On structures of celestial OPE and algebras

Lecheng Ren

Brown University

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based on 2211.11356 with J. Mago, A. Y. Srikant, A. Volovich
2206.08322 with A. Y. Srikant, M. Spradlin, A. Volovich
2306.04630 with A. Sharma, A. Schreiber, D. Wang

Introduction

- Celestial holography aims to formulate holography in asymptotic flat space. [Pasterski, Shao, Strominger, 17']
- A key observation is that collinear limits $\leftrightarrow$ OPE in celestial CFT $\leftrightarrow$ current algebra. [Fan, Fotopoulos, Taylor, 19'], [Guevara, Himwich, Pate, Strominger, 21'], [Strominger, 21']
- Effect of higher dimension operators: add constraints on bulk amplitudes.
- We also compute the all-order OPE in the MHV sector of gluon and graviton amplitudes at tree-level, using inverse soft recursion.

Outline

- Brief review of Celestial Amplitudes
- Collinear limits $\leftrightarrow$ OPE $\leftrightarrow$ algebra
- Influence from Jacobi identity
- Rational terms in OPE

- Only talk about gravity, while for YM theory there is a similar story
- Only talk about tree level amplitudes.
- Only talk about massless amplitudes.

- Scattering amplitudes in 4d flat space are labelled by spinor helicity variables. For massless particles:

$$p_{\alpha\dot{\alpha}} := \sigma_{\mu\alpha\dot{\alpha}}p^{\mu} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}}$$

- We parametrize them by

$$\lambda_{\alpha} = \sqrt{\omega}\begin{pmatrix}1\\ z\end{pmatrix} \qquad \tilde{\lambda}_{\dot{\alpha}} = \epsilon\sqrt{\omega}\begin{pmatrix}1\\ \bar{z}\end{pmatrix} \qquad \omega \in \mathbb{R}^{+}$$

- Celestial amplitudes for massless particles is constructed from Mellin transformation:

$$\begin{aligned}\tilde{\mathcal{A}}_n\left(\{\Delta_i, z_i, \bar{z}_i, s_i\}\right) &= \int \frac{d\omega_1}{\omega_1}\dots\frac{d\omega_n}{\omega_n}\omega_1^{\Delta_1}\dots\omega_n^{\Delta_n}\mathcal{A}_n\left(\{\omega_i, z_i, \bar{z}_i, s_i\}\right)\\ &=: \left\langle \mathcal{O}_{h_1,\bar{h}_1}\left(z_1,\bar{z}_1\right)\dots\mathcal{O}_{h_n,\bar{h}_n}\left(z_n,\bar{z}_n\right)\right\rangle\end{aligned}$$

[Pasterski, Shao, Strominger, 17']

- The Lorentz invariants are

$$\langle ij\rangle = \det\left(\lambda_i \tilde{\lambda}_j\right) \qquad [ij] = \det\left(\tilde{\lambda}_i \tilde{\lambda}_j\right)$$
$$2p_i \cdot p_j = -\langle ij\rangle\,[ij] = 2\epsilon_i\epsilon_j\omega_i\omega_j\, z_{ij}\, \bar{z}_{ij}$$

- As $z_i \longrightarrow z_j$, the insertions $\mathcal{O}_{h_i,\bar{h}_j}\left(z_i,\bar{z}_i\right), \mathcal{O}_{h_j,\bar{h}_j}\left(z_j,\bar{z}_j\right)$ approach each other.
- OPE is controlled by the collinear limit, where the amplitudes get factorized:

$$\mathcal{A}_n\left(1^{s_1},\ldots n^{s_n}\right)$$
$$\xrightarrow{1||2} \mathcal{A}_3\left(1^{s_1},2^{s_2},I^{-s_I}\right) \times \frac{1}{(p_1+p_2)^2}\mathcal{A}_{n-1}\left(I^{s_I},3^{s_3},4^{s_4},\ldots n^{s_n}\right)$$

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- We will only consider the holomorphic pole: on $z_{1,2}$. This comes from anti-holomorphic 3-pt amplitudes.
- Each type of 3pt amplitude will contribute a conformal family of states in OPE.

Collinear and OPE limits

- For an arbitrary 3-pt amplitudes,

$$\mathcal{A}_3\left(1^{s_1},2^{s_2},3^{s_3}\right)=\kappa_{s_1,s_2,s_3}\,[12]^{s_1+s_2-s_3}[23]^{s_2+s_3-s_1}[31]^{s_3+s_1-s_2}$$

- Contribution to the collinear limit:

$$\mathcal{A}_n\xrightarrow{1||2}\;-\frac{\kappa_{s_1,s_2,s_3}}{2}\frac{[12]^p}{\langle 12\rangle}\frac{\langle n1\rangle^{p+1-2s_1}}{\langle n2\rangle^{p+1-2s_1}}\exp\left[\frac{\langle 12\rangle}{\langle n2\rangle}\tilde{\lambda}_1\tilde{\partial}_n+\frac{\langle n1\rangle}{\langle n2\rangle}\tilde{\lambda}_1\tilde{\partial}_2\right]\mathcal{A}_{n-1}$$

$$p=s_1+s_2+s_3-1$$

- Corresponding OPE:

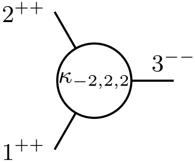
$$\mathcal{O}_{h_1,\bar{h}_1}\left(z_1,\bar{z}_1\right)\mathcal{O}_{h_2,\bar{h}_2}\left(z_2,\bar{z}_2\right)\sim\frac{1}{z_{12}}\sum_p\sum_{\alpha=0}^{\infty}C_p^{\alpha}\left(\bar{h}_1,\bar{h}_2\right)\frac{1}{\alpha!}\bar{z}_{12}^{\alpha+p}\bar{\partial}^{\alpha}\mathcal{O}_{h_1+h_2-1,\bar{h}_1+\bar{h}_2+p}\left(z_2,\bar{z}_2\right)$$

with

$$\begin{aligned}C_p\left(\bar{h}_1,\bar{h}_2\right)&=\kappa_{s_1,s_2,s_3}B\left(\Delta_1-s_1+p,\Delta_2-s_2+p\right)\\&=\kappa_{s_1,s_2,s_3}B\left(2\bar{h}_1+p,2\bar{h}_2+p\right)\end{aligned}$$

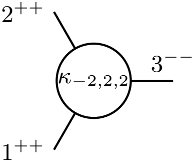
[Pate, Himwich, Singh, 21']

Einstein Gravity

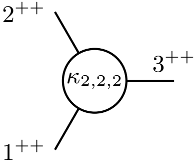


$$= \kappa_{2,2,-2} \frac{[12]^6}{[23]^2[13]^2} : \; p = 1$$

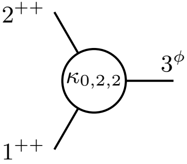
$$\mathcal{O}^{+2}_{\Delta_1}(z_1, \bar{z}_1)\, \mathcal{O}^{+2}_{\Delta_2}(z_2, \bar{z}_2) \sim \frac{1}{z_{12}} \left[\kappa_{2,2,-2} B(\Delta_1 - 1, \Delta_2 - 1)\, \bar{z}_{12} \mathcal{O}^{+2}_{\Delta_1+\Delta_2}(z_2, \bar{z}_2) + O(\bar{z}_{12}^2) \right] + \cdots$$



$$= \kappa_{2,2,-2} \frac{[12]^6}{[23]^2[13]^2}$$



$$= \kappa_{2,2,2} [12]^2 [23]^2 [13]^2$$



$$= \kappa_{0,2,2} [12]^4$$

$$\begin{aligned}
&\mathcal{O}^{+2}_{\Delta_1}(z_1, \bar{z}_1)\, \mathcal{O}^{+2}_{\Delta_2}(z_2, \bar{z}_2) \\
\sim &\frac{1}{z_{12}} \Big[\kappa_{2,2,-2} B(\Delta_1 - 1, \Delta_2 - 1)\, \bar{z}_{12} \mathcal{O}^{+2}_{\Delta_1+\Delta_2}(z_2, \bar{z}_2) + \ldots \\
&\quad + \kappa_{2,2,0} B(\Delta_1 + 1, \Delta_2 + 1)\, \bar{z}^3_{12} \mathcal{O}^{0}_{\Delta_1+\Delta_2+2}(z_2, \bar{z}_2) + \ldots \\
&\quad + \kappa_{2,2,2} B(\Delta_1 + 3, \Delta_2 + 3)\, \bar{z}^5_{12} \mathcal{O}^{-2}_{\Delta_1+\Delta_2+4}(z_2, \bar{z}_2) + \ldots \Big]
\end{aligned}$$

- Define single soft currents by take residues on the poles:

$$H^{k,s}(z,\bar{z}) \equiv \lim_{\epsilon \to 0} \epsilon\, \mathcal{O}^{s}_{k+\epsilon}(z,\bar{z})$$

[Donnay, Puhm, Strominger, 18'; Puhm, 19']

- Soft-soft OPE:

$$\begin{aligned} &H^{k_1,+2}(z_1,\bar{z}_1)\, H^{k_2,+2}(z_2,\bar{z}_2) \\ &\sim -\frac{\kappa_{2,2,-2}}{2}\frac{\bar{z}_{12}}{z_{12}} \binom{-2\bar{h}_1 - 2\bar{h}_2 - 2}{-2\bar{h}_2 - 1} H^{k_1+k_2,2} + O(\bar{z}_{1,2}^2) \\ &\quad -\frac{\kappa_{2,2,0}}{2}\frac{\bar{z}_{12}^3}{z_{12}} \binom{-2\bar{h}_1 - 2\bar{h}_2 - 6}{-2\bar{h}_2 - 3} H^{k_1+k_2+2,0} + O(\bar{z}_{1,2}^4) \\ &\quad -\frac{\kappa_{2,2,2}}{2}\frac{\bar{z}_{12}^5}{z_{12}} \binom{-2\bar{h}_1 - 2\bar{h}_2 - 10}{-2\bar{h}_2 - 5} H^{k_1+k_2+4,-2} + O(\bar{z}_{1,2}^6) \end{aligned}$$

- Mode expansion:

$$H^{k,2} = \sum_{\bar{m}=\bar{h}}^{-\bar{h}} \frac{1}{\bar{z}^{\bar{m}+\bar{h}}} H^{k,2}_{\bar{m}}(z) \qquad H^{k,2}_{\bar{m}}(z) = \oint_{|\bar{z}|=\epsilon} \frac{d\bar{z}}{2\pi i} \bar{z}^{\bar{m}+\bar{h}-1} H^{k,2}(z,\bar{z}).$$

- Redefinition of soft modes:

$$W^{q,2}_{\bar{m}}(z) = (-\bar{m}+q-1)!\,(\bar{m}+q-1)!\,H^{4-2q,2}_{\bar{m}}(z)$$

- Full commutator:

$$\left[W^{q_1,2}_{\bar{m}_1}, W^{q_2,2}_{\bar{m}_2}\right] = -\sum_{p=1,3,5} \frac{\kappa_{2,2,p-3}}{2} N\left(q_1,q_2,\bar{m}_1,\bar{m}_2,p\right) W^{q_1+q_2-p-1,3-p}_{\bar{m}_1+\bar{m}_2}$$

where

$$N\left(q_1,q_2,\bar{m}_1,\bar{m}_2,p\right) = \sum_{x=0}^{p} (-1)^{p-x} \binom{p}{x} [\bar{m}_1+q_1-1]_{p-x}[-\bar{m}_1+q_1-1]_x \times [\bar{m}_2+q_2-1]_x[-\bar{m}_2+q_2-1]_{p-x}.$$

$$[a]_n := a(a-1)\cdots(a-n+1)$$

[Mago, LR, Yelleshpur Srikant, Volovich, 21']

$$\left[W^{q_1,2}_{\bar{m}_1}, W^{q_2,2}_{\bar{m}_2}\right] = -\sum_{p=1,3,5} \frac{\kappa_{2,2,p-3}}{2} N\left(q_1, q_2, \bar{m}_1, \bar{m}_2, p\right) W^{q_1+q_2-p-1,3-p}_{\bar{m}_1+\bar{m}_2}$$

- For pure Einstein gravity, $\kappa_{2,2,p-3} \neq 0$ only when $p = 1$,

$$N(q_1, q_2, \bar{m}_1, \bar{m}_2, 1) = \bar{m}_1 \left(q_2 - 1\right) - \bar{m}_2 \left(q_1 - 1\right)$$

This forms the algebra $Lw_{1+\infty}$. [Strominger, 21'], [Adamo, Mason, Sharma, 21']

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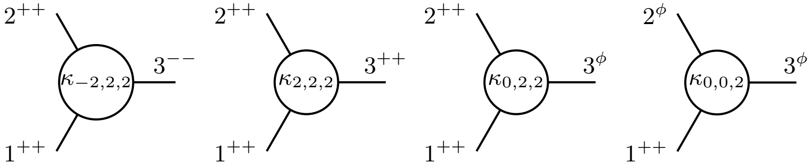
- For generic $\kappa_{2,2,p-3}$, Jacobi identity fails for generic values of the κ_{s_1,s_2,s_3}:

$$\left[\left[W^{q_1,2}_{\bar{m}_1}, W^{q_2,2}_{\bar{m}_2}\right], W^{q_3,2}_{\bar{m}_3}\right] + \text{cyclic} \neq 0$$

- This imposes constraints on the spectrum and the couplings.

Constraints from Jacobi identity



$$(\kappa_{-2,2,2} - \kappa_{0,0,2})\,\kappa_{0,2,2} = 0 \qquad (\kappa_{-2,2,2} - \kappa_{0,0,2})\,\kappa_{0,0,2} = 0$$

$$3\kappa_{0,2,2}^2 = 10\,\kappa_{-2,2,2}\,\kappa_{2,2,2}\,.$$

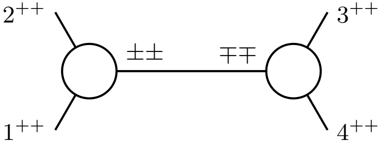
[Mago, LR, Yelleshpur Srikant, Volovich, 21']

- $\kappa_{-2,2,2} \neq 0$, while $\kappa_{0,2,2} = \kappa_{2,2,2} = \kappa_{0,2,2} = 0$: pure gravity
- $\kappa_{-2,2,2} = \kappa_{0,0,2} \neq 0$, while $\kappa_{0,2,2} = \kappa_{2,2,2} = 0$: pure gravity with scalar fields.
- $\kappa_{-2,2,2} = \kappa_{0,0,2} \neq 0$, $\kappa_{0,2,2} \neq 0$, $\kappa_{2,2,2} \neq 0$, with

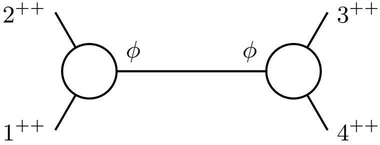
$$\kappa_{s_1,s_2,s_3} \sim \frac{q^{s_1+s_2+s_3-1}}{\Gamma(s_1+s_2+s_3)}$$

[See also Skinner's talk]

$$\mathcal{A}_4\left(1^{++}, 2^{++}, 3^{++}, 4^{++}\right) =$$



$+$



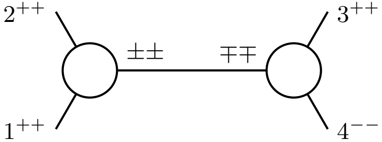
$+$ cyclic

[Broedel, Dixon,12']

$$\mathcal{A}_4\left(1^{++}, 2^{++}, 3^{++}, 4^{++}\right) = \left(10\kappa_{-2,2,2}\kappa_{2,2,2} - 3\kappa_{0,2,2}^2\right)\, s_{12}s_{13}s_{23}\,\frac{[12][23][34][41]}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$
$$= 0$$

[LR, Spradlin, Yelleshpur Srikant, Volovich, 22']

$$\mathcal{A}\left(1^{++},2^{++},3^{++},4^{--}\right) =$$

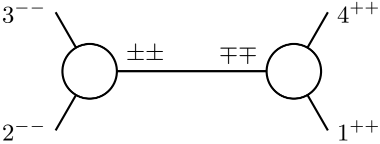


$$+ \text{ cyclic}$$

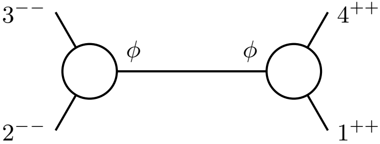
$$= \kappa_{2,2,2}\kappa_{-2,-2,2}(\langle 14\rangle[13]\langle 34\rangle)^2 \frac{[12][23][31]}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

$$\mathcal{A}\left(1^{++},2^{--},3^{--},4^{++}\right)$$

$$=$$



$$+$$



$$= \frac{(\langle 23\rangle\,[14])^4}{s_{14}}\left(\kappa_{2,2,2}\,\kappa_{-2,-2,-2}s_{12}\,s_{13} - \kappa_{2,2,0}\,\kappa_{-2,-2,0} + \kappa_{2,2,-2}\,\kappa_{-2,-2,2}\frac{1}{s_{12}\,s_{13}}\right)$$

Effect on Amplitudes

- Amplitudes that can be constructed *solely* out of anti-holomorphic vertices (or solely out of holomorphic vertices) vanish.

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- If we set

$$\kappa_{s_1,s_2,s_3} = 0, \quad \forall s_1 + s_2 + s_3 < 0$$

  then all tree-level amplitudes beyond 3-pt vanish: Moyal deformed self-dual gravity [Monteiro, 22'], [Bu, Heuveline, Skinner, 22']

- We only consider pure gravity theory.
- The whole amplitudes recursion relation is easier to see by inverse-soft limit:

$$\begin{aligned}&\mathcal{A}_n(1_+\,2\cdots n)\\ =&\sum_{i=3}^{n}\frac{[1i]}{\langle 1i\rangle}\frac{\langle i2\rangle^2}{\langle 12\rangle^2}\,\exp\left(\frac{\langle i1\rangle}{\langle i2\rangle}[1\tilde{\partial}_2]+\frac{\langle 12\rangle}{\langle i2\rangle}[1\tilde{\partial}_i]\right)\mathcal{A}_n(2\cdots i\cdots n)\,,\end{aligned}$$

OPE in soft descendants

- Soft generators: $H_{r,m}(\bar{z})$: mode expansion of soft current $H_r(z,\bar{z})$, whose OPE with the boost eigenstates can be obtained by contour integrals

$$H_{r,m}(\bar{z})O^{\varepsilon}_{\Delta,s}(w,\bar{w}) = \oint \frac{\mathrm{d}z}{2\pi i}\,\frac{1}{(z-w)^m}\,H_r(z,\bar{z})\,O^{\varepsilon}_{\Delta,s}(w,\bar{w})\,,$$

- Written explicitly:

$$\Big\langle \Big(H_{r,m}(\bar{z})O_{\Delta_2,s_2}(w_1,\bar{w}_2)\Big)O_{\Delta_3,s_3}(w_3,\bar{w}_3)\cdots O_{\Delta_n,s_n}(w_n,\bar{w}_n)\Big\rangle$$

$$= \sum_{i=3}^{n}\frac{\bar{z}_{1,i}}{z^{m}_{2,i}}e^{\partial_{\Delta_i}}\left[e^{-\partial_{\Delta_i}}\left(\bar{z}_{1,i}\frac{\partial}{\partial\bar{z}_i}-\Delta_i\right)\right]^{r+1}$$

$$\times\langle O_{\Delta_2,s_2}(w_1,\bar{w}_2)O_{\Delta_3,s_3}(w_3,\bar{w}_3)\cdots O_{\Delta_n,s_n}(w_n,\bar{w}_n)\rangle$$

$$\left\langle \left(H_{r,m}(\bar{z}) O_{\Delta_2,s_2}(w_1,\bar{w}_2) \right) O_{\Delta_3,s_3}(w_3,\bar{w}_3) \cdots O_{\Delta_n,s_n}(w_n,\bar{w}_n) \right\rangle$$

$$= \sum_{i=3}^{n} \frac{\bar{z}_{1,i}}{z_{2,i}^{m}} e^{\partial \Delta_i} \left[e^{-\partial \Delta_i} \left(\bar{z}_{1,i} \frac{\partial}{\partial \bar{z}_i} - \Delta_i \right) \right]^{r+1}$$

$$\times \langle O_{\Delta_2,s_2}(w_1,\bar{w}_2) O_{\Delta_3,s_3}(w_3,\bar{w}_3) \cdots O_{\Delta_n,s_n}(w_n,\bar{w}_n) \rangle$$

- $m = 0, r = -1$:

$$H_{-1,0}(\bar{z}) = \sum_{i=3}^{n} \bar{z}_{1,i} e^{\partial \Delta_i} = -\bar{z}_{1,2} e^{\partial \Delta_2}$$

- $m = 0, r = 0$:

$$H_{0,0}(\bar{z}) = \sum_{i=3}^{n} \bar{z}_{1,i} \left(\bar{z}_{1,i} \frac{\partial}{\partial \bar{z}_i} - \Delta_i \right) = -\bar{z}_{1,2} \left(\bar{z}_{1,2} \frac{\partial}{\partial \bar{z}_2} - \Delta_2 \right)$$

[Stieberger, Taylor, 18']

- The final hard OPE in all regular terms in Einstein gravity:

$$
\begin{aligned}
&O^{\varepsilon}_{\Delta_1,+}(z_1,\bar{z}_1)\,O^{\varepsilon}_{\Delta_2,s_2}(z_2,\bar{z}_2)\\
&=\sum_{m,\bar{m}=0}^{\infty}\sum_{r=0}^{m+1}\frac{\varepsilon^{r+1}}{\bar{m}!}\frac{B(2\bar{h}_1+r+\bar{m}+1,2\bar{h}_2)\,\Gamma(2\bar{h}_1+m+3)}{(m-r+1)!\,\Gamma(2\bar{h}_1+r+2)}\\
&\qquad\times z_{12}^{m-1}\,\bar{z}_{12}^{\bar{m}}\bar{\partial}_2^{\bar{m}}\,H_{r-1,m}(\bar{z}_1)O^{\varepsilon}_{\Delta_1+\Delta_2+r-1,s_2}(z_2,\bar{z}_2)
\end{aligned}
$$

[LR, Schreiber, Sharma, Wang, 22']

- At $m=0$, this exactly goes back to the OPE with only singular terms in z_{12}.
- $\bar{L}_{-1}$ descendants + full towers of soft graviton descendents.

OPE in all orders: Graviton and Gluon

- Similar story for YM theory: $\bar{L}$ descendants + full towers of soft gluon descendents
- Subleading level of $z_{1,2}$: $L_{-1} + \bar{L}_{-1}$ + leading soft gluons [Ebert, Sharma, Wang, 20']
- BG equation (null states) $\rightarrow$ soft + subleading soft [Banerjee, Ghosh 20']
- Null states also exists in celestial gravity [Banerjee, Ghosh, Paul, 20']

- We have computed the algebra of soft current modes due to the non-minimal couplings.
- Jacobi identity imposes constraints on the three point coupling constants and the spectrum.
- Such constraints makes pure-holomorphic or pure-anti-holomorphic amplitudes vanish.
- The regular terms in OPE are contributed by $\bar{L}_{-1}$ and full towers of soft descendants.

Thank you!