Wrapped Membranes, Supergravity, and U

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Non-Relativistic Strings and Beyond

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1. ॐ/Wrapped Membrane and U Duality

2. 11d Supergravity

Wrapped Membrane

Wrapped Membrane Limit

a.k.a. WM2 and ॐ (OM) Limit

[Gopakumar, Minwalla, Seiberg, Strominger 2000] [Bergshoeff, Berman, Schaar, Sundell 2000] [Garcia, Guijosa, Vergara 2002]

$$\ell_P = \varepsilon^{1/3} L_P \qquad C_{012} = 1$$

$$g_{\mu\nu} = \mathrm{diag}\big(-1,\, +1,\, +1,\, \varepsilon,\, \varepsilon,\, \cdots,\, \varepsilon\big)$$

$$\varepsilon \to 0$$

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- double dimensional reduction on 2:

$$\ell_s = \varepsilon^{1/2} L_s \qquad g_s = \varepsilon^{-1/2} G_s \qquad g_{\mu\nu} = (-1, +1, \varepsilon, \cdots, \varepsilon) \qquad B_{01} = 1$$

Closed M2 Brane

[Garcia, Guijosa, Vergara 2002]

Flat

$$S_{M2} = T_{M2} \int d^3\sigma \sqrt{-\det\, g_{\alpha\beta}} - T_{M2} \int C_3$$

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for a closed membrane with $X^1 \sim X^1 + w_1 R_1$ and $X^2 \sim X^2 + w_2 R_2$

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$$E = \frac{\mathbf{p}^2}{W A_{12} T_{M2}} + \text{oscillators}$$

$$W = w_1 w_2 > 0$$

$$A_{12} = (2\pi R_1)(2\pi R_2)$$

U Duality

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- M theory on a rectangular three-torus has the self-duality

$$U_{IJK}: \qquad R_I \to \tilde{R}_I = \frac{\ell_P^3}{R_J R_K} \qquad \ell_P^3 \to \tilde{\ell}_P^3 = \frac{\ell_P^6}{R_I R_J R_K}$$

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- exchanges KK ↔ wrapped M2: $\quad \dfrac{1}{R_{11}} \quad \leftrightarrow \quad (2\pi R_1)(2\pi R_2)\, T_{M2}$

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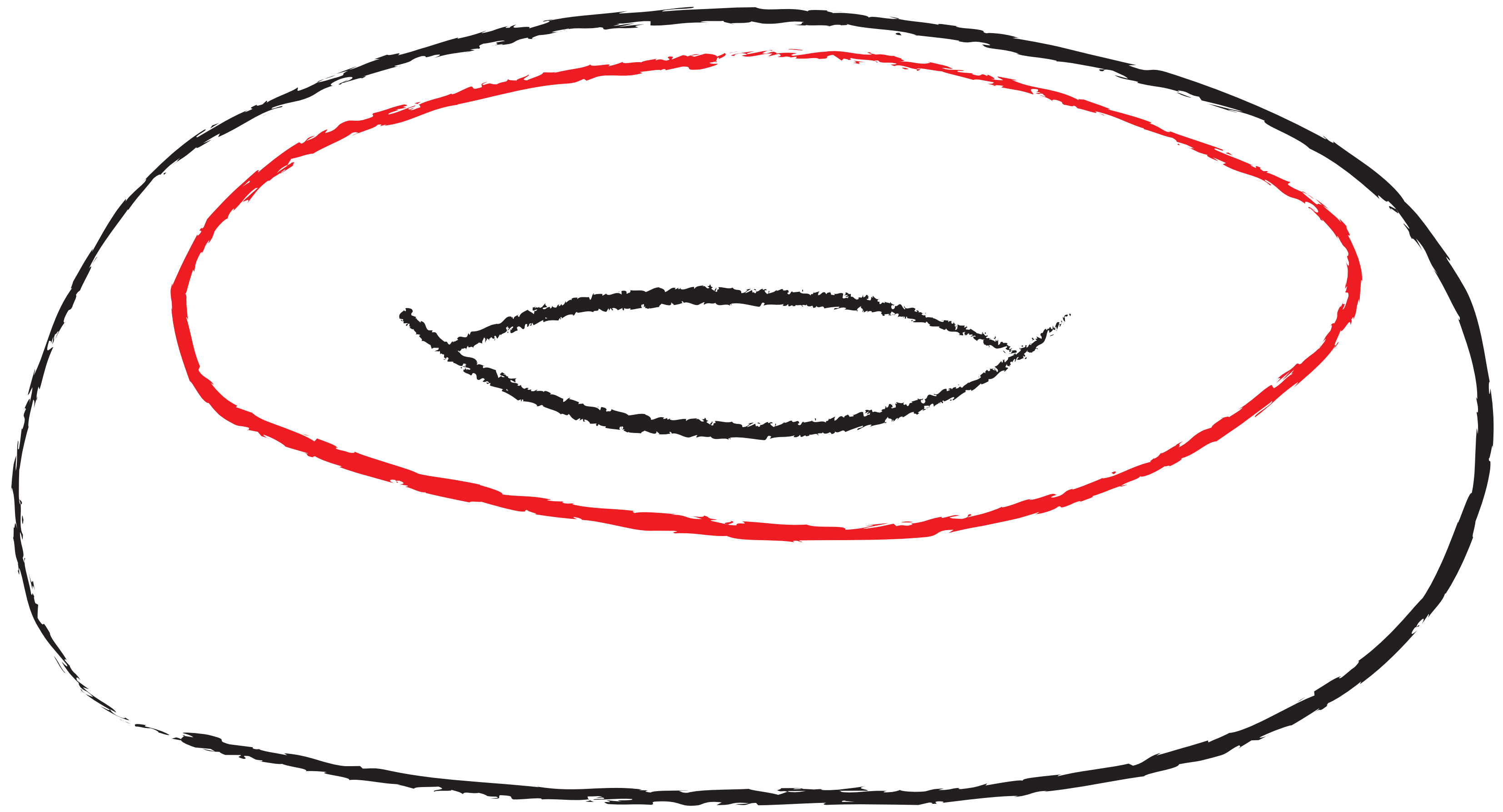
- Seiberg/Sen (1997) "proved" that proposal and justified the use of low-energy D0 brane dynamics by considering $p_{11} = N/R_{11}$ and

$$R_{11} \to 0\,, \qquad R_i \to 0\,, \qquad \ell_P \to 0\,, \qquad \text{with} \quad \left(\frac{R_{11}}{\ell_P^2}, \frac{R_i}{\ell_P}\right) \quad \text{fixed}\,.$$

WM2 Torus

or: how to inflate a donut

consider a rectangular three-torus that is mostly longitudinal and $R_I = (R_i, R_T)$



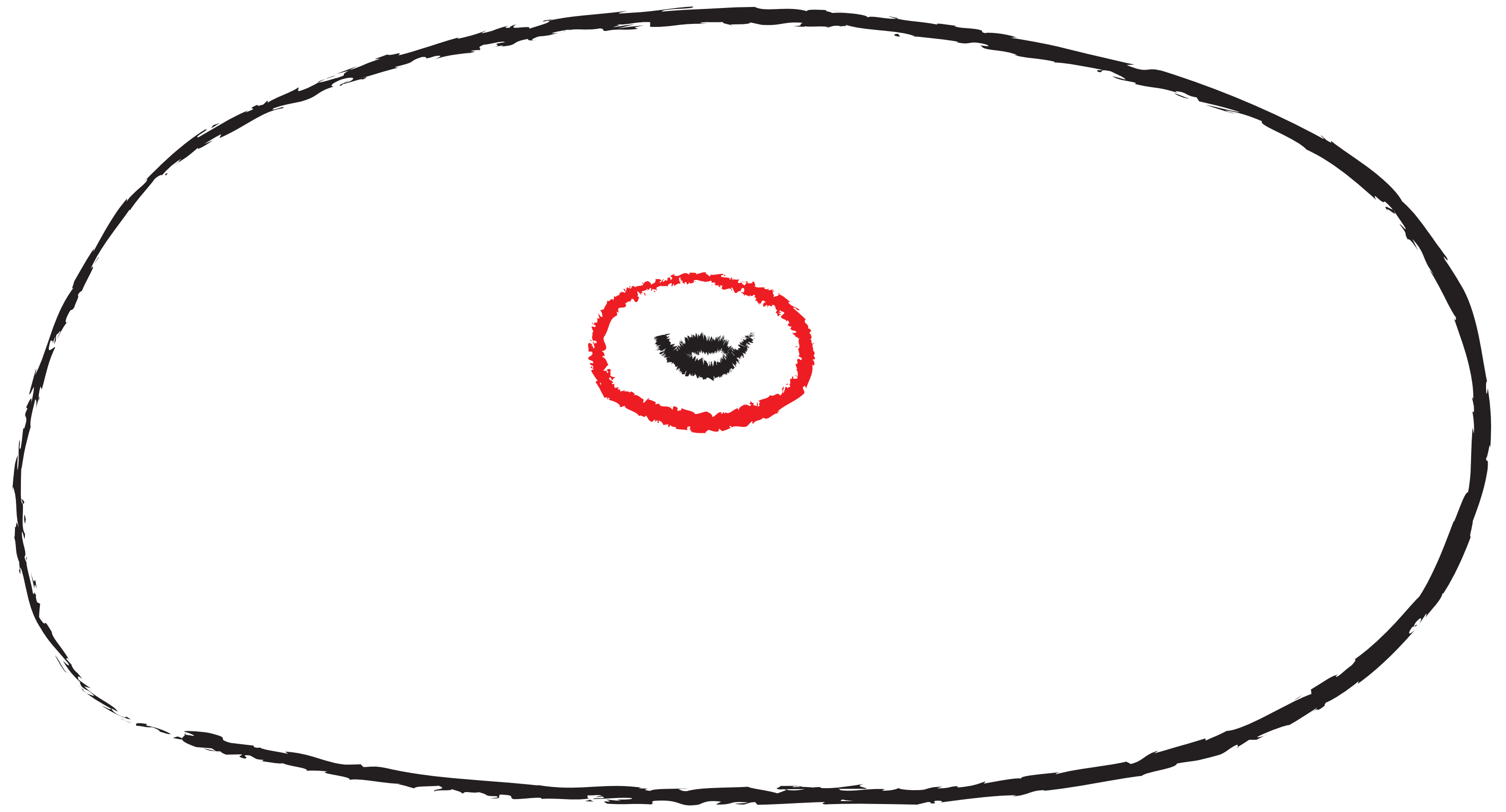
$R_T \to 0$

R_i fixed

$\ell_P \to 0$

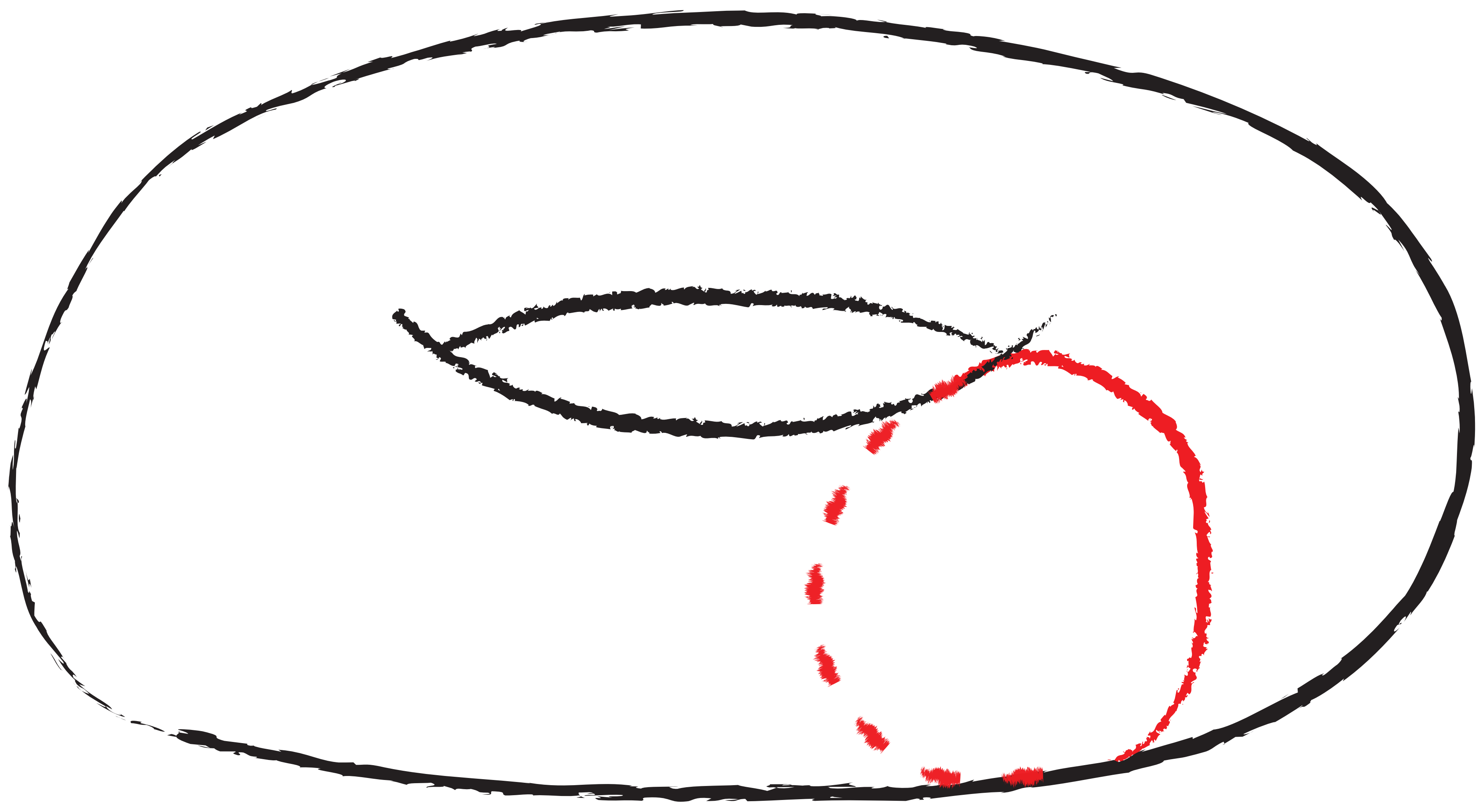
R_T^2/ℓ_P^3 fixed

$V = R_i R_j R_T/\ell_P^3 \to \infty$

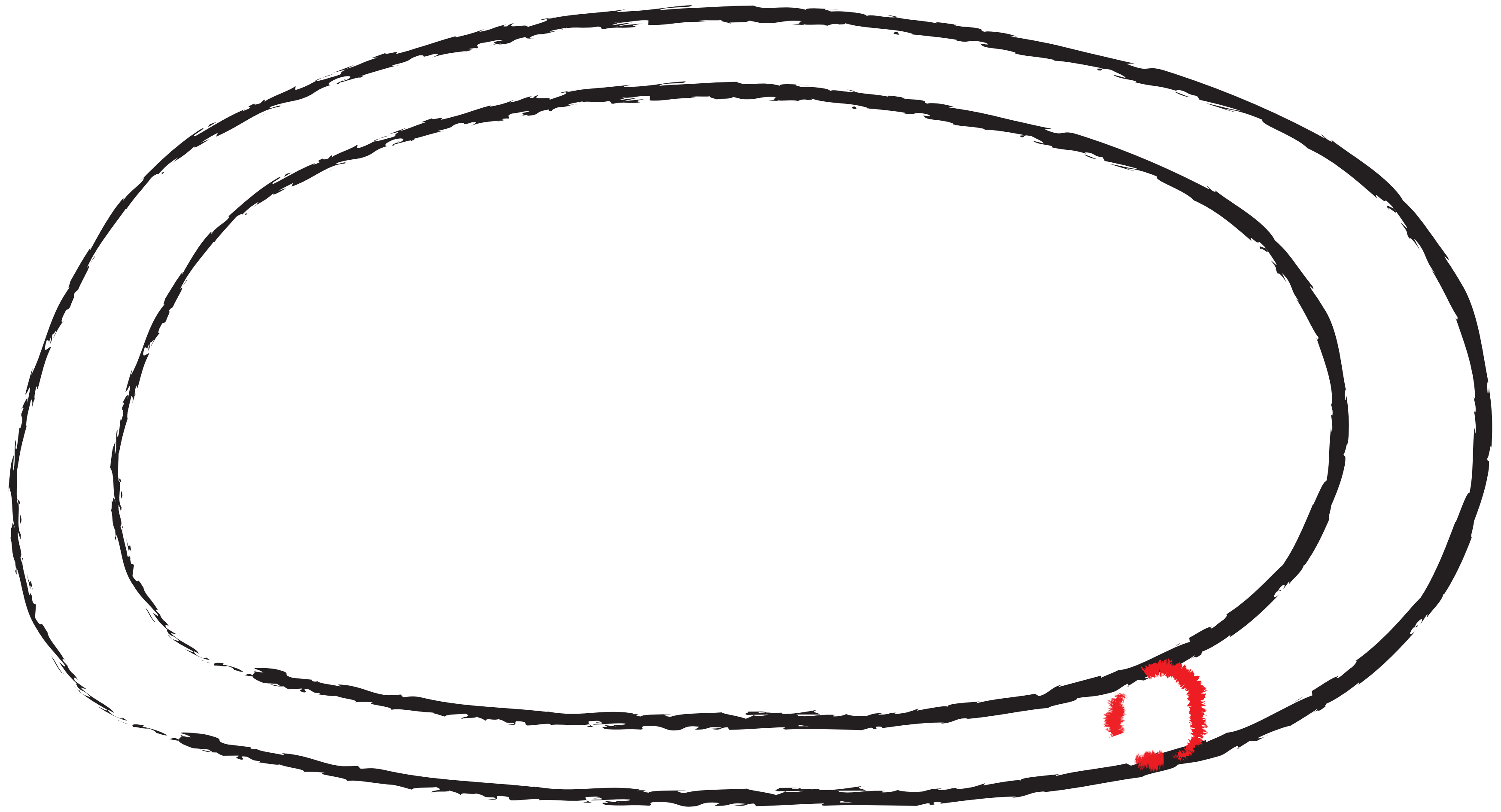


DLCQ Torus

or: how to strangle a donut



consider a rectangular three-torus with $g_{IJ} = R_I^2\,\delta_{IJ}$ and $R_I = (R_{11}, R_i)$



$R_{11} \to 0$

R_i/ℓ_P fixed

$\ell_P \to 0$

R_{11}/ℓ_P^2 fixed

$V = R_{11}R_iR_j/\ell_P^3 \to 0$

Conjecture [???]:

Closed wrapped membrane theory is U dual to matrix theory.

A Non-Lorentzian Supermultiplet in 11D

Wrapped Membrane Limit

[Blair, Gallegos, Zinnato 2021]

Curved

Recipe: 1: absorb ℓ_P , 2: go to frames ,
3: rescale anisotropically w/ A=0,1,2 and a=3, … , 10

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$$E_\mu{}^A = c\,\tau_\mu{}^A$$

$$E_\mu{}^a = c^{-1/2}\,e_\mu{}^a$$

$$C_3 = -\,c^3\,\tau^0 \wedge \tau^1 \wedge \tau^2 \;+\; c_3$$

$$c \to \infty$$

Membrane Newton-Cartan Geometry

G-structure, simultaneity, and intrinsic torsion

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- defines $(SO(1,2) \times SO(8)) \ltimes R^{24}$ structure with Galilean type boosts λ_{Aa}

$$\delta \tau_\mu{}^A = 0\,, \qquad \delta e_\mu{}^a = \lambda_A{}^a \tau_\mu{}^A\,, \qquad \delta c_3 = -\frac{1}{2}\epsilon_{ABC}\lambda^A{}_a e^a \wedge \tau^B \wedge \tau^C\,.$$

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- realizes anisotropic dilatations w/ $\Delta(\tau^A) = 1\,,\quad \Delta(e^a) = -\,1/2\,,\quad \Delta(c_3) = 0\,.$
- has generalized intrinsic torsion $(T^A = d\tau^A$ and $f_4 = dc_3)$

$$\mathfrak{T} \quad = \quad \left\{T_a{}^{\{AB\}}\,, \qquad T_{ab}{}^A\,, \qquad f_{Aabc}\,, \qquad f_{abcd}\right\}$$

Hunt for a Membrane NC Supermultiplet

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Hunt for a Membrane NC Supermultiplet

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- in the spinor representation of $SO(1,2) \times SO(8)$

- in a reducible, indecomposable boost rep: $\psi_{+\mu} \rightarrow \psi_{-\mu} \rightarrow 0$

- guess [...] susy rules for 2×16 supercharges (ϵ_+, ϵ_-)

$$\delta\, \tau_\mu{}^A = \bar{\epsilon}_- \gamma^A \psi_{-\mu}$$

$$\delta\, e_\mu{}^a = \bar{\epsilon}_+ \gamma^a \psi_{-\mu} + \bar{\epsilon}_- \gamma^a \psi_{+\mu}$$

$$\delta\, c_{\mu\nu\rho} = 6\, \epsilon_{ABC}\, \bar{\epsilon}_+ \gamma^A \psi_{+[\mu} \tau^B{}_\nu \tau^C_{\rho]} + \cdots$$

$$\delta\, \psi_{+\mu} = \nabla_\mu \epsilon_+ + \cdots$$

$$\delta\, \psi_{-\mu} = \nabla_\mu \epsilon_-$$

- impose geometric constraints

$$\mathfrak{T} \quad = \quad \left\{ T_a{}^{\{AB\}} \,, \quad T_{ab}{}^{A} \,, \quad f_{Aabc} \,, \quad f_{abcd} \right\} \quad = \quad 0$$

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- introduce a anti-selfdual four-form $\star_8 \lambda_{abcd} = -\lambda_{abcd}$

$$\delta\lambda_{abcd} = \frac{1}{4}\,\bar{\epsilon}_{-}\gamma^{e}\gamma_{abcd}\nabla\psi_{+e} \qquad \delta\psi_{+\mu} = \nabla_{\mu}\epsilon_{+} - \frac{1}{12}\tau_{\mu}{}^{A}\boldsymbol{\lambda}\gamma_{A}\epsilon_{+} - \frac{1}{8}\,e_{\mu}{}^{a}\boldsymbol{\lambda}\gamma_{a}\epsilon_{-}$$

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- take care of $\delta\,\mathfrak{T} = \bar{\epsilon}_-\mathfrak{r}_+ + \bar{\epsilon}_+\mathfrak{r}_-$ etc.

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- identify the multiplet of constraints $(\mathfrak{T}, \mathfrak{r}_+, \mathfrak{r}_-, \mathfrak{R}, \cdots)$

Summary and Outlook

- reviewed OM/WM2 limit as a regular limit of M theory with a discrete non-relativistic spectrum
- conjectured U duality between closed wrapped membrane and matrix theory
- presented WM2 backgrounds and a corresponding supermultiplet

- strengthen (or reject) the conjectured duality
- explore the large wrapping number W limit
- understand the role of the constraints
- wrapped supermembrane
- Killing spinor equations, solutions, etc. [cf. Rishi's talk]
- ...