Longitudinal Galilean and Carrollian limits of Non-relativistic Strings

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Non-relativistic Strings and Beyond

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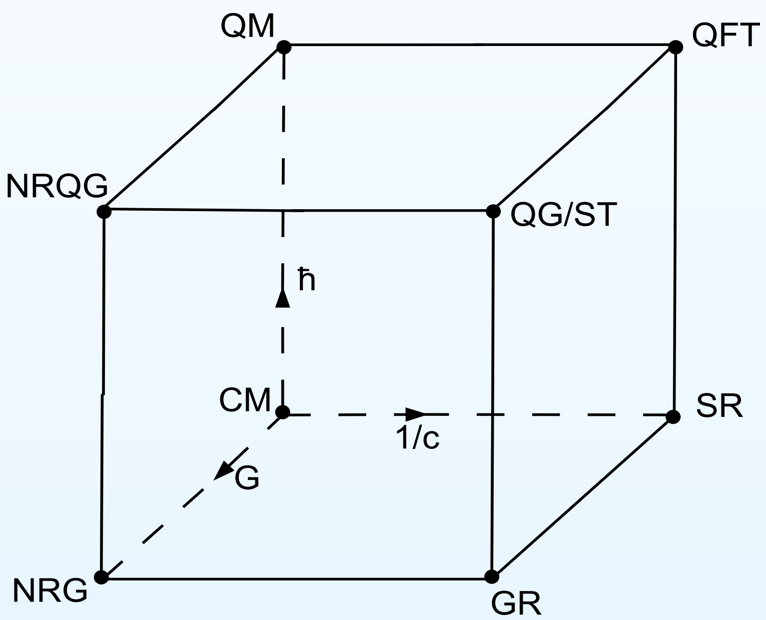
Introduction

- Bronstein cube

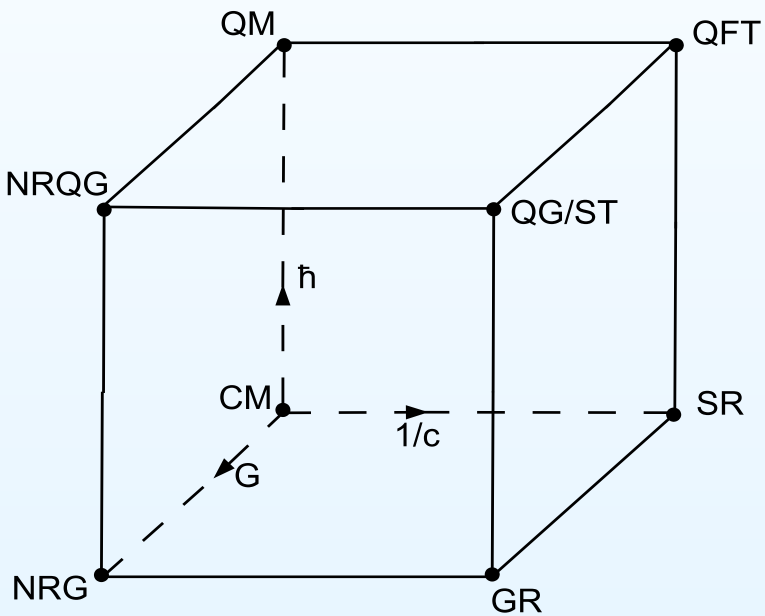


- NRG = non-relativistic gravity $\supset$ Newtonian gravity
- $1/c$ expansion $\supset$ post-Newtonian expansion
- NRQG = non-relativistic quantum gravity

- NRQG is a bit of a misnomer. There is no dynamical gravity to quantise. Think of quantum matter (described by quantum mechanics) backreacting with a background that reacts instantaneously.

Introduction

- Is there a well-defined non-relativistic limit of quantum gravity/string theory?



- In string theory there exist dualities between QG and specific QFTs (holography, AdS/CFT, ...).
- Do such dualities exist in the NR domain, i.e. can NR strings be dual to some NR field theory?

- Limits of AdS/CFT called Spin Matrix Theory give rise to NR strings dual to quantum mechanical limits of AdS/CFT.

- Can NR techniques be used to learn more about ordinary string theory?
- What is the landscape of UV complete non-Lorentzian string theories?
- Open strings and D-branes in non-Lorentzian string theory?
- Can we build explicit examples of holographic correspondences?
- Is there a well-defined corner called NRQG? Does it have a string theory description and if so why would it need one?

Outline

- NR strings in a nutshell
- Longitudinal Galilean limit
- Relation with the spin matrix limit
- Longitudinal Carrollian limit

Part I: Galilean worldsheets

- There are several ways to obtain a NR string (in the sense of Gomis–Ooguri and Danielsson–Guijosa–Kruczenski):
    - Limit in a near-critical B-field
    - String-$1/c$ expansion for an appropriate class of target spaces
    - A ‘lightlike T-duality’ along a non-compact null isometry (null reduction)

- The result is a target space described by string Newton–Cartan geometry: Riemannian geometry (transverse) fibered over a 2-dimensional Lorentzian base (longitudinal).

- The embedding map is such that the pullback of the longitudinal base is a Lorentzian metric on the woldsheet.

- Non-rel. string Lagrangian ($\tau = \det\left(\tau_\alpha^0, \tau_\beta^1\right)$)

$$\mathcal{L} = \frac{T_{\mathrm{NR}}}{2}\left[\tau\left(\tau_0^\alpha\tau_0^\beta - \tilde{c}^2\tau_1^\alpha\tau_1^\beta\right)h_{\alpha\beta} - \varepsilon^{\alpha\beta}m_{\alpha\beta}\right]$$

- Embedding is such that $(\tau_\alpha^0, \tau_\beta^1)$ is invertible. Target spacetime must admit a 2D Lorentzian submanifold.

- Target space fields: τ_M^A, h_{MN} $(0, 0, 1, \ldots, 1)$ and m_{MN}.

- Gauge symmetries:

$$\begin{aligned}
\delta\tau_M^A &= \lambda\varepsilon^A{}_B\tau_M^B \\
\delta h_{MN} &= -\eta_{AB}\left(\tau_M^A\lambda_N^B + \tau_N^A\lambda_M^B\right), \qquad \tau_A^M\lambda_M^B = 0 \\
\delta m_{MN} &= -\varepsilon_{AB}\left(\tau_M^A\lambda_N^B - \tau_N^A\lambda_M^B\right) + \partial_M\Lambda_N - \partial_N\Lambda_M
\end{aligned}$$

  Longitudinal Lorentz, transverse string Galilei boosts, 2-form gauge trafos

- Define $T_{\text{NR}} = T_G \tilde{c}^{-2}$ and $m_{MN} = \tilde{c}^2 \tilde{m}_{MN}$. Send $\tilde{c} \to \infty$

$$\mathcal{L} = -\frac{T_G}{2} \left(\tau \tau_1^\alpha \tau_1^\beta h_{\alpha\beta} + \varepsilon^{\alpha\beta} m_{\alpha\beta} \right)$$

- Target space fields now have the gauge symmetries:

$$\begin{aligned}
&\delta \tau_M^0 = 0 \,, \qquad \delta \tau_M^1 = \lambda \tau_M^0 \\
&\delta h_{MN} = \tau_M^0 \lambda_N^0 + \tau_N^0 \lambda_M^0 - \tau_M^1 \lambda_N^1 - \tau_N^1 \lambda_M^1 \,, \qquad \tau_A^M \lambda_M^B = 0 \\
&\delta m_{MN} = \tau_M^0 \lambda_N^1 - \tau_N^0 \lambda_M^1 + \partial_M \Lambda_N - \partial_N \Lambda_M
\end{aligned}$$

  Longitudinal Galilei, transverse string Galilei boosts, 2-form gauge trafos

- We can also write:

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{T_G}{2} \frac{1}{\tau} h_{MN} \tau_P^0 \tau_Q^0 \left(\dot{X}^M X'^P - X'^M \dot{X}^P \right) \left(\dot{X}^N X'^Q - X'^N \dot{X}^Q \right) \\
& - T_G m_{MN} \dot{X}^M X'^N \,, \qquad \tau = \tau_R^0 \tau_S^1 \left(\dot{X}^R X'^S - \dot{X}^S X'^R \right)
\end{aligned}$$

- When $\tau_\sigma^0 = \tau_M^0 X'^M \neq 0$ we get wrong sign kinetic terms. This is a gauge artefact.
- Also the relation between canonical momenta P_i and $\dot{X}^i$ is singular when $\tau_\sigma^0 = \tau_M^0 X'^M = 0$.
- We partially fix the gauge by $\tau_\sigma^0 = \tau_M^0 X'^M = 0$. We thus have fewer constraints to generate the residual gauge symmetries: $\tau \to \tilde{\tau}(\tau)$ and $\sigma \to \tilde{\sigma}(\tau, \sigma)$
- In general we cannot do better than

$$X^t = a\tau \,, \qquad X^v = wR_v\sigma + f(\tau)$$

  where $X^v \sim X^v + 2\pi R_v$ and $w > 0$ is the winding number.
- In any coordinate system σ runs from 0 to 2π. To preserve the origin we cannot shift σ by a function of τ only.
- The EOM of f is related to the level matching condition, the only constraint that is left.

- 2D diffeo invariance tells us that off shell

$$\left(\partial_\alpha \frac{\partial \mathcal{L}}{\partial \partial_\alpha X^M} - \frac{\partial \mathcal{L}}{\partial X^M}\right) \partial_\beta X^M = 0$$

  so EOMs are not all independent. In particular (in static gauge) the X^t and X^v EOM are automatically satisfied when the X^i EOM are obeyed.

- We will take $\tau^0_M dx^M = dt$ and $\tau^1_M dx^M = dv$. Split $X^M = (X^t, X^v, X^i)$ then in our gauge $(X'^t = 0)$ we find:

$$\begin{aligned} \mathcal{L} &= T\Big[m_{iv}\left(\dot{X}^v X'^i - \dot{X}^i X'^v\right) - \frac{\dot{X}^t}{2X'^v} h_{ij} X'^i X'^j \\ &\quad -\frac{1}{2}(h_{tt} + 2m_{tv})\dot{X}^t X'^v - (h_{vi} + m_{ti})\dot{X}^t X'^i - m_{ij}\dot{X}^i X'^j \Big] \end{aligned}$$

- Consider part that is quadratic in X^i. For constant m_{ij} that term is a total derivative and $\delta m_{ij} = 0$ implies $\delta m_{iv} = \partial_i \sigma$.
- We want a kinetic term for every X^i so m_{iv} must be such that its field strength is invertible: symplectic structure.
- Need $i = 1, \ldots, 2n$. Hence $m_{iv} = -\omega_{ij} X^j$ where ω_{ij} is a $2n \times 2n$ invertible antisymmetric matrix.
- WLOG the most general quadratic action that preserves the rotations leaving ω invariant is

$$\mathcal{L} = T\omega_{ij} X^j \left(\dot{X}^i - \dot{f}(\tau) X'^i \right) - \frac{T}{2} X'^i X'^i - \frac{\alpha^2}{2} X^i X^i$$

  where we used the strongest possible gauge fixing.
- n decoupled Schrödinger fields $\Phi^I = X^{2I-1} + iX^{2I}$ with $I = 1, \ldots, n$ and masses determined by eigenvalues of ω_{ij}.

- Theories of the form

$$\mathcal{L} = T\left[m_{iv}\left(\dot{X}^v X'^i - \dot{X}^i X'^v\right) - \frac{\dot{X}^t}{2X'^v} h_{ij} X'^i X'^j\right]$$

  can be obtained by starting with rel. strings on $\mathrm{AdS}_5 \times S^5$ and taking their spin matrix theory (SMT) limit.

- $\mathrm{AdS}_5 \times S^5$ admits a null Killing vector. We can null reduce the standard string and then take a large $\tilde{c}$ limit. This gives a particular SMT (the most general one). The others are special cases.

- $\tilde{c} = (4\pi g_s N)^{-1/2}$

- What is the dual field theory description after the null reduction but before taking the SMT limit?

- Unlike in standard string theory, here the gauge fixed NG action for the simplest backgrounds is quadratic in the fields.
- The Polyakov version of NR strings is

$$\mathcal{L} = -\frac{T_{\text{NR}}}{2}\Big[e\left(-e_0{}^\alpha e_0{}^\beta + \tilde{c}^2 e_1{}^\alpha e_1{}^\beta\right)h_{\alpha\beta} + \epsilon^{\alpha\beta}m_{\alpha\beta}$$
$$+\omega\epsilon^{\alpha\beta}\left(e^0{}_\alpha\tau^0{}_\beta + \frac{1}{\tilde{c}^2}e^1{}_\alpha\tau^1{}_\beta\right) + \psi\epsilon^{\alpha\beta}\left(e^0{}_\alpha\tau^1{}_\beta + e^1{}_\alpha\tau^0{}_\beta\right)\Big]$$

- Rescaling

$$T_{\text{NR}} = \tilde{c}^{-2}T_G\,, \qquad T_{\text{NR}}m_{MN} = T_G\tilde{m}_{MN}\,, \qquad \omega = \tilde{c}^2\tilde{\omega}\,, \qquad \psi = \tilde{c}^2\tilde{\psi}$$

and dropping the tildes gives:

$$\mathcal{L} = -\frac{T_G}{2}\Big[\left(e\, e_1{}^\alpha e_1{}^\beta h_{\alpha\beta} + \epsilon^{\alpha\beta}m_{\alpha\beta}\right)$$
$$+\omega\epsilon^{\alpha\beta}e^0{}_\alpha\tau^0{}_\beta + \psi\epsilon^{\alpha\beta}(e^0{}_\alpha\tau^1{}_\beta + e^1{}_\alpha\tau^0{}_\beta)\Big]$$

- To avoid negative sign kinetic terms we make the gauge choice $e_\sigma^0 = 0$.
- The worldsheet has a Galilean metric structure with Weyl symmetry.
- WIP comment: e_1^α must be nowhere vanishing and regular. Does this restrict the worldsheet topologies?
- In the gauge $X'^t = 0$ the algebra of first class constraints is Virasoro$\oplus\mathbb{R}$. This generates (under Hamiltonian flow) the gauge transformations $\tau \to \tilde{\tau}(\tau)$ and $\sigma \to \tilde{\sigma}(\tau, \sigma)$ when acting on the X^M.
- To do: add dilaton, critical dimension (reproduce target space symmetries in quantum theory), spectrum, beta functions, string interactions, supersymmetry, etc.
- Is there an open string version?

Part II: Carrollian worldsheets

- Non-rel. string Lagrangian ($\tau = \det\left(\tau^0_\alpha, \tau^1_\beta\right)$)

$$\mathcal{L} = \frac{T_{\mathsf{NR}}}{2}\left[\tau\left(\tau^\alpha_0\tau^\beta_0 - \tilde{c}^2\tau^\alpha_1\tau^\beta_1\right)h_{\alpha\beta} - \varepsilon^{\alpha\beta}m_{\alpha\beta}\right]$$

- Embedding is such that $(\tau^0_\alpha, \tau^1_\beta)$ is invertible. Target spacetime must admit a 2D Lorentzian submanifold.

- Target space fields: τ^A_M, h_{MN} $(0,0,1,\ldots,1)$ and m_{MN}.

- Gauge symmetries:

$$\delta\tau^A_M = \lambda\varepsilon^A{}_B\tau^B_M$$

$$\delta h_{MN} = -\eta_{AB}\left(\tau^A_M\lambda^B_N + \tau^A_N\lambda^B_M\right)\,, \qquad \tau^M_A\lambda^B_M = 0$$

$$\delta m_{MN} = -\varepsilon_{AB}\left(\tau^A_M\lambda^B_N - \tau^A_N\lambda^B_M\right) + \partial_M\Lambda_N - \partial_N\Lambda_M$$

  Longitudinal Lorentz, transverse string Galilei boosts, 2-form gauge trafos

- Without rescaling anything we can send $\tilde{c} \to 0$

$$\mathcal{L} = \frac{T_C}{2}\left(\tau\tau_0^\alpha\tau_0^\beta h_{\alpha\beta} - \varepsilon^{\alpha\beta}m_{\alpha\beta}\right)$$

- Target space gauge symmetries:

$$\delta\tau_M^0 = \lambda\tau_M^1\,, \qquad \delta\tau_M^0 = 0$$
$$\delta h_{MN} = \tau_M^0\lambda_N^0 + \tau_N^0\lambda_M^0 - \tau_M^1\lambda_N^1 - \tau_N^1\lambda_M^1\,, \qquad \tau_A^M\lambda_M^B = 0$$
$$\delta m_{MN} = -\tau_M^1\lambda_N^0 + \tau_N^1\lambda_M^0 + \partial_M\Lambda_N - \partial_N\Lambda_M$$

- The Lagrangian can also be written as

$$\mathcal{L} = \frac{T_C}{2}\frac{1}{\tau}h_{MN}\tau_P^1\tau_Q^1\left(\dot{X}^M X'^P - X'^M\dot{X}^P\right)\left(\dot{X}^N X'^Q - X'^N\dot{X}^Q\right) - T_C m_{MN}\dot{X}^M X'^N$$

- The Galilean and Carrollian cases are related by $0 \leftrightarrow 1$.

- Restricting to the quadratic theory and after gauge fixing

$$\mathcal{L} = \frac{T_C}{2}\left(\dot{X}^i - \dot{f}(\tau)X'^i\right)^2 + T_C\omega_{ij}X^jX'^i$$

  where we dropped a potential term.

- We can remove f and impose the constraint that the generator of σ translations $\oint d\sigma P_i X'^i$ is constant (and zero on states in the quantum theory: level matching) by hand.

- Before gauge fixing, local and global symmetries are $\delta X^M = \zeta^\alpha \partial_\alpha X^M - \Xi^M$ such that $\mathcal{L}_\Xi$ acting on τ_M^A, h_{MN}, m_{MN} gives zero up to a gauge transformation.

$$\mathcal{L} = \frac{T_C}{2}\dot{X}^i\dot{X}^i + T_C\omega_{ij}X^jX'^i$$

- After gauge fixing we have
  1. τ translation symmetry: $\delta X^i = \zeta^\tau \dot{X}^i$
  2. σ translation symmetry: $\delta X^i = \zeta^\tau X'^i$
  3. Carroll boost symmetry: $\delta X^i = b\left(\sigma\dot{X}^i - \tau\omega_{ij}X^j\right)$
  4. Internal symmetry: $\delta X^i = \epsilon\omega_{ij}X^j + \lambda_{ij}X^j + a^i(\tau,\sigma)$,

  where $\lambda_{ij} \neq \omega_{ij}$ is a rotation leaving ω_{ij} invariant and where $\ddot{a}^i + 2\omega_{ij}a'^j = 0$.
- Longitudinal symmetries: centrally extended 2D Carroll

$$\{Q_C\,, Q_P\} = -Q_H\,, \qquad \{Q_C\,, Q_H\} = Q_N\,.$$

- Infinitely many symmetries in the transverse directions with Noether charges $\oint d\sigma\,\left(P_i a^i - X^i\dot{a}^i\right)$ whose Poisson brackets lead to central extensions.

- n decoupled Carroll Schrödinger fields $X^{2I-1} + iX^{2I}$ obeying

$$i\partial_\sigma \left(X^{2I-1} + iX^{2I} \right) = \frac{1}{2\lambda_I} \partial_\tau^2 \left(X^{2I-1} + iX^{2I} \right)$$

  where λ_I are the eigenvalues of ω_{ij}.

- Ordinary Schrödinger equation disperson relation is $\omega \sim k^2$. Here we have $k \sim \omega^2$.

- Worldsheet is a cylinder so the k are discrete and real (positive and negative) so only $\omega^4 \geq 0$.

- Oops!

- The Polyakov formulation leads to a worldsheet with a Carrollian metric structure on it that is Weyl invariant.
- In the non-gauge fixed theory the algebra of first class constraints gives BMS_3.
- What kind of limit is this if we consider strings on $\text{AdS}_5 \times S^5$?
- Can we remove the modes with imaginary ω or is there interesting physics in them? Unitarity? Non-invertible ω_{ij}?

Non-Lorentzian closed strings

	standard string theory	Non-relativistic string theory	Galilean string	Carroll string
target spacetime geometry	Lorentzian	string Newton–Cartan	Symplectic & Riemannian geometry fibered over Galilean cylinder	Riemannian geometry fibered over Carrollian cylinder
worldsheet geometry	Lorentzian	Lorentzian	Galilean	Carrollian
algebra of first class constraints	Virasoro$\oplus$Virasoro	Virasoro$\oplus$Virasoro	Virasoro$\oplus U(1)$ GCA$_2$ without gauge fixing?	BMS$_3$

- As well as others (tensionless strings, ...) and many more to come!