Tensionless Tales

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Non-Relativistic Strings and Beyond, NORDITA

Based on work done over the last decade


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- AB 1303.0291 (JHEP)
- AB, Chakrabortty, Parekh 1507.04361 (JHEP)
- AB, Chakrabortty, Parekh 1606.09628 (JHEP)
- AB, Banerjee, Chakrabortty, Parekh 1710.03482 (JHEP)
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- AB, Mandlik, Sharma 2105.09682 (JHEP)
- AB, Banerjee, Chakrabortty, Chatterjee 2111.01172 (JHEP)
- AB, Grumiller, Sheikh Jabbari 2210.10794
- AB, Banerjee, Mandlik (upcoming)


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Null Strings?! What? Why?

* Massless point particles move on null geodesics. Worldlines are null.

* Null strings: extended analogues of massless point particles.
  Massless point particles => Tensionless strings.

* Tensionless or null strings: studied since **Schild** in 1970's.

* Tension $T = \dfrac{1}{2\pi\alpha'} \to 0$: point particle limit of string theory => Classical gravity.

* Tensionless regime: $T = \dfrac{1}{2\pi\alpha'} \to \infty$: ultra-high energy, ultra-quantum gravity!

Null strings are vital for:

A. Strings at **very high temperatures**: Hagedorn Phase.

B. Strings near **spacetime singularities:** Strings near Black holes, near the Big Bang.

C. Connections to **higher spin theory**.

Summary of Results

* 2d Conformal Carrollian (or BMS3) and its supersymmetric cousins arise on the worldsheet of the tensionless string replacing the two copies of the (super) Virasoro algebra.
* Classical tensionless strings: properties can be derived intrinsically or as a limit of usual tensile strings.
* Quantum tensionless strings: many surprising new results.
* A theory of Black hole microstates based on null strings!

Classical Tensionless Strings

Isberg, Lindstrom, Sundborg, Theodoridis 1993

AB 2013; AB, Chakrabortty, Parekh 2015.

Going tensionless

Isberg, Lindstrom, Sundborg, Theodoridis 1993

Start with Nambu-Goto action:

$$S = -T \int d^2\xi \sqrt{-\det \gamma_{\alpha\beta}}. \tag{1}$$

To take the tensionless limit, first switch to Hamiltonian framework.

- Generalised momenta: $P_m = T\sqrt{-\gamma}\gamma^{0\alpha}\partial_\alpha X_m$.
- Constraints: $P^2 + T^2\gamma\gamma^{00} = 0, \;\; P_m\partial_\sigma X^m = 0$.
- Hamiltonian: $\mathcal{H}_T = \mathcal{H}_\mathcal{C} + \rho^i(\text{constraints})_i = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho\, P_m\partial_\sigma X^m$.

Action after integrating out momenta:

$$S = \frac{1}{2}\int d^2\xi\, \frac{1}{2\lambda}\left[\dot{X}^2 - 2\rho\dot{X}^m\partial_\sigma X_m + \rho^2\partial_\sigma X^m\partial_\sigma X_m - 4\lambda^2T^2\gamma\gamma^{00}\right] \tag{2}$$

Identifying

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2T^2 \end{pmatrix},$$

action takes the familiar Weyl-invariant form

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g}g^{\alpha\beta}\partial_\alpha X^m\partial_\beta X^n\eta_{mn}. \tag{3}$$

Going Tensionless ...

Isberg, Lindstrom, Sundborg,Theodoridis 1993

- Tensionless limit can now be taken systematically.
- $T \to 0 \Rightarrow$

$$g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 \end{pmatrix}.$$

- Metric is degenerate. $\det g = 0$.
- Replace degenerate metric density $T\sqrt{-g}g^{\alpha\beta}$ by a rank-1 matrix $V^\alpha V^\beta$ where V^α is a vector density

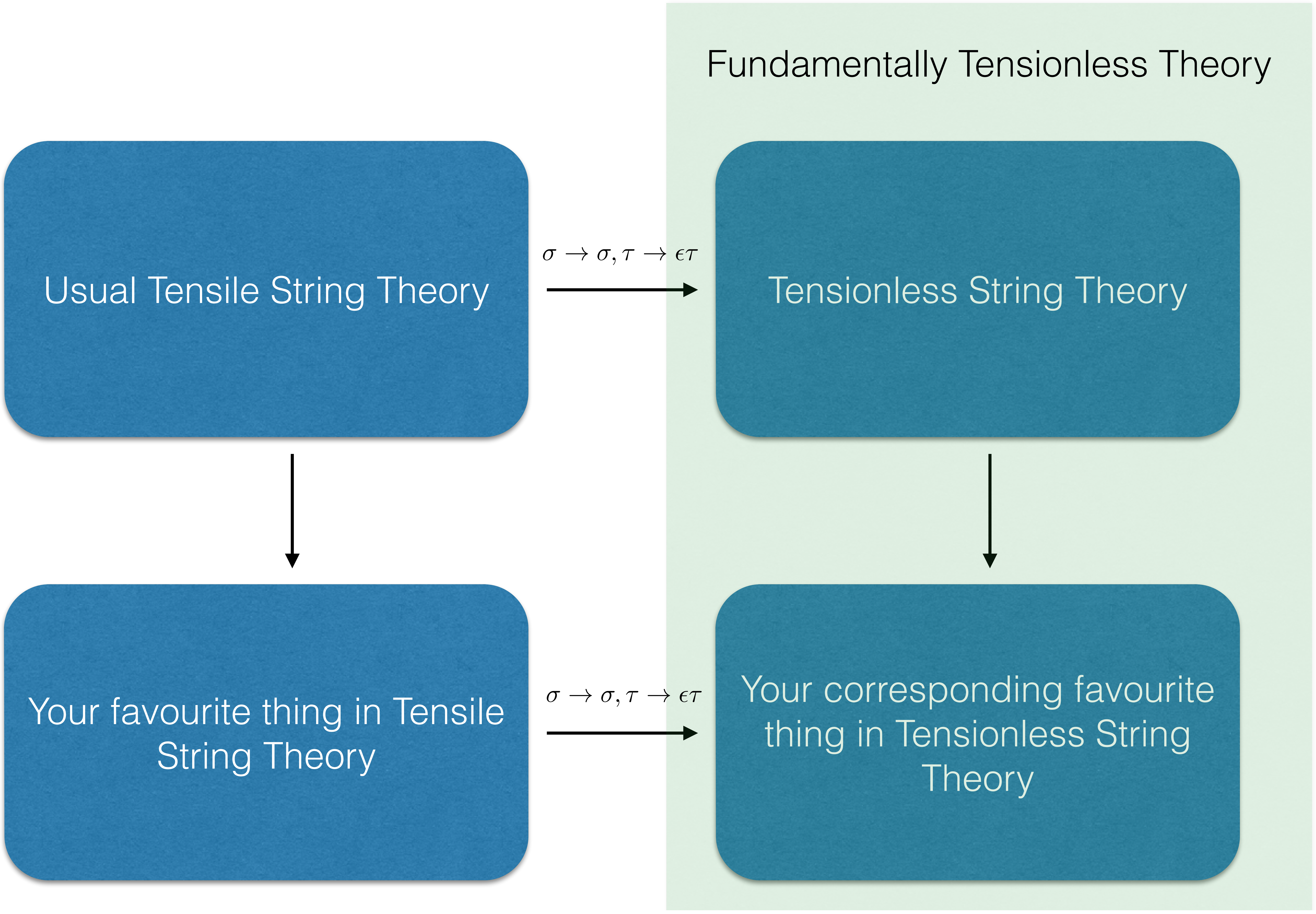
$$V^\alpha \equiv \frac{1}{\sqrt{2\lambda}}(1, \rho) \tag{4}$$

- Action in $T \to 0$ limit

$$\boxed{S = \int d^2\xi \; V^\alpha V^\beta \partial_\alpha X^m \partial_\beta X^n \eta_{mn}.} \tag{5}$$

- Starting point of tensionless strings.
- Need not refer to any parent theory. Treat this as action of fundamental objects.

Completing the square?



Gauge and Residual Gauge Symmetries

Tensionless action is invariant under world-sheet diffeomorphisms.

Fixing gauge: "Conformal" gauge: $V^\alpha = (v, 0)$ (v: constant).

Tensile: Residual symmetry after fixing conformal gauge = Vir $\otimes$ Vir. Central to understanding string theory.

Tensionless: Similar residual symmetry left over after gauge fixing.

For world-sheet diffeomorphism: $\xi^\alpha \to \xi^\alpha + \varepsilon^\alpha$, change in vector density: $\delta_\varepsilon V^\alpha = -V \cdot \partial\varepsilon^\alpha + \varepsilon \cdot \partial V^\alpha + \frac{1}{2}(\partial \cdot \varepsilon)V^\alpha$

Tensionless residual symmetries: for $V^\alpha = (v, 0)$, $\varepsilon^\alpha = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\}$

Define: $L(f) = f'(\sigma)\tau\partial_\tau + f(\sigma)\partial_\sigma$, $M(g) = g(\sigma)\partial_\tau$. Expand: $f = \sum a_n e^{in\sigma}$, $g = \sum b_n e^{in\sigma}$

$$L(f) = \sum_n a_n e^{in\sigma}(\partial_\sigma + in\tau\partial_\tau) = \sum_n a_n L_n, \quad M(g) = \sum_n b_n e^{in\sigma}\partial_\tau = \sum_n b_n M_n.$$

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c_L}{12}(m^3 - m)\delta_{m+n,0}, \quad [M_m, M_n] = 0.$$
$$[L_m, M_n] = (m-n)M_{m+n} + \frac{c_M}{12}(m^3 - m)\delta_{m+n,0}.$$

BMS3 or 2d Conformal Carroll Algebra

Tensionless Limit from the Worldsheet

A Bagchi 2013

- Tensile string: Residual symmetry in conformal gauge $g_{\alpha\beta} = e^{\phi}\eta_{\alpha\beta}$:

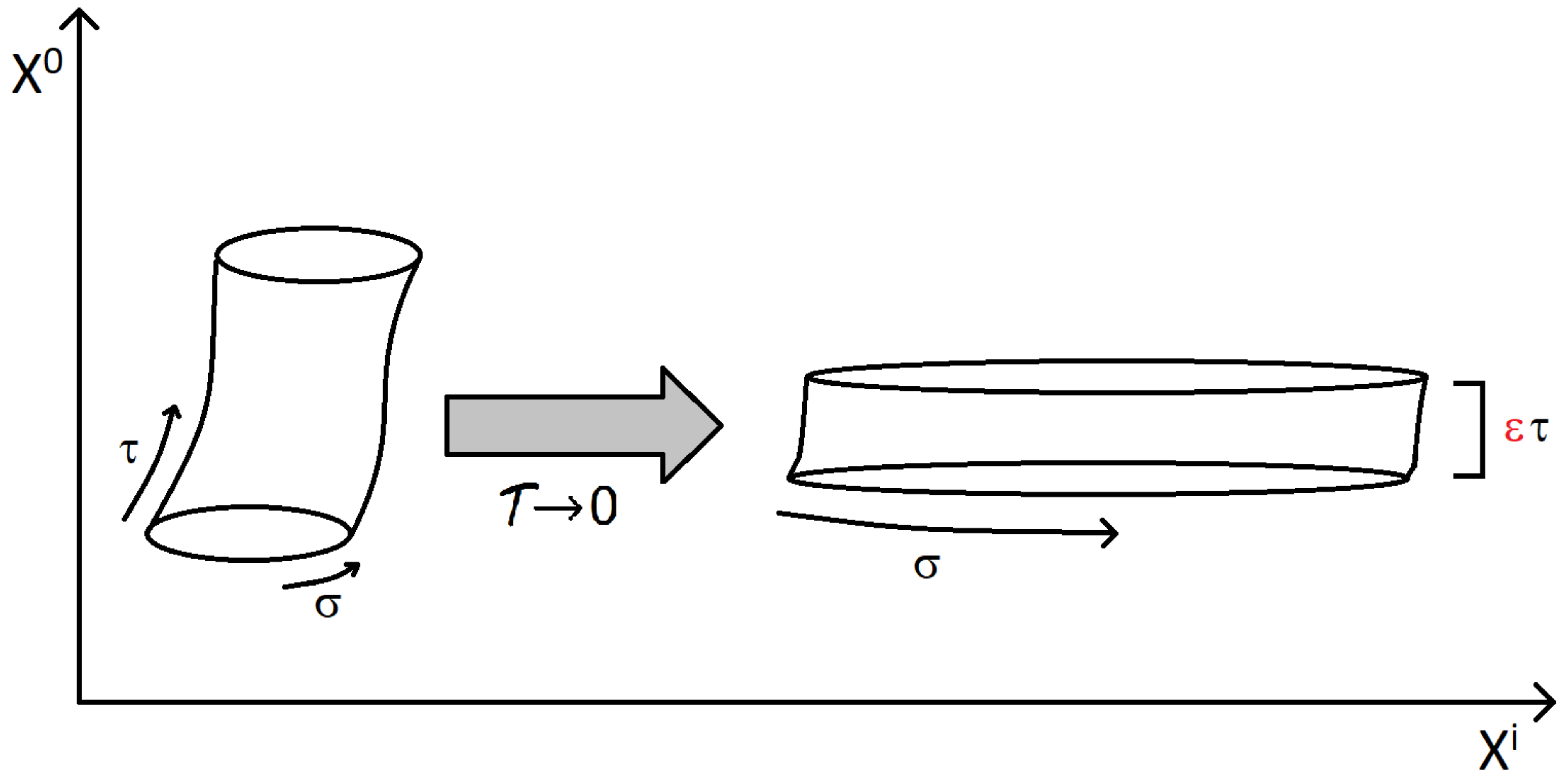
$$\begin{aligned} [\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\ [\mathcal{L}_m, \bar{\mathcal{L}}_n] &= 0, \quad [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0} \end{aligned}$$

- World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the cylinder.

$$\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}}$$

where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros.

- Tensionless limit $\Rightarrow$ length of string becomes infinite ($\sigma \to \infty$).
- Ends of closed string identified $\Rightarrow$ limit best viewed as ($\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$).



Tensionless Limit from the Worldsheet

A Bagchi 2013

- Define

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}).$$

- New vector fields (L_n, M_n) well-defined in limit and given by:

$$L_n = ie^{in\sigma}(\partial_\sigma + in\tau\partial_\tau), \quad M_n = ie^{in\sigma}\partial_\tau .$$

- These are *exactly the generators defined previously* . Close to form BMS_3.

$$[L_m, L_n] = (m-n)L_{m+n} \quad [L_m, M_n] = (m-n)M_{m+n} \quad [M_m, M_n] = 0.$$

- Tensionless limit on the worldsheet: $\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0$
- Worldsheet velocities $v = \frac{\sigma}{\tau} \to \infty$. Effectively, $\frac{v}{c} \to \infty$
- Hence worldsheet speed of light $\to 0$. Carrollian limit.
- Degenerate worldsheet metric.
- Riemannian tensile worldsheet $\to$ Carrollian tensionless worldsheet.

Tensionless EM Tensor and constraints

A Bagchi 2013

Spectrum of tensile string theory (in conformal gauge in flat space)

- Quantise worldsheet theory as a theory free scalar fields.
- Constraint: vanishing of EOM of metric (which is fixed to be flat).
- Op form: Physical states vanish under action of modes of E-M tensor.

EM tensor for 2d CFT on cylinder: $T_{cyl} = z^2 T_{plane} - \frac{c}{24} = \sum_n \mathcal{L}_n e^{in\omega} - \frac{c}{24}; \quad \bar{T}_{cyl} = \sum_n \bar{\mathcal{L}}_n e^{in\bar{\omega}} - \frac{\bar{c}}{24}$

Ultra-relativistic EM tensor

$$T_{(1)} = \lim_{\epsilon \to 0} \left(T_{cyl} - \bar{T}_{cyl} \right) = \sum_n (L_n - in\tau M_n) e^{in\sigma} - \frac{c_L}{24}$$

$$T_{(2)} = \lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \bar{T}_{cyl} \right) = \sum_n M_n e^{in\sigma} - \frac{c_M}{24}$$

- Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$

- Quantum version: physical spectrum of tensionless strings restricted by

$$\langle \text{phys}| T_{(1)} |\text{phys}'\rangle = 0, \quad \langle \text{phys}| T_{(2)} |\text{phys}'\rangle = 0.$$

Intrinsic Analysis: EOM and Mode Expansions

AB, Chakraborty, Parekh 2015

- Equation of motion in $V^a = (v, 0)$ gauge: $\ddot{X}^\mu = 0$.
- Solution: $X^\mu(\sigma,\tau) = x^\mu + \sqrt{2c'}A_0^\mu\sigma + \sqrt{2c'}B_0^\mu\tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left(A_n^\mu - in\tau B_n^\mu\right)e^{in\sigma}$
- Closed string b.c.: $X^\mu(\sigma,\tau) = X^\mu(\sigma+2\pi,\tau) \Rightarrow A_0^\mu = 0$.
- Constraints: $\dot{X}^2 = 2c'\sum_{m,n} B_{-m}\cdot B_{m+n}\,e^{in\sigma} = 0, \quad \dot{X}\cdot X' = 2c'\sum_{m,n}(A_{-m} - in\tau B_{-m})\cdot B_{m+n}\,e^{in\sigma} = 0$
- Define: $L_n = \sum_m A_{-m}\cdot B_{m+n}, \quad M_n = \sum_m B_{-m}\cdot B_{m+n}$
- Classical constraints in terms of modes: $\sum_n (L_n - in\tau M_n)\,e^{in\sigma} = 0 = T_{(1)}, \quad \sum_n M_n\,e^{in\sigma} = 0 = T_{(2)}.$

Familiar form obtained earlier from purely algebraic considerations.

- The algebra of the modes

$$\{A_m^\mu, A_n^\nu\} = 0, \quad \{B_m^\mu, B_n^\nu\} = 0, \quad \{A_m^\mu, B_n^\nu\} = -im\delta_{m+n,0}\,\eta^{\mu\nu}.$$

- The worldsheet symmetry algebra of tensionless strings, now constructed from the quadratics of the modes:

$$\{L_m, L_n\} = -i(m-n)L_{m+n},\ \{L_m, M_n\} = -i(m-n)M_{m+n},\ \{M_m, M_n\} = 0.$$

Quantization: $\{,\}_{PB} \to -\frac{i}{\hbar}[,]$ leads to the BMS_3 Algebra.

Limiting Analysis: EOM and Mode Expansions

AB, Chakrabortty, Parekh 2015

- Tensile string mode expansion: $X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{2\alpha'}\alpha_0^\mu \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)} + \alpha_n^\mu e^{-in(\tau-\sigma)}]$.
- The limiting procedure: $\tau \to \epsilon\tau,\ \sigma \to \sigma,\ \alpha' = c'/\epsilon$ with $\epsilon \to 0$

$$X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{\frac{2c'}{\epsilon}}\alpha_0^\mu \epsilon\tau + i\sqrt{\frac{2c'}{\epsilon}}\sum_{n\neq 0}\frac{1}{n}[\tilde{\alpha}_n^\mu e^{-in\sigma}(1-in\epsilon\tau) + \alpha_n^\mu e^{in\sigma}(1-in\epsilon\tau)],$$

$$= x^\mu + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^\mu \tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left[\frac{\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu}{\sqrt{\epsilon}} - in\tau\sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu)\right]e^{in\sigma}.$$

- Thus we get a relation between the tensionless and tensile modes:

$$A_n^\mu = \frac{1}{\sqrt{\epsilon}}(\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu), \quad B_n^\mu = \sqrt{\epsilon}(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu).$$

- The equivalent of the Virasoro contraints

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon\left[\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right]$$

Quantum Tensionless Strings

A summary of quantum results

* Novel closed to open string transition as the tension goes to zero.
  [AB, Banerjee, Parekh (PRL) 2019]
* Careful canonical quantisation leads to not one, but <u>**three different vacua**</u> which give rise to different quantum mechanical theories arising out of the same classical theory.
  [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]
* Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction on the other theory. All acceptable limits of quantum tensile strings.
  [AB, Mandlik, Sharma 2021]
* Interpretation in terms of Rindler physics on the worldsheet.
  [AB, Banerjee, Chakrabortty (PRL) 2021]
* Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

Tensionless Path From Closed to Open Strings

AB, Banerjee, Parekh, Physical Review Letters 123 (2019) 111601.

BMS Induced Representations

- An important class of BMS representations: Massive modules.
- The Hilbert space of these modules contains a wavefunction $|M, s\rangle$ satisfying:

$$\boxed{M_0|M,s\rangle = M|M,s\rangle, \quad L_0|M,s\rangle = s|M,s\rangle, \quad M_n|M,s\rangle = 0, \ \forall n \neq 0.} \tag{33}$$

- This defines a 1-d rep spanned by $\{L_0, M_n, c_L, c_M\}$. Can be used to define an *induced BMS module* with basis vectors
$$|\Psi\rangle = L_{n_1} L_{n_2} \ldots L_{n_k} |M,s\rangle .$$
- Limit from Virasoro $\times$ Virasoro to BMS3: $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \ M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$.
- Virasoro primary conditions:

$$\mathcal{L}_n|h,\bar{h}\rangle = 0 = \bar{\mathcal{L}}_n|h,\bar{h}\rangle \ (n > 0); \ \mathcal{L}_0|h,\bar{h}\rangle = h|h,\bar{h}\rangle, \ \bar{\mathcal{L}}_n|h,\bar{h}\rangle = \bar{h}|h,\bar{h}\rangle .$$

- This translates to

$$\left(L_n + \frac{1}{\epsilon}M_n\right)|h,\bar{h}\rangle = 0, \quad \left(-L_{-n} + \frac{1}{\epsilon}M_{-n}\right)|h,\bar{h}\rangle = 0, \ n > 0.$$

- In the limit, this gives (33), along with the identification: $M = \epsilon(h + \bar{h}), \ s = h - \bar{h}$.

Induced Reps and Tensionless String

- In term of oscillator modes, the induced modules: $B_n|M,s\rangle = 0, \ \forall n \neq 0$.
- We are interested in the vacuum module. Hence we have $B_n|I\rangle = 0$ where $|I\rangle$ is the induced vacuum.
- Wish to return to harmonic oscillator basis for the tensionless string. Define:

$$C^\mu_n = \frac{1}{2}(A^\mu_n + B^\mu_n), \quad \tilde{C}^\mu_n = \frac{1}{2}(-A^\mu_{-n} + B^\mu_{-n})$$

- The algebra: $[C^\mu_m, C^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}$, $[\tilde{C}^\mu_m, \tilde{C}^\nu_n] = m\delta_{m+n}\eta^{\mu\nu}$.
- The tensile and tensionless raising and lowering operators are related by

$$C^\mu_n(\epsilon) = \beta_+\alpha^\mu_n + \beta_-\tilde{\alpha}^\mu_{-n}, \text{ where: } \beta_\pm = \frac{1}{2}\left(\sqrt{\epsilon} \pm \frac{1}{\sqrt{\epsilon}}\right)$$

$$\tilde{C}^\mu_n(\epsilon) = \beta_-\alpha^\mu_{-n} + \beta_+\tilde{\alpha}^\mu_n.$$

- $|0\rangle_c$: $C^\mu_n|0\rangle_c = 0 = \tilde{C}^\mu_n|0\rangle_c \quad \forall n > 0$. Different from tensile vacuum: mixing of tensile raising & lowering op in $C, \tilde{C}$.
- In the C basis, the induced vacuum is given by $(C^\mu_n + \tilde{C}^\mu_{-n})|I\rangle = 0, \quad \forall n$.
- This is precisely the condition of a Neumann boundary state $|I\rangle = \mathcal{N} \exp\left(-\sum_n \frac{1}{n} C_{-n}\tilde{C}_{-n}\right)|0\rangle_c$

Worldsheet Bogoliubov Transformations

- The relation between operators is a Bogoliubov transformation

$$\alpha_n^\mu = e^{iG} C_n e^{-iG} = \cosh\theta\, C_n^\mu - \sinh\theta\, \tilde{C}_{-n}^\mu, \quad G = i\sum_{n=1}^{\infty}\theta\left[C_{-n}.\tilde{C}_{-n} - C_n.\tilde{C}_n\right]$$

$$\tilde{\alpha}_n^\mu = e^{iG}\tilde{C}_n e^{-iG} = -\sinh\theta\, C_{-n}^\mu + \cosh\theta\, \tilde{C}_n^\mu, \quad \tanh\theta = \frac{\epsilon - 1}{\epsilon + 1}$$

- Relation between the two vacua:

$$|0\rangle_\alpha = \exp[iG]|0\rangle_c = \left(\frac{1}{\cosh\theta}\right)^{1+1+\dots}\prod_{n=1}^{\infty}\exp[\tanh\theta C_{-n}\tilde{C}_{-n}]|0\rangle_c$$

- Using the regularisation: $1 + 1 + 1 + \dots\infty = \zeta(0) = -\frac{1}{2}$

$$|0\rangle_\alpha = \sqrt{\cosh\theta}\prod_{n=1}^{\infty}\exp[\tanh\theta\, C_{-n}\tilde{C}_{-n}]|0\rangle_c$$

- From the point of view of $|0\rangle_c$, $|0\rangle_\alpha$ is a squeezed state.

From Closed to Open Strings

- When $\epsilon = 1$, $\tanh\theta = 0$, and we have $|0\rangle_\alpha = |0\rangle_c$. This is the closed string vacuum.
- As ϵ changes from 1, from the point of view of the C observer, the vacuum evolves. It becomes a squeezed state as shown before.

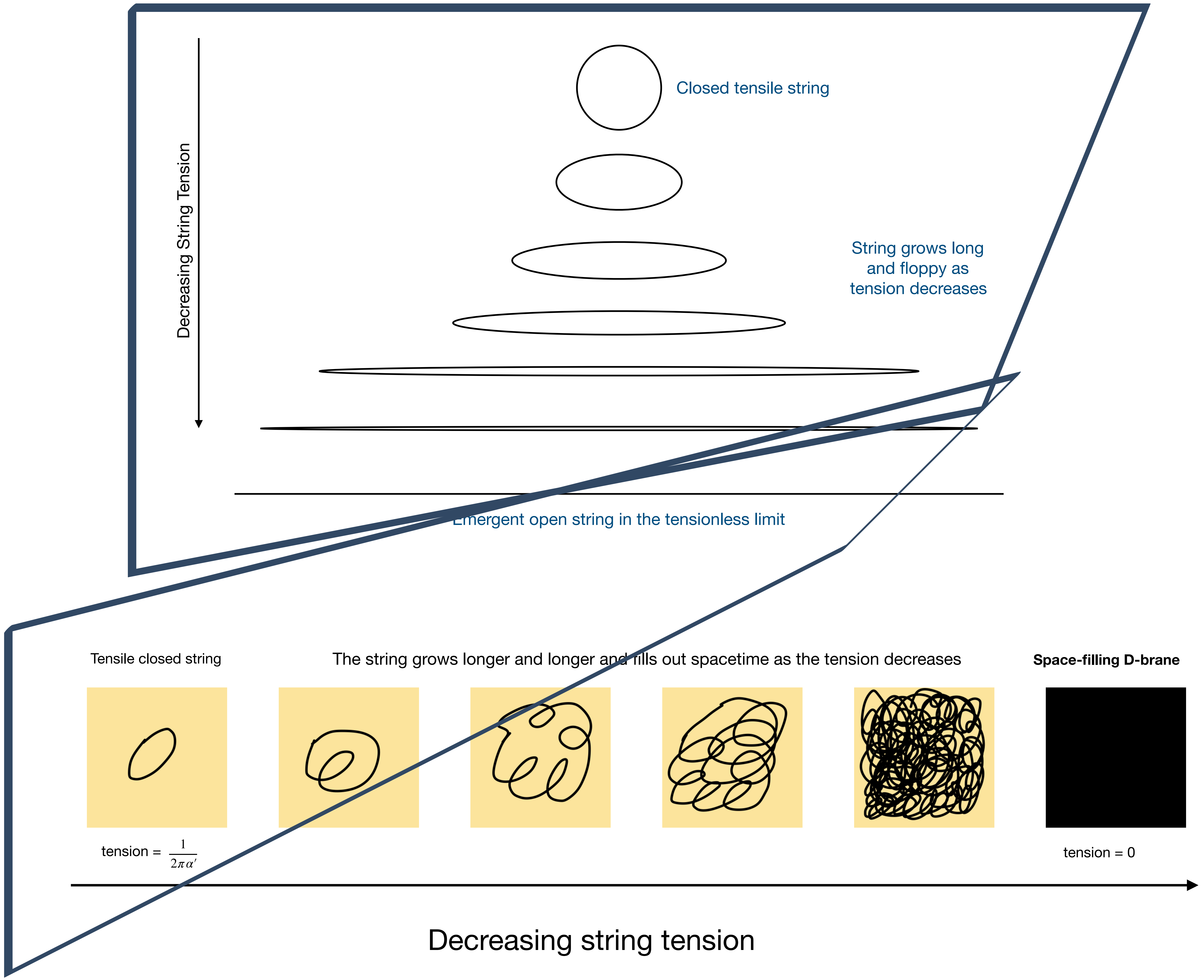
- In the limit where $\epsilon \to 0$, we have $\tanh\theta = -1$. The relation is thus:

$$|0\rangle_\alpha = \mathcal{N}\prod_{n=1}^{\infty}\exp[-C_{-n}\tilde{C}_{-n}]|0\rangle_c$$

  This is precisely the Induced vacuum $|I\rangle$ that we introduced before.
- As we said, this is a Neumann boundary state.
- This is thus an open string free to move in all dimensions (or a spacefilling D-brane).

We have thus obtained an open string by taking a tensionless limit on a closed string theory.

From Closed to Open Strings and D-branes



Bose-Einstein like Condensation on Worldsheet

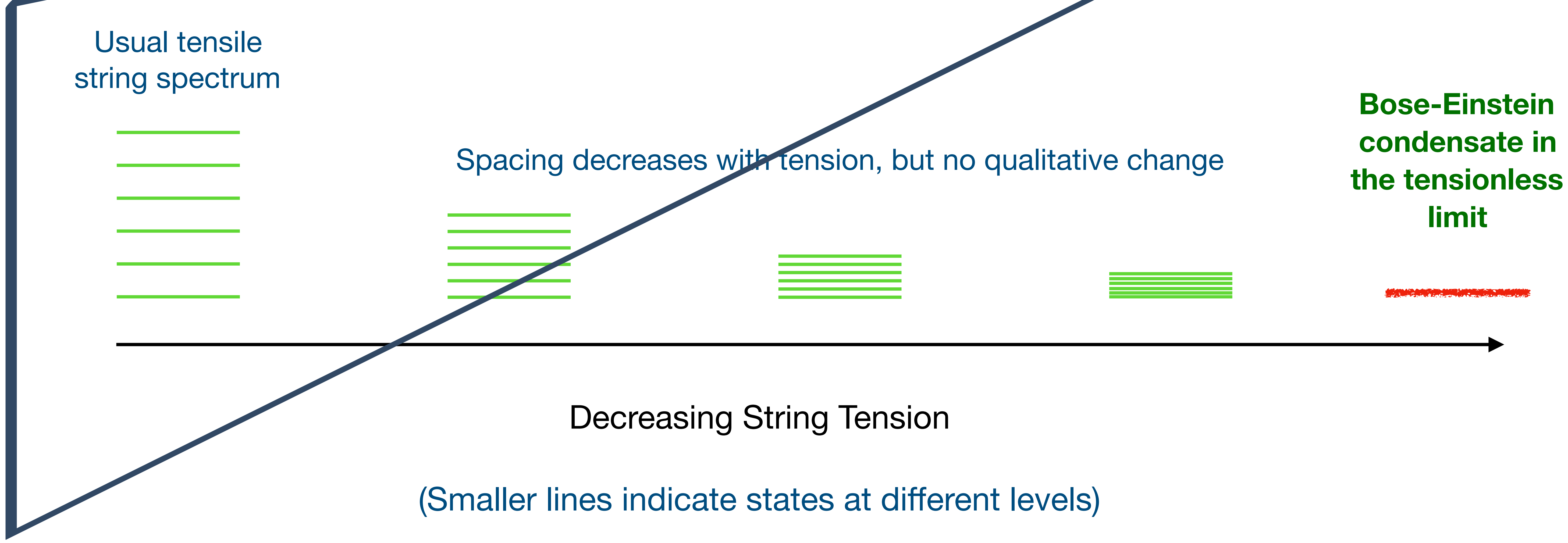
- Consider any perturbative state in the original tensile theory $|\Psi\rangle = \xi_{\mu\nu}\alpha^{\mu}_{-n}\tilde{\alpha}^{\nu}_{-n}|0\rangle_{\alpha}$ where $\xi_{\mu\nu}$ is a polarisation tensor. Let us attempt to understand the evolution of the state as $\epsilon \to 0$.
- Close to $\epsilon = 0$, the alpha vacuum can be approximated as follows: $|0\rangle_{\alpha} = |I\rangle + \epsilon|I_1\rangle + \epsilon^2|I_2\rangle + \ldots$
- In this limit, the conditions on the alpha vacuum translate to:

$$\alpha_n|0\rangle_{\alpha} = \tilde{\alpha}_n|0\rangle_{\alpha} = 0,\ n > 0$$
$$\Rightarrow \qquad B_n|I\rangle = 0, \forall n; \quad A_n|I\rangle + B_n|I_1\rangle = 0,\ A_{-n}|I\rangle - B_{-n}|I_1\rangle = 0,\ n > 0.$$

- One can now take this limit on the state:

$$\alpha_{-n}\tilde{\alpha}_{-n}|0\rangle_{\alpha} = \left(\frac{1}{\sqrt{\epsilon}}B_{-n} + \sqrt{\epsilon}A_{-n}\right)\left(\frac{1}{\sqrt{\epsilon}}B_{n} - \sqrt{\epsilon}A_{n}\right)(|I\rangle + \epsilon|I_1\rangle + \ldots). \to K|I\rangle$$

All perturbative closed string states condense on the open string induced vacuum.



Quantum Tensionless Strings II

Based on:

AB, Banerjee, Chakrabortty, PRL 2021.
AB, Banerjee, Chakrabortty, Dutta, Parekh, JHEP 2020.
AB, Mandlik, Sharma, JHEP 2021.
AB, Banerjee, Chakrabortty, Chatterjee, JHEP 2022.

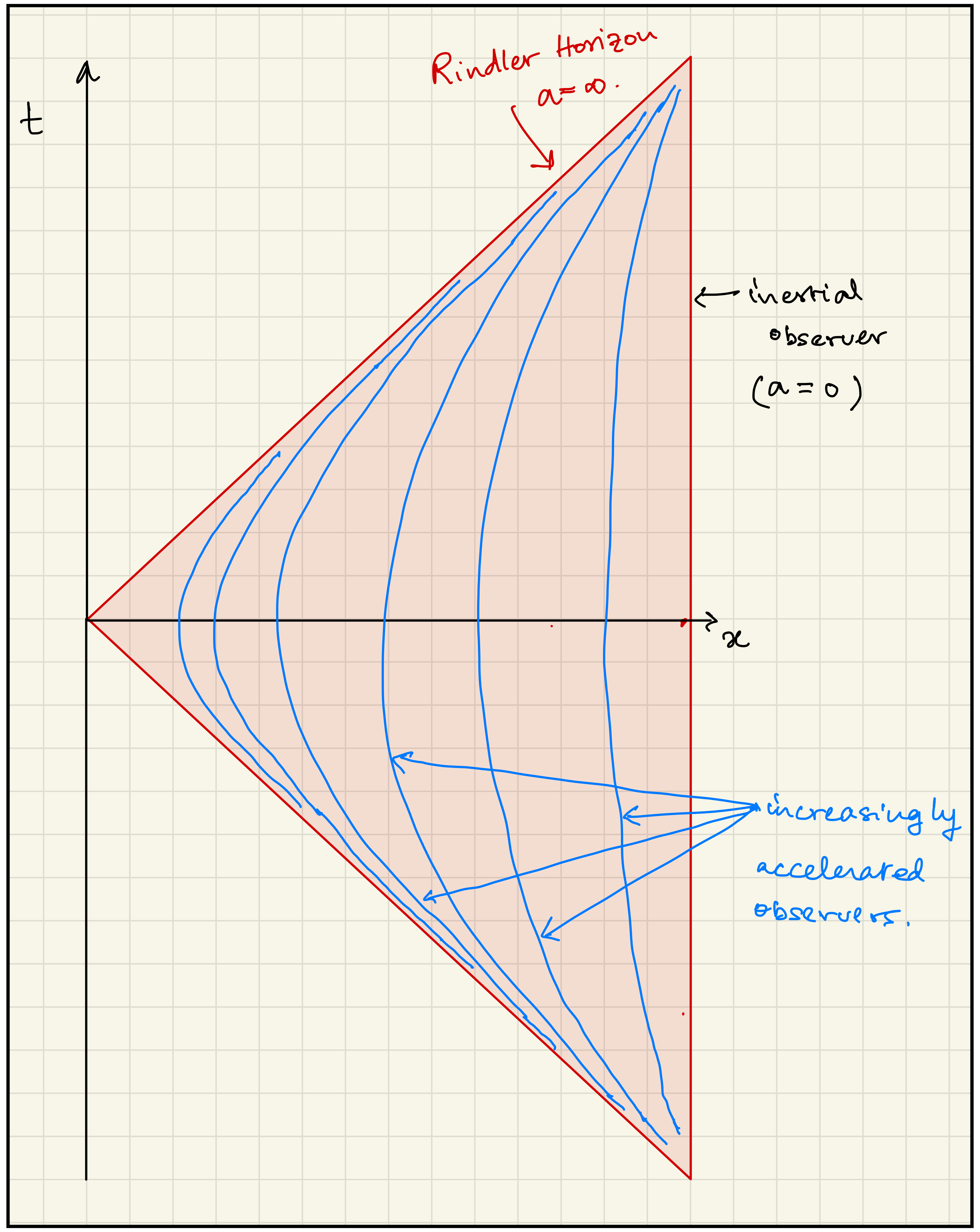
Tension and Acceleration

AB, Banerjee, Chakrabortty, Physical Review Letters 126 (2021) 3, 031601.

Tension as Acceleration

AB, Banerjee, Chakrabortty [PRL 2021]

- One of the most common occurrences of Bogoliubov transformations is in the physics of accelerated observers vis-a-vis inertial observers.
- Minkowski spacetime <-> Rindler spacetime.
- By identifying our Bogoliubov transformations to Rindler Bogoliubov transformations, we can recast the decrease of tension to the increase of acceleration.
- So, tensionless limit of string theory can be modelled as a series of worldsheet observers with increasing acceleration.
- The tensionless or null string emerges where the accelerated observer hits the Rindler horizon. This is where the acceleration goes to infinity.



A quick Rindler tour

- 2d Rindler metric: $ds_R^2 = e^{2a\xi}(-d\eta^2 + d\xi^2).$
- From Minkowski to Rindler $t = \frac{1}{a}e^{a\xi}\sinh a\eta,\ x = \frac{1}{a}e^{a\xi}\cosh a\eta$
- EOM: $\Box_{t,x}\phi = 0 = \Box_{\eta,\xi}\phi.$
- Minkowski mode expansion

$$\phi(\sigma,\tau) = \phi_0 + \sqrt{2\alpha'}\alpha_0\tau \ + \sqrt{2\pi\alpha'}\sum_{n>0}[\alpha_n u_n + \alpha_{-n}u_n^* + \tilde{\alpha}_n\tilde{u}_n + \tilde{\alpha}_{-n}\tilde{u}_n^*]$$

$$u_n = [ie^{-in(\tau+\sigma)}]/\sqrt{4\pi n}, \quad \tilde{u}_n = [ie^{-in(\tau-\sigma)}]/\sqrt{4\pi n}.$$

- Rindler mode expansion

$$\phi(\xi,\eta) = \phi_0 + \sqrt{2\alpha'}\beta_0\xi + \sqrt{2\pi\alpha'}\sum_{n>0}[\beta_n U_n + \beta_{-n}U_n^* + \tilde{\beta}_n\tilde{U}_n + \tilde{\beta}_{-n}\tilde{U}_n^*]$$

$$U_n = \frac{ie^{-in(\xi+\eta)}}{\sqrt{4\pi n}}, \quad \tilde{U}_n = \frac{ie^{-in(\xi-\eta)}}{\sqrt{4\pi n}}.$$

- The oscillators $\{\beta, \tilde{\beta}\}$ act on a new vacuum $|0\rangle_R$.
- U's act only in one wedge. To continue between them one defines smearing functions. Combinations for both wedges: $U_n^{(R)} - e^{-(\pi n/a)}U_{-n}^{(L)*}, \qquad U_{-n}^{(R)*} - e^{(\pi n/a)}U_n^{(L)}.$
- Relation between oscillators:

$$\beta_n = \frac{e^{\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\alpha_n - \frac{e^{-\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\tilde{\alpha}_{-n}, \quad \tilde{\beta}_n = -\frac{e^{-\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\alpha_{-n} + \frac{e^{\pi n/2a}}{\sqrt{2\sinh\frac{\pi n}{a}}}\tilde{\alpha}_n.$$

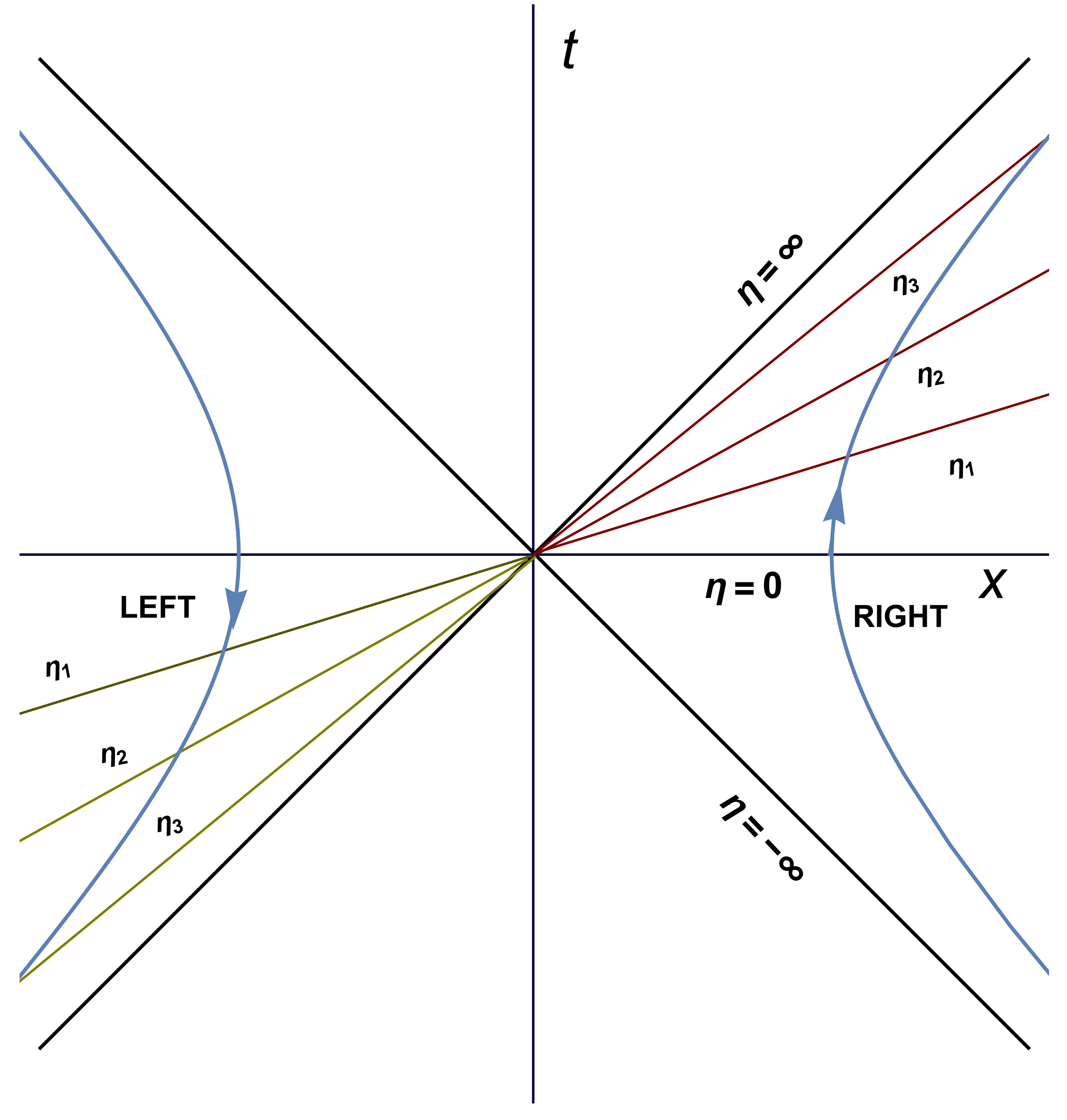
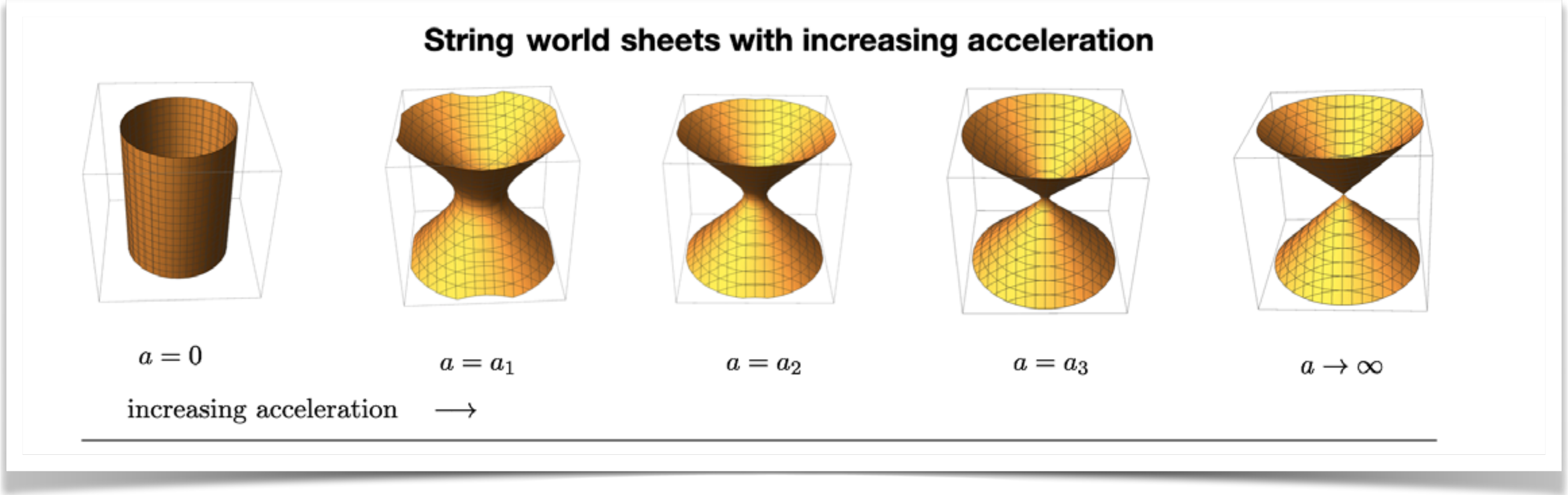


FIG. 1. Equal time slices in Rindler spacetimes.

Evolution in Acceleration

- String equivalent of Rindler observer hitting the horizon = increasingly accelerated world sheets.



- Rindler Bogoliubov transformation at large accelerations:

$$\beta_n^\infty = \frac{1}{2}\left(\sqrt{\frac{\pi n}{2a}}+\sqrt{\frac{2a}{\pi n}}\right)\alpha_n + \frac{1}{2}\left(\sqrt{\frac{\pi n}{2a}}-\sqrt{\frac{2a}{\pi n}}\right)\tilde{\alpha}_{-n}, \quad \tilde{\beta}_n^\infty = \frac{1}{2}\left(\sqrt{\frac{\pi n}{2a}}-\sqrt{\frac{2a}{\pi n}}\right)\alpha_{-n} + \frac{1}{2}\left(\sqrt{\frac{2a}{\pi n}}+\sqrt{\frac{\pi n}{2a}}\right)\tilde{\alpha}_n.$$

- Identification: $C_n = \beta_n^\infty, \qquad \tilde{C}_n = \tilde{\beta}_n^\infty, \qquad \epsilon = \dfrac{\pi n}{2a}.$

- The limit of zero tension is thus the limit of infinite acceleration: $\epsilon \to 0 \Rightarrow a \to \infty.$

- **Evolution:** $a=0$: $\{\beta_n, \tilde{\beta}_n\} \to \{\alpha_n, \tilde{\alpha}_n\}$, $0<a<\infty$: $\{\beta_n(a), \tilde{\beta}_n(a)\}$, $a\to\infty$: $\{\beta_n, \tilde{\beta}_n\} \to \{C_n, \tilde{C}_n\}$. Complete interpolating solution.

Hitting the Horizon: Evolution in Rindler Time

- We explored hitting the Rindler horizon by evolving in acceleration.
- The horizon can also be hit by evolving in Rindler time at constant acceleration.
- So the infinite time limit on the Rindler worldsheet would also generate the null string.

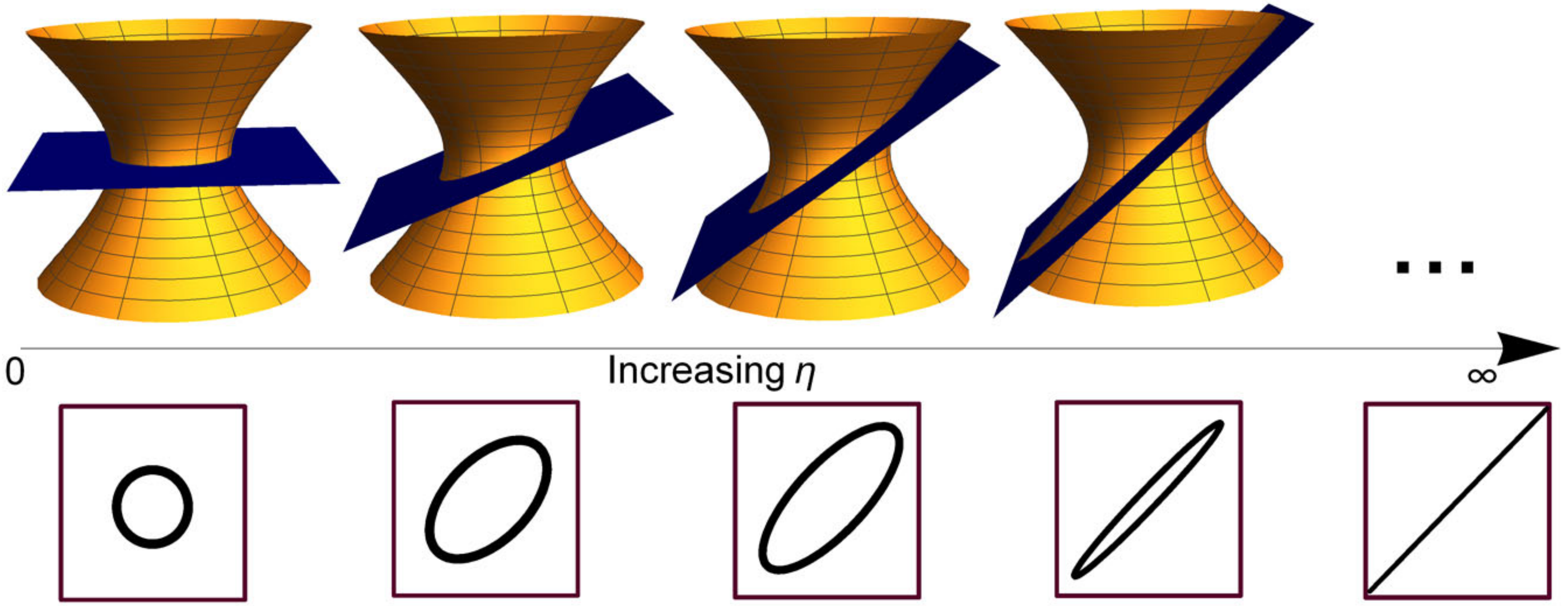


FIG. 3. Equal time slices of a Rindler worldsheet.

Hitting the Horizon: Evolution in Rindler Time

- Mathematically, this is the limit $\eta \to \infty$. Or equivalently,

$$\eta \to \eta, \qquad \xi \to \epsilon\xi, \qquad \epsilon \to 0.$$

- Conformal generators in Rindler: $\mathcal{L}_n, \bar{\mathcal{L}}_n = \pm \frac{i^n}{2} e^{n(\xi-\eta)}(\partial_\eta \mp \partial_\xi).$
- In the limit we get:

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} = i^n e^{-n\eta}(\partial_\eta - n\xi\partial_\xi),$$
$$M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}) = -i^n e^{-n\eta}\partial_\xi.$$

- These close to form the BMS algebra as expected and the null string emerges.


A Tale of Three
AB, Banerjee, Chakrabortty, Dutta, Parekh, JHEP 04 (2020) 061

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354

- From a single classical theory, several inequivalent quantum theories may emerge. This happens when we consider canonical quantisation of tensionless string theories.

- As we saw earlier Classical constraint on the tensionless string: $T_{(1)} = 0, \quad T_{(2)} = 0.$

  Quantum version: physical spectrum of tensionless strings restricted by $\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0.$

- This amounts to $\boxed{\langle phys|L_n|phys'\rangle = 0, \quad \langle phys|M_n|phys'\rangle = 0.}$

- For each type of oscillator F obeying $\langle phys|F_n|phys'\rangle = 0$, there can be three types of solutions.

$$
\begin{aligned}
&1.\quad F_n|phys\rangle = 0 \quad (n > 0),\\
&2.\quad F_n|phys\rangle = 0 \quad (n \neq 0),\\
&3.\quad F_n|phys\rangle \neq 0,\ \ but\ \langle phys'|F_n|phys\rangle = 0.
\end{aligned}
$$

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354

- Here $F_n = (L_n, M_n)$. Hence seemingly nine conditions:

$$L_m|phys\rangle = 0,\ (m>0),\ \begin{Bmatrix} M_n|phys\rangle = 0,\ (n>0) \\ M_n|phys\rangle = 0,\ (n\neq 0) \\ M_n|phys\rangle \neq 0,\ (\forall\ n) \end{Bmatrix};\ L_m|phys\rangle = 0,\ (m\neq 0),\ \begin{Bmatrix} M_n|phys\rangle = 0,\ (n>0) \\ M_n|phys\rangle = 0,\ (n\neq 0) \\ M_n|phys\rangle \neq 0,\ (\forall\ n) \end{Bmatrix};\ L_m|phys\rangle \neq 0,\ (\forall\ m),\ \begin{Bmatrix} M_n|phys\rangle = 0,\ (n>0) \\ M_n|phys\rangle = 0,\ (n\neq 0) \\ M_n|phys\rangle \neq 0,\ (\forall\ n) \end{Bmatrix}$$

- But the underlying BMS algebra also has to be satisfied. It turns out that only three of the nine choices lead to consistent solutions.

- These are three inequivalent vacua, leading to three inequivalent quantum theories.

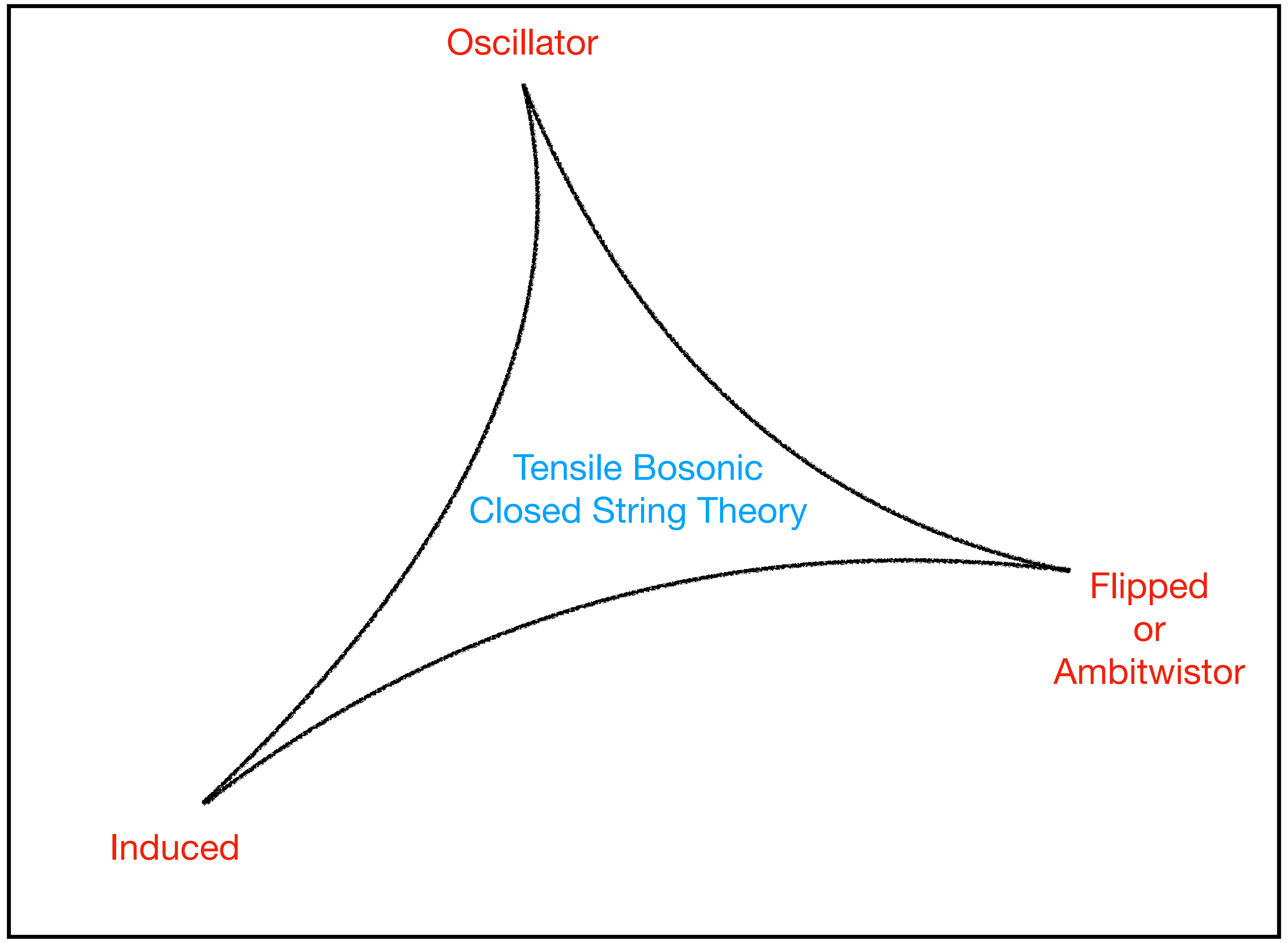
  - **Induced vacuum:** Theory obtained from the limit of usual tensile strings.

  - **Flipped vacuum:** Leads to ambitwistor strings. (See e.g. **Casali, Tourkine, (Herfray) 2016-17**)

  - **Oscillator vacuum:** Interesting new vacuum. Contains hints of huge underlying gauge symmetry.

Critical Dimensions

AB, Mandlik, Sharma. 2105.09682



Tensionless corners of Quantum Tensile String Theory

A summary of quantum results

* Novel closed to open string transition as the tension goes to zero.
  [AB, Banerjee, Parekh (PRL) 2019]

* Careful canonical quantisation leads to not one, but <u>**three different vacua**</u> which give rise to different quantum mechanical theories arising out of the same classical theory.
  [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]

* Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction on the other theory. All acceptable limits of quantum tensile strings.
  [AB, Mandlik, Sharma 2021]

* Interpretation in terms of Rindler physics on the worldsheet.
  [AB, Banerjee, Chakrabortty (PRL) 2021]

* Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

Other results

* Tensionless superstrings: Two varieties depending on the underlying Superconformal Carrollian algebra.

* **Homogeneous Tensionless Superstrings**: Fermions scale in same way.
  Previous construction: Lindstrom, Sundborg, Theodoridis 1991.
  Limiting point of view: AB, Chakrabortty, Parekh 2016.

* **Inhomogeneous Tensionless Superstrings**: Fermions scale differently.
  New tensionless string! AB, Banerjee, Chakrabortty, Parekh 2017-18.

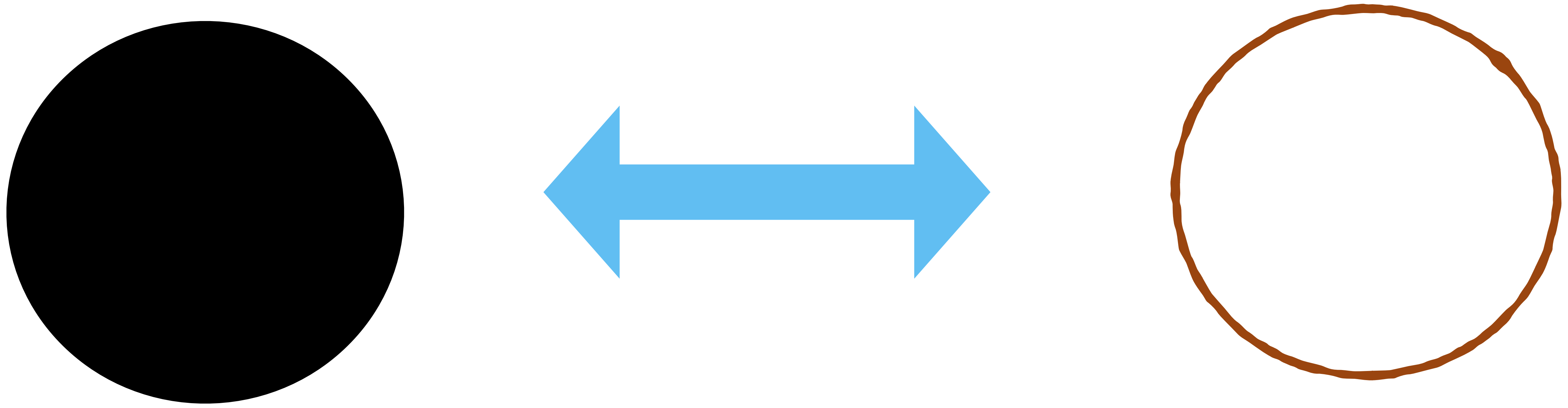
Open questions: Tensionless Strings

* Analogous calculation of beta-function=0. Consistent backgrounds?
* Linking up to Gross-Mende high energy string scattering from worldsheet symmetries.
* Attacking the Hagedorn transition from the Carroll perspective. Emergent degrees of freedom? [in progress with Banerjee, Mandlik]
* Strings near black holes, strings falling into black holes? [in progress with Banerjee, Hartong, Have, Kolekar, Mandlik]
* Extend "Tale of Three" to superstrings. Different superstring theories?
* Intricate web of tensionless superstring dualities?

Black hole Microstates from Null Strings

AB, Grumiller, Sheikh-Jabbari 2210.10794

Black holes from Null Strings?



- Event horizon of black holes are null surfaces.
- In d=3, consider BTZ black holes. Event horizon is a null circle.
- **Proposal:** A null string wrapping the event horizon contains in its spectrum the micro states of a BTZ black hole.
- We can reproduce **the Bekenstein-Hawking entropy** as well as its logarithmic corrections!
- Possible generalisations to higher dimensions.

Horizon Strings

* Proposal motivated by symmetries. Symmetries of event horizon same as symmetries of the null string worldsheet.
* Dynamic horizon on which d.o.f. live is then equivalent to a null string.
* Quantize the null string in **Oscillator Vacuum**. Use Lightcone gauge for convenience.
* Black hole states: a band of states with sufficiently high level.
* Mass is proportional to the radius of the horizon. Motivated by Near Horizon first law. [Donnay et al 2015, Afshar et al 2016].
* Complicated combinatorics leads to entropy and amazing the correct logarithmic corrections.
* Can be thought of as a precise formulation of the membrane paradigm.
* Generalization to d=4 with null membranes in progress and showing interesting signs.

Null strings to BTZ microstates

* ILST action: $\mathcal{S} = \frac{\kappa}{2}\int d\tau\, d\sigma\, (V^a\partial_a X^\mu)(V^b\partial_b X^\nu)\, G_{\mu\nu}(X)$. Metric: $G_{\mu\nu}\, dX^\mu\, dX^\nu = -2\, dx^+\, dx^- + R_h^2\, d\phi^2$
* Gauge fixed action: $\mathcal{S}_{\rm gf} = \frac{\kappa}{2}\int d\tau\, d\sigma\, \Big(-2(\partial_\tau X^+)(\partial_\tau X^-) + (\partial_\tau X^\varphi)^2\Big)$
* Allow strings to wind: $X^\varphi(\sigma + 2\pi, \tau) = X^\varphi(\sigma, \tau) + 2\pi R_h\, \omega$
* Null string states over oscillator vacuum: $|\Psi\rangle = |p^\mu, \{r_i\}, \{s_i\}, \omega\rangle$.

Vacuum: $|0, p^\mu, \omega\rangle \equiv |0\rangle$

Level 1: $J_{-1}|0\rangle, \tilde{J}_{-1}|0\rangle$

Level 2: $J_{-2}|0\rangle, J_{-1}^2|0\rangle, J_{-1}\tilde{J}_{-1}|0\rangle, \tilde{J}_{-1}^2|0\rangle, \tilde{J}_{-2}|0\rangle$

* Constraints: $s - r = \omega n$.
* Mass formula: $m^2 = (r + s)\kappa + \frac{n^2}{R_h^2}$. Notice **T-duality no more holds.**
* Now, microstates of the BTZ blackhole: $|m\rangle_{\rm BTZ} = |\{r_i\}, \{s_i\}, \omega, n\rangle$
* Near horizon first law: $m = \kappa R_h$. -> $\kappa R_h^2 = s + r + \frac{n^2}{\kappa R_h^2} := N + \frac{n^2}{\kappa R_h^2}$ $\quad N = r + s.$
* Approximation: **Large black hole.** $R_h \gg 1/\sqrt{\kappa}$

Entropy of BTZ black hole

* States from different sectors (complicated combinatorics):
  * Soft: string momentum vanishes $n = 0$
  * High momentum sector $n \gg N$. Exponentially suppressed. $n \approx N$ also exponentially suppressed.
  * Generic sector, typical microstates $N \gg n$.
  * Non-winding sector $\omega = 0$.
* Full partition function: $Z_{\text{BTZ}} = Z_{\text{soft}} + Z_{n \gg N} + Z_{n \approx N} + Z_{\text{generic}} + Z_{\omega=0}$.
* **Bekenstein-Hawking entropy:** $S = \ln Z_{\text{BTZ}} = 2\pi R_h \sqrt{\frac{\kappa}{3}} - \frac{3}{2} \ln R_h + o(\ln R_h)$.
* We fix $\kappa = \frac{3}{16G^2}$. This gives $S_{\text{BH}} = \frac{\text{Area}}{4G}$. We cannot get the 3/16 from anything yet. Only input!
* **But the coefficient of the log term is the real surprise. Unexpected! Must be something very deep!**

A theory of Black holes based on Null Membranes?

* Looks like the theory of null strings has something deep to say about BTZ microstates.
* Of course, there are questions. This is an effective theory.
* How can you make this quantum mechanically consistent? What about anomalies?
* Relatedly: Dimensions? Looks like D=3 for the moment. Add spectator D=23 dimensions? Wish away KK modes?
* But can we go further? Null 2-branes for D=4 Blackholes?
* Classical analysis [in progress AB, DG, MMS, others] seems to indicate that we do have an analogous infinite dimensional symmetry related to BMS4 at play here.
* Is it possible to quantise this? Can the infinite dimensional algebra work its magic again, unlike the relativistic case? We hope to come back with answers.


Thank you!