Tensionless Tales

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Non-Relativistic Strings and Beyond, NORDITA

Based on work done over the last decade

- AB, Mandlik, Sharma 2105.09682 (JHEP)
- AB, Banerjee, Chakrabortty, Chatterjee 2111.01172 (JHEP)
- AB, Grumiller, Sheikh Jabbari 2210.10794
- AB, Banerjee, Mandlik (upcoming)

Daniel Grumiller Shahin Sheikh-Jabbari

Aritra Banerjee Shankhadeep Chakrabortty

- AB 1303.0291 (JHEP)
- AB, Chakrabortty, Parekh 1507.04361 (JHEP)
- AB, Chakrabortty, Parekh 1606.09628 (JHEP)
- AB, Banerjee, Chakrabortty, Parekh 1710.03482 (JHEP)
- AB, Banerjee, Chakrabortty, Parekh 1811.03482 (PLB)
- AB, Banerjee, Parekh 1905.11732 (PRL)
- AB, Banerjee, Chakrabortty, Dutta, Parekh 2001.00354 (JHEP)
- AB, Banerjee, Chakrabortty 2009.01408 (PRL)

Pulastya Parekh Mangesh Mandlik Ritankar Chatterjee Punit Sharma Sudipta Dutta

Null Strings?! What? Why?

- Massless point particles move on null geodesics. Worldlines are null.
- * Null strings: extended analogues of massless point particles. Massless point particles => Tensionless strings.
- Tensionless or null strings: studied since Schild in 1970's.

Null strings are vital for:

A. Strings at very high temperatures: Hagedorn Phase.

- B. Strings near spacetime singularities: Strings near Black holes, near the Big Bang.
- C. Connections to higher spin theory.
- nit of string theory => Classical gravity.
-

Example 7
$$
T = \frac{1}{2\pi\alpha'}
$$
 \rightarrow 0: point particle lim

Tensionless regime: $T = \frac{1}{\Omega - \epsilon'} \to \infty$: ultra-high energy, ultra-quantum gravity! 1 $\overline{2\pi\alpha'}\rightarrow\infty$

Summary of Results

- 2d Conformal Carrollian (or BMS3) and its supersymmetric cousins arise on the worldsheet of the tensionless string replacing the two copies of the (super) Virasoro algebra.
- Classical tensionless strings: properties can be derived intrinsically or as a limit of usual tensile strings.
- Quantum tensionless strings: many surprising new results. \ast
- A theory of Black hole microstates based on null strings!

Classical Tensionless Strings

Isberg, Lindstrom, Sundborg, Theodoridis 1993 AB 2013; AB, Chakrabortty, Parekh 2015.

Going tensionless \blacksquare SYMMETRIES OF TENSIONLESS CLOSE EXPERIMENT IN A SYMMETRIES OF TENSIONLESS CLOSE EXPERIMENT IN A SYMMETRIES OF
Symmetries of the symmetries of the sy

Start with Nambu-Goto action:

To take the tensionless limit, first switch to Hamiltonian framework.

- **I** Generalised momenta: $P_m = T \sqrt{-\gamma} \gamma^{0\alpha} \partial_\alpha X_m$. $T = \frac{1}{\sqrt{2\pi}}$
- ▶ Constraints: $P^2 + T^2 \gamma \gamma^{00} = 0$, $P_m \partial_{\sigma} X^m = 0$. $\sum_{i=1}^{\infty} \frac{1}{2} \arctan 2 \arct$ Constraints: $P^{-} + I^{-} \gamma \gamma^{\circ} = 0$, $P_{m} O_{\sigma} X^{\gamma} = 0$
- ▶ Hamiltonian: $H_T = H_C + \rho^i(\text{constraints})_i = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho P_m \partial_\sigma X^m$. I Constraints: *P*² + *T*² ⁰⁰ = 0*, Pm*@ *X^m* = 0*.* **F I** Hammorian: $\pi T = \pi C + \rho$ (constraints)*i* = $\lambda (T + T \gamma)$ / $\tau \rho T_m \sigma \Delta$.

 \mathbf{A} and after integrating out momentain \mathbf{A} Action after integrating out momenta: Isborg, Lindstrom, Sundborg Isberg, Lindstrom, Sundborg,Theodoridis 1993

Isberg, Lindstrom, Sundborg, Lindstrom, Sundborg, Theodoridis 1993, Isberg, Theodoridis 1993

$$
(1)
$$

 $\sqrt{2}$

$$
S = \frac{1}{2} \int d^2 \xi \, \frac{1}{2\lambda} \left[\dot{X}^2 - 2\rho \dot{X}^m \partial_\sigma X_m + \rho^2 \partial_\sigma X^m \partial_\sigma X_m - 4\lambda^2 T^2 \gamma \gamma^{00} \right] \tag{2}
$$

Identifying Identifying

 $\alpha^{\alpha\beta}$ – $g^{\alpha\beta} =$

$$
S = -T \int d^2 \xi \sqrt{-\det \gamma_{\alpha\beta}}.
$$
 (1)

$$
\left(\begin{matrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2 T^2 \end{matrix}\right),
$$

action takes the familiar Weyl-invariant form action takes the familiar Weyl-invariant form

$$
S = -\frac{T}{2} \int d^2 \xi \sqrt{-g} g^{\alpha \beta} \partial_{\alpha} X^m \partial_{\beta} X^n \eta_{mn}.
$$
 (3)

 $S =$ Z *d*

- Starting point of tensionless strings.
- \blacktriangleright Need not refer to any parent theory. Treat this as action of fundamental objects.

$$
\blacktriangleright \hspace{0.2cm} T \to 0 \Rightarrow
$$

 \blacktriangleright Metric is degenerate. det $g = 0$.

density

 V^α

Action in $T \rightarrow 0$ limit

Going Tensionless … CLASSICAL TAPADAL Isberg, Lindstrom, Sundborg,Theodoridis 1993 SYMMETRIES OF TENSIONLESS CLOSED STRINGS

 \blacktriangleright Tensionless limit can now be taken systematically.

$$
g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 \end{pmatrix}.
$$

P Replace degenerate metric density $T\sqrt{-gg}^{\alpha\beta}$ by a rank-1 matrix $V^{\alpha}V^{\beta}$ where V^{α} is a vector

$$
V^{\alpha} \equiv \frac{1}{\sqrt{2}\lambda}(1,\rho) \tag{4}
$$

 $2\xi V^{\alpha}V^{\beta}\partial_{\alpha}X^{m}\partial_{\beta}X^{n}\eta_{mn}.$ (5)

Isberg, Lindstrom, Sundborg,Theodoridis 1993

Completing the square?

Your favourite thing in Tensile **String Theory**

Gauge and Residual Gauge Symmetries I Tensionless actions and a conformal symmetry after fixing conformal gauge = Vir a conformal gauge = Vir a co Gauge and Kesidual Gauge Symmetries I Fixing gauge: "Conformal" gauge: *V*↵ = (*v,* 0) (*v*: constant). I Fixing gauge: "Conformal" gauge: *V*↵ = (*v,* 0) (*v*: constant). Isberg, Lindstrom, Sundborg,Theodoridis 1993 Isberg, Lindstrom, Sundborg,Theodoridis 1993 I Tensionless actions action is invariant und

Tensionless action is invariant under world-sheet diffeomorphisms. Fixing gauge: "Conformal" gauge: $V^{\alpha} = (v, 0)$ (*v*: constant). I Tensionless: Similar residual symmetry left over after gauge fixing. ICHSIONICSS. OMINICI TOSICICAI SYMMICU Y I'LAIR gauge. Comonital gauge. $v = (v, v)$ (v. Constant). Tonsila: Rosidual symmatry after fivir I Tensionless: Similar residual symmetry left over after gauge fixing. INTRODUCTION CLASSICAL ASPECTS SUPERSTRINGS SUPERSTRINGS ASPECTS SUPERSTRINGS APPLICATIONS REMARKS ARE SERVICE Tensionless: Similar residual symmetry left over after gauge xing gauge: "Conformal" gauge: $V^{\alpha} = (v, 0)$ (v: constant). S^{111111} S^{111} S^{1111} $Symmetry$ after fixing conformal gauge $= \text{Vir } \otimes \text{ Vir } \; \text{Central to understand}$ f **t** over after gauge fixing. Tensile: Residual symmetry after fixin insionless: Similar residual symmetry left over after gauge fixing. ϵ insile: Kesidual symmetry after fixing conformal gauge = Vir \otimes Vir. Central to understanding string theories $\frac{1}{\sqrt{2}}$ Fixing gauge: "Conformal" gauge: $V^{\alpha} = (v, 0)$ (v: constant). $I^r \cap I^r \cap I^r$ Γ $\left(1 \right)$ $\left(1 \right)$ 'Iensionless action is invariant under world-sheet diffeomorp I Foreignless: Similar residual symmetry left over after gauge fiving Tensile: Residual symmetry after fixing conformal gauge = Vir \otimes Vir. I Tensionless: Similar residuel symmetry left over a function over a function over a function over a function
In the symmetry left over a function of the symmetry left over a function of the symmetry left over a function T_{α} is a property of the contract α is inversiont under world, choot diffeomorp T_{e} ile: R_{e} idual symmetry often fixing conformal gauge $\frac{V_{\text{e}}}{V_{\text{e}}} \otimes V_{\text{e}}$ Tensile: Residual symmetry after fixing conformal gauge = Vir \otimes Vir. Central to understanding string theory. Tensionless: Similar residual symmetry left over after gauge fixing. Γ I Tensionless: Similar residual symmetry left over after gauge fixing.

For world-sheet diffeomorphism: $\xi^{\alpha} \to \xi^{\alpha} + \varepsilon^{\alpha}$, change in vector density: Define: $L(f) = f'(\sigma)\tau \partial_{\tau} + f(\sigma)\partial_{\sigma}, \quad M(g) = g(\sigma)\partial_{\tau}.$ Expand: $f = \sum a_n e^{in\sigma}$, $+$ *in* τ $(\partial_{\tau}) = \sum_{n} a_{n} L_{n}, \quad M(g) =$ **1** \overline{a} $\sum_{n} b_{n}e^{in\sigma}\partial_{\tau}$ $[L_m, L_n] = (m - n)L_{m+n}$ m *f* \overline{a} $\int_{0}^{1} M_{m+n} + \frac{c_{M}}{m} (m^{3} - m) \delta_{m+n}$ لساسا
من $\frac{1}{2}$ $\varepsilon^{\alpha} = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\}$ י
1) (@ *·* ")*V*↵ $I(f)$ $I(f)$ $I(f)$ " = *f* $\frac{m}{n}$ Isberg, Lindstrom, Sundborg,Theodoridis 1993 **Define:** $L(f) = f'(\sigma)\tau \partial_{\tau} + f(\sigma)\partial_{\sigma}, \quad M(g) = g(\sigma)\partial_{\tau}$. Expand: $f = \sum a_n e^{in\sigma}, \quad g = \sum b_n e^{in\sigma}$ $I(f) = \sum_{p \in \mathcal{P}} a_p p^p$ $\frac{1}{2}$ $(\partial_z + i n \tau \partial_z)$ η $[L_n]$ = $(i$ $n (m - n)L_{m+n} + \frac{c_L}{4\pi}(m^3 - m)\delta_{m+n,0}, \quad [M_m, M_n] =$ $\left[\begin{array}{c} n\,,\,M_n \end{array} \right]$ = (*i n bne* $(m - m)\Delta_{m+n} + \frac{m}{12}(m - m)\delta_{m+n,0}.$ *n* $\sum_{i=1}^{\infty}$ $\int_l^{U_l I V I_l}$ *n* $[M_m, M_n] = 0.$ Iffeomorphism: $\xi^{\alpha} \rightarrow \xi^{\alpha} + \varepsilon^{\alpha}$, change in vector c $\alpha = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\}$ $\left\{\begin{array}{c}$ $\frac{1}{2}$ $\$ *i d i d i d i d i d i d i d i d i d i d i d i d j d j d j d j d j d j d j d j d j d j d j d j d j d j d j d j d* $L(f) = \sum$ *n ane* $e^{in\sigma}(\partial_{\sigma} + in\tau\partial_{\tau}) = \sum$ *n* a_nL_n , $M(g)$ = $\sum b_n e_n$ *bne* $[I_{\alpha\alpha} \quad I_{\alpha\alpha}]$ $\mu = (m - n)I_{m+1,n} + \frac{c_I}{m} (m^3 - m)\delta_n$ iism: $\xi^{\alpha} \to \xi^{\alpha} + \varepsilon^{\alpha}$, change in vector density: $\delta_{\varepsilon}V^{\alpha} = -V \cdot \partial \varepsilon^{\alpha} + \varepsilon \cdot \partial V^{\alpha} + \frac{1}{2}$ ries: for $V^\alpha=(v,0)$ $\partial^2 \partial^2 \phi$, Expand: $f = \sum a_n e^{in\sigma}$, $g = \sum b_n e^{in\sigma}$ *n* $b_n e^{in\sigma} \partial_\tau = \sum$ *n* $b_n M_n$. $(m^3 - m)M_{m+n} + \frac{c_M}{m^2}(m^3 - m)\delta_{m+n}$ ³ *^m*)*m*+*n,*0*,* [*Mm, Mn*] = ⁰*.* [*Lm, Mn*]=(*m n*)*Mm*+*ⁿ* + *cM* ¹² (*^m* ³ *^m*)*m*+*n,*0*.* (9) *n n* (σ) ∂ $M(\sigma) = \sigma$ *i b* θ *i i b i j d i f x nand: f* $\sigma = \sum a_n e^{in\sigma}$, $\sigma = \sum b_n e^{in\sigma}$ $U = \sum_{n} u_n e \quad (U_{\sigma} + III' U_{\tau}) = \sum_{n} u_n L_n, \quad IV(8)$ $[L_m, L_n] = (m - n)L_{m+n} +$ *cL* $\frac{12}{12}$ (*m*) $(3 - m)\delta_{m+n,0}, \quad [M_m, M_n] = 0.$ $[L_m, M_n]$ = $(m - n)M_{m+n}$ *cM* $\frac{1}{12}$ (*m*) $3 - m)\delta_{m+n,0}$. 1 2 $U = U(0) + 1 \delta(0)$, $U(f)$ $\partial \sigma$, $M(g) = g(\sigma) \partial_{\tau}$. Expand: $f = \sum u_n e$, $g = \sum v_n e$ α ^{whicm</sub>. ζ^{α} ζ^{α} + ζ^{α} change} \vdots (@ *·* ")*V*↵ Tensionless residual symmetries: for $V^{\alpha} = (v, 0)$, $\varepsilon^{\alpha} = \{f'(\sigma)\tau + g(\sigma), f(\sigma)\}$ ∂_{σ} , $M(g) = g(\sigma)\partial_{\tau}$. Expand: $f = \sum a_n e^{in\sigma}$, $g = \sum b_n e^{in\sigma}$ "*V*↵ ⁼ *^V ·* @"↵ ⁺ " *·* @*V*↵ ⁺ $+f(\sigma)\partial_{\sigma}$, $M(g) = g(\sigma)\partial_{\tau}$. Expan 2 $\frac{1}{\sqrt{2}}$ *inc* $\frac{1}{2}$ n $\frac{m}{m}$ $\overline{}$ $\left\{ \begin{array}{l}\n 0 \end{array} \right\}$ + $\frac{1}{12}$ (*M* $-$ *M*) $\sigma_{m+n,0}$, [*N*₁_m, *N*₁_n] = **U**. $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$ $\overline{1}$ $\mathrm{d}\cdot f=\sum a_i$ $L(y) = \sum_{n} u_n e \quad (O_{\sigma} + \mathcal{U} \cap \mathcal{O}_{\tau}) = \sum_{n} u_n L_n, \quad \mathcal{W} \cap \mathcal{W}$ \mathbf{m} $=$ *f* (*n*³ - *m*) δ_{m+n} [*M_m M_n*] = 0 IISILY. $\sigma_{\varepsilon} v = -v \cdot \sigma_{\varepsilon} + \varepsilon \cdot \sigma v + \frac{1}{2} (\sigma \cdot \varepsilon) v$ $\partial_{\tau} = \sum b_n M_n.$

$$
L(f) = \sum_{n} a_{n}e^{in\sigma} (\partial_{\sigma} + in\tau \partial_{\tau}) = \sum_{n} a_{n}L_{n}, \quad M(g) = \
$$

ER ENDS OF ZU CONTOR

BMS3 or 2d Conformal Carroll Algebra

m(*m* $n(m^2-1)\delta_{m+n,0}$ 12 *m*(*m* $[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \quad [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0}$ *m*(*m* $1)$ δ_{m+1} ⁰,0 \mathfrak{m} -1 ⁾ δ_{m+n} ⁰ $\mathcal{I} = \mathcal{I}$, $|\mathcal{L}_m, \mathcal{L}_n| = (m - n)\mathcal{L}_{m+n} + \mathcal{I}$

*n*essed as 2d conform

 $\bar{\cal L} = i e^{i n \omega} \partial_\omega, \quad \bar{\cal L}_n = i e^{i n \bar{\omega}} \partial_{\bar{\omega}}$

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m(*m*

Tensionless Limit from the Worldsheet Tensionless limit from the V \bullet TENSIONLESS LIMIT TROM THE V nition in (4.49) and put in them back into (4.33):

In Tensile string: Residual symmetry in conformal gauge $g_{\alpha\beta} = e$ $\mathbf{F} = \mathbf{F} \cdot \mathbf{F$ 2 *m ^C*°*^m ·Cm+ⁿ* °*C*˜°*^m ·C*˜*m*°*ⁿ* \mathbf{r}

nformal gauge
$$
g_{\alpha\beta} = e^{\phi} \eta_{\alpha\beta}
$$
:

$$
[\mathcal{L}_m, \mathcal{L}_n] = (m - n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0}
$$

$$
[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m - n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2 - 1)
$$

I World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the cylinder. cylinder. α *L*_{*M*} α *L*_{*J*} *IIIIder.* \mathbf{r} *m*(*m* I World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the \dot{u}

 $\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}}$ $\overline{}$

where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros. $L_n = le \t O_{\omega}, \t L_n = le \t O_{\overline{\omega}}$
where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoro where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros.

- \rightarrow **I** Tensionless limit \Rightarrow length of string becomes infinite ($\sigma \to \infty$). in the tensionless limit. This means that the square of the string length *Æ*⁰ ! 1. From α is integrated in Figure 4.1. In terms of worldsheet coordinated in α
- I LINS OF CRSCG STING IGENTIFICA \rightarrow INHIT DEST VIEWED as $(0 \rightarrow 0, 1 \rightarrow \epsilon_1, \epsilon_1 \rightarrow 0)$. If Ends of closed string identified \Rightarrow limit best viewed as $(\sigma \to \sigma, \tau \to \epsilon \tau, \epsilon \to 0)$.

- *n*
- $L_n = i e^{in\sigma} (\partial_\sigma + in\tau \partial_\tau), \quad M_n = i e^{in\sigma} \partial_\tau.$ *bnMn.* (8)
	- find previously. Close to form bivi₃. *reviously* . Close to

$$
L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}).
$$

$$
\Rightarrow \sigma, \tau \to \epsilon \tau, \epsilon \to 0
$$

ctively, $\frac{v}{c} \to \infty$

Tensionless Limit from the Worldsheet where \bm{r} is the contress \bm{r} . Vector fields generate centre-less \bm{r} I Tensionless Limit trom the Wa I Ends of closed string identified) limit best viewed as (! *,* ⌧ ! ✏⌧*,* ✏ ! 0). **Therefore Lang Lines & L** *L*(*f*) = X *ane in* (@ ⁺ *in*⌧@⌧) = ^X *i* Experience Limit TENSIONLESS LIMIT TROM THE

- **P** Tensionless limit on the worldsheet: $\sigma \to \sigma, \tau \to \epsilon \tau, \epsilon \to 0$ **•** Tensionless limit on the worldsheet: $\sigma \to \sigma, \tau \to \epsilon \tau, \epsilon$ If Tensionless limit on the worldsheet $\sigma \rightarrow \sigma \tau \rightarrow \epsilon \tau \epsilon \rightarrow 0$
	- \blacktriangleright Worldsheet velocities $v = \frac{\sigma}{\tau}$ $\frac{\sigma}{\tau} \rightarrow \infty$. Effectively, $\frac{v}{c}$ **I** WOITUSHEET VELOCILIES $\upsilon = \frac{\tau}{\tau} \to \infty$. Ellectively, $\frac{\tau}{c} \to \infty$ **Isberg et al.** Find \blacksquare worldsheet ve. ϕ locities $v = \frac{\sigma}{\phi} \to \infty$. Effectively. $\frac{v}{\phi} \to \infty$ \blacktriangleright Worldsheet velocities $v = \frac{\sigma}{\tau}$
	- \blacktriangleright Hence worldsheet speed of light \rightarrow 0. Carrollian limit. I TERRE WOHLSHEET SPEER OF HEAT 7 O. CALIONIAN MINIT. ► Hence worldsheet speed of light \rightarrow 0. Carrollian limit.
	- In Degenerate worldsheet metric. I Degenerate wondsneet method.
In the 3d flat space, Galilean field theories, Galilean field theories, Galilean field theories, Galilean field n_0 $\blacktriangleright \text{ Degenerate worldsheet metric.}$
- $\sum_{i=1}^{n}$ \blacktriangleright Riemannian tensile worldsheet \rightarrow Carrollian tensionless worldsheet. non-relativistic limit of Ademandian tensue \blacktriangleright Riemannian tensile worldsheet $-$ **I** Riemannian tensile worldsheet \rightarrow Carrollian tensionles

¹² (*^m*

anLn, (7) *n* A Bagchi 2013

$$
[m] = (m-n)M_{m+n} [M_m, M_n] = 0.
$$

cM

Tensionless EM Tensor and constraints I Speciess EM-Tensor and constraints I Spectrum of tensile string theory (in conformal gauge in flat space) in the original gauge in flat space) in
In the space of the I Spectrum of the spectrum of the space in the space in flat space in flat space in flat space in flat space I Spectrum of tensile string theory (in conformal gauge in flat space) IGUSIONIGSS EIVI IGUSOF ANU Tensionless EMA-Tensor and conservaing IGUANDINISSS EIM IGUAN GUN G TENSIONIESS EIM JENSOM AN

Spectrum of tensile string theory (in conformal gauge in flat space) I Spoctrum of tonsilo string theory (in conformal Specifium of tensue string theory (in conformal gauge in octrum of tensile string theory (in conformal gauge in flat sp In the space of the state of the state of the complete description of the space I ectrum of tensile string theory (in conformal gauge in flat si I Op form: Physical states vanish under action of modes of E-M tensor. \mathcal{L} operation of modes vanish under action of \mathcal{L} I ectrum of tensile string theory (in conformal gauge in Ω of the side strips theory (in conformal cause in flat engage)

- \blacktriangleright Quantise worldsheet theory as a theory free scalar fields. \blacktriangleright Constraint: vanishing of EOM of metric (which is fixed to be flat). \blacktriangleright Quantise worldsheet theory as a theory free s \triangleright Constraint: vanishing of EOM of metric (which is fixed to \blacksquare I Continue vanishing of EVIII of metric (which is media I Op form: Physical states vanish under action of modes of E-M tensor. **I Quantise** worldsheet theory as a theory free scalar field \Box Constraint: vanishing of \Box of \Box is fixed to be flat \Box **I Quantise** worldsheet theory as a theory free scalar fields. Constraint: vanishing of EON
- **Op form: Physical states vanish under action of modes of E-M tensor.** r<mark>m</mark>: Physical states vanish under actic \triangleright On form Physical states *Tcyl* = *z* $\overline{}$ al states va $\frac{1}{2}$ action c of modes of E-M tensor. *Tplane ^c in*! *^c* ► Constrant, vanishing of EOM of metric (which is fixed to the **Property of Strings in conformal gauge** *e*

CHOOT IOT 2d CT T ON Cymruch. I $cyl =$ EM tensor for 2d CFT on cylinder: $T_{cyl} = z$ 2 $T_{plane} - \frac{c}{24}$ $\overline{}$ *The CFT on cy* $\overline{\text{11}}$ $T_{cyl} = z^2 T_p$ *Tcyl* = *z Tcyl* = *z Tplane ^c* $\overline{}$ \overline{T} \mathcal{L} *n Lne in*! *^c* \overline{T} $=$ $X = 0$ *Lne in*! *^c n* I The Ultra-relativistic EM tensor *Tcyl* = *z* \prod nder: T $u_l = z^2$ $T_{plane} - \frac{c}{24}$ 24
24 Juni 10
24 Juni 10 $Z_{\textit{cul}} = z^2 T_{\textit{pla}}$ [*Lm, Ln*]=(*m n*)*Lm*+*ⁿ* + $-$ *z i pl*

Tcyl = *z* $\frac{1}{2}$ *c* $\frac{1}{2}$ *c* $\frac{1}{2}$ \boxed{T} $\overline{\pi}$ *n* $L_{\text{av}} = \lim_{\tau \to 0} \left(T_{1} - \overline{T}_{1} \right) T_{\Omega} = \lim_{\epsilon} \epsilon$ $T = \lim_{T \to \infty} \left(T - \bar{T} \right) = \bar{Y}$ *Lne in*! *^c* **T** $=\lim_{\epsilon\to 0}\epsilon\Big(T_{cyl} \left(\begin{array}{c} 1 \\ -1 \end{array} \right)$ \sum Ultra-relativistic EM tensor $T_{(1)} = \lim_{\epsilon \to 0}$ $T_{(2)} = \lim_{\epsilon \to 0} \epsilon \Big(1$ \bar{T}_x $T_{(2)} = \lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \overline{T}_{cyl} \right) = \sum M_n e^{in\sigma}$ $\epsilon \rightarrow 0$ $\left(T_{cyl}-\bar{T}_{cyl}\right)$ $T_{(1)} = \lim_{\epsilon \to 0} \left(T_{cyl} - \bar{T}_{cyl} \right) = \sum$ $\epsilon \rightarrow 0$ ϵ $T_{(2)} = \lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \overline{T}_{cyl} \right) = \sum$ $\epsilon \rightarrow 0$ $\left(T_{cyl} - \overline{T}_{cyl}\right)$ $\tau_{\text{cut}} = \lim_{l \to \infty} \left(T_{\text{cut}} - \bar{T}_{\text{cut}} \right) = \sum$ $\epsilon \rightarrow 0$ ✏ $\sum_{\epsilon \to 0}$ $\lim_{\epsilon \to 0} \epsilon \left(T_{cyl} + \overline{T}_{cyl} \right) = \sum$ $\begin{array}{c} \begin{array}{c} \end{array}$ $\overline{T}_{\text{cyl}}$ $\overline{Y}_{\text{cyl}}$ $\overline{Y}_{\text{cyl}}$ $\sqrt{2}$ y \overline{T}_{cyl} = \sum $I \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot I$ $\overline{1}$. ✓ *Tcyl ^T*¯*cyl*◆ $\epsilon \rightarrow 0$ τ _c \rightarrow 0 τ $\overline{1}$ $\left(\begin{array}{c} 1 - \bar{T}_{cyl} \end{array} \right) \, = \, \sum_n$ $\left(\begin{array}{c} \n + T_{cyl} \ \n \end{array} \right) = \sum_{n} P_{n}$ **Illtra-relativistic EM tensor** $T_{(1)} = \lim_{\epsilon \to 0} \left(T_{cyl} - \overline{T}_{cyl} \right) = \sum_n (L_n - in_{\epsilon})$ $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{\infty}$ cylinder. Symmetry best expressed as $\frac{1}{n}$

> $(\gamma \circ \gamma) = 0, \quad (\gamma \circ \gamma) = 1$
 $(\gamma \circ \gamma) = 0.$ i = 0*.* (17)

nsiol λ tensionless string: $T_{(1)} = 0$, cal constraint on the tensionless string: T₍ ✏!0 t on the tensionless string: \blacktriangleright Classical constraint on the tensionless string: $T_{(1)} = 0$, $T_{(2)} = 0$. **I** Classical constraint on the tensionless string: $T_{(1)} = 0$, $T_{(2)} = 0$. ✏!0

 \cup (*Ln in*⌧*Mn*)*e* (2) $\Gamma_{(1)} = 0,$ *n* ²⁴ (16) I Classical constraint on the tensionless string: *T*(1) = 0*, T*(2) = 0.

$$
+\bar{T}_{cyl}\bigg) = \sum_{n} M_n e^{in\sigma} - \frac{c_M}{24}
$$

 $\frac{1}{2}$ rum! of topejopless stripes rost \dot{M} *i* by $\langle \text{phys}|I_{(1)}| \text{phys} \rangle = 0, \quad \langle \text{phys}|I_{(2)}| \text{phys} \rangle$ I Classical constraint on the tensionless string: *T*(1) = 0*, T*(2) = 0. I Classical constraint on the tensionless string: *T*(1) = 0*, T*(2) = 0. I Quantum version: physical spectrum of tensionless strings restricted by **D** Quantum version: physical spectrum of tensionless strings restricted by $\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0.$ • Quantum version: physical spectrum of tensionless strings restricted by hphys*|T*(1)*|*phys⁰ i = 0*,* hphys*|T*(2)*|*phys⁰ $\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0.$ *^Lⁿ* ⁼ *iein*!@!*, ^L*¯*ⁿ* ⁼ *iein*!¯ @!¯ (11)

$$
\mu_{nl} = \sum_{n} (L_n - in\tau M_n) e^{in\sigma} - \frac{c_L}{24}
$$

$$
T_{plane} - \frac{c}{24} = \sum_{n} \mathcal{L}_n e^{in\omega} - \frac{c}{24}; \quad \bar{T}_{cyl} = \sum_{n} \bar{\mathcal{L}}_n e^{in\bar{\omega}} - \frac{\bar{c}}{24}
$$

A Bagchi 2013

= 2*c* $\frac{1}{2}$ *i* $\frac{1}{2}$ *b* $\frac{1}{2}$ *consider* $\frac{1}{2}$ Γ ² annia form obtained earlier *n*
Familiar form obtained earlier from purely algebraic considerations.

AB, Chakrabortty, Parekh 2015

AB, Chakrabortty, Parekh 2015 in (18) *in* (18)

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ⁿ in⌧*B^µ*

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Intrinsic Analysis: EOM and Mode Expansions AB, Chakrabortty, Parekh 2015 INTRINSIC ANALYSIS: EOM AN
Intrinsic analysis: Eom A INTENSIC ANAlysis. EVIVI ANU Intrincip Analysis: EOM a AB, Chakrabortty, Parekh 2015 INTRINSIC ANALYSIS: EOM AND SOLUTIONS AAA, VAAR \mathbb{R} and \mathbb{R} I Equation of motion in *V^a* = (*v,* 0) gauge: *X*¨*^µ* = 0. IMI FINSIC FINAIYSIS. EVI I Equation of motion in *V^a* = (*v,* 0) gauge: *X*¨*^µ* = 0. I Equation of motion in *V^a* = (*v,* 0) gauge: *X*¨*^µ* = 0. *Intrinsic Analysis: EOM and Mode Expari* 1 *n* $\bm{310} \bm{M3}$ I ANIA ILIANO PUNAICIPIAI INTRODUCTION CLASSICAL ASPECTS SUPERSTRINGS QUANTUM ASPECTS APPLICATIONS REMARKS

- **I** Equation of motion in $V^a = (v, 0)$ gauge: $\ddot{X}^{\mu} = 0$. Equation of motion in $V^a = (v, 0)$ gauge: $X^{\mu} = 0$. Γ suchiam of motion in $V^a = (v, 0)$ gauge: $\ddot{X}^\mu = 0$. ^I Closed string b.c.: *^Xµ*(*,* ⌧) = *^Xµ*(⁺ ²⇡*,* ⌧)) *^A^µ* $\text{Equation of motion in } V = (0,0) \text{ gauge.} \quad \text{A} = 0.$ **Interest Equation of motion in** $V^u = (v, 0)$ gauge: $X^{\mu} = 0$.
- Solution: $X^{\mu}(\sigma,\tau) = x^{\mu} + \sqrt{2c'}A_0^{\mu}\sigma + \sqrt{2c'}B_0^{\mu}\tau + i\sqrt{2c'}\sum_{n=1}^{\infty}$: $X^{\mu}(\sigma \tau) = r^{\mu} + \sqrt{2c'} A^{\mu} \sigma + \sqrt{2c'} R^{\mu} \tau +$ $\sum_{i=1}^{n}$ (*b*, *i*) = *x* | **v** 2*z*₁₁₀ *b* | **v** 2*c D*₀^{*i*} | *i* **v** \bullet Solution: $X^{\mu}(\sigma,\tau) = x^{\mu} + \sqrt{2c^{\prime}} A^{\mu}_{\sigma} \sigma + \sqrt{2c^{\prime}} B^{\mu}_{\sigma}$ ^I Closed string b.c.: *^Xµ*(*,* ⌧) = *^Xµ*(⁺ ²⇡*,* ⌧)) *^A^µ* $\sum_{i=1}^{n} a_i$ $\sqrt{2c'}A_0^{\mu}$ $\sigma + \sqrt{ }$ INTRINSIC ANALYSIS: EOM AND SOLUTIONS
	- $\int_0^1 \cosh(\sinh(\pi x)) \, dx = \int_0^1 \sin(\pi x) \, dx = \int_0^1 \cos(\pi x) \, dx$ $\lim g b.c.: X^{\mu}(\sigma, \tau) = X^{\mu}(\sigma + 2\pi, \tau) \Rightarrow A$ $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$ ▶ Closed string b.c.: $X^{\mu}(\sigma, \tau) = X^{\mu}(\sigma + 2\pi, \tau) \Rightarrow A_0^{\mu}$ I Closed string b.c.: $X^{\mu}(\sigma, \tau) = X^{\mu}(\sigma + 2\pi, \tau) \Rightarrow A_0^{\mu} = 0$ IIIIg D.C.. Λ^{\prime} (0, • Closed string $\frac{1}{2}$ \therefore $X^{\mu}(\sigma, \tau) = X^{\mu}$ = 2*c* $2\pi,\tau$ $\overline{}$ 2 see the strip $\overline{\text{ob}}$ $\therefore X^{\mu}(\sigma,\tau)=$
	- $\sum_{m,n} B_{-m} \cdot B_{m+n} e^{inS} = 0, \quad X \cdot$ $\dot{X}^2 = 2c'$, $\sum B_{-m} \cdot B_{m+n} e^{in\sigma} = 0$, $\dot{X} \cdot X' = 0$ **I** $\frac{1}{m,n}$ $\frac{1}{n}$ Constraints: **B** ⁼ ⁰*, ^X*˙ *· ^X*⁰ Γ Constraints $\overline{m}, \overline{n}$ *in* ⁼ ⁰*, ^X*˙ *· ^X*⁰ = 2*c* \blacktriangleright Constraints: $\dot{X}^2 = 2c$ $\sqrt{ }$ *m,n* $B_{-m} \cdot B_{m+n}$ *e* \overline{M} $\frac{1}{\sqrt{2}}$ $a^2 = 2c'$ $\sum B$ *<i>P*_{*n*} *in*^{*d*}</sup> \overline{X} *X* \overline{Y} \overline{Z} \overline{Z} \overline{Z} \overline{Z} \overline{Y} \overline{B} \overline{Z} \over I Define: m,n $\sum_{m,n}$

$$
\overline{m,n}
$$

\n- Solution:
$$
X^{\mu}(\sigma,\tau) = x^{\mu} + \sqrt{2c'}A^{\mu}_{0}\sigma + \sqrt{2c'}B^{\mu}_{0}\tau + i\sqrt{2c'}\sum_{n\neq 0}\frac{1}{n}\left(A^{\mu}_{n} - in\tau B^{\mu}_{n}\right)e^{in\sigma}
$$
\n- Chosed string b.c.: $X^{\mu}(\sigma,\tau) = X^{\mu}(\sigma+2\pi,\tau) \Rightarrow A^{\mu}_{0} = 0.$
\n- Constraints: $\dot{X}^{2} = 2c'\sum_{m,n} B_{-m} \cdot B_{m+n}e^{in\sigma} = 0$, $\dot{X} \cdot X' = 2c'\sum_{m,n} (A_{-m} - in\tau B_{-m}) \cdot B_{m+n}e^{in\sigma} = 0$
\n

- $Classical$ constraints *m*, *stra* **In Classical constraints in terms of modes:** Ω and Ω are Ω the modes are Ω *m* $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{n}$ $\frac{1}{2}$ *M* $\frac{1}{2}$ *M* $\frac{1}{2}$ *M* $\frac{1}{2}$ $\frac{1}{2}$
- *m,n* X *B^m · Bm*+*ⁿ e* e algebra of the modes *l* **<u>zebra</u>** of the mo $\sum_{i=1}^{n}$ **•** The algebra of the modes ${A_m^{\mu}, A_n^{\nu}\} = 0, \quad {B_m^{\mu}, B_n^{\nu}\} = 0, \quad {A_m^{\mu}, B_n^{\nu}\}$ des des $\left(\begin{array}{ccccc} \mu & \mu & \rho \end{array}\right)$ of $\left(\begin{array}{ccccc} \mu & \rho & \rho \end{array}\right)$ Note: this is *not* the algebra of harmonic oscillator modes. (More later.) Note: this is *not* the algebra of harmonic oscillator modes. (More later.) $I - m = n$ is $I - m = n$ of tensor $I - m = n$ is $I - m$, $I - n$,
	- I Familiar form obtained earlier from purely algebraic considerations. Note: this is *not* the algebra of harmonic oscillator modes. (More later.) Quantization: $\{ , \}_{PB} \rightarrow -\frac{i}{\hbar} [,]$ leads to the BMS₃ Algebra. Note: this is *not* the algebra of harmonic oscillator modes. (More later.) ▶ The worldsheet symmetry algebra of tensionless strings, now constructed from the quadratics of the modes: $(-m, -n)$ $(-m, -n)$ \overline{a} the modes: *{Lm, Ln}* = *i*(*m n*)*Lm*+*n, {Lm, Mn}* = *i*(*m n*)*Mm*+*n, {Mm, Mn}* = 0*.* (21)

$$
\sum_{m,n} B_{-m} \cdot B_{m+n}
$$
\n
$$
\sum_{m} A_{-m} \cdot B_{m+n}, \quad M_n = \sum_{m} B_{-m} \cdot B_{m+n}
$$
\n
$$
\sum_{m} C_{-m} \cdot B_{m+n}
$$

$$
= 0.
$$

 $\sqrt{ }$

$$
\sum_{n}^{m} (L_n - in\tau M_n) e^{in\sigma} = 0 = T_{(1)}, \sum_{n} M_n e^{in\sigma} = 0 = T_{(2)}.
$$

Family form obtained earlier from purely algebraic considerations.

n

$$
{n}^{\nu}\} = 0, \quad \{A{m}^{\mu}, B_{n}^{\nu}\} = -im\delta_{m+n,0} \eta^{\mu\nu}.
$$

$$
\{L_m, L_n\} = -i(m-n)L_{m+n}, \{L_m, M_n\} = -i(m-n)M_{m+n}, \{M_m, M_n\} = 0.
$$

n $\frac{1}{\sqrt{2}}$]*.*

Limiting Analysis: EOM and Mode Expansions LIMITING ANALYSIS AB, Champion <u>In the string mode</u> **I THE STRING MODE EXPANSION:** AB, Chakrabortty, Parekh 2015 Limiting Analysis[.] FOM and Mode Expansions r2*c*⁰ LIMITING RNALYSIS. EVIVI ANU IVIUUE LIMIT ANALYSIS CON A AB, Chakrabortty, Parekh 2015

- **Industring mode expansion:** \mathbf{X}^{μ} is the paradetermination: \mathbf{X}^{μ} (\mathbf{X}^{μ}) = \mathbf{x}^{μ} *n*6=0 $\frac{1}{\sqrt{2}}$ $+2$ \mathbf{A} $X^{\mu}(\sigma)$ μ ₁ *n*6=0 *n* I Eitsile string mode expansion: $X^{\mu}(\sigma, \tau) = x^{\mu} + 2\sqrt{2\alpha'}\alpha_0^{\mu}\tau + i$ $X^{\mu}(\sigma,\tau)=x^{\mu}+2$
- $\lim g$ procedure: $\tau \to \epsilon \tau$, $\sigma \to \epsilon$ \rightarrow $\sigma,$ α' $= c'/\epsilon$ with The limiting procedure: $\tau \to \epsilon \tau$, $\sigma \to \sigma$, $\alpha' = c'/\epsilon$ with $\epsilon \to 0$ I $\left(\begin{array}{c} 0, & \cdots \end{array}\right)$ if $\left(\begin{array}{c} 0, & \cd$ $= x^{\mu} + 2\sqrt{2}$ $\frac{1}{2}$ colorum $\sqrt{2}$ α_0^{μ} , $\int_{0}^{\mu} \tau + i \sqrt{2}$ $\overline{2c}$ $\frac{1}{2}$ \vert *n*6=0 α $= x^{\mu} + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^{\mu}\tau + i\sqrt{2c'}\sum_{n} \frac{1}{n} \left[\frac{\alpha_n^{\mu} - c}{\sqrt{\epsilon}} \right]$ $X^{\mu}(\sigma,\tau)=x^{\mu}+2$ $\sqrt{2c}$ ✏ α_0^{μ} $\epsilon \tau + i$ $\sqrt{2c}$ ✏ $\sqrt{ }$ $n{\neq}0$ 1 *n* $\sqrt{2c}$ $n{\neq}0$ 1 *n* $\int \alpha_n^{\mu} - \tilde{\alpha}_-^{\mu}$ $= x^{\mu} + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^{\mu}\tau + i\sqrt{2c'}\sum_{n=0}^{\infty}\frac{1}{n}\left[\frac{\alpha_0^{\mu} - \tilde{\alpha}_{-n}^{\mu}}{\sqrt{\epsilon}} - in\tau\right]$ **I** The limiting procedure: $\tau \to \epsilon \tau$, $\sigma \to \sigma$, $\alpha' = c'/\epsilon$ with $\epsilon \to 0$ $X^{\mu}(\sigma, \tau) = x^{\mu} + 2\Lambda$ $\sqrt{2c}$ $\overline{}$ µ
∩ ∈ 7 $\epsilon \tau + i \sqrt{\frac{2}{\pi}}$ $\overline{2c}$ ^{$\overline{2}$} ✏ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ η ⁿ $\frac{1}{n}$ $e^{-in\sigma}$ $\tau + i\sqrt{2}$ \overline{c} ¹ $\frac{1}{2}$ ϵ $\frac{\mu}{n} - \hat{C}$ $\frac{1}{\sqrt{2}}$. $\frac{1}{\sqrt{2}}$ XZ^{μ} λ μ + 2 λ r2*c*⁰ r2*c*⁰ $\chi_0^{\, \prime}$ $\frac{1}{2}$ $\frac{1}{2}$ ✏ $\mathfrak{g}_{\mathfrak{n}}$ *n* $= x^{\mu} + 2\sqrt{2c'}(\sqrt{\epsilon})\alpha_0^{\mu}\tau +$ $+ i\sqrt{2c'}$ $\sum_{i=1}^{\infty} \frac{1}{i} \left[\frac{\alpha}{i} \right]$ *n*6=0 \mathcal{L} I'lle limiting procedure: $\tau \to \epsilon \tau$, $\sigma \to \sigma$, $\alpha' = c'/\epsilon$ with $\epsilon \to 0$ $X^{\mu}(\sigma,\tau) = x^{\mu} + 2^{\mu}$ $\sqrt{\frac{2c'}{c}}$ ✏ χ_0^{μ} ϵ $\epsilon \tau + i \sqrt{}$ $\sqrt{\frac{2c'}{}}$ $\frac{1}{2}$ *n*6=0 $\overline{1}$ $\tilde{\alpha}_n^{\mu}e^{-\mu}$ $= x^{\mu} + 2\sqrt{2}c'(\sqrt{\epsilon})\alpha_0^{\mu}$ \overline{a} + \overline{a} \overline{b} $\frac{1}{2c}$ 1 *n* $e: \tau \to \epsilon \tau, \ \sigma \to \sigma, \ \alpha' = c'/\epsilon$ with $I = r^{\mu} + 2\sqrt{2c'} (\sqrt{\epsilon})\alpha^{\mu} \tau + i\sqrt{2c'} \sum \frac{1}{m} \left[\frac{\alpha^{\mu}_{n} - \tilde{\alpha}^{\mu}_{-n}}{m} - i n \tau \sqrt{\epsilon} (\alpha^{\mu} + \tilde{\alpha}^{\mu}) \right] e^{i n \sigma}$ The limiting pro $\int \frac{2c'}{\epsilon} \alpha_0^{\mu} \epsilon \tau + i \sqrt{\frac{2c'}{\epsilon}} \sum_i \frac{1}{n} [\tilde{\alpha}_n^{\mu} e^{-in\sigma} (1 - i\pi)$ $I = x^{\mu} + 2\sqrt{2}c'(\sqrt{\epsilon})\alpha_0^{\mu}\tau + i\sqrt{2}c' \sum_{n \neq 0} \frac{1}{n} \left[\frac{n}{\sqrt{\epsilon}} - in\tau\sqrt{\epsilon} \right]$ I The limitive: **1999**
- <u>—</u> $\int \tilde{\alpha}_n^{\mu} e$ $\sqrt{2c'}\sum_{n=1}^{\infty}\frac{1}{n}$ $\frac{n}{\sqrt{\epsilon}}$
- get a relation between t ⁰ ⌧ + *i* **p**
n
*c*000
*c*000 • Thus we get a relation between the tensionless and tensile modes: **Fig. 111 Thus we get a relation between the tensionless and tensile mode** $\frac{1}{2}$ *n* $SS \hat{c}$ $\frac{d}{dt}$ Thus we get a relation between the tensionless and tensile modes: zec a relation servedir are tendicities and t

s we get a relation between the tensions and tensine modes:
\n
$$
A_n^{\mu} = \frac{1}{\sqrt{\epsilon}} (\alpha_n^{\mu} - \tilde{\alpha}_{-n}^{\mu}), \quad B_n^{\mu} = \sqrt{\epsilon} (\alpha_n^{\mu} + \tilde{\alpha}_{-n}^{\mu}).
$$

$$
2\sqrt{2\alpha'}\alpha_0^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0} \frac{1}{n} [\tilde{\alpha}_n^{\mu}e^{-in(\tau+\sigma)} + \alpha_n^{\mu}e^{-in(\tau-\sigma)}].
$$

with $\epsilon \to 0$

$$
[\tilde{\alpha}_n^{\mu} e^{-in\sigma} (1 - in\epsilon\tau) + \alpha_n^{\mu} e^{in\sigma} (1 - in\epsilon\tau)],
$$

$$
\frac{\alpha_n^{\mu} - \tilde{\alpha}_{-n}^{\mu}}{\sqrt{\epsilon}} - in\tau\sqrt{\epsilon}(\alpha_n^{\mu} + \tilde{\alpha}_{-n}^{\mu})\bigg]e^{in\sigma}.
$$

 nd tensile mod usile m modes:

 $L_n - L_n - L_{-n},$ N_1 _n $- \epsilon$ L_n + $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon \left[\mathcal{L}_n + \bar{\mathcal{L}}_{-n} \right]$ $\frac{1}{2}$

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*n*6=0

 \blacktriangleright The equivalent of the Virasoro contraints \triangleright The equivalent of the Virasoro contraints \parallel \parallel $ARChals$ AB]*. i* AB, Chakrabortty, Parekh 2015

n)*, ^B^µ*

Quantum Tensionless Strings

A summary of quantum results

Careful canonical quantisation leads to not one, but three different vacua which give rise to

 \ast Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction

- * Novel closed to open string transition as the tension goes to zero. [AB, Banerjee, Parekh (PRL) 2019]
- different quantum mechanical theories arising out of the same classical theory. [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]
- on the other theory. All acceptable limits of quantum tensile strings. [AB, Mandlik, Sharma 2021]
- * Interpretation in terms of Rindler physics on the worldsheet. [AB, Banerjee, Chakrabortty (PRL) 2021]
- near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless

Tensionless Path From Closed to Open Strings

AB, Banerjee, Parekh, Physical Review Letters 123 (2019) 111601.

BRACIPAL PROPERTY I An important class of BMS representations: Massive modules. BMS INDUCED R

- An important class of BMS representations: Massive modules.
- \blacktriangleright The Hilbert space of these modules contains a wavefunction $|M, s\rangle$ satisfying:

BMS Induced Representations Campoleoni, Gonzalez, Oblak, Riegler 2016 Campoleoni, Gonzalez, Oblak, Riegler 2016

$$
\mathcal{L}_n|h,\bar{h}\rangle=0=\bar{\mathcal{L}}_n|h,\bar{h}\rangle\ (n>
$$

$$
M_0|M, s\rangle = M|M, s\rangle, \quad L_0|M, s\rangle = s|M, s\rangle, \quad M_n|M, s\rangle = 0, \ \forall n \neq 0.
$$
 (33)

▶ This defines a 1-d rep spanned by $\{L_0, M_n, c_L, c_M\}$. Can be used to define an *induced BMS* $|\Psi\rangle = L_{n_1} L_{n_2} \ldots L_{n_k} |M, s\rangle.$ \blacktriangleright Virasoro primary conditions: $\sum_{i=1}^{n} \binom{n}{i}$ *Lⁿ* + \vdash *Mⁿ* $-\gamma$ \int $\frac{1}{h}$, $\frac{1}{h}$ \int 0*,* $\frac{1}{h}$ \int 0*.* This defines a 1-d rep spanned by $\{L_0, M_n, c_L, c_M\}$. Can be used to define an *induced B |* i = *Ln*¹ *Ln*² *... Lnk |M, s*i*.* (34) • Limit from Virasoro \times Virasoro to BMS₃: $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}$, $M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$. $\mathcal{L}_n|h,\bar{h}\rangle=0=\bar{\mathcal{L}}_n|h,\bar{h}\rangle$ $(n>0)$; $\mathcal{L}_0|h,\bar{h}\rangle=h|h,\bar{h}\rangle$, $\bar{\mathcal{L}}_n|h,\bar{h}\rangle=\bar{h}|h,\bar{h}\rangle$. $\sqrt{2}$ $-L_{-n}$ + 1 ✏ M_{-n} ◆ $|h,\bar{h}\rangle = 0, n > 0.$ $\lambda = L_{m} I_{m}$ *Ln*₂ $|\mathcal{M} s \rangle$ $\langle \overline{h} \rangle = 0 = \overline{\mathcal{L}}_n | h, \overline{h} \rangle$ $(n > 0)$; $\mathcal{L}_0 | h, \overline{h} \rangle = h | h, \overline{h} \rangle$, $\overline{\mathcal{L}}_n | h, \overline{h} \rangle = \overline{h} | h, \overline{h} \rangle$. *Lⁿ* + $\frac{1}{\sqrt{2}}$

In the limit, this gives (33), along with the identification. $M = \epsilon (h + h), \ s = h - h$ In the limit, this gives (33), along with the identification: $M = \epsilon (h + \bar{h})$, $s = h - \bar{h}$.

*M*0*|M, s*i = *M|M, s*i*, L*0*|M, s*i = *s|M, s*i*, Mn|M, s*i = 0*,* 8*n* 6= 0*.* (33)

- *module* with basis vectors *module* with basis vectors
-
- I Virasoro primary conditions: I Virasoro primary conditions:

I This translates to \blacktriangleright This translates to I This translates to

$$
\left(L_n+\frac{1}{\epsilon}M_n\right)|h,\bar{h}\rangle=0,\quad \left(-L_{-n}+\frac{1}{\epsilon}M_{-n}\right)|h,\bar{h}\rangle=0,\ n\geq 0.
$$

1 ors are related
 n \log operators are relate $1\,\mathsf{b}$ $\frac{1}{2}$ by

 \overline{a} <mark>um: mixin</mark> mig $\ddot{}$ *n*)*|I*ⁱ ⁼ ⁰*,* ⁸*n.* (40) sile raising & lower

<u>1</u> *n* → *n n*
 n \overline{C} Different from tensile vacuum: mixing of tensile raising & lowering op in *C*, \tilde{C} .

$$
\tilde{C}_{n}^{\mu}(\epsilon) = \beta_{-}\alpha_{-n}^{\mu} + \beta_{+}\tilde{\alpha}_{n}^{\mu}.
$$

- $C_n^{\mu}|0\rangle_c$ $\forall n > 0$. Different from tensile v $|0\rangle_c$: $C_n^{\mu}|0\rangle_c = 0 = \tilde{C}_n^{\mu}|0\rangle_c \quad \forall n > 0.$ \blacktriangleright $|0\rangle_c$: $C^{\mu}_n|0\rangle_c = 0 = \tilde{C}^{\mu}_n|0\rangle_c$ $\forall n > 0$. Different from tensile vacuum: mixing of tensile
- *uced vacuum is given by* (C \tilde{C} ▶ In the *C* basis, the induced vacuum is given by $(C_n^{\mu} + \tilde{C}_{-n}^{\mu})|I\rangle = 0$, $\forall n$. \tilde{C}^{μ} + \tilde{C}^{μ}
- *r n n*^{*n*} *c*^{*n*} *c*^{*c*} *c*^{*n*} *c*^{*n*} *c*^{*n*} *c*^{*n*} *c*^{*n*} *c*^{*n*} *c*^{*n*} *c*^{*n*}

Induced Reps and Tensionless String Induced Keps and Jen AB et al 2019 [to appear]. I In term of oscillator modes, the induced modules: *Bn|M, s*i = 0*,* 8*n* 6= 0*.* I WE ARE IN THE VALUE IN THE VALUE OF A I In term of oscillator modes, the induced modules: *Bn|M, s*i = 0*,* 8*n* 6= 0*.* I In term of oscillator modes, the induced modules: *Bn|M, s*i = 0*,* 8*n* 6= 0*.* I We are interested in the vacuum module. Hence we have *Bn|I*i = 0 (37)

- In term of oscillator modes, the induced modules: $B_n|M, s\rangle = 0$, $\forall n \neq 0$. In term of osciliator modes, the in modes, the induced modules: $B_n|M, s\rangle = 0$, $\forall n \neq 0$. $\sum_{i=1}^{n}$ of oscillator modes the induced modules: $R \mid M \leq \theta \implies \theta$
- \blacktriangleright We are interested in the vacuum module. Hence we have $B_n|I\rangle = 0$ where $|I\rangle$ is the induced vacuum.
- ► Wish to return to harmonic oscillator basis for the tensionless string. Define: U

► In the C basis, the induced vacuum is given by
$$
(C_n^{\mu} + \tilde{C}_{-n}^{\mu})|I\rangle = 0
$$
, $\forall n$.
▶ This is precisely the condition of a Neumann boundary state $|I\rangle = \mathcal{N} \exp\left(-\sum_n \frac{1}{n}C_{-n}\tilde{C}_{-n}\right)|0\rangle_c$

$$
C_n^{\mu} = \frac{1}{2}(A_n^{\mu} + B_n^{\mu}), \quad \tilde{C}_n^{\mu} = \frac{1}{2}(-A_{-n}^{\mu} + B_{-n}^{\mu})
$$

- The algebra: $[C_m^{\mu}, C_n^{\nu}] = m \delta_{m+n} \eta^{\mu \nu}$, $[\tilde{C}_m^{\mu}, \tilde{C}_n^{\nu}] = m \delta_{m+n} \eta^{\mu \nu}$. $\sum_{\mu} \text{The algebra: } [C^{\mu} C^{\nu}] = m \delta \qquad m^{\mu\nu} \qquad [\tilde{C}^{\mu} \tilde{C}^{\nu}] = m \delta \qquad m^{\mu\nu}$ $\ddot{\cdot}$ $\left[C_m^{\mu}, C_n^{\nu} \right] = m \delta_{m+n} \eta^{\mu \nu}.$
- less ra .
51 \log and lowering oper \blacktriangleright The tensile and tensionless raising and lowering operators are related by **I** The tensile and tensionless raising and lowering operate *nd* $\sum_{i=1}^{n}$

$$
C_n^{\mu}(\epsilon) = \beta_+ \alpha_n^{\mu} + \beta_- \tilde{\alpha}_{-n}^{\mu}, \text{ where: } \beta_{\pm} = \frac{1}{2} \left(\sqrt{\epsilon} \pm \frac{1}{\sqrt{\epsilon}} \right)
$$

Cµ ^e (*A^µ ⁿ* ⁺ *^B^µ*

$$
\delta_{m+n} \eta^{\mu\nu}.
$$

Worldsheet Bogoliubov Transformations BOGOLIUBOV ON THE WORLDSHEET AB et al. 2019 and 2019 Worldshoot Roaali

 \blacktriangleright The relation between operators is a Bogoliubov transformation III Telation between operators is a Bogondbov transformation

$$
\alpha_n^{\mu} = e^{iG} C_n e^{-iG} = \cosh \theta C_n^{\mu} - \sinh \theta \tilde{C}_{-n}^{\mu}, \quad G = i \sum_{n=1}^{\infty} \theta \left[C_{-n} \tilde{C}_{-n} - C_n \tilde{C}_n \right]
$$

$$
\tilde{\alpha}_n^{\mu} = e^{iG} \tilde{C}_n e^{-iG} = -\sinh \theta C_{-n}^{\mu} + \cosh \theta \tilde{C}_n^{\mu}, \quad \tanh \theta = \frac{\epsilon - 1}{\epsilon + 1}
$$

 \blacktriangleright Relation between the two vacua: Relation between the two vacua:

 \blacktriangleright Using the regularisation: $1 + 1 + 1 + \ldots \infty = \zeta(0) = -\frac{1}{2}$ $|0\rangle_{\alpha}=\sqrt{\cosh\theta}\prod^{\infty}e^{-\frac{1}{2}\cosh\theta}$ ∞ *n*=1 $|0\rangle_{\alpha} = \sqrt{\cosh \theta} \prod_{n=1}^{\infty} \exp[\tanh \theta C_{-n} \tilde{C}_{-n}]|0\rangle_{c}$ ▶ Using the regularisation: $1 + 1 + 1 + \ldots$ ∞ = $\zeta(0) = -\frac{1}{2}$ ∞ *n*=1

▶ From the point of view of $|0\rangle_c$, $|0\rangle_\alpha$ is a squeezed state.

 $\exp[\tanh \theta C_{n} \tilde{C}_{n} | 0 \rangle_c$

$$
\alpha_n^{\mu} = e^{iG} C_n e^{-iG} = \cosh \theta C_n^{\mu} - \sinh \theta \tilde{C}_{-n}^{\mu}, \quad G = i \sum_{n=1}^{\infty} \theta \left[C_{-n} \tilde{C}_{-n} - C_n \tilde{C}_n \right]
$$

$$
\tilde{\alpha}_n^{\mu} = e^{iG} \tilde{C}_n e^{-iG} = -\sinh \theta C_{-n}^{\mu} + \cosh \theta \tilde{C}_n^{\mu}, \quad \tanh \theta = \frac{\epsilon - 1}{\epsilon + 1}
$$

$$
|0\rangle_{\alpha} = \exp[iG]|0\rangle_{c} = \left(\frac{1}{\cosh\theta}\right)^{1+1+\dots}\prod_{n=1}^{\infty}\exp[\tanh\theta C_{-n}\tilde{C}_{-n}]|0\rangle_{c}
$$

From Closed to Open Strings AB et al 2019 [to appear].

- As ϵ changes from 1, from the point of view of the *C* observer, the vacuum evolves. It becomes $\Delta s \epsilon$ changes from 1 from the point of
- \blacktriangleright When $\epsilon = 1$, tanh $\theta = 0$, and we have $|0\rangle_{\alpha} = |0\rangle_{c}$. This is the closed string vacuum. a squeezed state as shown before. a squeezed state as shown before. \blacktriangleright When $\epsilon = 1$, tanh $\theta = 0$, and we have $|0\rangle_{\alpha} = |0\rangle_{c}$. This is the closed string vacuum. As ϵ changes from 1, from the point of view of the *C* observer, the vacuum evolves. It becone
- In the limit where $\epsilon \to 0$, we have $\tanh \theta = -1$. The relation is thus: $|0\rangle_\alpha = {\cal N} \prod {\rm e}$ J
T *n*=1 *n*=1 $exp[-C_{-n}\tilde{C}_{-n}]|0\rangle_c$ \blacksquare In the limit where $\epsilon \to 0$ we have tank $\theta = -1$. The relation is thus: $\alpha = \mathcal{N} \prod_{i=1}^{\infty} \exp[-i\lambda_i]$ exp[*^CnC*˜*n*]*|*0i*^c* (45) In the limit where $\epsilon \to 0$, we have tanh $\theta = -1$. The relation is thus: $|0\rangle_\alpha = \mathcal{N} \prod$ ∞ $\exp[-C_{-n}\tilde{C}_{-n}]|0\rangle_c$

This is precisely the Induced vacuum $|I\rangle$ that we introduced before. I As we said, this is a Neumann boundary state. I As we said, this is a Neumann boundary state. I As we said, this is a Neumann boundary state. **P** This is thus an open string free to move in all dimensions (or a spacefilling D-brane). It This is thus an open string free to move in all dimensions (or a spacefilling D-brane).

We have thus obtained an open string by taking a tensionless limit on a closed string theory. We have thus obtained an open string by taking a tensionless limit on a closed string theory. We have thus obtained an open string by taking a tensionless limit on a closed string theory.

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From Closed to Open Strings and D-branes

Decreasing string tension

 $\zeta_{\mu\nu}$ is a poialisation tensor. Let us attempt to understand the evolution of the $\zeta_{\mu\nu}$ is a poialisation translate to: INTRODUCTION CLASSICAL ASPECTS SUPERSTRINGS QUANTUM ASPECTS APPLICATIONS REMARKS h_n the exiginal tensile theory $|J_n| = \zeta$ of ζ^n $|0\rangle$ ushere Ice in the original tensue theory $\frac{1}{2}r = \frac{1}{2} \mu v \alpha_{-n} \alpha_{-n} v / \alpha$ where $|0\rangle_{\alpha} = |I\rangle + \epsilon |I_1\rangle + \epsilon$ 2 $|I_2\rangle + \ldots$

I The mass of the open string state is zero.

$$
\Rightarrow B_n|I\rangle = 0, \forall n; \quad A_n|I\rangle + B_n|I_1\rangle = 0, \quad A_{-n}|I\rangle - B_{-n}|I_1\rangle = 0, \quad n > 0.
$$

▶ One can now take this limit on the state:

AB et al 2019 [to appear]. ^I Norm of the boundary state: *^N* = cosh ✓ ⁼ ¹ ⇣p✏ ⁺ ^p **THE** ! 1 as ✏ ! 0*.* ^I Norm of the boundary state: *^N* = cosh ✓ ⁼ ¹ 2 ⇣p✏ ⁺ ^p ✏ ! 1 as ✏ ! 0*.* **DOSE-BINSTEIN HKE CONGENSATION ON YVOI I** I Close to ✏ = 0, the alpha vacuum can be approximated as follows: *|*0i↵ = *|I*i + ✏*|I*1i + ✏ $\overline{}$ *|I*2i + *....* ^I Norm of the boundary state: *^N* = cosh ✓ ⁼ ¹ 2 ⇣p✏ ⁺ ^p ✏ ! 1 as ✏ ! 0*.* IM TIKE COMURMSATION ON AAOLIASURE I Bose-Einstein like Condensation on Worldsheet

- ▶ Consider any perturbative state in the original tensile theory $|\Psi\rangle = \xi_{\mu\nu} \alpha_{-n}^{\mu} \tilde{\alpha}_{-n}^{\nu} |0\rangle_{\alpha}$ where $\xi_{\mu\nu}$ is a polarisation tensor. Let us attempt to understand the evolution of the state as $\epsilon \to 0$.
- \blacktriangleright Close to $\epsilon = 0$, the alpha vacuum can be approximated as follows:
- ▶ In this limit, the conditions on the alpha vacuum translate to: $\frac{1}{2}$

One can now take this limit on the state:
\n
$$
\alpha_{-n}\tilde{\alpha}_{-n}|0\rangle_{\alpha} = \left(\frac{1}{\sqrt{\epsilon}}B_{-n} + \sqrt{\epsilon}A_{-n}\right)\left(\frac{1}{\sqrt{\epsilon}}B_{n} - \sqrt{\epsilon}A_{n}\right)(|I\rangle + \epsilon|I_{1}\rangle + \ldots) \rightarrow K|I\rangle
$$

 ll perturbativ \mathbf{z} CIO $\frac{1}{2}$ sed string st λ $rac{c}{c}$ *n*dense o All perturbative closed string states condense on the open string induced vacuum. on the open strin \mathbf{d} u red vacuu

$$
\alpha_n|0\rangle_{\alpha} = \tilde{\alpha}_n|0\rangle_{\alpha} = 0, n > 0
$$

) *Bn|I*i = 0*,* 8*n*; *An|I*i + *Bn|I*1i = 0*, An|I*i *Bn|I*1i = 0*, n >* 0*.* (*|I*i + ✏*|I*1i + *...*)*.* ! *K|I*i

Quantum Tensionless Strings II

Based on:

AB, Banerjee, Chakrabortty, PRL 2021. # AB, Banerjee, Chakrabortty, Dutta, Parekh, JHEP 2020. # AB, Mandlik, Sharma, JHEP 2021. # AB, Banerjee, Chakrabortty, Chatterjee, JHEP 2022.

Tension and Acceleration

AB, Banerjee, Chakrabortty, Physical Review Letters 126 (2021) 3, 031601.

Tension as Acceleration

AB, Banerjee, Chakrabortty [PRL 2021]

- ✤ One of the most common occurrences of Bogoliubov transformations is in the physics of accelerated observers vis-a-vis inertial observers. \vert_{\pm}
- ✤ Minkowski spacetime <-> Rindler spacetime.
- ✤ By identifying our Bogoliubov transformations to Rindler Bogoliubov transformations, we can recast the decrease of tension to the increase of acceleration.
- ✤ So, tensionless limit of string theory can be modelled as a series of worldsheet observers with increasing acceleration.
- ✤ The tensionless or null string emerges where the accelerated observer hits the Rindler horizon. This is where the acceleration goes to infinity.

A quick Rindler tour proper acceleration) describe Rindler space with the metric $\mathcal{L}^{\text{max}}(\mathcal{L}^{\text{max}})$ and which $\mathcal{L}^{\text{max}}(\mathcal{L}^{\text{max}})$ and $\mathcal{L}^{\text{max}}(\mathcal{L}^{\text{max}})$ we now consider a mass of the constant of the second theory in the second term in the sec β˜ $\overline{}$ **4 quick R**
= $e^{2a\xi}(-d\eta^2 + d\xi^2)$.
dler $t = \frac{1}{a}e^{a\xi}\sinh a\eta$, $x = \frac{1}{a}e^{a\xi}\cos\theta$. For the Minkowski solution, now defined on a cylinder ðσ; τÞ, the wave equation is solved by of our recapitulation is the action in the action in the action \mathcal{A} $\overline{}$ Z d2ξVαVβ∂α
2ξVαVβ∂αXμ∂βXνημιν: διαθέτερα Xμβ∂βXνημιν: διαθέτερα Xμβ∂βXνημιν: διαθέτερα Xμβ∂βXνημιν: διαθέτερα Xμβ∂βXνημιν quick Rindler
 $+ d\xi^2$).
 $\ddot{\xi}$ sinh *an*, $x = -e^{a\xi} \cosh a\eta$ ð og tveir tve
Tveir tveir tv ^ϕðσ; ^τÞ ¼ ^ϕ⁰ ^þ ffiffiffiffiffiffi ²α⁰ ^p ^α0^τ $ds_R^2 = e^{2a\xi}(-d\eta^2 + d\xi^2).$ For the Minkowski solution, now defined on a cylinder $\mathcal{O}(\mathcal{O}_\mathcal{A})$ the wave equation is solved by $\mathcal{O}_\mathcal{A}$ **TALU** Z er 100F **A** quick \mathbb{R}
 $e^{2a\xi}(-d\eta^2 + d\xi^2).$ ð seg einnig eða einni 21 Z Ω β quick Rindl
+d&2). Δ 010 r denne een $\overline{\mathbf{S}}$ **Example 10** A quick Rindler four n e[−]inðτ−σ^Þ ; ð4Þ where oscillations satisfy \mathbf{v} the Minkowski vacuum j0iM. To match with usual scalar the bosonic string as tension \mathbf{F} densities V^α replace the degenerate worldsheet metric. Like in Nites string theory, Eq. (10) enjoys worldsheet theory, Eq. (10) enjoys worldsheet of the string tension of A quick Rindler tour e−in $\overline{}$ where α is α and α and α not α not α not α not α not α $\mathbf{E} = \mathbf{E} \mathbf{E} \mathbf{E} + \mathbf{E} \mathbf{E} \mathbf{E}$ \mathbf{b} bosonic string as the \mathbf{b} dens is tieden van de generate word in tensile string theory, Eq. (10) enjoys worldsheet diffeo**p f**_i n>0 n anual anual beann

- ÷ 2d Rindler metric: $ds_R^2 = e^{2a\xi}(-d\eta^2 + d\xi^2)$. the Creative Commons Attribution 4.0 International license. \overline{a} $\frac{1}{R}$ \mathcal{R} $ds_R^2 = e^{2\alpha_5}(-d\eta^2 + d\xi^2)$ $\zeta - a$ 2 \overline{u} n αn $\left(\begin{array}{c} 1 \ 1 \end{array}\right)$ \mathbf{e} $\boldsymbol{\ell}$ $\overline{}$ l, $= e^{2a\xi}(-a)$ \overline{a} $\boldsymbol{\varUpsilon}$ $s_p^2 = e^{2a\xi}(-d\eta^2 + d\xi^2).$ \overline{c} \overline{a} $2a\zeta$ (– dn^2+d $\frac{1}{2}$ \mathbf{r} $\frac{1}{2}$ \boldsymbol{v} indlar motrie $v^2+d\xi^2$).
- ✤ From Minkowski to Rindler $\begin{array}{cc} a & a & b \\ c & b & c \end{array}$ are linked by $\begin{array}{cc} a & a \\ c & d \end{array}$ the author(s) and the published article's title, journal citation, $t =$ 1 \overline{a} $e^{a\xi}$ sinh an, $x = \frac{1}{a}e^a$ for the field are the field are the same of the same o b
britt \mathbf{d} n \overline{t} $\frac{1}{a^{\xi}}$. n $\frac{1}{\alpha^{\xi}}$ Sindler $t = -e^{as}$ sinh $a\eta$, $x = -e^{as}$ cosh $a\eta$ Ainkowski to Rindler $t=-e^{a\xi}\sinh\,an,\,x$ the Minkowski vacuum $v = a$ community with a_{ij} , α wski to Rindler $t=-e^{a\xi}\sinh\,a\eta, \; x=\frac{1}{\tau}e^{a\xi}$ the Minkowski vacuum $u = a$ ^c sinn u_1 , $v = a$ ^c $\int \int \text{d}v \, du = \frac{1}{2} e^{a\xi} \sinh \alpha w \cdot v = \frac{1}{2} e^{a\xi} \cosh \alpha w$ **field** $I = -e^{as}$ sinn p wski to <mark>R</mark> $t = -e^{a\varsigma}\sinh\alpha$ ر
2006 - مطر زام و مورد و مارد زام ا **KOWSKI TO KINGIEI** $t = -e^{a\varsigma}$ sin er $t = -\frac{1}{a}e^{a\xi} \sinh a\eta, x = -\frac{1}{a}e^{a\xi} \cosh a\eta$ 1 $\frac{1}{\sqrt{2}}$ $=\frac{1}{a}e^{a\xi}$ sinh an, $x=$ 1 \overline{a} $\sum_{i=1}^{a} a_i$ sinh $a\eta$, $x = -e^{a}$;
	- \cdot EOM: $\square_{t,x} \phi = 0 = \square_{\eta,\xi} \phi$. the Minkowski vacuum j σ . \Box $\phi = \Omega = \Box$ ϕ $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{v}$ 2α0 p α0τ
2α0τ
2α0τ $\Box_{\eta,\xi}\phi.$ $\overline{0}$
- ✤ Minkowski mode expansion 0031-0031-0031-0031601-031601-031601-031601-1 Published by the American Physical Society of American Physical S * MINKOWSKI MODE EXPANSION owski mode expansion ansion

unity of the set of th pde expansion $\boldsymbol{\theta}$ pansion
De $\overline{}$ kowski mode expansion Similarly, we write the Rindler model model with the Rindler model with the Rindler model with the Rindler mod inkowski mode expansion S^{S}

✤ Rindler mode expansion \overline{d} similarly mode expansion as α

where oscillators satisfy ½αn; αm& ¼ nδⁿþ^m and annihilate

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in tensile string theory, Eq. (10) enjoys worldsheet diffeo-

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consistency. This expansion also leads to the constraint of the constraints in the constraint significant of the constraints in the constraints in

We pictorially depict the above process in Fig. 2. The above process in Fig.

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• **EOM:**
$$
\Box_{t,x}\phi = 0 = \Box_{\eta,\xi}\phi.
$$

\n• **Minkowski mode expansion**

\n $\phi(\sigma,\tau) = \phi_0 + \sqrt{2\alpha'}\alpha_0\tau + \sqrt{2\pi\alpha'}\sum_{n>0} [\alpha_n u_n + \alpha_{-n} u_n^* + \tilde{\alpha}_n \tilde{u}_n + \tilde{\alpha}_{-n} \tilde{u}_n^*]$

\n $u_n = [ie^{-in(\tau+\sigma)}]/\sqrt{4\pi}n, \quad \tilde{u}_n = [ie^{-in(\tau-\sigma)}]/\sqrt{4\pi}n.$

\n• **Rindler mode expansion**

\n $\phi(\xi,\eta) = \phi_0 + \sqrt{2\alpha'}\beta_0\xi + \sqrt{2\pi\alpha'}\sum_{n>0} [\beta_n U_n + \beta_{-n} U_n^* + \tilde{\beta}_n \tilde{U}_n + \tilde{\beta}_{-n} \tilde{U}_n^*]$

\n $U_n = \frac{ie^{-in(\xi+\eta)}}{\sqrt{4\pi}n}, \quad \tilde{U}_n = \frac{ie^{-in(\xi-\eta)}}{\sqrt{4\pi}n}.$

\n□

$$
\phi(\xi,\eta) = \phi_0 + \sqrt{2\alpha'}\beta_0\xi + \sqrt{2\pi\alpha'}\sum_{n>0} [\beta_n U_n + \beta_{-n} U_n^* + \tilde{\beta}_n \tilde{U}_n + \tilde{\beta}_{-n} \tilde{U}_n^*]
$$

$$
U_n = \frac{ie^{-in(\xi+\eta)}}{\sqrt{4\pi n}}, \quad \tilde{U}_n = \frac{ie^{-in(\xi-\eta)}}{\sqrt{4\pi n}}.
$$

- \clubsuit The oscillators $\{\beta, \tilde{\beta}\}$ act on a new vacuum $|0\rangle_R$. \mathcal{F} the Usumaturs $\{\beta, \beta\}$ autum a new vacuum $\{\beta/R\}$. ϕ icillators $\{\beta,\tilde{\beta}\}$ act on a new vacuu $\textbf{ellators} \; \{\beta, \beta\} \; \; \textbf{act on a new vacuum} \; \; |0\rangle_R \; .$ $\textbf{oscillators} \; \{ \beta, \tilde{\beta} \} \; \; \textbf{act on a new vacuum} \; \; |0\rangle_R \; .$ $\mathcal{L}_{\mathcal{F}}$ is only defined in the L wedge (hence the L wedge (hence the L wedge (hence the L wedge (hence the L) e oscillators $\{\beta, \tilde{\beta}\}$ act on a new vacuum $\sum_{i=1}^{n}$ is only defined in the L wedge $\sum_{i=1}^{n}$ we define the L wedge $\sum_{i=1}^{n}$ \mathbf{r} $\frac{1}{\sqrt{2}}$ we encourance \int $\frac{1}{\sqrt{2}}$
- ✤ U's act only in one wedge. To continue between them one defines smearing functions. Combinations for both wedges: dge. To continue between them one define $\bullet\bullet$ U's act only in one wedge. To one wedge. To continue between them one defir ν_n . Combinations for both wedges: $U_n^{(R)} - e^{-(\pi n/a)} U_{-n}^{(L)*}, \qquad U_{-n}^{(R)*} - e^{(\pi n/a)} U_{-n}^{(R)}$ i euge lo contin ie
Dødsf e between thei $\frac{1}{2}$ by contribution and $\frac{1}{2}$ is the EQM can solved by $\frac{1}{2}$ ions for bot \mathbf{W} edaes: $U_n^{(R)} = e^{-(\pi n/a)} U^{(R)}$ inue[.] ie
ie⊢in etwe n them or at only in one wedge To continue hetween t space. To continue between wedges, one needs to define the $s=\frac{1}{2}$ tions. Combinations for both zakoulutu ononjodug To sonkinna hakusaan klassa ona defines ora s act only in one wedge. To continue betwee uetione Combinatione for he e wedge. To continue between them one o between them one defines smearing eag sinh and an intervalse sinh and an 1 tions. Combinations for both wedges: $U_n^{(R)} - e^{-(\pi n/a)} U_{-n}^{(L)*}$ $U_{-n}^{(R)*} - e^{-(\pi n/a)} U_{-n}^{(L)*}$ where wought to continued bet week them one ac Lorection tions for the there decen $\tau^{(R)}$ $-(\pi^{(a)}\tau)$ *Fombinations for both wedges:* $U_n^s = e^{-(\pi n/n)t}U_{-n}^s$, $U_{-n}^s =$

on between oscillators:
$$
\beta_n = \frac{e^{\pi n/2a}}{\sqrt{2 \sinh \frac{\pi n}{a}}} \alpha_n - \frac{e^{-\pi n/2a}}{\sqrt{2 \sinh \frac{\pi n}{a}}} \tilde{\alpha}_{-n}, \quad \tilde{\beta}_n = -\frac{e^{-\pi n/2a}}{\sqrt{2 \sinh \frac{\pi n}{a}}} \alpha_{-n} + \frac{e^{\pi n/2a}}{\sqrt{2 \sinh \frac{\pi n}{a}}} \tilde{\alpha}_n.
$$

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This is the 3d Bondi-Metric is the 3d Bondi-Metric (BMS) algebra (BMS3) algebra (BMS3) algebra (BMS3) algebra (here in algebra (here in algebra (here in algebra (here in algebra (here) algebra (here) algebra (here) algebra

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ued in both wedges take the form $\mathcal{O}(\mathcal{A})$. The form $\mathcal{O}(\mathcal{A})$ is the form $\mathcal{O}(\mathcal{A})$

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÷ Relation between oscillators: 2
Rolation hot In det ween oscillators. The $\beta_n = \frac{e^{-\alpha_n}}{2\pi}$ α Equation (2) is for the right Rindler wedge on the right Rin **E** BERTION DET MEEN OOM

for the field are the same:

✤ Identification: i $\frac{1}{2}$ λ \overline{a} β^{∞} \overline{a} $\tilde{\rho}$ $\epsilon =$ $\overline{\pi}$ $\overline{}$ $\mathcal{L}_n = \beta_n^{\infty}, \qquad \mathcal{L}_n = \beta_n^{\infty}, \qquad \varepsilon = \frac{1}{2a}.$ ϵ to ϵ $C_n = \beta_n^{\infty}, \qquad \tilde{C}_n = \tilde{\beta}_n^{\infty}, \qquad \epsilon =$ πn $\frac{2a}{}$ A l classical physics of the tensionless string can be tensionle $\epsilon=\frac{\pi n}{2}$. from Eq. (9), we see that $\frac{1}{\sqrt{2}}$ reduces to the tensile string $\frac{1}{\sqrt{2}}$ reduces to tensile string $\frac{1}{\sqrt{2}}$ **Identification:** $C_n = \beta_n^{\infty}$, $C_n = \beta_n^{\infty}$, $\epsilon =$ $\angle a$ where N and N and N and N stand for the Neumann and Dirichlette in the Neumann and Dirichl conditions, respectively. This can be solved explicitly to the solved ex $m = \frac{1}{2}$ or πn $c_n = \beta_n^{\infty}, \qquad \epsilon = \frac{1}{2a}.$

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cussion above, the string tension was put exactly to zero.

Now we describe a limiting procedure on the worldsheet

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limit, the worldsheet becomes a 2

 $\overline{}$ with a degenerate metric which is the definite metric which is t

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feature of a tensionless or a null string. The worldsheet

Evolution in Acceleration An evolution in boost can only lead to changes in physics (e.g., change in vacuum structure, changes in spectrum) in

 ϵ observer hitting the horizon = increasingly accelerated world sheets. parameter space is very naturally explained by acceleration of the space is very space in the space of the space is α $2 - 1$ nc 2a **sın**y accel $\overline{\mathbf{z}}$ wur
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- ÷ The limit of zero tension is thus the limit of infinite acceleration: $\epsilon \to 0 \Rightarrow a \to \infty$ $\sum_{i=1}^{\infty}$ and the tensile viral o tension is thus the limit of infinite acceleration Eq. (4)] and the tensionless expansions (13), we get \ast The limit of zero tension is thus the limit of infinite acceleration: $\epsilon \to 0 \Rightarrow a \to \infty.$ acceleration in the contract of limit is usually perceived as a limit on velocities and hence The livest of zero tension is thus the liveit of infinition $\mathbf{v} \sim \mathbf{v}$ and \mathbf{v} is the set of t where the contract and $\kappa: \epsilon \rightarrow 0 \Rightarrow a \rightarrow \infty$
- close to form B e and the tension service expansions expansions and the tension service expansion \sim $\beta \rightarrow {\alpha_n, \alpha_n}, 0 < a < \infty$: { β_{β} } \overline{a} \overline{a} \tilde{B} \tilde{B} \tilde{C} \tilde{D} \tilde{C} \tilde{D} \tilde{C} \tilde{C} \tilde{B} \tilde{C} \tilde{C} \tilde{D} \tilde{C} \tilde{D} \tilde{C} \tilde{C} \tilde{D} \tilde{C} \tilde{D} \tilde{C} \tilde{D} \tilde{C} \tilde{D} \tilde{D} $\tilde{$ \sim 6 reproduced by following this UR limit. The Carrollian $\left(\begin{array}{cc} n & n \end{array} \right)$ **a** Evolution: $a = 0: {\beta_n, \beta_n} \rightarrow {\alpha_n, \tilde{\alpha}_n}$, 0 $a = 0$: $\{\beta_n, \tilde{\beta}_n\} \rightarrow \{\alpha_n, \tilde{\alpha}_n\}, 0 < a < \infty$: $0 < a < \infty$: { $\beta_n(a), \tilde{\beta}_n(a)$ }, $a \to \infty$:

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The limit of zero tension is the limit of $\mathcal{L}_\mathcal{L}$ the limit of infinite of infinit

 Δ l classical physics of the tensionless string can be tensionl

reproduced by following this UR limit. The Carrollian

This is because the equivalence has to hold for all n \mathbb{R}^n and \mathbb{R}^n are to hold for all n \mathbb{R}^n

✤ String equivalent of Rindler observer hitting the horizon = increasingly accelerated world sheets. \clubsuit String equivalent ot Kindler observer nitting the l \mathbf{a} t of Rindlar ohearvar hitting tha horizon = inor μ , or entergy opportion the internet from the thought of as {
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|n now build the string equivalent of a Rindler of a Rindler
The string equivalent of a Rindler of a Rindl → String equivalent of Rindler observer hitting the horizon = inc 1 \mathbf{u}

> \Rightarrow $a \rightarrow \infty$. ⁿðaÞg; ð26bÞ

 $\bullet\bullet$ Evolution: $\alpha = 0: \{\beta_n, \tilde{\beta}_n\} \to \{\alpha_n, \tilde{\alpha}_n\}, 0 < a < \infty: \{\beta_n(a), \tilde{\beta}_n(a)\}, a \to \infty: \{\beta_n, \tilde{\beta}_n\} \to \{C_n, \tilde{C}_n\}.$ Complete interpolating solution. (a) , $p_n(a)$ $\},\,$ The limit of zero tension is thus the limit of infinite $a\rightarrow\infty$: $<\infty\colon\quad \{\beta_n(a),\tilde{\beta}_n(a)\},\quad a\to\infty\colon\quad \{\beta_n,\tilde{\beta}_n\}\to \{C_n,\tilde{C}_n\}.$ Complete interpola $\overline{10}$ $\{\tilde{C}_n\}$. Comple \mathbf{r} n¼1 ln **τ** $\boldsymbol{0}$ ing solution. $a \to \infty$: { β_n , $\tilde{\beta}_n$ } \to { C_n , \tilde{C}_n }. Complete rerpolating solution.

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boost on the Riemannian worldsheet of a tensile string that

turns it into a degenerate Carrollian worldsheet. The carrollian worldsheet is a degenerate Carrollian worldsh

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$$
\beta_n^{\infty} = \frac{1}{2} \left(\sqrt{\frac{\pi n}{2a}} + \sqrt{\frac{2a}{\pi n}} \right) \alpha_n + \frac{1}{2} \left(\sqrt{\frac{\pi n}{2a}} - \sqrt{\frac{2a}{\pi n}} \right) \tilde{\alpha}_{-n}, \quad \tilde{\beta}_n^{\infty} = \frac{1}{2} \left(\sqrt{\frac{\pi n}{2a}} - \sqrt{\frac{2a}{\pi n}} \right) \alpha_{-n} + \frac{1}{2} \left(\sqrt{\frac{2a}{\pi n}} + \sqrt{\frac{\pi n}{2a}} \right) \tilde{\alpha}_n.
$$

 \mathscr{L} Rindler Bogoliutrov transformation at large accelerations: $\overline{}$ This sends the worldsheet speed of light to zero and is a senade of light to zero and is a $\sqrt{2}$ Cimildr Bonoliutow transformation at Eq. (4) and the tension of the tension we get to decreasing tensiber tension, with a was and Kindler Bogoliutov transtormation at large acco complete interpretence interpretence interpretence in the string orientation string orientations. structure through the flow is described in terms of the flow is described in terms of

Hitting the Horizon: Evolution in Rindler Time Increase the constant of the constant of the constant hyperstructure of the constant of the co i ing the nortzon. Evolution in Expaigr i

at constant acceleration, the analogous picture is $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$ and $\mathcal{L}_\mathcal{A}$

shown in the boxes below. The circular closed string at the

- ✤ We explored hitting the Rindler horizon by evolving in acceleration. * We explored hitting the Rindler horizon by evolving in acceleration we get a point is what is what is the December of the December decreases, and the string gets longer and longer and longer and longer and longer and longer and longer as giv
The string gets longer as given by the string gets longer as given by the string gets longer and longer and lo
- \cdot The horizon can also be hit by evolving in Rindler time at constant acceleration.
- \cdot So the infinite time limit on the Rindler worldsheet would also generate the null string. mentary picture is that when viewed from the j0ic, j0i^α ite time limit on the Rindler worldsheet would also generate

- \cdot Mathematically, this is the limit $\eta \to \infty$. Or equivalently, FIG. 2. Increasing accelerated worldsheets. i n
	-
- ✤ Conformal generators in Rindler:

 \angle

✤ These close to form the BMS algebra as expected and the null string emerges. o form the BMS algebra as expected and the null string emerges.

✤ In the limit we get:

 L_n $=\mathcal{L}_n-\bar{\mathcal{L}}_{-n}$

 \overline{M}_n

Hitting the Horizon: Evolution in Rindler Time Reach Freeholms in Pindler Time reaching the Rindler and Constant and Constant acceleration by the Ring Constant and Constant acceleration by the Ring Constant and Constant acceleration by the constant acceleration by the constant acceleration by the con reaching the Rinder Time Company in the Rinder Time reaching the Rindler horizon at constant acceleration by

of spacetime to form a D-25 brane when the tension goes to form a D-25 brane when the tension goes to \mathcal{L}

gen FIG. 1. Equal time slices in Rindler spacetimes.

- $\epsilon \to 0$; $\epsilon \to 0$. $\eta \to \eta$, $\xi \to \epsilon \xi$, $\epsilon \to 0$. $\rightarrow \eta, \qquad \xi \rightarrow \epsilon \xi, \qquad \epsilon \rightarrow 0.$
- To simularity $\omega_n, \omega_n = \frac{1}{2}$ conformal $\omega_n + \omega_{\xi}$. ienerators in Rindler: $\mathcal{L}_n.\bar{\mathcal{L}}_n=\pm\frac{\iota}{2}e^{n(\xi-\eta)}(\partial_n\mp\partial_{\varepsilon}).$ $\frac{m}{2}$ x 1 $\mathcal{L}_n, \bar{\mathcal{L}}_n = \pm$ i n enerators in Rindler: $\mathcal{L}_n, \bar{\mathcal{L}}_n = \pm \frac{\iota}{2} e^{n(\xi-\eta)} (\partial_\eta \mp \partial_\xi).$

This close to form the close to form the classical part of the BMS algebra \mathcal{L}

 $\mathcal{N}(\mathcal{N})$ (i.e., c.e., c

null string, which we had expected. A detailed analysis of the string, which we had expected. A detailed analysis of

$$
L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} = i^n e^{-n\eta} (\partial_\eta - n \xi \partial_\xi),
$$

$$
M_n = \epsilon (\mathcal{L}_n + \bar{\mathcal{L}}_{-n}) = -i^n e^{-n\eta} \partial_\xi.
$$

 \bigwedge (i.e., c.e., c.e

null string, which we had expected. A detailed analysis of the string, which we had expected. A detailed analysis of

the aspects of Rindler physics on constant accelerated

the aspects of Rindler physics on constant accelerated

 W now present a particularly intriguing picture that \mathcal{N} is a particularly intriguing picture that \mathcal{N}

worldsheets will be presented elsewhere $\sum_{i=1}^n\sum_{j=1}^n\alpha_j$

A Tale of Three

AB, Banerjee, Chakrabortty, Dutta, Parekh, JHEP 04 (2020) 061

✤ This amounts to

hphys*|T*(1)*|*phys⁰ i = 0*,* hphys*|T*(2)*|*phys⁰ i = 0*,* hphys*|T*(2)*|*phys⁰ i = 0*.* (17) $\langle phys|M_n|phys'\rangle = 0.$

 \cdot For each type of oscillator F obeying $\langle phys|F_{n}|phys'\rangle=0$, there can be three types of solutions. v each type of oscillator E obeving $/h$ $_{hel}$ F/h $_{hel}$ $_{el}$ there can be three types of solutions t \mathbb{R}^n of the property of hermiticity can be broken down into the broken down into the set of three distinct cases:

 $3.$ $F_n|phys\rangle \neq 0$, but $\langle phys'|F_n|phys\rangle = 0$.

A Tale of Three AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354 *T*(1) = lim ✓ *Tcyl ^T*¯*cyl Tplane ^c* 24 $\frac{1}{2}$ *n Lne in*! *^c* ²⁴ ; *^T*¯*cyl* ⁼ ^X *n* I The Ultra-relativistic EM tensor I The Ultra-relativistic EM tensor *^Ln|phys*ⁱ ⁼ *^L*¯*n|phys*ⁱ =0 (*n >* 0)*.* (5.2) It should be noted that the same work via the sandwich conditions here work via the *right handed* action of the *right* handed action

- we consider canonical quantisation of tensionless string theories. **T**(1) = limited the limited to the limite 20KV *Tcyl ^T*¯*cyl*◆ z_{on} question
- **As we saw earlier** Classical constraint on the tensionless string: $T_{(1)} = 0$, $T_{(2)} = 0$. I Classical constraint on the tensionless string: *T*(1) = 0*, T*(2) = 0.

actional version. Physical speculum of tensioniess sul

I Classical constraint on the tensionless string: *T*(1) = 0*, T*(2) = 0. IC ICHSIONIICSS SUINES. $I(1) - 0$, $I(2) - 0$.

✤ From a single classical theory, several inequivalent quantum theories may emerge. This happens when ✓ ◆ **The Limits Company** 333 **Partival Trey**
Pring theories *zs* may eme *r*ge. This ²⁴ (16) $\mathcal{L} = \mathcal{L} \times \mathcal{L}$ *n* (*Ln in*⌧*Mn*)*e in cL* OR MARY OMOVOO ation of tensionless string the 244 (244) 244 (244) 244 ur ies may e **VA** 1 *Pr*ge. 1 h $\overline{\mathbf{e}}$ rom a single classical theory, several inequivalent quantum theories may emerge. This happens when
In consider concepted were that they of tourisuless chrisp, theories *^Ln|phys*ⁱ ⁼ *^L*¯*n|phys*ⁱ =0 (*n >* 0)*.* (5.3)

 $\frac{1}{1}$ $\frac{1}{1}$ Quantum version: physical spectrum of tensionless strings restricted by \langle phys $|T_{(1)}|$ phys^{\langle}) = 0, \langle phys $|T_{(2)}|$ phys \langle) = 0. Notice in the case above, the anti-holomorphic constraints actually impose a *left handed*

n

is amounts to
$$
\langle phys|L_n|phys'\rangle = 0
$$
, $\langle phys|M_n|phys'\rangle = 0$.

- - 1*.* $F_n|phys\rangle = 0$ $(n > 0)$,
	- 2. $F_n|phys\rangle = 0$ $(n \neq 0)$,
	-

AB, Ban erjee, Chakrabortty, Dutta, Parel AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354 ✏!0 ✓ *Telakrabor* ◆ Dutta, l Parekh. 2001.0035 t_{max} is the constraint $\frac{1}{2}$ of the case of the case of the $\frac{1}{2}$

A Tale of Three weight representation of the algebra (case 1), which is often the usual norm to study and usual norm to study a
The usual norm to study and usual norm In the case of the BMS3 algebra things are not the BMS3 algebra things are not the BMS3 algebra that simple, a for which the above classification of conditions are possible. It seems that we could have nine possible combinations in total through which we can impose the constraint on the states. These are dependent

 $\boldsymbol{\cdot}$ Here $F_n = (L_n, M_n)$. Hence seemingly nine conditions: $n = \frac{1}{\sqrt{L}}$ or $\frac{1}{\sqrt{L}}$ or $\frac{1}{\sqrt{L}}$ is the combination of the constraint on the cons states. These are dependent of the set of the m *inaly nine conditi*

AB, Banerjee, Chakrabortty, Dutta, Parekh. 2001.00354 9 *Lm|phys*i = 0*,* (*m >* 0)*,* ta,

- consistent solutions. *Mn|phys*i = 0*,* (*n >* 0) *Mn|phys*i = 0*,* (*n* 6= 0) **30** ru
M 1SISTENT SOIUTIONS. *Lm|phys*i = 0*,* (*m* 6= 0)*,* |
|- $\overline{\text{u}^{\text{r}}}\text{ is } \overline{\text{h}^{\text{r}}}\text{ is } \overline{\text{h}^{\text{$ *Mn|phys*i = 0*,* (*n >* 0) *ing BMS algebra* \overline{a} so has to be s *Lm|phys*i 6= 0*,* (8 *m*)*,*
- ✤ These are three inequivalent vacua, leading to three inequivalent quantum theories. *Mn|phys*i 6= 0*,* (8 *n*) >; 92 E *Lm|phys*i 6= 0*,* (8 *m*)*,* >: *Mn|phys*i = 0*,* (*n* 6= 0) \bm{p} inequivalent vacua, leading to th
- Induced vacuum: Theory obtained from the limit of usual tensile strings. *Mn|phys*i = 0*,* (*n* 6= 0) $\frac{1}{2}$ induced vacuum: Theory obtained from the limit of usual
- Flipped vacuum: Leads to ambitwistor strings. (See e.g. Casali, Tourkine, (Herfray) 2016-17) *Mn|phys*i = 0*,* (*n >* 0) \bigcirc
- *Mn|phys*i = 0*,* (*n* 6= 0) **Decillator vacuum: li**

Oscillator vacuum: Interesting new vacuum. Contains hints of huge underlying gauge symmetry.

✤ But the underlying BMS algebra also has to be satisfied. It turns out that only three of the nine choices lead to \mathbf{I} *Mn|phys*i = 0*,* (*n >* 0) *Mn|phys*i = 0*,* (*n* 6= 0) \overline{v} >; *.* (5.5c)

$$
L_m |phys\rangle = 0, (m > 0), \begin{cases} M_n |phys\rangle = 0, (n > 0) \\ M_n |phys\rangle = 0, (n \neq 0) \\ M_n |phys\rangle \neq 0, (\forall n) \end{cases}; L_m |phys\rangle = 0, (m \neq 0), \begin{cases} M_n |phys\rangle = 0, (n > 0) \\ M_n |phys\rangle = 0, (n \neq 0) \\ M_n |phys\rangle \neq 0, (\forall n) \end{cases}; L_m |phys\rangle \neq 0, (\forall m), \begin{cases} M_n |phys\rangle = 0, (n > 0) \\ M_n |phys\rangle \neq 0, (n \neq 0) \\ M_n |phys\rangle \neq 0, (\forall n) \end{cases}
$$

*Mn|phys*i = 0*,* (*n >* 0)

>: *Mn|phys*i = 0*,* (*n* 6= 0) ><

Critical Dimensions

AB, Mandlik, Sharma. 2105.09682

Figure 1. Tensionless corners of Bosonic String Theory. Tensionless corners of Quantum Tensile String Theory

A summary of quantum results

Careful canonical quantisation leads to not one, but three different vacua which give rise to

 \ast Lightcone analysis: spacetime Lorentz algebra closes for two theories for D=26. No restriction

- * Novel closed to open string transition as the tension goes to zero. [AB, Banerjee, Parekh (PRL) 2019]
- different quantum mechanical theories arising out of the same classical theory. [AB, Banerjee, Chakrabortty, Dutta, Parekh 2020]
- on the other theory. All acceptable limits of quantum tensile strings. [AB, Mandlik, Sharma 2021]
- * Interpretation in terms of Rindler physics on the worldsheet. [AB, Banerjee, Chakrabortty (PRL) 2021]
- near blackhole event horizons. [AB, Banerjee, Chakrabortty, Chatterjee 2021]

Carroll limit on spacetime induces tensionless limit on worldsheet. Strings become tensionless

- * Tensionless superstrings: Two varieties depending on the underlying Superconformal Carrollian algebra.
- Homogeneous Tensionless Superstrings: Fermions scale in same way. Previous construction: Lindstrom, Sundborg, Theodoridis 1991. Limiting point of view: AB, Chakrabortty, Parekh 2016.
- Inhomogeneous Tensionless Superstrings: Fermions scale differently. New tensionless string! AB, Banerjee, Chakrabortty, Parekh 2017-18.

Open questions: Tensionless Strings

- Analogous calculation of beta-function=0. Consistent backgrounds?
- Linking up to Gross-Mende high energy string scattering from worldsheet symmetries.
- Attacking the Hagedorn transition from the Carroll perspective. Emergent degrees of freedom? Lin progress with Banerjee, Mandlik1
- Strings near black holes, strings falling into black holes? [in progress with Banerjee, Hartong, Have, Kolekar, Mandlik]
- Extend "Tale of Three" to superstrings. Different superstring theories?
- * Intricate web of tensionless superstring dualities?

Black hole Microstates from Null Strings

AB, Grumiller, Sheikh-Jabbari 2210.10794

Proposal: A null string wrapping the event horizon contains in its spectrum the micro

- Event horizon of black holes are null surfaces.
- In d=3, consider BTZ black holes. Event horizon is a null circle.
- states of a BTZ black hole.
-
- Possible generalisations to higher dimensions.

Black hole Null String Wrapping Horizon

We can reproduce the Bekenstein-Hawking entropy as well as its logarithmic corrections!

- * Proposal motivated by symmetries. Symmetries of event horizon same as symmetries of the null string worldsheet.
- * Dynamic horizon on which d.o.f. live is then equivalent to a null string.
- Quantize the null string in Oscillator Vacuum. Use Lightcone gauge for convenience.
- Black hole states: a band of states with sufficiently high level.
- Mass is proportional to the radius of the horizon. Motivated by Near Horizon first law. [Donnay et al 2015, Afshar et al 2016].
- Complicated combinatorics leads to entropy and amazing the correct logarithmic corrections.
- * Can be thought of as a precise formulation of the membrane paradigm.
- \ast Generalization to d=4 with null membranes in progress and showing interesting signs.

Vacuum: $|0, p^{\mu}, \omega\rangle \equiv |0\rangle$ $\langle r_i \rangle$, $\langle s_i \rangle$, ω) Level 1: $J_{-1}|0\rangle$, $\tilde{J}_{-1}|0\rangle$ Level 2: $J_{-2}|0\rangle$, $J_{-1}^2|0\rangle$, $J_{-1}J_{-1}|0\rangle$, $J_{-1}^2|0\rangle$, $J_{-2}|0\rangle$ $\mathcal{L}(\mathbf{U}|\mathcal{L}, \mathbf{J}-2|\mathbf{U}/2, \mathbf{J}-1|\mathbf{U}/2, \mathbf{J}-1|\mathbf{U}/2, \mathbf{J}-1|\mathbf{U}/2)$ um: $|0, p^{\mu}, \omega\rangle \equiv |0\rangle$ VaC 2*p*⁺ *p* of $\text{Vacuum: } |0, p^{\mu}, \omega\rangle \equiv |0\rangle$ Level 2: $J_{-2}|0\rangle$, $J'_{-1}|0\rangle$, $J_{-1}J_{-1}|0\rangle$, J'_{-1} $I.e.$ $\overline{\mathbf{S}}$ \overline{P} $\begin{array}{c} 0, p, 0 \\ I, |0\rangle \end{array}$ $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ where *X*' = *Rh X*. Varying the gauge-fixed action (3.9) yields the equation of motion

where *p*
But it where *public* P. The level splits into two integers associated with each set of oscillator modes, in the set of oscillator modes, α *N* = *r* + *s*. For instance, the five states above at level *N* = 2 have, respectively, (*r*, *s*) = (2, 0), . Notice T-duality no more holds.

Null strings to BTZ microstates ... where *p*^µ = *B*^µ ⁰. The level splits into two integers associated with each set of oscillator modes, is given by arbitrary combinations of the creation operators *Jm*, *J ^m* acting on the oscillator vac-, !i. Physical states are a subclass of these generic states subject to the constraints (). We are interested in physical states with momentum in the radial direction, which is a radial direction, which in \mathbf{N} . Il. Physical states are a subject to the constraints (??). We are interested in physical states without momentum in the radial direction, which in our lightcone coordinates implies *p*⁺ = *p*, and we keep *p*' arbitrary, see (3.14). (for a more detailed treatment, see [63]). By virtue of the oscillator vacuum (3.22)-(3.24), the **ROLL STRUNGS TO DIZ WII** $B_{\rm T}$ black hole. Due to the near horizon first law, the latter scales linearly in the horizon radius σ *m* = *Rh* . (4.5) 3.2 Horizon Strings: The basic set up The metric that we pick for the target space is the BTZ metric evaluated in a suitable gauge and a The version of the ILST action that we would be interested in for our analysis is co-rotating frame zooming onto the horizon (with radius *Rh*) a ba RT7 minvanababan **Wull etringe to BTZ migroetat**

- ILST action: $S = \frac{k}{2}$ d $\sigma(V^a \partial_a X^\mu)(V^b \partial_b X^\nu) G_{\mu\nu}(X)$. Metric: Gauge fixed action: *N* = *r* + *s*. For instance, the five states above at level *N* = 2 have, respectively, (*r*, *s*) = (2, 0), (*r*, *s*) = (2, 0), (*r*, *s*) = (1, 1), (*r*, *s*) = (0, 2) and (*r*, *s*) = (0, 2). The levels *r*, *s* in turn can be decomposed into a collection of integers associated with individual creation operators, *r* = P lightcone coordinates implies *p*⁺ = *p*, and we keep *p*' arbitrary, see (3.14). WE ROUGHION $S = \frac{1}{2} \int d\tau d\sigma (V^{\sigma} \partial_a X^{\mu})(V^{\sigma} \partial_b X^{\sigma}) G_{\mu\nu}(X)$. WEITIG, $G_{\mu\nu} dx^{\sigma} dx^{\sigma}$ **conditions (2)** the physical definition $S = \frac{K}{a} \int d\tau d\tau \left(-2(a Y^+) (a Y^-) + (a Y^{\varphi})^2 \right)$ $K \int$ and the tensile string $K \int$ and impose the physical string σ in the physical state σ in the physical **c** is the probability $\sigma = \frac{1}{2} \int d\ell d\theta$ (*V* $\theta_a A^2$)(*V* $\theta_b A$) $\theta_{\mu\nu}(A)$. **Within**, $\theta_{\mu\nu}$ with ω \mathcal{L} *s r* = !*n* . (4.2) From $\sigma = 2 \int d\mathbf{r} d\mathbf{v}$ (*v* σ_{a} *x*) σ_{μ} σ_{μ} σ_{μ} , we deduce the mass σ_{μ} of σ_{μ} tixed action **in:** $S = \frac{1}{2} \int d\tau d\sigma (V^{\alpha} \partial_{\alpha} X^{\mu})(V^{\nu} \partial_{\beta} X^{\nu}) G_{\mu\nu}(X)$. **We the** $G_{\mu\nu}$ $d\Lambda'$ $d\Lambda' = -2 dx dX + K_h d\varphi$ mentum number *n*, subject to the level-matching (4.2) and the mass-shell condition (4.3). While $S = \frac{K}{\sigma} \int d\tau d\tau (V^a \partial V^{\mu}) (V^b \partial V^{\nu}) G(V^b)$ for the freedom to fix the coupling constant \mathcal{L} of \mathcal{L} Elaborate on the near horizon first law.... $S =$ $\overline{\mathcal{K}}$ 2 \mathbb{Z} $\int d\tau d\sigma (V^a \partial_a X^\mu) (V^b \partial_b X^\nu) G_{\mu\nu}(X)$. **Metric:** $G_{\mu\nu} dX^\mu dX^\nu = -2 dx^+ dx^- + R_h^2 d\phi^2$ **Example fixed action:** $S_{\alpha} = \frac{K}{\alpha} \int d\tau d\sigma \left(-2(\partial_{\tau}X^{+})(\partial_{\tau}X^{-}) + (\partial_{\tau}X^{\varphi})^{2} \right)$ $T^{\dagger}(\partial_z X^{-}) + (\partial_z X^{\varphi})^2$ X ¹ *A*etric: G $\int_{\mathcal{U}} u \, dX^{\mu} \, dX^{\nu} = -2 \, dx^{+} \, dx^{-} + C$ $R = \frac{1}{2}$ **SECTION:** $S = \frac{1}{2}$ for $(V^{\circ}O_aX^{\circ})(V^{\circ}O_bX^{\circ})G_{\mu\nu}(X)$. $\overline{\mathcal{K}}$ $\overline{}$ $\big)$
	- $\left(1 + \frac{1}{2} \right)$ **sed action:** $S_{\text{gf}} = \frac{1}{2} \int d\tau d\sigma \left(-2(\partial_{\tau}X^{+})(\partial_{\tau}X^{-}) + (\partial_{\tau}X^{\varphi})^{2} \right)$ $\frac{1}{2}$ and a formula for the theory. The remaining physical state conditions are automatically satisfied on $\frac{2}{3}$ or $\frac{1}{3}$ or $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ **THOM:** $S_{\text{gf}} = \frac{1}{2} \int d\tau d\sigma \left(-2(\theta_{\tau}X^{\dagger})(\theta_{\tau}X^{\dagger}) + (\theta_{\tau}X^{\dagger})^{\dagger} \right)$ co-rotating frame zooming onto the horizon (with radius *Rh*) real values, and the angular coordinate is periodic, _{in the} gauge-fixed version of the gauge-fixed version of the
The gauge-fixed version of the null line of the null line of the null line of the null line of the null l Fauye lixed activity, $\sigma_{gf} - \frac{1}{2} \int d\tau d\sigma \left(-\frac{2(\theta_{\tau}A_{\tau})(\theta_{\tau}A_{\tau}) + (\theta_{\tau}A_{\tau})}{2} \right)$ $\mathcal{S}_{\text{gf}}=% \begin{bmatrix} \omega_{\text{f}} & \omega_{\text{f}}\ \omega_{\$ 2 $d\tau d\sigma \left(-2(\partial_\tau X^+)(\partial_\tau X^-) + (\partial_\tau X^\varphi)^2 \right)$
	- Allow strings to wind: $2\pi R_h(t)$ respectively, into *r*² = 1 and *r*¹ = 2 (with all other *ri* = 0 in each case). A generalization of $A^T(\sigma + 2\pi, \tau) = A^T(\sigma)$ ***** Allow strings to wind: $X^{\varphi}(\sigma + 2\pi, \tau) = X^{\varphi}(\sigma, \tau) + 2\pi R_h \omega$ $f(x) = \frac{f(x)}{2}$. By virtue of the oscillator vacuum (3.23) of the oscillator vacuum (3.22)-(3.24) σ **Requirement in the sum of the solution of the solution of** $\Delta^r(\sigma + 2\pi, \tau) = \Delta^r(\sigma, \tau) + 2\pi K_h \omega$ **

	Measure the** n^{μ} **(s) = 10)** of the horizon radius *Rh*. Since *m* is the mass of our string states at the horizon, on dimensional **11011 311 11199 Gallerings to wind:** $X^{\varphi}(\sigma + 2\pi, \tau) = X^{\varphi}(\sigma, \tau) + 2\pi R_h \omega$ τ τ) + $2\pi R_{b}$ (i) $X^{\varphi}(\sigma + 2\pi, \tau) = X^{\varphi}(\sigma, \tau) + 2\pi R_h \omega$ Allow strings to wind: $X^{\varphi}(\sigma+2\pi,\tau) = X^{\varphi}(\sigma,\tau) + 2\pi R_h \omega$
	- Null string states over oscillator vacuum: $|\Psi\rangle = |p^{\mu}$ ***** Null string states over oscillator vacuum: $|\Psi\rangle = |p^\mu, \{r_i\}, \{s_i\}, \omega\rangle$ Level 1: J **R**₂ *h* **R22** *h* * Null string states over oscillator vacuum: $|\Psi\rangle = |p^{\mu}, \{r_i\}, \{s_i\}, \omega\rangle$ Vacuum: 10

		- Constraints: $s r = \omega n$. $s - r = \omega n$. $\textbf{\textit{x}}$ Constraints: $s - r = \omega n$. mentum number *n*, subject to the level-matching (4.2) and the mass-shell condition (4.3). T versioning is to $T = \omega n$. *n*2 string and the target space $s-r=0.8$ *X*µ(⌧,) = *x*^µ + *A*^µ
		- Mass formula: $m^2 = (r + s)\kappa + \frac{h^2}{R^2}$. Notice T-duality no more holds. $\frac{1}{2}$, $\frac{1$ () where $m^2 = (r + s)\kappa + \frac{n^2}{r}$. Notice T-duality no more holds, R_h^2 * Mass formula: $m^2 = (r + s)\kappa + \frac{n^2}{D^2}$. Notice **T-duality no more** $m^2 = (r + s)\kappa + \frac{n^2}{n^2}$. Notice **T-duality no more holds.** *n*2 R_h^2 *h* n^2 below n^2 call physicates $m = (r + s)\kappa + \frac{1}{R^2}$. Indice physical string Hilbert space. **of Mass formula:** $m^2 - (r + s)x + \frac{n^2}{\sqrt{r^2}}$ Notice Lduality no more holds R_h^2 Ω factor by the free order to the free frequency in the constant $m^2 = (r+s)\kappa + \frac{n^2}{2}$. Notice f-duality no more holds. K_h^2 *N* n^2 *h* $\frac{1}{2}$ $m a$. η' $\frac{d\mathbf{r}}{d\mathbf{x}} + \frac{d\mathbf{r}}{d\mathbf{x}}$ and the \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} and \mathbf{r} n^2 Mass for *<i>b n* ⁰ ⁺ *^B*^µ ⁰⌧ + *i n*,0 R_h^2
			- Now, microstates of the BTZ blackhole: $|m\rangle_{\rm BTZ} = |\{r_i\},\{s_i\},\omega,\,n\rangle$ *reading the p* $\overline{2}$ 1 ***** Now microstates of the BTZ blackhole: $|m\rangle_{\text{max}} = |\{r_i\}, \{s_i\}, \omega, n\rangle$ $|m\rangle^{\text{BTZ}}$ * Now, microstates of the BTZ blackhole: $|m\rangle_{\text{max}} = |\{r_i\}, \{s_i\}, \omega, n\rangle$ where $\frac{n}{\sqrt{2}}$. $\frac{n}{\sqrt{2}}$. $\frac{n}{\sqrt{2}}$. $\frac{n}{\sqrt{2}}$
				- Near horizon first law: $m = \kappa R_h$. $\rightarrow \kappa R_h^2 = s + r + \frac{h}{R^2} := N + \frac{h}{R^2}$ $N = r + s$. solved as before by the model we can be model the Near horizon first law: $m = \kappa R$, $\rightarrow \kappa R^2 = s$.
- Approximation: Large black hole. $R_h \gg 1/\sqrt{\kappa}$ wation (4.2) Announcement Condition (4.3) $\textbf{\textit{*}}$ Approximation: Large black hole. $R_h \gg 1/\sqrt{K}$ M_n humparinum arbitrary numerical factor in the relation $P_n \times 1/\sqrt{n}$ factor by the freedom to fix the free \mathcal{L} ***** Approximation: Large black hole. $R_h \gg 1/$ $\overline{\mathbf{A}}$ $\overline{\mathcal{K}}$ *Mation: Large black hole.* R_h $\gg 1/\sqrt{\kappa}$ $\sqrt{\kappa}$ *A*' ⁰ = *Rh*!, *A*[±]

★ Now, microstates of the BTZ blackhole:
$$
|m\rangle_{BTZ} = |\{r_i\}, \{s_i\}, \omega, n\rangle
$$

\n**★** Near horizon first law: $m = \kappa R_h$. **→** $\kappa R_h^2 = s + r + \frac{n^2}{\kappa R_h^2} := N + \frac{n^2}{\kappa R_h^2}$ $N = r + s$.

- States from different sectors (complicated combinatorics): *N*0=*N* **x** STATES TP sign compiled to combinations would prove the partition of order units in the partition of order units in the partition of the partition Thus, difformation that the contribution of the constructions of *n*, and the contributions to the contributions to the contract of the contra partition function *zeuters* in *algebra in article for large* masses. Consider the opposite of the soft sector: The mass is dominated by high momentum, *n N*. **Level and in primitive vanishing induced vanishing winding winding winding the same result as formation** with a
Formation in primitive and the same result as formation as formation in same result as formation in same resu of the partition function (5.9) is the main result of our counting. It is valid for large horizon radii, which permits comparing with semiclassical results for the black hole entropy. 6 Bekenstein–Hawking law 16 *x* States trou
	- \quad Soft: string momentum vanishes $n=0$ $\text{\textdd{F}}$ Soft: string momentum vanishes $n = 0$ 5.3 Generic sector $\text{Soft-erimal momentum vanishes } n = 0$ Therefore, this sector is suppressed exponentially as compared to the soft sector and, as we SOTT: STYING MOMENTUM VANISNES $n = 0$ 7 Concluding remarks 16 T , and generic microstates subject to the fluctuations (5.5) is then given by the fluctuations (5.
- t_{min} is the momentum cooform $r > N$ function fully ounnecessed res infinite degeneracy from the winding modes: no amount of winding changes anything about the $\textbf{High momentum sector } n \gg N.$ \quad High momentum sector $n \gg N$. Exponentially suppressed. $n \approx N$ \ast then momentum sector $n \gg n.$ Exponentially sup M OMentum X $n \gg N$. Exp
- Generic sector, typical microstates **Feneric sector typical** numbers.
1980 - Santo Carlos Barcos (b. 1980).
1980 - Santo Carlos Barcos (b. 1980). For fixed (large) mass *m* the counting is now straightforward. The total level *N* must be large * Generic sector, typical microstates $N \gg n$. Thus, we conclude that typical microstates require large levels, *N n*, and the contributions to the **Example 7 5 and 7 are negligible function function function** \mathbf{z} **and** \mathbf{z} **are negligible for the formulation of the fo** Generic sector, typical microstates r
List
List $\sqrt{2}$ Non-winding sector

But the coefficient of the log term is the real surprise. Unexpected! Must be something very deep! 3 ⇣*N* ⌘ **SUI PI ISE.** r*N* ◆ typical microstates require *N n*. Generically, there are no further constraints on winding or \ast -but the coethcient ot the log term is time being, we fix the level *N* but permit varying the mass by changing the momentum number. The partition function in the generic sector for fixed *N* \ast But the coefficient of the log term is the real surprise. Unexpected! Must be something very deep! sub-subleading terms are small as compared to ln *Rh* but still infinite for *Rh* ! 1. In particular, gravity, given the paucity of experimental data (see e.g. See e.g. See e.g. Sec. 10.2 in [24]). That is why the
That is why the hep-th is why the hep-0t the log to the real ournries Uporn 100 varijiv
21 esteknica

Entropy of BTZ black hole) ⇧2 ⇣*N*⁰ 2 0 1 16K ✓ IO $\overline{\mathbf{a}}$ all of them and determine their respective combinatorics. In all cases, we assume *Rh* 1/ Therefore, this section is suppressed exponentially as compared to the soft sector and, as we computed t **ENTYOPY OT DIZ DIACK NOIC** 5.3 Generic sector 14 5.4 Motropy of 51.4 highlenders in the sector 15 μ While it was pure computed to consider the consideration of the constant \mathcal{L}_1

High momentum sector $n~\gg~N.$ Exponentially suppressed. $n~\approx~N$ also exponentially suppressed. \ast - thyn momentum sector $n \gg n$. Exponentially suppressed, $n \approx n$ also exponentially suppres T_{t} the her the stronger the exponential enhancement in the integer partitions (5.1). Thus, the integer partitions (5.1). Thus, the integer partitions (5.1). Thus, thus, the integer partitions (5.1). Thus, the integ High momentum sector $n \gg N$. Exponentially suppressed, $n \approx N$ also exponentially suppressed. nentially SUI $\ddot{}$ $2.6. n \approx N$ also expone

(5.9)

$$
\textbf{W} = \textbf{W} \cdot \textbf{W}
$$

- Full partition function: $Z_{\text{BTZ}} = Z_{\text{soft}} + Z_{n \gg N} + Z_{n \approx N} + Z_{\text{generic}} + Z_{\omega=0}$. integers. The number of integer partitions, ⇧(*N*), is given by the Hardy–Ramanujan formula [72] \sim 1000 por 111 \mathbf{r} full nartition function: $Z_{\text{max}} = Z_{\text{max}}$ partition function *Zn^N* + *Zn*⇡*^N* are negligible for large masses. ***** Full partition function: $Z_{\text{BTZ}} = Z_{\text{soft}} + Z_{n \gg N} + Z_{n \approx N} + Z_{\text{generic}} + Z_{\omega=0}$ $t \cdot$ t is the partition of the partition function τ . a well-known numerical factor ³ ***** Full partition function: $Z_{\text{BTZ}} = Z_{\text{soft}} + Z_{n \gg N} + Z_{n \approx N} + Z_{\text{generic}} + Z_{\omega=0}$.
	- Bekenstein-Hawking entropy: ***** Bekenstein-Hawking entropy: $S = \ln Z_{\text{B}TZ} = 2\pi R_h$, $\sqrt{\frac{K}{\pi}} - \frac{5}{\pi} \ln R_h + o(\ln R_h)$. g IVES $D_{BH} = \frac{1}{4G}$. We cannot get the py: 5 = $\frac{1}{2}$ $Area$ \mathbf{p} **Thus, thus, thus, th**e cannot get the soft on any thing yet. **EIN HAWNING CHILOPY.** $D = \text{III } \angle B T Z = 2\pi R R R$ $\sqrt{3}$ $2 \frac{\text{III } R R + O(\text{III } R)}{2}$. ***** dekenstein-mawking entropy: $\beta = \text{III}$ $\text{Z}_{\text{BTZ}} = 2\pi K h$ $\sqrt{\frac{2}{3}}$ $\frac{1}{2}$ $\frac{\text{III}}{\text{N}}$ π of M_0 $\zeta = 3$ This since $S = A$ Ca M_0 counsel set the 2/1 G free ***** Bekenstein-Hawking entropy: $S = \ln Z_{\text{BTZ}} = 2\pi R_h$ sub-subleading terms are small as compared to ln *Rh* but still infinite for *Rh* ! 1. In particular, $\kappa =$ 3 $\frac{3}{16G^2}$. This gives $S_{BH} = \frac{1}{4G}$. We cannot (BH) entropy Area $\textbf{\textit{x}}$ bekenstein-hawking entropy: $\texttt{D} = \text{IN} \space \textbf{Z}_{\text{B1}}$ *Z*!=0(*N*) = *N*+O($\overline{ }$ $\frac{1}{2}$ gives $S_{\,\mathrm{B}}$ \overline{X})
|-⇣*N*⁰ ⌘

We fix $\kappa = \frac{1}{16C^2}$. This gives $S_{BH} = \frac{1}{16C}$. We cannot get the 3/16 from anything yet. Only input! \cdot 2⇡ *Rh* om ar * We fix $\kappa = \frac{3}{16G^2}$. This gives $S_{BH} = \frac{12.02}{16G}$. We cannot get the 3/16 from anything yet. Only input! IX $K = \frac{1}{16G^2}$. This gives $D_{\rm BH} - \frac{1}{4G}$. We cannot get the 37TO trom anything yet. Unly input: $\frac{1104}{4G}$. We cannot get the 3/16 from anything ye r*N* ◆

N+O(m *N*) ^O(*N*3/⁴ 0 \mathbf{r} guarantee the validity of the semiclassical approximation. and subleading O(ln *N*) contributions, while dropping terms subleading to these. This simplification has the added benefit that we can assume non-negative ! and *n* since considering all possible where we used the relation (4.7) between the level *N* and the horizon radius *Rh*. The approximation 5.5 Full partition function function $\mathcal{L}(\mathcal{L})$ function $\mathcal{L}(\mathcal{L})$ function $\mathcal{L}(\mathcal{L})$ *^m* ⁼ ^O(1) \$ *^N* ⁼ ^O ^p tions may have been anticipated on physical grounds, as we are in an ensemble of fixed temperature

\n- \n * Full partition function:
$$
Z_{\text{BTZ}} = Z_{\text{soft}} + Z_{n \gg N} + Z_{n \approx N} + Z_{\text{generic}} + Z_{\omega=0}
$$
.\n
\n- \n * Bekenstein-Hawking entropy: $S = \ln Z_{\text{BTZ}} = 2\pi R_h \sqrt{\frac{K}{3}} - \frac{3}{2} \ln R_h + o(\ln R_h)$.\n
\n

A theory of Black holes based on Null Membranes?

- Looks like the theory of null strings has something deep to say about BTZ microstates. *
- Of course, there are questions. This is an effective theory.
- How can you make this quantum mechanically consistent? What about anomalies? *
- Relatedly: Dimensions? Looks like D=3 for the moment. Add spectator D=23 dimensions? * Wish away KK modes?
- But can we go further? Null 2-branes for D=4 Blackholes? *
- Classical analysis Lin progress AB, DG, MMS, others1 seems to indicate that we do have an * analogous infinite dimensional symmetry related to BMS4 at play here.
- \ast Is it possible to quantise this? Can the infinite dimensional algebra work its magic again, unlike the relativistic case? We hope to come back with answers.

Thank you!

