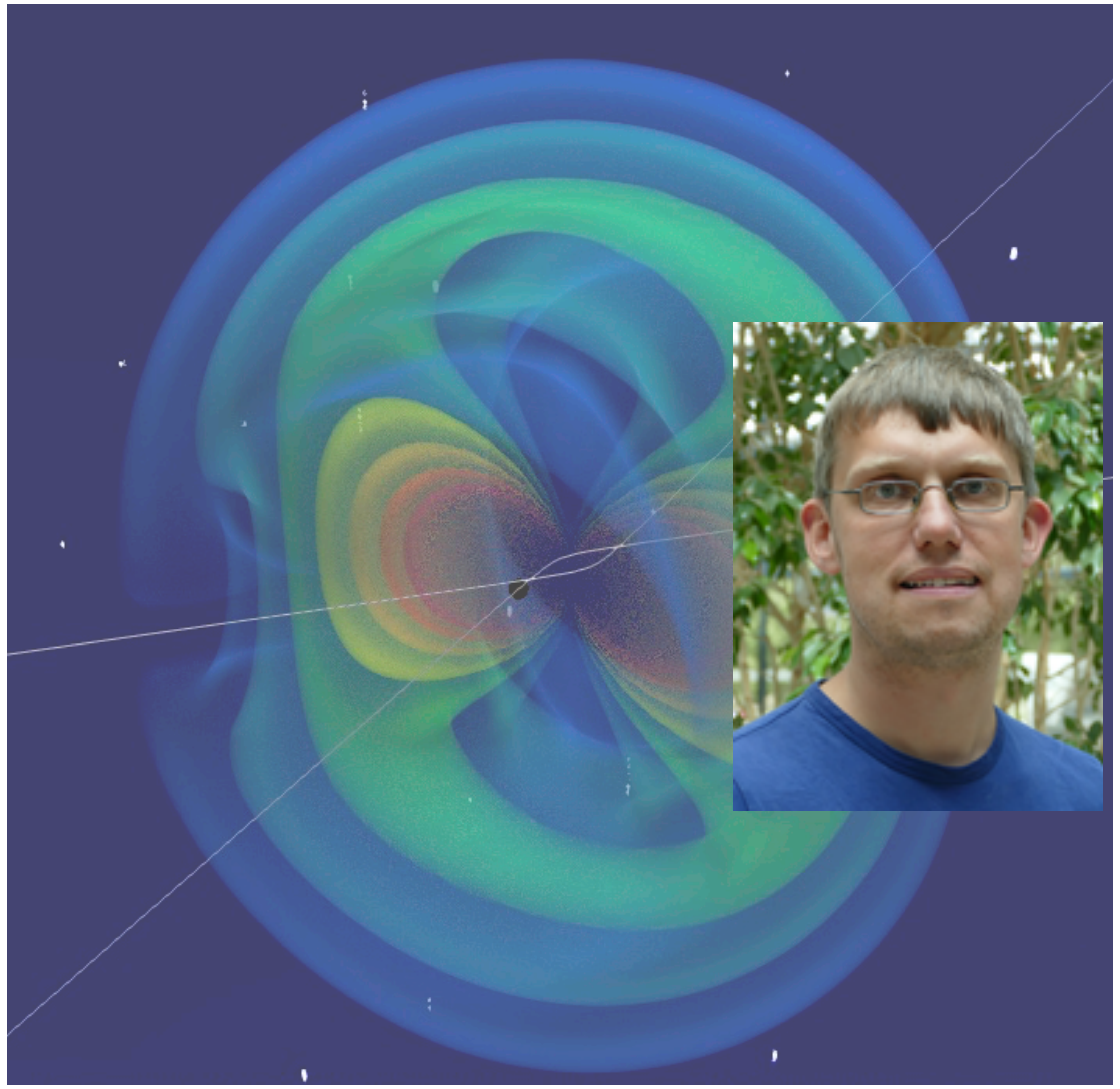
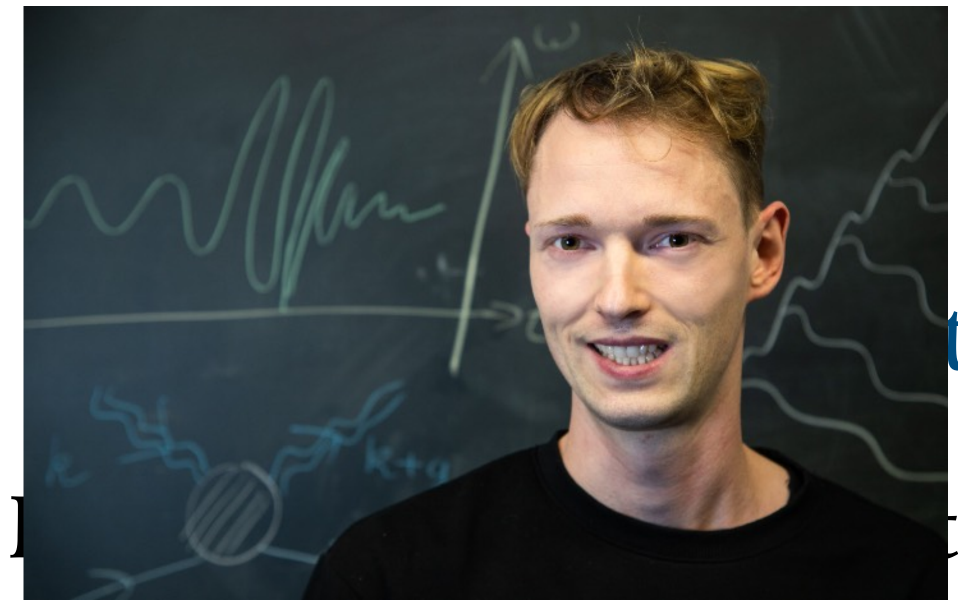
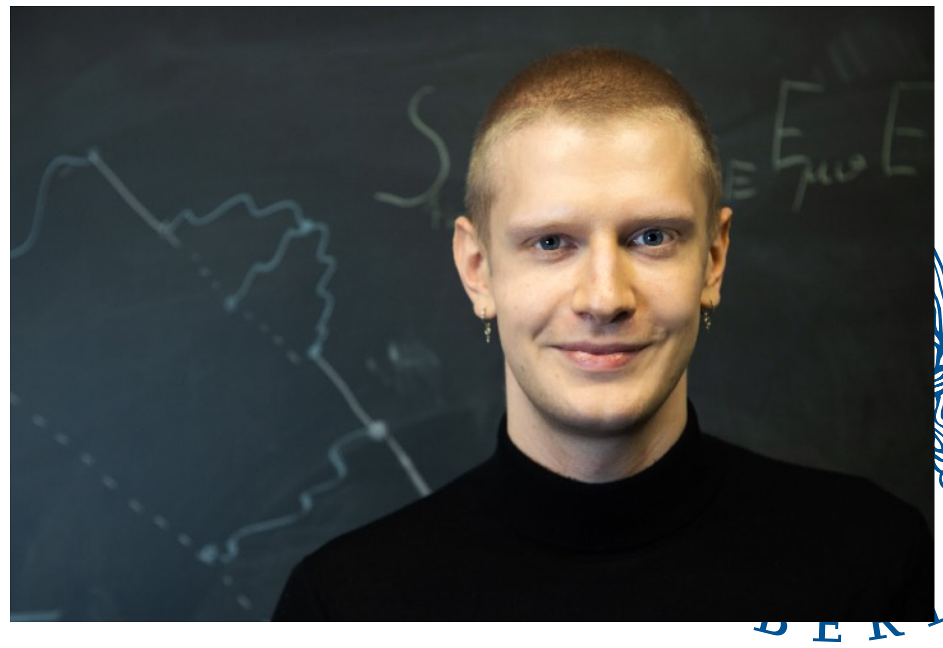
HIGH PRECISION GRAVITATIONAL WAVE PHYSICS FROM A WORLDLINE QUANTUM FIELD THEORY







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Jan Steinhoff (AEI)

2010:02865, *JHEP* 02 (2021) 048
2101.12688, ***PRL*** 126 (2021) 20
2106.10256, ***PRL*** 128 (2022) 1
2109.04465, *JHEP 01 (2022) 027*
2207.00569, *JHEP* 10 (2022) 128
2306.01714

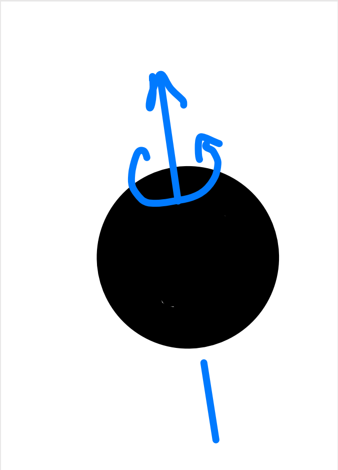
2201.07778, ***PRL*** 128 *(2022) 14*
2210.06451, *PRD 107 (2023) 4*

RTG 2575:
Rethinking Quantum Field Theory


$\gamma = 1.1, \ \frac{m_1}{m_2} = 1, \ \frac{b}{m_1} = 50$
Time: -5.840000

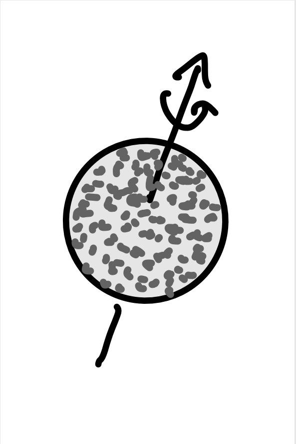
GRAVITATIONAL TWO-BODY PROBLEM

Black Hole



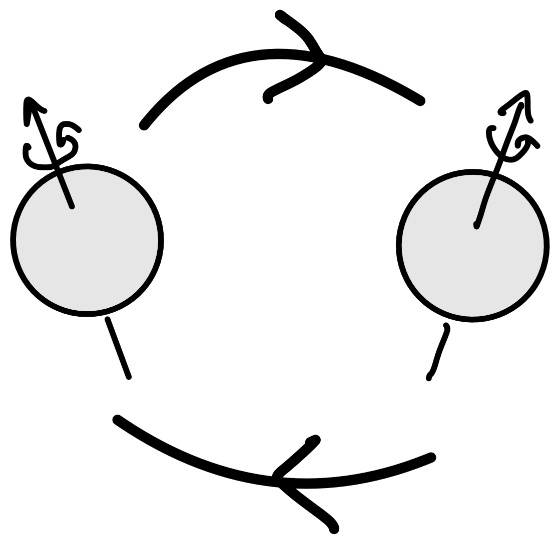
mass, spin

Neutron Star

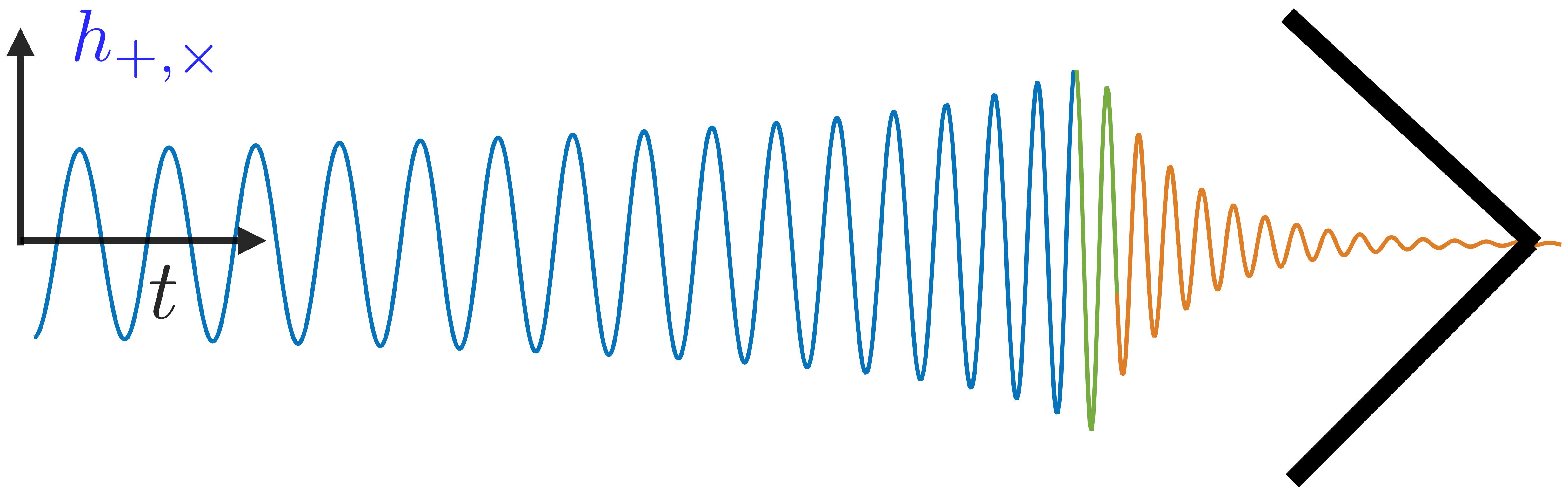


mass, spin, radius, tidal deformability

Black Hole/Neutron Star Binaries:



Bound state



inspiral

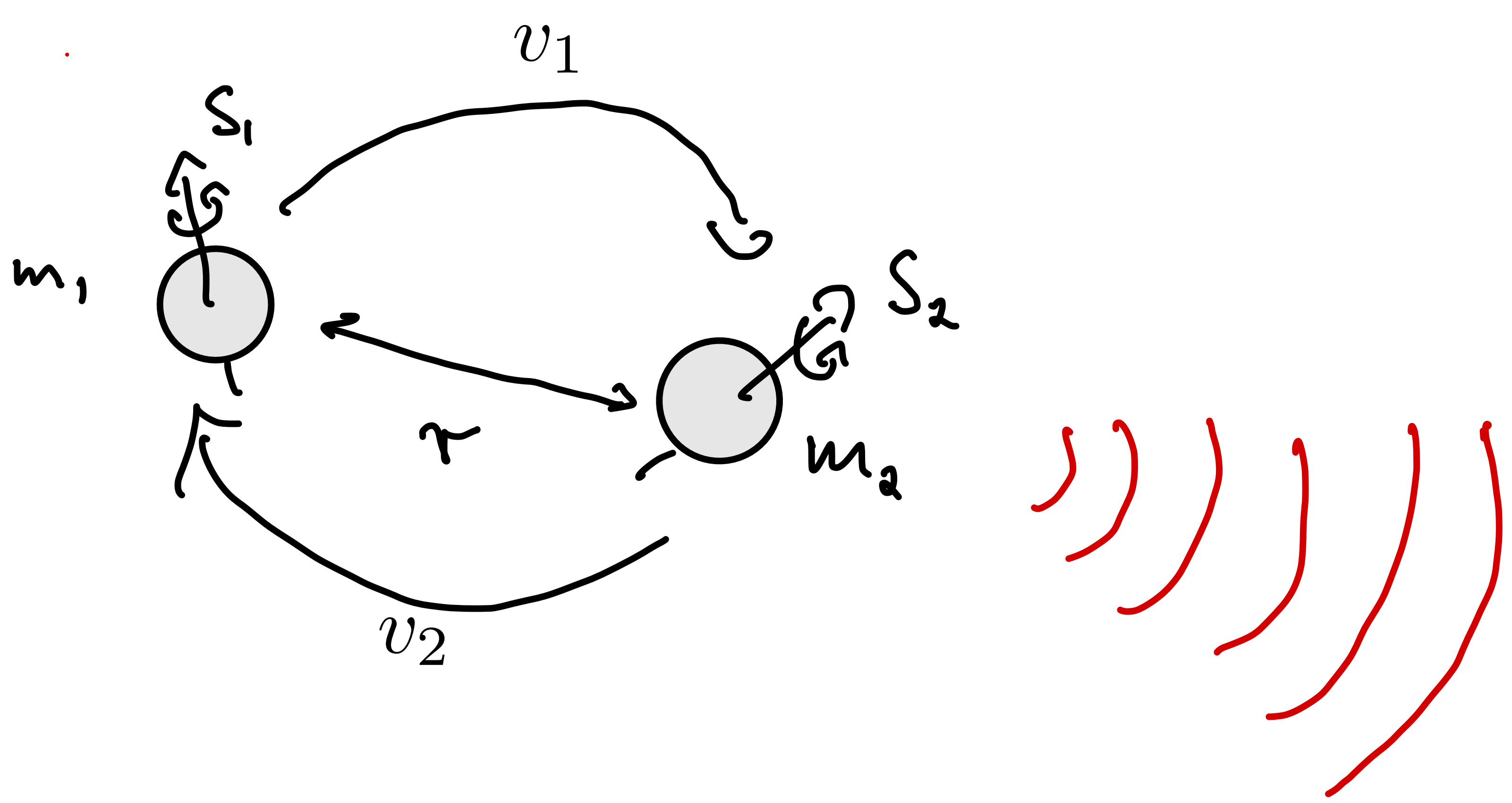
merger

- During **inspiral**: weak gravitational fields $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
- **Quantum** field theory formalism for **classical** two-body problem:

WORLDLINE QUANTUM FIELD THEORY

THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



Inspiral of 2 black holes or neutron stars:

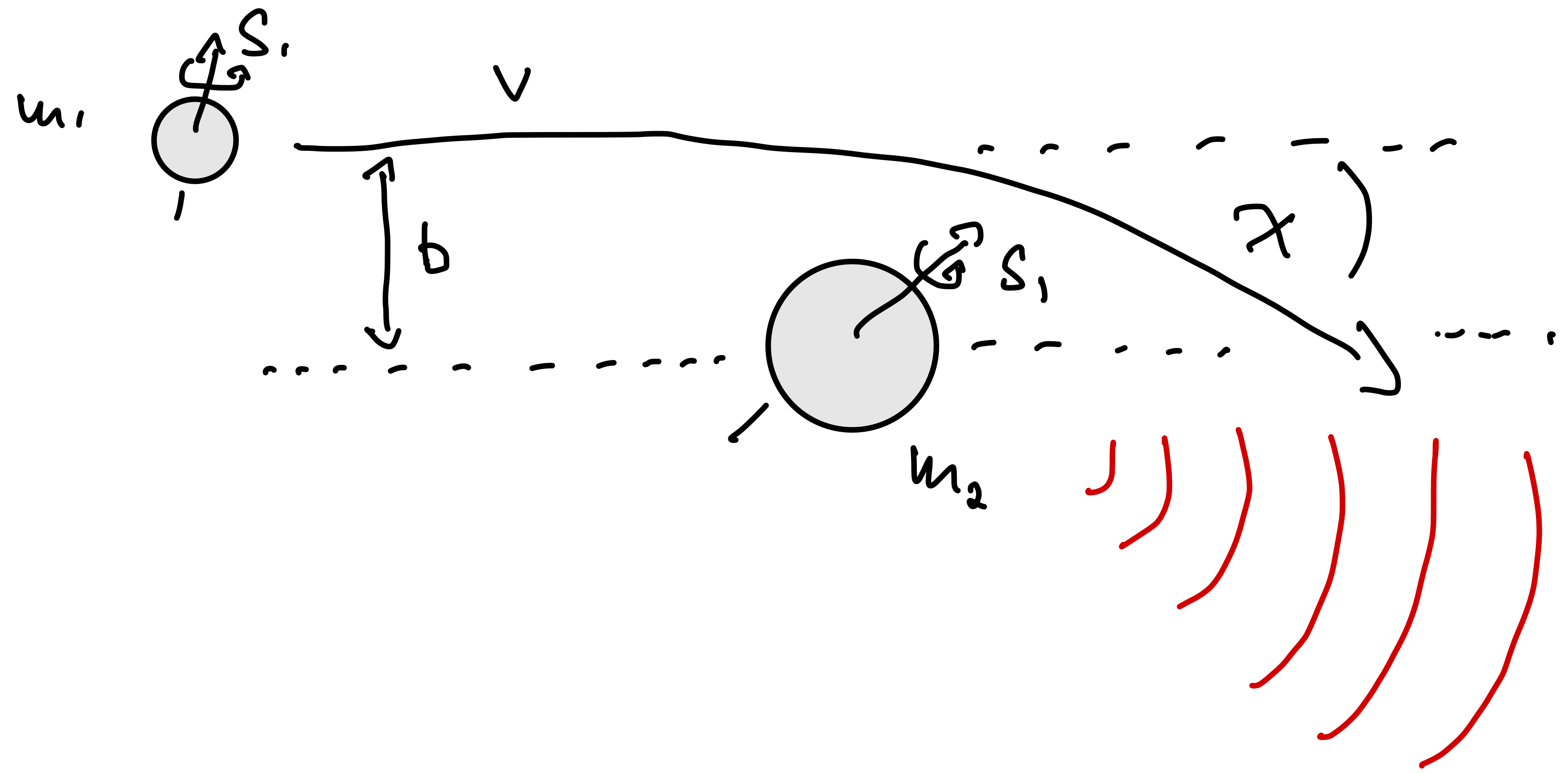
Virial-theorem: $\frac{GM}{r} \sim v^2 \qquad (c = 1)$

post-Newtonian (PN) expansion in G & v^2

$$\boxed{\text{Weak field expansion:} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}}$$

$\kappa = \sqrt{32\pi G}$

Newton's constant

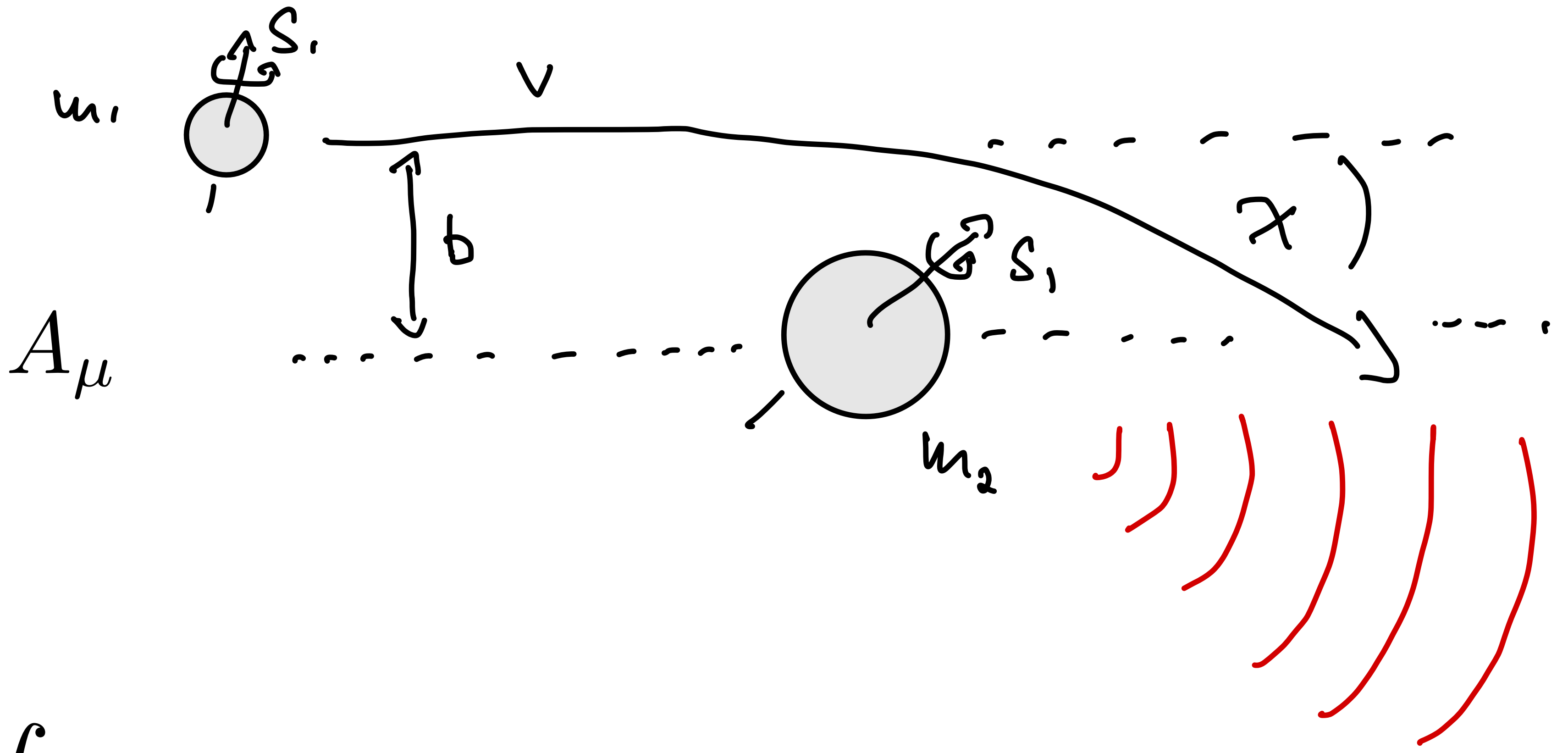


Scattering of 2 black holes or neutron stars:

Weak field (G), but exact in v^2

post-Minkowskian (PM) expansion

WARMUP: ELECTROMAGNETISM



Scattering of charged particles in Maxwell's theory

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu := \frac{\partial}{\partial x^\mu}$$

$$S = -\sum_{i=1}^{2} \int d\tau_i \left(m_i \sqrt{\eta_{\mu\nu} \dot{x}_i^\mu(\tau_i)\, \dot{x}_i^\nu(\tau_i)} - q_i\, A_\mu(x)\, \dot{x}_i^\mu(\tau_i) \right) - \frac{1}{16\pi} \int d^4x F_{\mu\nu}(x)\, F^{\mu\nu}(x) + S_{\mathrm{g.f}}$$

Charged particles

Maxwell theory & gauge fixing

Equations of motion:

$$\partial_\mu F^{\mu\nu}(x) = 4\pi j^\nu(x)$$

Maxwell's eqs.

$$\frac{d^2 x_i^\mu}{ds^2} = \frac{q_i}{m_i} F^\mu{}_\nu \frac{dx_i^\nu}{ds}$$

Lorentz eqs.

Solve perturbatively in q_i

$$A_\mu(x) = \sum_{n=1}^{\infty} q^n\, A_\mu^{(n)}(x)$$

emitted radiation

$$x_i(\tau) = b_i^\mu + v_i^\mu \tau + \sum_{n=1}^{\infty} q^n z_i^{(n)\,\mu}(\tau)$$

straight line: „in“ state

deflections

Non trivial due to back reaction !

WARMUP: ELECTROMAGNETISM

1) Worldline simplification: Introduce „Einbein“ $e(\tau)$

$$S_p = -m \int d\tau \sqrt{\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \longrightarrow \tilde{S}_p = -\frac{m}{2}\int \left(e^{-1}\,\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + e\right)$$

Algebraic equations of motion for e: $\frac{\delta \tilde{S}}{\delta e} = 0 \longrightarrow e^2 = \eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$

Plug this back into $\tilde{S}_p$ to find S_p

Proper time gauge: $e = 1 \leftrightarrow \dot{x}^2 = 1$

2) Space-time simplification: Go to Lorenz gauge: $\partial \cdot A = 0$

3) Gauge fixed theory:

$$\tilde{S} = -\sum_{i=1}^{2} \int d\tau_i \left(\frac{m_i}{2}\dot{x}_i^2(\tau_i) - q_i\, A_\mu[x_i(\tau_i)]\, \dot{x}_i^\mu(\tau_i)\right) + \frac{1}{16\pi}\int d^4x A_\mu(x)\Box A^\mu(x)$$

Worldline　　　　Space-time

TRICK: QED WITHOUT THE Q

$$\tilde{S} = -\sum_{i=1}^{2} \int d\tau_i \left(\frac{m_i}{2} \dot{x}_i^2(\tau_i) - q_i \, A_\mu[x_i(\tau_i)] \, \dot{x}_i^\mu(\tau_i) \right) + \frac{1}{16\pi} \int d^4x A_\mu(x) \Box A^\mu(x)$$

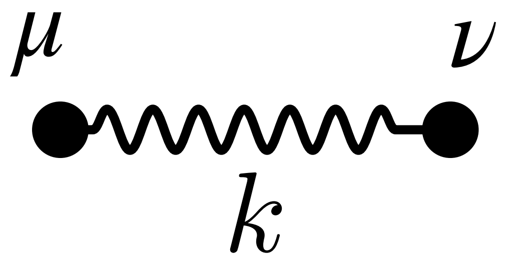
To set up efficient perturbation theory: Quantize and take $\hbar \to 0$ limit! $x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z^\mu(\tau)$

$$\langle \mathcal{O} \rangle = \int D[A, z] \mathcal{O} \, e^{-\frac{i}{\hbar} \tilde{S}[A,z]} \xrightarrow{\hbar \to 0} \begin{array}{l} \lim\limits_{\hbar \to 0} \langle z^\mu(\tau) \rangle = z^\mu_{\text{classical}}(\tau) \\ \lim\limits_{\hbar \to 0} \langle A^\mu(x) \rangle = A^\mu_{\text{classical}}(x) \end{array}$$

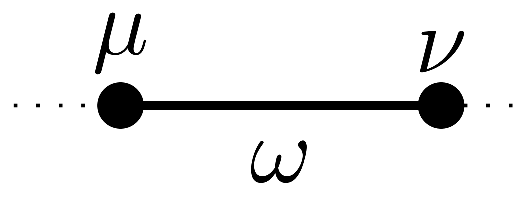
Tree-level one-point functions solve classical equations of motion

FEYNMAN DIAGRAMMATIC EXPANSION

Propagators:

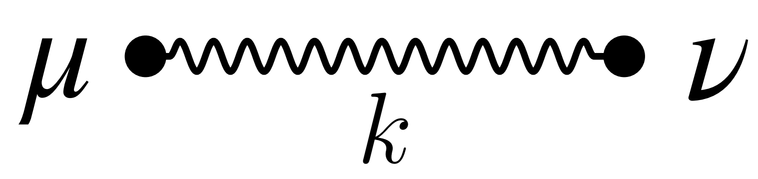


$$\langle A^\mu(k) A^\nu(-k) \rangle = i \frac{\eta_{\mu\nu}}{(k^0 + i0)^2 - \vec{k}^2}$$

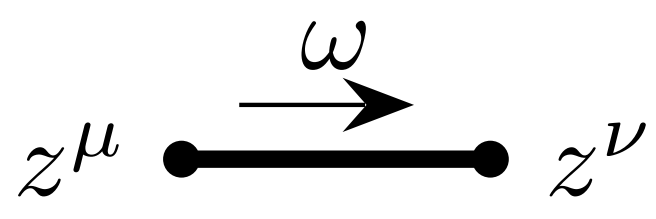


$$\langle z^\mu(\omega) z^\nu(-\omega) \rangle = -\frac{i}{m} \frac{\eta_{\mu\nu}}{(\omega + i0)^2}$$

FEYNMAN DIAGRAMMATIC EXPANSION

- Propagators: Photon 

$$\langle A^\mu(k) A^\nu(-k)\rangle = i\frac{\eta_{\mu\nu}}{(k^0+i0)^2-\vec{k}^2}$$

Worldline 

$$\langle z^\mu(\omega) z^\nu(-\omega)\rangle = -\frac{i}{m}\frac{\eta_{\mu\nu}}{(\omega+i0)^2}$$

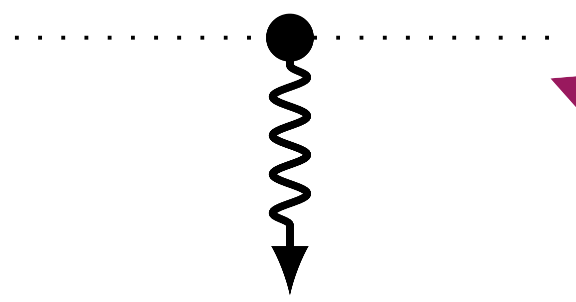
- Vertices: Arise from interaction term:

$$S_{\text{int}} = q\int d\tau\, A_\mu[x(\tau)]\,\dot{x}^\mu(\tau)$$

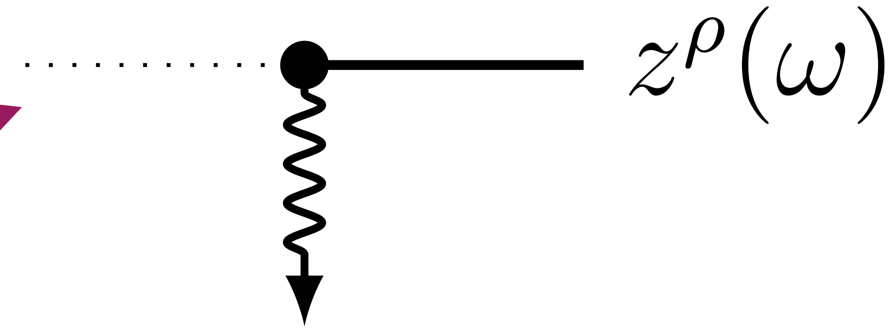
$$A_\mu(x(\tau)) = \int_k e^{ik\cdot(b+v\tau+z(\tau))} A_\mu(-k) = \sum_{n=0}^{\infty}\frac{i^n}{n!}\int_k e^{ik\cdot(b+v\tau)}\,(k\cdot z(\tau))^n\, A_\mu(-k)$$

Worldline-Photon Vertices

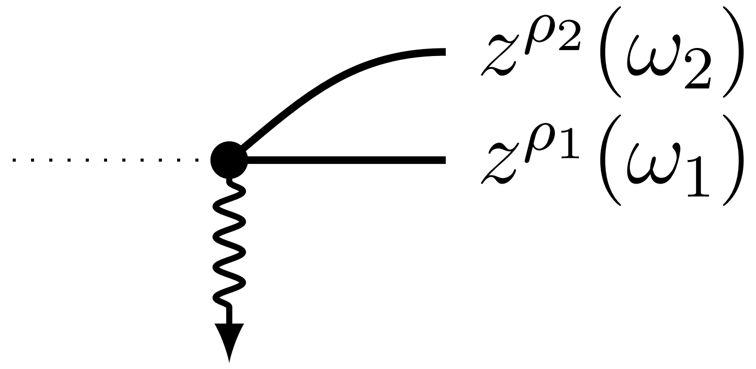
straight line background b^μ & v^μ



$\sim q\, e^{ik\cdot b}\delta(k\cdot v)\, v^\mu$



$\sim q\, e^{ik\cdot b}\delta(k\cdot v+\omega)\,(\omega\eta^{\mu\rho}+v^\mu k^\rho)$

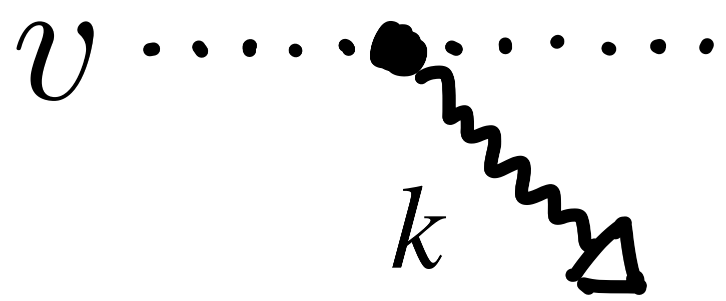


arbitrary # of $z^{\rho_i}(\omega_i)$'s

...

COULOMB POTENTIAL

- Single charge at rest:



$$= \quad \langle A^\mu(k)\rangle = \frac{q\, v^\mu \delta(v\cdot k)}{k^2}\, e^{ik\cdot b}$$

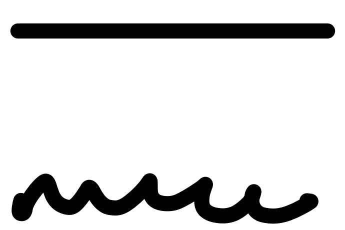
Put particle at rest & origin:

$$v^\mu = (1,\vec{0}) \qquad b^\mu = 0 \qquad \Rightarrow \qquad \vec{A} = 0$$

$$A_0(x) = \phi(x) = \int d^4k \langle A_0(k)\rangle = q\int d^3k \frac{1}{\vec{k}^2} e^{-i\vec{k}\cdot\vec{x}} = \frac{q}{4\pi|\vec{x}|}$$

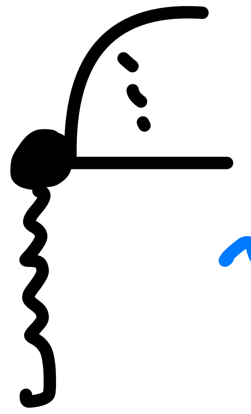
- $\hbar$ counting trivialised:

$$\langle \mathcal{O}\rangle = \int D[A,z]\,\mathcal{O}\, e^{-\frac{i}{\hbar}\tilde{S}[A,z]}$$



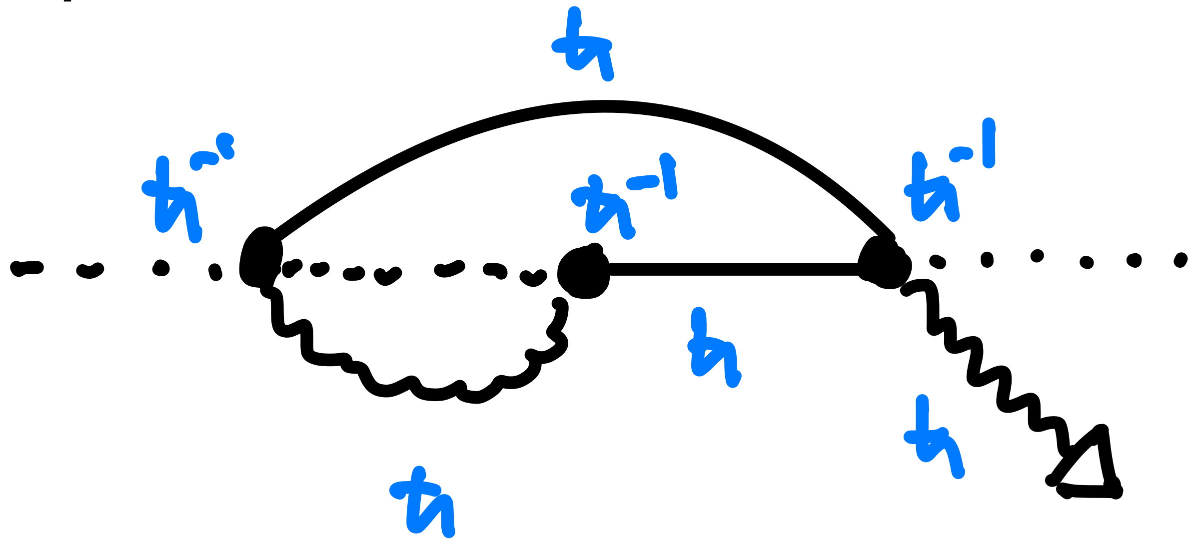
$\sim \hbar$

$\sim \hbar$



$\sim \frac{1}{\hbar}$

E.g:



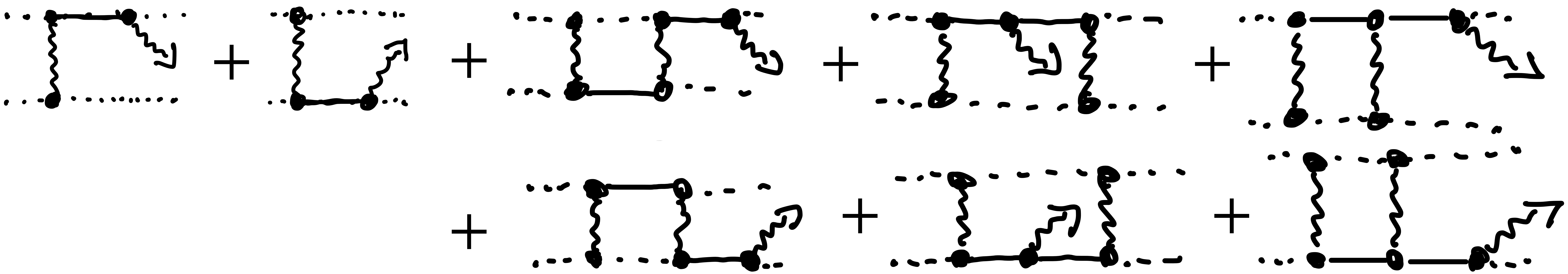
$\sim \hbar$

$\Rightarrow$ Loops are quantum effects!

- Two worldlines

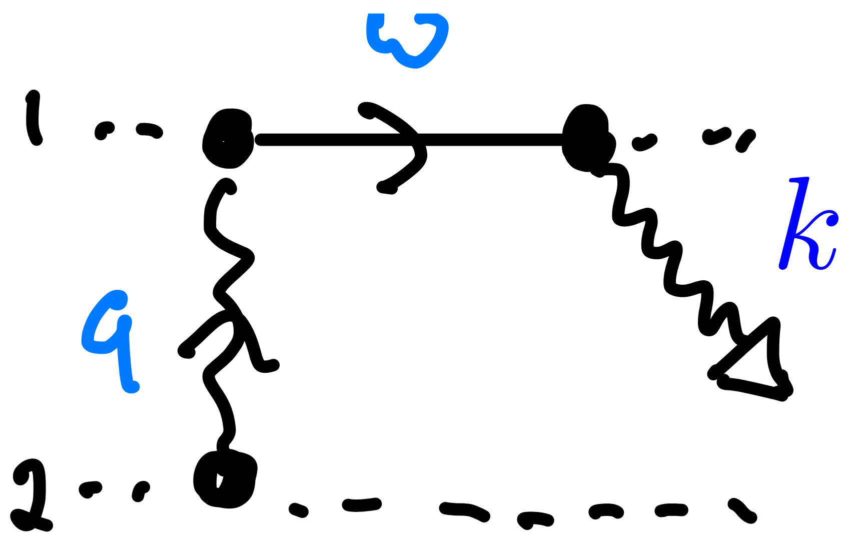
q^3 q^5

$\langle A^\mu(k)\rangle =$



$+\ldots$

Leading order:

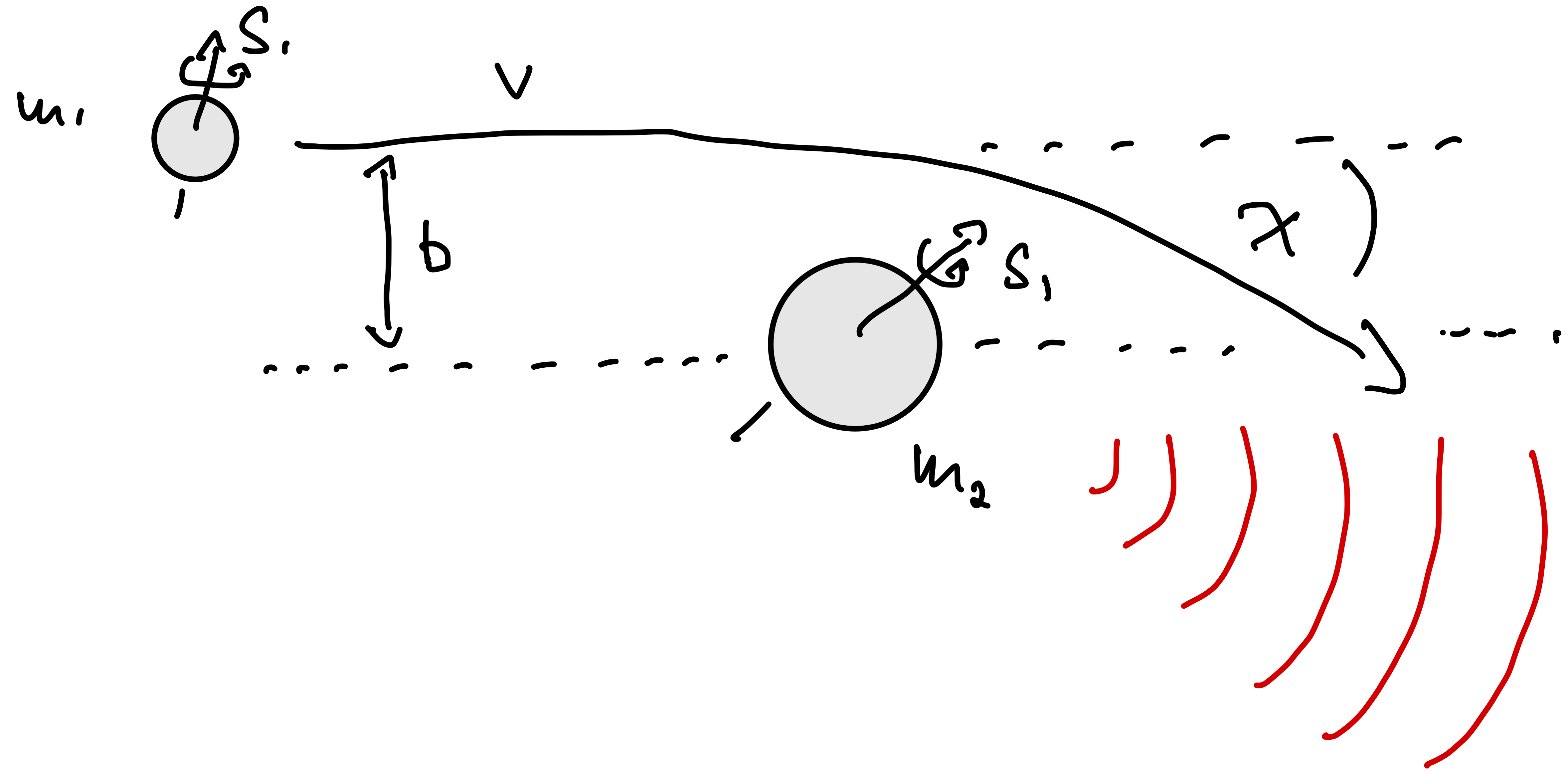


$$= \int_{q,\omega} \frac{\delta(v_2\cdot q)\,\delta(v_1\cdot q-\omega)\delta(\omega - v_1\cdot k)}{q^2\,\omega^2\,k^2} \times [\text{Numerator}]$$

4+1 integrations
versus
3 delta-functions

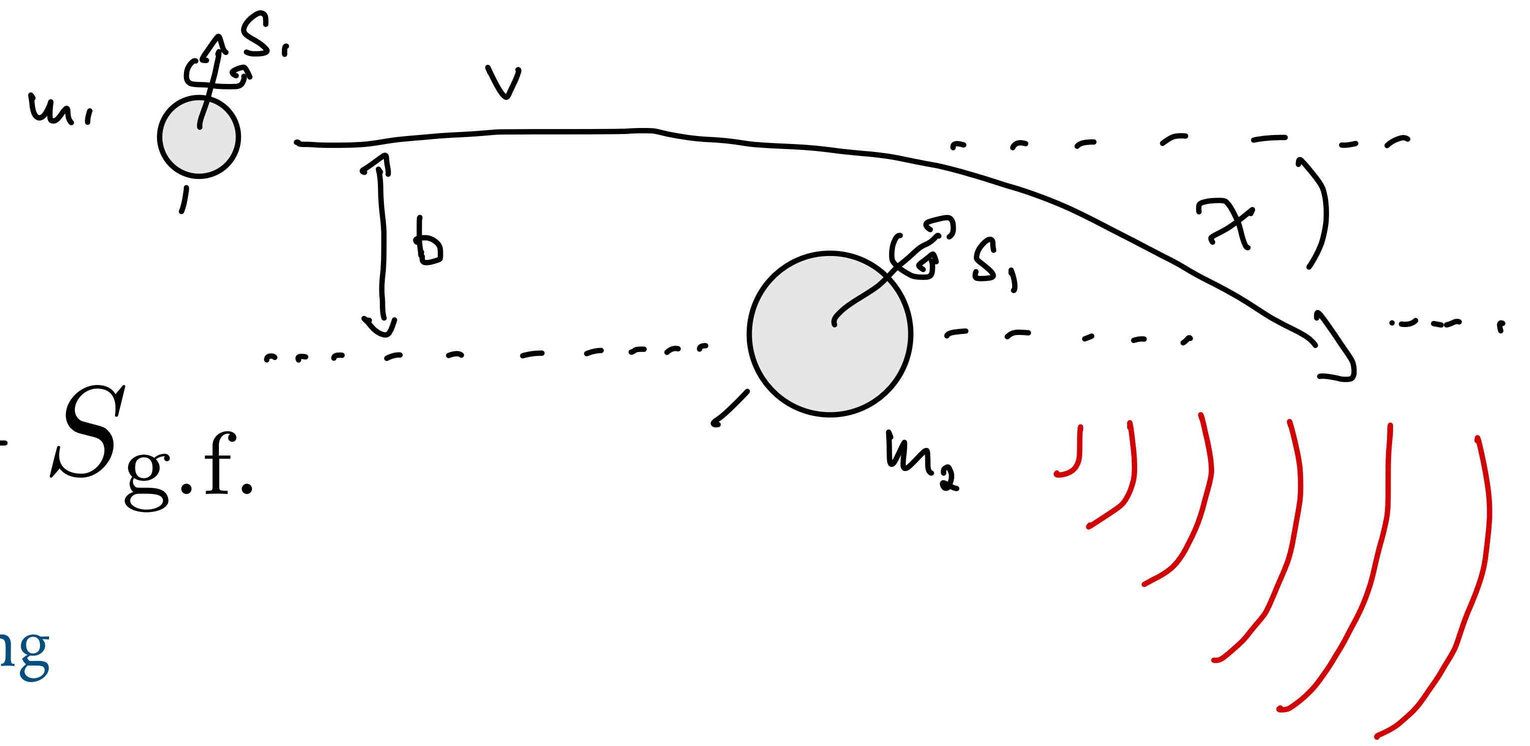
$\Rightarrow$ Non-trivial **loop** integral due to hybrid (4d-1d) theory

THE GENERAL RELATIVISTIC 2-BODY PROBLEM



RELATIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH

Point-particle approximation for BHs (or NSs)



$$S = -\sum_{i=1}^{2} m_i \int d\tau_i \sqrt{g_{\mu\nu}\dot{x}_i^\mu(\tau_i)\dot{x}_i^\nu(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\text{g.f.}}$$

Point particle approximation — Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu}$$

Einstein's eqs.

$$\ddot{x}_i^\mu + \Gamma^\mu{}_{\nu\rho}\dot{x}_i^\nu \dot{x}_i^\rho = 0$$

Geodesic eqs.

2) Solve iteratively in G

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\sum_{n=0}^{\infty} G^n h_{\mu\nu}^{(n)}(x)$$

emitted radiation

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu\,\tau + \sum_{n=1}^{\infty} G^n z_i^{(n)\,\mu}(\tau)$$

straight line: „in" state — deflections

3) Construct observables

Far field waveform:

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}(\frac{1}{r^2})$$

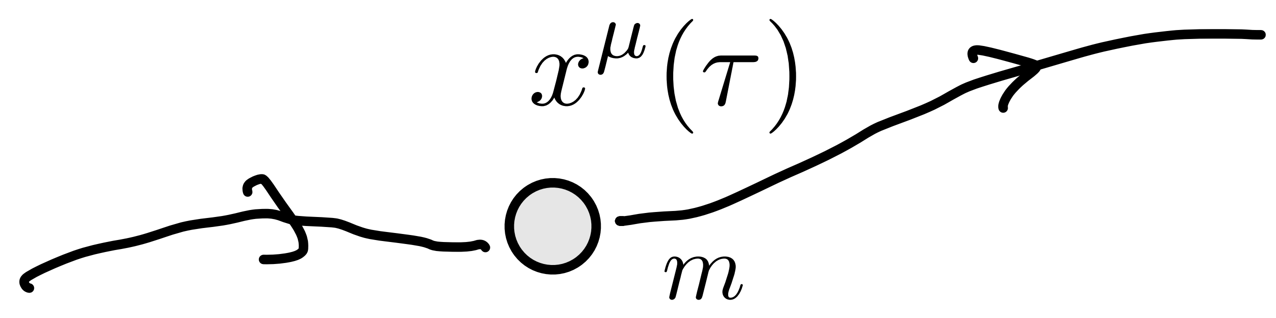
„Impulse" (change in momentum):

$$\Delta p_i^\mu = m_i \dot{x}_i^\mu\Big|_{\tau=-\infty}^{\tau=+\infty} = m_i\int d\tau\, \ddot{x}_i^\mu(\tau)$$

WORLDLINE QUANTUM FIELD THEORY

Mogull, **JP**, Steinhoff *JHEP* 02 (2021) 048

- Model Black Holes/Neutron Stars as a point particles



$$S_{\mathrm{BH/NS}} = -\frac{m}{2}\int d\tau g_{\mu\nu}\,\dot{x}^\mu(\tau)\,\dot{x}^\nu(\tau) \quad + \text{[spin \& tidal effects]}$$

They interact through Einstein's gravity: $$S = S_{\mathrm{BH/NS}} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^\mu(\tau) = b_i^\mu + v_i^\mu\tau + z^\mu(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$

- Path integral quantisation perturbative in Newton's constant G but exact in velocity

$$\langle\mathcal{O}\rangle_{\mathrm{WQFT}} = \int D[h,z]\mathcal{O}\,e^{-\frac{i}{\hbar}S[z,h]} \quad \xrightarrow{\hbar\to 0}$$

Tree-level one-point functions $\langle h_{\mu\nu}\rangle$ and $\langle z^\mu\rangle$ solve classical equations of motion

$\Rightarrow$ Advanced quantum field theory technology for classical gravitational wave physics

WORLDLINE QUANTUM FIELD THEORY: PERTURBATIVE SETUP

$$S_{\mathrm{WQFT}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot{x}^{\mu}(\tau)\,\dot{x}^{\nu}(\tau) + \frac{1}{16\pi\, G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$

- Worldline propagators: $z^{\mu} \xrightarrow{\omega} z^{\nu}$ $\qquad \langle z^{\mu}(\omega) z^{\nu}(-\omega)\rangle = -\frac{i}{m}\frac{\eta_{\mu\nu}}{(\omega + i0)^2}$

- Perturbative (quantum) gravity:

$$\sqrt{-g}\,R(g) \xrightarrow[g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}]{} \boxed{-\frac{1}{2}h_{\mu\nu}(P^{-1})^{\mu\nu;\rho\sigma}\Box h_{\rho\sigma} + \sqrt{G}[\partial^2 h^3] + \sqrt{G}^2[\partial^2 h^4] + \sqrt{G}^3[\partial^2 h^5] + \dots}$$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

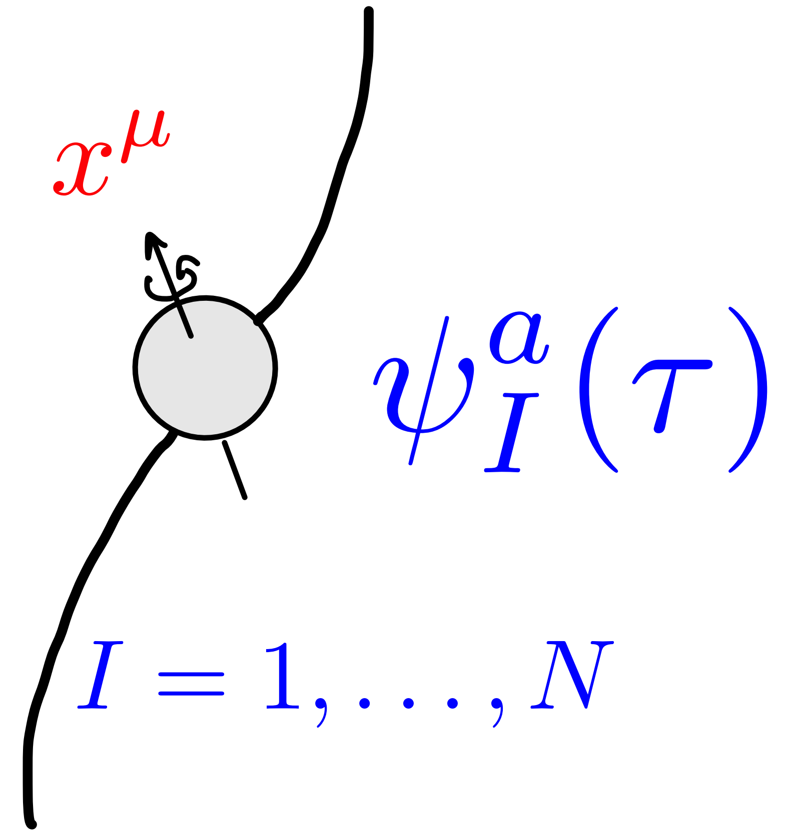
$\Rightarrow$ graviton propagator:

$$\underset{-}{\mu\nu} \overset{\rightarrow}{\underset{k}{\leadsto}} \underset{+}{\rho\sigma} = i\frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i0)^2 - \mathbf{k}^2}$$

N.B. need to take retarded propagator (in-in formalism)

PUTTING SPIN ON THE WORLD-LINE

[Jakobsen,Mogull,JP,Steinhoff]



- Hidden **supersymmetry** of spinning black holes!

  Add N anti-commuting fields $\psi_I^a(\tau)$: N-extended superparticle

  Flat space: $$S_{\text{susy}} = -m \int d\tau \left[\tfrac{1}{2}\eta_{\mu\nu}\dot{x}^\mu\,\dot{x}^\nu + i\eta_{ab}\psi_I^a\dot{\psi}_I^b\right]$$

  [Howe,Penati,Pernici,Townsend]

  Equivalent to massive, spin N/2 particle.

- Coupling to curved space-time: ($N = 1$) $\quad D_\tau\psi^a = \dot{\psi}^a + \dot{x}^\mu\,\omega_\mu^{ab}\psi_b$

$$S_1 = -m \int d\tau \left[\tfrac{1}{2}g_{\mu\nu}\dot{x}^\mu\,\dot{x}^\nu + i\psi_a D_\tau\psi^a\right]$$

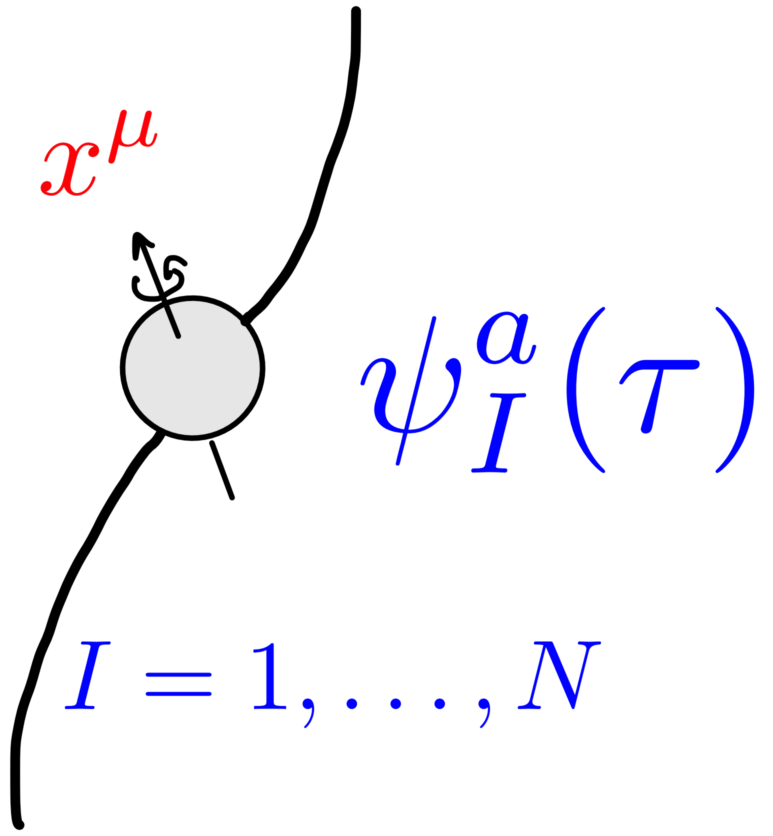
$$\boxed{S^{ab} = -2im\psi^{[a}\psi^{b]}}$$

spin tensor

Captures linear in spin (spin-orbit) interactions of BHs & NSs

PUTTING SPIN ON THE WORLD-LINE

[Jakobsen,Mogull,JP,Steinhoff]



- Spin-orbit & spin-spin interactions ($\mathcal{N} = 2$ SUSY) [Bastianelli,Benincasa,Giombi]
- Augmented by NS-term: spin-induced quadropol moment

$$S_{\mathrm{BH/NS}} = -m \int d\tau \Big[\tfrac{1}{2} g_{\mu\nu} \dot{x}^\mu \, \dot{x}^\nu + i\bar{\psi}_a D_\tau \psi^a + \tfrac{1}{2} R_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d + C_E R_{a\dot{x}b\dot{x}} \bar{\psi}^a \psi^b \, \bar{\psi} \cdot \psi \Big]$$

N=2 superparticle

neutron star term

Approximate SUSY persists.

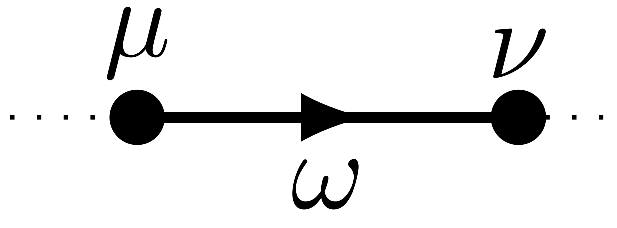
Spin tensor of BHs/NSs

$$\boxed{S_i^{ab} = -2im \bar{\psi}_i^{[a} \psi_i^{b]}}$$

- Scattering scenario:

$$\boxed{\begin{aligned} x_i^\mu(\tau) &= b_i^\mu + v_i^\mu \, \tau + z_i^\mu(\tau) \\ \psi_i^a(\tau) &= \Psi_i^a + \psi_i'^a(\tau) \end{aligned}}$$

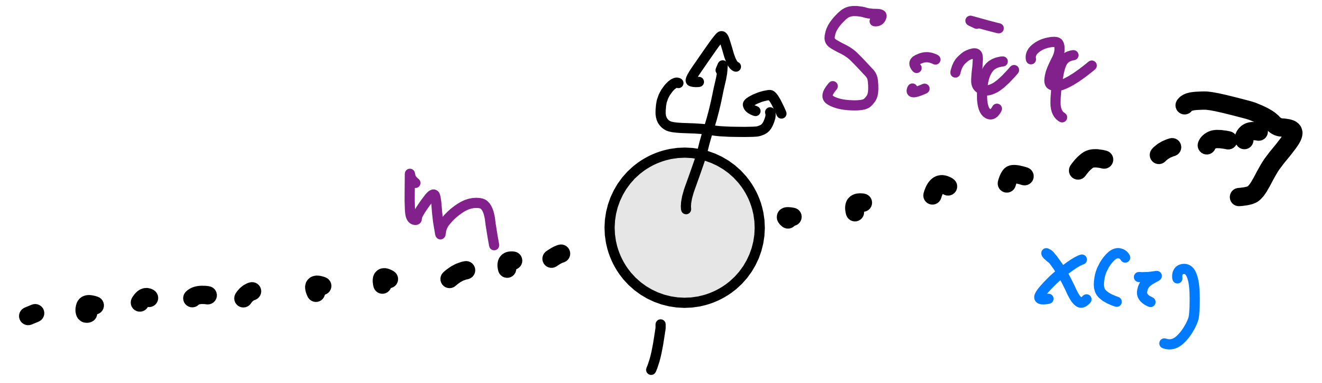
Quantize $z_i^\mu, \psi_i'^a, \bar{\psi}_i'^a$



$$\langle \psi^a(\omega) \bar{\psi}^b(-\omega) \rangle = \frac{-i\eta^{ab}}{m(\omega + i0)}$$

PHYSICAL INTERPRETATION OF SUSY

- Traditional approach: [Vines,Kunst,Steinhoff,Hinderer][Steinhoff][Porto][Levi]



  Spin tensor $S_i^{\mu\nu}(\tau)$ & co-moving frame $\Lambda_i^{A\,\mu}(\tau)$

  Eoms: $\frac{Dp^\nu}{D\tau} + \frac{1}{2} S^{\mu\rho} R_{\mu\rho\nu\kappa} \dot{x}^\kappa = 0$ $\quad \frac{DS^{\mu\nu}}{D\tau} + 2\dot{x}^{[\mu}\, p^{\nu]} = 0$ [Matthisson-Papapetrou-Dixon]

  Freedom of imposing a Spin-Supplementary Condition (SSC): $p_\mu\, S^{\mu\nu} = 0$

- Our approach: Spinning super-particle $\boxed{S_i^{\mu\nu} = -2i\bar{\psi}_i^{[\mu}\psi_i^{\nu]}}$

  Background SUSY transformations:

$$\delta b_i^\mu = i\bar{\epsilon}\Psi_i^\mu + i\epsilon\bar{\Psi}_i^\mu\,, \quad \delta v_i^\mu = 0\,, \quad \delta\Psi_i^\mu = -\epsilon v_i^\mu$$

$$\Rightarrow \quad \delta S_i^{\mu\nu} = v_i^\mu\, \delta b_i^\nu - v_i^\nu\, \delta b_i^\mu$$

- Interpretation of SUSY:

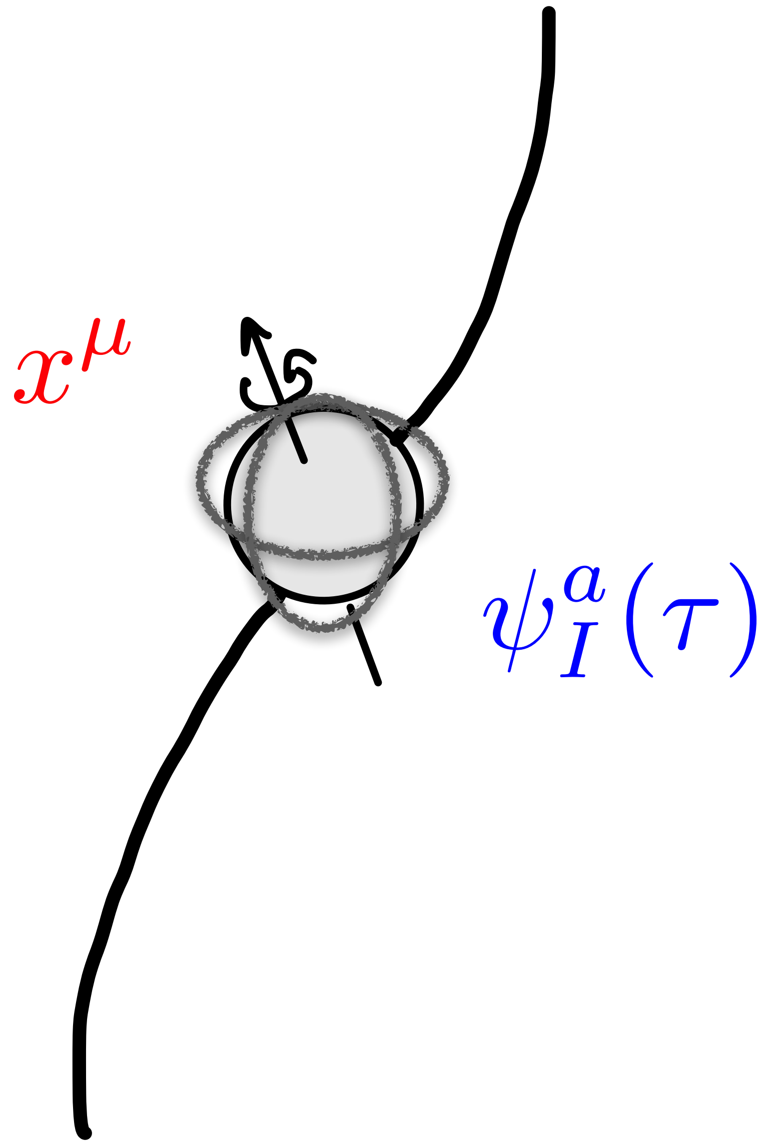
  SUSY = Conservation of SSC

  Covariant SSC:

$$v_i \cdot \Psi_i = 0 \Rightarrow v_{i,\mu} S_i^{\mu\nu} = 0$$

- Need of understanding relation to dual amplitudes approach [Bern,Luna,Roiban,Shen,Zeng] [Bern,Kosmopoulos,Luna,Roiban,Teng]

TIDAL INTERACTIONS



- First layer of tidal & finite size effects:

$$S_{\text{tidal}} = m \int d\tau \left[c_{E^2} E_{\mu\nu} E^{\mu\nu} + c_{B^2} B_{\mu\nu} B^{\mu\nu} \right]$$

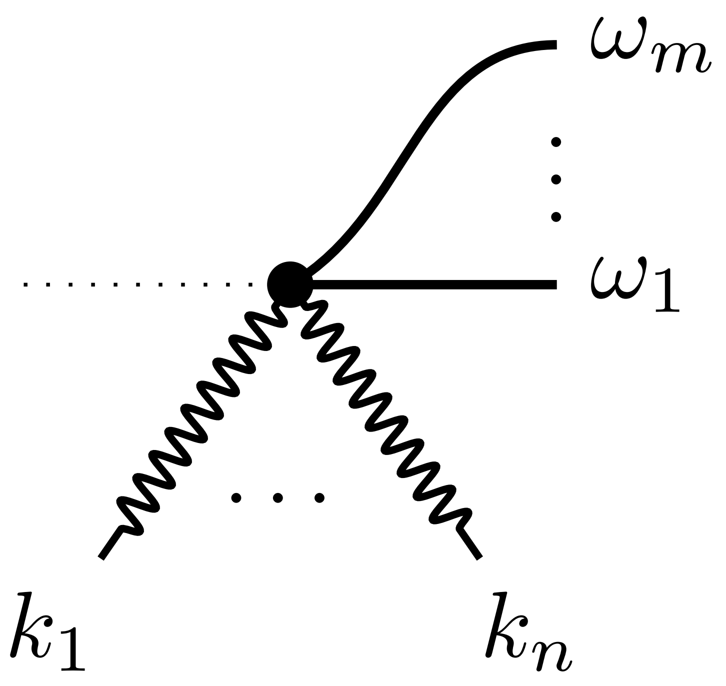
Electric and magnetic curvature:

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta \qquad B_{\mu\nu} := R^*_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta$$

Wilson coefficients (or „Love numbers“): c_{E^2} & c_{B^2} (vanish for black holes)

WORLDLINE QUANTUM FIELD THEORY: VERTICES

- Worldline vertices: n-gravitons & m world-line fluctuations

$V_{n|m} =$ 

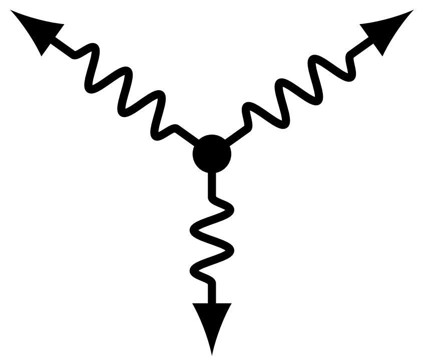
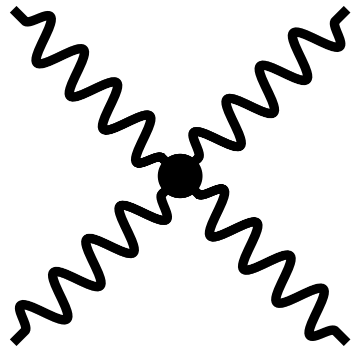
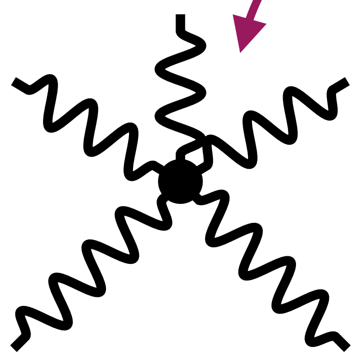
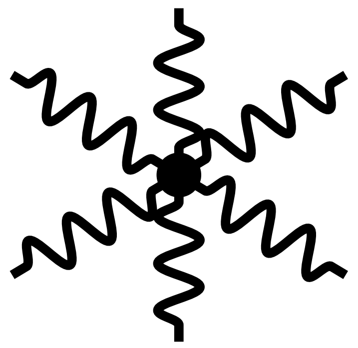
$$= m\sqrt{G}^{\,n}\, e^{ib\cdot\sum_j k_j}\,\delta\Big(v\cdot\sum_{j=1}^{n} k_j + \sum_{i=1}^{m}\omega_i\Big) \times \begin{pmatrix}\text{polynomial in } \omega_i, k_j \\ \text{of degree } 2n+m \\ \text{depending on } v^\mu,\ S^{\mu\nu}\end{pmatrix}$$

+ C_E, c_{E^2}, c_{B^2} for neutron stars

Energy conservation on worldline

- „Bulk“ graviton vertices:

Four-momentum conservation in bulk $\delta^4(\sum p)$

 $\sim \sqrt{G}\,k^2$  $\sim \sqrt{G}^2 k^2$,  $\sim \sqrt{G}^3 k^2$  $\sim \sqrt{G}^4 k^2, \ldots$

N-PM order needs: $V_{n|m}$ for $n = 1,\ldots,N, m = 0,\ldots,(N+1-n)$ & graviton vertices up to $N+1$

CLAIM:

$$\langle \mathcal{O} \rangle_{\text{WQFT}} = \int D[h, z]\mathcal{O}\, e^{-\frac{i}{\hbar} S[z,h]} \xrightarrow{\hbar \to 0}$$

Tree-level one-point functions $\langle h_{\mu\nu} \rangle$ and $\langle z^{\mu} \rangle$ solve classical equations of motion

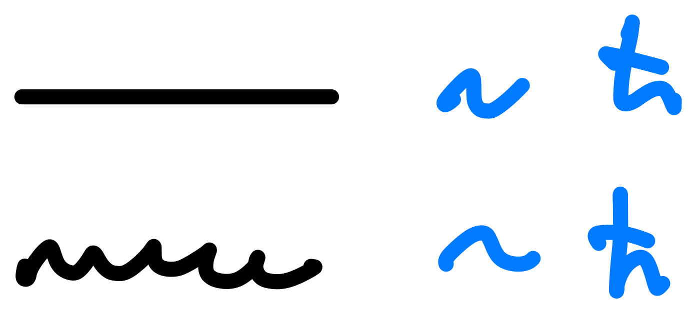
CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

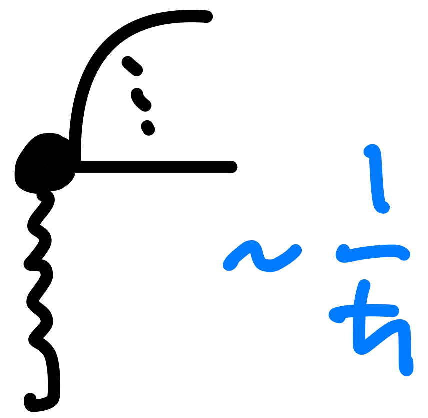
[Boulware,Brown,'68]

- Action: $S[\Phi_A]$ with fields $\Phi_A(x_A) = \{h_{\mu\nu}(x), z^\mu(\tau)\}$ and coordinates $x_A = \{x^\mu, \tau\}$
- Partition function in the presence of sources

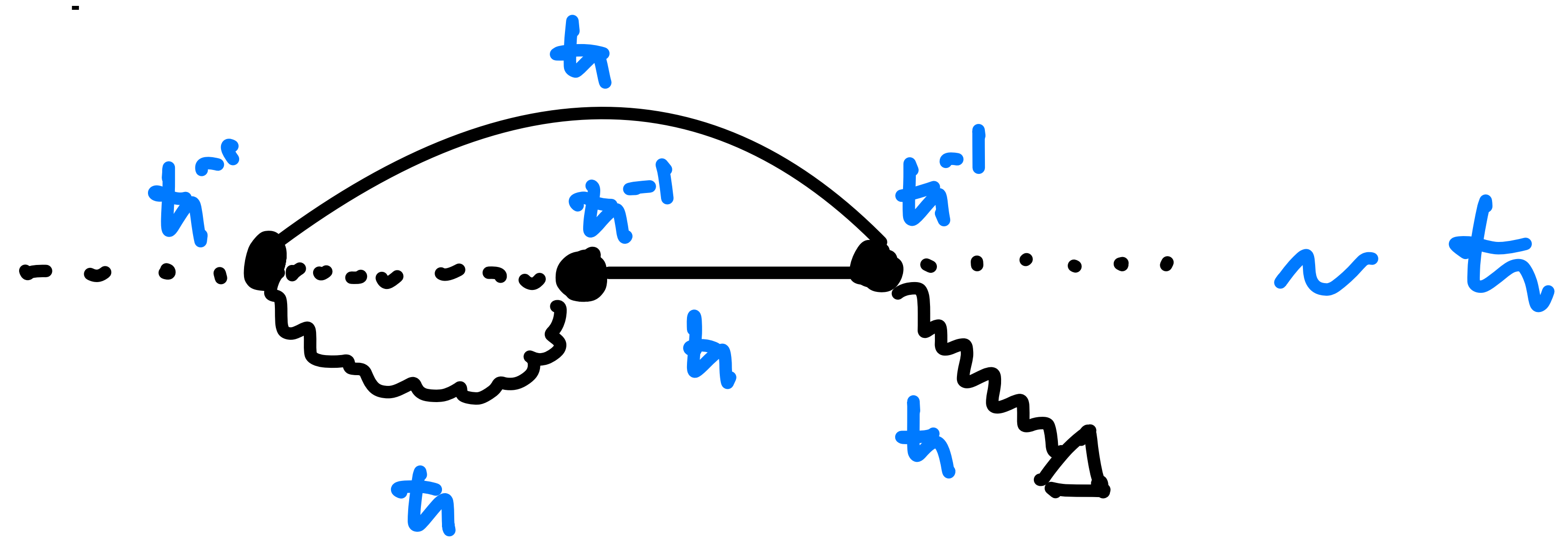
$$Z[J_A] = \int D[\Phi_A] \exp\left[\frac{i}{\hbar}\left(S[\Phi_A] + \sum_A \int dx_A J_A(x_A)\, \Phi_A(x_A)\right)\right]$$

- $\hbar$ counting:





E.g:



$\Rightarrow$ Loops are quantum effects!

- Scalings of **connected** n-point functions:

$$\langle \Phi_{A_1} \ldots \Phi_{A_n} \rangle_{\text{conn}} \sim \sum_L \hbar^{-1+n+L} \quad \text{(L-loop connected n-point diagrams)}$$

Well defined classical limit only for n=1 and L=0: **Tree-level one-point functions**

CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

[Jakobsen]

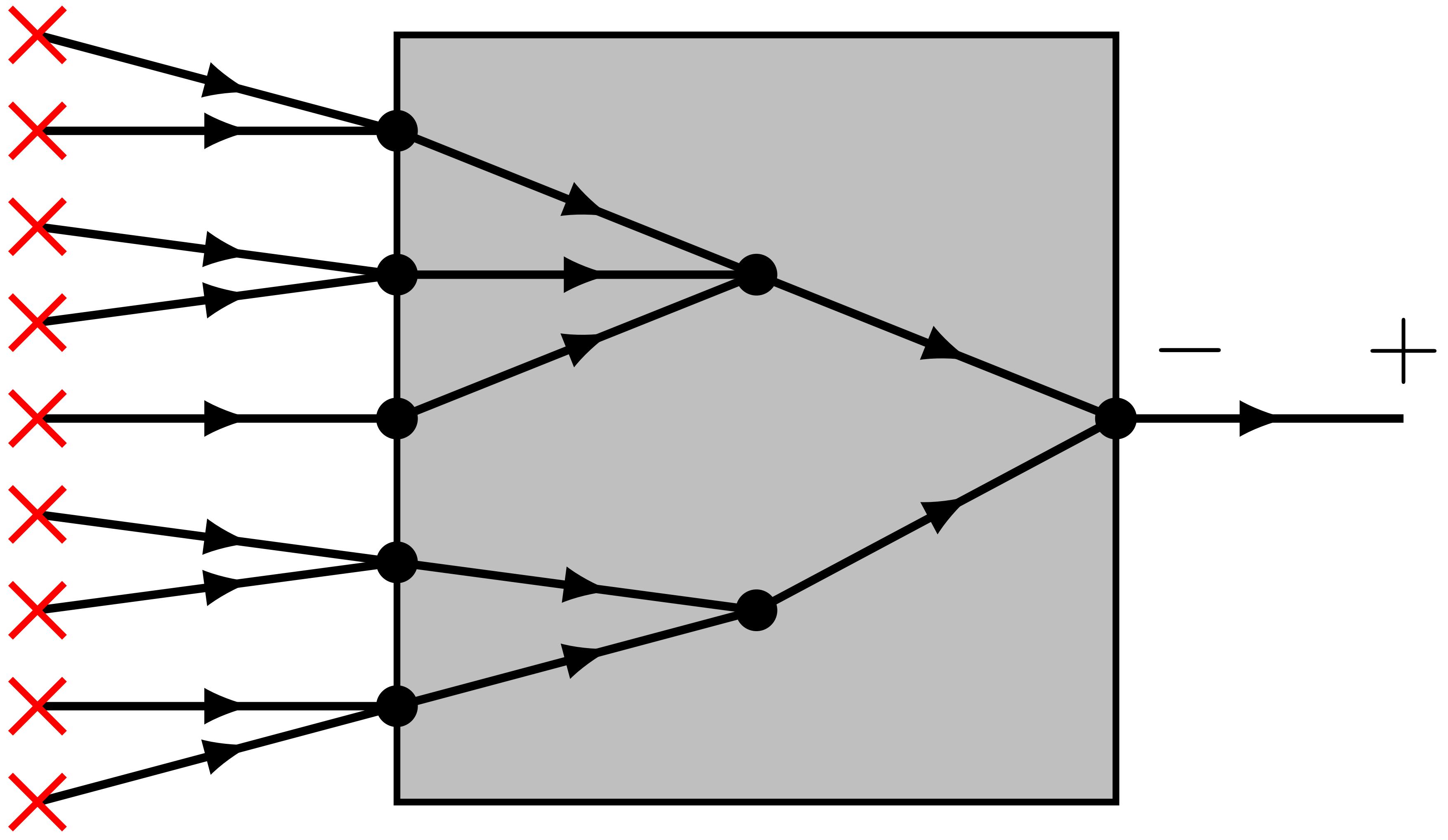
- Factorization $$\lim_{\hbar\to 0} \langle \Phi_{A_1}\Phi_{A_2}\dots\Phi_{A_n}\rangle_{\text{discon}} = \langle \Phi_{A_1}\rangle^{\text{tree}}_{\text{con}}\,\langle \Phi_{A_2}\rangle^{\text{tree}}_{\text{con}}\dots\langle \Phi_{A_n}\rangle^{\text{tree}}_{\text{con}}$$
- Consequence for Schwinger-Dyson equations: $\Leftrightarrow$ Ehrenfest theorem in QM

$$\left\langle \frac{\delta S[\Phi_A]}{\delta \Phi_A} \right\rangle = 0 \quad \xrightarrow{\hbar\to 0} \quad \boxed{\frac{\delta S[\langle \Phi_A\rangle_{\text{tree}}]}{\delta \Phi_A} = 0}$$

Tree-level one-point functions solve classical equations of motion

- Importantly $S[\Phi_A]$ must be independent of $\hbar$ (not the case in amplitudes approach - massive field!) -> Key advantage of WQFT approach (no „super classical" terms)
- Need non-trivial background field configurations for non-vanishing one-point functions

THE IN-IN (SCHWINGER-KELDYSH) FORMALISM FOR WQFT



OR WHY RETARDED PROPAGATORS?

[Jakobsen,Mogull,JP,Sauer]

IN-IN (SCHWINGER-KELDYSH) FORMALISM

[Galley,Tiglio] [Jordan]

- Standard Feynman path integral yields $\langle \phi_H(x) \rangle_{\text{in}-\text{out}} = {}_{\text{out}}\langle 0 | \phi_H(x) | 0 \rangle_{\text{in}}$ but want

$$\boxed{\langle \phi_H(x) \rangle_{\text{in}-\text{in}} = {}_{\text{in}}\langle 0 | \phi_H(x) | 0 \rangle_{\text{in}} = \langle 0 | U(-\infty, t)\, \phi_0(t, \mathbf{x})\, U(t, -\infty)\, | 0 \rangle}$$

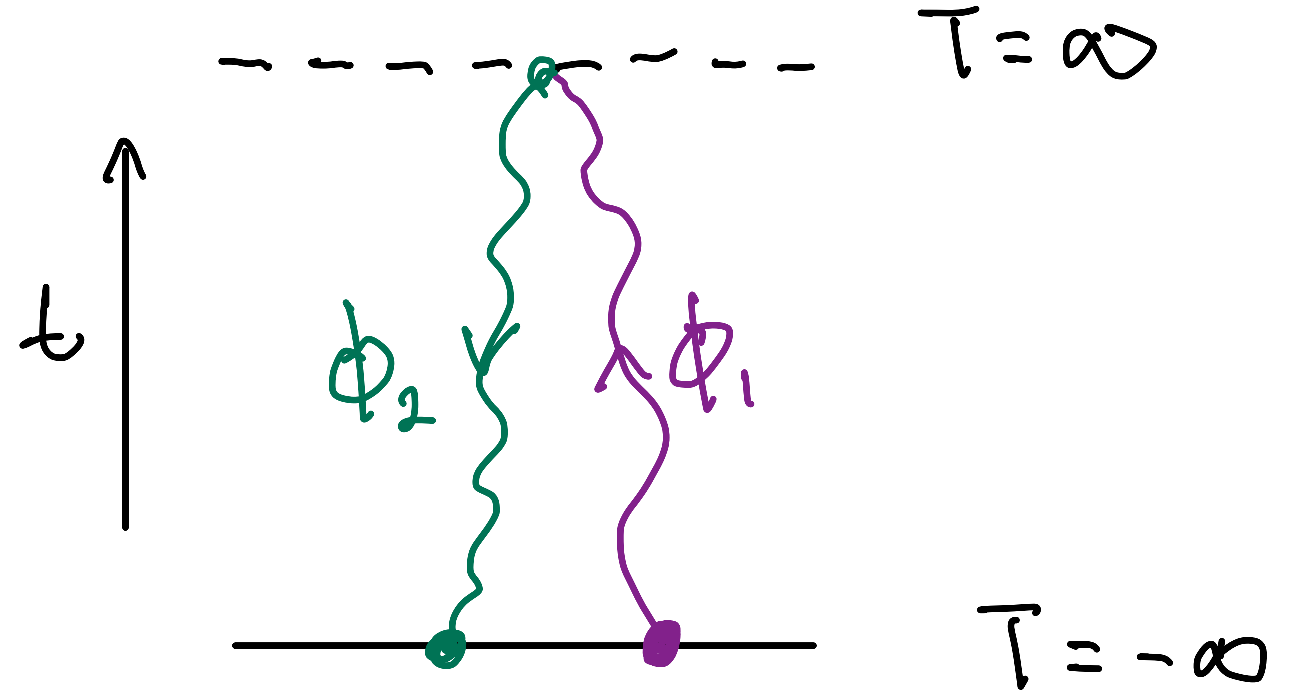
$\Rightarrow$ need two time-evolution operators: Double the fields!

$$Z[J_1, J_2] = \langle 0 |\, U_{J_1}(-\infty, \infty)\, U_{J_1}(\infty, -\infty)\, | 0 \rangle$$

$$= \int D[\phi_1, \phi_2] \exp\left[\frac{i}{\hbar} \left(S[\phi_1] - S[\phi_2] + \int_x J_1(x)\, \phi_1(x) - J_2(x)\, \phi_2(x) \right) \right]$$

$$\boxed{\frac{1}{Z[0,0]} \frac{\delta Z[J_1, J_2]}{\delta J_1(x)} \bigg|_{J_i = 0} = \langle \Phi_H(x) \rangle_{\text{in}-\text{in}}}$$

$$\phi_1(t = +\infty, \mathbf{x}) = \phi_2(t = +\infty, \mathbf{x})$$



Boundary conditions:

$$\phi_1(t = -\infty, \mathbf{x}) = 0 = \phi_2(t = -\infty, \mathbf{x})$$

KELDYSH BASIS: $\phi_+ = \frac{1}{2}(\phi_1 + \phi_2)$ $\phi_- = \phi_1 - \phi_2$

- This yields

$$Z[J_\pm] = \int D[\phi_+, \phi_-] \exp\left[\frac{i}{\hbar}\left(S[\phi_+ + \frac{1}{2}\phi_-] - S[\phi_+ - \frac{1}{2}\phi_-] + \int d^4x (J_+\phi_- + J_-\phi_+)\right]$$

- Propagator matrix from free part:

$$\langle \phi_a(x)\,\phi_b(y)\rangle = \begin{array}{cc} \begin{array}{cc} \quad + & \qquad\qquad - \end{array} & \\ \begin{pmatrix} \frac{1}{2}D_H(x,y) & D_{\mathrm{ret}}(x,y) \\ -D_{\mathrm{adv}}(x,y) & 0 \end{pmatrix} & \begin{array}{c} + \\ - \end{array} \end{array}$$

irrelevant @ tree-level

$$\widetilde{D}_{\mathrm{ret}}(k) = \underset{-}{\bullet}\!\!\longrightarrow\!\!\underset{+}{\bullet} = \frac{-i}{(k^0 + i0)^2 - \mathbf{k}^2}\,,$$

$$\widetilde{D}_{\mathrm{adv}}(k) = \underset{+}{\bullet}\!\!\longleftarrow\!\!\underset{-}{\bullet} = \frac{-i}{(k^0 - i0)^2 - \mathbf{k}^2}\,,$$

- Vertices from:

$$S_{\mathrm{int}}[\phi_+ + \frac{1}{2}\phi_-] - S_{\mathrm{int}}[\phi_+ - \frac{1}{2}\phi_-] = \phi_- \left(\frac{\delta S_{\mathrm{int}}[\phi]}{\delta\phi}\right)\Bigg|_{\phi\to\phi_+} + \mathcal{O}(\phi_-^3)$$

$\Rightarrow$ only **odd** number of ϕ_- legs!

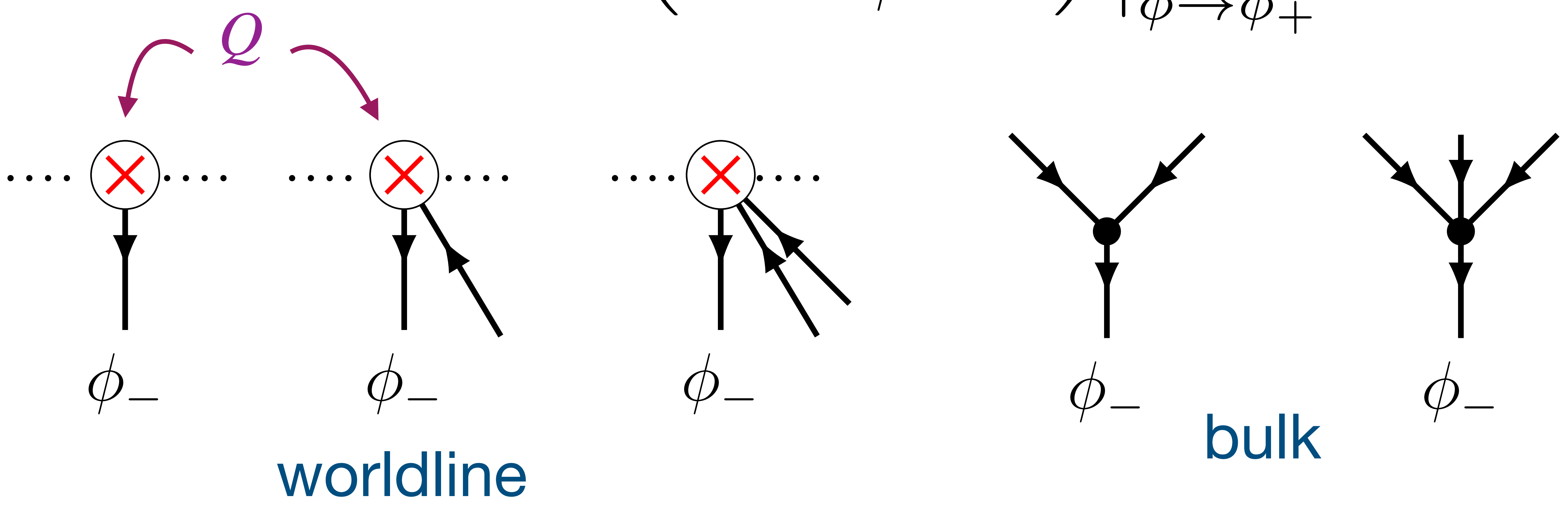
WQFT ONE POINT FUNCTIONS @ TREE-LEVEL

[Jakobsen,Mogull,JP,Sauer] [Kälin,Neef,Porto]

- Vertices including background field Q

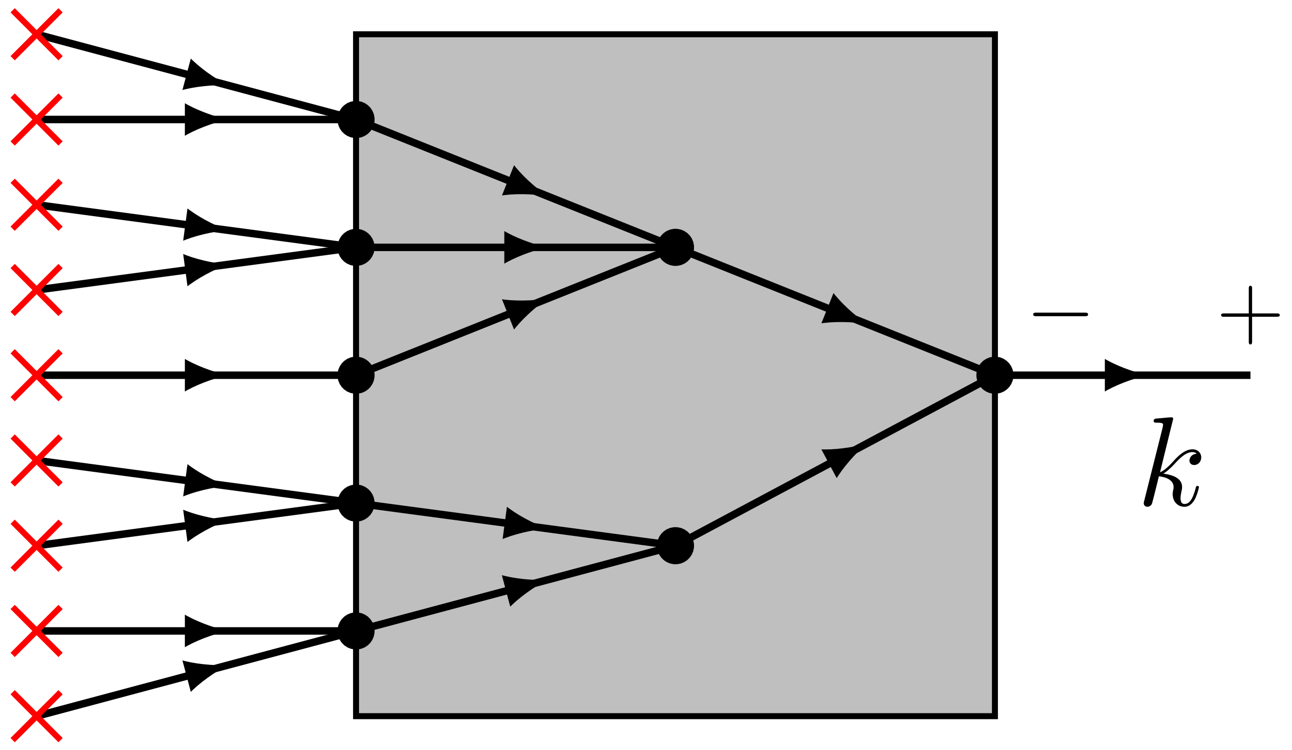
$$S_{\text{int}}[\phi; Q] \to \phi_- \left(\frac{\delta S_{\text{int}}[\phi, Q]}{\delta \phi} \right)\Bigg|_{\phi \to \phi_+} + \mathcal{O}(\phi_-^3)$$

- Yields **same vertices** as in-in theory!



- In tree-level one-point functions:

$$\langle \phi(k) \rangle_{\text{in-in}} =$$



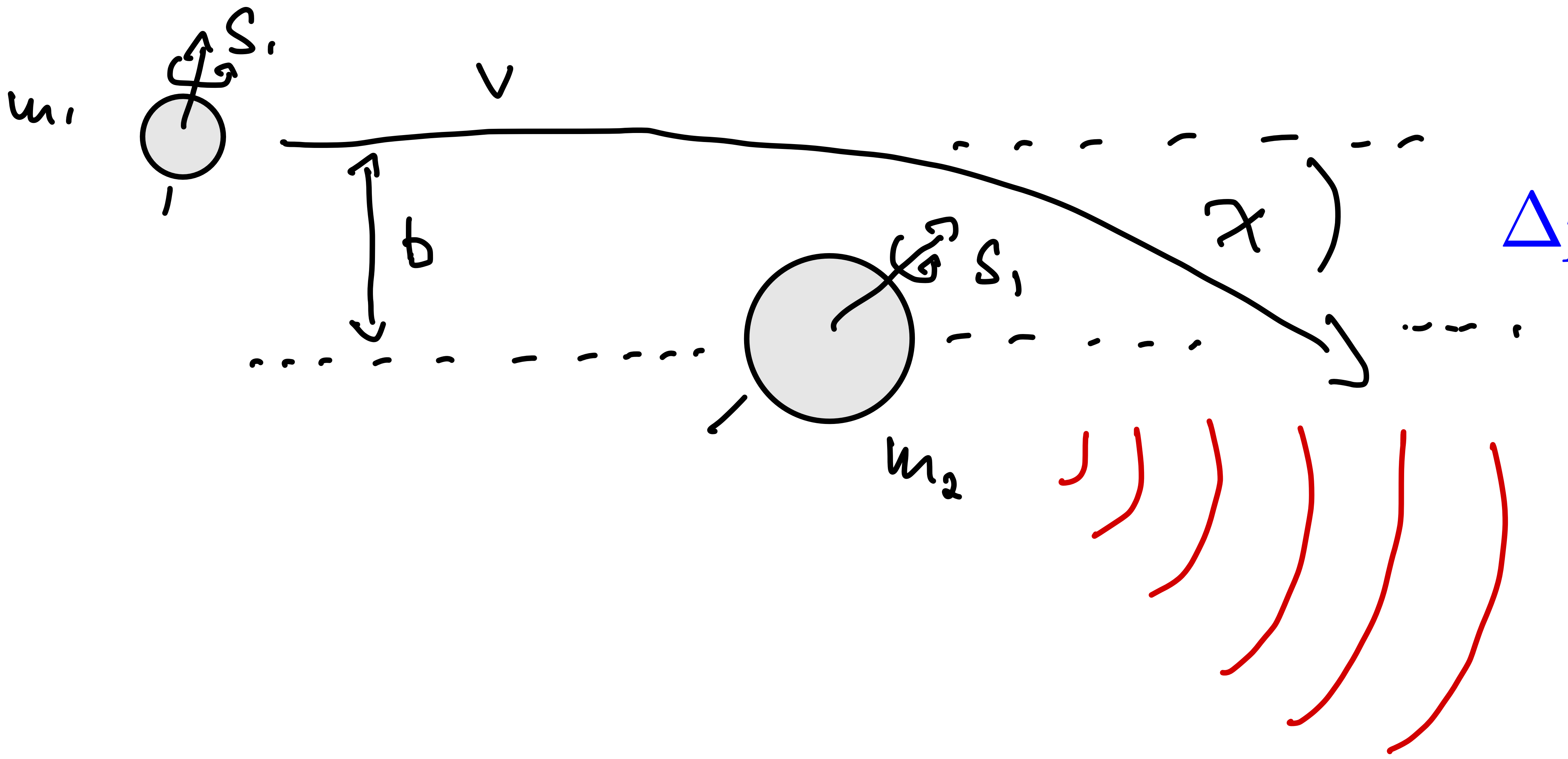
Only retarded propagators contribute!

- Absorbs the cuts present in **KMOC** formalism!

[Kosower,Maybee,O'Connell]

$$\frac{1}{k^2 + \text{sgn}(k^0) i0} = \frac{1}{k^2 + i0} + 2i\pi\theta(-k^0)\delta(k^2)$$

WQFT OBSERVABLES

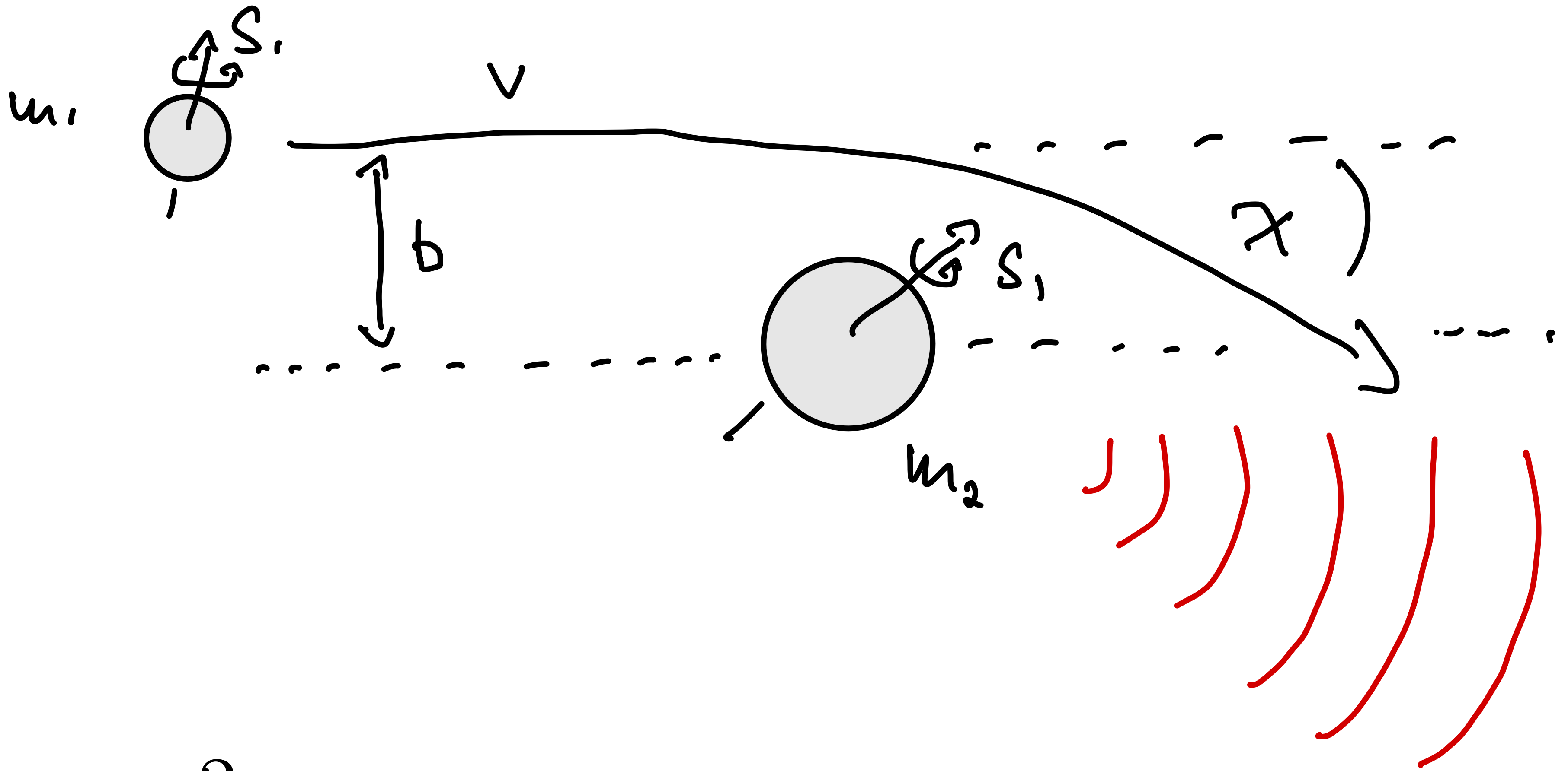


$$\Delta p_1^\mu = \sum_{n=1}^{\infty} G^n \, \Delta p_1^{(n)\,\mu}$$

$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

Spin-less BH/NS scattering:

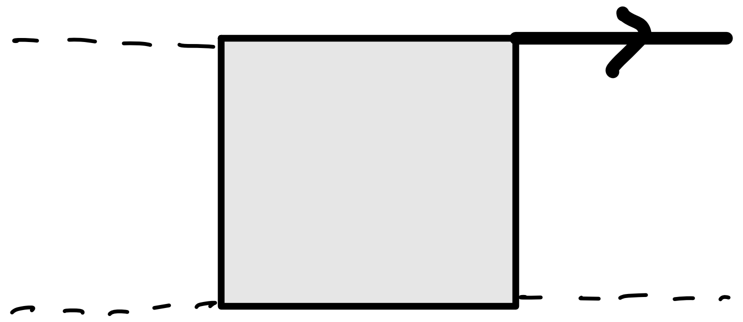
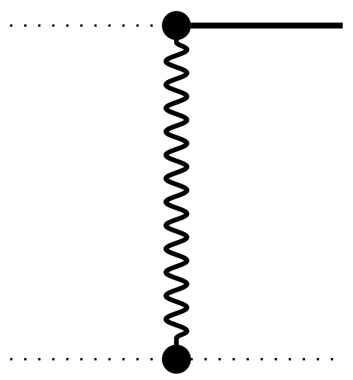
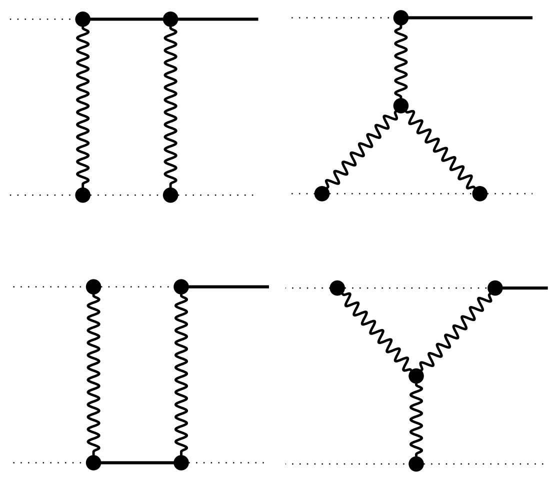
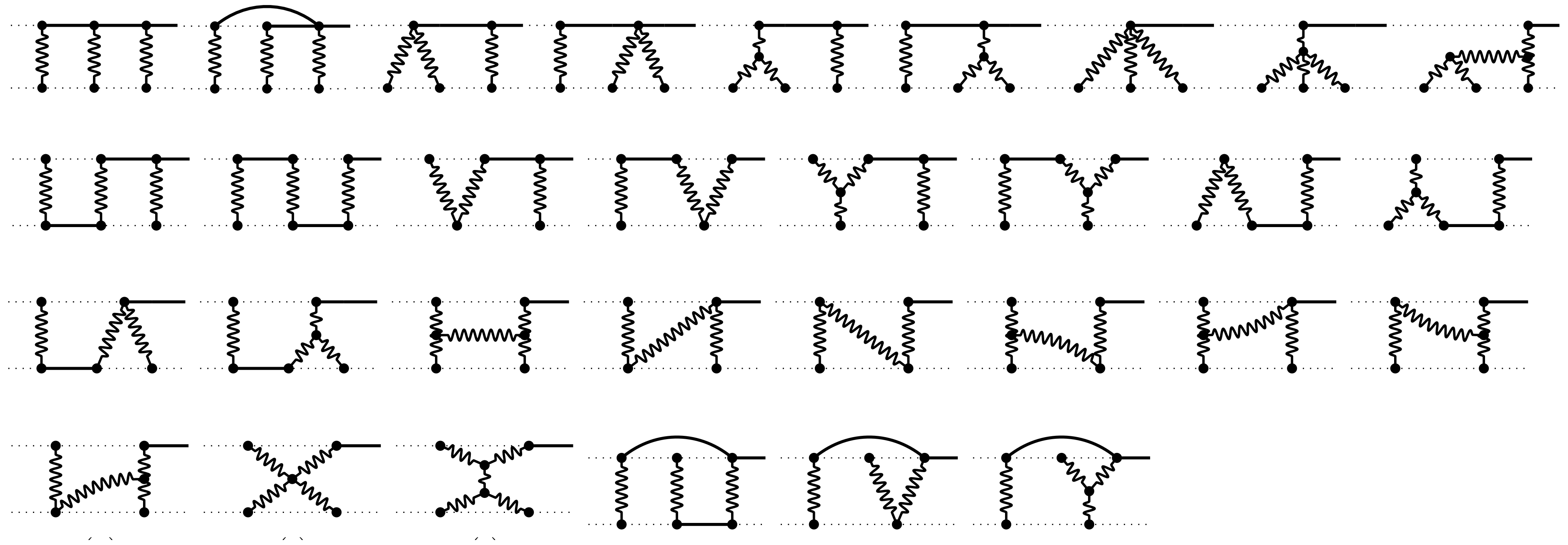


1) Impulse (change of momentum)

$$\Delta p_i^\mu = m_i \langle \dot{x}_i^\mu \rangle \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau \langle \ddot{x}_i^\mu(\tau) \rangle = m_i \int d\tau \frac{d^2}{d\tau^2} \langle z_i^\mu(\tau) \rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega\to 0}$$

Fourier trans.

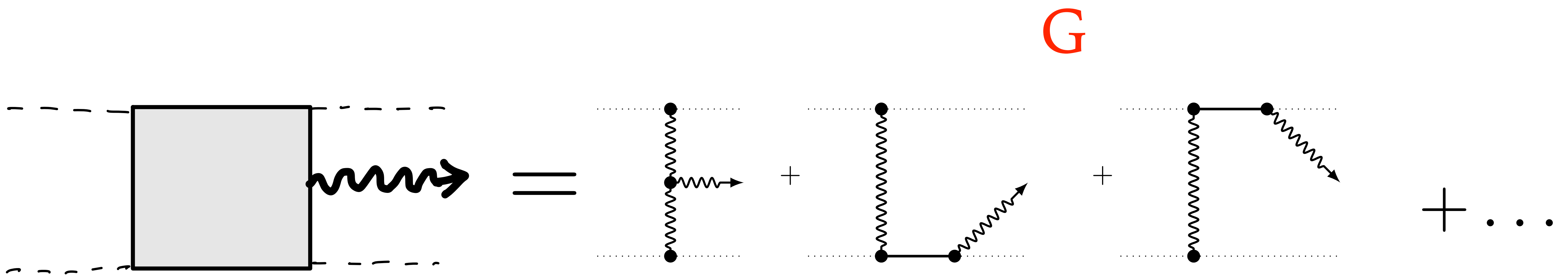
Needs sum of all graphs with outgoing z-line:

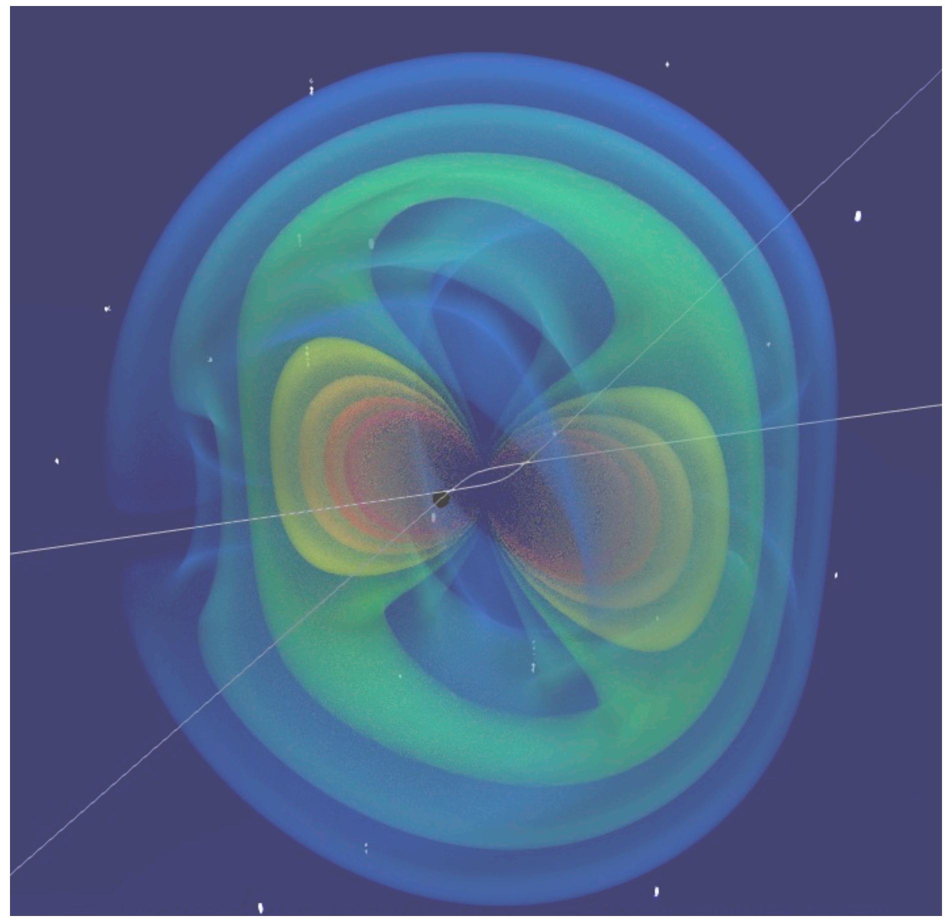
$\langle \Delta p^\mu \rangle =$  $=$  $+$  $+$ 

G G^2 G^3

[Jakobsen,Mogull,JP,Steinhoff] [Jakobsen,Mogull] [Jakobsen,Mogull,JP,Sauer]

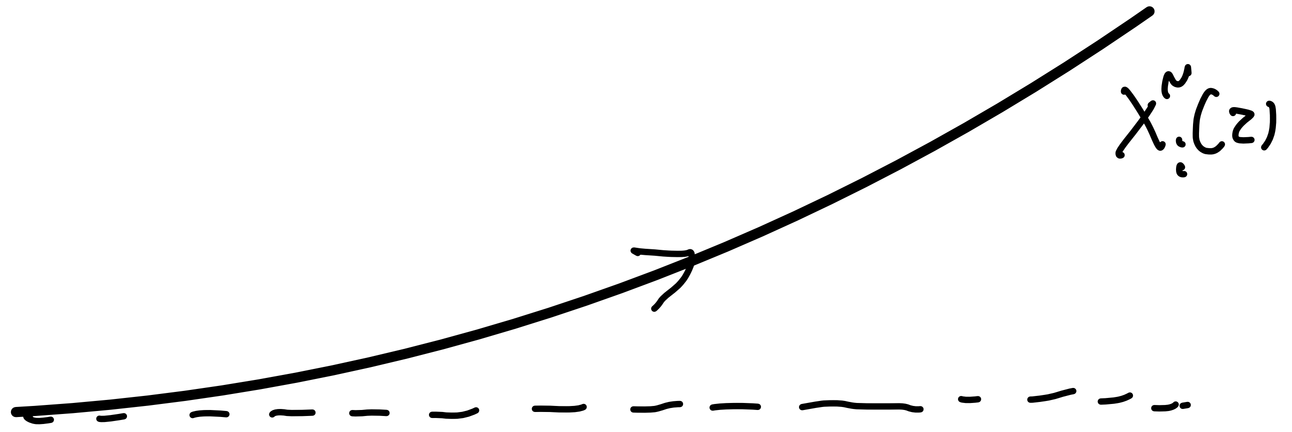
2) **Emitted Waveform** (Gravitational Bremsstrahlung) [Jakobsen,Mogull,JP,Steinhoff]

$$\langle h_{\mu\nu} \rangle =$$ 



3) **Trajectory!**

$$x^\mu(\tau) = b^\mu + v^\mu \tau + \int_\omega e^{i\omega\tau} \langle z^\mu(\omega) \rangle$$

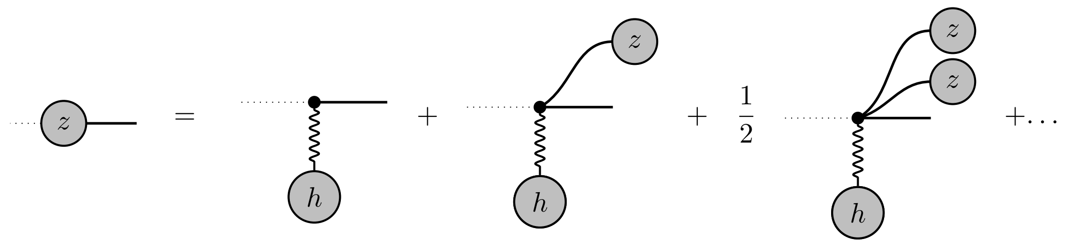


WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



Insert Feynman rules

Form script
(4PM: 5g vertex)

Tensor reduction to scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i\epsilon$

IBPs: Basis of Master Integral

Fire/Kira
Reduced symmetries
due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions
(potential & radiation gravitons)

$i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

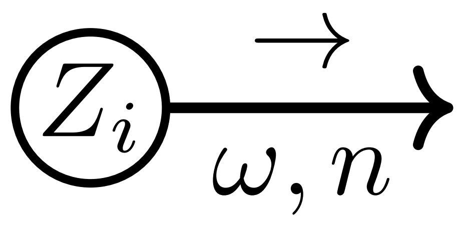
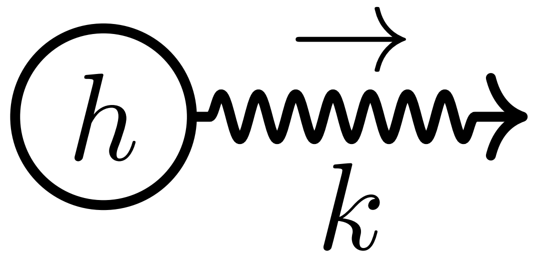
ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

Generate graphs

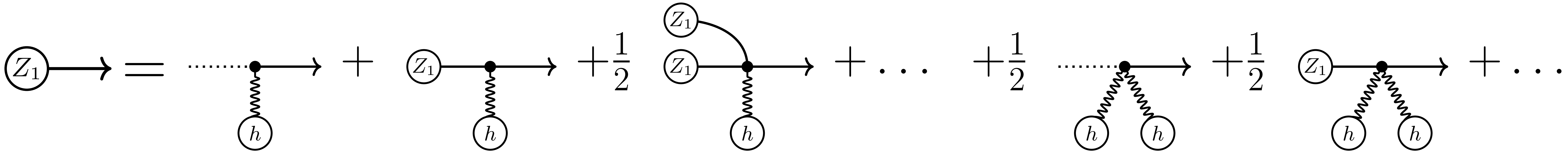
BERENDS-GIELE TYPE RECURSION

[Jakobsen,Mogull,JP,Sauer]

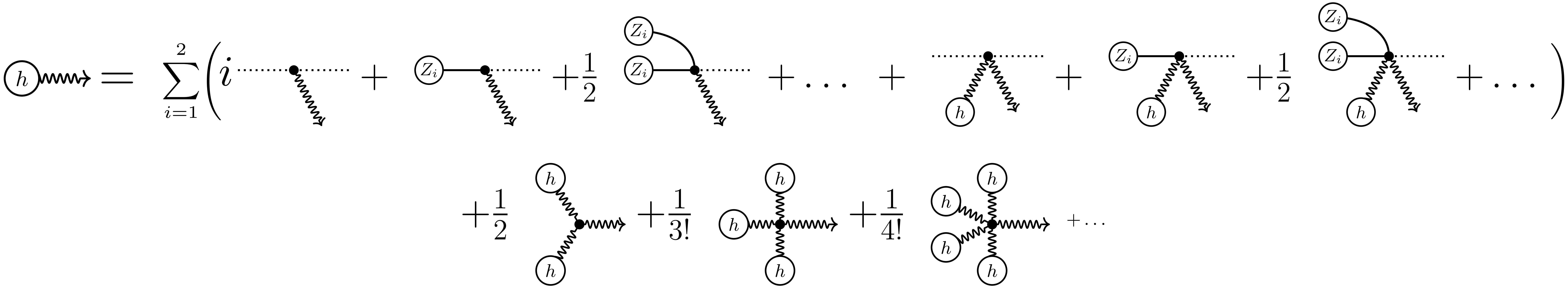
- One-point functions: $\langle Z_i(\omega)\rangle =$  $\langle h_{\mu\nu}(k)\rangle =$ 

- Recursion $\Leftrightarrow$ equations of motion:

Geodesic:



Einstein:



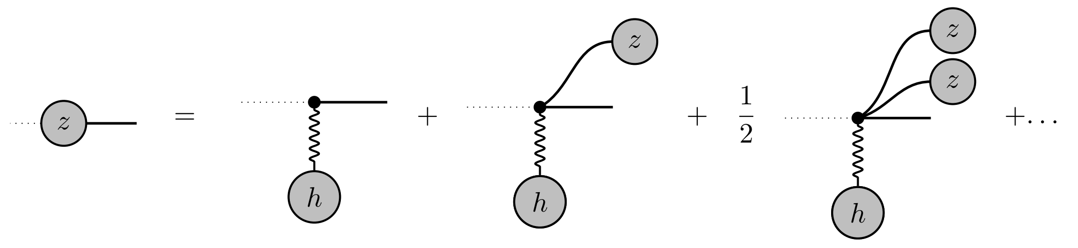
Causality flow implemented

WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



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Find same set of Master integrals for spin and tidal effects@ 3PM

Tensor reduction to scalar integrals

STRUCTURE OF WQFT INTEGRALS: IMPULSE & SPIN KICK

[Jakobsen,Mogull,JP,Sauer]

- Order **n-PM** : Single scale **(n-1)-loop** integral

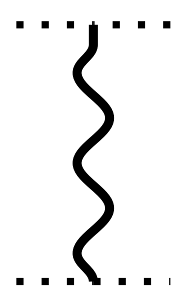
$$I_{\mathrm{nPM}} = \int_q e^{-q\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{l_1,l_2\ldots l_{n-1}} \frac{\mathrm{num}[l_i]}{D_1\ldots D_j}\delta(l_1\cdot v_*)\delta(l_2\cdot v_*)\ldots\delta(l_{n-1}\cdot v_*)$$

$$v_* \in \{v_1, v_2\}$$

Retarded propagators $D_i(l_i, q, v_*)$ are worldline $(l_i\cdot v_*)^{(1,2)}$ or quadratic $(l_i + q)^2$

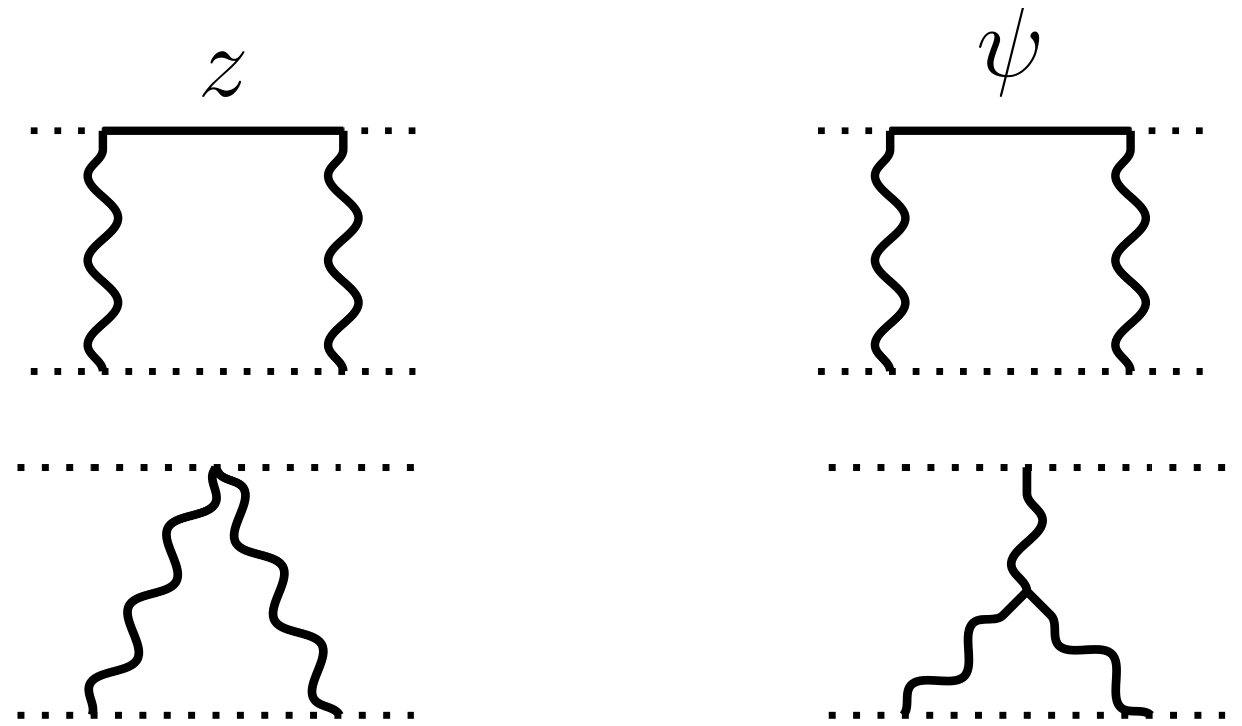
As $v_*^2 = 1$ scale q factors out, left with single parameter integral $\gamma = v_1 \cdot v_2$

- **1PM: Trivial - pure Fourier transform**



$$\sim G m_1 m_2$$

- **2PM: 1-loop**



$$\sim G^2 m_1 m_2^2$$

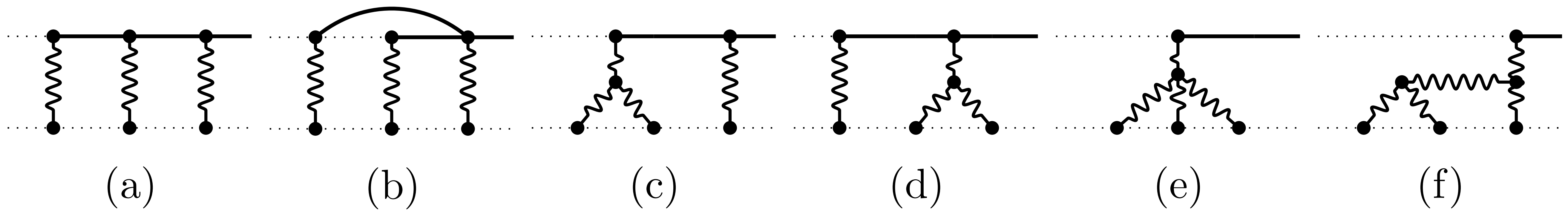
$$\begin{aligned} D_1 &= l\cdot v_1 \pm i\epsilon\ , \\ D_2 &= l^2\ , \\ D_3 &= (l+q)^2\ . \end{aligned}$$

$$\int_l \frac{\mathrm{num}[l]}{D_1 D_2 D_3}\delta(l\cdot v_*)$$

3PM

[Jakobsen,Mogull,JP,Sauer]

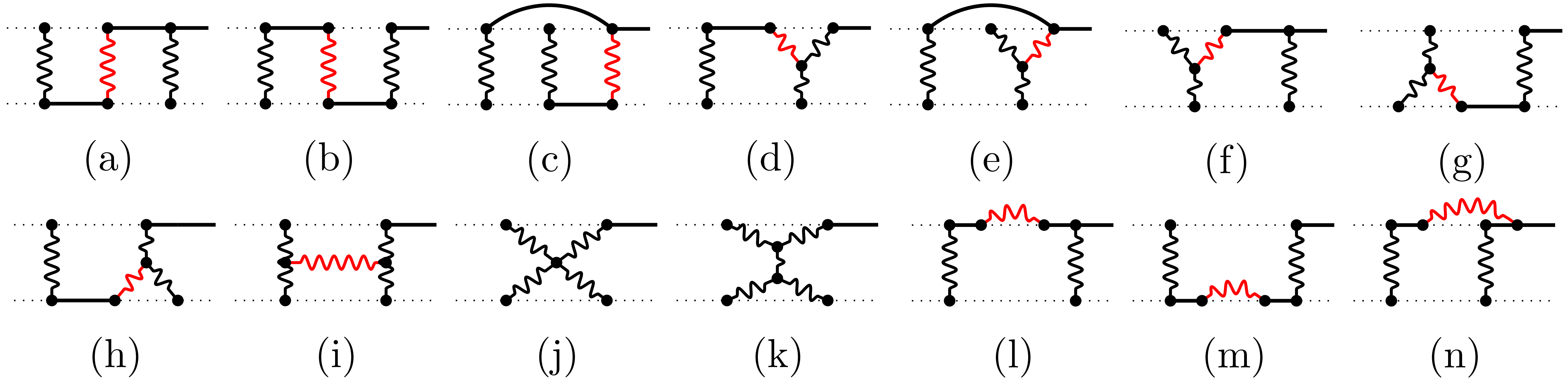
1) Test body diagrams (geodesic motion in Schwarzschild background):



$\sim G^3 m_1\, m_2^3$

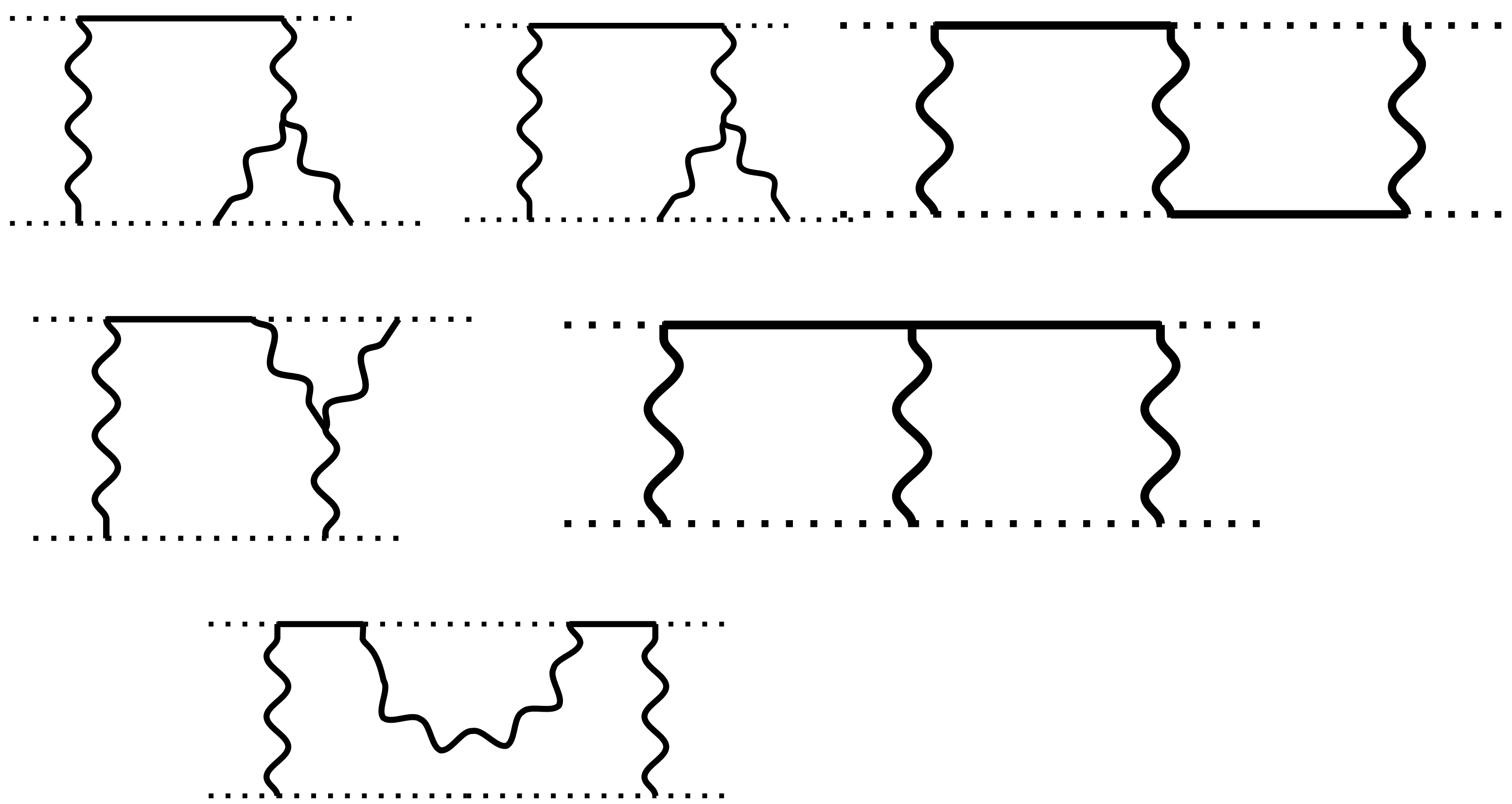
A 2-loop computation!

2) Comparable mass diagrams (i0 prescription relevant for red propagators):



$\sim G^3 m_1^2\, m_2^2$

6 top sectors integrals:



Integral family

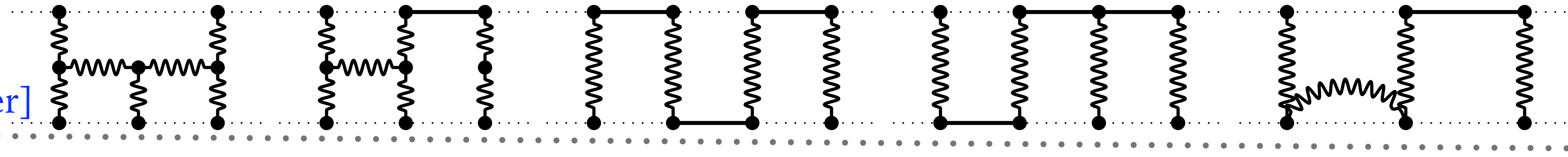
$$\boxed{\int_{l_1 l_2} \frac{\delta(l_1\cdot v_{\bar{i}_1})\delta(l_2\cdot v_{\bar{i}_2})}{\prod_{j=1}^{7} D_j^{n_j}}}$$

$$\boxed{D_3 = (k^0+\sigma_3 i0)^2 - \boldsymbol{k}^2}$$

$$D_1 = l_1\cdot v_{i_1} + \sigma_1 i0\ , \qquad D_2 = l_2\cdot v_{i_2} + \sigma_2 i0\ ,$$
$$D_4 = l_1^2\ , \qquad D_5 = l_2^2\ ,$$
$$D_6 = (l_1+q)^2\ , \qquad D_7 = (l_2+q)^2\ .$$

Tensor reduction to scalar integrals

4PM: [Jakobsen,Mogull,JP,Sauer]



- Two integral families:

$$I^{[i](\sigma_1,\sigma_2,\ldots,\sigma_8)}_{n_1,n_2,\ldots,n_{12}} = \int_{l_1,l_2,l_3} \frac{\delta(l_1\cdot v_1)\delta(l_2\cdot v_1)\delta(l_3\cdot v_i)}{D_1^{n_1}D_2^{n_2}\ldots D_{12}^{n_{12}}}$$

$$\begin{aligned}
D_j &= l_j\cdot v_{i_j} + i0^+\sigma_j\\
D_4 &= -(l_1+l_2+l_3+q)^2 - i0^+\sigma_4 v\cdot(l_1+l_2+l_3)\\
D_5 &= -(l_1+l_2+q)^2 - i0^+\sigma_5 v\cdot(l_1+l_2)\\
D_{5+k} &= -(l_k+l_3)^2 - i0^+\sigma_{6+k} v\cdot(l_k+l_3)\\
D_{7+j} &= -l_j^2\,, \qquad D_{10+k} = -(l_k+q)^2\,.
\end{aligned}$$

$$J^{(\sigma_1,\sigma_2,\ldots,\sigma_5)}_{n_1,n_2,\ldots,n_{12}} := \int_{\ell_1,\ell_2,\ell_3} \frac{\delta(\ell_1\cdot v_1)\delta(\ell_2\cdot v_1)\delta(\ell_3\cdot v_2)}{D_1^{n_1}D_2^{n_2}\ldots D_{12}^{n_{12}}}$$

$$\begin{aligned}
D_j &= \ell_j\cdot v_i + i0^+\sigma_j\\
D_4 &= -(\ell_1-\ell_3)^2 - i0^+\sigma_4 v\cdot(\ell_1-\ell_3)\\
D_5 &= -(\ell_2-\ell_3)^2 - i0^+\sigma_5 v\cdot(\ell_2-\ell_3)\\
D_6 &= -(\ell_1-\ell_2)^2\,, \quad D_{6+j} = -\ell_j^2\,, \quad D_{9+j} = -(\ell_j+q)^2
\end{aligned}$$

- Number of master integrals: I-type 23+23, J-type 64+66 (even & odd # worldline propagators)

- Function space:

$$F^{(b)}_\alpha(\gamma) = \Big\{1, \mathrm{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_\pm}{2}\right], \mathrm{arccosh}^2[\gamma], \mathrm{arccosh}[\gamma]\log\left[\frac{\gamma_\pm}{2}\right], \log\left[\frac{\gamma_+}{2}\right]\log\left[\frac{\gamma_-}{2}\right], \log^2\left[\frac{\gamma_+}{2}\right],$$
$$\mathrm{Li}_2\left[\pm\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \mathrm{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{K}\left[\frac{\gamma_-}{\gamma_+}\right]\mathrm{E}\left[\frac{\gamma_-}{\gamma_+}\right]\Big\}$$

Elliptic functions appear!

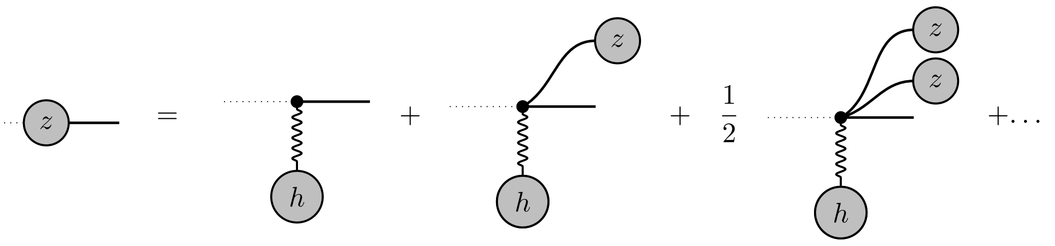
$$\gamma = v_1\cdot v_2 \qquad (\gamma_\pm := \gamma\pm 1)$$

WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



Insert Feynman rules

Form script
(4PM: 5g vertex)

Tensor reduction to scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \ldots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i0$

IBPs: Basis of Master Integral

Fire/Kira
Reduced symmetries
due to retarded prop.

$l \to -l \qquad i0 \to -i0$

Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions
(potential & radiation gravitons)

$i0$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

RESULTS SPIN-LESS IMPULSE @ 3PM PRECISION:

[Jakobsen,Mogull,JP,Sauer]

$$\boxed{\Delta p_1^\mu = p_\infty \sin\theta \frac{b^\mu}{|b|} + (\cos\theta - 1)\frac{m_1 m_2}{E^2}[(\gamma m_1 + m_2)v_1^\mu - (\gamma m_2 + m_1)v_2^\mu] - v_2 \cdot P_{\text{rad}}\, w_2^\mu}$$

$$w_1^\mu = \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2 - 1}$$

$$\gamma = v_1 \cdot v_2$$

- Scattering angle:

$$\frac{\theta}{\Gamma} = \underbrace{\frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1}}_{1\text{PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^2 \frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)}}_{2\text{PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^3 \left(2\frac{64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4 - 12\gamma^2 - 3)}{(\gamma^2-1)}\frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)}_{3\text{PM conservative}}$$

$$+ \underbrace{\left(\frac{GM}{|b|}\right)^3 \frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3} + \frac{1}{v^2} + \frac{(3v^2-1)}{v^3}\text{arccosh}\gamma\right)}_{3\text{PM radiation-reaction}}$$

$$\Gamma = E/M = \sqrt{1 + 2\nu(\gamma - 1)}$$

$$\nu = \frac{m_1 m_2}{M^2}$$

- Radiated 4-momentum: $P_{\text{rad}}^\mu = -\Delta p_1^\mu - \Delta p_2^\mu$ Dissipation! Need for retarded propagators

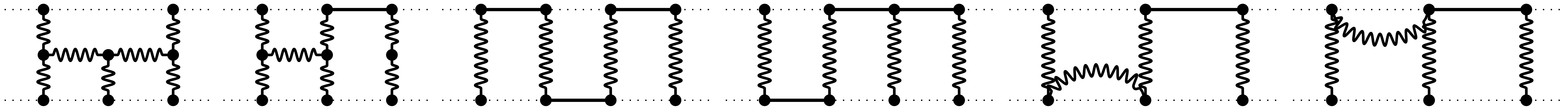
$$P_{\text{rad}}^\mu = \frac{G^3 m_1^2 m_2^2 \pi}{|b|^3}\frac{v_1^\mu + v_2^\mu}{\gamma + 1}\left[e_1 + e_2 \log\left(\frac{\gamma+1}{2}\right) + e_3 \frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right]$$

$$e_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48(\gamma^2-1)^{3/2}}$$

$$e_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}},$$

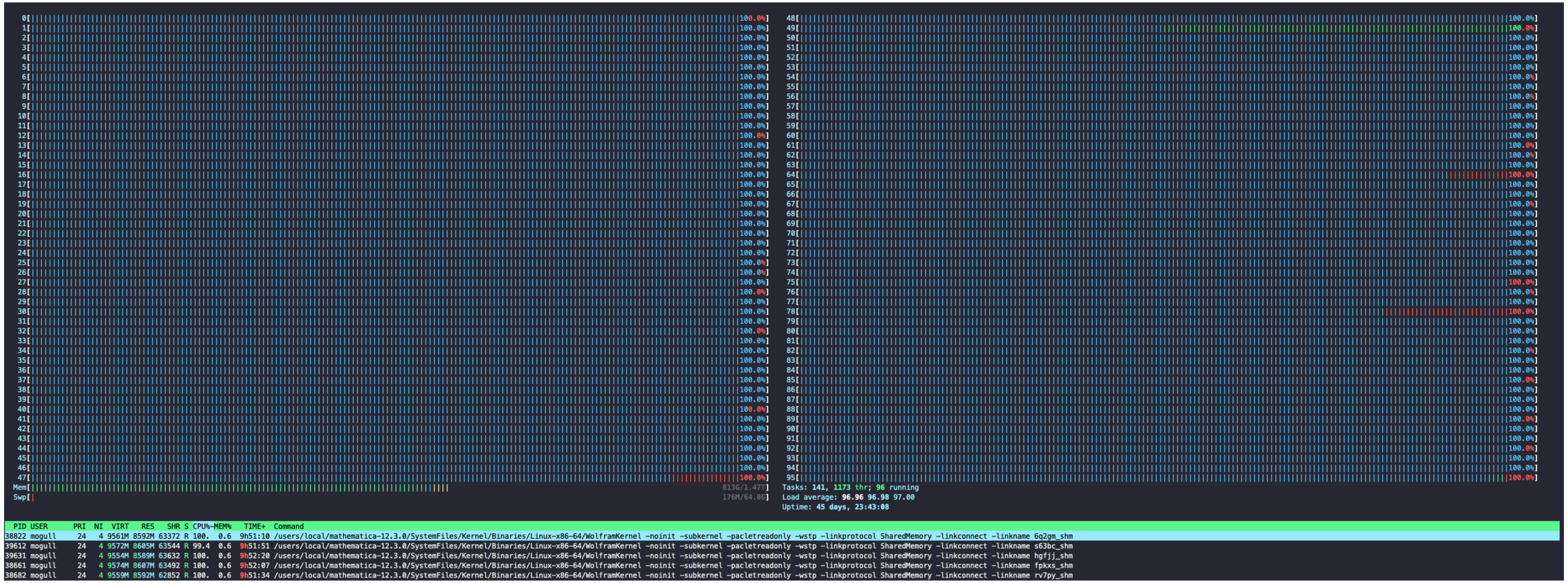
$$e_3 = \frac{\gamma(2\gamma^2-3)(35\gamma^4 - 30\gamma^2 + 11)}{16(\gamma^2-1)^{3/2}}.$$

STATS FOR 4PM:



[Jakobsen,Mogull,JP,Sauer]

- High Performance Computing Needed:



Pepsi: (with Uwer)
Xeon Gold 48 physical Cores with 1.5 TByte Main Memory

Deepthought:
Xeon Gold 18 physical Cores with 0.5 TByte Main Memory

- Number of diagrams: 201 Non-spinning, 529 spin-orbit
- Some 10.000 integrals need to be reduced to the master integrals (I-type 23+23, J-type 64+66)
- Analytic! Using Mathematica, FORM, Fire & KIRA

RESULT SPINNING IMPULSE @ 4PM PRECISION: CONSERVATIVE SECTOR

[Jakobsen,Mogull,JP,Sauer]

- Scattering angle:

$$\boxed{\theta^{(4,1)}_{\rm cons} = \sum_{\alpha=1}^{16} \pi\nu\left(s_+ h^{(+)}_\alpha(\gamma) + \delta\, s_- h^{(-)}_\alpha(\gamma)\right) F^{(b)}_\alpha(\gamma) - \frac{21\pi\gamma\left(33\gamma^4-30\gamma^2+5\right)\left(13s_+ - 3\delta s_-\right)}{32\left(\gamma^2-1\right)^{5/2}}}$$

$$\gamma = v_1\cdot v_2$$

$$\delta = (m_2 - m_1)/M$$

$$\nu = m_1 m_2/M^2$$

Spin-orbit coupling: $s_\pm = -(a_1 \pm a_2)\cdot\hat{L}$

Function basis:

$$F^{(b)}_\alpha(\gamma) = \Big\{1, \text{arccosh}[\gamma], \log[\gamma], \log\left[\frac{\gamma_+}{2}\right], \log\left[\frac{\gamma_-}{2}\right], \text{arccosh}^2[\gamma], \text{arccosh}[\gamma]\log\left[\frac{\gamma_+}{2}\right], \text{arccosh}[\gamma]\log\left[\frac{\gamma_-}{2}\right],$$
$$\log\left[\frac{\gamma_+}{2}\right]\log\left[\frac{\gamma_-}{2}\right], \log^2\left[\frac{\gamma_+}{2}\right], \text{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right], \text{Li}_2\left[-\frac{\gamma_-}{\gamma_+}\right], \text{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right], \text{K}^2\left[\frac{\gamma_-}{\gamma_+}\right], \text{E}^2\left[\frac{\gamma_-}{\gamma_+}\right], \text{K}\left[\frac{\gamma_-}{\gamma_+}\right]\text{E}\left[\frac{\gamma_-}{\gamma_+}\right]$$

Coefficients:

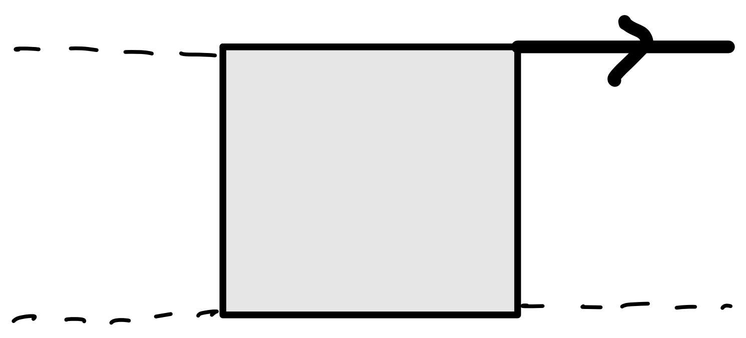
$$h^{(+)}_1 = \frac{3\pi^2(\gamma+1)^2(1225\gamma^8+1225\gamma^7-1875\gamma^6-1875\gamma^5+795\gamma^4+3035\gamma^3-3601\gamma^2+1775\gamma-448)}{192(\gamma+1)^3\sqrt{\gamma^2-1}} - \frac{1}{192\gamma^8(\gamma+1)(\gamma^2-1)^{5/2}}\Big(22050\gamma^{19}$$
$$+33075\gamma^{18}-71725\gamma^{17}-123397\gamma^{16}+186555\gamma^{15}+67503\gamma^{14}-89885\gamma^{13}-190167\gamma^{12}+181103\gamma^{11}+137042\gamma^{10}-506830\gamma^9$$
$$+407004\gamma^8-33671\gamma^7-33671\gamma^6+8501\gamma^5+8501\gamma^4-1885\gamma^3-1885\gamma^2+315\gamma+315\Big)$$

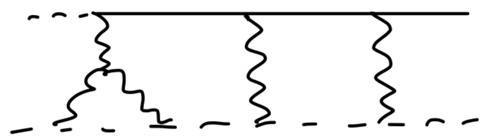
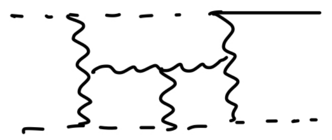
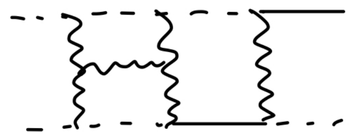
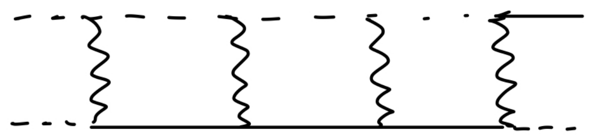
$$h^{(-)}_1 = \frac{3\pi^2(-1225\gamma^8-1225\gamma^7+1875\gamma^6+1875\gamma^5-795\gamma^4-1115\gamma^3+401\gamma^2-111\gamma+64)(\gamma+1)^2}{192(\gamma+1)^3\sqrt{\gamma^2-1}} + \frac{1}{192\gamma^8(\gamma+1)(\gamma^2-1)^{5/2}}\Big(22050\gamma^{19}$$
$$+33075\gamma^{18}-71725\gamma^{17}-115333\gamma^{16}+96699\gamma^{15}+140871\gamma^{14}-56261\gamma^{13}-73191\gamma^{12}-6593\gamma^{11}+27498\gamma^{10}-3718\gamma^9$$
$$+9004\gamma^8-1491\gamma^7-1491\gamma^6+313\gamma^5+313\gamma^4+95\gamma^3+95\gamma^2-105\gamma-105\Big) \quad \ldots.$$

4PM RADIATIVE SPINNING IMPULSE: CHECKS

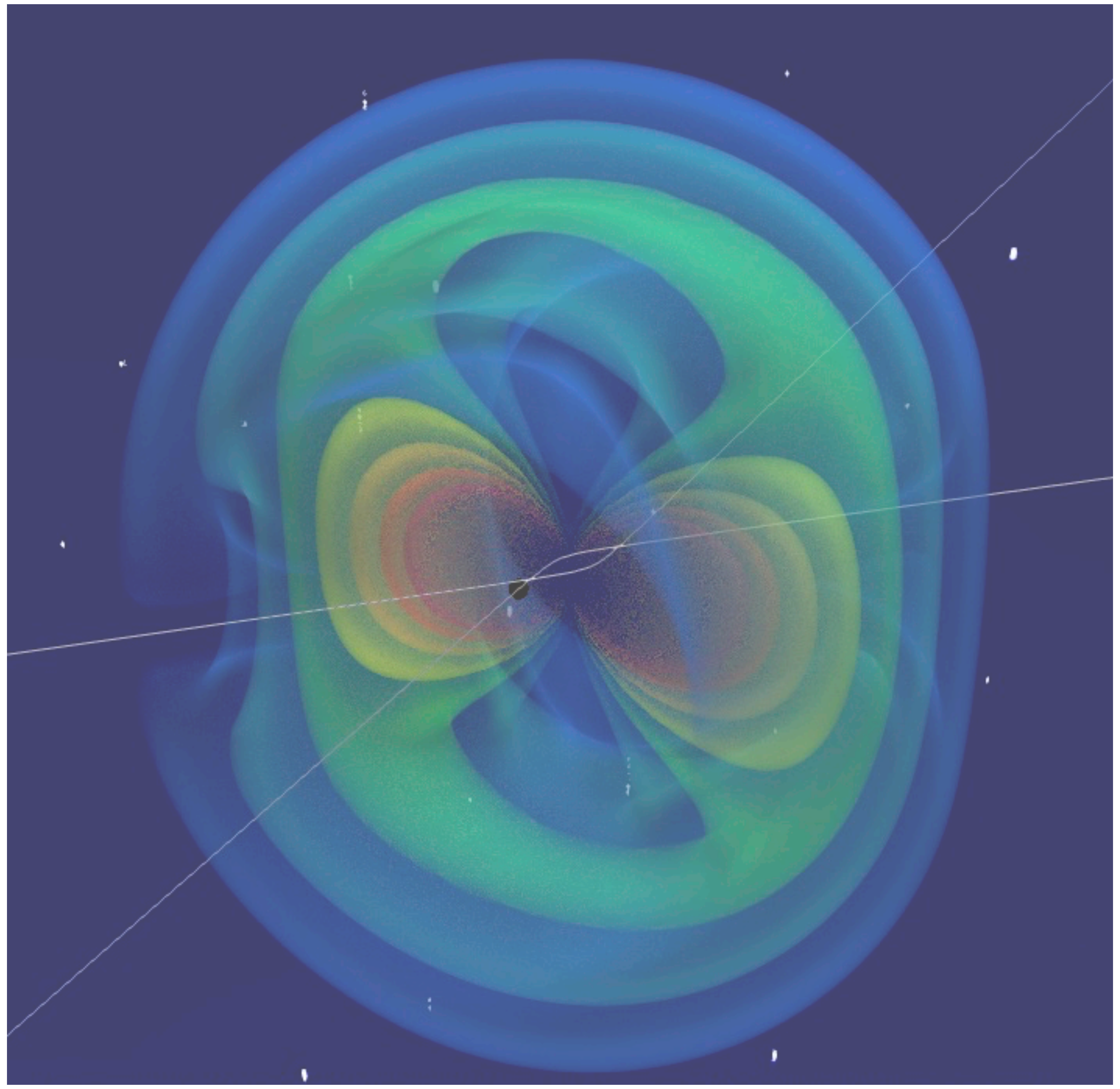
[Jakobsen,Mogull,JP,Sauer]

- Established radiation reacted impulse, spin kick, scattering angle (unpublished)

$\langle \Delta p^\mu \rangle =$ 

$\langle \Delta p \rangle_{4PM}$	 $+ \cdots$ $m_1 m_2^4$	 $+ \cdots$ $m_1^2 m_2^3$	 $+ \cdots$ $m_1^3 m_2^2$	 $+ \cdots$ $m_1^4 m_2$
w/o	Schwarzschild Geodesic	confirms PM result [Dlapa,Källin,Liu,Neef,Porto]	confirms PM result	Schwarzschild Geodesic
S_1	Probe mass limit (spinning particle in Kerr Background) [Damgaard,Hoogeveen, Luna,Vines]	agrees in PN limit	agrees in PN limit	Probe mass limit (spinning particle in Kerr Background) [Damgaard,Hoogeveen, Luna,Vines]
S_2	Probe mass limit (spinning particle in Kerr Background)	agrees in PN limit [Mandal,Mastroia,Patil,Steinhoff]	agrees in PN limit	Probe mass limit (spinning particle in Kerr Background)

GRAVITATIONAL BREMSSTRAHLUNG



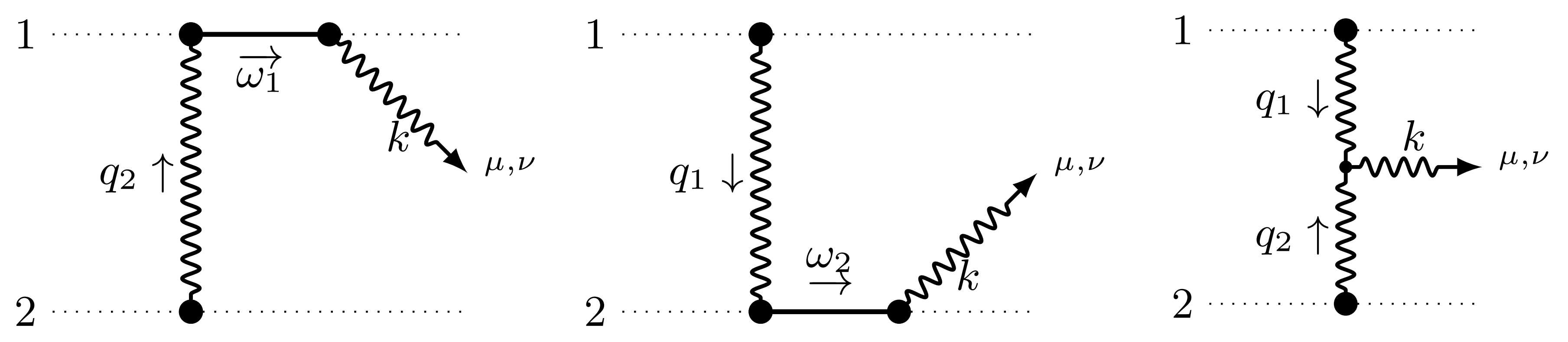
[Jakobsen,Mogull,JP,Sauer]

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum of diagrams with outgoing graviton:

$$\langle h_{\mu\nu}(k)\rangle =$$



For time-domain waveform needs to integrate over outgoing energy : Ω

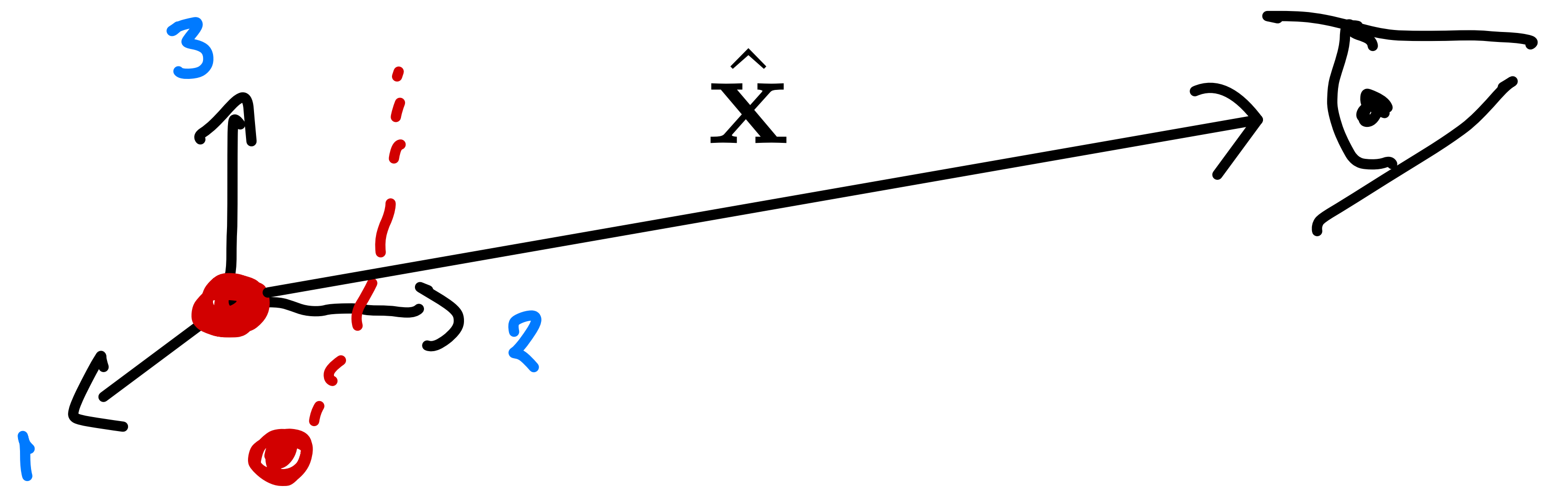
$$\frac{f_{+,\times}(t-r,\hat{\mathbf{x}})}{r} = \frac{4G}{r}\int d\Omega\, e^{-i\Omega(t-r)}\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega(1,\hat{\mathbf{x}}))\rangle$$

where unit vector $\hat{\mathbf{x}}$ points towards the observer

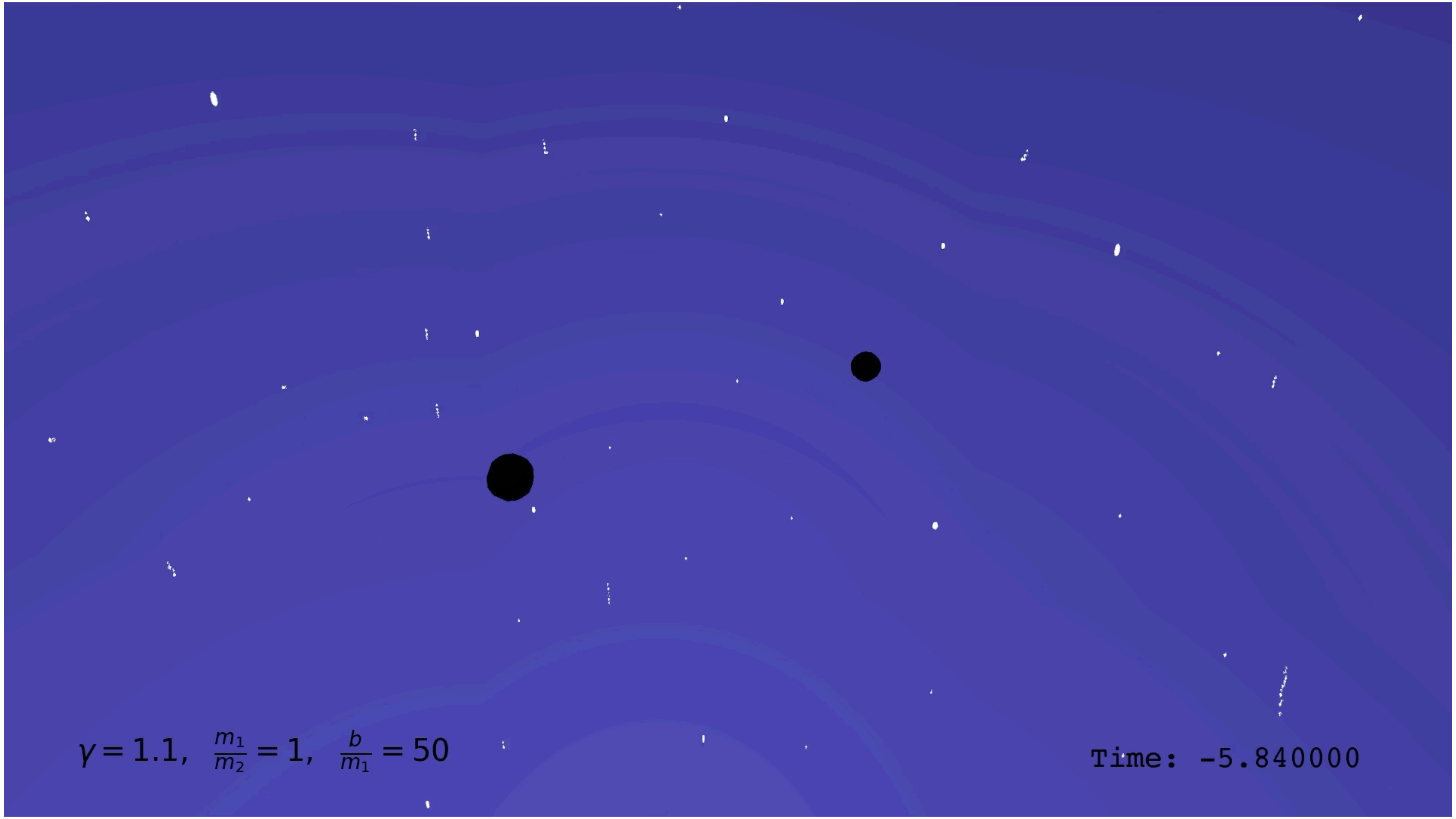
The waveform has two polarizations

$$f_{+,\times}(\underbrace{t-r}_{u},\underbrace{\theta,\phi}_{\hat{\mathbf{x}}};v,|b|,m_1,m_2)$$

retarded time



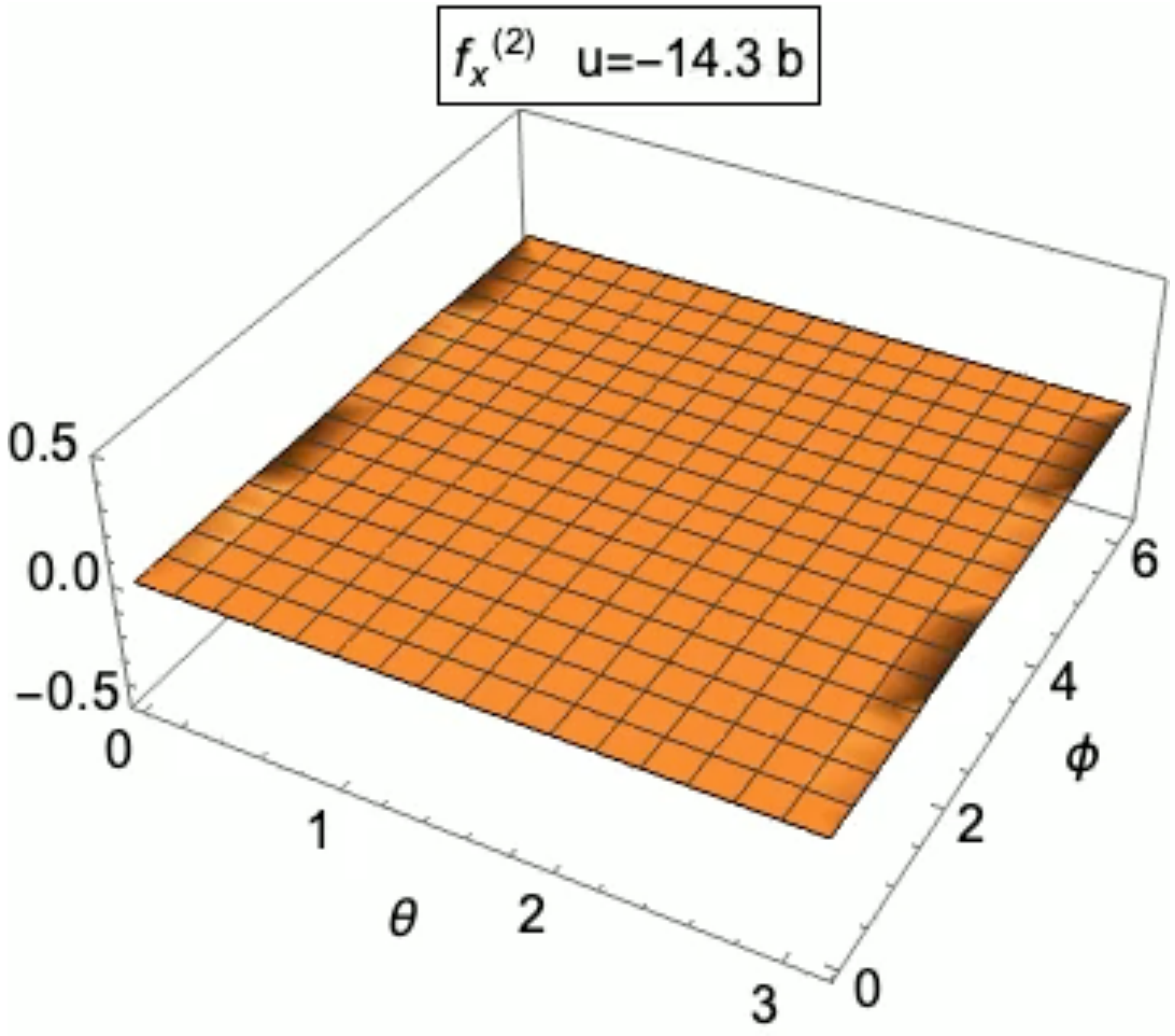
Visualization: Plus-Polarization $f_{+}^{(2)}$

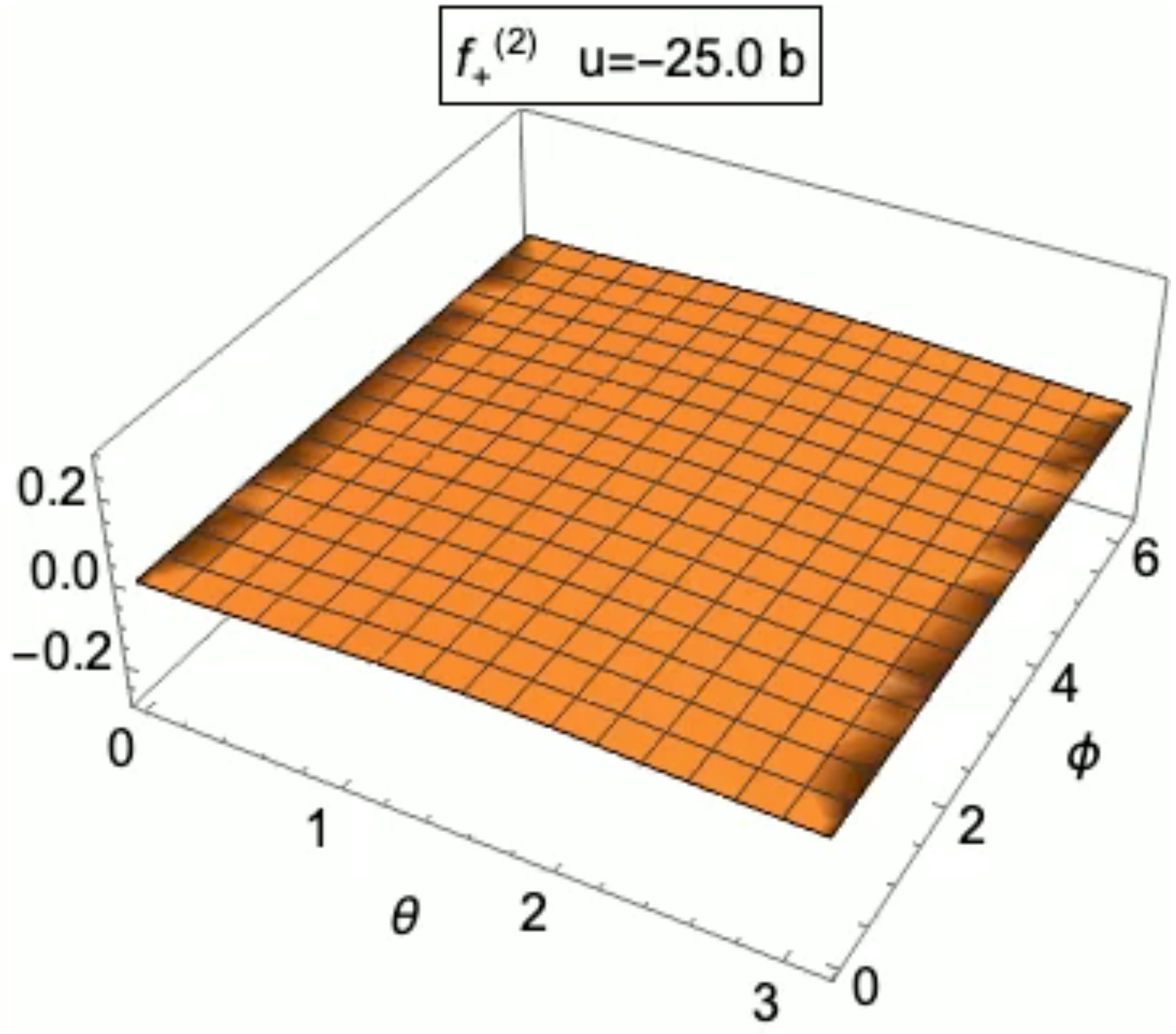


$v = 0.4$

[HU/AEI BSc thesis Babayemi]

Memory effect



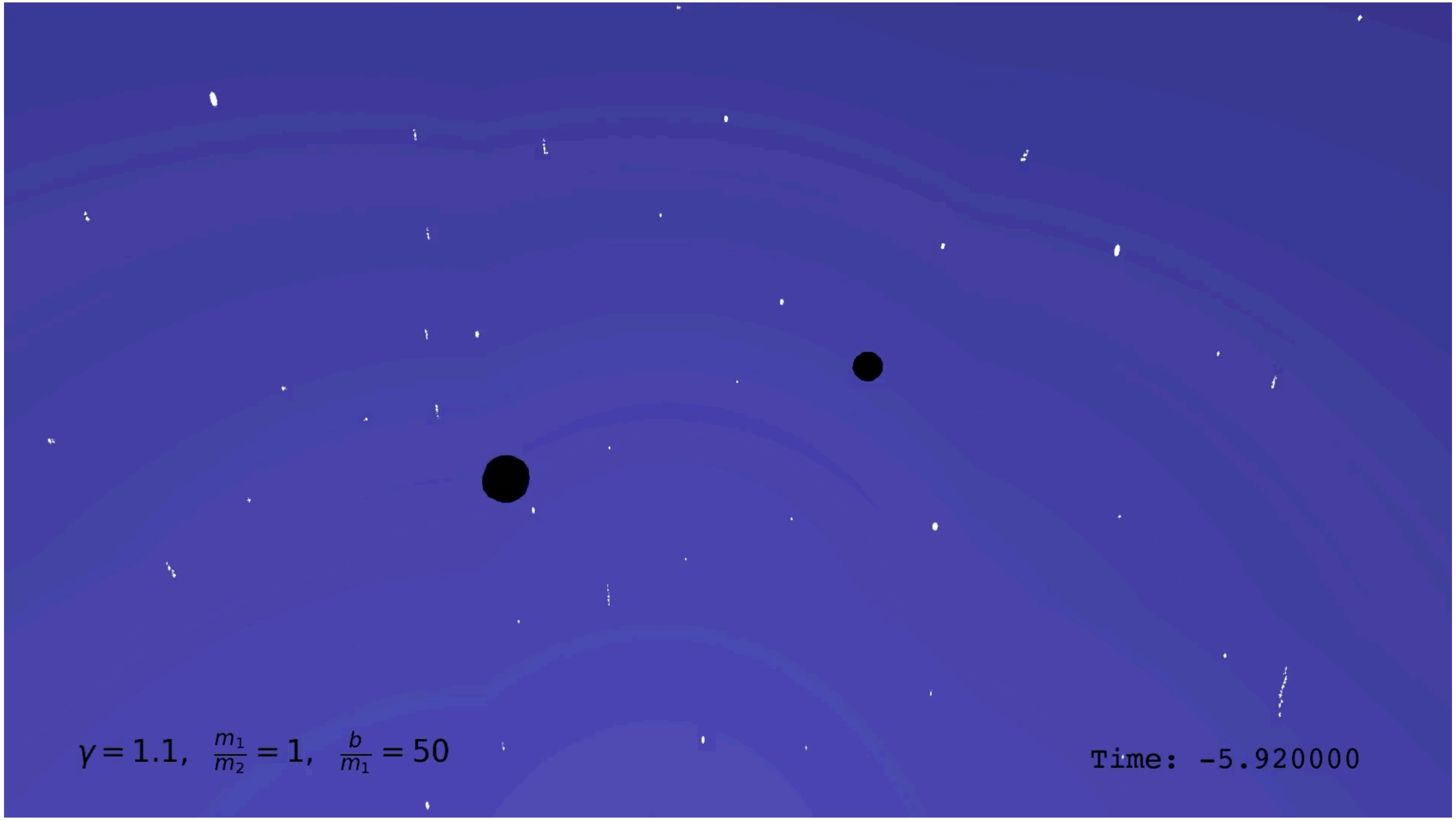


$$\frac{\Delta f^{(2)}_{\mathcal{S}=0}}{m_1 m_2} = \frac{4(2\gamma^2-1)\epsilon\cdot v_1(2b\cdot\epsilon\,\rho\cdot v_1 - b\cdot\rho\,\epsilon\cdot v_1)}{|b|^2\sqrt{\gamma^2-1}(\rho\cdot v_1)^2}$$

$$\rho = (1, \hat{\mathbf{x}})$$

$$\gamma = v_1\cdot v_2$$

Visualization: Plus-Polarization curvature



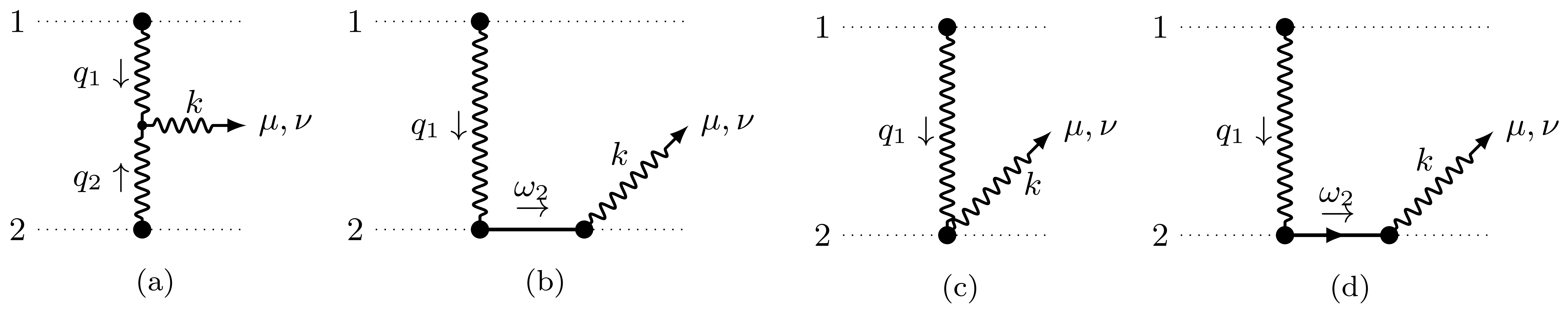
$v = 0.4$

More visualisations at: https://www.youtube.com/channel/UC5UVcydoMoG7ILkJo9bik1w

SPINNING WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

- Update Kovacs-Thorne with spin.



$$\frac{f^{(2)}}{m_1 m_2} = \sum_{s=0}^{2} \frac{1}{|\tilde{\mathbf{b}}|_1^{2s+1}} \left[\alpha_1^{(s)} + \frac{\beta_1^{(s)}}{|\tilde{b}|^{2s+2}}\right] + (1 \leftrightarrow 2)$$

- The spinning wave memory:

$$\Delta f^{2)} = f^{(2)}(u = +\infty) - f^{(2)}(u = -\infty)$$

$$\Delta f^{(2)} = \left(1 + \frac{2v|a_3|}{b(1+v^2)} + \frac{|a_3|^2}{|b|^2} - \sum_{i=1}^{2} \frac{C_{E,i}|a_i|^2}{|b|^2}\right) \Delta f^{(2)}_{\mathcal{S}=0}$$

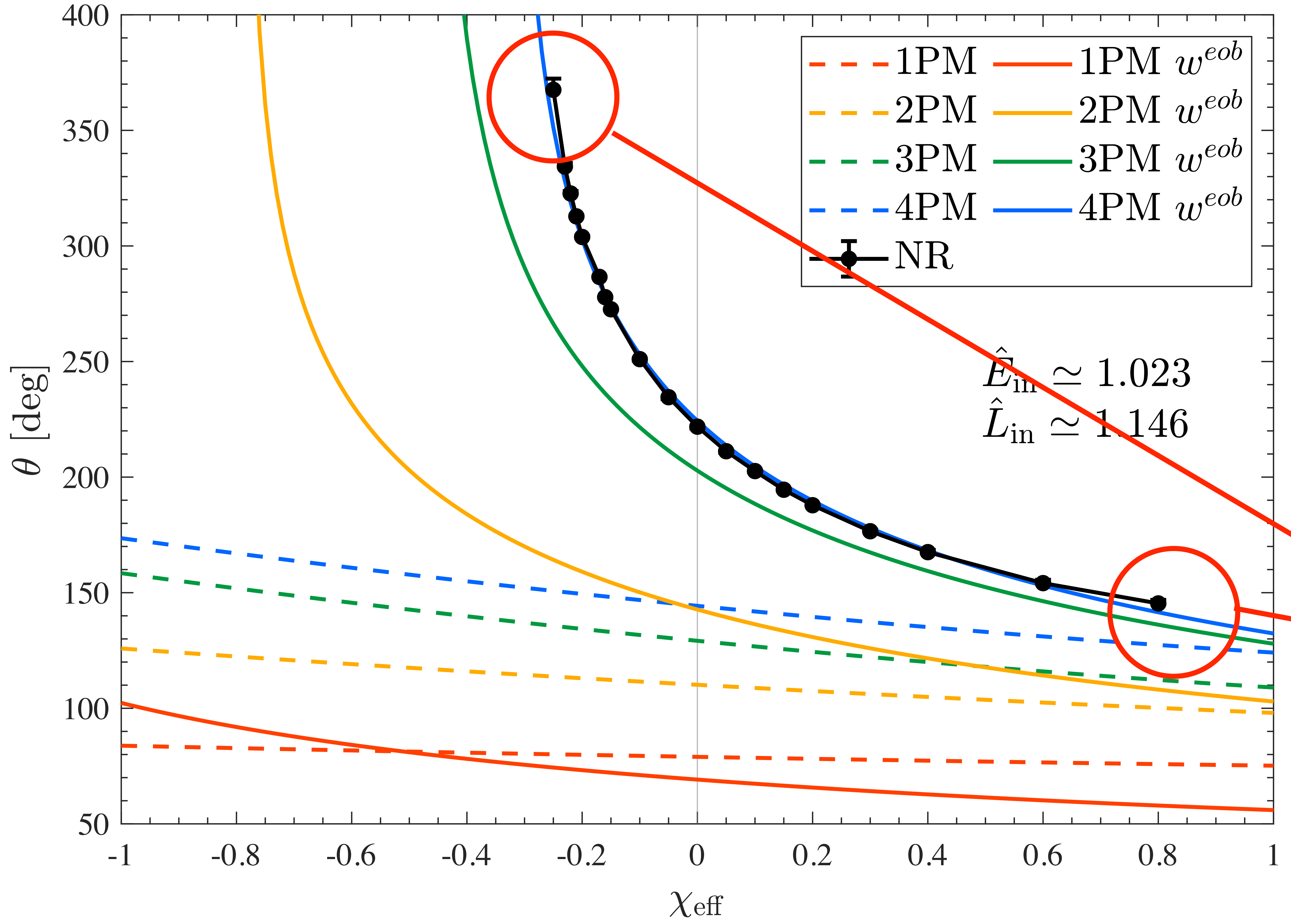
Using Pauli-Lubanski vector: $\mathcal{S}_i^{\mu\nu} = \epsilon^{\mu\nu}{}_{\rho\sigma} v_i^\rho a_i^\sigma \qquad a_3^\mu = a_1^\mu + a_2^\mu$

- Radiated angular momentum in COM:

$$\frac{J^{\rm rad}_{xy} + iJ^{\rm rad}_{zx}}{J^{\rm init}_{xy}\big|_{\mathcal{S}=0}} = \frac{4G^2 m_1 m_2}{|b|^2} \frac{(2\gamma^2-1)}{\sqrt{\gamma^2-1}} \mathcal{I}(v)$$
$$\times \left(1 - \frac{2iv\,\mathbf{a}_3 \cdot \mathbf{l}}{|b|(1+v^2)} - \frac{(\mathbf{a}_3 \cdot \mathbf{l})^2}{|b|^2} + \sum_{i=1}^{2} \frac{C_{E,i}}{|b|^2} (\mathbf{a}_i \cdot \mathbf{l})^2\right)$$

COMPARISON TO NUMERICAL RELATIVITY AND RESUMMATION TECHNIQUES

- Spinning BH scattering angle: **Equal** mass & **equal** spin [Rettengo,Pratten,Thomas,Schmidt,Damour]



Analytic 4PM input includes:

4PM: no-spin [Dlapa,Källin,Liu,Neef,Porto] [Daamgaard,Vanhove,Hansen,Plante]

3PM: S^1, S^2 [Jakobsen,Mogull]

2PM: S^1, S^2, S^3, S^4 [Cordero,Ruf,Kraus,Lin,Zeng] [Bern,Kosmopoulos,Luna,Roiban,Teng] [Aoude,Haddad,Helset]

Missing spin our fresh 4PM spin contributions!

Interesting to see an update!

PM STATE-OF-THE-ART

WQFT [Jakobsen,Mogull,Plefka,Sauer,Steinhoff] [Bastianelli,Comberiati,de la Cruz] [Comberiati,Shi] [Wang]

WEFT **Worldline effective theory** [Källin,Porto,Dlapa,Cho,Liu,..] [Riva,Vernizzi,Mougiakakos..]

HEFT **Heavy BH effective theory** [Aoude,Haddad,Helset,Damgaard] [Brandhuber,Travaglini,Chen]

Amps **Scattering amplitudes** [Bern,Roiban,Shen,Parra-Martinez,Ruf,Zeng..] [Bjerrum-Bohr,Damgaard,Plante,Vanhove,..] [Di Vecchia,Veneziano,Heissenberg,Russo] [Solon,Cheung,..] [Huang,..] [Guevera,Ochirov,Vines,…] [Johansson,Pichini [Kosower,O'Connell,Maybee, Cristofoli, Gonzo…]

order	deflection & spin kick					waveform			
	plain	spin-orbit	spin-spin	spin>2	tidal	plain	spin-orbit spin-spin	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	WQFT WEFT Amps	WQFT WEFT Amps HEFT	WQFT WEFT	WQFT WEFT	~ 1-loop
3PM w/o r-r	WQFT WEFT Amps HEFT	WQFT Amps	WQFT (Amps)		WQFT WEFT	Amps HEFT			~ 2-loop
3PM r-r	WQFT WEFT Amps HEFT	WQFT	WQFT		WQFT WEFT				
4PM w/o r-r	WQFT WEFT Amps	WQFT							~ 3-loop
4PM w r-r	WQFT WEFT Amps	WQFT							

r-r: Radiation-reaction (…) : partial results

SUMMARY

Worldline Quantum Field Theory: Highly efficient technology for classical scattering in GR

- „Quantize“ world-line degrees of freedom
- One-point functions = observables
- Classical theory = tree-level diagrams
- IN-IN Formalism: All propagators retarded.
- Include spin degrees of freedom through world-line supersymmetry

OUTLOOK

WQFT still needs to be extended:

- Higher precision (4PM Spin-Spin, 5PM)
- Higher spin (beyond Spin-Spin)
- Bound orbits? Resummation in Effective-one-body Formalism
- Contact to self force expansion?

WE ARE HIRING!

- Fall 23: Long Term Postdoc (5y), ~~2 PhD~~
- Fall 24: Postdoc (4y), 1 PhD

European Research Council
Established by the European Commission

Thank you for your attention!

BACKUP

POST-NEWTONIAN VS POST-MINKOWSKIAN EXPANSIONS

Conservative non-spinning 2-body dynamics:

		0PN	1PN	2PN	3PN	4PN	5PN		Integration complexity
0PM [Einstein]	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	…	
1PM [Westpfahl]		G/r [Newton]	$G\, v^2/r$ [EIH]	$G\, v^4/r$	$G\, v^6/r$	$G\, v^8/r$	$G\, v^{10}/r$	…	~ tree-level
2PM [many]			$G^2\, 1/r^2$	$G^2\, v^2/r^2$	$G^2\, v^4/r^2$	$G^2\, v^6/r^2$	$G^2\, v^8/r^2$	…	~ 1-loop
3PM				$G^3\, 1/r^3$	$G^3\, v^2/r^3$	$G^3\, v^6/r^3$	$G^3\, v^8/r^3$	…	~ 2-loop

[Bern,Cheung,Roiban,Shen, Solon,Zeng][Kälin, Liu, Porto][Di Vecchia, Heissenberg, Russo,Veneziano][Bjerrum-Bohr,Vanhove,Damgaard][Brandhuber,Chen,Travaglini,Wen][Jakobsen,Mogull,JP,Sauer]

(4PM)	PM state-of-the-art		→	$G^4\, 1/r^4$	$G^4\, v^2/r^4$	$G^4\, v^6/r^4$		…	~ 3-loop

[Bern,Parra-Martinez,Roiban,Ruf,Shen,Solon,Zeng][Dlapa,Källin,Liu,Porto]

....						$G^5\, 1/r^5$	↑ :	

PN state-of-the-art