Supersymmetric Galilean Electrodynamics

Silvia Penati, University of Milano-Bicocca and INFN

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An example of non-relativistic susyQFT

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S. Baiguera, L. Cederle, SP, JHEP 09 (2022) 237 [arXiv:2207.06435]

Motivations

- In models describing CM systems SUSY has been observed to be an emergent symmetry, that is it appears in the effective theory describing the low-energy modes. On the other hand, at these scales the system is typically in a non-relativistic regime.

  Therefore, it is physically relevant to construct NR SUSY models

- Non-relativistic holography: Non-relativistic generalisation of the AdS/CFT is of interest for the holographic description of CM systems.

D.T. Son, PRD78 (2008)
K. Balasubramanian, J. McGreevy, PRL101 (2008)
W.D. Goldberger, JHEP03 (2009)
S. Kachru, X. Liu, M. Mulligan, PRD78 (2008)
S. Janiszewski, A. Karch, JHEP02 (2013)
M. H. Christensen, J. Hartong, N. A. Obers and B. Rollier, PRD89, JHEP01 (2014)
M. Taylor, CQG33 (2016)
A. Bagchi, R. Gopakumar, JHEP07 (2009)
A. Bagchi, R. Basu, D. Grumiller, M. Riegler, PRL114 (2015)

Which is the role of supersymmetry in NR holography?

- NR limits allow to improve our understanding of the relativistic theory:

  - NR corners of $\mathcal{N}=4$ SYM to go beyond the planar limit in AdS/CFT

    T. Harmark, N. Wintergerst, PRL124 (2020)
    S. Baiguera, T. Harmark, N. Wintergerst, JHEP02 (2021)

  - NR corners of M-theory: Null M5-branes on S^1 described by (4+1)d non-Lorentzian SUSY Lagrangians

    N. Lambert, A. Lipstein, P. Richmond, JHEP10 (2018)
    N. Lambert, A. Lipstein, R. Mouland, P. Richmond, JHEP01 (2020); JHEP03 (2021)

  - Supermembrane in M-theory might allow for a UV completion in terms of a non-relativistic membrane

    P. Horava, JHEP03 (2009); Z. Yan, JHEP03 (2023)

- NR SYM theories should describe the low energy degrees of freedom of D-branes in NR open string theory

  J. Gomis, Z. Yan, M. Yu, JHEP03 (2021); G. Oling, Z. Yan, Front. in Phys. 10 (2022)

We are going to study (2+1)D gauge field theories with $\mathcal{N} = 2$ Super-Bargman symmetry

From a QFT point of view important open questions are:

- Which are the renormalization properties of NR SUSY theories?
- Does SUSY conspire with the NR space-time symmetry to mild UV divergences?
- Do non-renormalization theorems still work ?

Plan of the talk

1. Construction of the non-relativistic (galilean) $\mathcal{N} = 2$ Superspace
2. Non-relativistic selection rules
3. Supersymmetric Galilean Electrodynamics (SGED): A renormalizable non-linear sigma model
4. Summary and future directions

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Non-relativistic superalgebras

Bargmann algebra $(H, \vec{P}, \vec{J}, \vec{G}, M)$ $\quad x' = Rx + vt + a, \quad t' = t + b$
$M = U(1)$ central extension

There are different ways to obtain the Super-Bargmann algebra in (d+1)D

- Completing the Bargmann algebra with a set of fermionic generators and impose constraints on the algebra
- Taking the Inönü-Wigner contraction of the (d+1)D super-Poincaré $\otimes\, U(1)$ algebra in the $c \to \infty$ limit
- By dimensionally reducing the ((d+1)+1)D relativistic SUSY algebra along a **null direction**

To construct a NR Superspace the most convenient approach is **null reduction**

Null reduction

- We start from the (3+1)D super-Poincarè algebra realized on the spacetime

$$(x^+, x^-, x^{i=1,2}) \qquad\qquad x^\pm = \frac{x^3 \pm x^0}{\sqrt{2}} \qquad \text{light} - \text{cone coords.}$$

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- compactify x^- on a circle of radius R and rescale $x^+ \to x^+/R,\, x^- \to Rx^-$

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$\Rightarrow P_+ \equiv H,\ P_{i=1,2},\ M_{12} \equiv J,\ M_{i-} \equiv G_i$ bosonic subalgebra

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- Define $P_- \equiv M$ and select only the generators which commute with M
$\Rightarrow P_+ \equiv H,\ P_{i=1,2},\ M_{12} \equiv J,\ M_{i-} \equiv G_i$ bosonic subalgebra
- In the fermionic sector, write the (3+1)D anticommutator $\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\beta}\} = i\sigma^\mu_{\alpha\dot\beta}\partial_\mu$ in terms of the light-cone coordinates

$$\{\mathcal{Q}, \bar{\mathcal{Q}}\} = i \begin{pmatrix} \sqrt{2}\partial_+ & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\sqrt{2}\partial_- \end{pmatrix}$$

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- Identify $\partial_+ \to \partial_t\ ,\ \partial_- \to im \qquad \mathcal{Q}_\alpha \to Q_\alpha, \quad \bar{\mathcal{Q}}_{\dot\alpha} \to Q^\dagger_\alpha \equiv \bar{Q}_\alpha$

Super-Bargmann & Super-Schroedinger algebras

$\mathcal{N}=2$ Super-Bargmann algebra

$$[P_j, G_k] = i\delta_{jk} M\,, \qquad [H, G_j] = iP_j\,,$$
$$[P_j, J] = -i\epsilon_{jk} P_k\,, \qquad [G_j, J] = -i\epsilon_{jk} G_k\,, \qquad j,k = 1,2$$

$$[Q_1, J] = \tfrac{1}{2} Q_1\,, \qquad \{Q_1, Q_1^\dagger\} = \sqrt{2} H\,,$$
$$[Q_2, J] = -\tfrac{1}{2} Q_2\,, \quad [Q_2, G_1 - iG_2] = -iQ_1\,, \quad \{Q_2, Q_2^\dagger\} = \sqrt{2} M\,,$$
$$\{Q_1, Q_2^\dagger\} = -(P_1 - iP_2)\,, \quad \{Q_2, Q_1^\dagger\} = -(P_1 + iP_2)$$

$\mathcal{N}=2$ Super-Schroedinger obtained by null reduction of the relativistic superconformal algebra $SU(2,2|1)$.

Additional generators: D = dilatations
K = special conformal transformations
$S, S^\dagger$ = superconformal transformations

Non-relativistic Superspace

$(3+1)\ \mathcal{N}=1$ relativistic superspace $\Longrightarrow$ $(2+1)$ $\mathcal{N}=2$ NR superspace

$$(x^+, x^-, x^i, \theta^\alpha, \bar\theta^{\dot\alpha}) \Longrightarrow (t, x^i, \theta^1, \theta^2, \bar\theta^1, \bar\theta^2)$$

$$[t]=-2,\ [x^i]=-1,\ [\theta^1]=-1,\ [\theta^2]=0$$

⋆ Reduction of a generic superfield

$$\Phi(x^M, \theta^\alpha, \bar\theta^{\dot\alpha}) = e^{imx^-}\,\tilde\Phi(t, x^i, \theta^1, \theta^2, \bar\theta^1, \bar\theta^2) \qquad m \to M - \text{eigenvalue}$$

⋆ Covariant derivatives

$$\begin{cases} \mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{i}{2}\bar\theta^{\dot\beta}\partial_{\alpha\dot\beta} \\ \bar{\mathcal{D}}_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \frac{i}{2}\theta^{\beta}\partial_{\beta\dot\alpha} \end{cases} \Longrightarrow \begin{cases} D_1 = \frac{\partial}{\partial\theta^1} - \frac{i}{2}\bar\theta^2(\partial_1 - i\partial_2) - \frac{i}{\sqrt2}\bar\theta^1\partial_t \\ \bar D_1 = \frac{\partial}{\partial\bar\theta^1} - \frac{i}{2}\theta^2(\partial_1 + i\partial_2) - \frac{i}{\sqrt2}\theta^1\partial_t \\ D_2 = \frac{\partial}{\partial\theta^2} - \frac{i}{2}\bar\theta^1(\partial_1 + i\partial_2) - \frac{1}{\sqrt2}\bar\theta^2 M \\ \bar D_2 = \frac{\partial}{\partial\bar\theta^2} - \frac{i}{2}\theta^1(\partial_1 - i\partial_2) - \frac{1}{\sqrt2}\theta^2 M \end{cases}$$

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = -i\partial_{\alpha\dot\alpha} \Longrightarrow \{D_\alpha, \bar D_\beta\} = -i\partial_{\alpha\beta} \qquad [D_1]=1 \quad [D_2]=0$$

⋆ (Anti)chiral superfields $\bar{D}_\alpha \Sigma = 0, \qquad D_\alpha \bar{\Sigma} = 0$

$$\Sigma(x_L, \theta^\alpha) = \varphi(x_L) + \theta^\alpha \psi_\alpha(x_L) - \theta^2 F(x_L)$$
$$\bar{\Sigma}(x_R, \bar{\theta}^\beta) = \bar{\varphi}(x_R) + \bar{\theta}_\gamma \bar{\psi}^\gamma(x_R) - \bar{\theta}^2 \bar{F}(x_R) \qquad x_{L,R}^{\alpha\beta} = x^{\alpha\beta} \mp i\,\theta^\alpha \bar{\theta}^\beta$$

⋆ Berezin and spacetime integrations

$$\int d^4x d^4\theta\, \Psi = \int d^4x\, \mathcal{D}^2 \bar{\mathcal{D}}^2 \Psi \Big|_{\theta=\bar{\theta}=0} \qquad (\Psi = e^{imx^-}\, \tilde{\Psi})$$

$$\longrightarrow \underbrace{\int d^3x D^2 \bar{D}^2 \tilde{\Psi}\Big|_{\theta=\bar{\theta}=0}} \times \frac{1}{2\pi} \int_0^{2\pi} dx^-\, e^{imx^-}$$

$\downarrow \qquad \Downarrow$

$$\equiv \int d^3x\, d^4\theta\, \tilde{\Psi} \qquad \text{Non-vanishing result only if } \mathrm{M}(\Psi) = 0$$

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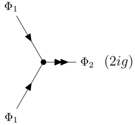
Selection rules

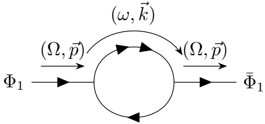
Selection rule I: Particle number conservation at each vertex



Number of incoming arrows = Number of outgoing arrows

Selection rule II: Arrows inside a Feynman diagram cannot form a closed loop

$$\langle \Phi\bar{\Phi}\rangle = \frac{i}{2m\omega - \vec{p}^{\,2} + i\varepsilon}\,\delta^{(4)}(\theta_1 - \theta_2) \qquad a = 1, 2$$



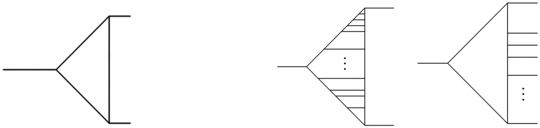
O. Bergman, PRD 46 (1992) 5474

Example: NR Wess-Zumino model

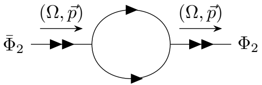
$$S_{NR} = \int d^3x d^4\theta \left(\bar{\Phi}_1 \Phi_1 + \bar{\Phi}_2 \Phi_2\right) + g \int d^3x d^2\theta \, \Phi_1^2 \Phi_2 + \text{h.c.}$$

$$M(\Phi_1) = m, \; M(\Phi_2) = -2m$$

- Selection rule 1 - Triangle diagrams are ruled out



- Selection rule 2 - Only divergent diagram



$$(\Phi_2)_{\text{bare}} = Z_2^{1/2} \Phi_2 \qquad Z_2 = 1 - \frac{|g|^2}{4\pi m} \frac{1}{\varepsilon}$$

NR WZ model is one-loop exact

R. Auzzi, S. Baiguera, G. Nardelli, SP, JHEP06 ('19)

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Scalar Galilean Electrodynamics (GED) in d=2+1

M. L. Bellac, J. M. Levy-Leblond, Nuovo Cim. B14 (1973)
A. Bagchi, R. Basu, A. Mehra, JHEP11 (2014)

Null reduction of 4d scalar QED: $A_\mu(x) = A_\mu(t, x_{i=1,2})$ coupled to
$\phi(x) = e^{imx^-}\phi(t, x_{i=1,2})$

$$A_\mu \to (A_- \equiv \varphi, A_t, A_{i=1,2})$$

$$S_{GED} = \int dt d^2x \left[\frac{1}{2}(\partial_t\varphi)^2 + E^i\partial_i\varphi - \frac{1}{4}f_{ij}f^{ij} + \frac{i}{2}\bar{\phi}\nabla_t\phi - \phi\nabla_t\bar{\phi} - \frac{1}{2\mathcal{M}}\nabla_i\bar{\phi}\nabla^i\phi\right]$$

where $E_i = \partial_t A_i - \partial_i A_t \quad f_{ij} = \partial_i A_j - \partial_j A_i, \qquad \mathcal{M} \equiv m - e\varphi$

$$\nabla_t\phi = (\partial_t - ieA_t)\phi \qquad \nabla_i\phi = (\partial_i - ieA_i)\phi$$

G. Festuccia, D. Hansen, J. Hartong, N. Obers, JHEP11 (2016)

At classical level:

- The real scalar field φ is invariant under gauge and galilean transformations. Moreover, $[\varphi] = 0$

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G. Festuccia, D. Hansen, J. Hartong, N. Obers, JHEP11 (2016)

At classical level:

- The real scalar field φ is invariant under gauge and galilean transformations. Moreover, $[\varphi] = 0$
- The theory exhibits Schroedinger symmetry
- There are no propagating gauge dof

At quantum level: S. Chapman, L. Di Pietro, K.T. Grosvenor, Z. Yan, JHEP10 (2020)

- Non-renormalization of the coupling e

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- Non-renormalization of the coupling e
- The theory is not renormalizable: infinitely many marginal couplings get turned on along the RG flow

$$\Delta S_{\rm GED} = \int dt\, d^2x \Big(\mathcal{J}[\mathcal{M}]\, \partial^i \mathcal{M} \partial_i \mathcal{M}\, \bar\phi\phi - \frac{1}{4}\lambda \mathcal{V}[\mathcal{M}]\, (\bar\phi\phi)^2 - \mathcal{E}[\mathcal{M}]\, (\partial^i\partial_i \mathcal{M} - e^2 \bar\phi\phi)\, \bar\phi\phi \Big)$$

$$\mathcal{M} = m - e\varphi$$

Renormalizable theory is $S_{GED} + \Delta S_{\rm GED}$

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$$\mathcal{M} = m - e\varphi$$

Renormalizable theory is $S_{GED} + \Delta S_{\rm GED}$

- Conformal manifold of fixed points where the theory exhibits Schroedinger symmetry. This is peculiar of the theory in d=2+1

SuperGalilean Electrodynamics (SGED)

$\mathcal{N} = 2$ SUSY generalization of the Galilean scalar electrodynamics in d=2+1 by null reduction in superspace

$$S_{\rm nSGED} = \frac{1}{2}\int d^3x d^2\theta \, W^\alpha W_\alpha + \int d^3x d^4\theta \, \bar{\Phi} e^{gV} \Phi$$

$$W_\alpha = i\bar{D}^2 D_\alpha V \qquad\qquad \bar{D}_\alpha \Phi = D_\alpha \bar{\Phi} = 0$$

At classical level:

- $U(1)_M$ assignment: $M(V) = 0 \quad M(\Phi) = m$

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At classical level:

- $U(1)_M$ assignment: $M(V)=0 \quad M(\Phi)=m$
- Super-Schroedinger $SU(2,2|1)$ and $U(1)$ supergauge invariance
- Gauge field components

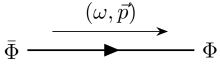
$$A_t = \frac{1}{2}[\bar{D}_1, D_1]V\Big| \qquad A_1 + iA_2 = \frac{1}{\sqrt{2}}[\bar{D}_1, D_2]V\Big| \qquad \varphi = D_2\bar{D}_2 V\Big|$$

The superfield $D_2\bar{D}_2V \equiv D\bar{D}V$ is supergauge invariant and dimensionless

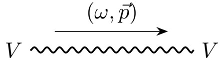
At quantum level

Feynman rules in Superspace

Propagators:



$$= \frac{i}{2m\omega - \vec{p}^{\,2} + i\epsilon}\, \delta^{(4)}(\theta' - \theta)$$



$$= -\, \frac{i}{-\vec{p}^{\,2} + i\epsilon}\, \delta^{(4)}(\theta' - \theta)$$

Vertices: $\bar{\Phi} V^n \Phi$

$\Rightarrow$ Background field method, $V \to V_0 + V$

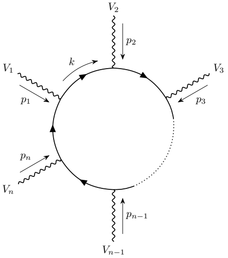
$$\nabla_\alpha = e^{-gV} \boldsymbol{\nabla}_\alpha e^{gV}\,, \qquad \boldsymbol{\nabla}_\alpha = e^{-gV_0/2} D_\alpha e^{gV_0/2}\,, \qquad \bar{\nabla}_\beta = \bar{\boldsymbol{\nabla}}_\beta = e^{gV_0/2} \bar{D}_\beta e^{-gV_0/2}$$

$$\tilde{\Phi} = e^{gV_0/2}\, \Phi \qquad \bar{\tilde{\Phi}} = \bar{\Phi}\, e^{gV_0/2}$$

Selection rules & non-renormalization theorems

1. Any 1PI Feynman diagram with negative superficial degree of divergence in the ω variable, vanishes identically.

2. All loop corrections to the effective action with purely vector external lines vanish, $\Gamma^{(n)}(V) = 0$.



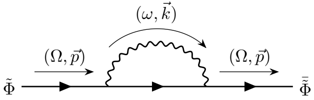
Therefore, V does not renormalize and for gauge invariance g does not renormalize

Non-renormalizability of nSGED

At one-loop the action S_{nSGED} acquires infinite UV divergent contributions

$$\Gamma^{(1L)} \to -\frac{ig^2}{16\pi m \varepsilon} \int d^3x d^4\theta \; \bar{\Phi} e^{gV} \Phi \; \frac{1}{1 - \frac{g}{\sqrt{2}m} \bar{D} D V} \qquad\qquad d = 2 - 2\varepsilon$$

Technical explanation:



$$\frac{1}{\square_c} = \frac{1}{\square} + \frac{1}{\square}\sqrt{2}g(\bar{D}DV)\omega\frac{1}{\square} + \frac{1}{\square}\sqrt{2}g(\bar{D}DV)\omega\frac{1}{\square}\sqrt{2}g(\bar{D}DV)\omega\frac{1}{\square} + \dots$$

Infinitively marginal couplings turn on. The model is not renormalizable! :-(

A renormalizable SGED

Consider the more general non-linear sigma model

$$S_{SGED} = \frac{1}{2}\int d^3x d^2\theta \, W^\alpha W_\alpha + \int d^3x d^4\theta \, \bar{\Phi} e^{gV} \Phi \, \mathcal{F}(\bar{D}DV)$$

$$\mathcal{F}(\bar{D}DV) = \sum_n \frac{1}{n!} \mathcal{F}^{(n)} (\bar{D}DV)^n$$

We have infinite new $n$pt vertices with couplings $\mathcal{F}^{(n)}$

Non-trivial renormalization

$$\delta\Phi = \frac{g}{16\pi m}\left(g - 2\sqrt{2} m \mathcal{F}^{(1)}\right)\frac{1}{\varepsilon} + \cdots \qquad \delta\mathcal{F}^{(n)} = \frac{g^{n+2}\, n!}{16\pi m (\sqrt{2} m)^n}\frac{1}{\varepsilon} + \cdots$$

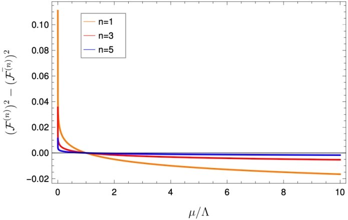
RG flows & conformal manifold

- One-loop beta functions for the couplings

$$\beta_{\mathcal{F}}^{(n)} = \frac{d\mathcal{F}^{(n)}}{d\log\mu} = -g^{n+2}\frac{n!\,n}{16\pi m(\sqrt{2}m)^n\mathcal{F}^{(n)}} \qquad \beta_g = 0$$

Solutions

$$(\mathcal{F}^{(n)})^2(\mu) - (\bar{\mathcal{F}}^{(n)})^2 = -\,g^{n+2}\frac{n!\,n}{16\pi m(\sqrt{2}m)^n}\log\left(\frac{\mu}{\Lambda}\right)$$



- Anomalous dimensions

$$\gamma_\Phi = \frac{1}{2}\frac{d\log(1+\delta_\Phi)}{d\log\mu} = \frac{g}{8\sqrt{2}\pi}\mathcal{F}^{(1)}$$

• IR interacting fixed point at $g = 0$

$$\gamma_\Phi = 0 \qquad\qquad \beta_{\mathcal{F}}^{(n)} = 0$$

At the fixed point the gauge-matter minimal coupling disappears, but the model contains an infinite number of gauge-matter couplings driven by $\bar{D}DV$

$$S_{SGED} = \frac{1}{2}\int d^3x d^2\theta\, W^\alpha W_\alpha + \int d^3x d^4\theta\, \bar{\Phi}\Phi\, \mathcal{F}(\bar{D}DV)$$

Matrix of anomalous dimensions, $I, J \in \{g, \mathcal{F}^{(1)}, \mathcal{F}^{(2)}, \cdots, \mathcal{F}^{(n)}, \cdots\}$

$$\partial_I \beta^J \to \frac{1}{16\pi m}\left(0, \frac{1}{\sqrt{2}m}\frac{g^3}{(\mathcal{F}^{(1)})^2}, \ldots, \frac{n!n}{(\sqrt{2}m)^n}\frac{g^{n+2}}{(\mathcal{F}^{(n)})^2}, \ldots\right)$$

At $g = 0$, infinite number of exactly marginal couplings $\Rightarrow$ infinite dimensional superconformal manifold

Plan of the talk

1. Construction of the non-relativistic (galilean) $\mathcal{N} = 2$ Superspace
2. Non-relativistic selection rules
3. Supersymmetric Galilean Electrodynamics (SGED): A renormalizable non-linear sigma model
4. Summary and future directions

Summary

We have studied quantum properties of NR $\mathcal{N} = 2$ SUSY models in d=(2+1) formulated in a NR Superspace. In particular,

The SGED model which is consistent at quantum level is a non-linear sigma model

$$S_{SGED} = \frac{1}{2}\int d^3x d^2\theta \, W^\alpha W_\alpha + \int d^3x d^4\theta \, \bar{\Phi} e^{gV} \Phi \, \mathcal{F}(\bar{D}DV)$$

The topology of the conformal manifold is peculiar of one-loop approximation. At higher loops we expect a richer spectrum of fixed points and further constraints on $\mathcal{F}$ for the existence of a conformal manifold.

Future directions

1. What is the meaning of the coupling $\mathcal{F}(\bar{D}DV)$? Is there a string interpretation?
2. Generalization to non-abelian theories
   A. Bagchi, R. Basu, M. Islam, K.S. Kolekar, A. Mehra, JHEP04 (2022)
3. Coupling to supergravity. Models coupled to Newton-Cartan supergravity as null reduction of relativistic SUSY models coupled to Poincaré supergravity
   E.Bergshoeff, A.Chatzistavrakidis, J.Lahnsteiner, L.Romano, J.Rosseel, JHEP07 ('20)
4. NR localization
5. Coupling to Chern-Simons terms and theories with more SUSY.
   Example: NR ABJM
   Y. Nakayama, M. Sakaguchi, K. Yoshida, JHEP04 (2009)
   Y. Nakayama Lett. Math. Phys. 89 (2009)
   K.-M. Lee, S. Lee, S. Lee JHEP09 (2009)
   Y. Nakayama, S-J. Rey, JHEP08 (2009)
6. Role of these theories in NR AdS/CFT correspondence