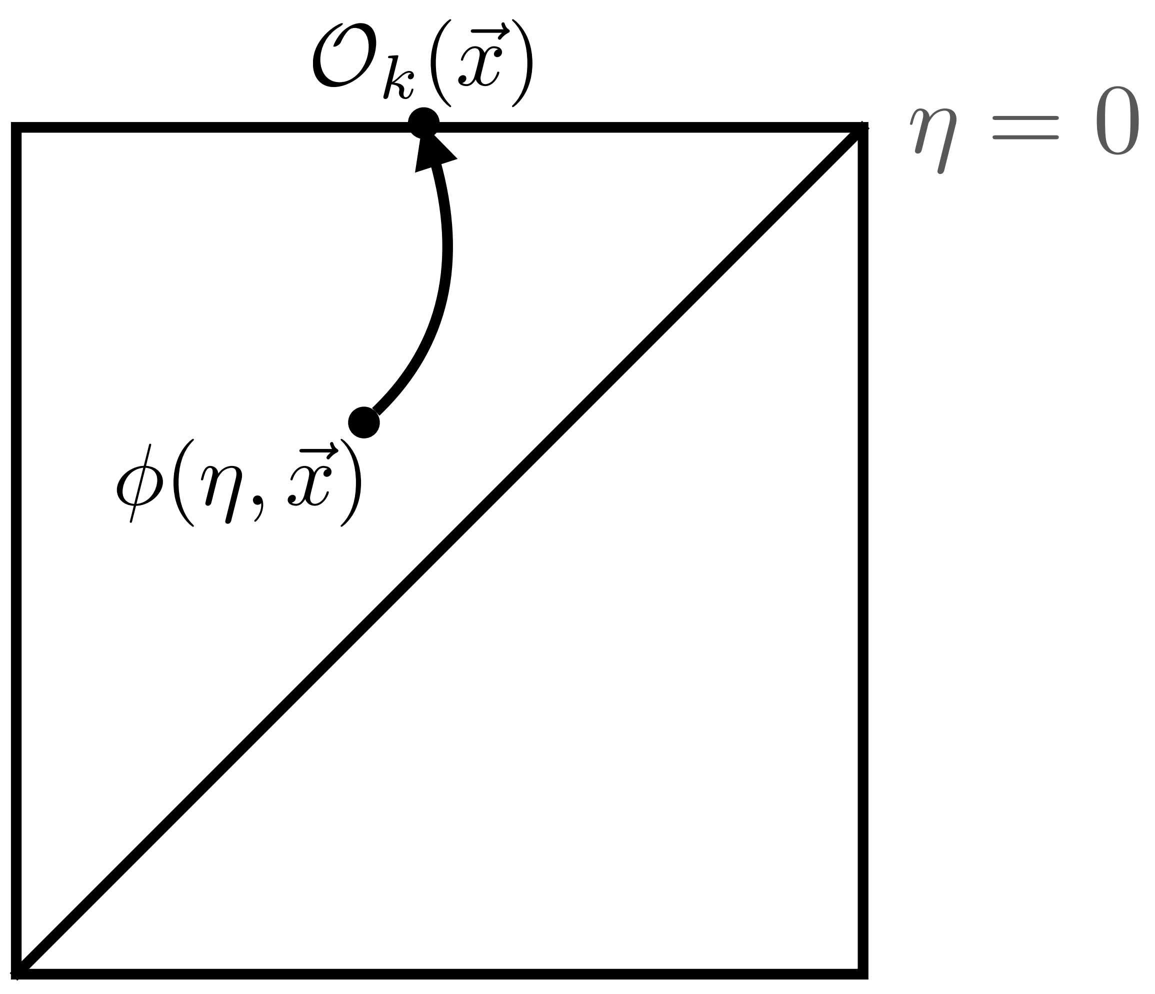
$$\phi(\eta, \vec{y}) = \sum_i (-\eta)^{\Delta_i}\, b_i \left[\mathcal{O}_i(\vec{y}) \;+\; \text{des}\right]$$

Spectral density and boundary operators:

$$\langle\phi(\eta,\vec{y}_1)\phi(\eta,\vec{y}_2)\rangle = \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta\,\rho(\Delta)\,G_{\text{free}}(\eta,\vec{y}_{12})$$

$\eta \to 0$ contour deformation

$$\sum_i b_i^2\,\langle\mathcal{O}_i(\vec{y}_1)\mathcal{O}_i(\vec{y}_2)\rangle \sim \sum_{j=\text{poles}} \text{Res}[\rho(\Delta_j)]\,\langle\mathcal{O}_j(\vec{y}_1)\mathcal{O}_j(\vec{y}_2)\rangle$$



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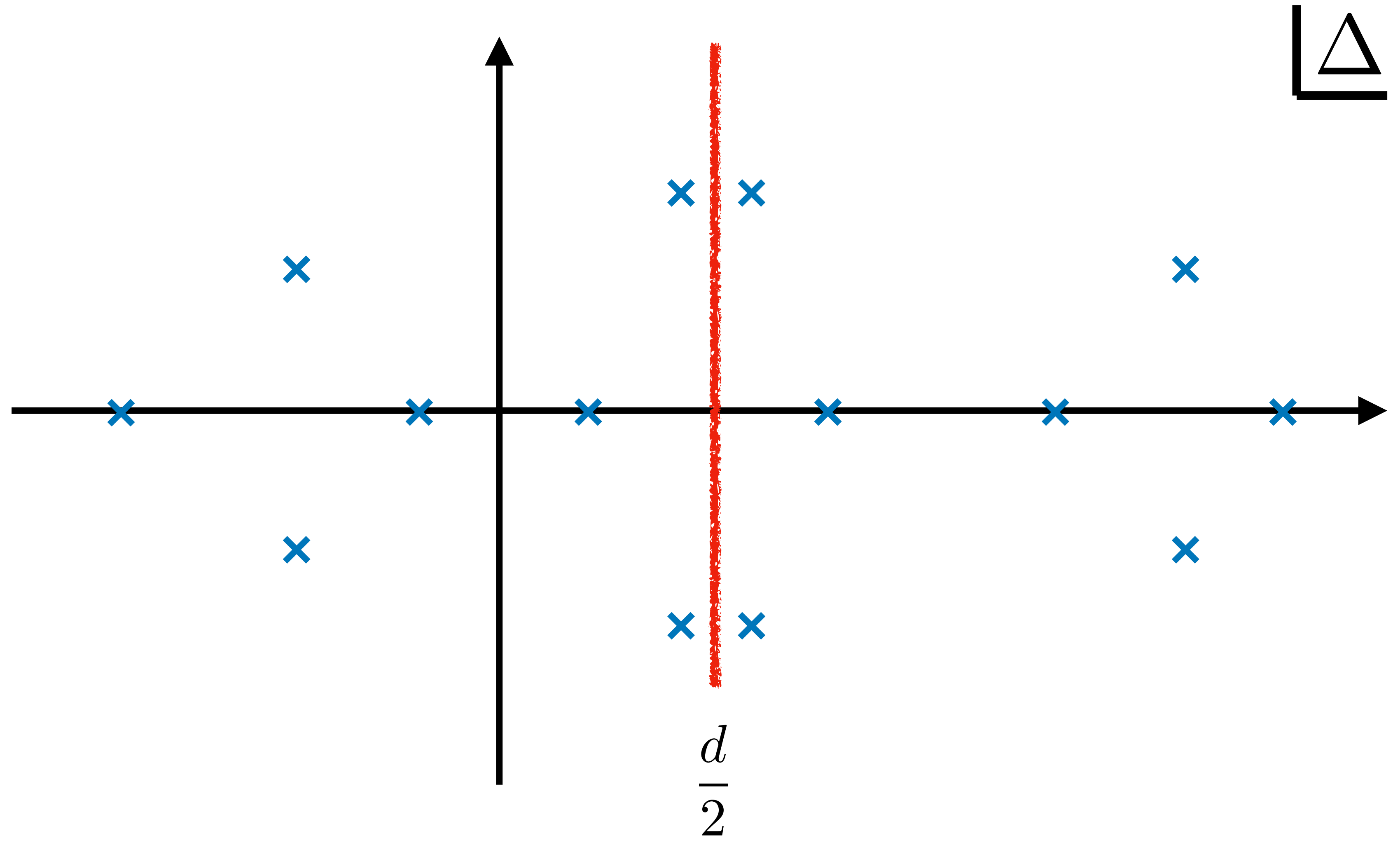


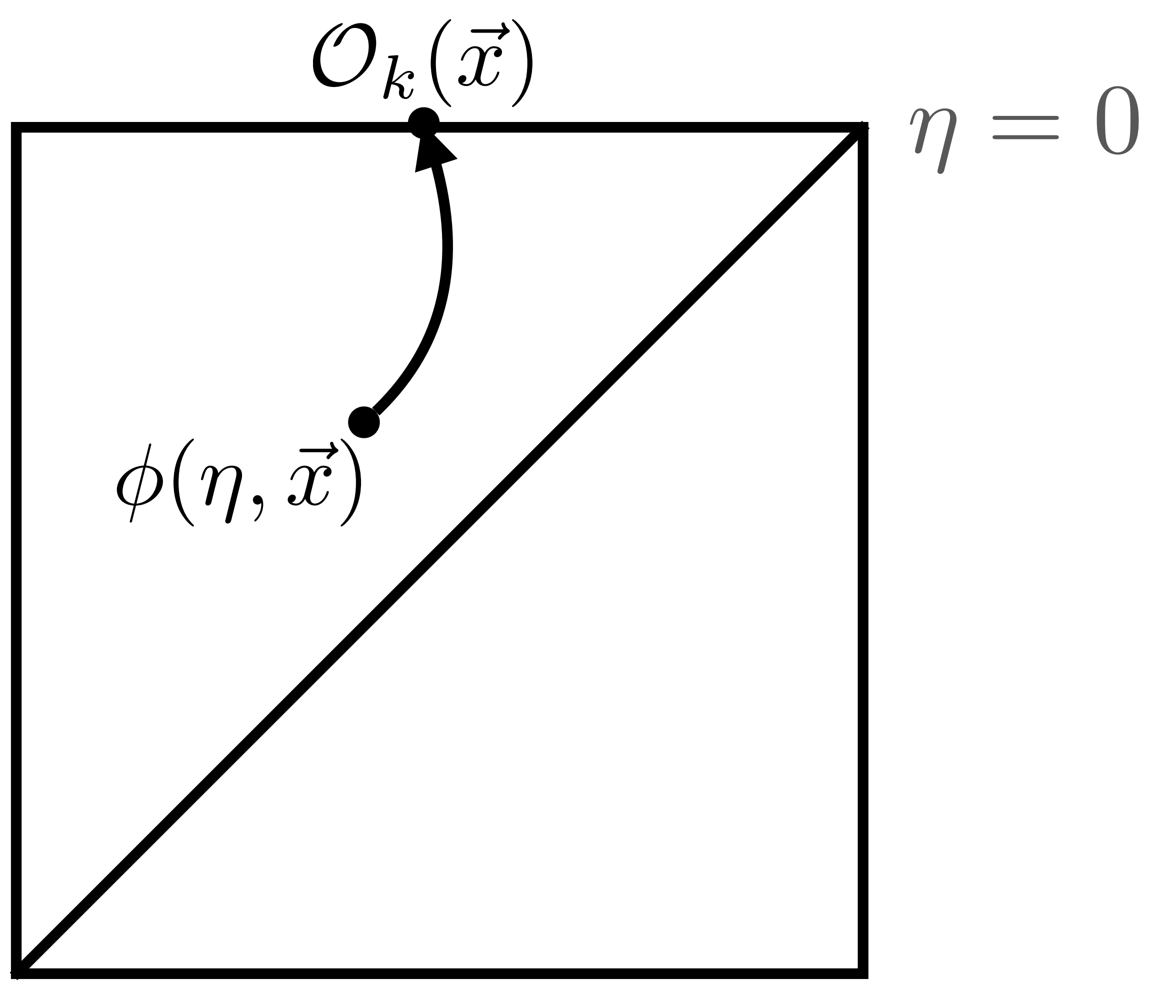
Spectral density and boundary operators:

$$\langle \phi(\eta, \vec{y}_1)\phi(\eta, \vec{y}_2)\rangle = \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta\, \rho(\Delta)\, G_{\text{free}}(\eta, \vec{y}_{12})$$

$\eta \to 0$ contour deformation

$$\sum_i b_i^2 \langle \mathcal{O}_i(\vec{y}_1)\mathcal{O}_i(\vec{y}_2)\rangle \sim \sum_{j=\text{poles}} \text{Res}[\rho(\Delta_j)]\, \langle \mathcal{O}_j(\vec{y}_1)\mathcal{O}_j(\vec{y}_2)\rangle$$

$$\Delta_i \sim \text{pole of } \rho(\Delta) \qquad \sum_i b_i^2 \sim \text{Res}[\rho(\Delta_i)]$$



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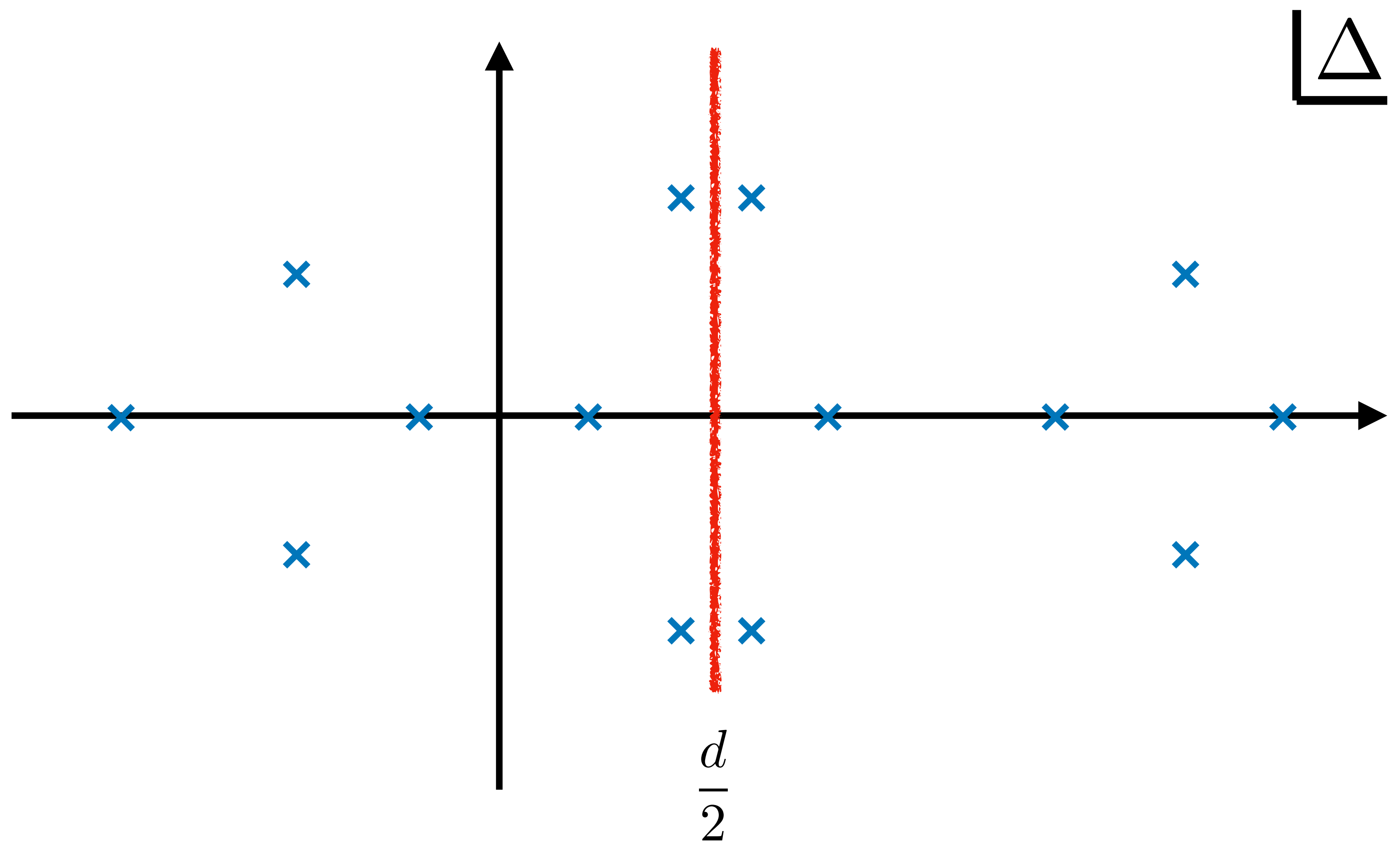


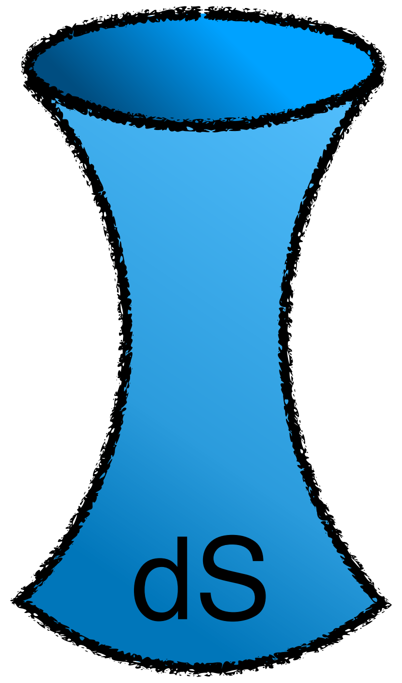
4. Can we invert the KL decomposition
and find explicit expression
for the spectral densities?

How to find spectral density? An inversion formula

$$\rho(\Delta) \overset{?}{=} \int_{\text{space}} \mathcal{F}(\Delta)\ \langle\phi\phi\rangle$$

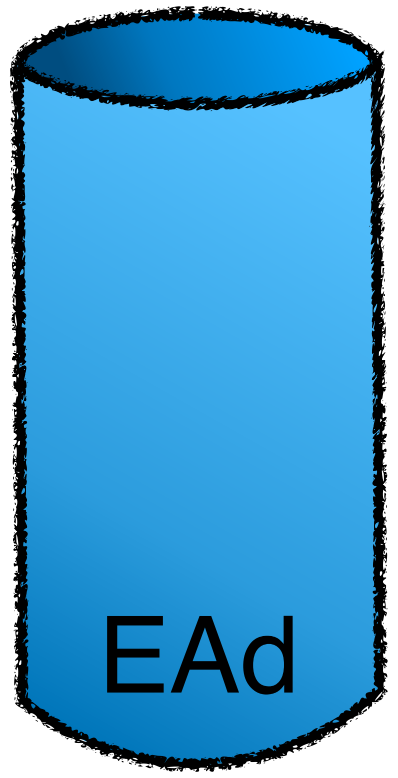
How to find spectral density? An inversion formula

Analytic continuation (Wick Rotation) to EAdS



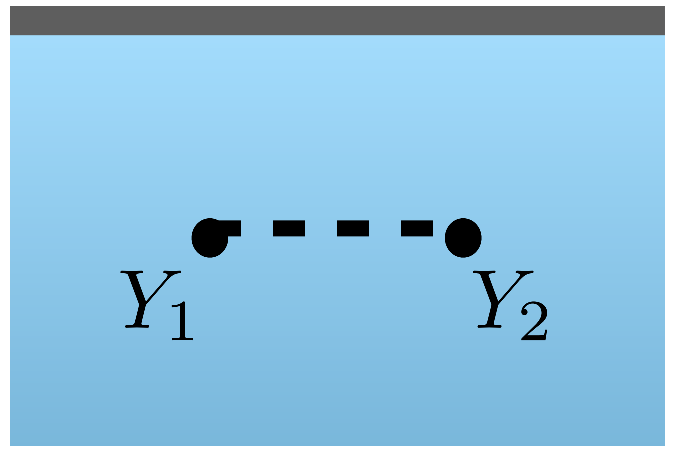
$$Y = (\eta, \vec{y}) \quad \longrightarrow \quad X = (\pm i z, \vec{x})$$

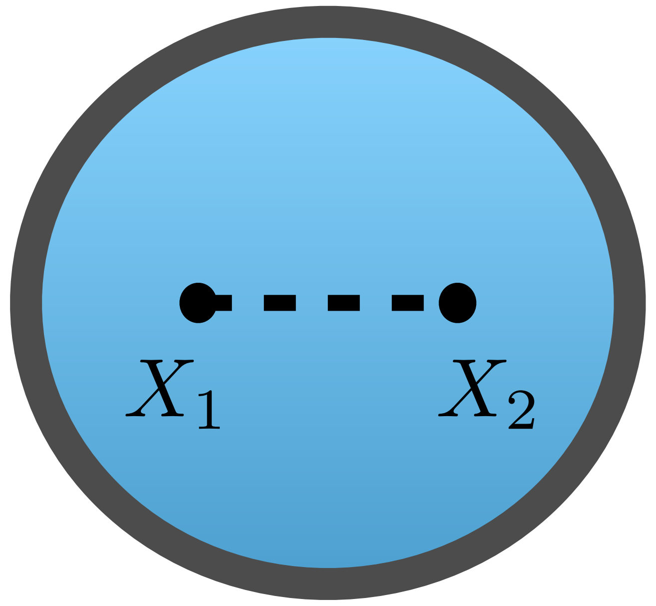
$$ds^2 = \frac{-d\eta^2 + d\vec{y}^2}{\eta^2} \qquad\qquad ds^2 = \frac{dz^2 + d\vec{x}^2}{z^2}$$



The propagators in dS translate to Harmonic functions in EAdS

$$G_{\text{free}}(Y_{12}, \Delta) \quad \longrightarrow \quad \Omega_\Delta(X_1, X_2)$$





Harmonic functions: Orthogonal

$$\int_X \Omega_\Delta(X_1, X)\Omega_{\Delta'}(X, X_2) = \delta(\Delta - \Delta')\,\Omega_\Delta(X_1, X_2)$$

Power of analytic continuation to EAdS

$$\langle\phi(Y_1)\phi(Y_2)\rangle \sim \int_{\text{reps}} \rho(\Delta)\; G_{\text{free}}(Y_{12},\Delta) \longrightarrow \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} d\Delta\; \rho_\ell(\Delta)\; \Omega_\Delta(X_1,X_2)$$

Completeness of principal series for square-integrable two-point functions

Should Decay fast enough at large distances

$$\int_{\text{EAdS}} |G|^2 < \infty$$

- Orthogonality of harmonic functions helps us to invert the KL decomposition of any spin to a one variable integral over (space-like) chordal distance!
  - For example for spin 0:

$$\rho(\Delta) \sim \int_{-\infty}^{-1} d\sigma\,(\sigma^2-1)\;{}_2F_1(\Delta, d-\Delta, \frac{d+1}{2}, \frac{1+\sigma}{2})\langle\phi\phi\rangle$$

Spinning KL:

$$\langle T^{(J)}(Y_1)T^{(J)}(Y_2)\rangle \sim \sum_{\ell=0}^{J}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}=i\infty} \rho_\ell(\Delta)\,\nabla_1^{J-\ell}\nabla_2^{J-\ell}\,G_\ell(Y_{12},\Delta)$$

$$\rho_\ell^{(J)}(\Delta) = \int_{X_1} \Omega_{\Delta,\ell}(X_1,X_2)\nabla_1^{(J-\ell)}\nabla_2^{(J-\ell)}\langle T^{(J)}(X_1)T^{(J)}(X_2)\rangle$$

Examples:

Explicit expressions for spectral densities: the expected boundary operator content, manifestly positive and match with the flat-space limit

- Free theory composite operators two-point functions:

1. $\langle \phi_1\phi_2(Y_1)\phi_1\phi_2(Y_2)\rangle = \langle\phi_1\phi_1\rangle\langle\phi_2\phi_2\rangle$
2. $\langle V_\mu\phi(Y_1)V_\nu\phi(Y_2)\rangle = \langle V_\mu V_\nu\rangle\langle\phi\phi\rangle$
3. $\langle \phi\nabla_\mu\phi(Y_1)\phi\nabla_\nu\phi(Y_2)\rangle = \langle\nabla_\mu\phi\nabla_\nu\phi\rangle\langle\phi\phi\rangle$

- Bulk CFT spin 0, 1, 2 $\qquad \rho^{(2)}_{\mathrm{CFT},\ell=2}(\Delta) \sim \dfrac{|\Gamma(\Delta_T-\Delta)|^2}{|\Gamma(\Delta-\frac{d}{2})|^2}$

Spinning KL:

$$\langle T^{(J)}(Y_1)T^{(J)}(Y_2)\rangle \sim \sum_{\ell=0}^{J}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}=i\infty} \rho_\ell(\Delta)\,\nabla_1^{J-\ell}\nabla_2^{J-\ell}\; G_\ell(Y_{12},\Delta)$$

$$\rho_\ell^{(J)}(\Delta) = \int_{X_1} \Omega_{\Delta,\ell}(X_1,X_2)\nabla_1^{(J-\ell)}\nabla_2^{(J-\ell)}\langle T^{(J)}(X_1)T^{(J)}(X_2)\rangle$$

Examples:

Explicit expressions for spectral densities: the expected boundary operator content, manifestly positive and match with the flat-space limit

- Free theory composite operators two-point functions:

$$\rho_{V\phi}^{\mathcal{P},0}(\lambda) = \frac{2^{-1}\pi^{-3-\frac{d}{2}}\lambda\sinh(\pi\lambda)}{(\Delta_V-1)(\bar{\Delta}_V-1)(d^2+4\lambda^2)\Gamma(\frac{d}{2})\Gamma(\frac{d}{2}\pm i\lambda+1)}\prod_{\pm,\pm,\pm}\Gamma\left(\frac{\frac{d}{2}+1\pm i\lambda\pm i\lambda_V\pm i\lambda_\phi}{2}\right)$$

$$\rho_{V\phi}^{\mathcal{P},1}(\lambda) = \frac{2^{-12}\pi^{-3-\frac{d}{2}}\lambda\sinh(\pi\lambda)f_{\lambda,\lambda_V,\lambda_\phi}}{\Gamma(\frac{d+2}{2})(\Delta_V-1)(\bar{\Delta}_V-1)\Gamma(\frac{d}{2}\pm i\lambda+1)}\prod_{\pm,\pm,\pm}\Gamma\left(\frac{\frac{d}{2}\pm i\lambda\pm i\lambda_\phi\pm i\lambda_V}{2}\right)$$

Spinning KL:

$$\langle T^{(J)}(Y_1)T^{(J)}(Y_2)\rangle \sim \sum_{\ell=0}^{J}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}=i\infty} \rho_\ell(\Delta)\,\nabla_1^{J-\ell}\nabla_2^{J-\ell}\; G_\ell(Y_{12},\Delta)$$

$$\rho_\ell^{(J)}(\Delta) = \int_{X_1} \Omega_{\Delta,\ell}(X_1,X_2)\nabla_1^{(J-\ell)}\nabla_2^{(J-\ell)}\langle T^{(J)}(X_1)T^{(J)}(X_2)\rangle$$

Examples:

Explicit expressions for spectral densities: the expected boundary operator content, manifestly positive and match with the flat-space limit

- Bulk CFT spin 0, 1, 2

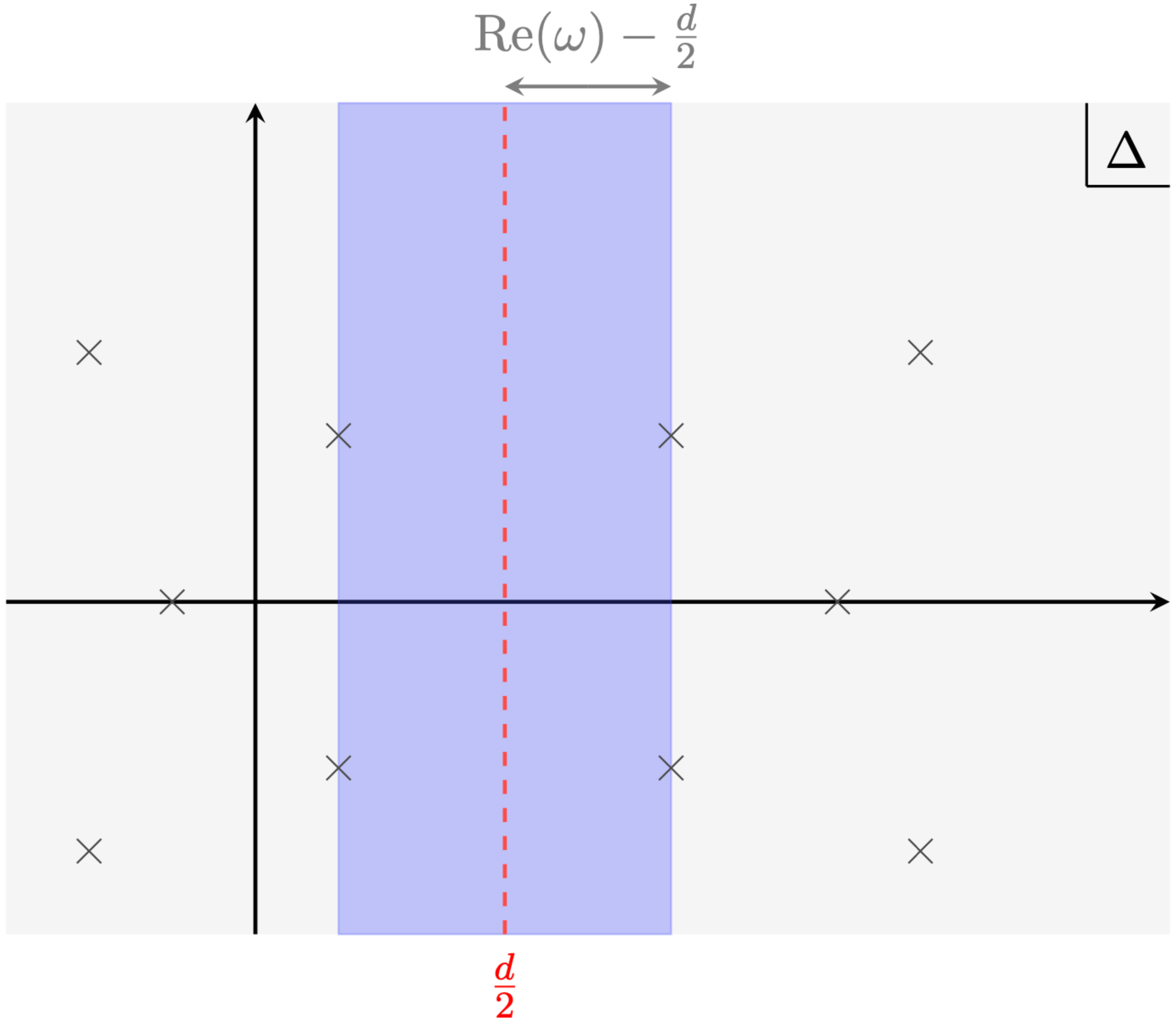
$$\rho_T^{\mathcal{P},0}(\lambda) = c_T \frac{2^{5+d-2\boldsymbol{\Delta}}(d+1)\pi^{\frac{d-1}{2}}(d-\boldsymbol{\Delta})(d+1-\boldsymbol{\Delta})\Gamma(-\frac{d}{2}+\boldsymbol{\Delta}\pm i\lambda)}{d(d^2+4\lambda^2)((d+2)^2+4\lambda^2)\Gamma(\boldsymbol{\Delta}+2)\Gamma(\frac{1-d}{2}+\boldsymbol{\Delta})}\lambda\sinh(\pi\lambda)\,,$$

$$\rho_T^{\mathcal{P},1}(\lambda) = c_T \frac{2^{4+d-2\boldsymbol{\Delta}}\pi^{\frac{d-1}{2}}(1-\boldsymbol{\Delta})(d+1-\boldsymbol{\Delta})\Gamma(-\frac{d}{2}+\boldsymbol{\Delta}\pm i\lambda)}{((d+2)^2+4\lambda^2)\Gamma(\boldsymbol{\Delta}+2)\Gamma(\frac{1-d}{2}+\boldsymbol{\Delta})}\lambda\sinh(\pi\lambda)\,,$$

$$\rho_T^{\mathcal{P},2}(\lambda) = c_T \frac{2^{1+d-2\boldsymbol{\Delta}}\pi^{\frac{d-1}{2}}(\boldsymbol{\Delta}-1)\boldsymbol{\Delta}\Gamma(-\frac{d}{2}+\boldsymbol{\Delta}\pm i\lambda)}{\Gamma(\boldsymbol{\Delta}+2)\Gamma(\frac{1-d}{2}+\boldsymbol{\Delta})}\lambda\sinh(\pi\lambda)\,.$$

5. Can we understand what controls
the analytic properties
of the spectral densities?

Strip of analyticity

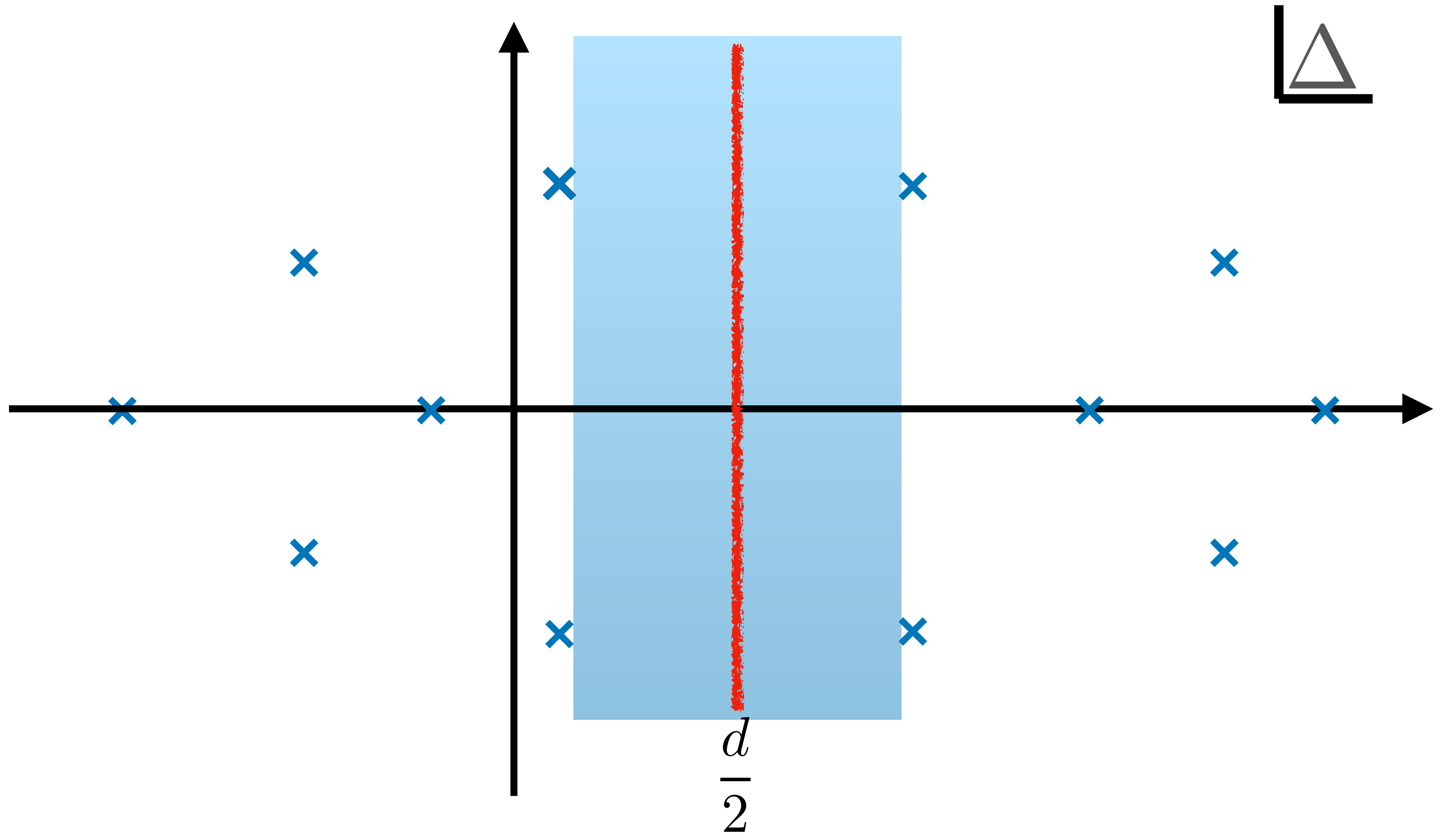


$$\langle \phi(Y_1)\phi(Y_2) \rangle \sim \sigma^{-\omega}$$

Fun facts:

- Anomalous dimensions of boundary operators from the spectral density

- Complementary series:
  pole crossing over principal series. A discrete sum?!



- Discrete series for d=1:
  Analysis for d=1 is Complete!

- Another way: analytic continuation from/to sphere. It is an integral of discontinuity of two-point function over time-like separated points — equivalent to the EAdS one!

Summary and open questions:

Summary

- Do we have a KL decomposition for dS? For any spin and any spacetime dimensions
- Does unitarity imply positivity for dS? Positivity of the spectral density
- Can we find boundary theory / boundary operators? Poles of the spectral density
- Can we invert the KL decomposition and find explicit expression for spectral densities? The inversion formula
- Can we understand what controls the analytic properties of spectral densities? Large distance behaviour of two-point function

Future direction

- Bounds on EFT coefficient in dS. Role of the Hubble scale?
- Making sense of bulk-to-boundary expansion: What is the boundary operators definition
- Flat-space limit?
- Bootstraping four-point functions in higher dimensions! Where to look at?

Thank You!

Unitary irreducible representations SO(1,d+1) $\{\Delta, s\}$

Casimir: $\Delta(d-\Delta) - s(d+s-2)$

- **Principal series** $\mathcal{P}_{\Delta,s}$: $\Delta \in \frac{d}{2} + i\mathbb{R}$ and $s \geq 0$.

  Heavy massive scalars fields

- **Complementary series** $\mathcal{C}_{\Delta,s}$: $0 < \Delta < d$ when $s = 0$ and $1 < \Delta < d-1$ when $s \geq 1$.

  Light massive scalars fields

- **Type I exceptional series** $\mathcal{V}_{p,0}$: $\Delta = d + p - 1$ and $s = 0$ for $p \geq 1$.

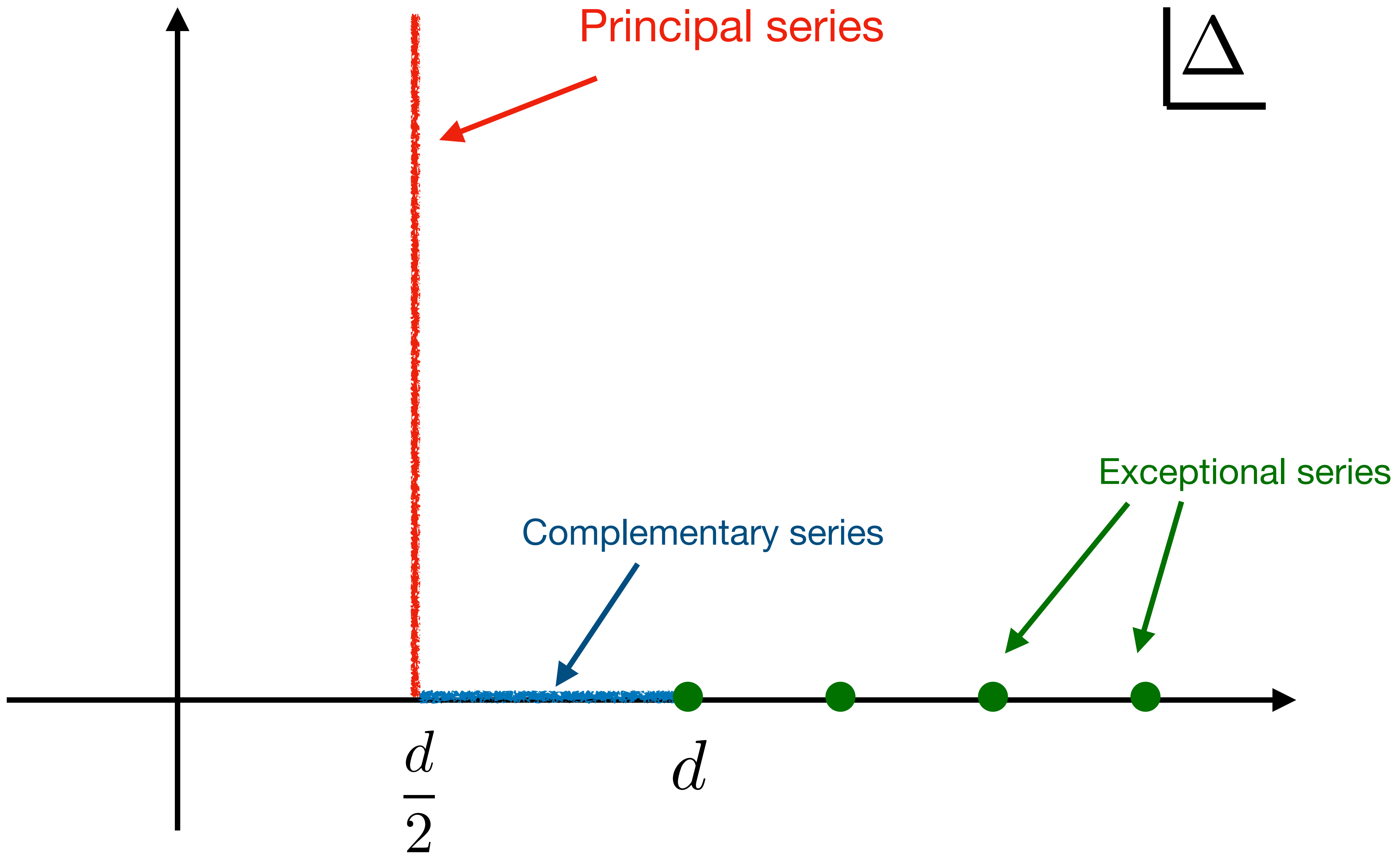
  Shift symmetric scalars in dS_{d+1}

- **Type II exceptional series** $\mathcal{U}_{s,t}$: $\Delta = d + t - 1$ and $s \geq 1$ with $t = 0, 1, 2 \cdots, s-1$.

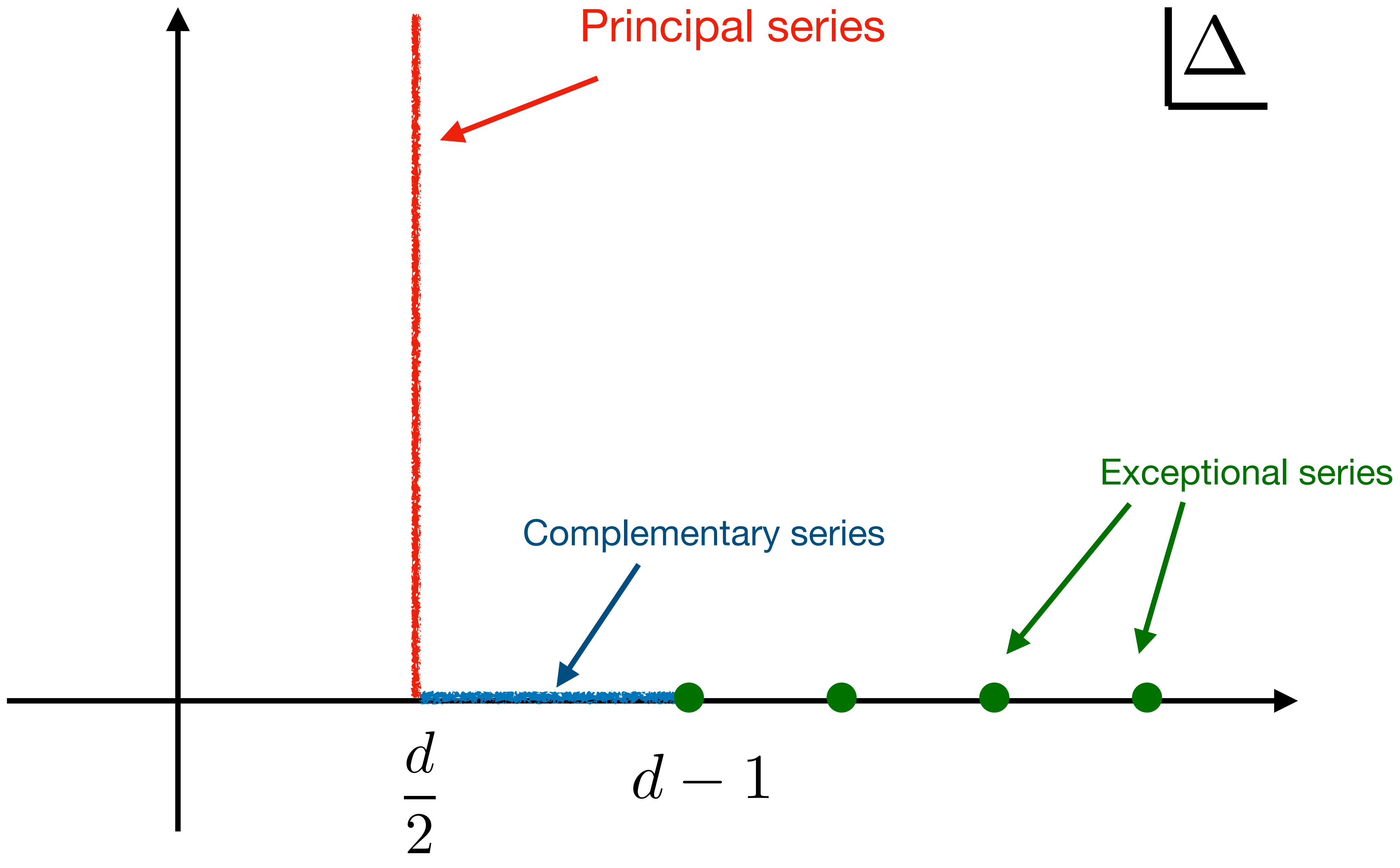
  Partially massless field of spin s and depth t in dS_{d+1}

Unitary irreducible representations SO(1,d+1) $\{\Delta, s\}$

Shadow symmetry: $\Delta \longleftrightarrow d - \Delta$



Scalar



Spinning

KL decomposition in Minkowski:

A spectral decomposition of the two-point function into a sum/integral over free propagators (kinematical functions)

≥ 0

$$\langle \phi(x_1)\phi(x_2)\rangle = \int d\mu^2 \rho(\mu^2)\, G_{\text{free}}(x_{12}, \mu^2)$$

Is it useful? Yes! Some examples:

1. No higher derivative terms in the Lagrangian

This yields our spectral representation:[9]

$$\Delta'(p) = \int_0^\infty \rho(\mu^2)\,\frac{d\mu^2}{p^2 + \mu^2 - i\epsilon}\,. \qquad (10.7.16)$$

One immediate consequence of this result and the positivity of $\rho(\mu^2)$ is that $\Delta'(p)$ cannot vanish for $|p^2| \to \infty$ faster** than the bare propagator $1/(p^2 + m^2 - i\epsilon)$. From time to time the suggestion is made to include higher derivative terms in the unperturbed Lagrangian, which would make the propagator vanish faster than $1/p^2$ for $|p^2| \to \infty$, but the spectral representation shows that this would necessarily entail a departure from the positivity postulates of quantum mechanics.

KL decomposition in Minkowski:

A spectral decomposition of the two-point function into a sum/integral over free propagators (kinematical functions)

$$\langle \phi(x_1)\phi(x_2)\rangle = \int d\mu^2 \, \overset{\geq 0}{\rho(\mu^2)} \, G_{\text{free}}(x_{12}, \mu^2)$$

Is it useful? Yes! Some examples:

1. No higher derivative terms in the Lagrangian
2. Bounds on EFT coefficients

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}\phi\left[\Box + \lambda_1 \frac{\Box^2}{\Lambda^2} + +\lambda_2 \frac{\Box^4}{\Lambda^4} + \cdots\right]\phi$$

$$\lambda_1 = \Lambda^2 \int_\Lambda^\infty dm^2 \, \frac{\rho_\Lambda(m^2)}{m^2} \geq 0$$

$$\lambda_1^2 - \lambda_2 = \Lambda^4 \int_\Lambda^\infty dm^2 \, \frac{\rho_\Lambda(m^2)}{m^4} \geq 0$$

$$\lambda_1^3 - 2\lambda_2\lambda_1 + \lambda_3 = \Lambda^6 \int_\Lambda^\infty dm^2 \, \frac{\rho_\Lambda(m^2)}{m^6} \geq 0$$

d=1

Spin indices $T_{\mu_1\cdots\mu_J}$ $\longrightarrow$ chirality $T_{\underbrace{\pm\cdots\pm}_{J}}$

$$\langle T^{(J)}(Y_1)T^{(J)}(Y_2)\rangle \sim \sum_{\ell=0,1}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \rho_\ell(\Delta)\,\nabla_1^{J-\ell}\nabla_2^{J-\ell}\;G_\ell(Y_{12},\Delta)$$